Anthropometric Model of Man

Hanavan Model and Wooley's Simplification

The following assumptions are made in the design of a mathematical model of man:

- The human body can be represented by a set of rigid bodies of simple geometric shape and uniform density, called segments, which each closely approximate the size, shape, mass, and centre of mass of the segment;
- The regression equations for segment weights from research literature are valid for the participants in the study;
- The limbs move about fixed pivot points when the body changes position. They are often the base of the neck, the arm-shoulder, the elbow joints, the wrist joints, the leg-pelvis sockets, the knee joints, and ankle joints.

Hanavan (1964) designed a 15-segmental model shown in Figure 1; for later use we have indicated the orientation of the coordinate system:

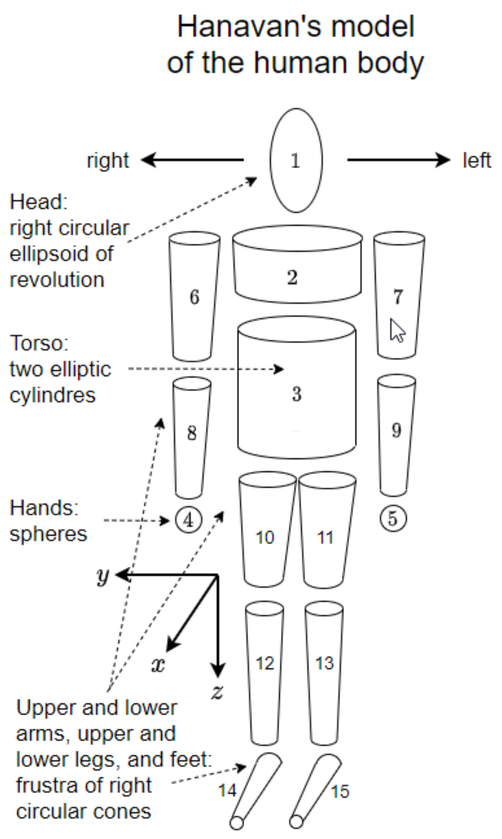


Figure 1, Hanavan's model of man

The following geometric shapes of body segments are used:

- the head (segment 1) is represented by a right circular ellipsoid;
- the upper and lower torso (segments 2 and 3) are represented by right elliptic cylindres
- the hands (segments 4 and 5) are represented by spheres;

- the upper arms (segments 6 and 7), the forearms (segments 8 and 9), the upper legs (segments 10 and 11), the lower legs (segments 12 and 13), and the feet (segments 14 and 15) are represented by frustra of right circular cones.

Woolley (1972) reduced the number of segments in the body model to nine by combining the head, upper and lower torso (segments 1, 2, and 3) into a single body segment, and by combining on the left and right body side the forearm with the hand (4+8 and 9+5) and the lower leg with the feet (12+14 and 13+15) into single body segments (See figure 2).

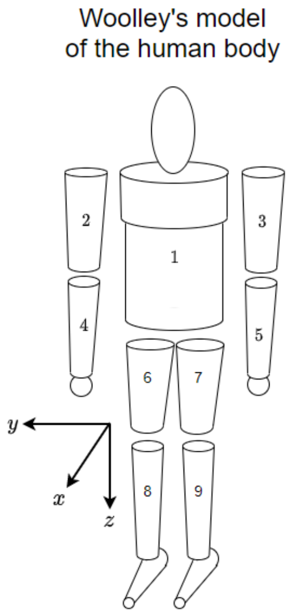


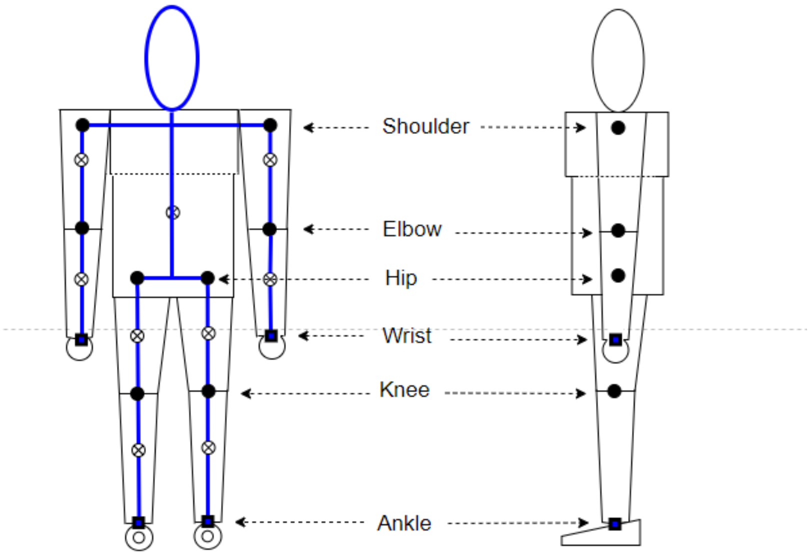
Figure 2. Woolley's model of man.

This also means that no account is taken of wrist, ankle or head motion about the joints, nor of bending of the spine. In the context in which this model was developed (astronauts with limited head and spine motion due to their suit), these assumption were made and suitable. Furthermore, the hands and feet are relatively small masses when compared to other body segments and provide minimal disturbance effects if moved vigorously.

In our study of springboard diving we will mainly use Woolley's model for the divers. But in case of a dive in pike position we will simplify it further by combining the upper arm on the left and right body side the upper and lower arm (including the hand) and the upper and lower leg (including the foot) into single body segments. This further reduces the number of segments in the model to five. Because we will only consider somersaults with no twist, the motion can be considers planar. When the arms and legs have the same anthropometric properties on the left- and right-hand side of the body and move parallel (to avoid reduction in the score of the dive), our models essentially boil down to a 5-segmental model (Woolley's model) or to a 3-segmental model (Woolley's model reduced to a model with upper and lower extremities), in case the arms and legs are kept in stretched position. In order to check if the bending of the spine can be neglected, we will also compare Woolley's model with the extended 10-segmental model in which the torso is split into an upper and lower torso.

For computations of (local) centres of mass and (local) mass moments of inertia for every segment and the total body we use the following stick model for Woolley model of man.

Front and side view of Woolley's model with segmental joints ● and centres of mass ⊗, and the wrist and ankle as distal points ■ of the lower arm and lower leg, and a stickmodel.



Mass Distribution

The man's total body mass is distributed over each of the body segments using regression equations. Popular choices are the regression equations of Barter (1957) and the regression equations of Clauser et al (1969). Like Hanavan (1964) and Woolley (1972) we will use Barter's regression equations:

Mass of head, neck, trunk = *HNT* = $0.47m'_T + 5.44$ kg
Mass of both upper arms = *BUA* = $0.08m'_T - 1.32$ kg
Mass of both forearms = *BFO* = $0.04m'_T - 0.23$ kg
Mass of both hands = *BH* = $0.01m'_T + 0.32$ kg
Mass of both upper legs = *BUL* = $0.18m'_T + 1.45$ kg
Mass of both lower legs = *BLL* = $0.11m'_T - 0.86$ kg
Mass of both feet = *BF* = $0.02m'_T + 0.68$ kg

where

$$m'_T = \frac{m_T - 5.48}{0.91}.$$

The corrected total mass of the model man m'_T distributes the total body mass m_T proportionately to the segment masses.

