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phase. Thus, the *dispersed phase* of the normal dispersion consists of ^4He -rich droplets, which may or may not be superfluid, depending on their size and temperature.

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Atomic Hydrogen in Contact with a Helium Surface: Bose Condensation, Adsorption Isotherms, and Stability

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The energy of interacting spin-polarized atomic hydrogen (H_1) on a helium surface and the density dependence of the adsorption energy have been calculated, permitting a realistic determination of the adsorption isotherms. Surface interactions are shown to allow Bose-Einstein condensation in the gas. At the Bose-Einstein condensation the surface coverage attains a maximum which is essentially independent of the isotherm temperature. Also calculated is the stability condition for surfaces in magnetic fields. Easily met requirements are found.

PACS numbers: 67.70.+n, 67.40.-w

A gas of spin-polarized atomic hydrogen (H_1) has recently been stabilized to densities of order 10^{16} – 10^{17} atoms/cm³.¹ This gas is only stable in a container with helium-covered walls. At densities a few orders of magnitude higher it is expected that the critical temperature [$T_c \approx 3.31 \times \hbar^2/mk_B(n_3/g)^{2/3}$; n_3 is the gas density, g the degeneracy, and m the mass] for Bose-Einstein condensation (BEC) will be within the range of current cryogenic techniques. The combination of the statistical attraction of bosons and the H_1 - H_1 interactions leads to important phenomena. In the absence of interactions the surface coverage, n_2 , becomes infinite at a critical density and no BEC will occur in the gas-surface system.² In this Letter we point out how interactions change this picture so that even at $T=0$ K surface coverages are strongly limited. We calculate the $T=0$ adsorption isotherm and extend this to finite temperatures. We find that for temperatures and densities of current experimental interest ($0 \leq T \leq 0.5$ K, $n_3 \leq 10^{21}$ atoms/cm³) there is a maximum surface coverage which is essentially independent of temperature. Moreover, our results show that

this maximum coverage must be reached on the surface before BEC can be observed at the critical density $n_3 = n_{3c}$. For experimental efforts to observe BEC in H_1 , there is thus no optimum temperature to minimize recombination to H_2 on the surface. We also calculate the magnetic field value required to provide static stability against recombination both in the gas and on the helium surface. We find that experimentally available fields are sufficient to achieve static stability of H_1 at n_{3c} and the predicted maximum value of n_2 .

We first show that another of the unique properties of H_1 in equilibrium with a helium surface is that it has a nontrivial isotherm at $T=0$ K; i.e., above a critical coverage the gas has a finite pressure and density. We consider a two-dimensional (2D) gas of H_1 with surface density n_2 adsorbed on helium in equilibrium with a 3D gas of H_1 with density n_3 . The isotherm is obtained by requiring the equality of the chemical potentials μ_2 and μ_3 of the surface-gas system. To determine the energy ϵ as a function of coverage we assume that motions within and normal to the surface are uncoupled so that they are additive: ϵ

$= -\epsilon_a + \epsilon_2^{\text{int}}$ where ϵ_a is the single-particle adsorption energy and ϵ_2^{int} is the interaction energy of a two-dimensional gas of H_4 interacting within the $^3\Sigma_u^+$ potential of Kolos and Wolniewicz³ which has been fitted to an analytical form for computational purposes.⁴ Recent calculations^{5,6} of the binding energy of a single H atom interacting with a ^4He surface have yielded the existence of a bound state with an adsorption energy ϵ_a/k_B ranging from 0.04 to 0.6 K.

We calculate the $T=0$ K pair distribution functions and the interaction energies ϵ_2^{int} and ϵ_3^{int} for the 2D and 3D systems in the hypernetted chain approximation using a Jastrow-type trial wave function of the form $\exp[-(b/r)^n]$, with b and n as variational parameters. The 2D model for the surface system allows the particles to move around each other on the surface but not to hop over each other. This treatment may slightly overestimate the kinetic energy on the surface leading to slightly lower surface coverages.

For moderate densities, of interest for H_4 , the resulting interaction energies per particle could be fitted to the linear forms $\epsilon_2^{\text{int}} = \alpha_2 n_2$ and $\epsilon_3^{\text{int}} = \alpha_3 n_3$, in 2D and 3D, respectively, with $\alpha_2 = 6.9 \times 10^{-15}$ K cm^2 and $\alpha_3 = 2.6 \times 10^{-22}$ K cm^3 . The $T=0$ isotherm is determined by equating the Gibbs free energy per particle, μ , in the two phases:

$$-\epsilon_a + 2\alpha_2 n_2 = 2\alpha_3 n_3$$

or

$$n_2 = \epsilon_a / 2\alpha_2 + (\alpha_3 / \alpha_2) n_3.$$

Thus at $T=0$ as particles are introduced the surface first loads up to coverage $\epsilon_a / 2\alpha_2$; additional particles go into the gas phase also. Note that because the slope α_3 / α_2 ($\sim 3.7 \times 10^{-8}$ cm) is almost zero, surface coverages remain of the order of $\epsilon_a / 2\alpha_2$ for all gas densities of interest here.

