UvA-DARE (Digital Academic Repository)

UPt3: Specific heat, critical fields, and the Ginzburg-Landau models for the superconducting phase diagram

Vorenkamp, T.; de Visser, A.; Wester, R.; Menovsky, A.A.; Franse, J.J.M.; Knetsch, E.A.

DOI

[10.1103/PhysRevB.48.6385](https://doi.org/10.1103/PhysRevB.48.6385)

Publication date

1993

Published in

Physical Review. B, Condensed Matter

[Link to publication](#)

Citation for published version (APA):

Vorenkamp, T., de Visser, A., Wester, R., Menovsky, A. A., Franse, J. J. M., & Knetsch, E. A. (1993). UPt3: Specific heat, critical fields, and the Ginzburg-Landau models for the superconducting phase diagram. *Physical Review. B, Condensed Matter*, 48(9), 6385-6399. <https://doi.org/10.1103/PhysRevB.48.6385>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UPt₃: Specific heat, critical fields, and the Ginzburg-Landau models for the superconducting phase diagram

T. Vorenkamp, A. de Visser, R. Wester, A. A. Menovsky, and J. J. M. Franse

Van der Waals-Zeeman Laboratorium, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

E. A. Knetusch

Kamerlingh Onnes Laboratorium der Rijksuniversiteit Leiden, Nieuwsteeg 18, 2300 RA Leiden, The Netherlands

(Received 8 March 1993; revised manuscript received 12 May 1993)

The superconducting phase diagram of UPt₃ has three important features that can be described by a Ginzburg-Landau model. These features are (i) the double transition in zero field, observable as two peaks in the specific heat; (ii) the kink in the lower-critical-field curve $H_{c1}(T)$; (iii) the kink in the upper-critical-field curve $H_{c2}(T)$. We present the observation of all three features for one and the same high-quality sample. The resultant data are used to test the internal consistency of two recently published models for the superconducting phase diagram.

I. INTRODUCTION

In the 1980s heavy-fermion metals¹ became known as a new class of intermetallic compounds. At liquid-helium temperatures they exhibit highly correlated electronic behavior with conduction-electron masses that are some 100 times the free-electron mass. At present, six heavy-fermion compounds with a superconducting ground state are known. Some of them attracted wide attention due to claims of unconventional superconductivity.² UPt₃ is the heavy-fermion compound for which most evidence is available that unconventional superconductivity is realized. This evidence consists of the following.

(i) The observation of power laws in the temperature dependence of specific heat,^{3–5} thermal conduction,⁴ longitudinal and transversal ultrasonic attenuations,^{6–8} nuclear spin-lattice relaxation time,⁹ and penetration depth^{10–13} in the superconducting state, which is in contrast with the exponential temperature behavior for normal superconductors with isotropic pairing.

(ii) The observation of a double superconducting transition at zero field in the specific heat,^{5,14,15} the thermal expansion,¹⁶ and the ultrasound velocity.^{17,18}

(iii) The existence of a multicomponent phase diagram in the (H - T) plane of at least three different superconducting phases, observed in measurements of ultrasound velocity.^{17,18} Parts of this phase diagram have been observed in other properties.¹⁹

The question whether the pairing is d -wave-like or p -wave-like is still subject to controversy.² (By p -wave-like we mean anisotropic pairing with an odd-parity order parameter of which the p wave is the analog in a system with rotational freedom such as superfluid ³He. By d -wave-like we mean anisotropic pairing with an even-parity order parameter of which the d wave is the analog in a system with rotational freedom.) The power laws in the transverse ultrasound attenuation α^T for different polarization directions $\hat{e}$ of the polarization of the sound, $\alpha^T(\hat{e} \parallel c) \propto T^3$ and $\alpha^T(\hat{e} \perp c) \propto T$ as measured by Shivarum *et al.*,⁷ strongly suggest that the gap structure is

dominated by gap nodes in the basal plane. According to the classification by Volovik and Gor'kov²⁰ of possible superconducting order parameters for a hexagonal lattice, with strong spin-orbit coupling, the only gap with such a structure is one which is dominated by a line node in the basal plane. This leads to a (d -wave-like) order parameter of the two-dimensional (2D) E_{1g} representation, which has a "hybrid gap" (with a line node in the basal plane and point nodes at the poles). This order parameter has been proposed by several authors.^{21,22,12} A second possibility has been proposed by Machida and Ozaki.²³ According to Miyake²⁴ it cannot be completely excluded that the effective spin-orbit coupling for Cooper pairs is weak, although individual quasiparticles near the Fermi level are subject to strong spin-orbit coupling. A classification by Ozaki and Machida²⁵ of possible order parameters for hexagonal symmetry and neglecting spin-orbit coupling shows that in this case p -wave-like states can have line nodes. The condition of a gap dominated by nodes in the basal plane leads in this case to the A_{2u} order parameter which also has a hybrid gap.

For the superconducting order parameter of UPt₃ there are, therefore, two possible candidates: (i) a two-dimensional E_{1g} (d -wave-like) order parameter under the assumption of strong spin-orbit coupling and (ii) a one-dimensional A_{2u} (p -wave-like) order parameter under the assumption of negligible spin-orbit coupling.

Given the symmetry of the order parameter, the superconducting phase diagram can be described with a model that makes use of Ginzburg-Landau (GL) theory. In the last few years a number of models have been developed for UPt₃. An overview of available models can be found in a recent paper of Joynt.²⁶

Two types of approaches to explain the double superconducting transition can be considered.

(i) The accidental near degeneracy of two order parameters belonging to representations of different symmetry. In this case the Ginzburg-Landau free-energy expansion should be extended to contain both representations, including the coupling terms between them. Two

distinct transition temperatures and a mixed representation phase at low temperatures emerge from the analysis of such a free-energy functional. Two models using this approach have been proposed by Joynt *et al.*²⁷ These models are, however, not compatible with a hybrid-gap state.

(ii) The split transition-temperature models, which assume a coupling between the superconducting order parameter and a symmetry-breaking field which is assumed to be the small ($0.02\mu_B$ per U atom) antiferromagnetic moment of which the ordering has been observed below 5 K.^{28,29} The results of neutron-diffraction measurements in magnetic field by Aeppli *et al.*,³⁰ specific-heat measurements under pressure by Trappmann, Lohneysen, and Taillefer³¹ and neutron measurements under pressure by Hayden *et al.*³² suggest that this coupling exists. The assumption of this coupling leads to the E -representation model,^{33–37} which is compatible with an order parameter of the E_{1g} representation (with spin-orbit coupling) and the 1D-representation model^{23,37} which is compatible with an order parameter of the one-dimensional A_{2u} representation (neglecting spin-orbit coupling).

To the remarkable properties of the phase diagram that should be explained by a model belong (i) two superconducting transitions at T_{c1} and T_{c2} in zero field (with $T_{c2} < T_{c1}$ and $\Delta T_c = 55$ mK) visible as two peaks in the specific heat,^{5,14} (ii) a kink in $H_{c2}(T)$,^{38,39} and (iii) a kink in $H_{c1}(T)$.⁴⁰ In this paper we report measurements of these three features on the same high-quality sample. Brief accounts of the critical-field measurements have been presented earlier.^{41,42}

In the recent work of Midgley *et al.*⁴³ it is shown that the presence of an incommensurate structural modulation in UPt_3 , which is visible after annealing the sample at 1200 °C for 6 days, is correlated with the presence of a sharp double-peak structure in the specific heat. In an unannealed sample the modulation is not developed and the double-peak structure is not visible. We note that (i) according to our experience, in a sample without a visible double-peak structure, the double transition in the specific heat is only hidden due to the broadness of the transitions. The sample can still have a kink in $H_{c2}(T)$ and in $H_{c1}(T)$ and, derived from this, a distance between T_{c1} and T_{c2} of around 60 mK. (ii) Using the formation temperature of the modulation and the compressibility one can deduce a critical pressure that destroys the modulation. As has been claimed by Midgley *et al.*,⁴³ because of the high formation temperature, it is likely that the structural modulation still exists above pressures that were observed to destroy both the splitting of T_c and the ordered antiferromagnetic moment.^{31,32}

We conclude that the presence of a modulation is not necessarily correlated with the presence of a splitting of T_c . Models which give an explanation of the splitting of T_c in terms of a breaking of symmetry caused by the ordered antiferromagnetic moment are still relevant.

In this paper thorough analyses are given of measurements of the above-mentioned three features, within both the E -representation model and the 1D-representation model. The fact that all features are measured on one and the same sample and that the characteristic temper-

atures that follow from the three types of measurements can be expressed in the same model parameters provides the possibility of testing the internal consistency of the models.

II. EXPERIMENTAL DETAILS

Measurements of the specific heat and the upper and lower critical field have been performed on single-crystalline UPt_3 with boron addition. Additionally the specific heat of single-crystalline pure UPt_3 has been measured. All samples mentioned in this work were annealed at 950 °C for a period of 7 days to reduce strains. To prevent reactions of the sample with air or container material, the sample was wrapped in tantalum foil and put in an evacuated quartz ampoule which was gettered with a piece of uranium.

Recent substitution studies in UPt_3 (Ref. 44) stress the importance of the purity of single-crystalline samples. The height of the peaks in the specific heat and the values of the transition temperatures were both seen to decrease due to pair breaking by controlled amounts of impurities. Therefore, pair breaking by inadvertently introduced impurities leads to different values for model parameters calculated for nominally pure samples. High values of the transition temperatures and a low residual resistivity (ρ_0) indicate high purity. Earlier measurements have shown that the addition of small amounts of boron to UPt_3 leads to samples possessing higher transition temperatures and lower residual resistivities.⁴⁵ This is now interpreted as a purification effect in which at least part of the boron probably reduces the oxygen content by the formation of volatile products under plasma heating conditions.⁴⁶

In the present study, use was made of a crystal of Czochralski-grown UPt_3B_x with a concentration of added boron of 11 at.%. The average electron microprobe is not able to detect the long wavelength x rays from elements with low atomic numbers such as boron. However, in the case that the microprobe is optimized with respect to several factors such as the detection angle and when it is carefully calibrated, measurements on boron are possible.⁴⁷ By using a calibration with UB_4 the presence of boron in the as-grown crystal could be established and the homogeneity of the boron distribution in the crystal could be confirmed by a qualitative measurement of the boron content. The quantitative boron content, however, could not be determined accurately enough, since for this a standard of boron in UPt_3 would be needed. The notation UPt_3B_x is used to denote that the final content of boron in the sample is not known.