Besides the total body mass m_T, twenty-four other individual body parameters are measured. They are listed in Tabel 1. The abbreviations will be used in mathematical formulas and computer programs about the centres of mass and the local moments of inertia.

Measurement	*Variable name*	*Description*
Ankle circumference	*ANKC*	Minimum circumference of left/right ankle
Axillary arc circumference	*AXILC*	Horizontal circumference of upper arm at armpit
Buttock depth	*BUTTD*	Front-to-rear dimension of body at buttocks
Chest breadth	*CHESB*	Width of body (less arms) at level of nipples
Elbow circumference	*ELBC*	Circumference of arm at elbow (arms extended)

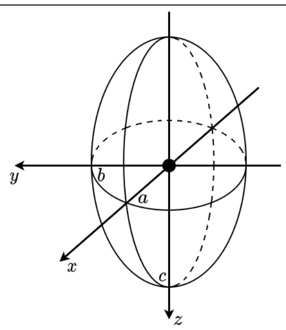
Fist circumference	*FISTC*	Circumference of fist measured over thumb and knuckles
Forearm length	*FOARL*	Length of extended forearm between elbow and wrist
Foot length	*FOOTL*	Maximum length of foot from heel to toe
Knee circumference	*GKNEC*	Circumference of left/right genu (knee) at kneecap (standing)
Head circumference	*HEADC*	Maximum circumference of head above eyebrows
Hip breadth	*HIPB*	Maximum width of body at hips
Shoulder (acromial) height	*SHLDH*	Distance from floor to left/right shoulder (acromion), standing erect
Sitting height	*SITH*	Height from seat to top of head, subject sitting erect with knees bent 90 degrees
Sphyrion height	*SPHYH*	Distance from floor to sphyrion (ankle), subject standing erect
Stature	*STAT*	Distance from floor to top of head, subject standing erect
Substernale height	*SUBH*	Distance from floor to substernale (lower breastbone), subject standing erect
Thigh circumference	*THIHC*	Horizontal circumference of upper left/right thigh, subject standing erect
Tibiale height	*TIBH*	Distance from floor to tibiale (knee), subject standing erect
Trochanteric height	*TROCH*	Distance from floor to trochanterion (hip), subject standing erect
Upper arm length	*UPARL*	Distance from acromion to radiale (shoulder to elbow), arms at side
Waist breadth	*WAISB*	Minimum width of waist, subject standing erect
Waist depth	*WAISD*	Front-to-rear dimension of body at waist
Wrist circumference	*WRISC*	Minimum circumference of wrist, arm extended

Table 1. Anthropometric measurements

Segmental centres of mass and mass moments of inertia

In this section we follow the expositions of Hanavan (1964) and Woolley (1972) about centres of mass and local moments of inertia of the segments in the model of man.

The local moments of inertia for the geometric shapes used as segments in the mathematical model of man are listed in Table 2. Each coordinate system is placed in the segmental centre of mass.

Description	Figure	I_{xx}	I_{yy}	I_{zz}
Solid ellipsoid with semi-axes a, b, c and mass m		$\frac{1}{5}m(b^2 + c^2)$	$\frac{1}{5}m(a^2 + c^2)$	$\frac{1}{5}m(a^2 + b^2)$

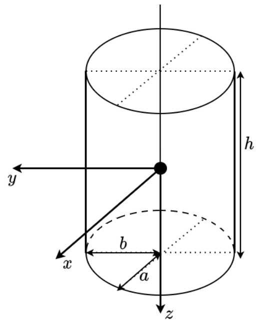
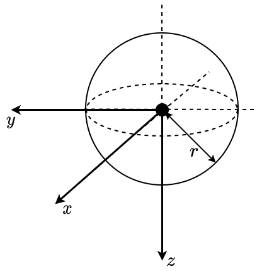
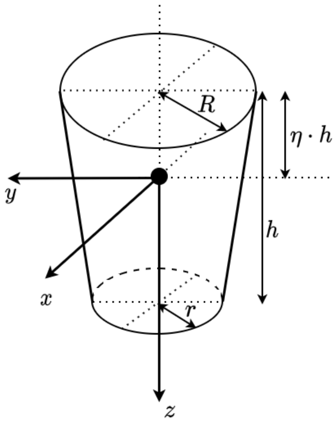
Solid right elliptical cylindre with semi-axes a, b, height h, and mass m		$\frac{1}{12}m(3b^2 + h^2)$	$\frac{1}{12}m(3a^2 + h^2)$	$\frac{1}{4}m(a^2 + b^2)$
Solid sphere with radius r and mass m		$\frac{2}{5}mr^2$	I_{xx}	I_{xx}
Frustrum of a right circular cone with radii R and r, height h, and mass m. Coordinate system in centre of mass at height $\eta \cdot h$ from the base with radius R		$A\frac{m^2}{\rho h} + Bmh^2$ with parameter definitions below the table	I_{xx}	$\frac{3}{10}m\frac{R^5 - r^5}{R^3 - r^3}$ This can also be written as $\frac{2Am^2}{\rho h}$

Table 2. Mass moments of inertia for geometrical shapes present in the mathematical model of man. Here, the parameters A, B, η and the density ϱ for the frustum of a circular cone are defined as follows:

$$A = \frac{9(1 + \mu + \mu^2 + \mu^3 + \mu^4)}{20\pi\sigma^2}$$

$$B = \frac{3(1 + 4\mu + 10\mu^2 + 4\mu^3 + \mu^4)}{80\sigma^2}$$

$$\eta = \frac{1 + 2\mu + 3\mu^2}{4\sigma}$$

where $\mu = r/R$ and $\sigma = 1 + \mu + \mu^2$.

$$\rho = \frac{3m}{\pi(r^2 + rR + R^2)h}$$

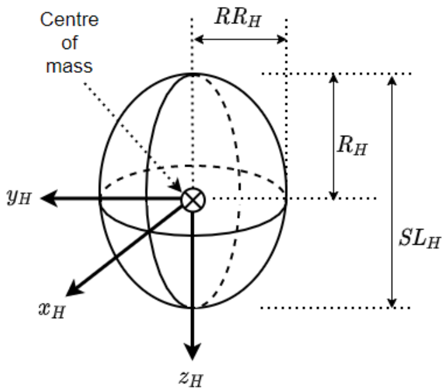
In the following subsections we will use the above formulas with variable names matching mostly those of Woolley (1972). Figures of segments are not on the right scale.

Head and torso

We consider anthropometric quantities of the head, the upper and lower torso, and combinations of these segments.

Head

The head in the model of man is a right circular ellipsoid of revolution. So it is a solid ellipsoid with semi-axes $a = b = RR_H$, $c = R_H$ and mass $m = m_H$.



