



UvA-DARE (Digital Academic Repository)

Seasonal reproduction leads to population collapse and an Allee effect in a stage-structured consumer-resource biomass model when mortality rate increases

Sun, Z.; de Roos, A.M.

DOI

[10.1371/journal.pone.0187338](https://doi.org/10.1371/journal.pone.0187338)

Publication date

2017

Document Version

Other version

Published in

PLoS ONE

[Link to publication](#)

Citation for published version (APA):

Sun, Z., & de Roos, A. M. (2017). Seasonal reproduction leads to population collapse and an Allee effect in a stage-structured consumer-resource biomass model when mortality rate increases. *PLoS ONE*, 12(10), Article e0187338.
<https://doi.org/10.1371/journal.pone.0187338>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

Supplementary material to Sun and De Roos: Seasonal reproduction leads to population collapse and an Allee effect in a stage-structured consumer-resource biomass model when mortality rate increases

Here we present the derivation of R_0 shown in Eq. 2 in the main text.

For $\theta = 1$ juveniles and adults have the same net biomass productivity. To simplify the discussion we denote the net biomass productivity of the consumer individuals as $\nu(R)$, defined as:

$$\nu(R) = \sigma I_{\max} R - Q, \quad (1)$$

Because consumers never starve during the season (Figure 4a), the mortality rates of juveniles and adults are also the same and equal to μ . The probability, denoted by $P_j(t)$, that the consumer individual is still in the juvenile stage at time t follows a dynamics described by the following ordinary differential equation:

$$\frac{dP_j}{dt} = (-\gamma(\nu(R(t)), \mu) - \mu) P_j, \quad P_j(0) = 1. \quad (2)$$

This ODE can be formally solved to yield as solution:

$$\begin{aligned} P_j(t_m) &= \exp\left(\int_0^{t_m} -\mu - \gamma(\nu(R(\tau)), \mu) d\tau\right) \\ &= \exp(-\mu t_m) \exp\left(-\int_0^{t_m} \gamma(\nu(R(\tau)), \mu) d\tau\right), \end{aligned} \quad (3)$$

The maturation rate at time t_m out of the juvenile stage is then given by

$$h(t_m) = \gamma(\nu(R(t_m)), \mu) P_j(t_m). \quad (4)$$

If we would consider a cohort of N newly produced offspring, $h(t_m) \cdot N$ would represent the expected rate of maturation of these individuals at time t_m .

The underlying assumption in the model about consumer life history [1, 2] is that juvenile individuals are born at the beginning of a season with body size $s(0) = s_b$, which we set without loss of generality equal to 1. They subsequently grow following:

$$\frac{ds}{d\tau} = \nu(R(\tau)) s, \quad s(0) = s_b.$$

Individual body size at t_m hence equals:

$$s(t_m) = \exp\left(\int_0^{t_m} \nu(R(\tau)) d\tau\right). \quad (5)$$

In the period from t_m to the end of the season an individual maturing at t_m accumulates and stores reproductive energy at a rate $s(t_m) \nu(R(\tau))$, which is proportional to its size at maturation. The contribution to the reproductive output at the end of the season by the individuals that mature at time t_m in the growing season is thus given by

$$\begin{aligned} R_0(t_m) &= h(t_m) \int_{t_m}^1 s(t_m) \nu(R(\tau)) d\tau \cdot \exp(-\mu(1-t_m)) \\ &= h(t_m) s(t_m) \int_{t_m}^1 \nu(R(\tau)) d\tau \cdot \exp(-\mu(1-t_m)). \end{aligned} \quad (6)$$

Using Equations (3)-(5) this expression can be rewritten as:

$$R_0(t_m) = \gamma(\nu(R(t_m)), \mu) \exp\left(-\mu + \int_0^{t_m} \nu(R(\tau)) - \gamma(\nu(R(\tau)), \mu) d\tau\right) \int_{t_m}^1 \nu(R(\tau)) d\tau. \quad (7)$$

References

- [1] De Roos AM et al. 2008 Simplifying a physiologically structured population model to a stage-structured biomass model. *Theor. Popul. Bio.* **73**, 47–62. (doi:10.1016/j.tpb.2007.09.004)
- [2] Soudijn FH, de Roos AM. 2017 Approximation of a physiologically-structured population model with seasonal reproduction by a stage-structured biomass model. *Theor. Ecol.* **10**, 73–90. (doi:10.1007/s12080-016-0309-9)