Three samples were cut from this crystal and annealed in the same way, as described above. X-ray analysis reveals a single phase of the hexagonal UPt_3 structure in all three samples, with some mosaic but not more than 1° or 2°. In the following c denotes the hexagonal axis, a the basal plane axis which points along the uranium atoms, and b is defined as the basal-plane axis which is perpendicular to a and points along platinum atoms. The high purity of the samples is evidenced by the low ρ_0 value

of $0.46 \mu\Omega \text{ cm}$ for $\mathbf{I} \parallel b$ and of $0.30 \mu\Omega \text{ cm}$ for $\mathbf{I} \parallel c$ and by the high residual resistivity ratio of 560. The crystal showed a high transition temperature, in both specific-heat ($T_{c+} = 533 \text{ mK}$) and resistivity ($T_c = 558 \text{ mK}$) measurements. (The difference between these two values of T_c results from the fact that the transition in the specific heat starts at the temperature where the transition in the resistance has ended, a feature which has been noted before by Hasselbach, Taillefer, and Flouquet.¹⁴)

The samples were cut in the shape of a bar with the longest dimension in the direction of the basal-plane a axis (samples 1, 3) or the basal-plane b axis (sample 2). Typical dimensions were $6 \times 1 \times 1 \text{ mm}^3$ (samples 2, 3) and $5 \times 1.9 \times 0.25 \text{ mm}^3$ (sample 1). Specific heat, $H_{c1}(T)$, and $H_{c2}(T)$ both for $\mathbf{H} \parallel b$ have been measured on sample 2. $H_{c2}(T)$ for $\mathbf{H} \parallel a$, b , and c has been measured on sample 1. $H_{c1}(T)$ for $\mathbf{H} \parallel c$ has been measured on sample 3. All three samples were cut from adjacent parts of the same single-crystalline rod.

Specific-heat measurements have been performed in a dilution refrigerator using a relaxation method. The estimated contribution of the addenda to the measured specific heat is less than 1% throughout the whole temperature range. It is therefore not subtracted from the data.

The upper critical field was determined from the resistive transition in a magnetic field. Resistance was measured at stabilized temperatures with a transformer coupled bridge. The temperature was measured with a RuO₂ thick-film resistor, of which the field effect was calibrated using a field-insensitive capacitance thermometer.

The lower critical field has been measured at the Kamerlingh Onnes Laboratory with a flux-gate magnetometer in a ³He cryostat. The samples were cooled in zero field, and the virgin magnetization was recorded as function of magnetic field at fixed temperature. The first penetration of flux was inferred from a 1% deviation from linearity or perfect diamagnetism. A small correction for demagnetizing effects due to the sharp edges of the sample was made ($D \approx 0.05$). As the sample dimension in the direction of the applied magnetic field was large compared to the pickup coil, no further corrections were necessary. Also another method for the determination of $H_{c1}(T)$, based on the Bean critical-state model, was used, giving the same results.⁴²

III. RESULTS AND DISCUSSION

A. Upper critical field

$H_{c2}(T)$ has been derived from the depression of resistive transitions by a magnetic field as shown in Fig. 1 for sample 2. The transition temperatures have been determined from the 50% value of the transition, the 100% value being defined as the point where the resistance initially deviates from the normal-state T^2 behavior. The transition width defined as the difference between the temperatures of the 10% and 90% values has been observed to be constant up to a field of 15 kOe.

In Fig. 2 it is demonstrated that for the field along the two basal-plane axes the upper critical field is isotropic, in contrast to the anisotropy for $\mathbf{H} \parallel c$ and $\mathbf{H} \perp c$. The

FIG. 1. Temperature dependence of the resistance for UPt₃B_x sample 2 with applied field directed along the b axis ($H = 0.0, 1.04, 2.08, 3.11, 4.2, 5.2, 6.2, 7.3, 8.3, 11.4, 12.5$ kOe from right to left).

$H_{c2}(T)$ curves for field along the b axis are nearly the same for sample 1 and sample 2 (Figs. 2 and 3), the only difference being that the kink of sample 2 is sharp, while the kink of sample 1 is slightly rounded. The origin of this rounding is not clear.

The kink in $H_{c2}(T)$ for the field along the c axis has been measured before by Hasselbach *et al.*¹⁶ In their analysis it has been shown that instead of a linear function a better description of the data is given by a crossing of two curved lines. Also in our data a weak kink of two curved lines becomes evident by fitting two quadratic polynomials to $H_{c2}(T)$ as shown in Fig. 2. With this fit a kink is obtained at a field of about 9 kOe, which is consistent with the phase diagram as obtained from ultrasound ve-

FIG. 2. $H_{c2}(T)$ of UPt₃B_x (sample 1) for the field directed along the c axis (Δ), basal-plane b axis ($\circ$), and basal-plane a axis ($\square$). Solid lines are guides to the eye in the case of $\mathbf{H} \parallel a$ and b , and a fit described in the text in case of $\mathbf{H} \parallel c$. The dashed line is the extrapolation of the low-temperature part to zero field, which indicates the presence of a kink. The dot-dashed lines are extrapolations of the fit that indicate the presence of a kink.

FIG. 3. $H_{c2}(T)$ of UPt_3B_x sample 2 for the field along the b axis. The solid lines indicate the slopes at the kink in $H_{c2}(T)$. The dashed lines are extrapolations of the slopes at the kink.

locity measurements by Adenwalla *et al.*¹⁸ and Bruls *et al.*¹⁷

In zero-field neutron-diffraction measurements the small antiferromagnetic moment was observed to be along the b axis.²⁸ In both the E -representation model^{34–37} and the 1D-representation model²³ the $H_{c2}(T)$ for the field in the basal plane is not expected to be isotropic in the case of $\mathbf{M} \parallel b$. It is, however, reasonable to assume that the anisotropy energy is very small²³ so that already in a small field the moments rotate to be perpendicular to the field. This assumption is consistent with the observation of a transverse magnetoresistance that is isotropic for fields in the basal plane. The data (measured on sample 1) are shown in Fig. 4.

The numerical data obtained from $H_{c2}(T)$ with $\mathbf{H} \parallel b$ are presented in Table I. In Table II the parameters derived from the upper critical field with $\mathbf{H} \perp c$ are compared with those obtained from the data of other authors.

FIG. 4. Transverse magnetoresistance of single-crystalline UPt_3B_x (sample 1) for the field direction along the b axis ($\circ$) and a axis ($\square$) at $T = 0.608$ K, for fields of 1 up to 80 kOe. The solid line is a guide to the eye.

TABLE I. Parameters derived from the upper- and lower-critical-field measurements on boron-doped UPt_3 . The T_{c+} denotes the transition temperature in zero field; T_{extrap} denotes the temperature that is obtained by extrapolation of the low-temperature part of H_c to zero field. The ratio of slopes R at the temperature of the kink T_{kink} is defined as $R(H_c) = H'_c(-)/H'_c(+)$, with $H'_c(-)$ the lower-temperature slope, $H'_c(+)$ the higher-temperature slope, and the prime denoting the derivative with respect to temperature.

	$H_{c2}(T)$ sample 2 $\mathbf{H} \parallel b$	$H_{c1}(T)$ sample 2 $\mathbf{H} \parallel b$	$H_{c1}(T)$ sample 3 $\mathbf{H} \parallel c$
T_{c+} (K)	0.558 (1)	0.540 (1)	0.540 (1)
T_{extrap} (K)	0.522 (1)	0.521 (1)	0.515 (2)
T_{kink} (K)	0.453 (4)	0.480 (5)	0.485 (7)
$H'_c(-)$ (kOe/K)	67.4 (10)	0.279 (1)	0.195 (1)
$H'_c(+)$ (kOe/K)	44.7 (10)	0.195 (4)	0.107 (5)
Slope ratio R	1.51 (3)	1.43 (5)	1.81 (10)

We conclude that samples of different origin give results that agree very well.

B. Lower critical field

The measured temperature dependence of H_{c1} for the boron-doped UPt_3 is shown in Fig. 5. For both field directions $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$, $H_{c1}(T)$ reveals a kink. The numerical data obtained from the critical-field measurements are presented in Table I. We note that the lower critical field is clearly anisotropic for $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$ both in the absolute value of $H_{c1}(T)$ as in the ratio of the slopes at the kink.

Measurements of $H_{c1}(T)$ for $\mathbf{H} \parallel c$ and $\mathbf{H} \perp c$ have been published before by Zhao *et al.*⁴⁸ and Vincent *et al.*⁴⁰ In the measurements of Zhao *et al.* a feature was observed at about 160 mK below T_{c+} . It is difficult to relate this to the second transition since this would imply a value of ΔT_c which is roughly 3 times the value of ≈ 60 mK, which is well established by specific-heat, ultrasound, and thermal-expansion measurements on samples of nominally pure UPt_3 of different sources.

