Optical studies of massive X-ray binaries
Zuiderwijk, E.J.

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CHAPTER II

THE RADIAL VELOCITY

ORBIT OF HD 77581
II A  
Mass determination for the X-ray binary system Vela X-1

The 6.9 mag B0.5 Ib supergiant HD77581 has been identified as the optical counterpart of the X-ray eclipsing binary system 3U0900-40 (Vela X-1), which eclipses with a period of 8.95 ± 0.02 d. The discovery of regular X-ray pulsations in Vela X-1 has also been reported, with a mean pulse period of 282.9 s, modulated because of the radial velocity variation of the X-ray pulsar in its orbital motion. This makes Vela X-1 the third X-ray binary system in which the orbits of both the optical and the X-ray component can be studied, and the following orbital parameters have been derived$^4$, $e_x = 0.15 ± 0.05, \omega_x = 157° ± 24°$ and $K_x = 268 ± 12$ km s$^{-1}$. Analyses of the light curve of HD77581 have shown that the heating effect is too small to be detected$^5$. Furthermore, the star is bright and the spectral lines are not very broad. This means that here the first relatively accurate direct mass determination of both the X-ray and the early-type supergiant components of an eclipsing X-ray binary becomes possible.

Earlier studies of the radial velocity variation of HD77581 have given contradictory results$^7$. For the semi-amplitude $K$ of the orbit, values between 19 and 40 km s$^{-1}$ and for the eccentricity from 0.00 to 0.54 have been found. According to Wallerstein$^7$, the radial velocity data of HD77581 do not allow a consistent solution of the orbit, because of mass transfer in the system. Here we show that a consistent solution can be obtained, however, provided that the lines of hydrogen—expected to be most sensitive to gas motions in the system—are excluded from the analysis. Our analysis of the radial velocity variations of HD77581 is based on 26 coudé spectrograms, obtained with the 152-cm telescope of the European Southern Observatory, La Silla, Chile, in 4 observing runs between April 1973 and June 1975. The spectra were taken on Ia-0 emulsion, over wavelengths from 3,600-4,950 Å.

The spectra were measured for line positions with the Grant comparator of the Kapteyn Astronomical Laboratory of the University of Groningen. All absorption features visible in the spectrum were measured, without selecting beforehand a particular set of lines. Some very weak lines were thus missed on some plates, but on the other hand, especially for the weak lines, it was considered an advantage to measure without a predetermined expectation of where lines should be found. Lines of H I, He I, O II, N III, N II, Si III and Si IV were present in at least half of the spectra. From the measurements of all individual line radial velocity measurements will be the homogeneity of the data we reduced the measurements of the radial velocity of the interstellar Ca II K-line, we found $v_{Ca II} = 16.1 ± 0.6$ (m.e.) and $13.7 ± 1.0$ km s$^{-1}$ for the 12 Å mm$^{-1}$ and the 20 Å mm$^{-1}$ spectra, respectively. To increase the mean velocity obtained from two such measurements of $l/a$ were weighted according to $l/a$ the best fitting radial-velocity curve through the points was computed. This was done for all lines of He I and the heavier ion lines were made. The radial velocity measurements were weighted according to $v_{He I} = 1/\sigma_r$, where $\sigma_r$ is the s.d.m. of the measurements. The average orbital period $P$ determined in these solutions is $8.966 ± 0.005$ d, which agrees well with Hutchings' result$^8$ ($P = 8.966 ± 0.001$ d), based on radial velocity determinations by several observers, over a time interval of 17 yr. We therefore subsequently used this period as a fixed parameter in our calculations. The orbital parameters derived from all lines together—except the hydrogen lines—are given in Table 2. The errors quoted in Table 2 are $\sigma_r$ (68% confidence) limits. The separate solutions for He I and the heavier ions are in good agreement with the mean solution (see Table 2), and give good agreement with the X-ray pulsar data, which for convenience are also given in Table 2. In particular the values of the eccentricity and the angle of periastron, which should be 180° apart for the optical and the X-ray solution, agree well (within the quoted accuracy intervals).

### Table 1  Journal of observations

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* Phase zero corresponds to mid-eclipse time JD2441446.54 + $a_x$ 8.966 d.
In our opinion this agreement lends confidence to the orbital character of the optical radial velocity variations; the non-orbital (gas streams, stellar wind fluctuations) component does not seem to have much effect on the results from He I and heavier elements. Using $a_{\sin i} = 2.5 \pm 0.2 \times 10^4$ (1o) as a compromise value, we find the following values for the system parameters:

$$m_{\text{ratio}} : M_{\odot} / m_\text{X} = (a_\odot / a_{\sin i}) = 13.1 \pm 1.65$$

$$m_{\text{total}} : (M_{\odot} + M_\text{X}) = 21.6 \pm 2.2 M_\odot$$

$$M_{\sin i}(i) = 1.52 \pm 0.22 M_\odot \text{ and } M_{\text{opt}} \sin i(i) = 20.0 \pm 2.1 M_\odot. \$$

From a detailed analysis of the optical-light variations, Avni and Bahcall\(^8\) have found that the inclination $i$ should be $> 74^\circ$ to get a consistent picture of both the observed light curve and the duration of the X-ray eclipse. Taking $74^\circ$ and $90^\circ$ as lower and upper limits of the angle of inclination we get:

$$M_{\odot} = 1.61 \pm 0.27 M_\odot \text{ and } M_{\text{opt}} = 21.2 \pm 2.6 M_\odot. \$$

All quoted errors in the mass parameters are $90\%$ confidence limits. This result shows that the compact component is very probably too heavy to be a white dwarf.

If it is a white dwarf, its evolutionary history implies that it should consist mainly of carbon and oxygen\(^8\); the upper mass limit\(^9\) for such white dwarfs is $\sim 1.4 M_\odot$. Its most probable mass of $1.61 M_\odot$ is just consistent with the presently allowed theoretical masses of neutron stars\(^7\), $M < 1.6 M_\odot$.

The mass determination of the supergiant allows a test of the theoretically computed evolution of massive stars, through a comparison with theoretical evolutionary tracks. The luminosity of HD77581 can be inferred in two ways. The spectral type and luminosity class (B5.5 Iib) provide, in principle, the absolute magnitude $M_\odot$, bolometric correction and effective temperature $T_{\text{eff}}$. Using the luminosity calibration of Blaauw\(^8\) and Keenan\(^1\) we derive $M_{\odot} = 5.9 \pm 0.4$ mag. For the bolometric correction, values between 2.5 and 2.6 mag have been given\(^1,8\). This gives $M_{\text{bol}} = -8.4 \pm 0.5$ mag. For the orbital parameters given here and by Rappaport and McClintock\(^1\), and assuming a minimum observed ellipse angle of $34^\circ$, Avni and Bahcall\(^8\) derive for the radius of HD77581 $R = 30 R_\odot$. For the effective temperature, values ranging from 22,000 K (ref. 17)–29,000 K (ref. 18) have been given for early B-type supergiants. Adopting $T_{\text{eff}}$ 25,000 $\pm$ 4,000 K we find $M_{\text{bol}} = -9.0 \pm 0.75$ mag. From a comparison with evolutionary tracks\(^1,8\, 28\) we then find the following values of the evolutionary mass: $M_{\odot} = 22 \pm 7$ and $30 \pm 10$, respectively; these values, although not very accurate, are consistent with the presently determined value of $21.2 M_\odot \pm 2.4 M_\odot$. Therefore, the early-type supergiant in this X-ray binary system does not seem to be particularly undermassive for its spectral type (as has been suggested\(^8\) for Cygnus X-1).

