A Fine-Grained Global Analysis of Implicatures

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Introduction

According to any (neo-)Gricean approach to quantity implicatures, we start with the semantic meaning of a sentence and then employ a pragmatic mechanism from the worlds that make the sentence true to
the ones where nothing more is true than needs to be. There are various ways how such a pragmatic mechanism could be spelled out (as in Gazdar 1979; Sauerland 2004; Soames 1982, and many others), but if spelled out in terms of exhaustive interpretation (e.g. Spector 2003; van Rooij and Schulz 2004) the following inferences are accounted for (below, \( \phi \leadsto \psi \) means that \( \psi \) is a conversational implicature of \( \phi \), if all that is mentioned is relevant):

\[
\begin{align*}
\text{(1) a. } & \quad p \lor q \quad \leadsto \quad \text{not both } p \text{ and } q \text{ (scalar implicature)} \\
\text{b. } & \quad \text{Two students passed} \quad \leadsto \quad \text{exactly two students passed (scalar implicature)} \\
\text{c. } & \quad p \rightarrow q \quad \leadsto \quad p \text{ if and only if } q \text{ (conditional perfection)} \\
\text{d. } & \quad p \lor q \lor r \quad \leadsto \quad \text{only one of } p, q, \text{ and } r \text{ is true} \\
\text{e. } & \quad (p \lor q) \land (r \lor s) \quad \leadsto \quad \text{only one of } (p \land r), (p \land s), (q \land r), \text{ or } (q \land s) \text{ holds}
\end{align*}
\]

If the (neo-)Gricean, or global approach, towards implicatures takes only semantic meaning as input, it follows immediately that two sentences with the same semantic content cannot give rise to different implicatures. In the above mentioned global accounts of implicatures, the semantic meaning of a sentence is modeled by a set of possible worlds. But this assumption immediately leads to the problem of how to account for implicatures like (2-a) and (2-b) (on the assumption that numerals receive an “at least” reading) and the lack of implicature in (2-c).

\[
\begin{align*}
\text{(2) a. } & \quad p \lor (p \land q) \quad \leadsto \quad \text{only } p \text{ or (only) } p \land q \\
\text{b. } & \quad \text{Two or three students passed} \quad \leadsto \quad \text{Exactly two or exactly three students passed} \\
\text{c. } & \quad \text{At least two students passed} \quad \nleftrightarrow \quad \text{Exactly two students passed}
\end{align*}
\]

Confronted with the problem posed by (2-a) and (2-b), a number of researchers (including Chierchia et al. 2012) have concluded that what's wrong with the standard account is that implicatures are calculated
globally.\textsuperscript{1} But there are other assumptions that might be questioned. Schulz and van Rooij (2006) have argued, for instance, that to account for implicatures we should give up the assumption that semantic meanings are as coarse-grained as sets of possible worlds, or in general that classical logical equivalents have the same semantic content. I will show in section “Implicatures and Exhaustive Interpretation” how they proposed to account for examples (2-a)–(2-c) making use of the more fine-grained notion of meaning in dynamic semantics. One might also question the assumption that to calculate implicatures one should rely on semantic input only: perhaps one can calculate implicatures globally if one also takes in addition, for instance, other alternatives into account.\textsuperscript{2}

Kratzer (2007) has suggested that one might account for many of the inferences in (1) by using situations or facts. We will take up her idea, using van Fraassen’s (1969) conception of verifying facts (or exact truth-makers, as Fine 2012, 2014 calls them), which provides a more fine-grained notion of meaning than standard possible-world semantics does. I will show in section “Facts and Implicatures” that in terms of this alternative semantics, we can account for the problematic examples (2-a)–(2-c) without giving up the global approach towards implicatures. I will argue that this approach is to be preferred to the dynamic approach because it is less dependent on the exact way the sentence is represented in logical form and that it can solve some other problems faced by the exhaustivity approach due to the latter’s reliance on predicate minimization. We will see that there are still some problems for the fact-based analysis in case a scalar item is embedded under a universal quantifier. I will argue that these problems can be overcome if we also take alternatives into account. On the other hand, I will show in section “Cancellation, Exhaustivity and Gricean Motivation” that the fact-based analysis can rather

\textsuperscript{1}Gazdar (1979) accounts for example (2-a), but his mechanism cannot account for the very similar (2-b). Récanati (2003, p. 303) explicitly reports the argument why Gricean implicatures cannot fall within the scope of a logical operator (see also Salvatore Pistoia-Reda 2014). More than Récanati himself, perhaps, I want to stick to this conclusion.

\textsuperscript{2}I am not aware of any worked out analysis for examples like (2-a)–(2-c), but that doesn’t mean that no such analysis can exist.
straightforwardly account for the exhaustivity and “cancellation”-effects for which the exhaustivity approach is so well-suited.

In Chierchia et al. (2012) non-global analysis of conversational implicatures, Hurford’s constraint, according to which no disjunct may entail another, plays an important role to account for (2-a). My global analysis doesn’t need Hurford’s constraint to account for the inference, but in section “Implicatures and Hurford’s Constraint” of this chapter we will discuss a closely related appropriateness condition for disjunctions and show how it accounts for a phenomenon brought up by Singh (2008). Singh argued that this phenomenon shows that we need a processing perspective on interpretation. I will argue, instead, that a more standard appropriateness condition is able to account for the phenomenon but that to state this condition the use of fine-grained semantic meanings is essential.

Implicatures and Exhaustive Interpretation

Exhaustive Interpretation

It is well-known that one can account for many scalar implicatures in terms of exhaustive interpretation, which can often be paraphrased in terms of “only.” For example, in a context in which it is relevant which students passed, (3-a) gives rise to the scalar implicature that not all students passed. This inference can also be derived from (3-b)—though now it follows from the semantic meaning of the sentence.

(3) a. Some of the students passed.
   b. Only \([\text{some} ]_F\) of the students passed.\(^3\)

To account for the scalar implicatures of “\(\phi\),” one could assume that a sentence should be pragmatically interpreted in terms of “\(\text{Prag}\),” which is modeled after Rooth’s (1996) analysis of “only”\(^4\):

\(^3\)The notation \([\cdot ]_F\) is used to convey that the relevant item receives focal stress.

\(^4\)Krifka (1995) introduces our \(\text{Prag}\) under the name “\(\text{Scal.Assert}\).”
In case one of the alternatives of (3-a) is "All of the students passed," the desired scalar implicature is indeed accounted for. The reader can easily see that the same correct prediction (5-b) is made for (5-a) if $Alt(\phi \lor \psi) = \{\phi \land \psi\}$.

What is pleasing about rule (4) as well is that it seems almost immediately to be motivated by Grice’s maxim of quantity “Say as much as you can” in terms of which standard scalar implicatures like (3-a)–(3-b) and (5-a)–(5-b) are standardly accounted for.\(^5\)

Unfortunately, as McCawley (1993) noticed, in case one scalar item is embedded under another—as in (6) of the form "A $\lor (B \lor C)\)，\(^6\)—a rule like Prag does not give rise to the desired prediction that only one of Alice, Bob, and Cindy passed if $Alt(\phi \lor \psi) = \{\phi \land \psi\}$ and we analyse (6) in terms of binary disjunction.

Worse, Prag does not even give rise to the desired prediction (5-b) for (5-a) if the set of alternatives also contains “Alice passed” and “Bob passed.” The reason is that one cannot infer from the semantic meaning of (5-a) that any of the alternatives is true. Therefore (4) predicts that both these alternatives are false, resulting in the impossible proposition. Assuming that in these cases the alternatives are closed under disjunction (and conjunction) obviously doesn’t help: the original alternatives remain alternatives when we make this shift and the problems remain.

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\(^5\)On the further assumption that the speaker is knowledgeable.

\(^6\)Landman (2000) discusses a structurally similar example, “Mary is either working at her paper or seeing some of her students.”
A popular way to solve these problems is to account for scalar implicatures by a rule of exhaustive interpretation. According to it, pragmatic interpretation excludes worlds where more of the relevant alternative propositions are true than are required to verify the sentence. This intuition is directly expressed in the following interpretation rule. In this rule it is assumed that the set of relevant alternatives to $\phi$, $\text{Alt}(\phi)$, is determined either by a question under discussion or as the focus alternatives, as in Rooth (1996) alternative semantics. Sentences like “Alice passed,” “Bob passed,” and “Cindy passed” are normally taken to be alternatives to sentences like (5-a) and (6).

$$(7) \quad \text{Exh}(\phi, \text{Alt}(\phi)) \overset{\text{def}}{=} \{w \in \llbracket \phi \rrbracket | \exists v \in \llbracket \phi \rrbracket : \{\psi \in \text{Alt}(\phi) | v \models \psi\} \subset \{\psi \in \text{Alt}(\phi) | w \models \psi\}\}$$

Notice that (7) doesn’t give rise to any of the (potential) problems discussed above for sentences like (5-a) or (6). It correctly predicts, in the case of (6), for instance, that only one of Alice, Bob, and Cindy passed.