From our H_{c1} data we obtain a ΔT_c of 60(6) mK, which is consistent with the data of Vincent *et al.*,⁴⁰ who reported a value for ΔT_c of 53(6) mK (for $\mathbf{H} \parallel b$). The absolute values of H_{c1} reported here are significantly lower than the values reported by Vincent *et al.* For $\mathbf{H} \parallel b$ and $T = 0.35$ K they report a value of H_{c1} of 64 Oe and for $\mathbf{H} \parallel c$ and $T = 0.40$ K a value of H_{c1} of 38 Oe, compared to 48 Oe and 23 Oe, respectively, for this work. This must be attributed to a different criterion for determining H_{c1} (Vincent *et al.* did not measure the change in field dependence of M due to a first penetration of the flux, but the temperature dependence of the shielding signal, and used a 15% loss-in-shielding criterion). In Table III the parameters derived from $H_{c1}(T)$ for $\mathbf{H} \parallel b$ are compared with those of Vincent *et al.* We conclude that both the slope ratio and the temperature differences derived from the data are in satisfactory agreement. The observed

TABLE II. Ratio of slopes near the kink and the temperature differences derived from the upper-critical-field data obtained from Taillefer (Ref. 39), Shivaram, Gannon, and Hinks (Ref. 38), and this work. The T_{c+} denotes the transition temperature in zero field; T_{extrap} denotes the temperature that is obtained by extrapolation of the low-temperature part of H_c to zero field. The R denotes the ratio of slopes at the temperature of the kink T_{kink} with $H'_c(-)$ the lower-temperature slope, $H'_c(+)$ the higher-temperature slope.

$H_{c2}(T)$	Taillefer	Shivaram, Gannon, and Hinks	UPt ₃ B _x sample 2
$R = H'_{c2}(-)/H'_{c2}(+)$	1.46	1.50	1.51 (3)
$T_{c+} - T_{\text{extrap}}$ (mK)	20	19 (7)	18 (2)
$T_{c+} - T_{\text{kink}}$ (mK)	≈ 100	102	105 (5)

anisotropy in $H_{c1}(T)$, however, is clearly in contradiction with the measurements of Vincent *et al.* One could, of course, argue that the observed anisotropy is only due to a sample dependence. It is, however, difficult to understand how two samples, cut from adjacent parts of the same crystal, can be so different that the slopes of $H_{c1}(T)$ at T_c differ a factor 2 (see Table I), and still can have the

same transition temperature, knowing that the value of T_c is sensitively dependent on residual resistivity,³⁹ mechanical stress,⁴⁹ and nonmagnetic impurities.⁵⁰ Several criteria have been used (1%, 2%, and 3%) for deviation from linearity of the M vs H curve,⁴² and although the absolute value of H_{c1} is influenced, the ratio of slopes remains the same.

C. Specific heat

The specific heat data of sample 2 clearly show a double peak (Fig. 6), in the C_p/T vs T plot (Fig. 7), visible as a peak and a shoulder. The measured transition in the specific heat of finite width δT_c can be replaced by a sharp transition by constructing an idealized peak. From this construction the specific-heat jump ΔC and the value of the transition temperature can be determined. For a single transition the procedure is straightforward. At $T = 0$ K, the entropy is zero, and going from $T = 0$ K to the same point in the (S, T) diagram along two different paths (I and II) we have $\Delta S_I = \Delta S_{II}$. Taking for path I the normal state and for path II the superconducting state and going from $T = 0$ K to the same temperature in the normal state, we have the condition $\Delta S_s = \Delta S_n$. With $dS = (C/T)dT$ this leads to the condition of entropy balance:

$$\int_0^{T_c^{\text{onset}}} dT \left(\frac{C_s - C_n}{T} \right) = 0. \quad (3.1)$$

TABLE III. Ratio of slopes at T_{kink} and the temperature differences derived from the lower-critical-field data for $\mathbf{H} \parallel b$, obtained from Vincent *et al.* (Ref. 40) and this work. The T_{c+} denotes the transition temperature in zero field; T_{extrap} denotes the temperature that is obtained by extrapolation of the low-temperature part of H_c to zero field. The R denotes the ratio of slopes at the temperature of the kink T_{kink} with $H'_c(-)$ the lower-temperature slope, $H'_c(+)$ the higher-temperature slope.

$H_{c1}(T)$ $\mathbf{H} \parallel b$	Vincent <i>et al.</i>	UPt ₃ B _x sample 2
$R = H'_{c1}(-)/H'_{c1}(+)$	1.40 (5)	1.43 (5)
$T_{c+} - T_{\text{extrap}}$ (mK)	≈ 16	19 (2)
$T_{c+} - T_{\text{kink}}$ (mK)	53 (6)	60 (6)

FIG. 5. Temperature dependence of the lower critical field of UPt₃ doped with boron for (a) the field in the b -axis direction (sample 2) and (b) the field in the c -axis direction (sample 3). The solid lines indicate the slopes at the kink in $H_{c2}(T)$. The dashed lines are extrapolations of the slopes at the kink.

FIG. 6. Double peak in the temperature dependence of the specific heat of UPt_3 doped with boron (sample 2).

In the case that C_s and C_n are known, with this condition an idealized sharp peak can be constructed.

For a double transition the construction is more complex and different authors^{5,14} use different approaches for their construction.

In the following we will denote the higher transition temperature by T_{c+} and the lower transition temperature by T_{c*} . For the low-temperature behavior of the specific heat of UPt_3 in the superconducting state a quadratic power law in addition to a $\gamma(0)T$ term is reported,^{5,14} where the value of $\gamma(0)$ is sample dependent. This behavior finds a possible explanation in the Hirschfeld model.⁵¹ In this model the power law is explained by the existence of nodes in the superconducting gap and the $\gamma(0)T$ term by gapless behavior due to resonant impurity scattering in nominally pure samples. The specific-heat data in the superconducting state therefore are represented by

$$C_s = \gamma(0)T + BT^2, \quad (3.2)$$

FIG. 7. Specific heat for the Czochralski-grown single-crystalline sample of UPt_3B_x (sample 2) plotted as C/T vs T . The solid lines and dashed line give the idealized double-peak construction which is described in the text.

within the measured temperature interval. The normal-state electronic specific heat below $T = 5$ K can be fitted to⁵²

$$C_n = \gamma T + \delta T^3 \ln T + \epsilon T^3. \quad (3.3)$$

As has been noted by Franse *et al.*,³ extrapolation of Eqs. (3.2) and (3.3) to $T = 0$ K leads to an entropy unbalance at T_c . The superconducting-state entropy is larger than the normal-state entropy. Therefore, it is important to know what happens below 100 mK. Recent measurements of the specific heat in a field $H > H_{c2}$ down to 15 mK by Schuberth *et al.*⁵³ show, however, an increase of the specific heat above $\gamma_n T$ at temperatures below 70 mK. This upturn is regarded as solving the discrepancy in entropy balance. Because of this upturn, the normal-state specific heat cannot simply be extrapolated to $T = 0$ K. Therefore, we use for our idealized double-peak construction the condition that the entropy of the superconducting path remain constant:

$$\int_0^{T_c^{\text{onset}}} dT \left(\frac{C_s - C_s^{\text{ideal}}}{T} \right) = 0. \quad (3.4)$$

This leads to the condition, illustrated in Fig. 8, that area $A_1 + \text{area } B_1 = \text{area } A_2 + \text{area } B_2$. We assume the broadening of the two transitions δT_{c+} and δT_{c*} to be smaller than the distance between the two transition temperatures $\Delta T_c = T_{c+} - T_{c*}$. The two peaks then can be treated separately, giving two conditions: area $A_1 = \text{area } A_2$ and area $B_1 = \text{area } B_2$.

In Fig. 8, line a is a fit to the data. Line b is determined by two unknown points: the $\gamma_b(0)$ value (being the extrapolation of γ to 0 K, for the higher temperature peak) and the temperature where line b crosses the data. The second peak results from a sudden increase of quasiparticle excitations due to the addition of a new component to the order parameter at T_{c*} , which has at this temperature a magnitude $\Delta \approx 0$ and starts to grow very rapidly below T_{c*} . Therefore we expect the line b of

FIG. 8. Specific-heat data of UPt_3 after Fisher *et al.* (Ref. 5) with the idealized double peak constructed as described in the text. The different areas and parameters used in the entropy balance considerations are indicated.

the construction to cross the data at T_{c*}^{onset} . Going from $T = 0$ K to $T = T_{c*}$ via the entropy path of phase 1 (the phase below T_{c+}) and thus via line *b*, or via the entropy path of phase 2 (the phase below T_{c*}), and thus via line *a*, we have again $\Delta S_+ = \Delta S_*$. The entropy balance

$$\int_0^{T_{c*}} dT \left(\frac{C_{s+}^{\text{ideal}} - C_{s*}^{\text{ideal}}}{T} \right) = 0 \quad (3.5)$$

leads to the condition illustrated in Fig. 8 that area $C_1 = \text{area } C_2$. In this construction, it has been assumed that the power of the temperature dependence remains the same in both phases. Anticipating the analyses of data with the *E*-representation model and the 1D-representation model, we have to note that this assumption is consistent with these models and the use of either an order parameter of the E_{1g} representation with spin-orbit coupling (*E*-representation model) or the A_{1u} representation without spin-orbit coupling (1D-representation model). In both cases the two superconducting phases are characterized by a superconducting gap that contains a line of nodes on the Fermi surface which is consistent with the observed low-temperature power laws. Recent measurements of the specific heat in the superconducting state of Schubert, Strickler, and Andres⁵⁴ reveal a transition at 18 mK, which is attributed to the formation of a more complete magnetic order. This will, however, occur in both scenarios and therefore hardly influence the amount of entropy contained in area C_1 .

This leaves us with three conditions and four unknowns [T_{c+} , T_{c*} , $\gamma_b(0)$, T_{c*}^{onset}]. In the case that the transition at T_{c+} is very sharp, the measurement directly supplies T_{c+} and the construction can be made. For a sample with less sharp transition at T_{c+} , the construction can be made by determining T_{c*}^{onset} from its definition as the temperature where the first peak evolves into the increase of the second peak. Results of both ways of making the construction have been checked to be in agreement with each other, using the data from Fisher *et al.*⁵ and Hasselbach, Taillefer, and Flouquet¹⁴ for two samples with a sharp transition at T_{c+} (one of the idealized double-peak constructions is shown in Fig. 8). For two Czochralski-grown single-crystalline samples of UPt₃ and UPt₃B_x (sample 2), both with less sharp transition at T_{c+} , the construc-

FIG. 9. Specific-heat data for the Czochralski-grown single-crystalline sample of UPt₃ with the idealized double-peak construction as described in the text.

tion has been made by determining T_{c*}^{onset} . The results are presented in Figs. 9 and 7. The parameters obtained from the constructions are collected in Table IV. To give an idea of the sensitivity of the parameters to changes in the input values for the construction, the uncertainty in the parameters due to an error of 5 mK in T_{c*}^{onset} , which is the least accurately known input value, has been included in the table.

