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### Laboratory tests of theories of strategic interaction

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# 4

## Credit Easing and R&D Expenditures

### 4.1 Introduction

Promoting innovation has become an important policy goal in many countries. Providing subsidies to relax the financial constraints that firms face in their R&D processes is one of the major policy instruments used by governments to stimulate research investments in targeted industries. It is an open question, however, whether such a policy always leads to an increase in aggregate R&D investment in the industry.

Intuition suggests that firms will invest more if not budget constrained. However, the matter is not as trivial as it may seem at the first glance, because the strategic interactions between firms need to be taken into account. To see why this matters, consider that the R&D investments by firms may be seen as a contest to win (temporary) monopoly power over the market concerned. Then, a firm with low prospects of winning the R&D race may see higher chances of winning the research contest if it expects other firms with better prospects to face financial constraints inhibiting a full realization of their research agenda. In this way, expectations of the winning probability may encourage a firm to invest more under a situation where all firms face potential budget constraints than when government policies relieve such constraints. In fact, Che and Gale (1998b) have used an all-pay auction with complete information to model a lobbying contest to show that, imposing a common (public) cap on each firm's lobbying expenditure may instead increase the total expenditures in equilibrium. But can this conclusion be directly generalized to our research question? Maybe not just yet. In the R&D contests, firms' budget constraints are usually heterogeneous and privately known. Therefore, in the study of this chapter, we model such R&D contests by firms as an all-pay auction with private information on both, each contestant's (i.e. firm's) valuation and their budget constraint.<sup>1</sup> For the case where the government offers financial aid to loosen firms' budgets, we take this to the theoretical limit by assuming that bids are unconstrained. To address our research question on how resolving firms' constraints for their R&D input will increase aggregate research investment, we then need to compare the expected revenue (aggregate bids) of the all-pay auctions with and without budget constraints.

For a set-up with complete information, Che and Gale (1998b) and Che and Gale (2006) show that a uniform cap on investments affects a high-value contestant

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<sup>1</sup>The all-pay auction is one of the most influential contest models in the literature, see e.g. Baye, Kovenock, and De Vries (1996), Che and Gale (2003) and Moldovanu and Sela (2006).

more severely than a low-value contestant, which hence implies higher expected aggregate investments in R&D despite the inefficient allocation. All-pay auctions under private valuations with an exogenous common budget (cap) are studied by Gavious, Moldovanu, and Sela (2002), who show that, if the cap is endogenized as the contest designer's choice variable, then convexity of contestants cost function could still justify the use of binding caps to enhance total expected investment on R&D, provided the number of contestants is sufficiently large.

In contrast to Che and Gale (1998b) who suggest that a cap (common constraint) can result in increased aggregate effort, the result from our theoretical model shows that private constraints unambiguously leads to lower total effort, which suggests that removing the private constraint will increase aggregate R&D investment.

To explore the magnitude of the stimulating impact of credit easing on R&D investments, we further consider two special cases of the general model. These allow us to illustrate the potential difference in various types of industries. For this, we consider two distinct joint distributions of the private valuation and private constraint. In the first, the supermodular case, a contestant with a higher valuation is more likely to face a higher constraint; while in the submodular case, a contestant with higher valuation is more likely to face a lower constraint. These two cases represent different types of industries, with the former representing an industry where the prospect of successful innovation is higher for well-established firms with sufficient capital (e.g., the pharmaceutical industry), and the latter representing an industry where innovations are mostly achieved by small new-comers (e.g. the IT industry). Using our specific numerical examples, we show that the investment-enhancing effect of providing liquidity is much larger in the latter than in the former.

We test our results in laboratory experiments. Our first observation is that the laboratory data support the theoretical prediction on revenue comparison between the all-pay auctions with and without constraints. The different effects on revenue between the two joint distributions are also supported by our experiment. Nevertheless, as found in many other experimental studies of auctions, observed individual bidding behavior systematically deviates from the theoretical predictions.<sup>2</sup> In particular, we observe overbidding by low-value bidders and underbidding by high-value bidders, compared to their predicted bids. This observation is similar to the finding by Schram and Onderstal (2009) in their experiment with an all-pay auction with

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<sup>2</sup>See related results on all-pay auctions in the laboratory by Rapoport and Amaldoss (2004), Gneezy and Smorodinsky (2006), Noussair and Silver (2006) and Schram and Onderstal (2009).

incomplete information and a charity. However, it is in contrast to Noussair and Silver (2006)'s finding in an all-pay auction with incomplete information and no constraints, that overbidding mainly happens when values are high, and underbidding mostly happens by bidders with low values.

The remainder of this chapter is organized as follows. Section 4.2 introduces the model and the theoretical results on revenue comparison between the environments with and without constraints. Section 4.3 gives two specific examples to illustrate that the revenue change caused by eliminating constraints differs as the market structure varies. Section 4.4 explains our experimental design, which is based on the two examples as illustrated in Section 4.3. The experimental results are presented in Section 4.5. Finally, Section 4.6 concludes the chapter.

## 4.2 The Contest Model and Predictions

### 4.2.1 The Model

We consider  $n$  contestants bidding for a single prize. Each contestant  $i$  has private information about his valuation  $v_i$  for the prize and his budget constraint  $w_i$ . The contestants are ex ante identical in that the vectors  $(v_i, w_i)$ , which we call *value-constraint pair*, are independently and identically distributed. Let  $f(v, w)$  denote the density function of  $(v, w)$ , assumed to be continuous on its support  $[0, 1] \times [0, 1]$ . The marginal distribution  $F(v) = \Pr(v_i \leq v)$  is given by

$$F(v) = \int_0^v \int_0^1 f(V, W) dW dV$$

Bids are submitted simultaneously and independently of each other in the contest. The contestant with the highest bid wins the prize, while all contestants incur their respective bidding costs. The payoff of contestant  $i$  who has value-constraint pair  $(v_i, w_i)$  and bids  $b_i$  is either  $v_i - b_i$  if he wins the prize, or  $-b_i$  if he does not win.

Suppose that there exists an equilibrium where contestants follow the symmetric bidding strategy  $b(v, w)$  defined by

$$b(v, w) = \min\{a(v), w\} \tag{4.1}$$

for some strictly increasing and differentiable function  $a(v)$  on  $[0, 1]$ .<sup>3</sup> If a contestant bids  $x$ , the probability of winning over one other contestant is denoted as  $H(x)$ , i.e.  $H(x) \equiv \Pr(\min\{a(v), w\} < x)$ . By further defining  $K(v) \equiv H(a(v))$ , we have the following proposition:

**Proposition 2.** *In the all-pay auction with private constraint, the equilibrium strategy  $b(v, w) = \min\{a(v), w\}$  is implicitly characterized by*

$$a(v) = \int_0^v x dK^{n-1}(x), \quad (4.2)$$

The proof of Proposition 2 is put in Appendix 4.7.1.1.

When government subsidies are provided adequately, firms no longer face any budget constraints. We model this case as a standard all-pay auction under incomplete information, where contestants' valuations are drawn independently from the identical distribution  $F(v)$ . It is well-known that the equilibrium strategy, denoted by  $e(v)$ , is given by<sup>4</sup>

$$e(v) = \int_0^v x dF^{n-1}(x) \quad (4.3)$$

### 4.2.2 Revenue and Comparison

Che and Gale (1998b) show that a potential cap or common budget constraint can lead to a higher total revenue for the seller in all-pay auction. Whether this observation holds true for the private budget constraints has not been documented, however. The question we address in this section, is how the revenue generated under the all-pay auction mechanisms with and without private constraints compare.