Obviously, if we define a (strict) partial ordering relation “$<_{\text{Alt}(\phi)}$” on worlds in terms of the sets of alternative sentences that are true in those worlds, where $v <_{\text{Alt}(\phi)} w$ if and only if $\{\psi \in \text{Alt}(\phi) : v \models \psi\} \subset \{\psi \in \text{Alt}(\phi) : w \models \psi\}$, we can define (7) equivalently as $\text{Exh}(\phi, \text{Alt}(\phi)) = \{w \in \llbracket \phi \rrbracket | \exists v \in \llbracket \phi \rrbracket : v <_{\text{Alt}(\phi)} w\}$. Suppose now that $\phi$ is of the form “$P([\alpha]_F)$” and that we define $\text{Alt}(\phi)$ in terms of predicate $P$ as follows:

$$\text{Alt}(\phi) \overset{\text{def}}{=} \{P(d) | d \in D\}, \text{ with } d \text{ a name for } d.$$ In that case (7) comes down to interpretation rule (8):

$$(8) \quad \text{Exh}(\phi, P) \overset{\text{def}}{=} \{w \in \llbracket \phi \rrbracket | \exists v \in \llbracket \phi \rrbracket : v <_P w\}$$

with $v <_P w$ iff $P(v) \subset P(w)$

In van Rooij and Schulz (2004) and Schulz and van Rooij (2006) it is explained that if in addition we assumed a ceteris paribus condition

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7 Another solution is to rule out the disjuncts as alternatives, because they are not “innocently excludable” (as in Chierchia et al. 2012).
for considering alternative worlds, (8) actually comes down to Groenendijk and Stokhof’s (1984) principle of exhaustive interpretation, or to McCarthy’s (1980) rule of predicate circumscription. In Spector (2003), van Rooij and Schulz (2004), and Schulz and van Rooij (2006) it is shown how the exhaustive interpretation rules (7) and (8) can be inferred and thus motivated by Gricean maxims of quality and quantity and assumptions of (maximizing) competence. 8

Prospects and Problems of the Standard Exhaustivity Account

Quite a number of conversational implicatures (including scalar ones) can be accounted for in terms of exhaustive interpretation. Except for the obvious result that from the answer “Alice passed” to the question “Who passed?” we derive that Alice is the only one who passed, we can also account for all of the following implicatures already mentioned in the introduction:

(9) a. \( p \lor q \) \( \sim \) not both \( p \) and \( q \) (scalar implicature)
    b. Two students passed \( \sim \) exactly two students passed (scalar implicature)
    c. \( p \rightarrow q \) \( \sim \) \( p \) if and only if \( q \) (conditional perfection)
    d. \( p \lor q \lor r \) \( \sim \) only one of \( p \), \( q \), and \( r \) is true
    e. \( (p \lor q) \land (r \lor s) \) \( \sim \) only one of \( p \land r \), \( p \land s \), \( q \land r \), or \( q \land s \) holds

Moreover, we derive the implicature that not everybody passed from the answer that most did, and we derive the so-called conversion inference that only men passed, if the answer is “Every man passed.” 9 Another pleasing

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8 For an approach based on similar ideas involving Gricean maxims and maximizing competence, see Sauerland (2004).

9 If we exchange “\( \lor \)” with “\( \exists \)” and “\( \land \)” with “\( \forall \),” things also work for quantified variants of e.g. (10e)’ \( \forall x(Px \lor Qx) \sim \neg \exists x(Px \land Qx) \). In van Rooij and Schulz (2004) it is shown, moreover, how in terms of exhaustive interpretation one can also account for the intuition that sentences represented
property of an exhaustivity analysis of implicatures is that it predicts that it depends on the context, or question-predicate, as to whether we observe these inferences.\footnote{Schulz and van Rooij (2006) suggest that some obvious problems of standard pragmatic interpretation rules (such as the rule given in (7) of exhaustive interpretation) can be solved when we take the minimal models into account. They propose, for instance, that to account for the context-dependence of exhaustive interpretation, the beliefs and preferences of agents are relevant to determine the ordering relation between worlds required to define the minimal models. In this way they get a better grasp of the context (and relevance) dependence of implicatures, and can account for, among other things, both mention-all and mention-some readings of answers (which Groenendijk and Stokhof 1984, could not).} For instance, in terms of exhaustification one can immediately account for the intuition that pragmatic inferences (9-a) and (9-c) are canceled (or better, not generated) if the statements are given as answers to yes–no questions of the form \("(p \lor q)\)?" and \("(p \to q)\)?," respectively.

Unfortunately, accounting for implicatures in terms of exhaustive interpretation as given in (7) or (8) also gives rise to some serious problems. The first problem—most clearly visible under formulation (8)—is due to the fact that interpreting by exhaustive interpretation adopts the strategy to look only for worlds where the extension of the relevant question-predicate under discussion is minimized. But as already seen by Groenendijk and Stokhof (1984), this gives rise to the highly unwelcome prediction that nobody passed, if the following negative, or better \textit{monotone decreasing}, answers to the question \"Who passed?\" are given.

\begin{align*}
\text{(10) a. Jo did not pass.} \\
\text{b. At most Jo passed.}
\end{align*}

von Stechow and Zimmermann (1984) (in a weakened form followed by Schulz and van Rooij 2006) proposed that in such cases we should not minimize the extension of the question-predicate \textquote{Pass,} but rather the \textit{negation} of the question-predicate, and thus \textit{maximize} the extension of the original question-predicate. It is disputable whether the new prediction for (10-a), that is everybody but Jo passed, is correct, but the prediction by something like "\(\square(p \lor q)\)" typically give rise to the implicatures that in all the accessible worlds only \(p\) or only \(q\) holds.
given for (10-b), that is Jo passed, is certainly wrong. Equally problematic are examples like (11-a) and (11-b) that are neither monotonic increasing, nor monotonic decreasing.

(11)  
  a. Between two and five students passed.  
  b. Jo but not Bo passed.

The extension of which predicate should be minimized by pragmatic interpretation? No single answer seems appropriate. This, then, I take to be the **first problem** of exhaustive interpretation: it is not clear how to account in a systematic way for the correct predictions for downward monotone sentences like (10-b) and non-monotone sentences like (11-a) and (11-b).

The **second problem** for an analysis of implicatures in terms of exhaustive interpretation as defined by (7) and (8), and the problem that I will mostly focus on in this chapter, is the fact that the following patterns also already mentioned in the introduction cannot be predicted:

(12)  
  a. \( p \lor (p \land q) \)  \( \leadsto \) only \( p \) or (only) \( p \land q \)  
  b. Two or three students passed  \( \leadsto \) Exactly two or exactly three students passed  
  c. At least two students passed  \( \not\leadsto \) Exactly two students passed

In Schulz and van Rooij (2006) this was called the **functionality problem**. The functionality problem follows from the fact that exhaustive interpretation works immediately on the **semantic meaning** of an expression. It follows that if two sentences have the same semantic meaning, they are predicted to give the same implicatures as well. It is, for instance, standardly assumed that “Alice passed or Bob passed” has the same semantic meaning as “Alice passed, Bob passed, or both passed,” and that “Two students passed” has the same semantic meaning as both “Two or three students passed” and “At least two students passed.” But sentences in which the former examples occur give rise to the “scalar” implicatures that Alice and Bob did not both pass, and that at most two students passed, respectively, while the latter do not.
Confronted with the problem posed by (25-a) and (25-b), a number of researchers (e.g. Chierchia et al. 2012) have concluded that what’s wrong with the standard account is that implicatures are calculated *globally*. In Schulz and van Rooij (2006) it had already been argued, instead, that another assumption should be blamed: the assumption that semantic meanings should be modeled as coarse-grained as by sets of possible worlds.\(^{11}\) Schulz and van Rooij (2006) proposed that instead of thinking of semantic meanings as sets of *possible worlds*, we should think of them as being *more fine-grained* as sets of *world-assignment pairs*, as is standard in dynamic semantics.\(^{12}\)

**Dynamic Exhaustification**

In dynamic semantics (e.g. Kamp and Reyle 1993) it is assumed that, to account for the anaphoric dependencies, we should represent (13-a), (14-a), and (15-a), respectively, by (13-b), (14-b), and (15-b) (on the simplifying assumption that the domain consists of students only).

(13) a. Two students passed.
    b. \(\exists X (P(X) \wedge \text{card}(X) = 2)\)

(14) a. Two or three students passed.
    b. \(\exists X (P(X) \wedge (\text{card}(X) = 2 \vee \text{card}(3)))\)

(15) a. At least two students passed.
    b. \(\exists X (P(X) \wedge \text{card}(X) \geq 2)\)

Although all three sentences are (standardly taken to be) true in exactly the same possible worlds, in dynamic semantics they give rise to different sets of dynamic meanings, thought of as sets of verifying world-assignment

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\(^{11}\)The well-known problem of “logical omniscience” is closely related, and adopting a more fine-grained notion of meaning has been suggested by various authors to solve this problem as well.

\(^{12}\)Sevi (2006), instead, proposed making use of plural semantics to account for part of the functionality problem. It turns out that this bears some similarity to what I will propose here.
pairs: each world-assignment \((w, g)\) that is an element of the dynamic meaning of (13-b), \(\llbracket (13-b) \rrbracket\), assigns to variable X a set of exactly two students that passed. For (14-b) and (15-b) this does not have to be the case: X can also be assigned to a set of exactly three students that passed for (14-b), while any set of students that passed with a cardinality of at least two is possible for (15-b).