To extend the isotherms to finite temperature for interacting particles we modify the Bose-Einstein distribution function $N_i/g = [\exp(\epsilon_i - \mu)/k_B T - 1]^{-1}$ by adding to ϵ_i the term $\partial[n\epsilon^{\text{int}}(n)]/\partial n$.⁷ Here N_i is the usual population of the i th state, g the degeneracy, μ the chemical potential, and ϵ_i the single-particle energy spectrum which is assumed to be unaltered by the interactions. In 3D, this yields $\epsilon_i = p_i^2/2m + 2\alpha_3 n_3$ and in 2D, $\epsilon_i = -\epsilon_a + p_i^2/2m + 2\alpha_2 n_2$. Since calculations⁶ show that the H_4 atoms sit on top of the helium surface we take m to be the true H mass. The chemical potential is determined by the total number of

particles $N = n_2 A + n_3 V$, where

$$n_2 = \frac{-1}{\lambda^2} \ln \left(\frac{1 - \exp[(\mu_2 + \epsilon_a - 2\alpha_2 n_2)/k_B T]}{1 - \exp[(\mu_2 - \epsilon_{\text{max}})/k_B T]} \right), \quad (1)$$

$$n_3 = \frac{\bar{N}_0}{V} + \frac{1}{\lambda^3} g_{3/2} \{ \exp[(\mu_3 - 2\alpha_3 n_3)/k_B T] \}. \quad (2)$$

Here $g_{3/2}$ is a Bose integral,⁸ $\lambda = (2\pi\hbar^2/mk_B T)^{1/2}$ the thermal wavelength, $\bar{N}_0/V$ the Bose condensate density, and ϵ_{max} the maximum kinetic energy of a surface particle. By letting $\epsilon_{\text{max}} \rightarrow \infty$ we include quasibound surface states⁹ and the denominator in Eq. (1) goes to unity. Equating the chemical potentials μ_2 and μ_3 in Eqs. (1) and (2) we obtain the adsorption isotherms; representative cases are plotted in Fig. 1. For $T > 0$ K the isotherms start at the origin with a finite slope $\lambda \exp(\epsilon_a/k_B T)$ and as more particles are introduced the surface loads up to just above the $T=0$ adsorption coverage when $n_3 = n_{3c}$; all further particles added to the system go into $\bar{N}_0$ of Eq. (2), the condensate. An important result of this analysis is that for experimental conditions of interest the surface will always load up to approximately the same coverage ($\sim 10^{13}$ – 10^{14} atoms/ cm^2) before reaching BEC, no matter what the critical temperature. We note that if $\alpha_2 = 0$, then n_2 diverges when $\mu = -\epsilon_a$ and we get a "macroscopic" population of the surface: μ never reaches the value $2\alpha_3 n_3$ at which BEC occurs. A study of the condensate fraction for $n_3 \gg n_{3c}$ would require a more sophisticated model, such as the hard-sphere approach.¹⁰

Since the surface will always load up to $n_2 \approx \epsilon_a / 2\alpha_2$ for BEC it is important to know what applied magnetic field is necessary to achieve static stability.^{11,12} This is the field B_s for which the Zeeman energy is greater than the exchange energy so that the energy of the gas is lower with all spins polarized antiparallel to B_s and can be expressed for both 2D and 3D as

$$\mu_e B_{st} = \frac{1}{2} n_i \int [V_t(\vec{r}) - V_s(\vec{r})] g_i(\vec{r}) d^3 r,$$

where μ_e is the magnetic moment of the electron, n_i the density, V_t and V_s are the potentials for the H_4 - H_4 interaction in the triplet ($^3\Sigma_u^+$) and singlet ($^1\Sigma_g^+$) states, respectively,³ and $g_i(r)$ is the pair distribution function. For densities considered here, we find that the stabilizing fields can be well approximated by $B_{s2} = \gamma_2 n_2$ and $B_{s3} = \gamma_3 n_3$ with $\gamma_2 = 1.46 \times 10^{-14}$ T cm^2 and $\gamma_3 = 3.1 \times 10^{-21}$ T cm^3 . The line $n_2 = (\gamma_3/\gamma_2) n_3$ gives the relation between the surface density n_2 and gas density n_3 stabilized by a given field and is also plotted in

FIG. 1. Adsorption isotherms of H^1 on a helium surface for two values of the single-particle adsorption energy ϵ_a . Isotherms are shown for three temperatures, including 0 K for each ϵ_a . An isotherm for zero interaction energy ($\alpha_2 = 0$) is shown for $\epsilon_a = 0.6$, $T = 0.1$ K (dashed line). The dash-dotted curve is the line of static stability which depends parametrically on the magnetic field—a few field values are shown. Vertical arrows indicate the critical density for BEC.