Lemma 3 presents an alternative representation of the ex ante expected expenditure.

**Lemma 3.** *Under the situation with budget constraint, for each contestant, the ex ante expected expenditure is equal to*

$$E[b(v, w)] = \int_0^1 a(v) dK(v) \quad (4.4)$$

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<sup>3</sup>See Che and Gale (1998a) for equilibrium existence in this model. It suffices here to assume that  $a$  is differentiable almost everywhere on  $[0, 1]$ .

<sup>4</sup>Please refer to Krishna (2009).

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See the proof in Appendix 4.7.1.2.

A new insight from Lemma 3 is that the expected revenue under private budget can be calculated *as though* every contestant were unconstrained and were originally endowed with a value distribution function  $K$ . Reducing the original multi-dimensional problem to a standard single-dimension allows one to examine properties such as stochastic dominance with regard to the induced distribution function  $K$  rather than the original density  $f(v, w)$ . This greatly simplifies the problem, allowing one to derive relevant conclusions straightforwardly.

Under the situation without constraint, the expected expenditure for each contestant is

$$E[e(v)] = \int_0^1 e(v) dF(v) \quad (4.5)$$

Thus the formal conclusion that private budget constraints reduce expected total expenditures follows as shown in the next proposition.

**Proposition 3.** *The expected total expenditure from the all-pay auction under private budget constraints is always lower than that without constraint.*

The proof can be found in Appendix 4.7.1.3.

Recall that Che and Gale (1998a) shows that a potential cap or common bidding constraint can lead to a higher *ex ante* expected total expenditures. However, the above proposition shows that such a conclusion does not extend to the case with private bidding constraints.

Although we know from the prediction by Proposition 3 that easing bidders' private constraints yields higher expected total bids in the all-pay auction, the magnitude of the revenue enhancement may differ across situations. In the following section, we consider two distinctive joint distributions of  $(v, w)$  of the bidders, which capture the specific characteristics of two typical types of industries.

### 4.3 Two Different Types of Industries

We consider two different types of industries, in which the firms' R&D prospects and their private values and constraints,  $(v, w)$ , follow distinct patterns. The first type consists of industries where innovation is based heavily on a pre-existing knowledge, equipment, etc., as for example, in the pharmaceutical industry. In such

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### 4.3 Two Different Types of Industries

types, firms' innovative ability and their financial status are usually positively associated. The other type of industry we consider is where firms' innovative power and financial budgets are negatively associated. Here, innovations primarily require newly-developed knowledge, relying more on researchers' new ideas than on traditional money-consuming research. An example of this type is the IT industry. Many start-ups in the IT industry are based on brilliant ideas and compared to established large-scale companies, these small firms usually have very limited funding for their R&D.

We use a supermodular and a submodular density functions to represent the above two types of industries. The reason will become clear shortly after we explain the two types of joint density functions.

We say that the density function  $f(v, w)$  is *supermodular* if for all  $v' > v$  and  $w' > w$ ,

$$f(v', w') + f(v, w) \geq f(v', w) + f(v, w')$$

and  $f(v, w)$  is strictly supermodular if the inequality is strict.

Analogously, we say that the density function  $f(v, w)$  is *submodular* if for all  $v' > v$  and  $w' > w$ ,

$$f(v', w') + f(v, w) \leq f(v', w) + f(v, w')$$

and  $f(v, w)$  is strictly submodular if the inequality is strict.

When a density function is supermodular and the values of  $v$  and  $w$  are simultaneously increased, the resulting increase in the density is larger than the sum of the increases that would result from each increase in isolation. This means that a larger  $v$  is more likely to occur when  $w$  is also larger.<sup>5</sup> In contrast, for a submodular density function, the marginal increase in the density for an increase in  $v$  is larger, the lower is  $w$ . This means that  $v$  and  $w$  are negatively associated, in the sense that a high  $v$  is more likely to take place when  $w$  is lower.

Next, we use two specific examples to illustrate the predicted influence of removing private budget constraints on the increase in aggregate R&D expenditures for each type of industry. For this comparison, we define the following ratio.

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<sup>5</sup>For details, please refer to Meyer and Strulovici (2013).

## 4. CREDIT EASING AND R&D EXPENDITURES

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**Definition 1.** The *Relative Revenue Increase Ratio (RRIR)* is defined as

$$\rho = \frac{R_{WOC} - R_{W/C}}{R_{W/C}}. \quad (4.6)$$

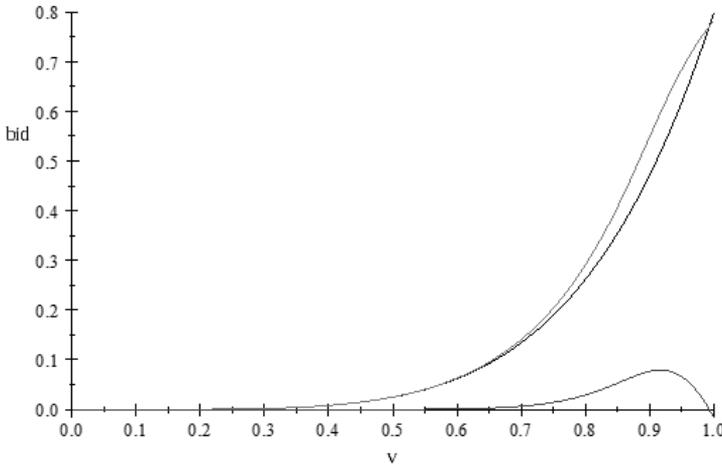
where  $R_{W/C}$  and  $R_{WOC}$  denote the revenue yielded from the all-pay auction with and without constraints, respectively.

We apply the model described above assuming that private values and constraints follow either a supermodular density function or a submodular density function that we will study in our experiment. For both cases, we assume  $n = 3$ , i.e. there are three contestants participating in the auction.

**The Supermodular Case** We consider the case where the joint density function

$$f^{SUP}(v, w) = 4vw \quad (4.7)$$

defined on  $[0, 1] \times [0, 1]$ . It can be readily verified that  $f(v, w)$  is strictly supermodular. Figure 4.1 displays the equilibrium bidding strategies for the situations with and without constraints as  $a(v)$  (red curve) and  $e(v)$  (black curve), respectively.