It is important to realize that the notion of meaning adopted in dynamic semantics is finer-grained than the notion of meaning in standard possible world semantics. Indeed, we can recover for any sentence \(\phi\), the standard semantic meaning, \(\llbracket \phi \rrbracket\), in terms of the dynamic semantic meaning, \(\llbracket \phi \rrbracket\), as follows:

\[
(16) \quad \llbracket \phi \rrbracket = \{w \in W | \exists g : \langle w, g \rangle \in \llbracket \phi \rrbracket\}.
\]

Schulz and van Rooij (2006) propose to make use of the differences in dynamic semantic meanings between (13-a), (14-a), and (15-a) in their dynamic exhaustivity operator, \(Exh_{dyn}(\phi, P)\). Recall that, according to (8), exhaustive interpretation is formulated in terms of a predicate-dependent ordering relation between worlds, \(v <_P w\). The definition of the order \(<_P\) comparing the extensions of the question-predicate used in (8) is now extended to an ordering on world-assignment pairs simply by adding the condition that the assignments have to be identical to make possibilities comparable. Thus, \(\langle w, g \rangle <_P \langle v, h \rangle\) iff \(def\ g = h\) and \(w <_P v\). Dynamic exhaustive interpretation is then defined as a function that selects minimal possibilities instead of worlds.

\[
(17) \quad Exh_{dyn}(\phi, P) \overset{def}{=} \{i \in \llbracket \phi \rrbracket | \neg \exists j \in \llbracket \phi \rrbracket : j <_P i\}.
\]

This straightforward extension of standard exhaustification rule (8) to dynamic semantics accounts for the different implicatures of (13-a), (14-a), and (15-a). The reason is that the extension of variable X can no longer be varied freely when the extension of the question-predicate is minimized. Notice that although each world-assignment pair \((w, g)\) in \(\llbracket (13-a) \rrbracket\) assigns to X a set of exactly two students that passed, it might still be the case that in the world \(w\) of that possibility more than two students passed. Such world-assignment pairs are eliminated, however, by means
of dynamic exhaustification rule (17). So far, nothing new. But for (14-a) and (15-a) things are different: a world-assignment pair \( \langle w, g \rangle \) that assigns to \( X \) a set of three students that passed can no longer be eliminated, even though there is another pair \( \langle v, h \rangle \) where in \( v \) only two students passed. The reason is that those two possibilities are now considered to be incomparable, because \( g(X) \neq h(X) \). As a result, (14-a) is predicted to give rise to the implicature that exactly two or exactly three students passed, while (15-a) is predicted not to give rise to an implicature concerning the number of students that passed at all.

Schulz and van Rooij (2006) proposed solving the problem posed by example (25-a) in a similar vein. To do so, they propose representing disjunctive sentences in terms of existential sentences as well. But whereas “John passed” is represented by something like \( \exists q : \forall q \land (q = \land P(j)) \) (where “\( q \)” is a propositional variable and “\( \lor \)” and “\( \land \)” have their usual Montagovian meanings), the sentence “John or John and Mary passed” is represented by \( \exists q : \forall q \land (q = \land P(j) \lor q = \land (P(j) \land P(m))) \). Although also on these representations they give rise to the same static semantic meanings, their dynamic meanings are predicted to be different: the latter allows for a verifying world-assignment pair where the assignment maps \( q \) to the proposition that both John and Mary passed the examination, while the former formula does not. In almost exactly the same way as for the examples (13-a) and (14-a), this difference in dynamic semantic meaning has the effect that the two formulas give rise to different exhaustive interpretations: the former, \( Exh_{dyn}(\exists q : \forall q \land (q = \land P(j)), P) \), allows only for possibilities (and thus worlds) in which only John passed the examination; the latter, \( Exh_{dyn}(\exists q : \forall q \land (q = \land P(j) \lor q = \land (P(j) \land P(m))), P) \), allows for possibilities where John and Mary passed the examination.

\[ ^{13} \text{Accounting for scalar implicatures in terms of exhaustive interpretation predicts that a sentence like “Some students passed,” represented by a formula like } \exists X(P(X) \land \text{card}(X) > 0), \text{ gives rise to the implicature that at most one student passed. That seems too strong a prediction. Dynamic exhaustification can be used to solve this problem as well, by representing the sentence as } \exists X(P(X) \land \text{card}(X) > 0 \land \text{card}(X) \leq \text{few}). \text{ This formula is true in a world exactly if the earlier formula was true, and in particular in worlds in which many or even all students passed. Still, a world-assignment pair } \langle w, g \rangle \text{ in the dynamic meaning of the formula in which in } w \text{ more than a few students passed is eliminated by dynamic exhaustification. World-assignment pairs in which in the world only a few students passed, however, are not eliminated.} \]
Appealing as this analysis of the functionality problem is, it still gives rise to two new problems. The first problem is that, at least so far, no Gricean motivation has been given for dynamic exhaustification in the same way that (7) has been motivated in the work of Spector (2003) and van Rooij and Schulz (2004). A second, and more serious, problem is that in order to predict that (13-a)–(15-a), and that “John passed” and “John or John and Mary passed” will give rise to different implicatures, it is essential to represent them differently, and in particular, to represent disjunctions in terms of existential quantifiers. Although this seems very natural for (13-a)–(15-a) (cf. Kadmon 1985; Kamp and Reyle 1993), this seems unnatural for the other example. This, together with the problem of how to account for examples like (10-b), is sufficient for finding an alternative.

Facts and Implicatures

Facts and Truth Makers

So-called “donkey sentences” like If a farmer owns a donkey, he beats it pose a problem for standard possible-worlds analyses of meaning. Dynamic semantics (Groenendijk and Stokhof 1991; Heim 1982; Kamp 1981) was developed to account for such sentences. But it soon became clear that one could solve the problem just as well by making use of minimal situations (e.g. Heim 1990). What both approaches have in common is that they adopt a finer-grained conception of semantic meaning than standard possible-worlds semantics. We have seen in the previous section that if we want to have a global approach towards implicatures, fine-grainedness seems to be an essential key to the solution. This gives rise to the expectation that a situation-based approach might be able to account for some conversational implicatures as well. Indeed, this has been suggested explicitly by Kratzer (2007), who proposed, however, that these “implicatures” follow from the semantic meaning already. In the rest of this chapter I will take up Kratzer’s suggestion, though I will work out the idea in a rather different way. First, I will not talk in terms of situations, but in terms of facts, or states of affairs. Nothing hangs on
this, but it does reflect what my approach is based on. The conception of facts that I will use comes from van Fraassen (1969). What is distinctive about his conception is that there are negative and conjunctive facts that figure as *exact truth-makers*, but there are no disjunctive ones. Whereas a simple conjunctive sentence has only one truth maker, an equally simple disjunctive sentence will have at least two truth makers. It turns out that this extra structure compared to what is standard in possible-worlds semantics is important for what follows.\(^{14}\) In contrast to Kratzer (2007), I will also treat the inferences dealt with in this chapter in terms of pragmatics rather than semantics. Whether this is crucial or not, it does clearly point to the similarity between the (global) possible-worlds-based analysis of implicatures discussed so far and the fact-based one to be discussed henceforth.

With van Fraassen, we will start with a set of basic *states of affairs*, \(\text{SOA}\).\(^{15}\) It is assumed that for every element \(p\) of \(\text{SOA}\) there is also its complement \(\overline{p} \in \text{SOA}\) for which it holds that \(\overline{\overline{p}} = p\). For simplicity we assume a close correspondence between atomic sentences of the language and the SOAs: with each literal (atomic sentence or its negation) of the language there corresponds exactly one SOA: the state of affairs that makes this literal true.\(^{16}\) The set of *facts*, \(\mathcal{F}\), is just \(\wp(\text{SOA}) - \emptyset\), so any non-empty subset of \(\text{SOA}\) is thought of as a fact. If \(p, q \in \text{SOA}\), then \(\{p\}\) and \(\{q\}\) are atomic facts, and \(\{p, q\}\) is a conjunctive fact. A fact is what makes a sentence true. But, of course, a sentence might have more than one truth maker.

Atomic sentence \(p\) is not only made true by atomic fact \(\{p\}\), but also by

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\(^{14}\)van Fraassen uses these truth makers to give a semantics for the notion of “tautological entailment” introduced by Anderson and Belnap (1962). For recent work on truth makers using this framework, see, for instance, Fine (2012, forthcoming). For use of the same framework for quite different purposes, see van Rooij (2000, 2014). In van Rooij (2017) and Cobreros et al. (2015) the framework is used to account for pragmatic inferences involving knowability and vagueness, respectively. The use of the framework to account for some of the problems that are central in this chapter was first sketched in van Rooij (2013). Fine developed a very similar analysis, but hasn’t published it yet.

\(^{15}\)If one doesn’t like facts, one can always think of them in a purely linguistic way simply as a set of literals, where a literal is an atomic sentence, \(p\), or its negation, \(\neg p\).