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J. A. van Paradijs
G. Hammerschlag-Hensberge
E. P. J. van den Heuvel
R. J. Taens
E. J. Zuidervijk

Astronomical Institute, University of Amsterdam, Roetersstraat 15, Amsterdam, The Netherlands, and European Southern Observatory, La Silla, Chile.

C. De Loore

Astrophysical Institute, Free University of Brussels, Adolph Buyllaan 105, Brussels, Belgium, and European Southern Observatory, La Silla, Chile.

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II B
Systematic Distortions of the Radial Velocity Curve of HD 77581 (Vela X-1) Due to Tidal Deformation

J. van Paradijs, R. Takens and E. Zuiderwijk
Astronomical Institute, University of Amsterdam, Roetersstraat 15, NL-1004 Amsterdam, The Netherlands

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Summary. The effect of the deformation of the primary star in the Vela X-1 X-ray binary system (HD 77581) on its radial velocity variation has been studied, using theoretically predicted line profiles and radial velocity curves for lines of several ions, computed for a range of system parameters. It is found that in fitting the theoretical curves with the observed radial velocity curves of HD 77581 (within the observational accuracy) appreciable systematic effects occur, differing from ion to ion. Especially the apparent eccentricity and velocity amplitude of the orbit may be strongly affected. However, it appears difficult to separate this effect from effects due to real orbital motion.

Key words: X-ray binaries — radial velocity — close binary systems

Introduction

The 6.9 mag B0.5 Ib star HD 77581 is the optical counterpart of the X-ray source 3U0900-40 (Vela X-1) (Hiltner et al., 1972; Forman et al., 1973; Jones and Liller, 1973). The X-ray source is eclipsed every 8.95 days (Forman et al., 1973) and shows a pulsed intensity variation with a period of 283 s (McClintock et al., 1976). HD 77581 shows photometric and radial velocity variations with a period of 8.966 days (Hutchings, 1974; Van Paradijs et al., 1976). A review of the optical studies of HD 77581 has been given by Bahcall (1976).

The radial velocity variation of the X-ray emitting compact object has been found from the variation of the arrival times of the X-ray pulses (Rappaport et al., 1976). The system, therefore, can be treated as a double-lined spectroscopic binary. As information on the orbital inclination is available from the analysis of the optical photometric variations and the duration of the X-ray eclipse, a mass determination of both components in the system is possible.

Analyses of the orbits and mass determinations have been made by van Paradijs et al. (1976) and Rappaport et al. (1976). The orbital parameters as obtained from the X-ray emitting object and the optical primary were found to be in good agreement ($e_x \approx e_{opt}$; $\omega_x - \omega_{opt} \approx 180^\circ$). This has given confidence in the reliability of the results, which is in particular important for the mass determination.

However, it has been known for a long time that radial velocity data for spectroscopic binaries may suffer from systematic effects due to gas streams or reflection effects (see Batten, 1973, for references on this problem). Another systematic effect, due to the distortion of the primary may be quite important in the case of X-ray binaries with a small mass ratio, such as the Vela X-1 system. In such a system the radial velocities of certain individual surface elements of the primary are much larger than the orbital velocity of the center of mass of this star, in the case of synchronous rotation. When the primary is tidally distorted and has a variation of effective temperature and gravity across its surface, it is by no means clear that the observed radial velocity, which is given by some spectrophotometric average over the surface, can be identified with the motion of the center of mass of the primary. An analysis of this problem was made by Wilson and Sofia (1976), who assumed that the contribution to the radial velocity from each point on the stellar surface is proportional to the local continuum intensity. They did not take into account the variation of the equivalent width over the stellar surface. Their results show quite large deviations from the radial velocity curve of the assumed circular orbit. The distortion effect may well introduce a spurious observed eccentricity, which is not really present in the orbit, masking the distortion effect for a large part.

Milgrom and Avni (1976) have presented arguments, showing that also orbital parameters as derived from the pulse arrival-time data for Vela X-1 may be influenced by systematic effects. Due to the relatively long pulse period the X-rays reflected by the surface of the primary do not average out to form a time-independent signal, and in this way cause a systematic distortion of the shape of the
The shape of the primary is given by a Lagrangian equipotential surface, i.e. we assume co-rotation. Also the heating of one side of the primary by the absorption of X-rays from the compact companion has not been taken into account. In the case of Vela X-1 the latter approximation is justified, as the ratio of the incident X-ray flux to the optical flux is less than $10^{-4}$.

2. The effective temperature and gravity acceleration vary together across the stellar surface according to Von Zeipel's law. The absolute values of the mean temperature are set by the luminosity of the star, whose size is fixed by spectroscopic and photometric data.

3. Numerical integration over the visible part of the stellar surface is performed using a network of 1730 points, distributed uniformly over the surface. Quantities needed at each such point (e.g. continuum intensity, limb-darkening coefficient, line strength) are obtained by interpolation in pre-calculated tables, which were made using the grid of model atmospheres of Kurucz, Peytremann and Avrett (1972; KPA).

4. In the calculation of continuum and line data we have throughout assumed local thermodynamic equilibrium.

For the KPA models with $12000 \leq T_{\text{eff}} \leq 35000$ K profiles of the lines O ii $\lambda$3467, Si iv $\lambda$4089 and He i $\lambda$4387 were calculated at a number of $\mu$-values ($\mu = \cos \theta$, $\theta$ being the angle at which the radiation emerges from the stellar atmosphere). For the calculation of the ionization and excitation equilibria through the model atmosphere we used subroutines from Kurucz's computer programme ATLAS (1973). We assumed a depth-independent microturbulent velocity of 10 km s$^{-1}$. In the calculation of the damping constant we included radiative damping (dominant at the low gravity values applicable to HD 77581) and damping due to collisions with electrons (quadratic Stark broadening). The relevant data for the radiative damping were taken from Wiese et al. (1966, 1969) and for the collisional damping from Griem (1974).

b) Check on the Reliability of the Model Calculation

A check on the basic correctness of the calculated profiles can be obtained from a comparison with observational data. We made such a comparison for the case of the Si iv $\lambda$4089 line. The observed values of the equivalent widths were taken from the compilation made by Kamp (1973). As stellar equivalent widths refer to the total flux, and not to specific intensity, our calculated equivalent widths—from the KPA models—refer to the total flux. In Figure 1 we show the result of the comparison. The observations have been made for main-sequence stars; therefore, we made the comparison for the log$g=4.5$ models. The transformation from spectral type to effective temperature was made using the calibration given by Code (1975). The variation with effective temperature of the equivalent width of the Si iv $\lambda$4089 line appears to be well reproduced by our calculations. Not too much weight should be attached to the agreement between the values of the equivalent widths themselves, as these clearly depend strongly on the assumed microturbulence, which in this case happened to have been chosen fortunately.