With the values of T_c and $\Delta C (= C_s - C_n)$ at T_c , derived with the idealized double-peak construction, the ratio of Ginzburg-Landau parameters β_2/β_1 can be obtained. For both the *E*-representation and the 1D-representation model it is given by the same expression

$$\frac{\beta_2}{\beta_1} = \left[\frac{(\Delta C_*/T_{c*})}{(\Delta C_+/T_{c+})} - 1 \right]. \quad (3.6)$$

For comparison, the same analysis was performed on data obtained by Fisher *et al.*⁵ and Hasselbach, Taillefer, and Flouquet¹⁴ and that of a Czochralski-grown single-crystalline sample of undoped UPt₃ and UPt₃B_x. The resultant parameter ratios β_2/β_1 are collected in Table IV. In contrast to the parameters derived from the upper and lower critical fields, there is a large variation in the values for different samples. This can be attributed to pair

TABLE IV. Parameters derived from the specific-heat data obtained from Fisher *et al.* (Ref. 5), Hasselbach, Taillefer, and Flouquet (Ref. 14), and this work. The parameters are derived using the idealized double-peak construction described in the text. The value in between brackets gives the uncertainty due to an error of ± 5 mK in the value for T_{c*}^{onset} . The T_{c+} denotes the higher transition temperature, T_{c*} the lower transition temperature, ΔT_c the difference between the two, and ΔC the jump in the specific heat at T_c . The β_2/β_1 is the ratio of GL parameters that is derived from the values of $\Delta C/T$ at the transition.

$C_p(T)$	Fisher <i>et al.</i> sample 1	Hasselbach, Taillefer, and Flouquet	UPt ₃	UPt ₃ B _x sample 2
T_{c+} (mK)	494 (1)	494 (1)	441 (1)	533 (1)
T_{c*} (mK)	431 (1)	436 (1)	365 (2)	479 (2)
ΔT_c (mK)	63 (0)	58 (0)	76 (1)	54 (1)
$\Delta C_+/T_{c+}$ (J/mol K ²)	0.228 (4)	0.231 (3)	0.205 (5)	0.208 (9)
$\Delta C_*/T_{c*}$ (J/mol K ²)	0.283 (2)	0.307 (1)	0.270 (3)	0.312 (3)
β_2/β_1	0.24 (2)	0.33 (2)	0.32 (6)	0.50 (8)

breaking by impurities. The height of the peaks in the specific heat and the value of the transition temperature are both depressed by pair breaking.^{55,56} Substitution studies have shown that the value of β_2/β_1 is largely influenced by pair breaking.⁴⁴ In contrast to this, the influence of pair breaking on the parameters derived from $H_{c1}(T)$ and $H_{c2}(T)$ can be expected to be very small, since the transition, and therefore the whole kink, is only shifted to lower temperature. This will not cause any difference in the ratio of slopes nor in the temperature differences $T_{c+} - T_{\text{extrap}}$ and $T_{c+} - T_{\text{kink}}$, which are given in Tables II and III.

IV. ANALYSIS OF THE DATA WITH THE E -REPRESENTATION MODEL

In the E -representation model^{34–37} a singlet order parameter belonging to the two-dimensional E_{1g} representation is assumed. For a singlet representation the gap function $\Delta(\mathbf{k})$ fully characterizes the order parameter. The order parameter as an expansion of the simplest basis functions of the E_{1g} symmetry is given by²⁰

$$\Delta(\mathbf{k}) \propto \eta_x k_z k_x + \eta_y k_z k_y, \quad (4.1)$$

with x and y orthogonal axes in the basal plane and z the hexagonal axis. Since a symmetry transformation of the basis functions again gives a linear combination of these, the transformation operates only on the coefficients η_i . Therefore the free-energy expansion of Landau theory can be given in terms of η_i .

Although an E_{1g} order parameter is assumed, the E -representation model does not distinguish between E_{1u} , E_{1g} , E_{2u} , and E_{2g} for the following reasons: The triplet and singlet versions belong to the same symmetry class, and because only the coefficients (η_1, η_2) are used in the expansion of F , both give the same functions. The E_1 and E_2 representations have different symmetries, but their Landau functionals have the same form. The notation for E_1 will be used, but the results for E_2 may be obtained from those by straightforward mapping.⁵⁷

The model is based on the principle of a symmetry-breaking term in the free energy that lifts the orbital degeneracy of the two components of a two-dimensional order parameter and in this way causes a splitting of T_c . The symmetry-breaking distortion results from a coupling between the order parameter $\boldsymbol{\eta}$ and a symmetry-breaking field (SBF), denoted with the vector $\mathbf{N}$, which is assumed to be the small antiferromagnetic moment.

We have now the following free-energy expression for E_{1g} symmetry in the absence of a SBF:

$$\frac{\Delta F}{V} = \alpha(T) |\boldsymbol{\eta}|^2 + \beta_1 |\boldsymbol{\eta} \cdot \boldsymbol{\eta}^*|^2 + \beta_2 |\boldsymbol{\eta} \cdot \boldsymbol{\eta}|^2. \quad (4.2)$$

The $\alpha(T) = \alpha_0(T - T_{c0})$, with $\alpha_0 > 0$ and T_{c0} being the transition temperature in absence of a SBF. By normalizing the components η_x and η_y by their magnitude four different types of order parameters can be defined: two one-component ones [the (0,1) and (1,0) types] and two two-component ones [the real (1,1) type and the complex (1, i) type]. In the case that $\mathbf{N}$ is taken to be parallel with the y axis the SBF term is given by³⁴

$$\left(\frac{\Delta F}{V}\right)_{\text{SBF}} = \varepsilon |\eta_x|^2 - \varepsilon |\eta_y|^2, \quad \varepsilon > 0. \quad (4.3)$$

After addition of this term to the free-energy density it can be rewritten³⁴ by absorbing ε into the coefficient α :

$$\frac{\Delta F}{V} = \alpha_- |\eta_x|^2 + \alpha_+ |\eta_y|^2 + \beta_1 |\boldsymbol{\eta} \cdot \boldsymbol{\eta}^*|^2 + \beta_2 |\boldsymbol{\eta} \cdot \boldsymbol{\eta}|^2, \quad (4.4)$$

with

$$(\alpha + \varepsilon) = \alpha_0 \left(T - T_{c0} + \frac{\varepsilon}{\alpha_0}\right) = \alpha_0 (T - T_{c-}) \equiv \alpha_-, \quad (4.5)$$

$$(\alpha - \varepsilon) = \alpha_0 \left(T - T_{c0} - \frac{\varepsilon}{\alpha_0}\right) = \alpha_0 (T - T_{c+}) \equiv \alpha_+. \quad (4.6)$$

The SBF lifts the degeneracy between the (0,1) and the (1,0) order parameters which have now a transition temperature of, respectively, T_{c+} and T_{c-} . Because of the asymmetry between η_x and η_y , the transition from the normal to the superconducting state is a transition to the (0,1) phase. It can be shown³⁴ that at a lower temperature T_{c*} (with $T_{c*} < T_{c-}$) a transition takes place to the (1, i) phase. This lower-temperature transition appears as a second peak in the specific heat and a kink in $H_{c1}(T)$.³⁴ In a magnetic field, ΔF should be written as an integral over the volume, and the following integrand, accounting for the gradient terms in the free energy,³⁴ is added:

$$(\Delta F)_{\text{grad}} = \kappa_1 (D_i \eta_j)(D_i \eta_j)^* + \kappa_2 (D_i \eta_i)(D_j \eta_j)^* + \kappa_3 (D_i \eta_j)(D_j \eta_i)^* + \kappa_4 (D_z \eta_i)(D_z \eta_i)^*, \quad (4.7)$$

in which $\kappa_1, \kappa_2, \kappa_3$, and κ_4 are the “bending coefficients.” The $D_i = \delta_i - (ie/c)A_i$ is the gauge-invariant derivative, and A_i is the vector potential. In the following, the shorthand notation κ_{23} denotes $\kappa_2 + \kappa_3$ and κ_{123} denotes $\kappa_1 + \kappa_2 + \kappa_3$. The gradient terms can be rewritten to obtain the kinetic-energy term that represents the diamagnetic coupling of the supercurrent with the field.⁵⁸ For $\mathbf{H} \perp \mathbf{N}$ and assuming $\kappa_{123} < \kappa_1$, the kinetic-energy term is asymmetric in η_x and η_y in such a way that at high field the (1,0) phase is favorable. Since at low field the (0,1) phase is favorable and since the favorable phase has a higher value of $H_{c2}(T)$, the two $H_{c2}(T)$ solutions cross each other and a kink in $H_{c2}(T)$ results.

To test the consistency of the E -representation model with the obtained experimental results, the following data can be deduced from experiment.

A. Ratio of the slopes near the kink in $H_{c1}(T)$

The ratio of the slopes of $H_{c2}(T)$ at the temperature of the kink T_H is given by

$$R(H_{c2}) \equiv \frac{H'_{c2}(T_H - \delta)}{H'_{c2}(T_H + \delta)} = \frac{\sqrt{\kappa_1 \kappa_4}}{\sqrt{\kappa_{123} \kappa_4}}, \quad (4.8)$$

with $H'_{c2}(T_H - \delta)$ being the lower-temperature slope, $H'_{c2}(T_H + \delta)$ the higher-temperature slope, and the prime denoting the derivative with respect to temperature. With this, an expression for the bending coefficients is found:

$$\frac{\kappa_{23}}{\kappa_1} = \left(\frac{1}{R(H_{c2})} \right)^2 - 1. \quad (4.9)$$

With $R(H_{c2}) = 1.51(3)$ we obtain $\kappa_{23}/\kappa_1 = -0.55(1)$. With Eq. (3.6) the specific heat provides for the GL parameters the value $\beta_2/\beta_1 = 0.50(8)$. With these values known, the ratio of the slopes of $H_{c1}(T)$ for $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$ can be predicted. As follows from the calculations by Hess, Tokuyasu, and Sauls³⁴ these slopes are given by

$$\begin{aligned} R_c(H_{c1}) &= \frac{1}{4} \left(1 + \frac{\beta_2}{\beta_1} \right) \frac{(\kappa_1 + \kappa_{123})^2}{\kappa_1 \kappa_{123}}, \\ R_b(H_{c1}) &= \frac{1}{4} \left(1 + \frac{\beta_2}{\beta_1} \right) \left(\frac{3\kappa_1 + \kappa_{123}}{\kappa_1} \right), \end{aligned} \quad (4.10)$$

with R_c (R_b) the ratio of slopes for field along the c axis (b axis). This leads to the prediction for H_{c1} that $R_b(H_{c1}) = 1.30(10)$ and $R_c(H_{c1}) = 1.75(8)$. These values agree reasonably well with the measured values: $R_b(H_{c1}) = 1.43(5)$ and $R_c(H_{c1}) = 1.81(9)$.