The dimensions and properties of the head are given by:

$$R_H = \frac{1}{2}(STAT - SHLDH)$$

$SR_H = R_H$ (distance from apex of head to centre of mass)

$$RR_H = \frac{HEADC}{2\pi}$$

Segmental length: $SL_H = STAT - SHLDH = 2R_H$

Volume of the head: $V_H = \frac{4}{3}\pi \cdot R_H \cdot {RR_H}^2$

$m_H = 0.079 m_T$ (i.e., mass of head m_H vs total body mass m_T according to Dempster [1955])

Density of the head: $\rho_H = {}^{m_H}/_{V_H}$

Local mass moments of inertia of the head:

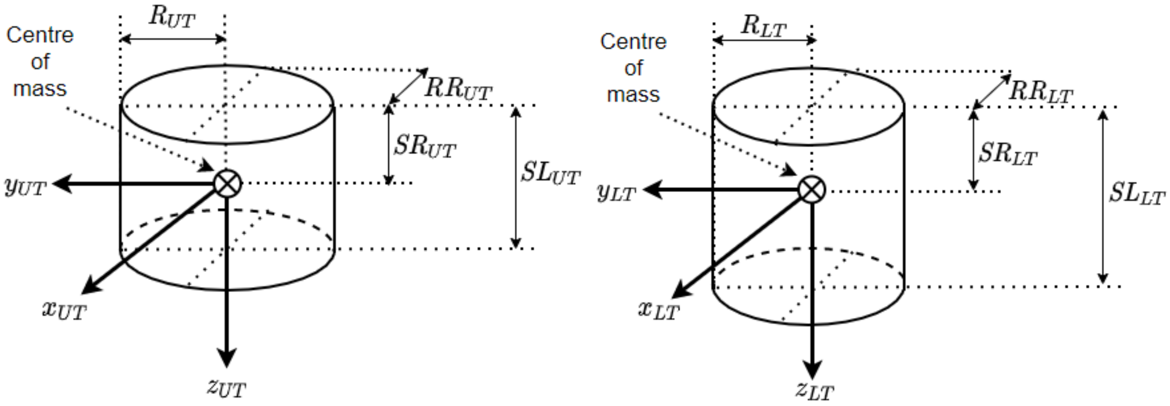
$$I_{xx,H} = \frac{1}{5} m_H \left({R_H}^2 + {RR_H}^2\right)$$

$$I_{yy,H} = I_{xx}$$

$$I_{zz,H} = \frac{2}{5} m_H {RR_H}^2$$

Upper and lower torso

The upper and lower torso in the model of man are right elliptic cylindres with semi-axes $a = RR_{UT}$, $b = R_{UT}$, height $h = SL_{UT}$, and mass m_{UT} for the upper torso, and with semi-axes $a = RR_{LT}$, $b = R_{LT}$, height $h = SL_{LT}$, and mass m_{LT} for the lower torso.



The dimensions and properties of the **upper torso** are given by:

$$R_{UT} = \frac{1}{2} CHESB$$

$$RR_{UT} = \frac{1}{4}(CHESD + WAISD)$$

Segmental length: $SL_{UT} = SHLDH - SUBH$

Distance from the top of the upper torso to the centre of mass: $SR_{UT} = \frac{1}{2} SL_{UT}$

Volume of the upper torso: $V_{UT} = \pi \cdot R_{UT} \cdot RR_{UT} \cdot SL_{UT}$

Volume of the lower torso: $V_{LT} = \pi \cdot R_{LT} \cdot RR_{LT} \cdot SL_{LT}$

Density of the upper torso: $\rho_{UT} = {(HNT - m_H)}\big/{(V_{UT} + r \cdot V_{LT})}$

where

$$r = \frac{\text{specific gravity of lower torso}}{\text{specific gravity of upper torso}} = \frac{1.01}{0.92} \text{ (according to Dempster, [1955])}$$

Mass of the upper torso: $m_{UT} = \rho_{UT} \cdot V_{UT}$

Local mass moments of inertia of the upper torso:

$$I_{xx,UT} = \frac{1}{12} m_{UT}\left(3{R_{UT}}^2 + {SL_{UT}}^2\right)$$

$$I_{yy,UT} = \frac{1}{12} m_{UT}\left(3{RR_{UT}}^2 + {SL_{UT}}^2\right)$$

$$I_{zz,UT} = \frac{1}{4} m_{UT}\left({R_{UT}}^2 + {RR_{UT}}^2\right)$$

The dimensions and properties of the **lower torso** are given by:

$$R_{LT} = \frac{1}{2} HIPB$$

$$RR_{LT} = \frac{1}{4}(WAISD + BUTTD)$$

Segmental length: $SL_{LT} = SITH - (STAT - SUBH)$

Distance from the top of the lower torso to the centre of mass: $SR_{LT} = \frac{1}{2} SL_{LT}$

Volume of the lower torso: $V_{LT} = \pi \cdot R_{LT} \cdot RR_{LT} \cdot SL_{LT}$

Mass of the lower torso: $m_{LT} = HNT - m_H - m_{UT}$

Density of the lower torso: $\rho_{LT} = {m_{LT}}/{V_{LT}}$

Local mass moments of inertia of the lower torso:

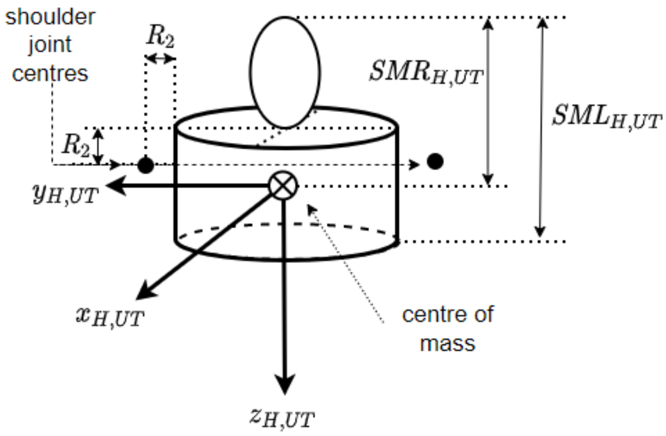
$$I_{xx,LT} = \frac{1}{12} m_{LT} \left(3{R_{LT}}^2 + {SL_{LT}}^2\right)$$

$$I_{yy,LT} = \frac{1}{12} m_{LT} \left(3{RR_{LT}}^2 + {SL_{LT}}^2\right)$$

$$I_{zz,LT} = \frac{1}{4} m_{LT} \left({R_{LT}}^2 + {RR_{LT}}^2\right)$$

Combination of head and upper torso

We combine the head and upper torso as a single compound segment about the common z-axis.