The non-LTE calculations of Kamp (1976) for this line enable us to get an impression of the influence of deviations from LTE on the results. In Figure 2 we represent, as a function of effective temperature, the ratio of the equivalent widths for the cases of NLTE and LTE. For the log$g=4$ models the ratio hardly changes with effective temperature. In the case of the log$g=3$ models, however, the temperature dependence of the ratio is appreciable. As the gravity of HD 77581 is not far from
log\(g = 3\) the use of LTE input data in the calculations of radial velocity curves will underestimate the effect of line strength variations across the surface of the distorted primary star. It is clear that the assumption of LTE limits the scope of the present results. However, as we have made the calculations for lines with very different temperature behaviour (see Fig. 3), we think that our results give a good representation of the effects of the relative variations in strength of different lines across the stellar surface, and are at least indicative of what is to be expected from the distortion of the primary.

c) Computation of the Theoretical Radial Velocity Curves

From a table of line profiles (35 KPA models, 12 \(\mu\)-values) we have constructed simple ad hoc interpolation subroutines giving:

1. Central line depth for perpendicularly emerging radiation as a function of \(T_{\text{eff}}\) and \(\log g\).

2. The limb-darkening law for the continuum intensity, and for the product continuum intensity times central line depth (a function of \(T_{\text{eff}}\) and \(\lambda\)).

3. A line-shape function, depending only on the central line depth; this approximation is not too bad, as the damping wings are not strongly developed in the spectra of low-gravity stars like HD 77581.

A composite line profile is then obtained by adding the contributions to the total flux of all network points on the stellar surface (visible to the observer) at a number of closely spaced wavelengths (\(\Delta \lambda = 0.2 \, \text{Å}\))

\[
F_{\lambda} = \sum_{i} I_{i}(0, \mu_{i}) \mu_{i} \Delta \sigma_{i}.
\]

Each such contribution is shifted in wavelength due to the radial velocity of the surface element, arising from orbital motion and the assumed co-rotation of the primary

\[
I_{i}(0, \mu_{i}) = I_{i}(0, 0) L_{1}^{1}(\mu_{i}) - L_{i}(0, 0) r_{i}(0) L_{2}^{2}(\mu_{i}) S(\lambda - \lambda_{0}(1 - v_{i}/c))
\]

In these expressions \(i\) denotes the surface elements, \(\Delta \sigma_{i}\) is their size, \(\mu_{i}\) is the cosine of the angle between the normal to the local stellar surface and the line of sight, \(L_{1}^{1}(\mu_{i})\) is the limb-darkening coefficient for continuum intensity, \(r_{i}(\mu)\) is the central depth of the spectral line, \(L_{2}^{2}(\mu)\) is the limb-darkening coefficient for the product \(I_{i}(0, \mu_{i}) r_{i}(\mu)\), and \(v_{i}\) is the radial velocity of surface element \(i\). The functions \(L_{1}^{1}(\mu)\) and \(L_{2}^{2}(\mu)\) are also dependent on wavelength and effective temperature. \(S(\Delta \lambda)\) is the line-shape function.

The net radial velocity of the primary has been obtained from the expression

\[
V_{\text{rad}} = c/\lambda_{0} \int \frac{r(\lambda - \lambda_{0})(\lambda - \lambda_{0}) \text{d} \lambda}{\int r(\lambda - \lambda_{0}) \text{d} \lambda}.
\]

\(\lambda_{0}\) is the rest wavelength of the spectral line, \(r(\lambda - \lambda_{0})\) the relative depth at wavelength \(\lambda\).

We performed the calculation of the radial velocity curves for a model, representing the Vela X-1 system, with a primary mass of \(21.2 \, M_{\odot}\) and a mass ratio \(q = 0.076\) (van Paradijs et al., 1976). As the luminosity is not well known due to the uncertain distance, we did the calculations for three values of the luminosity, corresponding to absolute bolometric magnitudes \(M_{\text{bol}} = -8.4, -9.0\) and \(-9.6\). An important quantity determining the final results is the amount of distortion of the primary. In the Roche geometry this distortion is conveniently described by the dimensionless potential parameter \(\Omega\) (Kopal, 1959). From the analysis of photometric variations of HD 77581 Avni and Bahcall (1975) and Zuiderwijk et al. (1976) have concluded that the star is nearly completely filling its critical Roche lobe. We have done the calculations for three values of \(\Omega\), one of which corresponds to the critical Roche surface. For the inclination \(i\) we adopted the values 90° and 75°, in accordance with the results from optical and X-ray brightness variations (Avni and Bahcall, 1975; Zuiderwijk et al., 1976).
Results, Discussion and Conclusions

Some results of the calculations are summarized in Figures 4–8. In Figure 4 we show the apparent radial velocity curve for the Si IV λ4089 line in the case that the Roche lobe is completely filled, together with its variation with the luminosity of the primary. For all values of the luminosity a hump appears on the radial velocity curve just before and after phase 0.0 (phase 0.5 taken at X-ray eclipse), as was also found by Wilson and Sofia (1976). This hump is caused by the dominant contribution of the relatively bright back-end side of the primary, which has a radial velocity amplitude much larger than that of the center of mass of the star, and is rapidly approaching us after X-ray conjunction (phase 0.0). After quadrature the radial velocity is somewhat smaller than the orbital velocity, due to the larger apparent surface on the side of the star facing the compact companion, a factor which outweighs the temperature difference.

For increasing values of the luminosity the hump on the curve becomes less pronounced. This is due to the fact that the effective temperature at each point of the surface is raised (the size of the star remains fixed). This causes the variation of the equivalent width over the surface to shift to another portion of the temperature-equivalent width relationship of the Si IV line. As the increase in equivalent width is largest on the cooler side of the star, the contrast in the relative contributions of different parts of the stellar surface is decreased.

In Figure 5 we show the same result for the O I λλ367 line. This line reaches its maximum strength at a lower value of the effective temperature than the Si IV λ4089 line. At the highest luminosity value the hotter parts of the stellar surface fall already on the decreasing branch of the temperature-equivalent width relationship. This is
reflected in the theoretical radial velocity curve by the disappearance of the hump at high luminosity, and a suggestion of a reversal in the phase dependence of the appearance of a hump, due to the fact that the line profile is dominated by contributions from the cooler part of the stellar surface.