B. Model temperatures

In Fig. 10(b) we see how, according to the model, certain characteristic temperatures can be derived from the measurements and how they are related to the model temperatures. The temperature dependence of the specific heat reveals two transitions. The higher transition temperature is T_{c+} , and the lower one is T_{c*} . The critical field has three characteristic temperatures: (i) the transition temperature in zero field, which is T_{c+} in the case of the upper as well as the lower critical field; (ii) the temperature of the kink T_{kink} , which is T_H in the case of the upper critical field and T_{c*} in the case of the lower critical field; (iii) the extrapolation of the low-temperature part of the critical field provides the temperature T_{extrap} , which is T_{c-} for the upper critical field as follows from the calculations of Hess, Tokuyasu, and Sauls.^{34,35}

As can be seen from the expression for H_{c1} and the expression for the penetration depth in the work of Hess, Tokuyasu, and Sauls,³⁴ the extrapolated value of the low-temperature part of $H_{c1}(T)$ to zero field for $\mathbf{H} \parallel b$ is determined by

$$\rho_{zz}\rho_{xx} = 0, \quad (4.11)$$

with ρ_{xx} and ρ_{zz} being components of the superfluid density tensor.³⁴ The condition $\rho_{zz} = 0$ leads to

$$\kappa_4 (|\eta_x|^2 + |\eta_y|^2) = 0. \quad (4.12)$$

For $T < T_{c*}$ we have the order parameter of the (1, i) phase, given by

$$\begin{aligned} (1, i) \text{ phase: } \quad |\eta_x|^2 &= -\frac{1}{4} \left(\frac{\alpha}{\beta_1} + \frac{\varepsilon}{\beta_2} \right), \\ |\eta_y|^2 &= -\frac{1}{4} \left(\frac{\alpha}{\beta_1} - \frac{\varepsilon}{\beta_2} \right). \end{aligned} \quad (4.13)$$

Substitution of this order parameter results in

$$-\frac{\kappa_4}{4} \left(\frac{2\alpha}{\beta_1} \right) = 0 \quad \text{or} \quad \alpha = \alpha_0(T - T_{c0}) = 0. \quad (4.14)$$

From this equation the extrapolated temperature T_{c0}

FIG. 10. (a) Overview of $C(T)/T$, $H_{c2}(T)$ for $\mathbf{H} \parallel b$ and $H_{c1}(T)$ for $\mathbf{H} \parallel b$ (all three measured on sample 2) and the temperature distances between characteristic temperatures that are derived from it. (b) The E -representation model predicts two jumps in $C(T)/T$ at the transition temperatures, a kink in $H_{c2}(T)$ for $\mathbf{H} \parallel b$ and a kink in $H_{c1}(T)$. For a value of $\beta_2/\beta_1 = 0.5$ and $\kappa_{23}/\kappa_1 = -0.55$ the E -representation model predicts the presented temperature differences between the characteristic temperatures. Note that the detailed temperature dependences of $C(T)$, $H_{c1}(T)$, and $H_{c2}(T)$ are not predicted.

emerges. The other possibility $(\rho_s)_{xx} = 0$ leads with $\kappa_{123} < \kappa_1$ to an unphysical temperature which is higher than T_{c0} .

C. Value of the parameter ratio β_2/β_1

This parameter ratio can be derived from the specific heat [see Eq. (3.6)]. Furthermore, a value for β_2/β_1 can be derived from the three model temperatures obtained from $H_{c1}(T)$ with $\mathbf{H} \parallel b$:

$$\frac{\beta_1}{\beta_2} = \left[\frac{(T_{c+} - T_{c*})}{(T_{c+} - T_{c0})} - 1 \right]. \quad (4.15)$$

The derived values are, from the specific heat (sample 2), $\beta_2/\beta_1 = 0.50(8)$ and, from the lower critical field (sample 2), $\beta_2/\beta_1 = 0.46(10)$, which is in good agreement.

D. Predicted temperature differences

Within the E -representation model the temperature difference between the various T_c values can be expressed in terms of ε/α_0 :

$$\begin{aligned} T_{c+} - T_{c0} &= \varepsilon/\alpha_0, \\ T_{c+} - T_{c-} &= 2(\varepsilon/\alpha_0), \\ T_{c+} - T_{c*} &= (1 + \beta_1/\beta_2)(\varepsilon/\alpha_0). \end{aligned} \quad (4.16)$$

From the jumps in the specific heat at the transition temperatures the GL-parameter ratio β_2/β_1 can be determined [see Eq. (3.6)]. Using the measured values of $\Delta C/T_c$ (for sample 2), we obtain $\beta_2/\beta_1 = 0.50(8)$. With substitution of this value in Eq. (4.16), this leads to a temperature difference

$$(T_{c+} - T_{c*}) = 3.0(4) \left(\frac{\varepsilon}{\alpha_0} \right). \quad (4.17)$$

For the temperature difference $T_{c+} - T_H$ we proceed in the following way. The ratio of the slopes of H_{c2} at the temperature of the kink can be expressed in temperature differences

$$R \equiv \frac{H'_{c2}(T_H - \delta)}{H'_{c2}(T_H + \delta)} = \frac{T_H - T_{c+}}{T_H - T_{c-}}. \quad (4.18)$$

From this equation follows an expression for the temper-

ature T_H :

$$T_H = \frac{R(H_{c2})T_{c-} - T_{c+}}{R(H_{c2}) - 1}. \quad (4.19)$$

With substitution of $T_{c-} = T_{c+} - 2(\varepsilon/\alpha_0)$ in Eq. (4.19) we obtain an expression in terms of ε/α_0 :

$$T_{c+} - T_H = \left(\frac{2R(H_{c2})}{R(H_{c2}) - 1} \right) \left(\frac{\varepsilon}{\alpha_0} \right). \quad (4.20)$$

Using the measured ratio $R(H_{c2}) = 1.51(3)$ we finally obtain

$$T_{c+} - T_H = 5.9(2) \left(\frac{\varepsilon}{\alpha_0} \right). \quad (4.21)$$

As is illustrated in Fig. 10 and listed in Table V, the ratio of the measured temperature differences is 1 : 1.9 (3) : 2.9 (6) : 3.2 (6) : 5.5 (8), which is in close agreement with the predicted ratios of 1 : 2 : 3.0 (4) : 3.0 (4) : 5.9 (2).

E. Conclusion

In conclusion, there are three possibilities to test the consistency of the E -representation model with the obtained experimental data.

(i) With the value for β_2/β_1 derived from the specific heat and the value for κ_{23}/κ_1 from the upper critical field, the slopes of the lower critical field for both field directions can be predicted.

(ii) From both specific heat and lower critical field for $\mathbf{H} \parallel b$ the value of the parameter β_2/β_1 can be derived.

(iii) From all three different types of experiments, differences between model temperatures can be determined. If β_2/β_1 is known, the model can predict all temperature differences expressed in the parameter ε/α_0 that describes the coupling of the superconducting order parameter with the symmetry-breaking field. With the ratio of slopes of $H_{c2}(T)$ known, also the temperature difference $T_{c+} - T_H$, T_H being the temperature of the kink in $H_{c2}(T)$ for $\mathbf{H} \parallel b$, can be expressed in ε/α_0 .

We noted already that (i) the measured slopes of $H_{c1}(T)$ are in good agreement with the predicted values. (ii) Both values for β_2/β_1 , derived from different measurements, are close to 0.5, which is the weak-coupling prediction. (iii) The predicted temperature differences

TABLE V. E -representation model: consistency of temperature differences ΔT between model temperatures. The ratios of the ΔT that result from measurement are compared with the model predictions. All measurements are performed on the same sample (sample 2). The T_{c+} denotes the higher transition temperature, T_{c*} the lower transition temperature, and T_H the temperature of the kink in H_{c2} . The T_{c0} represents in this model the transition temperature in the case that no SBF is present, and T_{c-} the transition temperature of the (1,0) component of the order parameter.

Measurement	ΔT	Result	Ratio	Predicted
$H_{c1}(T)$	$T_{c+} - T_{c0}$	19 (2) mK	1	1 (ε/α_0)
$H_{c2}(T)$	$T_{c+} - T_{c-}$	37 (2) mK	1.9 (3)	2 (ε/α_0)
$C_p(T)$	$T_{c+} - T_{c*}$	54 (6) mK	2.9 (6)	3.0 (4) (ε/α_0)
$H_{c1}(T)$	$T_{c+} - T_{c*}$	60 (6) mK	3.2 (6)	3.0 (4) (ε/α_0)
$H_{c2}(T)$	$T_{c+} - T_H$	105 (5) mK	5.5 (8)	5.9 (2) (ε/α_0)

are in close agreement with experiment. We conclude that all three predictions of the E -representation model are consistent with the presented experimental data.