The dimensions and properties of this compound segment are given by:

Segmental length: $SL_{H,UT} = SL_H + SL_{UT} = STAT - SUBH$

Segmental mass: $m_{H,UT} = m_H + m_{UT}$

Volume of the compound segment: $V_{H,UT} = V_H + V_{UT}$

Density of the compound segment: $\rho_{H,UT} = {m_{H,UT}}/{V_{H,UT}}$

The distance $SMR_{H,UT}$ from the top of the head to the centre of mass of the compound segment, which is a two-body system, is given by:

$$SMR_{H,UT} = \frac{m_H R_H + m_{UT}(SL_H + SR_{UT})}{m_{H,UT}}$$

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

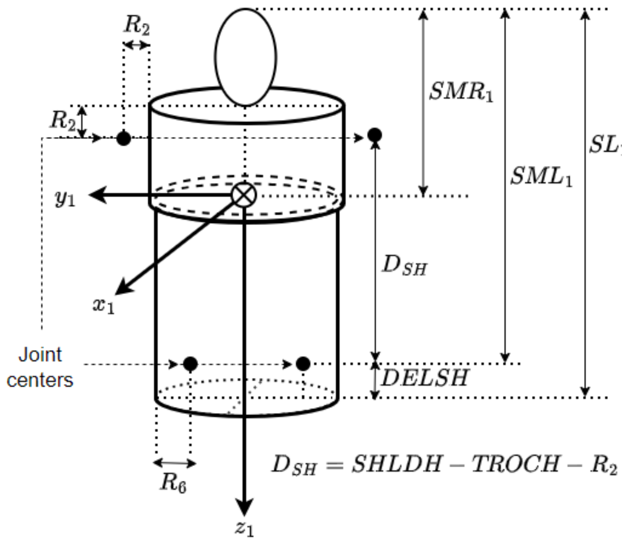
$$I_{xx,H,UT} = I_{xx,H} + I_{xx,UT} + m_H\left(SMR_{H,UT} - SR_H\right)^2 + m_{UT}\left(SL_H + SR_{UT} - SMR_{H,UT}\right)^2$$

$$I_{yy,H,UT} = I_{yy,H} + I_{yy,UT} + m_H\left(SMR_{H,UT} - SR_H\right)^2 + m_{UT}\left(SL_H + SR_{UT} - SMR_{H,UT}\right)^2$$

$$I_{zz,H,UT} = I_{zz,H} + I_{zz,UT}$$

Combination of head, upper and lower torso

We combine the head, the upper and lower torso as a single compound segment about the common z-axis, i.e. in a stretched position.



The dimensions and properties of this compound segment, numbered as segment 1 in the model of Woolley (1972) are given by:

Segmental length: $SL_1 = SL_H + SL_{UT} + SL_{LT} = SITH$

Distance from bottom of lower torso to hinge point: $DELSH = SITH - (STAT - TROCH)$

Distance from top of the head to the hip joint centre: $SML_1 = SL_1 - DELSH$

Distance from shoulder to hip joint centre: $D_{SH} = SHLDH - TROCH - R_2$

Segmental mass: $m_1 = m_H + m_{UT} + m_{LT} = HNT$

Volume of the compound segment: $V_1 = V_H + V_{UT} + V_{LT}$

Density of the compound segment: $\rho_1 = {}^{m_1}/_{V_1}$

The distance SMR_1 from the top of the head to the centre of mass of the compound segment, which is a three-body system, is given by:

$$SMR_1 = \frac{m_H R_H + m_{UT}(SL_H + SR_{UT}) + m_{LT}(SL_H + SL_{UT} + SR_{LT})}{m_1}$$

Location of centre of mass with respect to shoulder-hip distance, from hip (proximal end) to shoulder (distal end): $R_{\text{distal},1} = {}^{(SMR_1 - SL_H - R_2)}/_{D_{\text{SH}}}$ and $R_{\text{proximal},1} = 1 - R_{\text{distal},1}$

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

$$I_{xx,1} = I_{xx,H} + I_{xx,UT} + I_{xx,LT} + m_H(SMR_1 - SR_H)^2 + m_{UT}(SL_H + SR_{UT} - SMR_1)^2 + m_{LT}(SL_H + SL_{UT} + SR_{UT} - SMR_1)^2$$

$$I_{yy,1} = I_{yy,H} + I_{yy,UT} + I_{yy,LT} + m_H(SMR_1 - SR_H)^2 + m_{UT}(SL_H + SR_{UT} - SMR_1)^2 + m_{LT}(SL_H + SL_{UT} + SR_{UT} - SMR_1)^2$$

$$I_{zz,1} = I_{zz,H} + I_{zz,UT} + I_{zz,LT}$$

Gyration radii (segment length = distance D_{SH} from shoulder joint centre to hip joint centre):

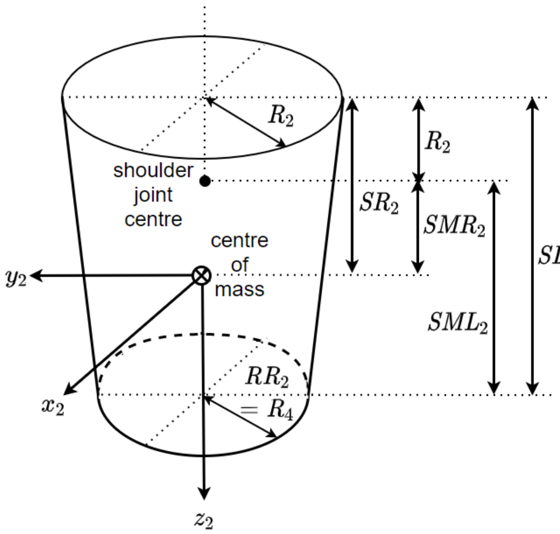
$$k_{xx,1} = \frac{\sqrt{\frac{I_{xx,1}}{m_1}}}{D_{SH}}, \qquad k_{yy,1} = \frac{\sqrt{\frac{I_{yy,1}}{m_1}}}{D_{SH}}, \qquad k_{zz,1} = \frac{\sqrt{\frac{I_{zz,1}}{m_1}}}{D_{SH}}$$

Upper extremity

We consider anthropometric quantities of the upper and lower arm, the hand, and combinations of these segments.

Upper arm

The upper arm in the model of man is a frustrum of a right circular cone with radii $R = R_{UA}$ and $r = RR_{UA}$, height $h = SL_{UA}$, and mass $m = m_{UA}$. Instead of using UA as index for denoting the upper arm we choose for the right upper arm index 2.



The dimensions and properties of the upper arm are given by:

$$R_2 = \frac{AXILC}{2\pi}$$

$$RR_2 = \frac{ELBC}{2\pi}$$

$$SL_2 = UPARL$$

$$m_2 = \frac{1}{2}BUA$$

$$A_2 = \frac{9(1 + \mu_2 + {\mu_2}^2 + {\mu_2}^3 + {\mu_2}^4)}{20\pi{\sigma_{UA}}^2}$$

$$B_2 = \frac{3(1 + 4\mu_2 + 10{\mu_2}^2 + 4{\mu_2}^3 + {\mu_2}^4)}{80{\sigma_2}^2}$$

$$\eta_2 = \frac{1 + 2\mu_2 + 3{\mu_2}^2}{4\sigma_2}$$

where $\mu_2 = RR_2/R_2$ and $\sigma_2 = 1 + \mu_2 + {\mu_2}^2$.