The effect of varying the degree of filling of the Roche lobe is illustrated in Figures 6 and 7, where the radial velocity curves for Si iv λ4089 and O ii are shown for different values of Ω. For Si iv the spike decreases strongly when the star becomes somewhat smaller than its Roche lobe, although the asymmetry of the radial velocity curve is still present. In the case of O ii λ4367 the hump disappears completely, and the radial velocity curve becomes quite symmetrical. These differences show that for a quantitative discussion of these systematic effects the variation in equivalent width of the spectra! line over the stellar surface should certainly be taken into account. Changing the inclination of the orbital plane from 90° to 75° results in a softening of the spike (see Fig. 8). This can be understood as a result of the moderating effect due to the radiation from the constantly visible relatively hot pole of the star on the contrast between the front and back-end side.

It would seem that the appearance of a spike on the radial velocity curve would make it in principle possible to distinguish between the asymmetry in radial velocity produced by the tidal distortion, from that due to orbital eccentricity. Also the differences in the behaviour of the radial velocity curves of lines with a different equivalent width versus temperature dependence would add to such a distinction. We have investigated this point by analyzing the theoretical radial velocity curves for orbital solutions, using the computer programme ORBIT, written by one of us (RT). This programme is based on a global least squares fit to input radial velocities, and will be described elsewhere (Takens, 1976). Of special interest are the eccentricity e and the velocity amplitude K, as these are directly related to the asymmetry of the radial velocity curve and, hence, to the mass determination of the compact object. In Table 1 we present the results of these solutions for e and K. Inspection of this Table shows that in some cases large spurious values of the eccentricity can be caused by the distortion of the primary, and furthermore, that the radial velocity amplitude may be increased by up to 30%. Due to the strong sensitivity of these results to the assumed luminosity it seems premature to quote best values for the spurious eccentricity and for a correction factor to be applied to the observed radial velocity amplitude.

The possibility of proving the existence of a spike in the radial velocity curve will strongly depend on the deviations of this curve from a fitted orbital solution. These deviations should show a systematic variation with phase. A typical example of what may be expected for a large spike on the radial velocity curve is shown in Figure 9. It appears that even for a large distortion effect the amplitude of the deviations never becomes larger than about 3 km s^{-1}; in most cases calculated here they are smaller than 2 km s^{-1}. It seems that these phase dependent deviations are on the verge of detectability for the following reasons.
orbital parameters, obtained from different ions seems to
van Paradijs et al., 1976b).
night-to-night variations with an amplitude of a few
night are sometimes systematically shifted due to erratic
many plates taken within a short time interval during one
rather rapid variation with phase of the deviations.
3. Well determined average radial velocities using
many plates taken within a short time interval during one
night are sometimes systematically shifted due to erratic
night-to-night variations with an amplitude of a few
km s$^{-1}$ (van Paradijs et al., 1976b).

Investigation of systematic differences between the
orbital parameters, obtained from different ions seems to
be a more promising way to discover the presence of the
distortion effect in radial velocity data. Again, the
approximations used in the present calculations (es-
pecially that of LTE), and the strong sensitivity of the
results to variations in the system parameters (e.g.
luminosity of the primary) make it premature to consider
the numerical results derived here as more than just
examples. However, the following general features of the
present results, which may be checked observationally,
are worth mentioning.

1. Their amplitude is small.
2. A very good phase coverage is needed to follow the
rather rapid variation with phase of the deviations.
3. Well determined average radial velocities using
many plates taken within a short time interval during one
night are sometimes systematically shifted due to erratic
night-to-night variations with an amplitude of a few
km s$^{-1}$ (van Paradijs et al., 1976b).

Table 1. (Spurious) orbital parameters for distorted radial velocity
curves (circular orbit assumed)

<table>
<thead>
<tr>
<th>Eccentricity, $i = 90^\circ$</th>
<th>$M_{bol} =$</th>
<th>-8.4</th>
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* The circular velocity amplitude of the center of mass of the primary
equals 20.17 ($i = 90^\circ$) or 19.48 ($i = 75^\circ$) km s$^{-1}$

Fig. 9. Deviation between the theoretical radial velocity curve and the
fitted orbital solution, as a function of phase. The theoretical curve is
for Si IV A4089, $M_{bol} = -9.6$, $\Omega = 1.885$ and $i = 90^\circ$ (see Fig. 4)

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CDC Cyber-73 of the Academic Computing Centre Amsterdam
(SARA).

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J. van Paradijs et al.: Effects of Distortion on the Radial Velocity Curve of HD 77581

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II C


J. VAN PARADIJS, E.J. ZUIDERWIJK, R.J. TAKENS, G. HAMMERSCHLAG-HENSBERGE
Astronomical Institute of the University of Amsterdam, The Netherlands
and
European Southern Observatory, La Silla, Chile

E.P.J. VAN DEN HEUVEL
Astronomical Institute of the University of Amsterdam and
European Southern Observatory, La Silla, Chile,
Astrophysical Institute of the Free University of Brussels, Belgium
and
C. DE LOORE
Astrophysical Institute of the Free University of Brussels, Belgium

Received March 28, 1977

Radial velocity measurements are presented of the B0.5 Ib supergiant HD 77581, the optical counterpart of the binary X-ray system 3U0900-40 (Vela X-1).

A description is given of a new computer programme for the determination of orbital parameters of spectroscopic binaries from both radial velocity and pulse-time delay data. New orbital parameters for the Vela X-1 system were determined from spectral lines of different ions as measured on 82 high-dispersion spectrograms.

The results for all lines together yield:

\[ P = 8.9681 \pm 0.0016 \text{ days} \]
\[ e = 0.136 \pm 0.046 \]
\[ K = 21.75 \pm 1.15 \text{ km s}^{-1} \]
\[ \omega = 355 \pm 20^\circ \]

This leads to the following mass parameters:

\[ m_x \sin^3 i = 1.67 \pm 0.12 \text{ m}_e \]
\[ m_\text{tot} \sin^3 i = 20.5 \pm 0.9 \text{ m}_e \]

All quoted errors are 1\(\sigma\).

The differences between the orbital elements obtained from lines of different ions are in some cases not insignificant.

The small discrepancy between the orbital parameters of HD 77581 and those derived from X-ray data is still within the range expected from the distortion of the radial velocity curve of the primary due to its tidal and rotational deformation.

Clear evidence is found for correlated night-to-night variations of the radial velocity with an amplitude of up to 5 km s\(^{-1}\).

The implications of these non-orbital effects on the mass determination are discussed.

