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### Seasonal reproduction leads to population collapse and an Allee effect in a stage-structured consumer-resource biomass model when mortality rate increases

Sun, Z.; de Roos, A.M.

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### Supplementary material to Sun and De Roos: Seasonal reproduction leads to population collapse and an Allee effect in a stage-structured consumer-resource biomass model when mortality rate increases

Here we present the derivation of  $R_0$  shown in Eq. 2 in the main text.

For  $\theta = 1$  juveniles and adults have the same net biomass productivity. To simplify the discussion we denote the net biomass productivity of the consumer individuals as  $\nu(R)$ , defined as:

$$\nu(R) = \sigma I_{\max} R - Q, \quad (1)$$

Because consumers never starve during the season (Figure 4a), the mortality rates of juveniles and adults are also the same and equal to  $\mu$ . The probability, denoted by  $P_j(t)$ , that the consumer individual is still in the juvenile stage at time  $t$  follows a dynamics described by the following ordinary differential equation:

$$\frac{dP_j}{dt} = (-\gamma(\nu(R(t)), \mu) - \mu) P_j, \quad P_j(0) = 1. \quad (2)$$

This ODE can be formally solved to yield as solution:

$$\begin{aligned} P_j(t_m) &= \exp\left(\int_0^{t_m} -\mu - \gamma(\nu(R(\tau)), \mu) d\tau\right) \\ &= \exp(-\mu t_m) \exp\left(-\int_0^{t_m} \gamma(\nu(R(\tau)), \mu) d\tau\right), \end{aligned} \quad (3)$$

The maturation rate at time  $t_m$  out of the juvenile stage is then given by

$$h(t_m) = \gamma(\nu(R(t_m)), \mu) P_j(t_m). \quad (4)$$

If we would consider a cohort of  $N$  newly produced offspring,  $h(t_m) \cdot N$  would represent the expected rate of maturation of these individuals at time  $t_m$ .

The underlying assumption in the model about consumer life history [1, 2] is that juvenile individuals are born at the beginning of a season with body size  $s(0) = s_b$ , which we set without loss of generality equal to 1. They subsequently grow following:

$$\frac{ds}{d\tau} = \nu(R(\tau)) s, \quad s(0) = s_b.$$

Individual body size at  $t_m$  hence equals:

$$s(t_m) = \exp\left(\int_0^{t_m} \nu(R(\tau)) d\tau\right). \quad (5)$$

In the period from  $t_m$  to the end of the season an individual maturing at  $t_m$  accumulates and stores reproductive energy at a rate  $s(t_m) \nu(R(\tau))$ , which is proportional to its size at maturation. The contribution to the reproductive output at the end of the season by the individuals that mature at time  $t_m$  in the growing season is thus given by

$$\begin{aligned} R_0(t_m) &= h(t_m) \int_{t_m}^1 s(t_m) \nu(R(\tau)) d\tau \cdot \exp(-\mu(1-t_m)) \\ &= h(t_m) s(t_m) \int_{t_m}^1 \nu(R(\tau)) d\tau \cdot \exp(-\mu(1-t_m)). \end{aligned} \quad (6)$$

Using Equations (3)-(5) this expression can be rewritten as:

$$R_0(t_m) = \gamma(\nu(R(t_m)), \mu) \exp\left(-\mu + \int_0^{t_m} \nu(R(\tau)) - \gamma(\nu(R(\tau)), \mu) d\tau\right) \int_{t_m}^1 \nu(R(\tau)) d\tau. \quad (7)$$

## References

- [1] De Roos AM et al. 2008 Simplifying a physiologically structured population model to a stage-structured biomass model. *Theor. Popul. Bio.* **73**, 47–62. (doi:10.1016/j.tpb.2007.09.004)
- [2] Soudijn FH, de Roos AM. 2017 Approximation of a physiologically-structured population model with seasonal reproduction by a stage-structured biomass model. *Theor. Ecol.* **10**, 73–90. (doi:10.1007/s12080-016-0309-9)