Volume of the upper arm: $V_2 = \frac{1}{3}\pi\,\sigma_2 {R_2}^2 SL_2$

Density of the upper arm: $\rho_2 = {}^{m_2}/_{V_2}$

Distance from the top of the upper arm to the centre of mass: $SR_2 = \eta_2 \cdot SL_2$

Distance from the bottom of the upper arm to the shoulder joint centre: $SML_2 = UPARL - R_2$

Distance from the shoulder joint centre to the centre of mass: $SMR_2 = SR_2 - R_2$

Local mass moments of inertia of the upper arm:

$$I_{xx,2} = A_2 \frac{{m_2}^2}{\rho_2 SL_2} + B_2 m_2 {SL_2}^2$$

$$I_{yy,2} = I_{xx,2}$$

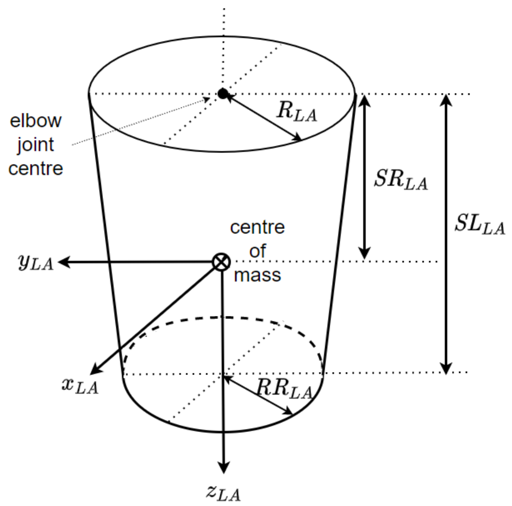
$$I_{zz,2} = \frac{2A_2 {m_2}^2}{\rho_2 SL_2}$$

Gyration radii (segment length = distance SML_2 from shoulder joint centre to elbow joint centre):

$$k_{xx,2} = k_{yy,2} = \frac{\sqrt{\frac{I_{xx,2}}{m_2}}}{SML_2}, \qquad k_{zz,2} = \frac{\sqrt{\frac{I_{zz,2}}{m_2}}}{SML_2}$$

Lower arm

The lower arm in the model of man is a frustrum of a right circular cone with radii $R = R_{LA}$ and $r = RR_{LA}$, height $h = SL_{LA}$, and mass $m = m_{LA}$.



The dimensions and properties of the lower arm are given by:

$$R_{LA} = \frac{ELBC}{2\pi}$$

$$RR_{LA} = \frac{WRISC}{2\pi}$$

$$SL_{LA} = FOARL$$

$$m_{LA} = \frac{1}{2} BFO$$

$$A_{LA} = \frac{9(1 + \mu_{LA} + {\mu_{LA}}^2 + {\mu_{LA}}^3 + {\mu_{LA}}^4)}{20\pi{\sigma_{LA}}^2}$$

$$B_{LA} = \frac{3(1 + 4\mu_{LA} + 10{\mu_{LA}}^2 + 4{\mu_{LA}}^3 + {\mu_{LA}}^4)}{80{\sigma_{LA}}^2}$$

$$\eta_{LA} = \frac{1 + 2\mu_{LA} + 3{\mu_{LA}}^2}{4\sigma_{LA}}$$

where $\mu_{LA} = RR_{LA}/R_{LA}$ and $\sigma_{LA} = 1 + \mu_{LA} + {\mu_{LA}}^2$.

Volume of the lower arm: $V_{LA} = \frac{1}{3}\pi({RR_{LA}}^2 + RR_{LA} \cdot R_{LA} + {R_{LA}}^2)SL_{LA}$

Density of the lower arm: $\rho_{LA} = {}^{m_{LA}}/_{V_{LA}}$

Distance from the elbow joint of the lower arm to the centre of mass: $SR_{LA} = \eta_{LA} \cdot SL_{LA}$

Local mass moments of inertia of the lower arm:

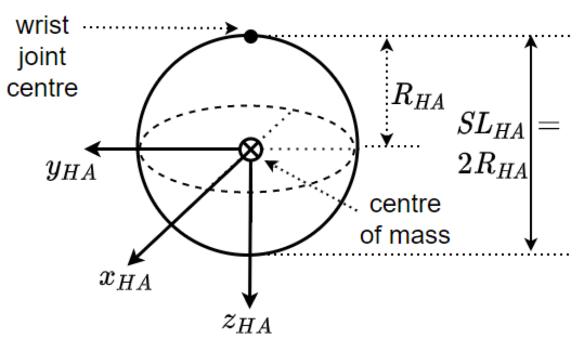
$$I_{xx,LA} = A_{LA}\frac{{m_{LA}}^2}{\rho_{LA} SL_{LA}} + B_{LA} m_{LA} {SL_{LA}}^2$$

$$I_{yy,LA} = I_{xx,LA}$$

$$I_{zz,LA} = \frac{2A_{LA}{m_{LA}}^2}{\rho_{LA} SL_{LA}}$$

Hand

The hand in the model of man is a solid sphere with radius $r = R_{HA}$ and height $h = SL_{HA} = 2R_{HA}$, and mass $m = m_{HA}$.



The dimensions and properties of the hand are given by:

$$R_{HA} = \frac{FISTC}{2\pi}$$

$m_{HA} = \frac{1}{2}BH$

Distance from the wrist joint centre of the hand to the centre of mass: $SR_{HA} = R_{HA}$

Volume of the hand: $V_{HA} = \frac{4}{3}\pi {R_{HA}}^3$

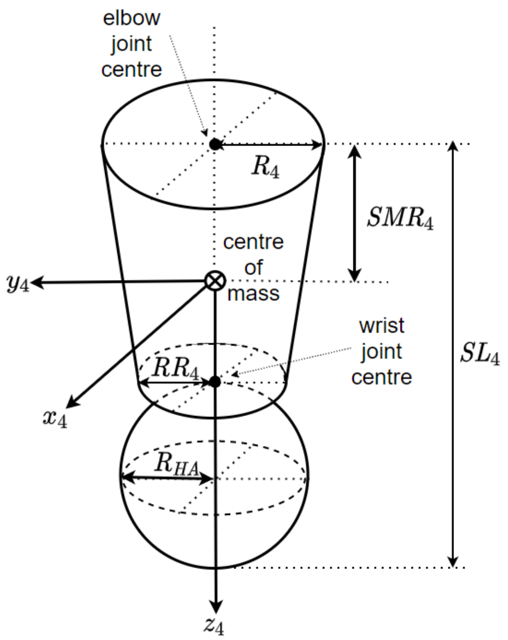
Density of the hand: $\rho_{HA} = {}^{m_{HA}}/_{V_{HA}}$

Local mass moments of inertia of the lower arm:

$I_{xx,HA} = I_{yy,HA} = I_{zz,HA} = \frac{2}{5}m_{HA} \cdot {R_{HA}}^2$

Combination of lower arm and hand

We combine the lower arm and hand as a single compound segment about the common z-axis. It is segment 4 (and 5)



The dimensions and properties of this compound segment, numbered as segment 4 in the model of Woolley (1972) are given by:

Segmental length: $SL_4 = SL_{LA} + SL_{HA}(= FOARL + 2R_{HA})$

Segmental mass: $m_4 = m_{LA} + m_H \left(= \frac{1}{2}(BFO + BH)\right)$

Volume of the compound segment: $V_4 = V_{LA} + V_{HA}$

Density of the compound segment: $\rho_4 = {}^{m_4}/_{V_4}$

The distance SMR_4 from the elbow joint centre to the centre of mass of the compound segment, which is a two-body system, is given by:

$$SMR_4 = \frac{m_{LA}SR_{LA} + m_{HA}(SL_{LA} + SR_{HA})}{m_4}$$

Location of centre of mass with respect to elbow-wrist distance, from elbow (proximal end) to wrist (distal end): $R_{\text{proximal},1} = {SMR_4}/{SL_{LA}}$ and $R_{\text{distal},4} = 1 - R_{\text{proximal},4}$. Recall that $SL_{LA} = FOARL$.