Key words: X-ray binaries - radial velocities - supergiants - orbital solutions for spectroscopic binaries

I. INTRODUCTION

Several X-ray binaries contain a pulsating X-ray source. From the optical spectrum and the Doppler delays of the arrival times of the X-ray pulses it is possible, in principle, to derive the radial velocity variations of both components in the system. Of these "double-lined" X-ray binaries the systems Vela X-1 and SMC X-1 are the only ones for which at present reasonably good classical mass determinations have been made. (van Paradijs et al. 1976, Rappaport et al. 1976, Hutchings et al. 1977). For Vela X-1 van Paradijs et al. (1976) found masses for both components which are in reasonable agreement with expectations generated from the theory of stellar evolution. The mass of the primary (21.2 m\(_e\)) may be somewhat low for its spectral type and luminosity class (B0.5 Ib). For the compact object a mass of 1.61 m\(_e\) was found, consistent with the theoretical upper mass limit of non-rotating neutron stars (Malone et al. 1975, Bowers et al. 1976). The radial velocity variation of the compact object was determined by Rappaport et al. (1976). Radial velocity data for the primary star were obtained by Hiltner et al. (1972), Zuiderwijk et al. (1974), Hutchings (1974), Wallerstein (1974), Petro and Hiltner (1974) and van Paradijs et al. (1976). It is known that non-orbital effects...
may affect the radial velocity curve of the primary, such as gaseous streams in the system and the deformation of the primary due to rotation and tidal forces. In order to improve the accuracy of the radial velocity data of HD 77581 and to investigate the possible presence of the distortion effect predicted by van Paradijs et al. (1977), and to possibly correct for it, we analyzed new spectrograms obtained during two recent observing runs at the European Southern Observatory, which increased the total number of plates available for analysis from 26 to 92.

2. OBSERVATIONS AND REDUCTION

This radial velocity study of HD 77581 is based on 92 blue spectrograms taken with the 152 cm spectrographic telescope of the European Southern Observatory. All spectrograms are on Kodak IIaO plates, developed in MWP-2 under standard conditions. The plates have reciprocal dispersions of 12 and 20 Å/mm, and a useful wavelength range from \( \lambda = 3700 \) to 4900 Å. With very few exceptions all spectrograms were widened to at least 0.5 mm. The observations were performed in five observing runs between April 1973 and May 1976. Basic data on the spectrograms are given in table 1. One of us (JvP) measured spectral line positions on all plates in three separate measuring runs, with the Grant Comparator of the Kapteyn Astronomical Laboratory (University of Groningen). In the last two runs a number of plates were measured for a second time, in order to obtain information on the accuracy of the individual measurements. The spectral line positions have been converted into wavelengths using a cubic dispersion relation, based on the positions of 32 lines in an Fe-arc comparison spectrum.

The accuracy of the wavelength determination of these Fe comparison lines is typically 0.01 Å.

Stellar lines from the following spectra were measured: HI, HeI, NII, NIII, OII, SiIII and SiIV. Table 2 lists the spectral lines used in the radial velocity study.

The position of a line was always determined by setting on the slopes of the line profile, each measurement being the average of two settings. The plates were always measured in the same direction. As the lines in the spectrum of HD 77581 are rather wide due to stellar rotation this setting on the line position was sometimes difficult, especially for the fainter lines.

The heliocentric radial velocities of the individual absorption lines are given in table 3. When a spectrogram was measured more than once the average value of the derived velocities is given.

2.1. Accuracy of the Radial Velocity Data

We have estimated the accuracy of the radial velocity of a single spectral line in two ways.

First we compared the results for the plates which were measured twice. We found that the mean value \( \Delta \) per plate of the differences \( \delta \) between the two results for one line vary from 0.2 to 13.6 kms\(^{-1}\). A typical value thus found is \( \Delta = \frac{\sum \Delta^2}{N} = 7.2 \) kms\(^{-1}\). The summation extends over all plates that were measured twice, \( N \) is their total number.

This corresponds to a typical accuracy for the average radial velocity as obtained from one plate of \( \sqrt[\sqrt{2}]{2 \times 7.2} = 5.1 \) kms\(^{-1}\). The standard deviation \( \sigma \) of the distribution of the differences \( \delta \) per plate varies between 6.4 and 22.3 kms\(^{-1}\), with a typical value \( \sigma = \sqrt{\frac{\sum \sigma^2}{N}} = 12.6 \) kms\(^{-1}\). Again the summation extends over all plates measured twice. From this we get a typical error in the measurement of a single line of \( \sqrt[\sqrt{2}]{2 \times 12.6} = 8.9 \) kms\(^{-1}\).

Another method to obtain information on the accuracy of the radial velocity from a single line measurement involves the comparison (for all plates) of the results for a particular line with the average radial velocity of the plate as derived from all lines. The standard deviation of this distribution gives a good estimate of the accuracy of a single velocity measurement.
For different lines these standard deviations range from 7.52 to 19.7 km s\(^{-1}\). These values are generally larger than those obtained in the first estimate, as not only the accuracy of a line measurement on one plate is involved but also variations from plate to plate. It is this determination of the accuracy of the radial velocity from a single line which will be used below in the estimate of the accuracy of the mean radial velocity.

### 2.2 Mean Radial Velocities per Plate

For each spectrogram we determined the mean radial velocity for the lines of HI, HeI, OII, SiIII and SiIV separately, and for all lines together (except the hydrogen lines). We noticed that for some individual lines the systematic deviations from the mean are quite large, up to 20 km s\(^{-1}\). A comparison with the detailed line identification list of e Ori (B0Ia) (Lamers 1972) shows that a reasonable explanation for these large systematic deviations is the disturbance of the line profile by a neighbouring spectral line of comparable strength.

In the calculation of the final mean velocity of a plate we corrected each line for the systematic deviations. This is a permissible procedure, as the mean error of these systematic deviations, averaged over all plates (1–2 km s\(^{-1}\)) is much smaller than the error in the results for an individual line.

In this way we obtain a better determination of the accuracy of the mean radial velocities per plate, not affected by systematic effects (such as whether or not a particular line has been included in the average). We estimated the accuracy \(\sigma\) of such a mean radial velocity per plate from the expression:

\[
\sigma = \frac{S^2 + PS\sqrt{N} + P^2/N}{S + P\sqrt{N}}
\]  

(1)

In this expression \(S\) is the standard deviation of the distribution of individual radial velocities and \(P\) is the mean accuracy of an individual radial velocity measurement per line: \(P = \sqrt{\sum P_i^2/N}\), where the summation extends over all lines in the group over which the average is taken. These values of \(P\) are given in table 4.

Expression (1) for \(\sigma\) gives a realistic estimate of the accuracy of the mean radial velocity: for a small number of lines the errors in the individual line measurements dominate, whereas for a large number of lines the accuracy is controlled by the standard deviation of distribution of radial velocities of individual lines.

In this way we introduce a reasonable bottom value \(P\sqrt{N}\) and avoid unreasonably small errors in the case where only a small number of lines is available (e.g. for the two SiIV lines), which happens to give very concordant results.

In figures 1a to 1e the variation of the mean radial velocity as a function of phase is shown.

### 3. DETERMINATION OF THE ORBITAL ELEMENTS

#### 3.1. Method

The orbital elements, given in this paper were derived with a computer programme that searches for the best possible solutions from data with observational scatter and produces realistic error estimates.