**Figure 4.1:** Predicted Investment Levels, Supermodular

*Note: The red (black) curve shows the equilibrium bidding curve with(out) constraint;; the blue curve shows the difference between the two curves.*

### 4.3 Two Different Types of Industries

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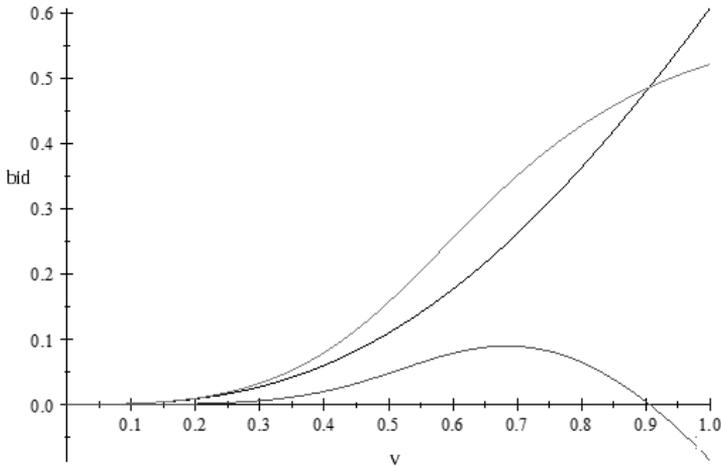
The expected revenues (sums of the bids) with and without private constraints are  $R_{W/C} = 0.675$  and  $R_{W/OC} = 0.685$ , respectively. Details of the derivations are offered in Appendix 4.7.2.1. For the supermodular case, (4.6), then yields for the the RRIR:  $\rho^{SUP} = \frac{0.685-0.675}{0.675} = 1.59\%$ .

**The Submodular Case** For this case, we use for  $(v, w)$  the joint density function

$$f^{SUB}(v, w) = \frac{4}{3}(1 - vw), \quad (4.8)$$

which can be verified to be strictly submodular.

Comparing (4.8) with (4.7), shows that the terms following the coefficients 4 and  $\frac{4}{3}$  in each representation are simply opposite to each other, with the 1 added in the submodular case is to make sure the density function is non-negative, and takes the same range of values on  $[0,1]$ , as in the supermodular case. The coefficients 4 and  $\frac{4}{3}$  normalizations ensuring that the density functions integrate to 1.



**Figure 4.2:** Predicted Bidding Strategies, Submodular

*Note: The red (black) curve shows the equilibrium bidding curve with(out) constraint;; the blue curve shows the difference between the two curves.*

Using the derivations in Appendix 4.7.2.2, we numerically solve for the bidding strategies in the situations with private constraints,  $a(v)$ , and without,  $e(v)$ , as displayed in Figure 4.2. The expected revenues (sums of the bids) in the situations

## 4. CREDIT EASING AND R&D EXPENDITURES

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with and without private constraints, are, respectively,  $R_{W/C} = 0.375$  and  $R_{WOC} = 0.435$ , yielding for this submodular case, an RRIR of  $\rho^{SUB} = \frac{0.435-0.375}{0.375} = 15.61\%$ .

These two specific examples were chosen such that the density functions (4.7) and (4.8) are very similar to each other, except for minimal changes necessary to characterize them as supermodular and submodular, respectively. Given the minimal differences, the influences of removing the private constraints on the expected aggregate bids in the all-pay auction differ dramatically. In comparison to the relative revenue increase (1.69%) in the supermodular case, the increase of 15.61% in the submodular case is a factor ten higher. This suggests that even though subsidies always increase aggregate R&D investments in an industry, the magnitude of this effect may strongly depend on the structure of the market.

Next, we will test the theoretical results of Section 4.2 in a laboratory experiment using the two specific cases introduced in this section. In particular, we aim to study the following questions: first, whether the predicted investment strategies under the situations with and without private budget constraints are supported by the laboratory observations; second, whether as the theory predicts, aggregate expenditure is higher without private constraints than with; third, whether the observed total investment increase in the supermodular case is indeed lower than in the submodular case. The experimental design is explained in the following section.

### 4.4 Experimental Design

We implement the experiment with 15 rounds of auctions. Each subject's total payoff equals a 7 Euro upfront show-up fee plus EUR 0.35 times the subject's payoffs in two rounds that are to be randomly chosen among all the 15 at the end of the entire session.<sup>6</sup> In each round, we implement two simultaneously run all-pay auctions, each framed as a single-prized *Contest* (i.e., Contest  $\tau$ , with  $\tau \in \{A, B\}$ ). In order to win the prize, the subjects, must compete on the number of points they are willing to input. Contest *A* corresponds to the situation where subjects face private constraints; Contest *B* corresponds to the situation without private constraints. In each round, the same group of three subjects participates in both Contests *A* and

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<sup>6</sup>In case a subject ended up with a negative total payoff from the randomly selected rounds, the deficit is deducted from his 7-Euro show-up fee, which means he would end up leaving with having earned less than 7 Euro from our experiment.

## 4.4 Experimental Design

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$B$ . We call this a *contest round*. Before each round, we rematch subjects in new groups of three to prevent possible collusive behavior.<sup>7</sup>

Prior to every contest round  $j$ , each subject  $i$  receives a private endowment  $w_{ij}$  (with  $w_{ij} \in \{0, 1, \dots, 19\}$ ) and a signal indicating his private value  $v_{ij}$  (with  $v_{ij} \in \{0, 1, \dots, 19\}$ ) to the prize. Consistent with the assumptions underlying our theory, both  $v_{ij}$  and  $w_{ij}$  are only known to subject  $i$ . For every contest round, the *value-endowment pair*  $(v_{ij}, w_{ij})$  is independently and identically drawn from a commonly known joint distribution  $f(v, w)$ , which differs across treatments. We distinguish between two treatments in this experiment, namely, *treatment SUP* and *treatment SUB*, featuring two distinct joint distributions  $f(v, w)$ . More specifically, in *treatment SUP*,  $(v_{ij}, w_{ij})$  is drawn from a *supermodular* joint distribution specified by the density function  $f^{SUP}(v, w) = \frac{vw}{40000}$ ; while in *treatment SUB*,  $(v_{ij}, w_{ij})$  is drawn from a *submodular* joint distribution specified by the density function  $f^{SUB}(v, w) = \frac{1}{300}(1 - \frac{vw}{400})$ . For both treatments,  $(v_{ij}, w_{ij}) \in [0, 20) \times [0, 20) \forall i$ .<sup>8</sup> When normalized to the domain of  $(v, w)$  on  $[0, 1) \times [0, 1)$ , these two density functions are exactly the same as those discussed in Section 4.3. Thus, the predictions derived in Section 4.3 can be directly mapped to the domain on  $[0, 20) \times [0, 20)$  as the predictions for the set-ups used in our experiment.

During an arbitrary contest round  $j$ , subject  $i$  has the same value-endowment pair  $(v_{ij}, w_{ij})$  for both Contests  $A$  and  $B$ . Upon knowing his  $(v_{ij}, w_{ij})$ , contestant  $i$  must then decide on his *investment*  $b_{ij}^\tau$  (with  $\tau \in \{A, B\}$ ,  $b_{ij}^\tau \in \{0, 1, \dots, 19\}$ ) in term of points. Contests  $A$  and  $B$  differ as follows. In Contest  $A$ , the points invested by subject  $i$ ,  $b_{ij}^A$ , must be weakly lower than his endowment  $w_{ij}$ ; whereas in Contest  $B$ , his investment  $b_{ij}^B$  is not subjected to such constraint. In other words, we employ a within-subject design for the behavior comparison between the two Contests. In either Contest, while the contestant who invests the most points wins the prize,<sup>9</sup> the investment  $b_{ij}^\tau$  made by any contestant  $i$  is forfeited regardless.<sup>10</sup> Furthermore, only one of the two Contests is (randomly) selected for payoff to a subject at the end of a contest round. If selected for payment, subject  $i$ 's payoff  $p_{ij}^\tau$  in an individual contest

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<sup>7</sup>To increase the number of independent observations in our experiment, rematching takes place in matching groups of nine subjects.