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

$I_{xx,4} = I_{xx,LA} + I_{xx,HA} + m_{LA}(SR_{LA} - SMR_4)^2 + m_{HA}(SL_{LA} + SR_{HA} - SMR_4)^2$

$I_{yy,4} = I_{xx,4}$

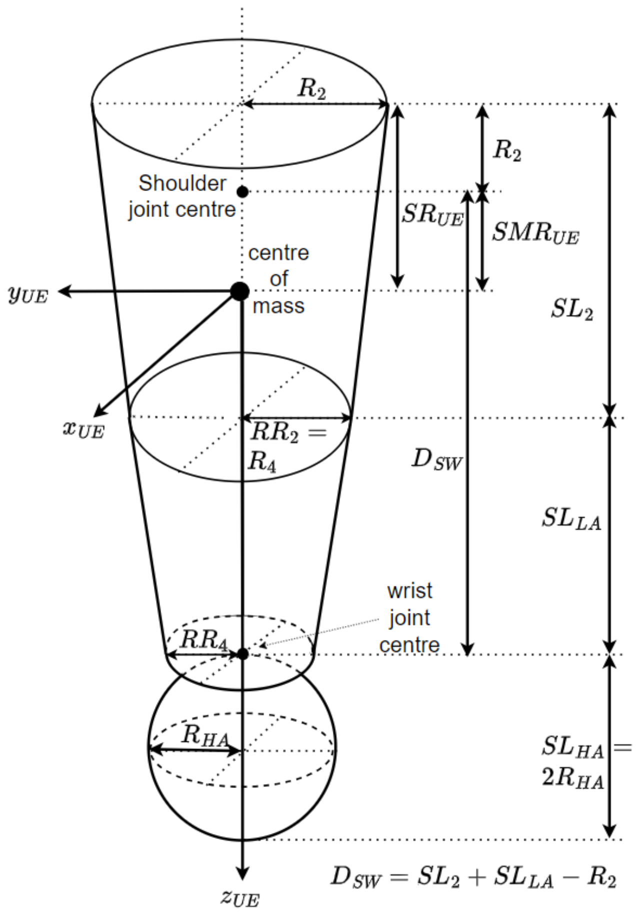
$I_{zz,4} = I_{zz,LA} + I_{zz,HA}$

Gyration radii (segment length = distance SL_{LA} $(= FOARL)$ from elbow joint centre to wrist joint centre):

$$k_{xx,4} = k_{yy,4} = \frac{\sqrt{\frac{I_{xx,4}}{m_4}}}{SL_{LA}}, \qquad k_{zz,1} = \frac{\sqrt{\frac{I_{zz,4}}{m_4}}}{SML_{LA}}$$

Upper extremity: a combination of upper arm, lower arm, and hand

We combine the upper arm, the lower arm, and the hand torso as a single compound segment about the common z-axis, i.e. the arm is in a stretched position.



The dimensions and properties of this compound segment, which we refer to as the (stretched) upper extremity are given by:

Segmental length: $SL_{UE} = SL_2 + SL_{LA} + SL_{HA}$

Distance from the top of the upper arm to the shoulder joint centre: R_2

Segmental mass: $m_{UE} = m_2 + m_{LA} + m_{HA} \left(= \frac{1}{2}(BUA + BFO + BH) \right)$

Volume of the compound segment: $V_{UE} = V_2 + V_{LA} + V_{HA}$

Density of the compound segment: $\rho_{UE} = {m_{UE}}/{V_{UE}}$

The distance SR_{UE} from the top of the upper extremity to the centre of mass of the compound segment, which is a three-body system, is given by:

$$SR_{UE} = \frac{m_2 SR_2 + m_{LA}(SL_2 + SR_{LA}) + m_{HA}(SL_2 + SL_{LA} + SR_{HA})}{m_{UE}}$$

Distance from the shoulder joint centre to the centre of mass: $SMR_{UE} = SR_{UE} - R_2$

Distance between shoulder joint centre and wrist joint centre is equal to $D_{SW} = SL_2 + SL_{LA} - R_2$. Recall that $SL_{LA} = FOARL$.

Location of centre of mass with respect to shoulder-wrist distance, from shoulder (proximal end) to wrist (distal end): $R_{\mathrm{proximal},UE} = {SMR_{UE}}/{D_{SW}}$ and $R_{\mathrm{distal},UE} = 1 - R_{\mathrm{proximal},UE}$.

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

$$I_{xx,UE} = I_{xx,2} + I_{xx,LA} + I_{xx,HA} + m_2(SR_2 - SR_{UE})^2 + m_{LA}(SL_2 + SR_{LA} - SR_{UE})^2 + m_{HA}(SL_2 + SL_{LA} + SR_{HA} - SR_{UE})^2$$

$$I_{yy,UE} = I_{xx,UE}$$

$$I_{zz,UE} = I_{zz,2} + I_{zz,LA} + I_{zz,HA}$$

Gyration radii (segment length = distance D_{SW} from shoulder joint centre to wrist joint centre):

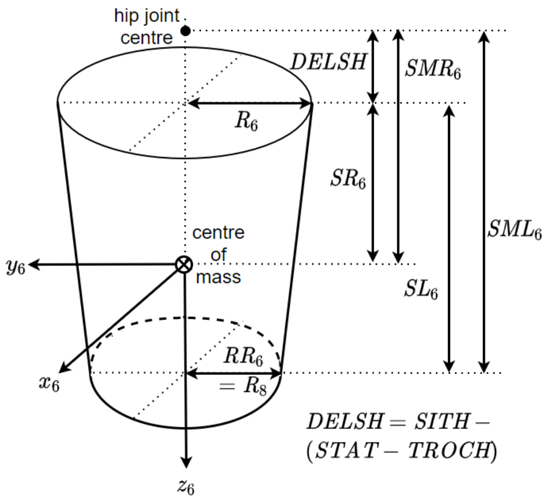
$$k_{xx,UE} = k_{yy,UE} = \frac{\sqrt{\frac{I_{xx,UE}}{m_{UE}}}}{D_{SW}}, \qquad k_{zz,UE} = \frac{\sqrt{\frac{I_{zz,UE}}{m_{UE}}}}{D_{SW}}$$

Lower extremity

We consider anthropometric quantities of the upper and lower leg, the foot, and combinations of these segments.

Upper leg

The upper leg in the model of man is a frustrum of a right circular cone with radii $R = R_{UL}$ and $r = RR_{UL}$, height $h = SL_{UL}$, and mass $m = m_{UL}$. Instead of using UL as index for denoting the upper leg we choose for the right upper leg index 6.