The parameters derived by the programme are:

- \(P\): binary star period (days)
- \(e\): eccentricity
- \(\Pi\): periastron phase
- \(\bar{\omega}\): periastron position
- \(V_0\): system velocity (km s\(^{-1}\))
- \(K\): radial velocity amplitude (or delay amplitude) (km s\(^{-1}\))
- \(P_p\): pulsar period (seconds)
- \(\dot{P}_p\): time derivative of the pulsar period (only in delay mode)

In order to fix \(\Pi\) the programme needs, apart from the observational data, a zero point of phase. For (nearly) circular orbits only a combination of \(\Pi\) and \(\bar{\omega}\) is meaningful; this situation is properly recognized. The programme analyzes either radial velocities or pulse arrival-time delays. The basic equations are well known:
with
\[ V_r = V_0 + K_r \cos \omega + K_r \cos (v + \omega) \]  
(2)
\[ K_r = \frac{2\pi a \sin i}{P\sqrt{1 - e^2}} \]
where \( v \) and \( E \) are the true and eccentric anomaly, respectively. With given \( P \), \( \Pi \) and \( e \) the epoch of an observation is transformed to either \( v \) (for radial velocities) or \( E \) (for delays). Then a direct least squares solution is possible for the elements \( \omega \), \( K \) and \( V_0 \) (or delay zero point). The mathematical formalism of this solution is given in the appendix to this paper. This solution also gives a value for \( L \), defined by:
\[ L = \Sigma g_i (O - C)^2_i / \Sigma g_i \]  
(4)
The explicit expression for \( L \) can be differentiated directly with respect to \( P \), \( \Pi \) and \( e \). This gradient is used to find a minimum value of \( L \). Then the programme is used in a mapping mode, to find out whether the minimum is sufficiently unique.

In order to improve the performance of the programme, grad \( L \) is calculated with respect to a Riemannian geometry:
\[ ds^2 = \frac{de^2 + e^2 d\Pi^2}{(1 - e^2)} + \frac{\gamma}{P^2} dP^2 \]  
(5)
Reasons for this choice of the metric are:
1. The poor definition of \( \Pi \) at \( e = 0 \) does not inhibit rapid convergence.
2. The eccentricity never becomes negative. The jump in \( \Pi \) needed in this case arises naturally.
3. \( e = 1 \) is at infinite distance and cannot be reached.
4. The factor \( \gamma \) is a complicated function of \( K, L \) and the timespan of the observations, and is defined so as to make the scale of structure in the function \( L(P, \Pi, e) \) comparable in all directions.

The factor \( \gamma \) is checked at every step, but is adjusted only if the deviations become too large. The reason for this is that in that case the tables with information of previous steps (\( e, P, \) etc., and their gradients) have to be transformed.

It should be kept in mind, that the assumed geometry (equation 5) influences the path along which one reaches the final solution, but that the solution itself is independent of the geometry.

3.2. Error Estimates
The error estimates are based on \( L \). Let \( L_0 \) be the square sum of residuals for the final solution. Then one defines:
\[ L(p) = L_0 \left( 1 + \frac{p^2}{n - m} \right) \]  
(6)
Here \( n \) is the total number of observations, and \( m \) is the number of parameters determined in the final solution. All solutions giving \( L < L(p) \) fall within a \( q \) percent confidence limit. The relation between \( p \) and \( q \) is given by a \( \chi^2 \) distribution:
\[ 2^{k/2} \Gamma \left( \frac{k}{2} \right) q \frac{p^2}{100} = \int_0^{p^2} \frac{1}{u^{3/2}} \frac{1}{e^{-u/2}} du \]  
(7)
Here \( k \) is the number of interesting parameters. This method was derived by Avni (1976). The somewhat surprising dependence on the number of interesting parameters is illustrated in figure 2.

All errors quoted in this paper have been derived as single parameter errors. If one is interested in confidence limits, this is a questionable approach, in two respects:
1. One would like to give a confidence level to the eccentricity, as compared to the circular orbit. The value of \( m \) for the eccentric orbit is two higher than for the circular orbit.
The correction for this difference is always a questionable procedure.

2. The method is correct for linear problems; this means that the meaningfulness of one parameter does not depend on the value of another parameter within its confidence limits. In our problem this complication arises for example for $e$ and $\Pi$. For very small eccentricities $\Pi$ loses its meaning. Now one wishes to give limiting values of $e$ between which the true solution is expected to be located with 90 percent probability. Then somehow one has to count the possible solutions. If 10 values of $\Pi$ are counted for every $e$, then the circular solution (only one possibility) is counted 10 times. In order to avoid this, a straightforward solution would be to define a metric like equation (5), and to sample the possible solutions per volume element. It is clear, however, that the resulting confidence limits depend on the metric defined in parameter space. An extreme example of this dependence is equation (5) itself: an orbital solution with $L_0 \neq 0$ does not differ at any confidence level ($>0$) from a constant radial velocity. (This arises from the behaviour at $e=1$).

Our confidence limits have been derived by taking the extreme values of a quantity over the confidence region with $L<L(p)$. For $p=1$ we call this the 68 percent confidence limit, and for $p=1.645$ we call this the 90 percent confidence limit. Although three parameters ($P, e, K$) are needed in order to derive a $\sin i$, one is interested only in a $\sin i$ (i.e. one parameter) if a mass determination is made, so the single parameter confidence limits should be used.

3.3. Solutions for the Orbital Elements

Using the observed pulse-arrival times given by Rappaport et al. (1976) we have determined the orbital elements for the compact object. The results are given in table 5. This solution differs slightly from the original solution given by Rappaport et al. (1976), due to the inclusion of $P_p$ as one of the parameters of the system. The value $P_p$ derived here corresponds to a spin-up of the compact object, with a time scale $P_P/P_p = 1500$ years.

Using the radial velocity data for HD 77581 we have made a solution for the elements $P, e, \bar{\omega},$ and $K$ for the lines of HeI, OII, SiIII and SiIV separately, and for all lines together. In order to increase the homogeneity of the input data, we used the results from the 12 Å/mm plates only. Each plate gives us one average value of the radial velocity as determined above, which is weighted according to $g=1/\sigma^2$ (see equation (1)). It appears, however, that the radial velocities obtained from plates taken during one night do not deviate from the fitted orbital solution in a random fashion, but show correlated deviations. For this reason the treatment of all plates independently will give a wrong estimate of the errors. Therefore, we have lumped together the data obtained during one night, making a grand average for that night: $V^\prime=\Sigma g_i V_i/\Sigma g_i$, with weight $g_i=\Sigma g_i$.

This leaves us with 29 independent radial velocity data. In figure 1e we show the phase variation of these nightly averages (taken for all lines), together with the best-fit solution.