V. ANALYSIS OF THE DATA WITH THE 1D-REPRESENTATION MODEL

The one-dimensional representation model of Machida and Ozaki^{23,37} also works with a breaking of symmetry by the antiferromagnetic moment; however, in this model the spin degeneracy is lifted instead of the orbital degeneracy. For this mechanism it is necessary to assume that spin-orbit coupling is weak and that spins have separate freedom of rotation. A classification of order parameters for hexagonal symmetry without spin-orbit coupling has been made by Ozaki and Machida.²⁵

The model uses a one-dimensional representation with an odd-parity (p -wave like) order parameter:²³

$$\Delta_{s_1 s_2}(\mathbf{k}) = \sum_{\lambda=x,y,z} \eta_{\lambda} l(\mathbf{k}) \tau_{\lambda}, \quad (5.1)$$

with $\tau_{\lambda} = i\sigma_y \sigma_{\lambda}$. The complex coefficient η_{λ} is characterized by the spin index λ . The orbital part $l(\mathbf{k})$ belongs to a one-dimensional odd-parity representation A_{2u} or B_{1u} or B_{2u} . The free-energy functional is the same as before [see Eq. (4.2)], but the symmetry-breaking term has a different form:²³

$$\left(\frac{\Delta F}{V}\right)_{\text{SBF}} = -\varepsilon (2|\eta_x|^2 - |\eta_y|^2 - |\eta_z|^2), \quad (5.2)$$

with $\varepsilon = cM^2$. M is the magnetic order parameter, and it is assumed that $\mathbf{M} \parallel x$. The coupling breaks the spin-space isotropy and, using the same notation for the transition temperatures as before, T_{c0} is split in two transition temperatures²³

$$T_{c+} = T_{c0} + 2\frac{\varepsilon}{\alpha_0}, \quad T_{c-} = T_{c0} - \frac{\varepsilon}{\alpha_0}. \quad (5.3)$$

In a similar way as in the E -representation model the (1,0,0) phase appears at T_{c+} with an order parameter given by²³

$$|\eta_x|^2 = -\frac{(\alpha - 2\varepsilon)}{\beta_1 + \beta_2}. \quad (5.4)$$

At a temperature T_{c*} ($T_{c*} < T_{c-}$), η_y and η_z appear in the background of the former but with a 90° phase difference. This happens via a second-order transition. The order parameter of the (1, i , i) phase is given by

$$|\eta_x|^2 = \frac{1}{4} \left(\frac{-2\alpha}{\beta_1} + \frac{\varepsilon}{\beta_1} + \frac{3\varepsilon}{\beta_2} \right), \quad (5.5)$$

$$|\eta_{yz}|^2 = |\eta_y|^2 + |\eta_z|^2 = \frac{1}{4} \left(\frac{-2\alpha}{\beta_1} + \frac{\varepsilon}{\beta_1} - \frac{3\varepsilon}{\beta_2} \right).$$

The resulting ratio for the specific-heat jumps is the same as for the E -representation model:²³

$$\left(\frac{\Delta C_*/T_{c*}}{\Delta C_+/T_{c+}} \right) = \left(1 + \frac{\beta_2}{\beta_1} \right). \quad (5.6)$$

An important difference from the E -representation model

is that there are two gradient terms for the one-dimensional representation of D_{6h} corresponding to the uniaxial nature of the effective mass around the z axis:²³

$$\left(\frac{\Delta F}{V}\right)_{\text{grad}} = \sum_{\nu=x,y,z} [\kappa_1 (|D_x \eta_{\nu}|^2 + |D_y \eta_{\nu}|^2)] + \sum_{\nu=x,y,z} [\kappa_2 |D_z \eta_{\nu}|^2]. \quad (5.7)$$

Furthermore, because the representation is one dimensional, there is no possibility of a reorientation of the order parameter to explain the kink in $H_{c2}(T)$. The odd-parity state, however, has another important property that might explain the crossing of two $H_{c2}(T)$ lines: It is characterized by a nonvanishing normal-state spin susceptibility χ . As a result of this, the free-energy functional in the case of applied field has a H^2 term which is quadratic in η :²³

$$\left(\frac{\Delta F}{V}\right)_Q = -\chi \sum_{\mu\nu} H_{\mu} H_{\nu} \eta_{\mu}^* \eta_{\nu}, \quad (5.8)$$

where the Greek subscripts refer to Cartesian components. This H^2 term plays the important role of raising the second transition temperature T_{c*} when the applied field $\mathbf{H}$ is directed along the y or z axis. It is assumed that the small antiferromagnetic moment $\mathbf{M}$ is oriented by the applied field in the basal plane, to be $\mathbf{M} \perp \mathbf{H}$. This means that with the field direction in the basal plane, always the situation ($\mathbf{M} \parallel x$) and ($\mathbf{H} \parallel y$) is measured. The two solutions for $\mathbf{H} \parallel y$ are given by²³

$$H_{c2}^{(1)}(T) = \frac{-\phi_0 \alpha_0 (T - T_{c+})}{2\pi \hbar^2 \sqrt{\kappa_1 \kappa_2}}, \quad (5.9)$$

$$H_{c2}^{(2)}(T) = \frac{-\phi_0 \alpha_0 [T - T_{c*}(H_{c2})]}{2\pi \hbar^2 \sqrt{\kappa_1 \kappa_2}}.$$

The two solutions for $\mathbf{H} \parallel z$ are given by²³

$$H_{c2}^{(1)}(T) = \frac{-\phi_0 \alpha_0 (T - T_{c+})}{2\pi \hbar^2 \kappa_1}, \quad (5.10)$$

$$H_{c2}^{(2)}(T) = \frac{-\phi_0 \alpha_0 [T - T_{c*}(H_{c2})]}{2\pi \hbar^2 \kappa_1}.$$

Because of the term $T_{c*}(H_{c2}) = T_{c*} + \chi H^2$, the two transition lines starting at T_{c+} and T_{c*} intersect at a finite field. The solutions for $\mathbf{H} \parallel y$ and $\mathbf{H} \parallel z$ are similar and differ only in scale. For the lower critical field the ratio of slopes can be derived, and it turns out to be the same for all field directions.²³

To test the consistency of the 1D-representation model with the obtained experimental results, the following data can be deduced from experiment.

A. Ratio of slopes in $H_{c1}(T)$ near the kink

For the lower critical field, the ratio of the slopes near the kink for $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$ are predicted to be identical:²³

$$\frac{H'_{c1}(T_{c+} - \delta)}{H'_{c1}(T_{c+} + \delta)} = \frac{H'_{c1}(T_{c+} - \delta)}{H'_{c1}(T_{c+} + \delta)} = \left(1 + \frac{\beta_2}{\beta_1}\right). \quad (5.11)$$

The parameter β_2/β_1 can be determined from the same expression as in the E -representation model [see Eq. (3.6)]. This leads to a predicted value for the ratio of slopes R of 1.50(8), which is close to the measured value for R of 1.43(5) in the case of $\mathbf{H} \parallel b$. However, with a measured value for R of 1.80(7) in the case of $\mathbf{H} \parallel c$ the measured ratios for $H_{c1}(T)$ are clearly not equal as the model predicts.

B. Model temperatures derived from experiment

The characteristic temperatures that are derived from the specific heat are again the transition temperatures. In the model we will denote the upper transition temperature again with T_{c+} and the lower transition temperature with T_{c*} . In this model only two model temperatures can be identified from $H_{c2}(T)$: The transition temperature in zero field gives us T_{c+} , and the extrapolation of the low-temperature part of $H_{c2}(T)$ is in this model equal to T_{c*} as follows from Eq. (5.9). As can also be seen from Eq. (5.9) the temperature of the kink in $H_{c2}(T)$ depends on the spin susceptibility and is therefore not predicted. From the lower-critical-field data, the transition temperature in zero field supplies T_{c+} and the temperature of the kink in $H_{c1}(T)$ gives us T_{c*} . The extrapolation of the low-temperature part of $H_{c1}(T)$ to zero field, T_{extrap} , can be calculated as before using $\rho_{zz}\rho_{xx} = 0$. From the gradient terms of Eq. (5.7) the components of the superfluid density tensor are obtained:

$$\begin{aligned} \rho_{xx} &= \kappa_1(|\eta_x|^2 + |\eta_{yz}|^2), \\ \rho_{yy} &= \kappa_1(|\eta_x|^2 + |\eta_{yz}|^2), \\ \rho_{zz} &= \kappa_2(|\eta_x|^2 + |\eta_{yz}|^2). \end{aligned} \quad (5.12)$$

Using this and the expression for the $(1, i, i)$ order parameter of Eq. (5.5), it emerges that the extrapolated temperature is given by

$$T_{\text{extrap}} = T_{c0} + \frac{1}{2} \left(\frac{\varepsilon}{\alpha_0} \right). \quad (5.13)$$

C. Predicted temperature differences

As before in the E -representation model, the second transition can take place if the sum of α_- and the $\beta_1 + \beta_2$ term changes sign:

$$\alpha_-(T_{c*}) + (\beta_1 + \beta_2) |\eta_x(T_{c*})|^2 = 0. \quad (5.14)$$

With the order parameter of the $(1, 0, 0)$ phase,

$$|\eta_x|^2 = -\frac{\alpha_0(T - T_{c+})}{\beta_1 + \beta_2}, \quad (5.15)$$

substituted in Eq. (5.14) the transition temperature T_{c*} is obtained. It must be noted that in Ref. 23 an incorrect expression for T_{c*} is given, in which β_1 and β_2 are interchanged in the numerator. The correct expression reads

$$T_{c*} = T_{c0} - \left(\frac{3\beta_1 - \beta_2}{2\beta_2} \right) \left(\frac{\varepsilon}{\alpha_0} \right). \quad (5.16)$$

Using this expression and $T_{c+} = T_{c0} + 2(\varepsilon/\alpha_0)$, we find the temperature difference

$$T_{c+} - T_{c*} = \left[\frac{3}{2} + \frac{3}{2} \left(\frac{\beta_1}{\beta_2} \right) \right] \left(\frac{\varepsilon}{\alpha_0} \right). \quad (5.17)$$

With $\beta_2/\beta_1 = 0.50(8)$, as derived from the specific heat, it follows that

$$T_{c+} - T_{c*} = 4.5(4) \left(\frac{\varepsilon}{\alpha_0} \right). \quad (5.18)$$

Finally, with $T_{c+} = T_{c0} + 2(\varepsilon/\alpha_0)$ and the expression for T_{extrap} of Eq. (5.13), for the temperature difference $T_{c+} - T_{\text{extrap}}$ we obtain

$$(T_{c+} - T_{\text{extrap}}) = \frac{3}{2} \left(\frac{\varepsilon}{\alpha_0} \right). \quad (5.19)$$

From the data of $H_{c1}(T)$ we obtain the temperature differences $T_{c+} - T_{c*}$ and $T_{c+} - T_{\text{extrap}}$. From both $C_p(T)$ and $H_{c2}(T)$ we obtain the temperature difference $T_{c+} - T_{c*}$. The results and predictions are compared in Table VI. If we take the ratio between the temperature differences, as shown in the table, the results are more easily compared with the predictions. Although the results and predictions for C_p and H_{c1} agree, the result for H_{c2} is clearly not consistent with the prediction.