The dimensions and properties of the upper leg are given by:

$$R_6 = \frac{THIHC}{2\pi}$$

$$RR_6 = \frac{GNEKC}{2\pi}$$

$$SL_6 = STAT - SITH - TIBH$$

$$m_6 = \frac{1}{2}BUL$$

$$A_6 = \frac{9(1 + \mu_6 + {\mu_6}^2 + {\mu_6}^3 + {\mu_6}^4)}{20\pi{\sigma_6}^2}$$

$$B_6 = \frac{3\left(1 + 4\mu_6 + 10{\mu_6}^2 + 4{\mu_6}^3 + {\mu_6}^4\right)}{80{\sigma_6}^2}$$

$$\eta_6 = \frac{1 + 2\mu_6 + 3{\mu_6}^2}{4\sigma_6}$$

where $\mu_6 = RR_6/R_6$ and $\sigma_6 = 1 + \mu_6 + {\mu_6}^2$.

Volume of the upper leg: $V_6 = \frac{1}{3}\pi\,\sigma_6 {R_6}^2 SL_6$

Density of the upper leg: $\rho_6 = {}^{m_6}/_{V_6}$

Distance from the top of the upper leg to the centre of mass: $SR_6 = \eta_6 \cdot SL_6$

Distance from hip joint centre of upper leg to top of the upper leg: $DELSH = SITH - (STAT - TROCH)$

Distance from hip joint centre to bottom of upper leg: $SML_6 = SL_6 + DELSH$

Distance from hip joint centre to centre of mass of upper leg: $SMR_6 = SR_6 + DELSH$

Location of centre of mass with respect to hip-knee distance, from hip (proximal end) to knee (distal end): $R_{\text{proximal},6} = {}^{SMR_6}/_{SML_6}$ and $R_{\text{distal},6} = 1 - R_{\text{proximal},6}$.

Local mass moments of inertia of the upper leg:

$$I_{xx,6} = A_6 \frac{{m_6}^2}{\rho_6 SL_6} + B_6 m_6 {SL_6}^2$$

$$I_{yy,6} = I_{xx,6}$$

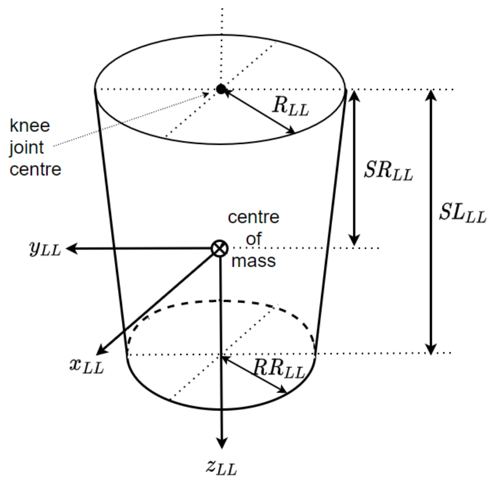
$$I_{zz,6} = \frac{2A_6 {m_6}^2}{\rho_6 SL_6}$$

Gyration radii (segment length = distance SML_6 from hip joint centre to knee joint centre):

$$k_{xx,6} = k_{yy,6} = \frac{\sqrt{\frac{I_{xx,6}}{m_6}}}{SML_6}, \qquad k_{zz,6} = \frac{\sqrt{\frac{I_{zz,6}}{m_6}}}{SML_6}$$

Lower leg

The lower leg in the model of man is a frustrum of a right circular cone with radii $R = R_{LL}$ and $r = RR_{LL}$, height $h = SL_{LL}$, and mass $m = m_{LL}$.



The dimensions and properties of the lower leg are given by:

$$R_{LL} = \frac{GKNEC}{2\pi}$$

$$RR_{LL} = \frac{ANKC}{2\pi}$$

$$SL_{LL} = TIBH - SPHYH$$

$$m_{LL} = \frac{1}{2} BLL$$

$$A_{LL} = \frac{9(1 + \mu_{LL} + {\mu_{LL}}^2 + {\mu_{LL}}^3 + {\mu_{LL}}^4)}{20\pi {\sigma_{LL}}^2}$$

$$B_{LL} = \frac{3(1 + 4\mu_{LL} + 10{\mu_{LL}}^2 + 4{\mu_{LL}}^3 + {\mu_{LL}}^4)}{80{\sigma_{LL}}^2}$$

$$\eta_{LL} = \frac{1 + 2\mu_{LL} + 3{\mu_{LL}}^2}{4\sigma_{LL}}$$

where $\mu_{LL} = RR_{LL}/R_{LL}$ and $\sigma_{LL} = 1 + \mu_{LL} + {\mu_{LL}}^2$.

Volume of the lower leg: $V_{LL} = \frac{1}{3}\pi\, \sigma_{LL} {R_{LL}}^2 SL_{LL}$

Density of the lower leg: $\rho_{LL} = {m_{LL}}/{V_{LL}}$

Distance from the elbow joint centre to the centre of mass: $SR_{LL} = \eta_{LL} \cdot SL_{LL}$

Local mass moments of inertia of the lower leg:

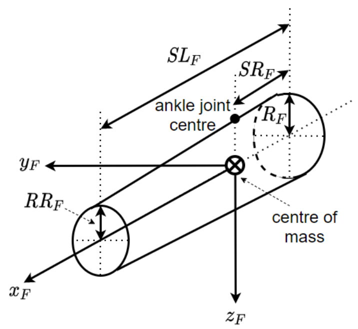
$$I_{xx,LL} = A_{LL}\frac{{m_{LL}}^2}{\rho_{LL} SL_{LL}} + B_{LL} m_{LL} {SL_{LL}}^2$$

$$I_{yy,LL} = I_{xx,LL}$$

$$I_{zz,LL} = \frac{2A_{LL}{m_{LL}}^2}{\rho_{LL} SL_{LL}}$$

Foot

The foot in the model of man is man is a frustrum of a right circular cone with radii $R = R_F$ and $r = RR_{FF}$, height $h = SL_F$, and mass $m = m_F$. The orientation is such as to produce a circular cross section in a plane parallel to the frontal plane and a trapezoidal cross section in planes parallel to the transverse sagittal planes; see the figure below and notice that the x_F-axis is now the main axis of the frustrum of the spherical cone. The hinge point is at the distance of R_F from this main axis



The dimensions and properties of the foot are given by: $R_F = \frac{1}{2} SPHYH$

Distance from the heel to the centre of mass: $SR_F = 0.429 SL_F$ according to Dempster (1955), p. 192.