In table 6 we present the orbital elements $P, e, K$ and $\bar{\omega}$ for the different groups of lines, together with their error estimates. We did not use the hydrogen lines in obtaining the orbital elements, as some previous radial velocity studies of HD 77581 (Wallerstein 1974, van Paradijs et al. 1976) showed rather discordant results from these lines. The hydrogen lines are expected to be particularly sensitive to the presence of large-scale gas flows in the system, as contributions to the line profile can be made over a large volume of space by recombinations and cascade processes. Contrary to previous results, the present hydrogen line data furnish us with orbital elements which agree quite well with those from the heavier elements ($P=8.9683$ days; $e=0.189$; $K=23.50$ km s$^{-1}$; $\bar{\omega}=29^\circ$). This somewhat variable behaviour of the hydrogen line results may reflect long time-scale variations of the gas flow parameters in the Vela X–1 system. But perhaps a more mundane explanation in terms of statistical fluctuations is to be sought. The most obvious way to investigate this point is a long-term study of the time variation of the Ha profile in the spectrum of HD 77581.
4. POSSIBLE NON-ORBITAL CONTRIBUTIONS TO THE OBSERVED RADIAL VELOCITY VARIATIONS

Before using the orbital parameters in table 6 for a mass determination we have to investigate whether they represent real orbital motion, i.e. to what extent they may have been influenced by non-orbital effects on the radial velocity curve due, for example, to deformation of the primary, stellar wind, and erratic variations of the primary.

We will discuss each of these effects separately.

4.1. Effects of Deformation of the Primary

In a previous paper we presented a numerical study of the effect of the deformation of the primary on its apparent radial velocity curve (van Paradijs et al. 1977). Appreciable effects on the apparent eccentricity and the radial velocity amplitude were found for a reasonable range of system parameters. In the case of a circular orbit a spurious eccentricity of the apparent orbit can be produced, which may vary considerably from one ion to another and e.g. for an orbit based on lines of Si IV can be as high as 0.45. The apparent radial velocity amplitude can in some cases increase by up to 30 percent, thereby increasing the apparent mass of the compact object by approximately the same amount. These theoretical deviations of the apparent orbital motion strongly depend on the assumed parameters for the system, especially on the luminosity of the primary and its degree of filling of the critical Roche lobe. Independently of these quantities we find the following general theoretically predicted behaviour of the distortions of the radial velocity curve:

a) For both the eccentricity and the radial velocity amplitude the deviation from the real orbital values is largest for the Si IV lines and smallest for the He I lines; for the O II lines it is intermediate in between these extremes.

b) The apparent orbit as derived from He I lines is not much different from the real orbit; in this case the orbital parameters are not much affected by the distortion of the primary.

Inspection of the results in table 6 shows that the observational evidence for the presence of the predicted distortion effects is not very strong. Comparing the X-ray orbit and the optical orbit (as derived from all lines together) we find:

1) $\tilde{\omega}_{\text{opt}} - \tilde{\omega}_x = 166 \pm 25^\circ$.

2) The eccentricity of the orbit of the primary is slightly larger than that derived for the orbit of the neutron star. The X-ray radial velocity curve is expected to give a nearly undisturbed picture of the orbital velocity of the compact object (Milgrom and Avni 1976, Milgrom 1976).

3) The expected differences between the values of the eccentricity as derived from lines of He I, O II and Si IV do not show up. Also the radial velocity amplitudes do not clearly show the predicted effects. The amplitude derived from He I lines is somewhat smaller than those derived from both the O II and Si IV lines. However, the result for the Si III lines is unexpectedly deviating. Because Si III lines have about the same temperature sensitivity as the O II lines (Underhill 1966) we expect the apparent Si III radial velocity amplitude to agree with the O II amplitude. Contrary to these expectations the O II and Si IV lines give the same result for the amplitude $K$, whereas Si III deviates upward by some 4 kms$^{-1}$. Inspection of the radial velocity curve for Si III lines shows that this difference is due to large upward deviations of just a few plates around phase 0.0 (see figure 1b). In brief, the apparent distortion of the radial velocity curve does not show up to the predicted extent.

However, there is still much room to reconcile theory with the observations. Some of the possibilities are:

4.1.1. Distortion effects in an elliptic orbit

It is possible that the combination of an elliptic orbit with the distortion due to tidal deformation of the primary produces the observed results. In order to investigate this we calculated radial velocity curves for a deformed primary in an elliptic orbit with eccentricity $e = 0.15$, and $\tilde{\omega}$ running from 0° to 330° in
steps of 30°. We assumed instantaneous co-rotation of the primary and took the deformation correction to
that radial velocity equal to the value calculated for a circular orbit, at the same value of the true anomaly.
As the distortion effects on radial velocities in eccentric orbits are still unexplored, this seems an appropriate
first approximation for estimating these effects on the apparent radial velocity curve. The calculations show
that the relative values for the radial velocity amplitudes for lines of He I, OII and SiIV do not change appreciably by the introduction of a non-circular orbit.

In the case of He I the contribution of the deformation of the primary to the apparent eccentricity is small
compared with the orbital contribution. So the spurious eccentricity is to be considered as a small pertur-
bation of the orbital eccentricity. For typical values of the system parameters ($M_{bol} = -9.0$ and complete
Roche lobe filling) we obtain values of $e$ in the range 0.06 to 0.19. On the other hand, in the case of the
SiIV lines the distortion of the radial velocity curve dominates the effect of the orbital eccentricity. Starting
with a spurious eccentricity (for a circular orbit) of 0.36 we obtain values of $e$ in the range 0.25 to 0.48,
depending on $\omega$. We see, therefore, that due to the ellipticity of the orbit a significant decrease of the
spurious eccentricity is quite possible. There is a tendency also, to bring the values of $e$, as deduced from
lines of different elements, closer together than in the case of a circular orbit.

4.1.2. A modified “Barr effect”?

A further interesting result of these trial calculations is that due to the deformation of the primary the radial
velocity curve tends to have the asymmetric shape characteristic for orbits with $\omega \approx 90°$ (see van Paradijs
et al. 1977, figures 4-8). In the case of a small distortion of the radial velocity curve, e.g. for the He I lines,
the distribution of $\omega$ is peaked around 90° and 270°; for a strong distortion of the radial velocity curve like
for the SiIV lines $\omega$ never deviates more than 35° from 90°. This tendency resembles the classical Barr effect
(Batten 1973), but differs from it by producing an excess number of orbits, not with $\omega \approx 0°$, but with
$\omega \approx 90°$. However, we feel that the reality of the classical Barr effect is to be doubted. Using Batten's (1967)
catalogue of spectroscopic binary orbits, and following his advice by taking into account only orbits qualified
A, B or C, we find no trace at all of the classical Barr effect, but instead, a marginally significant excess of
the number of orbits with $\omega$ values around 90°. This, of course, is not a proof for the existence of a
modified Barr effect, rotated over 90°, but it certainly shows that one should be careful in using a large,
inhomogeneous set of data for deriving systematic effects such as Barr's.

4.1.3. Effects of the luminosity of the primary

From our calculations it appears that the observed results can be reconciled with the theoretical predictions
easier for large values of the primary's luminosity. In that case the spurious eccentricity due to the distortion
of the primary is so small as to be a small perturbation of the orbital parameters. This could then explain
why the observed values of the eccentricity tend to be only slightly larger than the X-ray eccentricity of
0.096, and are—within the observational accuracy—not much different from one element to another. For $M_{bol} = -9.6$ for example, we expect the observed eccentricity derived from the He I and OII lines not to
deviate from the real eccentricity by more than a few times 0.01. The eccentricity for the SiIV lines is
expected to be in the range from 0.15 to 0.35.