Fig. 1. A given field B_s is represented by a point on this line; densities to the left (3D) and below (2D) this point are stable. For a field of 10 T densities up to $n_3 = 2.2 \times 10^{21}$ atoms/cm³ and $n_2 = 1.8 \times 10^{14}$ atoms/cm² are stable.¹³ Taking $\epsilon_a = 0.6$ K we find maximum predicted coverages to be 4.3×10^{13} atoms/cm². Edwards¹⁴ has also calculated the surface interaction energy using a different formalism and finds a coverage about twice our value. Even with his value our calculations indicate that the surfaces will have static stability in a 10 T field. We have also used the Lennard-Jones form¹⁵ for the triplet interaction and find in three dimensions $B_s(L-J) \approx \frac{3}{4}B_s$ (expt.-dispersion) for densities up to 150 cm³/mole. Lantto and Nieminen¹² have also computed B_s in 3D with the Lennard-Jones potential¹⁶ using a different method for $g(r)$. Our present Lennard-Jones values for B_s are larger than theirs by a factor of roughly 1.4. We also find that the present 3D values for B_s differ only slightly (up to 15% at higher densities) from those obtained earlier¹¹ by solving the two-body, zero-energy, s-wave (unreduced) equation for the Jastrow function.

Our results show that in experimental efforts to achieve BEC, there is no optimum temperature for minimizing the static instability of H^1 on the helium surface. Since higher densities are more likely to be limited by dynamical three-body recombination rates which increase with the cube

of the density, experimental work can best be aimed at low values of n_{3c} and thus low temperatures.

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Convection in Dilute Solutions of ^3He in Superfluid ^4He

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Thermal convection in dilute solutions of ^3He in superfluid ^4He has been studied in a novel cell of unity aspect ratio which permits inhomogeneity in the heat flux to be measured. At least three different convecting states have been observed, including a time-dependent state with diminished thermal conductance which can exist very close to the onset of convection.

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Dilute solutions of ^3He in superfluid ^4He constitute an important model system for studying nonlinear thermohydrodynamics¹⁻³ in general and thermal convection in particular, because of the advantages both of the cryogenic environment for measurement⁴ and of the superfluid for its unique properties. In this work we have studied thermal convection in a unity aspect ratio, vertical-axis, cylindrical cell with insulating side walls and constant-temperature horizontal bounding planes, with the novel feature that horizontal inhomogeneity in the heat flux out of one bounding plane can be measured in addition to the usual total thermal conductance. Using this arrangement, we have observed a convective instability in a cell heated from above and have found several convective structures. These include two steady-state structures, probably corresponding to toroidal flow with center either falling or rising, and time-dependent flow structures. The latter includes a spectacular structure with a characteristic frequency which displays a critical-slowing-down effect, with a diminished overall thermal conductance, and with a horizontal inhomogeneity in the heat flux which oscillates between that for center fluid rising and that for center fluid falling.

The experimental fluid was confined to a cylindrical volume with axis within 0.5 degree of vertical and bounded radially by a fused quartz annulus ground to 4.14 cm i.d. and 2.07 cm height, with faces parallel and perpendicular to the axis, and

typical tolerances of 1–2 μm . Against each face of the quartz annulus was held, by means of springs, a 2-cm-thick cylinder of copper. These copper cylinders bounded the fluid above and below. The faces in contact with the quartz and the fluid were mechanically polished until flat to the same tolerance. This entire assembly was contained in a close-fitting vessel with stainless-steel walls and low longitudinal conductance.

The bottom copper slab was placed in thermal contact with a ^3He refrigerator. The temperature of this slab, indicated by a GRT-200A-100 germanium thermometer calibrated by Lakeshore Cryotronics, was maintained constant electronically. The upper copper slab was constructed out of two pieces, a ring-shaped piece with a central tapered hole of approximately 1.4 cm smallest diameter and a plug-shaped tapered cylinder. These were pressed together with a 12- μm sheet of Mylar plastic carefully fitted between them so as to provide some thermal isolation of the plug from the ring. The surfaces of this assembly were then machined and polished. A germanium thermometer and a four-wire Manganin heater were slid into close-fitting holes in each of the plug and the ring. These two heaters and two thermometers were used to maintain the temperature of the ring (T_R) *equal* to the temperature of the plug (T_P) and *greater* than that of the lower slab by a known amount (ΔT). The total heat flow out of the plug ($\dot{Q}_P$) was measured separately