D. Ratios of the slopes of phase lines

In Eqs. (5.9) and (5.10) the solutions of the transition lines of $H_{c2}^{(1)}$ and $H_{c2}^{(2)}$ for $\mathbf{H} \perp c$ and $\mathbf{H} \parallel c$ were given. In these equations the $H_{c2}^{(1)}$ denotes the phase line that emerges from T_{c+} and the $H_{c2}^{(2)}$ denotes the phase line that emerges from T_{c*} . From the Eqs. (5.9) and (5.10) it follows that the ratio of slopes,²³

$$R^{(1)} \equiv \frac{H'_{c2}^{(1)}(y)}{H'_{c2}^{(1)}(z)} = R^{(2)} \equiv \frac{H'_{c2}^{(2)}(z)}{H'_{c2}^{(2)}(y)} = \sqrt{\frac{\kappa_1}{\kappa_2}}. \quad (5.20)$$

Since in our experiment the phase lines that emerge from T_{c*} have not been measured, for this comparison, we make use of the values of slopes that are obtained from the precise measurement of phase lines by the specific heat, as have been reported by Hasselbach *et al.*¹⁶ The reported values are, for $\mathbf{H} \perp c$, $H'_{c2}^{(1)}(y) = -43$ kOe/K and $H'_{c2}^{(2)}(y) = -95$ kOe/K, and for $\mathbf{H} \parallel c$, $H'_{c2}^{(1)}(z) = -75$ kOe/K and $H'_{c2}^{(2)}(z) = -100$ kOe/K. This leads to a value for $R^{(1)}$ of 0.57 and a value for $R^{(2)}$ of 0.95, which is not consistent with the prediction $R^{(1)} = R^{(2)}$.

E. Conclusion

In conclusion, there are three possibilities to test the consistency of the 1D-representation model with experimental data.

(i) With the value of β_2/β_1 derived from the specific heat, the ratio of slopes near the kink in the lower critical

TABLE VI. 1D-representation model: consistency of temperature differences ΔT between model temperatures. The ratios of the ΔT that result from measurement are compared with the model predictions. All measurements are performed on the same sample (sample 2). The T_{c+} denotes the higher transition temperature, T_{c*} the lower transition temperature, and T_{extrap} the temperature that is obtained by extrapolation of the low-temperature part of $H_{c1}(T)$ to zero field.

Measurement	ΔT	Result	Ratio	Predicted	Ratio
$H_{c1}(T)$	$T_{c+} - T_{\text{extrap}}$	19 (2) mK	1	$3/2(\varepsilon/\alpha_0)$	1
$H_{c2}(T)$	$T_{c+} - T_{c*}$	37 (2) mK	1.9 (3)	$4.5(4)(\varepsilon/\alpha_0)$	3.0 (3)
$C_p(T)$	$T_{c+} - T_{c*}$	54 (6) mK	2.9 (6)	$4.5(4)(\varepsilon/\alpha_0)$	3.0 (3)
$H_{c1}(T)$	$T_{c+} - T_{c*}$	60 (6) mK	3.2 (6)	$4.5(4)(\varepsilon/\alpha_0)$	3.0 (3)

field can be predicted.

(ii) For all three different types of experiments the temperature differences between certain model temperatures can be measured. If β_2/β_1 is known, the model can predict all temperature differences expressed in ε/α_0 .

(iii) From the calculations of Machida and Ozaki²³ it follows that the ratio of slopes of the transition lines of the upper critical field, $R^{(1)} = H'_{c2}(y)/H'_{c2}(z)$, should be equal to the ratio $R^{(2)} = H'_{c2}(y)/H'_{c2}(z)$ with the prime indicating the derivative with respect to temperature and the H_{c2} phase lines defined in Eqs. (5.9) and (5.10).

We conclude that for the lower critical field the ratio of slopes near the kink is clearly not isotropic for $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$ as is predicted by the model. From Table VI we conclude that the measured temperature differences are not at all consistent with the model. This inconsistency is due to the fact that the low-temperature part of $H_{c2}(T)$ for $\mathbf{H} \parallel b$ clearly does not extrapolate to the model temperature T_{c*} , as can be seen from the last three entries of the table. Finally, the ratios of slopes $R^{(1)}$ and $R^{(2)}$ are not equal as is predicted.

VI. CONCLUSION

Data derived from measurements of the jumps in the specific heat at the transition temperatures, the kink in the upper critical field for a field direction in the basal plane, and the kink in $H_{c1}(T)$, for a field perpendicular as well as parallel to the c axis, are compared with the predictions of two models: the E -representation model^{34–37} and the 1D-representation model.^{23,37} For both models we use the prediction of temperature differences between characteristic temperatures that are derived from measurements of $C_p(T)$, $H_{c2}(T)$, and $H_{c1}(T)$ on one and the same high-quality sample. In the case of the 1D-representation model we use two additional predictions, one concerning a ratio of slopes for the lower critical field and the other concerning a ratio of slopes for the phase lines of the upper critical field.

For the E -representation model we also use two additional predictions: (i) the ratio of slopes for $H_{c1}(T)$ and (ii) the value of β_2/β_1 , which can be obtained from both $C_p(T)$ and $H_{c1}(T)$.

Neither of the three predictions of the 1D-representation model is consistent with the measurements. For the E -representation model, in contrast, the data are in good agreement with all three predictions.

We conclude that the E -representation model is able to explain consistently the jumps in the specific heat at the transition temperatures, the kink in $H_{c2}(T)$ for a field direction in the basal plane, and the kink in $H_{c1}(T)$ for $\mathbf{H} \parallel b$ and $\mathbf{H} \parallel c$.

Two experimental observations that also should find an explanation are (i) the kink in $H_{c2}(T)$ for field perpendicular to the c axis and (ii) the “crossing” of the upper-critical-field phase lines for two different orientations of the field, $\mathbf{H} \parallel c$ and $\mathbf{H} \perp c$, as has been observed by Shivaram, Gannon, and Hinks.³⁸

It might be possible to explain⁵⁹ these two observations with a c -axis phase diagram consisting of an E -representation (0,1), (1, i), and ($\alpha 1$, βi) phase (the latter phase being one which has been proposed by Sundaram and Joynt^{60,61} and Zhitomirsky⁶²). It would therefore be of interest to investigate whether such an explanation of the phase diagram is consistent with experiment. In the case of this explanation, one would expect a phase transition to occur from the ($\alpha 1$, βi) phase to the (1, i) phase if the field is rotated from the c -axis direction to the b -axis direction. This phase transition would be expected to appear as a kink in H_{c2} as function of the angle θ in the (b , c) plane. Such a measurement has been performed by Shivaram, Rosenbaum, and Hinks,⁶³ and no kink is visible. However, in these measurements, over the whole range of 90°, only seven equidistant points have been taken. Shivaram, Rosenbaum, and Hinks⁶³ also report data of H_{c2} as a function of temperature for a field in the c -axis direction. From these data it appears that even 14 equidistant points over the whole range of temperature (0, T_c) are not enough to reveal the weak kink in $H_{c2}(T)$ for $\mathbf{H} \parallel c$, which is known to be present from measurements of the phase diagram¹⁸ and measurements of $H_{c2}(T)$ by Hasselbach *et al.*¹⁶ Therefore, the presently existing data of the upper critical field as function of angle cannot exclude the possibility of a weak kink in H_{c2} , and the verification of a possible phase transition, as function of angle of the field in the (b , c) plane, remains of interest.

ACKNOWLEDGMENTS

We thank Professor P. de Châtel for useful discussions. This work was part of the research program of the foundation FOM. The group of Dr. Ir. Bastin from the University of Technology Eindhoven is acknowledged for performing the microprobe analysis.