\(^{16}\)A standard objection to this type of approach is that we don’t really have negative states of affairs. That \(\neg p\) is true is not because \(\overline{p}\) is the case, but rather because some \(q\) is the case that is incompatible with \(p, p \perp q\), where \(\perp\) is a primitive and symmetric incompatibility relation between SOAs. Thus, “This is green” has more than one false maker: “This is red,” “This is blue,” and so on. Although assuming that atomic sentences might have more than one basic false maker is appealing, for simplicity I will stick to van Fraassen’s (1969) original approach here.
conjunctive fact \{p, q\}, or by the maximally consistent fact that contains the SOA p. A fact is maximally consistent iff, for each SOA x, either x or \(\overline{x}\) and not both. We will call such maximally consistent sets worlds. The fact \{p\} is more minimal than the conjunctive fact \{p, q\} and the maximally consistent fact that contains p. More interestingly, disjunctive sentences might have several minimal truth makers. The disjunction \(p \lor q\), for instance, has two minimal truth makers: \{p\} and \{q\}. What we are after, however, is the notion of the exact truth makers for \(\phi\). We will say that the disjunction \(p \lor (p \land q)\) has—although it has only \{p\} as its minimal truth maker—two exact truth makers: \{p\} and \{p, q\}. We can give the following simultaneous recursive definition of the set of exact truth makers of \(\phi\), \(T(\phi)\), and the set of exact false-makers, \(F(\phi)\)\(^1\): for each sentence represented by a first-order formula \(\phi\), we define below its set of exact truth makers, \(T(\phi)\), and its set of false makers, \(F(\phi)\), by means of a simultaneous recursion:

- \(T(p) = \{\{p\}\}\)  \(F(p) = \{\{\overline{p}\}\}\) for atomic p.
- \(T(\lnot \phi) = \overline{T(\phi)}\)
- \(T(\phi \land \psi) = T(\phi) \otimes T(\psi) = \{X \cup Y | X \in T(\phi), Y \in T(\psi)\}\).
- \(F(\phi \land \psi) = F(\phi) \cup F(\psi)\)
- \(T(\phi \lor \psi) = T(\phi) \cup T(\psi)\)\(^2\)  \(F(\phi \lor \psi) = F(\phi) \otimes F(\psi)\).

Note that, according to these rules, \(T(p) = \{\{p\}\}\), \(T(\lnot p) = \{\{\overline{p}\}\}\), \(T(p \lor q) = \{\{p\}, \{q\}\}\) and \(T(p \land q) = \{\{p, q\}\}\). It will be important for

\(^{17}\)It is easy to see that the definition of \(T(\phi)\) is parallel to the construction of the disjunctive normal form of \(\phi\).

\(^{18}\)In recent work on facts by Fine and others, \(T(\phi \lor \psi)\) is defined as \(T(\phi) \cup T(\phi) \cup T(\phi \land \psi)\). Although Fine has good reasons to do so (it helps to validate \(T(\phi) = T(\phi \land \phi)\)), for our way of calculating the pragmatic interpretation this would have fatal consequences. I'll stick to van Fraassen's (1969) original approach.

\(^{19}\)Notice that facts might not only be incomplete (neither verify nor falsify a sentence), they might also be inconsistent and both verify and falsify a sentence. Indeed, we have not ruled out facts like \{p, \overline{p}\}. Indeed, this would be the exact truth maker of \(p \land \lnot p\). Such inconsistent facts won’t play a role in this chapter though, because we assume that all worlds are maximally consistent facts.
our analysis of implicatures that $T(p \lor (q \lor r)) = T((p \lor q) \lor r) = \{\{p\}, \{q\}, \{r\}\}$, $T((p \lor q) \land (r \lor s)) = \{\{p\}, \{r\}, \{q, s\}\}$ and $T(p \lor (p \land q)) = \{\{p\}, \{p, q\}\}$. We analyse conditionals like $\phi \rightarrow \psi$ as material implications, that is $p \rightarrow q \equiv \neg p \lor q$, and thus $T(p \rightarrow q) = \{\{\overline{p}\}, \{q\}\}$. Observe also that $T(\neg (p \land q)) = \{\{\overline{p}\}, \{q\}\}$.

To account for quantifiers, we first have to spell out how we interpret atomic sentences with predicates and individual terms. To do so, we assume that each atomic sentences with predicates and individual terms. To do so, we assume that each fact also contains all and only all identity SOAs of the form $\{d = d\}$. Identity statements deserve special treatment. We assume that the truth maker of an identity statement like $a = b$ is just $\{a = b\}$. We assume that each world contains all and only all identity states of affairs of the form $d = d$. More special, though, is that we assume that each fact also contains all and only all identity SOAs of the form $d = d$. Thus, if $D = \{a, b\}$, we assume that a fact like $\{a\}$ is really an abbreviation for the fact $\{a, a = a, b = b\}$.

Truth makers of modal statements like $\Box \phi$ and $\Diamond \phi$ can be given as well. The most straightforward—though not the only—way to do so (cf. van Fraassen 1969) is (i) to assume that the representation of each atomic sentence $\phi$ has an extra world slot, filled by a distinguished world variable $i$ and interpreted as the actual world, and (ii) to represent modal

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Note that the definition of $T(\phi)$ parallels the construction of the disjunctive normal form of $\phi$. 

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20 Note that the definition of $T(\phi)$ parallels the construction of the disjunctive normal form of $\phi$. 

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statements as formulas that explicitly quantify over worlds in the object language, which can shift the interpretation of the world variable. Where an atomic sentence like “It is pouring” is now represented as $P_i$ instead of $p$, the modal sentence “It is possible that it is pouring,” for instance, is now represented by $\exists j(P_j)$. On such an approach, states of affairs are world-dependent as well: they are of the form “It is pouring at $w$,” $P_w$, instead of simply “It is pouring,” $p$. On this analysis of modal statements it immediately follows that if $W = \{w, v, u\}$ is the set of “internal” worlds, $T(\exists j(P_j)) = \{P_w, P_v, P_u\}$ and $T(\forall j(P_j)) = \{P_w, P_v, P_u\}$.

There should be a relation between $P_v \in w$ and what holds in world $v$, obviously. The simplest would be the following: $\forall u, v, w : P_u \in v$ iff $P_u \in w$. Thus if $p$ is true in $u$ according to $v$, $P_u \in v$, then $p$ is true in $u$ according to all worlds, including $u$, $P_u \in u$. For readability, we will in this chapter only think of states of affairs as being world-dependent in case we explicitly deal with modal statements.

It is interesting to note that $T(\phi)$ can be thought of as a fine-grained semantic interpretation of $\phi$. This can be used to determine its standard truth-conditional meaning as given by possible-worlds semantics, if a world is taken to be a maximally consistent conjunctive fact. In that case the standard truth-conditional meaning of $\phi$, $[\phi]$, can be recovered as the set of worlds in which “$\phi$” has a truth maker.

$$[\phi] \overset{def}{=} \{w \in W | \exists f \in T(\phi) : f \subseteq w\}.$$
The Basic Fact-Based Approach Toward Implicatures

But we did not introduce truth makers just to recover a notion, $[[\cdot]]$, that we already had. Our purpose in introducing truth makers is to define a notion of pragmatic meaning in terms of which we can explain a number of pragmatic inferences. Indeed, we can use $T(\phi)$ to determine the pragmatic meaning $PRAG(\phi)$ which is then used to account for some implicatures. Our first step in this direction is to replace \textit{`there is an exact truth maker of $\phi$' ($\exists! f \in T(\phi)$) in the above definition of $[[\phi]]$, (18), into `there is a unique exact truth maker of $\phi$' ($\exists! f \in T(\phi)$)}:

\begin{equation}
PRAG(\phi) \overset{\text{def}}{=} \{ w \in W | \exists! f \in T(\phi) : f \subseteq w \}.
\end{equation}

Our rule immediately accounts for the standard scalar implicatures (20-a) and (20-b) as well as the examples (20-c), (20-d), and (20-e) mentioned in the introduction.

\begin{enumerate}
\item \( p \lor q \) \quad \sim \quad \text{not both } p \text{ and } q \text{ (scalar implicature)}
\item Two students passed \quad \sim \quad \text{exactly two students passed (scalar implicature)}
\item \( p \rightarrow q \) \quad \sim \quad p \text{ if and only if } q \text{ (conditional perfection)}
\item \( p \lor q \lor r \) \quad \sim \quad \text{only one of } p, q, \text{ and } r \text{ is true}
\item \( (p \lor q) \land (r \lor s) \) \quad \sim \quad \text{only one of } (p \land r), (p \land s), (q \land r), \text{ or } (q \land s)
\end{enumerate}

Moreover, from “Every boy kissed Mary or Sue” it can now be concluded via a scalar implicature that every boy kissed only Mary, or only Sue.

\footnote{Because we have only defined truth makers for sentences represented by first-order formulas, we will only discuss implicatures generated by such sentences here.}
Similarly, “John believes that \( p \lor q \),” represented by a quantified formula like \( \forall j (\text{DOX}(i, j) \rightarrow (P_j \lor Q_j)) \), is predicted to mean that, in all of John’s doxastic alternatives, exactly one of \( p \) or \( q \) is true. Observe that because \text{PRAG} predicts implicature (21-b) from (21-a), it also predicts that (22-a) gives rise to the implicature that at most two students passed, where we assume that (22-a) is represented by (22-b).

\[
\begin{align*}
(21) & \quad \text{a. } (p \land q) \lor (p \land r) \lor (q \land r) \\
& \quad \text{b. only one of } (p \land q), (p \land r) \text{ and } (q \land r) \\
(22) & \quad \text{a. Two students passed.} \\
& \quad \text{b. } \exists x, y(P_x \land P_y \land x \neq y)
\end{align*}
\]

So far, the predictions are very much the same as those predicted by standard exhaustivity approaches to implicatures discussed in sections “Exhaustive Interpretation” and “Prospects and Problems of the Standard Exhaustivity Account.” Interestingly enough, our fact-based analysis predicts correctly for some non-monotonic sentence statements that were problematic for the exhaustivity approach. Consider, for instance, a sentence like (23),

\[
(23) \quad \text{Jo but not Bo passed.}
\]

It is obvious that (23) is accounted for immediately: according to \text{PRAG} it does not give rise to any implicature because \( T((23)) = \{\{P_j, \overline{P_b}\}\} \). Unfortunately, the analysis does not predict correctly for another non-monotonic sentence like (24).

\[
(24) \quad \text{Between two and five students passed.}
\]

It seems natural that we represent this sentence as saying that either three or four students passed. But, of course, in worlds in which four students passed there will be more than one truth maker of the sentence. Thus,

\[24\text{Except for the fact that the exhaustivity approach also predicts further exhaustivity and “cancellation” effects. We will discuss how our fact-based analysis can account for these predictions as well in section “Cancellation, Exhaustivity and Gricean Motivation.”}\]
such worlds are by the current fact-based pragmatic interpretation (19) incorrectly ruled out.