<sup>8</sup>We round down the random values in our experiment to integers for simplicity of implementation.

<sup>9</sup>The allocation of the prize is determined randomly in the case of a tie.

<sup>10</sup>Therefore, in Contest  $B$ , if player  $i$ 's investment  $b_i^B$  is lower than his value  $v_i$  but higher than his endowment  $w_i$ , he could end up with a negative payoff if he loses in the contest.

round  $j$  can be described as

$$p_{ij}^\tau \equiv \begin{cases} w_{ij} - b_{ij}^\tau + v_{ij}, & \text{if he wins;} \\ w_{ij} - b_{ij}^\tau, & \text{if he loses;} \end{cases} \quad \forall i \forall j \forall \tau.$$

To facilitate the subjects' learning process of the distribution of  $(v, w)$  pairs, all three subjects in a group are informed about each other's value-endowment pair  $(v_{ij}, w_{ij})$  and the chosen investment  $b_i^\tau$  in both Contests  $A$  and  $B$  after a contest round concludes.

The experiment took place at the CREED laboratory at University of Amsterdam in March and April, 2014. We ran three sessions for each of our treatments, with 27 subjects in each session. The 162 subjects were recruited via the CREED lab subjects pool, which consists of over 2000 registered subjects. Most of them are university bachelor or master students. Each of the sessions lasted about an hour. The lowest and highest earnings in our experiment were 4.2 and 23.5 Euro, respectively, and the average earning was 14.41 Euro.

## 4.5 Experimental Results

### 4.5.1 Individual Investing Behavior

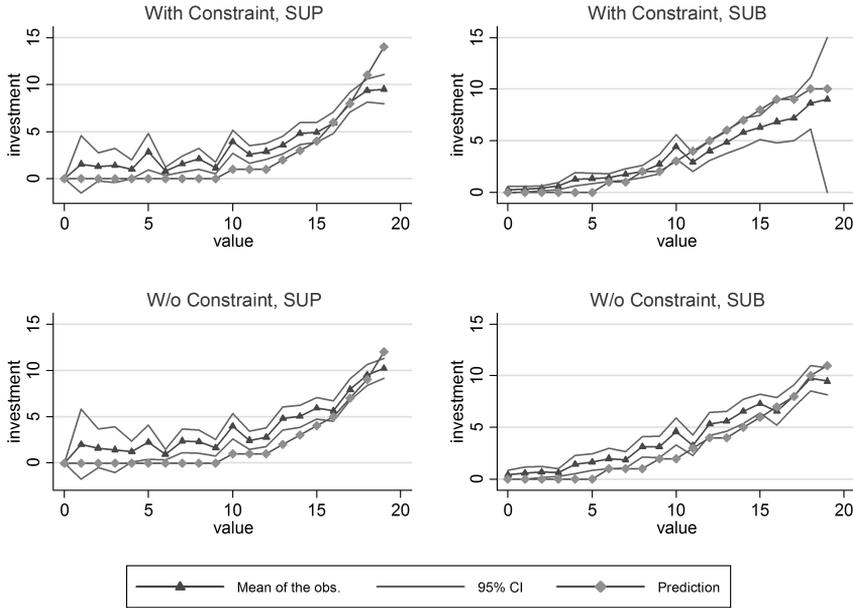
We use two different methods to test the theoretical predictions of the individual investing behavior with our laboratory observations. In the first, we compare the observations with the predicted investment levels (according to the numerical solutions of  $a(v)$  and  $e(v)$  derived in Section 4.3, as displayed in Figure 4.1 (SUP) and 4.2 (SUB)). Because for the contests with private constraints, the predictions given by  $a(v)$  only apply for unconstrained subjects, in the with-constraint contest  $A$ , we use only observations from subjects whose endowment is above their predicted investment level. For each private value, the subjects' average investment levels and their 95% confidence intervals are displayed in Figure 4.3. This is done separately for each combination of treatment and contests with and without constraints.<sup>11</sup> On average, subjects invest significantly more than the predicted level (indicated by the

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<sup>11</sup>The 95% confidence intervals for high private values in the contests with constraints in treatment SUB are much wider than in the other three cases, due to the low number of observations there. In that treatment, high private values are often associated with low endowments, which are more likely to be below the predicted investment levels given the high private values for the prize.

## 4.5 Experimental Results

prediction curve lying below the 95% confidence interval). When the private values reach a reasonably high level, they switch to investing less than the predicted levels (the under-investment comparing to the prediction is not significant, though, except for treatment SUP when the value reaches 19). The switch from over-investment to under-investment comes at lower values in the contest with constraint than without. The deviations from the predicted investment levels are more significant in treatment SUP than in SUB.



**Figure 4.3:** Average Investment Level and Average Prediction (Method 1)

The patterns of deviation from equilibrium observed in our experiment is the reverse of the findings reported by Noussair and Silver (2006) and Müller and Schotter (2010). In Noussair and Silver’s (2006) experimental analysis of an all-pay auction with incomplete information without budget constraints, they find bidders with low values bid lower than in equilibrium, while bidders with high values bid higher than the equilibrium prediction. Müller and Schotter (2010), find that in a tournament where workers compete for a prize, low-ability workers tend to bid too low comparing

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to equilibrium (hence they become “dropouts”) while high-ability workers tend to overbid (hence they become “workaholics”).

Our finding is, however, similar to the findings by Schram and Onderstal (2009), who also find an observed bidding curve crossing the prediction from above in their experiment of an all-pay auction with incomplete information.

Our observations may partially relate to the findings by Dufwenberg and Gneezy (2002) and Ockenfels and Selten (2005) in their experiments with repeated first-price sealed-bid auctions. The former find that when the losing bids from the previous round are revealed, prices stay higher than the theoretical prediction across rounds. The latter find that when feedback on losing bids is provided the auctions generate lower revenue than auctions where this feedback is not given. In our repeated auctions, we reveal previous round bids from all contestants. However, even if the findings by the above two experiments can extend to the all-pay auction, neither can explain the switch from overbidding to underbidding we observe.

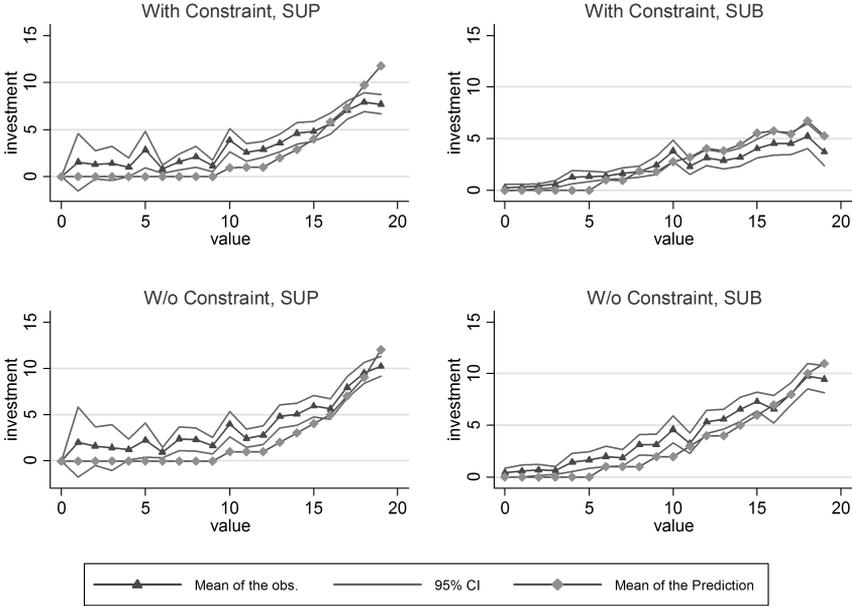
We also use a second method to compare the observed investment patterns to the theoretical predictions. In this method, we first generate the average of the predicted investment levels for each given private value for the prize, which is denoted as

$$\widehat{I}^{\tau,C}(v) = \frac{1}{n^{\tau}(v)} \sum_{i=1}^{n^{\tau,C}(v)} I_i^{\tau,C}(v, w_i),$$