- ¹For a general review on heavy-fermion compounds, see H.R. Ott, T.M. Rice, and J.L. Smith, *Nature* **320**, 124 (1986); Z. Fisk, D.W. Hess, C.J. Pethick, D. Pines, J.L. Smith, J.D. Thompson, and J.O. Willis, *Science* **239**, 33 (1988). For a recent review on heavy-fermion systems, see N. Grewe and F. Steglich, in *Handbook on the Physics and Chemistry of Rare Earths*, edited by K.A. Gschneidner and L. Eyring (North-Holland, Amsterdam, 1991), Vol. 14; and on uranium-based heavy-fermion superconductors, see A. de Visser and J.J.M. Franse, *J. Magn. Magn. Mater.* **100**, 204 (1991).
- ²For a recent review on unconventional superconductivity in heavy-fermion compounds, see M. Sigrist and K. Ueda, *Rev. Mod. Phys.* **63**, 239 (1991).
- ³J.J.M. Franse, A. Menovsky, A. de Visser, C.D. Bredl, U. Gortwick, W. Lieke, H.M. Mayer, U. Rauchschwalbe, G. Sparr, and F. Steglich, *Z. Phys. B* **59**, 15 (1985).
- ⁴A. Sulpice, P. Gandit, J. Chaussy, J. Flouquet, D. Jaccard, P. Lejay, and J.L. Tholence, *J. Low Temp. Phys.* **62**, 39 (1986).
- ⁵R. Fisher, S. Kim, B. Woodfield, N. Phillips, L. Taillefer, K. Hasselbach, J. Flouquet, A. Giorgi, and J. L. Smith, *Phys. Rev. Lett.* **62**, 1411 (1989).
- ⁶D.J. Bishop, C.M. Varma, B. Batlogg, E. Bucher, Z. Fisk, and J.L. Smith, *Phys. Rev. Lett.* **53**, 1009 (1984).
- ⁷B.S. Shivaram, Y.H. Jeong, T.F. Rosenbaum, and D.G. Hinks, *Phys. Rev. Lett.* **56**, 1078 (1986).
- ⁸V. Müller, D. Maurer, E.W. Scheidt, Ch. Roth, K. Lüders, E. Bucher, and H.E. Bommel, *Solid State Commun.* **57**, 319 (1986).
- ⁹Y. Kohori, T. Kohara, H. Shibai, Y. Oda, T. Kaneko, Y. Kitaoka, and K. Asayama, *J. Phys. Soc. Jpn.* **56**, 2263 (1987).
- ¹⁰F. Gross, B.S. Chandrasekhar, K. Andres, U. Rauchschwalbe, E. Bucher, and B. Luthi, *Physica C* **153&155**, 439 (1988).
- ¹¹B.S. Shivaram, J.J. Gannon, Jr., and D.G. Hinks, *Physica B* **163**, 141 (1990).
- ¹²C. Broholm, G. Aeppli, R.N. Kleiman, D.R. Harshman, D.J. Bishop, E. Bucher, D.L.I. Williams, E.J. Ansaldo, and R.H. Heffner, *Phys. Rev. Lett.* **65**, 2062 (1990).
- ¹³P.J.C. Signore, J.P. Koster, E.A. Knetsch, C.M.C.M. van Woerkens, W.M. Meisel, S.E. Brown, and Z. Fisk, *Phys. Rev. B* **45**, 10151 (1992).
- ¹⁴K. Hasselbach, L. Taillefer, and J. Flouquet, *Phys. Rev. Lett.* **63**, 93 (1989).
- ¹⁵T. Vorenkamp, Z. Tarnawski, H.P. van der Meulen, K. Kadowaki, V.J.M. Meulenbroek, A.A. Menovsky, and J.J.M. Franse, *Physica B* **163**, 564 (1990).
- ¹⁶K. Hasselbach, A. Lacerda, A. de Visser, K. Behnia, L. Taillefer, and J. Flouquet, *J. Low Temp. Phys.* **81**, 299 (1990).
- ¹⁷G. Bruls, D. Weber, B. Wolf, P. Thalmeier, B. Luthi, A. de Visser, and A.A. Menovsky, *Phys. Rev. Lett.* **65**, 2294 (1990).
- ¹⁸S. Adenwalla, S. Lin, Q. Ran, Z. Zhao, J. Ketterson, J. Sauls, L. Taillefer, D. Hinks, M. Levy, and B. Sarma, *Phys. Rev. Lett.* **65**, 2298 (1990).
- ¹⁹S. Adenwalla, Z. Zhao, J.B. Ketterson, M. Levy, and B.K. Sarma, *Physica B* **165&166**, 351 (1990).
- ²⁰G.E. Volovik and L.P. Gor'kov, *Zh. Eksp. Teor. Fiz.* **88**, 1412 (1985) [*Sov. Phys. JETP* **61**, 843 (1985)].
- ²¹S. Schmitt-Rink, K. Miyake, and C. M. Varma, *Phys. Rev. Lett.* **57**, 2575 (1986).
- ²²W.O. Putikka, P.J. Hirschfeld, and P. Wölfle, *Phys. Rev. B* **41**, 7285 (1990).
- ²³K. Machida and M. Ozaki, *Phys. Rev. Lett.* **66**, 3293 (1991).
- ²⁴K. Miyake, in *Theory of Heavy Fermions and Valence Fluctuations*, edited by T. Kasuya and T. Saso (Springer-Verlag, Berlin, 1986), p. 256.
- ²⁵M. Ozaki and K. Machida, *Prog. Theor. Phys.* **75**, 442 (1986).
- ²⁶For an overview of presently available models, see R. Joynt, *J. Magn. Magn. Mater.* **108**, 31 (1992).
- ²⁷J. Joynt, V.P. Mineev, G.E. Volovik, and M.E. Zhitomirsky, *Phys. Rev. B* **42**, 2014 (1990).
- ²⁸G. Aeppli, E. Bucher, C. Broholm, J.K. Kjems, J. Baumann, and J. Hufnagl, *Phys. Rev. Lett.* **60**, 615 (1988).
- ²⁹D.W. Cooke *et al.*, *Hyperfine Interact.* **31**, 425 (1986).
- ³⁰G. Aeppli, D. Bishop, C. Broholm, E. Bucher, K. Siemensmeyer, M. Steiner, and N. Stusser, *Phys. Rev. Lett.* **63**, 676 (1989).
- ³¹T. Trappmann, H. v. Lohneysen, and L. Taillefer, *Phys. Rev. B* **43**, 13714 (1991).
- ³²S.M. Hayden, L. Taillefer, C. Vettier, and J. Flouquet, *Phys. Rev. B* **46**, 8675 (1992).
- ³³R. Joynt, *Supercond. Sci. Technol.* **1**, 210 (1988).
- ³⁴D.W. Hess, T.A. Tokuyasu, and J.A. Sauls, *J. Phys. C* **1**, 8135 (1989).
- ³⁵D.W. Hess, T.A. Tokuyasu, and J.A. Sauls, *Physica B* **163**, 720 (1990).
- ³⁶K. Machida and M. Ozaki, *J. Phys. Soc. Jpn.* **58**, 2244 (1989); K. Machida, M. Ozaki, and T. Ohmi, *ibid.* **58**, 4116 (1989).
- ³⁷K. Machida, *Prog. Theor. Phys. Suppl.* **108**, 229 (1992).
- ³⁸B.S. Shivaram, J.J. Gannon, Jr., and D.G. Hinks, *Phys. Rev. Lett.* **63**, 1723 (1989).
- ³⁹L. Taillefer, *Physica B* **163**, 278 (1990).
- ⁴⁰E. Vincent, J. Hammann, L. Taillefer, K. Behnia, N. Keller, and J. Flouquet, *J. Phys. Condens. Matter* **3**, 3517 (1991).
- ⁴¹T. Vorenkamp, A. de Visser, R. Wester, A.A. Menovsky, and J.J.M. Franse, *Physica B* **165**, 363 (1990).
- ⁴²E.A. Knetsch, J.A. Mydosh, T. Vorenkamp, and A.A. Menovsky, *J. Magn. Magn. Mater.* **108**, 75 (1992).
- ⁴³P.A. Midgley, S.M. Hayden, L. Taillefer, B. Bogenberger, and H. v. Löhneysen, *Phys. Rev. Lett.* **70**, 678 (1993).
- ⁴⁴T. Vorenkamp, M.C. Aronson, Z. Koziol, K. Bakker, J.J.M. Franse, and J.L. Smith, preceding paper, *Phys. Rev. B* **48**, 6373 (1993).
- ⁴⁵T. Vorenkamp, K. Kadowaki, A. de Visser, P. Haen, V.J.M. Meulenbroek, M. van Sprang, and J.J.M. Franse, *J. Magn. Magn. Mater.* **76&77**, 531 (1988).
- ⁴⁶A.A. Menovsky, *J. Magn. Magn. Mater.* **76&77**, 631 (1988).
- ⁴⁷G.F. Bastin and H.J.M. Heijligers, *Quantitative Electron Probe Analysis of Boron in Binary Borides* (University Press, University of Technology Eindhoven, Eindhoven, The Netherlands, 1986).
- ⁴⁸Z. Zhao, F. Behroozi, J.B. Ketterson, Y. Guan, B.K. Sarma, and D.G. Hinks, *Physica B* **165&166**, 345 (1990).
- ⁴⁹G.R. Stewart, Z. Fisk, J.O. Willis, and J.L. Smith, *Phys. Rev. Lett.* **52**, 679 (1984).
- ⁵⁰M.C. Aronson, T. Vorenkamp, Z. Koziol, A. de Visser, K. Bakker, J.J.M. Franse, and J.L. Smith, *J. Appl. Phys.* **69**, 5487 (1991).
- ⁵¹P.J. Hirschfeld, P. Wölfle, and D. Einzel, *Phys. Rev. B* **37**, 83 (1988).
- ⁵²A. de Visser, J.J.M. Franse, A.A. Menovsky, and T.T.M. Palstra, *Physica B* **127**, 442 (1984).

- ⁵³E.A. Schuberth, G. Hofmann, F. Gross, K. Andres, and J. Hufnagl, *Europhys. Lett.* **11**, 249 (1990).
- ⁵⁴E.A. Schuberth, B. Strickler, and K. Andres, *Phys. Rev. Lett.* **68**, 117 (1992).
- ⁵⁵M.B. Maple, in *Magnetism*, edited by H. Suhl (Academic, London, 1973), Vol. 5, Chap. 10.
- ⁵⁶M.B. Maple, *J. Appl. Phys.* **9**, 179 (1976).
- ⁵⁷T.A. Tokuyasu, D.W. Hess, and J.A. Sauls, *Phys. Rev. B* **41**, 8891 (1990).
- ⁵⁸M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
- ⁵⁹T. Vorenkamp, Ph.D. thesis, University of Amsterdam, 1992.
- ⁶⁰S.K. Sundaram and R. Joynt, *Phys. Rev. B* **40**, 8780 (1989).
- ⁶¹S.K. Sundaram and R. Joynt, *Phys. Rev. Lett.* **66**, 512 (1991).
- ⁶²M. Zhitomirsky, *Pis'ma Zh. Eksp. Teor. Fiz.* **49**, 333 (1989) [*JETP Lett.* **49**, 379 (1989)].
- ⁶³B.S. Shivaram, T.F. Rosenbaum, and D.G. Hinks, *Phys. Rev. Lett.* **57**, 1259 (1986).