In fact, the problem of how to account for the pragmatic interpretation of (24) is really the functionality problem again, as discussed above. Indeed, PRAG also cannot account for the problematic cases discussed in the introduction, repeated here as (25-a), (25-b), and (25-c).

(25)  a. $p \lor (p \land q)$  \iff only if $p$ or (only) $p \land q$

     b. Two or three students passed  \iff Exactly two or exactly three students passed

     c. At least two students passed  \not\iff Exactly two students passed

Interestingly enough, other problems of the exhaustivity approach are also problematic for the fact-based analysis, though due to this same functionality problem. Let us look at a downward monotone sentence like (26-a), and assume it should be represented by (26-b).

(26)  a. At most Jo passed.

     b. $\forall x (Px \rightarrow x = j)$

Recall from section “Exhaustive Interpretation” that the standard exhaustivity approach proposes to minimize the extension of the background predicate—which we assume to be “passed”—resulting in the completely wrong prediction that (26-a) implies that nobody passed. von Stechow and Zimmermann (1984) (followed in a weakened form by Schulz and van Rooij 2006) propose that for “negative” answers the to-be-minimized predicate should be the negation of the background predicate. But in this case this would predict that Jo passed, which is equally wrong. Unfortunately, our interpretation rule PRAG gives rise to the same incorrect prediction. To show this, let us assume that the domain consists of just Jo, Bo, and Ko. In that case $T((26-b)) = \{\{Pj, Pb,Pk\} \{j = j, Pb, Pk\}\}$, if we forget about SOAs like $b = j$ that are assumed to be false. Remembering our assumption that each fact contains the SOA $j = j$, we see that although (26-b) has two truth makers, the second one is a subset of the other, meaning that according to the pragmatic interpretation rule (19) the pragmatic interpretation
strengthens the semantic meaning: it is incorrectly predicted that Jo passed, because worlds in which Jo did not pass would have two truth makers in \( T((26-a)) \).

For exactly the same reason, \((27-a)\) is also incorrectly predicted to give rise to the implicature that exactly two students passed.

\begin{itemize}
  \item \textit{a.} At most two students passed.
  \item \textit{b.} \( \forall x, y, z ((P(x) \land P y \land P z) \rightarrow x = y \lor x = z \lor y = z) \)
\end{itemize}

Fortunately, we now have the resources to account for such cases as well. We propose the following slight weakening of \( PRAG \) defined in (19) as \( PRAG^* \), defined in (28) by changing \( g = f \) to \( g \subseteq f \):

\begin{equation}
PRAG^*(\phi) \overset{\text{def}}{=} \{ w \in W | \exists f \in T(\phi) : f \subseteq w \land \forall g \in T(\phi) : g \subseteq w \rightarrow g \subseteq f \} \\
= \{ w \in W | \exists f \in T(\phi) : f \subseteq w \land \forall g \in T(\phi) : w \in (|g| \rightarrow |f|) \subseteq (|g|) \}
\end{equation}

Before we go on to discuss how this new rule deals with some examples, let us first note the similarity between this rule and the pragmatic interpretation rule \( Prag^* \), defined in section "Exhaustive Interpretation." Observe that \( w \in [\phi] \iff \exists f \in T(\phi) : f \subseteq w \) and that \( (|f|) \) is the proposition that corresponds with fact \( f \), and thus that the only difference with the earlier mentioned—and more standard—pragmatic interpretation rule \( Prag^* \) is that \( PRAG^* \) makes use of the facts that make \( \phi \) true, rather than the proposition expressed by \( \phi \), \( [\phi] \), itself. We will see below that this makes a crucial difference.

We have seen in section "Exhaustive Interpretation" that the pragmatic interpretation rule \( Prag \), (4), and thus also the almost identical \( Prag^* \), can almost immediately be motivated by Grice’s maxims of quality, and of quantity “Say as much as you can.” Because of the similarity between \( PRAG^* \) and \( Prag^* \), this motivation can be carried over to the case of \( PRAG^* \), with the only difference that things should now be restated into something like ‘Provide the strongest \textit{facts} you can’ instead of the strongest \textit{proposition}. We will come to the Gricean motivation of (a slight variant of) \( PRAG^* \) in section “Cancellation, Exhaustivity and Gricean Motivation.”
Let us see how our new interpretation rule accounts for example (25-a). Suppose \( W = \{ w, v, u \} \) and \( \llbracket p \rrbracket = \{ w, v \} \) and \( \llbracket q \rrbracket = \{ w, u \} \). Now, the sentences \( p \lor q \) and \( p \lor q \lor (p \land q) \) give rise to the same semantic meaning: \( \llbracket p \lor q \rrbracket = \llbracket p \lor q \lor (p \land q) \rrbracket = \{ u, v, w \} \), but to different sets of exact truth makers: \( T(p \lor q) = \{ \{ p \}, \{ q \} \} \), and \( T(p \lor q \lor (p \land q)) = \{ \{ p \}, \{ q \}, \{ p, q \} \} \). Although according to our old pragmatic interpretation rule it holds that \( PRAG(p \lor q) = \{ u, v \} = PRAG(p \lor q \lor (p \land q)) \), our new pragmatic interpretation rule predicts that \( p \lor q \) and \( p \lor q \lor (p \land q) \) also give rise to different pragmatic meanings: \( PRAG^*(p \lor q) = \{ u, v \} \), while \( PRAG^*(p \lor q \lor (p \land q)) = \{ u, v, w \} \). This is the desired interpretation, which we have now obtained without a special quantificational representation of disjunctive sentences as was required by the dynamic exhaustivity approach discussed in Schulz and van Rooij (2006).

As one might expect, our new fact-based pragmatic interpretation rule \( PRAG^* \) also accounts for the intuition that from (29-a) we infer that exactly two or exactly three students passed. The reason it does so is the same as why our new rule accounts for (25-a) as discussed above.

(29) a. Two or three students passed.
    b. \( \exists x, y(Px \land Py \land x \neq y) \lor \exists x, y, z(Px \land Py \land Pz \land x \neq y \land x \neq z \land y \neq z) \)

That we can represent (29-a) without making use of plural quantification is interesting, even if there is independent anaphoric evidence for such a pluralistic representation (cf. Kadmon 1985; Kamp and Reyle 1993). Obviously, the non-monotonic example (24) is now accounted for as well. More interestingly, perhaps, the downward monotone examples (26-a) and (27-a) do not give rise to the incorrect predictions anymore. It is predicted that the pragmatic interpretation does not strengthen the meaning of either sentence: \( PRAG((26-a)) = \llbracket (26-a) \rrbracket \) and \( PRAG((27-a)) = \llbracket (27-a) \rrbracket \).

**Intermediate Implicatures**

Fox and Spector (2008), Sauerland (2012, 2014) and others have put forward yet another challenge to any global analysis of scalar implicatures: the problem of so-called “intermediate implicatures.” A first challenge
posed for global analyses is to account for the intuition that (30) gives rise to the implicature that everybody read either some but not all of the books, or all the books.25

(30) Everybody read some of the books or everybody read all the books.

Indeed, the standard exhaustivity approach described in section “Exhaustive Interpretation” cannot account for this intuition. To see that our current analysis can account for this intuition, let’s assume that we have two individuals, Alice and Bob, and two books, 1 and 2. The facts that make (30) exactly true are then \{Ra1, Rb1\}, \{Ra1, Rb2\}, \{Ra2, Rb1\}, \{Ra2, Rb2\}, and \{Ra1, Ra2, Rb1, Rb2\}. It is easy to see that our new fact-based pragmatic interpretation rule PRAG* does indeed account for the intuition that everybody read either some but not all of the books, or that everybody read all the books. Similarly, our approach correctly predicts that (31) yields the implicature that Mary is allowed to read exactly three books, though she is also allowed to read more.

(31) Either Mary must read at least three of the books or she must read at least four of them.

Also (31) is discussed by Sauerland (2012) as an intermediate implicature which gives, according to him, decisive evidence in favor of a non-global approach to implicatures. But this is wrong: in terms of a more fine-grained analysis, a global approach can easily account for these data.

A more serious challenge is posed by sentences of the form (32-a) and (33-a), with the intended readings represented by (32-b) and (33-b) respectively:

(32) a. Every object is pink or round.
    b. \( \forall x (Px \lor Rx) \).

25The example actually discussed by Sauerland is slightly different from (30): it uses “most” instead of “some.” The issue of how to account for this is the same, though.
(33) a. Every object is pink, or pink and round.
    b. \( \forall x (P_x \lor (P_x \land R_x)) \).

The analysis given above does not predict via a scalar implicature for (32-a) that at least one object is pink and that at least one object is round. Similarly, our analysis does not (yet) predict that (33-a) conversationally implicates that there are at least some objects, though not all, that are both pink and round. Chierchia et al. (2012) argue that this is what should be predicted, though.