$I_i^{\tau,C}(v) = \widehat{b}(v, w) = \min\{a(v), w_i\}$  (for Contest A), or  $e(v)$  (for Contest B), is the theoretical prediction for the investment level when the private value for the prize is  $v$  and the endowment is  $w_s$  in contest  $C$  of treatment  $\tau$ , and  $n^{\tau,C}(v)$  is the total number of the observations with private value of  $v$  in contest  $C$  of treatment  $\tau$ ,  $\forall \tau \in \{SUP, SUB\}$ ,  $C \in \{A, B\}$ ,  $v \in \{0, 1, \dots, 19\}$ . In other words, for this average, we consider the realized constraint-draws for each value and determine the average predicted investment across those constraint values. Then, we calculate the average observed investment level for subjects with private value  $v$ , in contest  $C$  of treatment  $\tau$ , denoted as  $\widehat{a}^{\tau,C}(v)$ . Figure 4.4 compares these two for each contest under each treatment. This method differs from the first method in two ways, which are both relevant for the contests with constraint: first, the means of the predicted investment levels are lower than what is shown in Figure 4.3, because the constraint (endowment) is taken into account; second, the number of observations used to generate the mean of the observed investment levels are larger, because we also include

## 4.5 Experimental Results

data from subjects with  $a^{\tau, A}(v)$ ,  $\forall \tau \in \{SUP, SUB\}$ ,  $v \in \{0, 1, \dots, 19\}$ . The results of this second method support our previous findings in the sense that the observed investment curve crosses the average prediction curve from above, indicating over-investment in the low-value area and under-estimate in the high-value area, while observations in a large range of mid values are consistent with the prediction (with insignificant deviation from the prediction,  $p > 0.05$ ).



**Figure 4.4:** Prediction and the Average of the Observed Investment (Method 2)

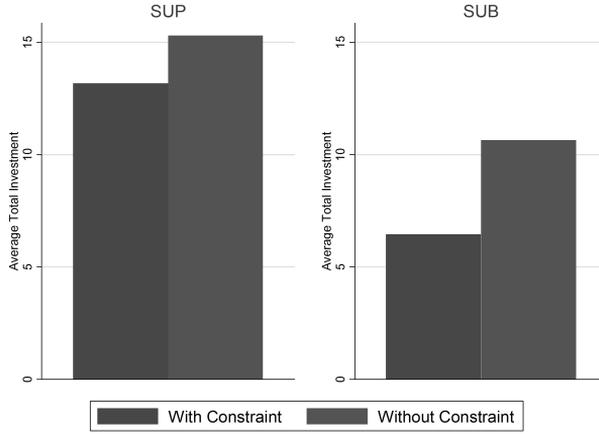
### 4.5.2 Aggregate Investment Comparison

Average aggregate investment per group of three in the contests with and without constraint are 13.17 and 15.10, respectively, in SUP. For SUB, these are 6.47 and 10.65, respectively, as shown in Figure 4.5. Differences in aggregate investment between the two different contest setups are significant in both treatments (t-tests,  $p=0.000$  for both SUP and SUB). This is consistent with our theoretical prediction in Proposition 3, that the expected aggregate investment in the contest

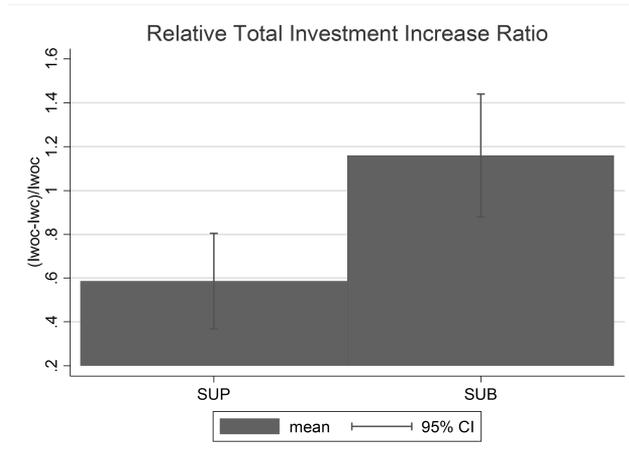
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without constraint is higher than in the contest with constraint, regardless of the joint distribution of private value and endowment.



**Figure 4.5:** The Average Total Investment (per small group) in the Contests with/without Constraint



**Figure 4.6:** Relative Investment Increase Ratio

We calculate the increase in aggregate investment in each group of three when con-

straints are released. In line with (4.6), the relative aggregate investment increase is  $\rho_s^\tau = \frac{I_{B,s}^\tau - I_{A,s}^\tau}{I_{A,s}^\tau}$ , where  $I_{C,s}^\tau$  denotes aggregate investment of group  $s$  in contest  $C \in \{A, B\}$  (A (B) for the contest with(out) constraint) in treatment  $\tau \in \{SUP, SUB\}$ . As shown in Figure 4.6, the average increase ratios in both SUP and SUB are significantly larger than 0. It is also worth noting that the mean of the increase ratio in SUB is significantly larger than in SUP, as predicted in Section 4.3.

### 4.5.3 Efficiency

We compare realized efficiency in the two contests and two treatments in two different ways. First, we check the fraction of contests where the prize is won by the contestant with the highest value. For this, we use an index  $r_{1,s}$ , where  $r_{1,s} = 1$  if in the group of three,  $s$ , the contestant with the highest value wins; otherwise  $r_{1,s} = 0$ ). The second method uses the ratio of realized value to maximal value in a contest's outcome, defined as

$$r_{2,s} = \frac{v_{win,s} - v_{min,s}}{v_{max,s} - v_{min,s}}, \tag{4.9}$$

where  $s$  is the index of a group of three,  $v_{max,s}$ ,  $v_{min,s}$  and  $v_{win,s}$ , are respectively the highest, lowest, and the winner's private value in group  $s$ .

**Table 4.1:** Fraction of Efficient Allocations and Efficiency Realization

	Eff Allocation (r1)		Eff Realization (r2)	
	SUP (405 obs.)	SUB (405 obs.)	SUP (405 obs.)	SUB (403 obs.)
With Constraint	57.28%	55.06%	74.19% (1.87%)	70.05% (1.96%)
Without Constraint	60.00%	70.12%	75.68% (1.86%)	80.95% (1.73%)

Note: For the second method, percentages in parentheses denote standard errors.