$RR_F = \mu_F R_F$ with μ according to Hanavan (1964) chosen such that centre of gravity of the foot is located at a distance $0.429 SL_F$ from the larger end of the foot. Woolley (1972) reported $\mu_F = 0.403$ but the formulas for the frustrum of a right circular cone would give $\mu = 0.644$. This difference in values of μ_F does not really influence our diving results as the largest contribution of the foot to the total moment of inertia is coming from an external term due to the parallel axis theorem.

$$SL_F = FOOTL$$

$$m_F = \frac{1}{2} BF$$

$$A_F = \frac{9(1 + \mu_F + {\mu_F}^2 + {\mu_F}^3 + {\mu_F}^4)}{20\pi {\sigma_F}^2}$$

$$B_F = \frac{3(1 + 4\mu_F + 10{\mu_F}^2 + 4{\mu_F}^3 + {\mu_F}^4)}{80 {\sigma_F}^2}$$

$$\eta_F = \frac{1 + 2\mu_F + 3{\mu_F}^2}{4\sigma_F}$$

where $\mu_F = RR_F / R_F$ and $\sigma_F = 1 + \mu_F + {\mu_F}^2$.

Volume of the lower leg: $V_F = \frac{1}{3}\pi\, \sigma_F {R_F}^2 SL_F$

Density of the lower leg: $\rho_F = {}^{m_F}/_{V_F}$

Local mass moments of inertia of the foot:

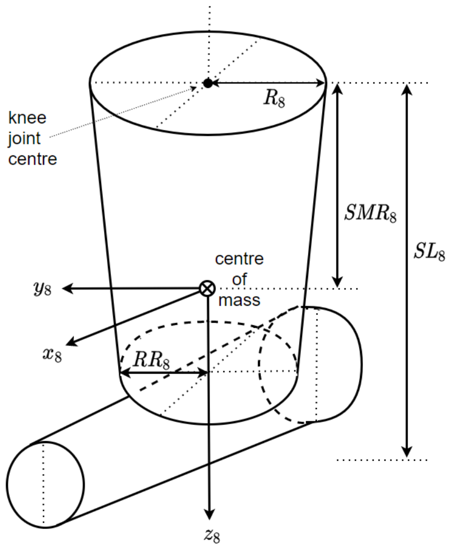
$$I_{xx,F} = \frac{2A_F {m_F}^2}{\rho_F SL_F}$$

$$I_{yy,F} = A_F \frac{{m_F}^2}{\rho_F SL_F} + B_F m_F {SL_F}^2$$

$$I_{zz,F} = I_{yy,F}$$

Combination of lower leg and the foot

We combine the lower leg and the foot as a single compound segment. We use index 8 for this segment



The dimensions and properties of this compound segment, numbered as segment 8 in the model of Woolley (1972) are given by:

Segmental length: $SL_8 = SL_{LL} + SPHYH (= TIBH)$

Segmental mass: $m_8 = m_{LL} + m_F \left(= \frac{1}{2}(BLL + BF)\right)$

Volume of the compound segment: $V_8 = V_{LL} + V_F$

Density of the compound segment: $\rho_8 = {}^{m_8}/_{V_8}$

The distance SMR_8 from the top of the lower leg (the knee joint centre) to the centre of mass of the compound segment, which is a two-body system, is given by:

$$SMR_8 = \frac{m_{LL}SR_{LL} + m_F(SL_{LL} + R_F)}{m_8}$$

Location of centre of mass with respect to knee-ankle distance, from knee joint centre (proximal end) to ankle joint centre (distal end): $R_{\text{proximal},8} = {}^{SMR_8}/_{SL_{LL}}$ and $R_{\text{distal},8} = 1 - R_{\text{proximal},6}$.

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

$$I_{xx,8} = I_{xx,LL} + I_{xx,F} + m_{LL}(SR_{LL} - SMR_8)^2 + m_F(SL_{LL} + R_F - SMR_8)^2$$

$$I_{yy,8} = I_{yy,LL} + I_{yy,F} + m_{LL}(SR_{LL} - SMR_8)^2 + m_F(SL_{LL} + R_F - SMR_4)^2$$

$$I_{zz,8} = I_{zz,LL} + I_{zz,F}$$

Gyration radii (segment length = distance SL_{LL} from knee joint centre to ankle joint centre, which is equal to the length of the lower leg):

$$k_{xx,8} = \frac{\sqrt{\frac{I_{xx,8}}{m_8}}}{SL_{LL}}, \qquad k_{xx,8} = \frac{\sqrt{\frac{I_{yy,8}}{m_8}}}{SL_{LL}}, \qquad k_{zz,8} = \frac{\sqrt{\frac{I_{zz,8}}{m_8}}}{SL_{LL}}$$

Lower extremity: a combination of upper leg, lower leg, and foot

We combine the upper leg, the lower leg, and the hand torso as a single compound segment about the common z-axis, i.e. the leg is in a stretched position.

The dimensions and properties of this compound segment, which we refer to as the (stretched) upper extremity are given by:

Segmental length: $SL_{LE} = SL_6 + SL_8$ $(= STAT - SITH)$

Distance from the top of the lower extremity to the hip joint centre: $DELSH$

Segmental mass: $m_{LE} = m_6 + m_8 \left(= \frac{1}{2}(BUL + BLL + BF)\right)$

Volume of the compound segment: $V_{LE} = V_6 + V_8$

Density of the compound segment: $\rho_{LE} = {}^{m_{LE}}/_{V_{LE}}$

The distance SR_{LE} from the top of the lower extremity to the centre of mass of the compound segment, which is a three-body system, is given by:

$$SR_{LE} = \frac{m_6 SR_6 + m_{LL}(SL_6 + SR_{LL}) + m_F(SL_6 + SL_{LL} + R_F)}{m_{LE}}$$

The distance SMR_{LE} from the hip joint centre to the centre of mass of the compound segment, which is a three-body system, is given by:

$$SMR_{LE} = SR_{LE} + DELSH$$

Distance from hip joint centre (proximal end) to ankle joint centre (distal end):

$SML_{LE} = TROCH - SPHYH$, which is the same as $SML_{UL} + SL_{LL}$ and the same as $SL_{LE} - SPHYH + DELSH$

Location of centre of mass with respect to hip-ankle distance, from hip joint centre (proximal end) to ankle joint centre (distal end): $R_{\text{proximal},LE} = {}^{SMR_{LE}}/_{SML_{LE}}$ and $R_{\text{distal},LE} = 1 - R_{\text{proximal},LE}$.

The local mass moments of inertia of the compound segment about the coordinates axes with their origin in the centre of mass of the compound segment can be computed with the parallel-axis theorem and are given by:

$$I_{xx,LE} = I_{xx,6} + I_{xx,8} + m_6(SR_6 - SR_{LE})^2 + m_8(SL_6 + SMR_8 - SR_{LE})^2$$

$$I_{yy,LE} = I_{yy,6} + I_{yy,8} + m_6(SR_6 - SR_{LE})^2 + m_8(SL_6 + SMR_8 - SR_{LE})^2$$

$$I_{zz,LE} = I_{zz,6} + I_{zz,8}$$

Gyration radii (segment length = distance SML_{LE} from hip joint centre to ankle joint centre):

$$k_{xx,LE} = \frac{\sqrt{\frac{I_{xx,LE}}{m_{LE}}}}{SML_{LE}}, \qquad k_{xx,8} = \frac{\sqrt{\frac{I_{yy,LE}}{m_{LE}}}}{SML_{LE}}, \qquad k_{zz,LE} = \frac{\sqrt{\frac{I_{zz,LE}}{m_{LE}}}}{SML_{LE}}$$

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