Perhaps even more challenging are examples of the form (34), also explicitly discussed by Chierchia et al. (2012):

(34) Everybody read some of the books or all of the books.

According to Chierchia et al. (2012), the preferred pragmatic reading has it that some people read only some of the books, while others read all of the books. I will argue below that this follows from our approach, together with another natural pragmatic mechanism that rules out stronger than intended readings of a sentence.

It is well known that natural language sentences can be ambiguous: “\( p \) and \( q \) or \( r \)” can be disambiguated as \( p \land (q \lor r) \) and as \( (p \land q) \lor r \), and “Every man loves a woman” can have the following logical forms \( \forall x (M(x) \rightarrow \exists y (W(y) \land L(x, y))) \) and \( \exists y (W(y) \land \forall x (M(x) \rightarrow L(x, y))) \). Although in principle ambiguous, these sentences are normally interpreted as intended, either because of the speaker’s use of intonation, and pause (for the first example), or by considering which of the stronger (and perhaps less stereotypical) readings could have been expressed more explicitly in the intended way. The second reason holds, presumably, for the second example: the speaker didn’t mean interpretation \( \exists y (W(y) \land \forall x (M(x) \rightarrow L(x, y))) \) of this sentence, because (i) this reading is logically stronger and/or less stereotypical, and (ii) the speaker could have expressed this reading without too much more effort by saying explicitly “There is a woman that everybody loves.”

26 This pragmatic reasoning could be accounted for by any mechanism that accounts for Horn’s famous division of pragmatic labor. For an up-to-date discussion of several types of approaches that can account for this, see Franke and van Rooij (2016).
Let us now look at examples (32-a), (33-a), and (34) again, and which readings we want to rule out. For example (32-a)—which is taken to have logical form $\forall x (Px \lor Rx)$—we would like to rule out $\forall x Px$ and $\forall x Rx$. For example (33-a)—with logical form $\forall x (Px \lor (Px \land Rx))$—we would like to rule out $\forall x Px$ and $\forall x (Px \land Rx)$. Finally, for example (34)—which is taken to have logical form $\forall x (\exists y (R(x, y) \lor \forall y R(x, y)))$—we would like to rule out $\exists y \forall x (R(x, y))$, $\forall y \forall x R(x, y)$, $\forall x \exists y R(x, y)$, and $\forall x \forall y R(x, y)$. The crucial observation is that all these alternatives to be ruled out are indeed predicted to be false, if the exact truth makers of the alternative readings of sentences (32-a), (33-a) and (34) that are logically stronger (and perhaps less stereotypical) than the intended readings are by an independent pragmatic reasoning already predicted to be false. The alternative stronger readings of (32-a) and (33-a) are $\forall x Px \lor \forall x Rx$ and $\forall x Px \lor \forall x (Px \land Rx)$, respectively. Example (34) has two stronger alternative readings than the intended $\forall x (\exists y (R(x, y) \lor \forall y R(x, y)))$, namely $\forall x \exists y (R(x, y) \lor \forall y \forall x R(x, y))$ (with ‘$\lor$’ scope over ‘$\forall$’), and $\exists y \forall x R(x, y) \lor \forall y \forall x R(x, y)$ (with “$\forall x$” scope below “$\lor$” and below the $y$-quantifier). If the exact truth makers of these alternative stronger readings are all predicted not to hold, the pragmatic readings of the three sentences are correctly predicted. This immediately gives rise to the following proposal: (i) collect all readings of the sentence that are logically stronger than the one to be considered (that could have been expressed more explicitly without too much effort), and (ii) assume that the exact truth makers of none of these stronger alternatives holds. If this procedure is followed, the initially problematic examples (32-a), (33-a), and (34) turn out to be unproblematic after all. Let’s illustrate this for example (33-a), assuming a model with domain $D = \{a, b\}$. Notice, first, that $T(\forall x (Px \lor (Px \land Rx))) = \{|Pa\}, \{|Pa, Ra\}\} \times \{|Pb\}, \{|Pb, Rb\}\} = \{|Pa, Pb\}, \{|Pa, Pb, Rb\}\}$, while $T(\forall x Px) = \{|Pa, Pb\}$ and $T(\forall x (Px \land Rx)) = \{|Pa, Ra, Pb, Rb\}\}$. By assuming that the exact truth makers of the stronger reading $\forall x Px \lor \forall x (Px \land Rx)$ do not hold, it follows that neither $\{|Pa, Pb\}$ nor $\{|Pa, Ra, Pb, Rb\}$ could (after pragmatic inference) be a truth maker of (33-a), and thus that it is ruled out that all objects are only pink, and that all objects are both pink and red, just as desired. We can conclude that our new rule also predicts correctly for example (33-a).
Of course, that our analysis can account for some so-called “intermediate” implicatures doesn’t mean it can for all of them. But it does mean that a global analysis of implicatures can be pushed further than is sometimes assumed.

Cancellation, Exhaustivity, and Gricean Motation

One of the great benefits of determining pragmatic meaning in terms of exhaustive interpretation is that one accounts not only for the fact that if a speaker says “ϕ or ψ” that ϕ ∧ ψ is not the case (as does the fact-based account), but also that this implication is canceled (or better, does not even arise) in case the assertion was given as an answer to the yes–no question as to whether ϕ ∨ ψ is true. Similarly, pragmatic interpretation in terms of exhaustivity not only gives rise to the prediction that ϕ → ψ is normally interpreted as ϕ ↔ ψ (as does the fact-based approach), it also predicts that this implication is canceled (or better, does not arise) in case it is given as an answer to the yes–no question as to whether ϕ → ψ is true. Neither of these cancelation effects have been predicted so far on the fact-based approach discussed above. Moreover, interpreting ϕ ∨ ψ exhaustively has the result that we conclude not only that ϕ ∧ ψ is not true, but that χ is not true as well, if χ is a relevant alternative. This exhaustivity effect is not accounted for by the fact-based approach discussed in the previous section either. This raises the question of whether we can accommodate our fact-based approach so as to account for these predictions as well. It turns out that this is rather straightforward.

27 Although I am not aware of an example of “intermediate” implicature for which it cannot account for.

28 But note that in the previous section we have already seen that some “cancelation” effects had already been predicted by the fact-based approach (just as on the dynamic exhaustivity account): ‘p ∨ q ∨ (p ∧ q)’ is predicted not to give rise to the implicature that ¬(p ∧ q), and “At least two students passed” does not implicate that at most two students passed. Of course, on the present analysis the term “cancelation” is not really appropriate, because we predict that the (potential) implicatures never arise. This has some interesting consequences of how implicatures are processed, but we won’t delve into those issues here.
Let us assume that a question gives rise to a set of alternatives, $ALT$. An alternative question like “$p, q$, or $r$?” gives rise to $ALT$, the set $\{[p], [q], [r]\}$ closed under intersection, while a yes–no question like “$\phi$?” gives rise to the set of alternatives $ALT = \{[\phi], [-\phi]\}$, just as expected. Now we will define the new pragmatic interpretation rule that also takes the set of alternatives under consideration:

$$\text{(35)} \quad \text{PRAG}_{ALT}^*(\phi) \overset{\text{def}}{=} \{w | \exists f \in T(\phi) : f \subseteq w \& \forall q \in ALT : w \in q : (\{f\} \subseteq q)$$

with $(\{f\}) \overset{\text{def}}{=} \{w \in W : f \subseteq w\}$

Before we see how this definition accounts for cancelation and exhaustification, let me first mention that this definition can be given a Gricean motivation. In Spector (2003), van Rooij and Schulz (2004), and Schulz and van Rooij (2006), exhaustive interpretation as discussed in section “Exhaustive Interpretation” was given a Gricean motivation: by quality it was assumed that the sentence is known to be true, by quantity it was derived that the speaker did not know more about relevant facts than he or she had to, and by maximizing competence, it was derived that of those things that are relevant and the speaker does not have to know, the speaker knows them not to be true. Although this result was very appealing, it obviously falsely predicted for a sentence like $p \_ (p ^ q)$. Fortunately, the Gricean motivation for $\text{PRAG}$ is only slightly different than for the standard exhaustive interpretation. The important difference is that we now assume that all $f \in T(\phi)$ have to be possible. Call this the principle of mentioning: whatever is mentioned as a possibility is taken to be possible. Let $S = \wp(W)$, the set of all possible knowledge states. Let us also for each $s \in S$ abbreviate $s \subseteq [\phi]$ and $s \cap [\phi] \neq \emptyset$ by $s \models \Box \phi$ and $s \models \Diamond \phi$, respectively. Finally, we state that $s \models \Diamond f$ iff $s \cap (\{f\}) \neq \emptyset$, if $f$ is a fact. Now we are going to define three functions:

$$\text{(36)} \quad a. \quad M(\phi) \overset{\text{def}}{=} \{s \in S | \forall f \in T(\phi) : s \models \Diamond f\}$$

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29 Also closed under intersection, though that is irrelevant here.
b. \( \text{Grice}(\phi) \overset{\text{def}}{=} \{ s \in M(\phi) \mid s \vdash \square \phi \land \forall s' \in M(\phi) : s' \vdash \square \phi \rightarrow \{ \psi \in \text{ALT} \mid s \vdash \square \psi \} \subseteq \{ \psi \in \text{ALT} \mid s' \vdash \square \psi \} \} \)

c. \( \text{Comp}(\phi) \overset{\text{def}}{=} \{ s \in \text{Grice}(\phi) \mid \forall s' \in \text{Grice}(\phi) : \{ \psi \in \text{ALT} \mid s' \vdash \Diamond \psi \} \not\subseteq \{ \psi \in \text{ALT} : s \vdash \Diamond \psi \} \} \)

It is important to observe that, except for \( M(\phi) \), the functions \( \text{Grice} \) (capturing Gricean quality, quantity, and relevance) and \( \text{Comp} \) (capturing maximizing competence) are defined exactly as in van Rooij and Schulz (2004) and Schulz and van Rooij (2006). Thus, they are sufficient to motivate exhaustive interpretation. The only thing we have added is the function \( M \), which is intended to capture that everything that is mentioned is taken to be possible. This function is also closely related to Gazdar’s (1979) idea of triggering clausal implicatures.