The averages of  $r_{1,s}$  and  $r_{2,s}$  in different contests of distinct treatments are as depicted in Table 4.1.<sup>12</sup> For both measures, contests without constraints are more efficient than those with constraints. When measured by  $r_1$ , the difference is significant (Fisher exact test,  $p < 0.01$ ) for both treatments. For  $r_2$ , the efficiency realization in a contest with constraint is significantly lower than without constraint

<sup>12</sup>For the measure of  $r_2$ , there are two observations missing, because in each of those two groups, the three subjects happen to have the same private values for the prize, which causes the denominator in the formula for  $r_2$  to be 0.

in treatment SUB (t-test,  $p < 0.01$ ) but in treatment SUP, the difference is insignificant ( $p = 0.46$ ). The efficiency improvement introduced by removing the budget constraint, is significantly lower in SUP than in SUB. ( $p < 0.01$  by the Fisher exact test, if the efficiency improvement is measured by the difference in  $r_1$  between Contest  $A$  and  $B$  and  $p < 0.01$  by the t-test, if measured by the difference in  $r_2$  of each group between the Contest  $A$  and  $B$ ).

### 4.6 Conclusion

This chapter investigates the influence of relaxing budget constraints in firms' R&D investments, e.g. by providing publically funded subsidies or loans, on an industry's aggregate research input. We study this question using both a theoretical model and a laboratory experiment, under the framework of an all-pay auction with incomplete information and private budget constraints. The theory suggests that removing firms' constraints will generally enhance the industry's aggregate R&D investment. Our laboratory experiments implement two distinct types of the joint-distribution of private values and constraints as representatives of two prototypical types of industry. The results support the theoretical findings. We also find consistent theoretical and experimental evidence that the magnitude of the aggregate investment increase realized by liquidizing firms differs with the structure of the market. In an industry where firms' innovating ability and their financial status are positively associated with each other, governmental co-funding will have less of an effect in aggregate investment than in an industry where firms' research prospects and their budgets are negatively associated. However, regarding individual bidding behavior in the all-pay auctions, the laboratory observations deviate from the theoretical predictions: bidders with low values bid higher than predicted, and those with high values bid lower than predicted. This finding is in line with the observations reported by Schram and Onderstal (2009). However, the underlying reasons remain to be understood in future research.

## 4.7 Appendix

### 4.7.1 Proofs

#### 4.7.1.1 Proof of Proposition 2

If a contestant bids  $x$ , the probability of winning over one other contestant is

$$\begin{aligned}
 H(x) &\equiv \Pr(\min\{a(v), w\} < x) \\
 &= \Pr(w < x) + \Pr(a(v) < x | w \geq x) \Pr(w \geq x) \\
 &= \int_0^1 \int_0^x f(V, W) dW dV + \int_0^{a^{-1}(x)} \int_x^1 f(V, W) dW dV \quad (4.10)
 \end{aligned}$$

The density function of  $H$  is then equal to

$$\begin{aligned}
 H'(x) &= \int_0^1 f(V, x) dV + \frac{1}{a'(a^{-1}(x))} \int_x^1 f(a^{-1}(x), W) dW - \int_0^{a^{-1}(x)} f(V, x) dV \\
 &= \int_{a^{-1}(x)}^1 f(V, x) dV + \frac{1}{a'(a^{-1}(x))} \int_x^1 f(a^{-1}(x), W) dW
 \end{aligned}$$

The contestant who bids  $x$  ( $\geq r$ ) has the expected payoff of

$$\pi = vH^{n-1}(x) - x,$$

where  $n$  denotes the total number of bidders in the all-pay auction.

If the contestant is unconstrained, he bids  $x = a(v)$  ( $\leq w$ ) which satisfies the first-order condition

$$\begin{aligned}
 \frac{d\pi}{dx} &= v(n-1)H^{n-2}(x)H'(x) - 1 = 0 \\
 (x \text{ equals } a(v)) &\Rightarrow v(n-1)K^{n-2}(v)k(v) - a'(v) = 0 \\
 &\Rightarrow a(v) = \int_0^v x dK^{n-1}(x) \quad \square
 \end{aligned}$$

4.7.1.2 Proof of Lemma 3

With private constraint, a contestant's expected bid is

$$\begin{aligned} E[b(v, w)] &= \int_0^1 \int_0^1 b(v, w) f(v, w) dw dv \\ &= \int_c^1 \left( \int_r^{a(v)} w f(v, w) dw \right) dv + \int_c^1 \left( \int_{a(v)}^1 a(v) f(v, w) dw \right) dv \end{aligned}$$

The expected payment

$$\begin{aligned} E[b(v, w)] &= \int_0^1 \int_0^1 b(v, w) f(v, w) dw dv \\ &= \underbrace{\int_0^1 \left( \int_0^{a(v)} w f(v, w) dw \right) dv}_I + \underbrace{\int_0^1 \left( \int_{a(v)}^1 a(v) f(v, w) dw \right) dv}_{II} \end{aligned}$$

$$\begin{aligned} F(v) &= \int_0^v \int_0^1 f(V, W) dW dV \\ dF(v) &= \int_0^1 f(v, W) dW dv \end{aligned}$$

Substituting  $a(y)$  for  $w$  we obtain

$$\begin{aligned} I &= \int_0^1 \int_r^{a(v)} w f(v, w) dw dv \\ &= \int_0^1 \left( \int_0^v a(y) f(v, a(y)) a'(y) dy \right) dv \\ &= \int_0^1 \left( a(y) \int_y^1 f(V, a(y)) a'(y) dV \right) dy \\ &= \int_0^1 \left( a(v) \int_v^1 f(V, a(v)) a'(v) dV \right) dv \end{aligned}$$

Therefore,

$$\begin{aligned} E[b(v, w)] &= \int_0^1 \left( a(v) \int_v^1 f(V, a(v)) a'(v) dV \right) dv + \int_0^1 \left( \int_{a(v)}^1 a(v) f(v, w) dw \right) dv \\ &= \int_0^1 a(v) \left( a'(v) \int_v^1 f(V, a(v)) dV + \int_{a(v)}^1 f(v, W) dW \right) dv \quad (4.11) \end{aligned}$$

From (4.10),  $K(v)$ , defined as  $H(a(v))$ , and its derivative function  $k(v)$ , can be written as

$$\begin{aligned} K(v) &\equiv H(a(v)) \\ &= \int_0^1 \int_0^{a(v)} f(V, W) dW dV + \int_0^v \int_{a(v)}^1 f(V, W) dW dV \end{aligned} \quad (4.12)$$

$$\begin{aligned} k(v) &\equiv K'(v) \\ &= H'(a)a'(v) = a'(v) \int_v^1 f(V, a(v)) dV + \int_{a(v)}^1 f(v, W) dW \end{aligned} \quad (4.13)$$

By comparing (4.11) with (4.13), we can further get

$$E[b(v, w)] = \int_0^1 a(v)k(v)dv = \int_0^1 a(v)dK(v) \quad \square$$

#### 4.7.1.3 Proof of Proposition 3

By comparing (4.2) and (4.4) with (4.3) and (4.5), respectively, the expected expenditure under the situation with private constraint as shown in (4.4), can be viewed as the expected expenditure under an all-pay auction without constraint, generated from a value distribution function  $K(v)$ .