To illustrate the working of this Gricean machinery, let’s look at a few simple examples. First, for a sentence like \( p \lor q \) the speaker takes both disjuncts to be possible, by the mentioning principle. By Gricean quality (part of \( \text{Grice} \)), the speaker knows that \( p \lor q \) is true, and by quantity (also captured by \( \text{Grice} \)) it is concluded that he or she knows none of \( p, q \), and \( p \land q \), if \( \text{ALT} = \{ (f) : f \in T(p \lor q) \} \) closed under intersection. By competence it is concluded that the speaker knows that \( p \land q \) is not true. If \( \text{ALT} \) also included \( (r) \), it was predicted that \( r \) is also known not to be true. For a sentence like \( p \land (p \land q) \), the speaker is by the mentioning principle predicted to take both \( p \) and \( p \land q \) to be possible. By Grice, we conclude that the speaker doesn’t know \( p \land q \), although the speaker is predicted to know \( p \), of course. As a result, the speaker is predicted to take both \( p \land q \) as well as \( p \land \neg q \) to be possible. Competence, in this example, doesn’t add anything, if \( \text{ALT} = \{ [f] : f \in T(p \lor (p \land q)) \} \) closed under intersection. Finally, from \( p \rightarrow q \) we still conclude \( p \leftrightarrow q \), because \( p \land \neg q \) is ruled out by quality, and \( \neg p \land q \) is ruled out by quantity, because it is an element of the set of alternatives, \( \{ [\neg p], [q], [\neg p \land q] \} \). Notice that these predictions are the same as the ones derived from function \( \text{Prag} \). Indeed, we can prove the following fact:
Theorem 4.1. \( \forall s \in \text{Comp}(\phi) : s \subseteq \text{PRAG}_{\text{ALT}}^*(\phi) \).

To prove this, suppose that the claim is false. This means that there is an \( s \in \text{Comp}(\phi) \) and a \( w \in s \) such that \( w \not\in \text{PRAG}_{\text{ALT}}^*(\phi) \). For the latter to be the case, either (i) \( \neg \exists f \in T(\phi) : w \in \{f\} \) or (ii) \( \exists f, g \in T(\phi) : w \in \{f\} \cap \{g\} \) & \( g \not\subseteq f \) and \( f \not\subseteq g \). Because each world in any element of \( \text{Comp}(\phi) \) must (due to quality implemented by \( \text{Grice}(\phi) \)) be an element of \( \{\phi\} \), option (i) is impossible. So suppose (ii) and take \( w \) to be such a world, and \( f \) and \( g \) to be such facts. Notice that there can be no disjunct \( \psi \) of \( \phi \) such that \( \{\psi\} = \{f\} \cap \{g\} \) with \( w \in \{\psi\} \), because otherwise \( w \) would have been an element of \( \text{PRAG}_{\text{ALT}}^*(\phi) \). Because there is no disjunct \( \psi \) of \( \phi \) such that \( \{\psi\} = \{f\} \cap \{g\} \), it doesn't follow by \( M(\phi) \) that \( s \vdash \Diamond (f \land g) \). By \( \text{Comp}(\phi) \) it then follows that \( s \not\vdash \Diamond (f \land g) \) and thus that our \( w \) cannot be an element of \( s \in \text{Comp}(\phi) \). We have reached a contradiction, and thus proved the theorem.

How does this new definition of pragmatic interpretation account for cancelation and exhaustive interpretation? Before explaining that, let us first observe that if \( \text{ALT} = \{\{f\} : f \in T(\phi)\} \), then \( \text{PRAG}_{\text{ALT}}^*(\phi) = \text{PRAG}^*(\phi) \). The reason is that (i) \( w \in \{f\} \) iff \( f \subseteq w \) and (ii) \( g \subseteq f \) iff \( \{f\} \subseteq \{g\} \). But now suppose that \( \text{ALT} \supset \{\{f\} : f \in T(\phi)\} \). For instance, let us assume that \( \phi = p \lor q \) was uttered, and thus that \( \{\{f\} : f \in T(\phi)\} = \{\{p\}, \{q\}\} = \{\{p\}, \{q\}, \{r\}\} \). In that case we immediately predict that the speaker implicated \( \neg r \) by his or her utterance “\( p \lor q \)”, because neither \( \{p\} \subseteq \{r\} \) nor \( \{q\} \subseteq \{r\} \). This explains the exhaustivity effect.\(^{31}\)

To account for the cancelation effect, assume that “\( p \lor q \)” was given as an answer to the yes–no question as to whether \( p \lor q \) is true, giving rise to \( \text{ALT} = \{\{p \lor q\}, \{\neg (p \lor q)\}\} \). Notice that in this case the alternative \( \{p \lor q\} \) in \( \text{ALT} \) is a proper superset of all the elements of \( \{\{f\} : f \in T(p \lor q)\} = \{\{p\}, \{q\}\} = \{\{p\}, \{q\}\} \), meaning, intuitively, that the

\(^{30}\)Closure under intersection is not crucial for \( \text{PRAG}^* \).

\(^{31}\)I assume that exhaustive interpretation on top of standard implicatures occurs only when the speaker indicates (via falling intonation) that the sentence given is meant as a total answer to the question. I assume also that if the question is \( p \lor q \lor r \), a negative answer like \( \neg p \) is normally not given with falling intonation, and that thus exhaustive interpretation won’t occur.
elements in the latter set are more fine-grained than required to resolve the issue under discussion. It is easy to see that according to our definition of $PRAG^*_ALT(\phi)$ we correctly predict that the implicature that $\neg(p \land q)$ is canceled (or better, does not arise) in these circumstances, because all that is required for all worlds $w$ such that $p \subseteq w$ or $q \subseteq w$ is that $[p] \subseteq [p \lor q]$ and $[q] \subseteq [p \lor q]$, respectively, if $w \in [p \lor q]$, which is trivially the case. Similarly, we can explain why “$p \rightarrow q$” is not interpreted pragmatically as $p \not\rightarrow q$, if it is given as an answer to the yes–no question as to whether $p \rightarrow q$ is true—giving rise to $ALT = \{[p \rightarrow q], [\neg(p \rightarrow q)]\}$.

**Implicatures and Hurford’s Constraint**

Hurford (1974) proposed a constraint—known as Hurford’s constraint—which bans disjunctions in which one of the disjuncts entails the other. This condition helps to explain the infelicity of

\[(37) \text{ *Jan is from (somewhere in) the Netherlands or Amsterdam.} \]

But we have seen already some examples, repeated below, where the constraint appears to be violated, even though the sentence is appropriate:

\[(38) \text{ a. Either Jo passed, or Jo and Bo passed.} \]
\[
\text{ b. Two or three students passed.} 
\]
\[
\text{ c. Everybody read some of the books or everybody read all the books.} 
\]

To account for (38-a),\(^3\) Gazdar (1979) claims that Hurford’s constraint should be weakened to: “$\phi \lor \psi$ is infelicitous if $\psi$ entails $\phi$, unless $\psi$ contradicts $\phi$ together with the implicatures of $\phi$.” Chierchia et al. (2012) argue, instead, that in contrast to (37), examples (38-a)–(38-c) do not violate Hurford’s constraint, because one of the disjuncts gives rise to an

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\(^3\)To account for some related data, Hurford (1974) argued that “or” is ambiguous between an inclusive and an exclusive reading. See Gazdar (1979) for a thorough criticism of this view.
embedded scalar implicature, and that because of that there actually is no entailment relation between the disjuncts.

(39) a. Either only Jo passed, or (only) Jo and Bo.
    b. Two and only two or three and only three students passed.
    c. Everybody read some but not all of the books or everybody read all the books.

I agree with Chierchia et al. (2012) that Gazdar’s weakening of Hurford’s constraint is rather ad hoc. Unfortunately for the purpose of this chapter, however, Chierchia et al. (2012) make crucial use of a local, or grammatical, notion of implicature: they would represent a sentence like (38-a) making use of (at least) one silent “only”: only(p) ∨ (p ∧ q). Although this analysis accounts for the data mentioned above, the question arises as to whether we cannot explain the same data making use of a global approach to implicatures.\(^{33}\) That is, to explain why, on the one hand, (37) is inappropriate, without being forced to say that, on the other hand, (38-a)–(38-c) are inappropriate as well.

In my opinion—and adopting a global perspective toward implicatures—the appropriateness of examples (38-a)–(38-c) shows that Hurford’s constraint is too strong. Even though the latter disjunct in each example entails the former one(s) (where entailment is thought of standardly), this doesn’t mean that there is anything wrong with them.\(^{34}\) But how, then, to account for the inappropriateness of (37)? I would like to claim that (37) is inappropriate, not so much because the first disjunct is entailed by the second, but rather because the second disjunct is redundant, since it has already been mentioned as a possibility in the first disjunct. I would like to represent (37) by a formula of the form ∃xPx ∨ Pa. Of course Pa entails ∃xPx, but more relevantly, I feel, is

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\(^{33}\)Although I prefer a traditional global approach toward implicatures to a local one as proposed by Chierchia et al. (2012), the aim of this chapter is just to show that the global approach can be pushed further than is sometimes assumed. I prefer to leave the, sometimes, heated discussions between globalists and localists to others (e.g. Geurts 2009).

\(^{34}\)Singh (2008) observes that whereas (38-a) is all right, the following type of example is inappropriate: “Either Jo and Bo passed, or Jo.” This suggests that perhaps Hurford’s constraint should still be in play for left to right inferences between disjuncts.
that $T(\exists x P x) = T(\exists x P x \lor P a)$: adding the latter disjunct doesn’t add a new way to make the sentence (exactly) true. Instead of claiming that $\phi \lor \psi$ is inappropriate if $[[\psi]] \subseteq [[\phi]]$, I suggest that one should propose the constraint that $\phi \lor \psi$ is inappropriate if $T(\psi) \subseteq T(\phi)$. Notice that because $T(P a) \subseteq T(\exists x P x)$ (or $T(p) \subseteq T(p \lor q)$), but $T(p \land q) \not\subseteq T(p)$, this constraint correctly predicts (37) to be inappropriate, without denying that examples like (38-a) can be appropriate. Because to be able to formulate the new constraint it seems essential to make use of truth makers $T(\cdot)$ rather than the standard semantic meaning $[\cdot]$, this can be seen as another motivation for using a finer-grained notion of meaning. 35

Although the constraint that $\phi \lor \psi$ is inappropriate if $T(\psi) \subseteq T(\phi)$ suggested above works for the examples discussed previously, there is reason to believe that it is not exactly what we are looking for. As observed by Singh (2008), disjunctive sentences are also inappropriate sometimes when two disjuncts are mutually consistent:

(40) *John is from (somewhere in) Russia or Asia.

This suggests that Hurford’s constraint is not only too strong, it is too weak as well. In addition, however, it also suggests that our newly suggested constraint on the appropriate assertion of disjunctions is too weak: (40) is not predicted to be inappropriate because both disjuncts have a truth maker that the other does not have. Of course, one can correctly predict (40) to be inappropriate by a constraint demanding an assertion of “$\phi \lor \psi$” that $[[\phi]] \cap [[\psi]] = \emptyset$. But such a constraint is much too strong: “Jo passed or Bo passed” is perfectly appropriate. 36 Using $T(\cdot)$, however, we can state the following much weaker demand for appropriateness of assertions with sentences of the form “$\phi \lor \psi$”:

(41) “$\phi \lor \psi$” is appropriate only if $T(\phi) \cap T(\psi) = \emptyset$. 35

Of course, one might suggest that the new constraint just follows from Hurford’s constraint, if Hurford’s constraint was formulated in terms of the appropriate fine-grained notion of semantic meaning, $T(\cdot)$. In a sense, this is the case. Notice, however, that formulating entailment in terms of a subset-relation of truth makers, i.e., $\phi \models^* \psi$ iff $T(\phi) \subseteq T(\psi)$, would have the (I take it) undesirable consequence that $p \land q \equiv^* p$. 36

Localists might suggest that this constraint should only hold after local strengthening. But in this chapter I adopt a globalist perspective.
Observe that none of the examples (38-a)–(38-c) are predicted to violate this constraint, intuitively because $T(p) \cap T(p \land q) = \emptyset$ (and for (38-a), $T(p) \cap T(q) = \emptyset$). Sentences like (40) of the form $(p \lor r) \lor (q \lor r)$ (or more generally of the form $\exists x_D P_x \lor \exists y_{D'} P_y$, with $D$ and $D'$ the domains of quantification of the two disjuncts and $D \cap D' \neq \emptyset$), on the other hand, are now correctly predicted to be ruled out because the two disjuncts share an exact truth maker, that is $r$.\textsuperscript{37} Again, to formulate this constraint, fine grainedness seems crucial.

### Conclusion and Outlook

I have argued in this chapter that fine grainedness is the key to accounting for quite a number of conversational implicatures and the statement of appropriateness conditions, if one adopts a global approach toward pragmatic inferences. In particular, I have argued in favor of a fact-based analysis, making use of truth makers. But we have seen that some of the data might also be accounted for using another fine-grained notion of meaning: the one adopted in dynamic semantics. It is noteworthy to observe that these two fine-grained notions of meaning have been used as well to account for other problems of the standard possible-worlds-based analysis of meaning posed by disjunctive and/or existential sentences. Indeed, both types of approaches have been used to account for (i) anaphoric dependencies in donkey sentences—as already mentioned at the beginning of section “Facts and Implicatures,” and (ii) simplification of disjunctive antecedents of counterfactual conditionals using a similarity-based semantics (van Rooij 2006 uses “dynamic” meanings while Fine (2012) uses facts).\textsuperscript{38}

\textsuperscript{37} Also a sentence like “Either she read at least three of the books or she read at least four of them” is predicted to be inappropriate as well. According to Sauerland (2012), this is as it should be (although he would account for it in terms of Hurford’s constraint.

\textsuperscript{38} Stating it roughly, Fine (2012) claims that the counterfactual conditional $\phi \rightsquigarrow \psi$ is true in $w$ iff $\forall f \in T(\phi) : s_w(\{f\}) \subseteq [\psi]$, where $s_w(A)$ picks out the worlds most similar to $w$ where $A$ holds. There are other problems of the standard possible-worlds approach posed by disjunctive sentences, e.g., the problem of free-choice permissions, and one might expect that the two fine-grained approaches would work here as well. Sentences that are naturally represented as being of
It can hardly be a coincidence that these approaches work for these types of examples and more: both approaches are more fine grained than the standard approach exactly if disjunctive and/or existential sentences are involved. Indeed, in terms of the dynamic meaning of an existential formula like \( \exists x P x \) one can easily recover not just its standard possible-worlds meaning \( \{ w \in W \mid \exists g : \langle w, g \rangle \in \exists x P x \} \), but also the (singular) propositions that correspond with its (exact) truth makers: \( \{ \langle f \rangle : f \in T(\exists x P x) \} = \{ \{ w \in W \mid \exists h : \langle w, h \rangle \in \exists x P x \} \& g(x) \in I_w(P) \& g(x) = h(x) \} \mid g \in G \}. \) The other way around seems to work similarly. The use of Roothan alternative semantics as the proper semantic treatment of disjunctive sentences (Alonso-Ovalle 2005), and the recently developed inquisitive semantics (cf. Ciardelli et al. 2013) have been used to solve some of these problems in similar ways as well (e.g. Brochhagen and Coppock 2013). Also here, a special treatment of disjunctive sentences is crucial. Naturally, this all asks for a formal proof that the fine-grained frameworks mentioned share a common core. It would be interesting to investigate what exactly this common core is.

In this chapter I have assumed that the inferences mentioned, for instance in the introduction, are \textit{pragmatic} in nature and show that a global approach can be adopted. Instead, as mentioned already, Kratzer (2007) has suggested that the inferences typically discussed under the heading “scalar implicatures” are really \textit{semantic} in nature. I am not sure how much substance is behind this different terminology: both Kratzer and I (and, actually, all those who treat the inferences in terms of exhaustive interpretation) would say that none of the implicatures

\[ \text{May} (\phi \lor \psi) \] seem to give rise to the free-choice inference \[ \text{May} (\psi) \]. The problem is that on the standard treatment of obligations and permission in possible-worlds semantics, (i) \( O(\phi) \models O(\phi \lor \psi) \) and (ii) \( O(\phi) \models P(\phi) \). Accounting for free-choice permissions as a semantic inference \( P(\phi \lor \psi) \models P(\psi) \) would have the absurd result that \( O(\phi) \models P(\psi) \) (by transitivity of inference). There are various ways to overcome this problem. According to almost all approaches, the free-choice permission inference is not semantic, but pragmatic in nature. Some have tried to account for this by Grice’s first maxim of quantity, “say as much as you can” (e.g., Fox 2007; Schulz 2005), while others have rather eluded to Grice’s second maxim of quantity, “don’t say more than you must,” perhaps in conjunction with an appeal to minimal complexity (e.g., Franke 2011). In contrast to these pragmatic approaches, van Rooij (2006) proposed a modification of the semantic performative approach adopted by van Rooij (2000), making crucial use of dynamic meanings. A similar move using (something as fine grained as) facts works as well (cf. Mastop 2005).
discussed in this chapter can be canceled, although the possibility of cancelation was according to Grice (1967) one of the distinctive features of pragmatic inferences. Still, in contrast to what Kratzer (2007) suggests, in my treatment the standard semantic possible-worlds meanings play a role. Indeed, although the exact truth makers are crucial to account for implicatures, I don’t think it would be appropriate to define entailment immediately in terms of them in the most straightforward way: as already noted above, if one demands entailment, $\phi \models^* \psi$, that $T(\phi) \subseteq T(\psi)$, a conjunctive sentence of the form $p \land q$ would not entail $p$, which seems absurd. More interestingly, perhaps: semantic and pragmatic approaches to account for the same data might suggest different processing loads (cf. Chemla and Singh 2014). Whether this speaks in favor of the pragmatic approach adopted here remains to be seen.

References