By Myerson's (1981) Lemma, when the value distribution follows  $K(v)$  ( $F(v)$ ), the total expected expenditure in the all-pay auction without private constraint is equal to the expected total expenditure in the second-price auction, which is the expected second highest (among the total  $n$ ) contestants value from the value distribution of  $K(v)$  ( $F(v)$ ).

Because  $\forall v \in (0, 1)$

$$\begin{aligned} K(v) &= \int_0^1 \left( \int_0^{a(v)} f(v, w) dw \right) dv + \int_0^v \int_{a(v)}^1 f(v, w) dw dv \\ &> \int_0^v \left( \int_0^{a(v)} f(v, w) dw \right) dv + \int_0^v \int_{a(v)}^1 f(v, w) dw dv \\ &= \int_0^v \int_0^1 f(v, w) dw dv = F(v) \end{aligned}$$

we know,  $K(v)$  stochastically dominates  $F(v)$ . Therefore, the expected second highest value from  $K(v)$  is lower than that from  $F(v)$ .  $\square$

### 4.7.2 Derivations for Section 4.3

#### 4.7.2.1 The Supermodular Case

From  $f(v, w) = 4vw$ , we can obtain

$$\begin{aligned} f_V(v) &= 4 \int_0^1 vwdw = 2v \\ f_W(w) &= 4 \int_0^1 vwdv = 2w \\ f_{W|V}(w|v) &= \frac{f(v, w)}{f_V(v)} = \frac{4vw}{2v} = 2w \end{aligned}$$

The marginal distribution of  $v$  is given by

$$F(v) = \int_0^v 2xdx = v^2,$$

So, in the case without constraint, a contestant with value  $v$  bids

$$\begin{aligned} e(v) &= vF(v)^{n-1} - \int_0^v F(x)^{n-1} dx \\ &= vF(v)^2 - \int_0^v F(x)^2 dx = \frac{4}{5}v^5 \end{aligned}$$

The expected revenue is then

$$\begin{aligned} R_{WOC} &= 3 \int_0^1 e(v)dF(v) \\ &= 3 \int_0^1 e(v)2v dv = 0.6857 \end{aligned}$$

When there are private constraints, we have

$$\begin{aligned} K(v) &= \int_0^1 \int_0^{a(v)} 4WV dW dV + \int_0^v \int_{a(v)}^1 4WV dW dV \\ &= \int_0^1 \int_0^{a(v)} dW^2 dV^2 + \int_0^v \int_{a(v)}^1 dW^2 dV^2 \\ &= a(v)^2 \int_0^1 dV^2 + \int_0^v (1 - a(v)^2) dV^2 \\ &= a(v)^2 + (1 - a(v)^2)v^2 \end{aligned}$$

$$k(v) = K'(v) = 2a(v)a'(v)(1 - v^2) + 2v(1 - a(v)^2)$$

Then by substituting  $K(v)$  and  $k(v)$  into

$$a'(v) = v(n - 1)K^{n-2}(v)k(v)$$

$a(v)$  can hence be solved numerically under the initial condition  $a(0) = 0$ .

The expected revenue when bidders are constrained under the super-modular density is

$$\begin{aligned} R_{W/C} &= 12 \left( \int_0^1 \left( \int_0^{a(v)} wvwdw \right) dv + \int_0^1 \left( a(v) \int_{a(v)}^1 vw dw \right) dv \right) \\ &= 12 \left( \int_0^1 \left( v \int_0^{a(v)} wdw \right) dv + \int_0^1 \left( a(v) \int_{a(v)}^1 vw dw \right) dv \right) \\ &= 12 \left( \int_0^1 v \frac{a(v)^3}{3} dv + \int_0^1 va(v) \frac{1 - a(v)^2}{2} dv \right) \\ &= 0.6750 \end{aligned}$$

#### 4.7.2.2 The Submodular Case

From  $f(v, w) = \frac{4}{3}(1 - vw)$ , we have

$$\begin{aligned} f_V(v) &= \frac{4}{3} \int_0^1 (1 - vw) dw = \frac{2}{3}(2 - v) \\ f_W(w) &= \frac{4}{3} \int_0^1 (1 - vw) dv = \frac{2}{3}(2 - w) \\ f_{W|V}(w|v) &= \frac{\frac{4}{3}(1 - vw)}{\frac{2}{3}(2 - v)} = \frac{2(1 - vw)}{2 - v} \end{aligned}$$

For the case with no constraint, we have

$$F(v) = \int_0^v \frac{2}{3}(2 - x) dx = \frac{1}{3}v(4 - v),$$

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So a bidder with value  $v$  will bid

$$\begin{aligned}
 b(v) &= vF(v)^{n-1} - \int_0^v F(x)^{n-1} dx \\
 &= vF(v)^2 - \int_0^v F(x)^2 dx \\
 &= \frac{2}{135}v^3 (6v^2 - 45v + 80)
 \end{aligned}$$

The expected revenue then equals:

$$\begin{aligned}
 R_{WOC} &= 3 \int_0^1 b(v) dF(v) \\
 &= 3 \int_0^1 b(v) \frac{2}{3} (2-v) dv \\
 &= 0.4339
 \end{aligned}$$

When there are potential constraints, we now have

$$\begin{aligned}
 K(v) &= \int_0^1 \int_0^{a(v)} f(V, W) dW dV + \int_0^v \int_{a(v)}^1 f(V, W) dW dV \\
 &= \int_0^1 \int_0^{a(v)} \frac{4}{3} (1 - WV) dW dV + \int_0^v \int_{a(v)}^1 \frac{4}{3} (1 - WV) dW dV \\
 &= \frac{1}{3}v (a(v) - 1) (v + a(v)v - 4) - \frac{1}{3}a(v) (a(v) - 4)
 \end{aligned}$$

$$k(v) = K'(v) = \frac{1}{3}v (a(v) - 1) (v + a(v)v - 4) - \frac{1}{3}a(v) (a(v) - 4)$$

By  $a'(v) = v(n-1)K^{n-2}(v)k(v)$ , the bid function  $a(v)$  can be solved numerically under the initial condition  $a(v) = 0$ .

The expected revenue when bidders are constrained can then be calculated according to the numerical solution of  $a(v)$ , by

$$\begin{aligned}
 R_{W/C} &= 4 \left( \int_0^1 \left( \int_0^{a(v)} w(1-vw) dw \right) dv + \int_0^1 \left( (a(v) + 1) \int_{a(v)}^1 (1-vw) dw \right) dv \right) \\
 &= 0.3753
 \end{aligned}$$

### 4.7.3 Instructions

Welcome to this experiment!

This experiment consists of 15 rounds. In each round, you will be participating in a three-player contest. Below you can find the details about the procedures and the rules of the contest.

**Group Matching** At the beginning of each round, you will be randomly assigned into groups of three. This means that in each round, the two other group members you are interacting with are very unlikely to be the same as in the previous round.

**Rules of the Contest** There is a single prize for each contest. This means that for each round, only one participant (out of the total three) in each group can win the prize.

At the beginning of each round, before the contest starts, each of you will receive a randomly generated private initial endowment, the amount of which is unknown to any other participants. Then each of the three participants in your group is asked to decide on an investment level for the contest. **The one who has chosen the highest investment level wins the contest** (in case more than one participant choose the same highest investment level, one of them will be randomly selected, with equal chance, to be the winner.) **No matter winning or losing the contest, you always need to pay for your investment.**

The winner of the contest will be rewarded with the single prize. The values of the prize for each participant are also randomly generated, which may be different for different participants. Each of these values of the prize is only privately known to each participant him/herself, but not to any of the other two contestants.

Regarding the restriction on the investment level, there are two situations, A and B. In **Situation A**, your highest possible investment level is your initial endowment of the round. In **Situation B**, you can invest any amount of points from 0 to 19, no matter how much endowment you have.

For both situation A and B, your payoff of that round is determined in the following way: If you win the contest, your payoff of the round will be:

$$\text{Initial endowment} - \text{investment} + \text{your private value of the prize}$$

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If you lose the contest, your payoff will be:

*Initial endowment – investment*

This rule can be illustrated by the following example. Three participants X, Y and Z are in the same group for a contest. Their initial endowments, private value of the prizes, and the three participants investment decisions under the two different situations are displayed in Table 1. Then we can see that under situation A, Xs investment is the highest, so she wins the contest. In this situation, Xs payoff is 23 (=18-9+14). Y and Z lose the contest, and each gets 0 (=7-7) and 6 (=9-3), respectively. Under situation B, where everyone can choose an investment level higher than their initial endowment, Y chooses to invest 12. In this situation, Y is the winner of the contest, and her payoff is 12 (=7-12+17). Please calculate the payoff for X and Z under situation B in this example.

	Initial endowment (w)	Private value of the prize (v)	Investment under Situation A	Investment under Situation B
X	18	14	9	8
Y	7	17	7	12
Z	9	7	3	3

Example

The number used in this example are just for illustration purpose. They are not necessarily representative amounts in this experiment.

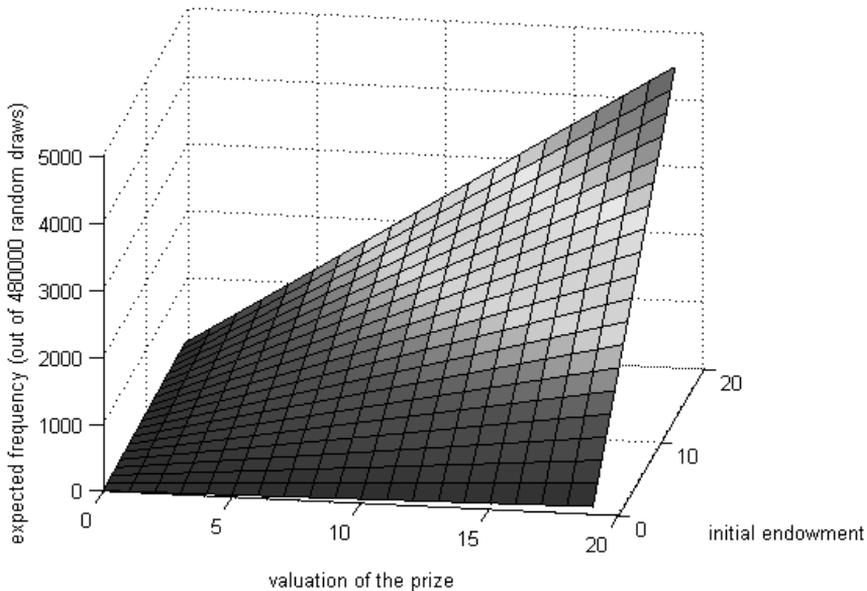
During the experiment, you need to make your decisions under each of these two situations. For each round, one situation out of A and B will be randomly selected to be realized, then all the participants' decisions under the realized situation will determine the outcome of the contest.

**Random Generation of the Private Values and the Endowments** (For SUP only)

For each of you, the private value of the prize ( $v$ ) and the private initial endowment ( $w$ ), are randomly generated from a joint distribution, with the density function being  $f(v, w) = \frac{vw}{40000}$ , in the area  $\{(v, w) | 0 \leq v < 20, 0 \leq w < 20\}$ . The property of this distribution is such that, **in general, a higher private value of**

**the prize is more likely to associate with a higher initial endowment.** The randomly generated values are always rounded down to the closest integer below. In this experiment, the values of the initial endowments, the private values of the prize, and the investment levels, can only take the values from 0, 1, 2, ..., 19.

To give you an intuitive impression about this joint distribution, Figure 1 shows the expected frequency of each combination of  $(v, w)$ , the private value of the prize and the initial endowment, out of 480000 random draws.



### Random Generation of the Private Values and the Endowments (for SUB only)

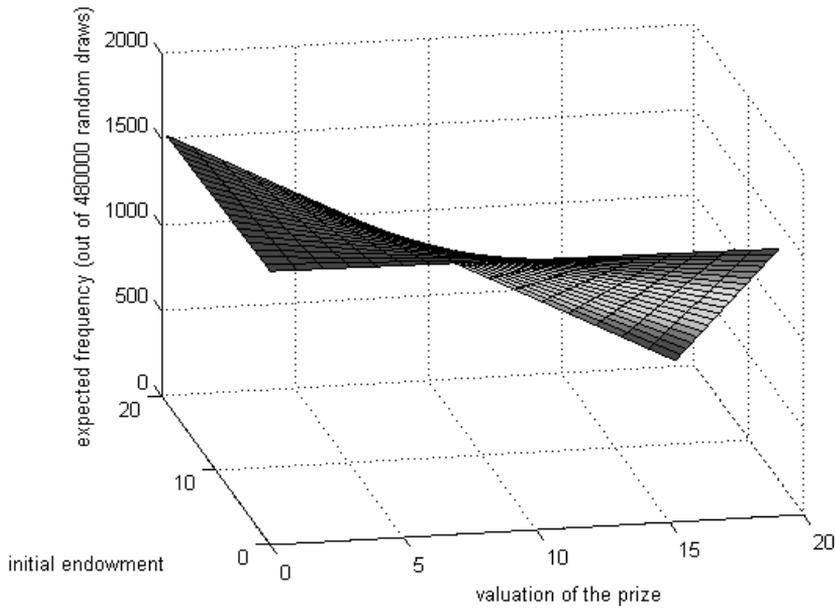
For each of you, the private value of the prize ( $v$ ) and the private initial endowment ( $w$ ), are randomly generated from a joint distribution, with the density function being  $f(v, w) = \frac{1}{300}(1 - \frac{vw}{400})$ , in the area  $\{(v, w) | 0 \leq v < 20, 0 \leq w < 20\}$ . The property of this distribution is such that, **in general, a high private value of the prize is more likely to associate with a low initial endowment, and a low private value of the prize is more likely to associate with a high initial endowment.** The randomly generated values are always rounded down to

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the closest integer below. In this experiment, the values of the initial endowments, the private values of the prize, and the investment levels, can only take the values from 0, 1, 2, ..., 19.

To give you an intuitive impression about this joint distribution, Figure 1 shows the expected frequency of each combination of  $(v, w)$ , the private value of the prize and the initial endowment, out of 480000 random draws.



**Payment of the experiment** Out of the total 15 rounds of the contests, only 2 rounds will be randomly selected to be paid. In the end of the whole experiment, you will be informed about which two rounds (under which situation, A or B) are randomly selected to be paid.