For this high value of the primary's luminosity the apparent radial velocity amplitude is not much affected. In the case of a circular orbit the He I lines yield a small underestimate of the amplitude $K$, by
between 0 and 10 percent, the OII lines yield an underestimate by between 0 and 8 percent and the SiIV
lines give values of $K$ between 99 and 108 percent of the true amplitude.

4.1.4. Deviation from co-rotation

Another possibility is a deviation of co-rotation of the primary, which is one of the basic assumptions
in our simulation of radial velocity curves. The observations available up to now give rotational velocities
from 90 to 135 kms$^{-1}$ (Wickramasinghe et al. 1974, Zuiderwijk et al. 1974), whereas the expected co-rotation
velocity is around 180 km s\(^{-1}\). As the distortion of the radial velocity curve is primarily due to phase dependent non-cancellation of the contributions of the different parts of the visible stellar surface to the average radial velocity, we expect a smaller distortion in the case of a slower rotation of the primary.

We conclude that although the agreement between theory and observation is not as expected, there is sufficient room in both the observations and in the assumptions entering the calculations, to accept their consistence.

4.2. Effects due to Stellar Wind

In many bright early-type stars a strong stellar wind has been observed, producing mass loss by up to \(10^{-6} m_\star\) per year (cf. Hutchings 1976). In case the wind is strong enough some spectral lines will be formed in the outflowing part of the atmosphere, thereby gaining an appreciable net shift in radial velocity. This effect is particularly visible in the radial velocities of the Balmer lines: the radial velocity increases along the Balmer sequence, and converges toward the stellar radial velocity for the higher Balmer lines ("Balmer progression").

When the stellar atmosphere is spherically symmetric no differences in the velocity stratification will exist in different directions. However, in case the wind parameters are not uniform across the stellar surface (which is not implausible for a deformed star) the Balmer progression may vary, and also other spectral lines may have a different radial velocity when observed from different directions. For a binary star this will result in a change of the radial velocity curve.

There are no indications for a really strong wind in the spectrum of HD 77581, but a small differential effect of a few km/sec is already of importance to the determination of \(K_{\text{opV}}\).

We have looked for indications of a non-isotropic stellar wind by comparing the radial velocities of individual Balmer lines with the mean value per plate. In ten phase intervals, each one tenth of a period wide, we averaged these differences for each line. We found no phase dependent deviations, indicative of a non-isotropic stellar wind, except for H\(\beta\). For this line a deviation of \(-15\) km s\(^{-1}\) (with a mean error of \(\pm 5\) km s\(^{-1}\)) was found near phase 0.5. Such a systematic shift in radial velocity was found before by Zuiderwijk et al. (1974).

The lines from the heavier ions, which we used for the determination of the spectroscopic orbit originate in much deeper layers than H\(\beta\), as shown by the radial velocity studies of stars with a strong stellar wind (see e.g. Hutchings 1975). As the deviation occurs near a phase where the orbital radial velocity is about zero, we expect the effect on the spectroscopic orbit, if any, to become visible in \(\omega\), and possibly in \(e\), but not in \(K_{\text{opV}}\). We conclude that non-isotropy of the stellar wind in HD 77581 is not present at a sufficiently significant level to influence the determination of the orbital parameters.

4.3. Erratic Variations of the Primary

We find in our data indications for correlated systematic variations of the radial velocity with respect to the average radial velocity curve.

— For the 13 nights in which more than one plate was obtained, eight nights give results which deviate by more than 1\(\sigma\) (mean error) from the average radial velocity curve. On statistical grounds about 4 are expected. These deviations are a property of the stellar velocity: they are not correlated with deviations of the radial velocity of the interstellar CaII \(K\) line. A plot of the average deviations for the interstellar line against the deviations of the stellar lines in one night reveals a scatter diagram.

— During the May 1976 observing run one complete orbital cycle of HD 77581 was covered, and after nine nights observations were made at the same orbital phase. The radial velocities for this phase (\(q \approx 0.03\)) are based on three and four plates respectively and differ by \(8.6 \pm 4.1\) km s\(^{-1}\), which seems significant.

Also, the photometric data published by Zuiderwijk et al. (1977) show clear indications for significant correlated variations of the brightness of HD 77581 during one night with respect to the average light curve. Inspection of their figure 1 shows that the brightness variations during one night are often quite large and systematic. These observations lead us to suspect that part of the photometric and radial velocity variations
of HD 77581 are due to a semi-regular pulsation of this star, as is also seen in many other early-type supergiants (Underhill 1966, Sterken 1977).

It is not a priori clear what will be the effect of the correlated variations on the orbital parameters. If we adopt the hypothesis that they are erratic on longer time scales, i.e. that the correlation length is not much longer than one orbital period, their influence on the average radial velocity curve should cancel, provided we have a sufficient amount of radial velocity data. The total number of independent radial velocity measurements (equal to the number of nights, so 29) is probably not large enough to feel completely safe in this respect, especially not since the amplitude of the correlated deviations is some 5 km/s, i.e. 25 percent of the orbital radial velocity amplitude.

In order to investigate the influence on the apparent radial velocity curve of correlated variations that are erratic on longer time scales, we performed the following simulation:

In each of the 29 independent radial velocity phase points we generated radial velocity values defined by the best fit solution for all lines plus a randomly chosen velocity deviation; for the amplitude of this random distribution a value of 5 km/s was taken. Each such set of 29 velocities was subjected to an orbital analysis. Taking together the results of 100 of these synthetic radial velocity curves we find that the average amplitude increases by only 0.04 km/s to 21.79 km/s, with a standard deviation of the distribution of K values of 0.8 km/s.

Also, the average eccentricity increases slightly from 0.136 to 0.143, with a standard deviation of 0.032. This small increase is to be expected since, for a circular orbit perturbed in this random fashion, we will always end up with an eccentricity ≥ 0. In the case of an eccentric orbit we also expect an increase, but one that is smaller for higher eccentricities.

We conclude that: a) the radial velocity amplitude is not significantly affected by the erratic variations; b) the observed eccentricity of 0.136 may quite well contain a contribution due to a superposed pulsation-like variation of the radial velocity.

4.4. Summary of the Possible Non-orbital Effects

In view of the many complicating factors it is not at present possible to make a really good estimate of the size of the individual non-orbital contributions to the observed orbital parameters. However, the absence of any indication for a strong systematic distortion of the radial velocity curve leads us to conclude that the amplitude of the radial velocity curve is not underestimated or overestimated by more than about 10 percent.

5. MASS DETERMINATION

From the orbital elements, obtained from all lines together (table 6) we obtain the following masses for the components in the Vela X-1 system:

\[ m_x \sin^3 i = 1.67 \pm 0.12 \, m_\odot \]
\[ m_{opt} \sin^3 i = 20.5 \pm 0.9 \, m_\odot \]

The errors are 1σ limits, determined as described above. This result implies a 2σ (95 percent confidence) lower limit of 1.43 m_\odot, which is larger than the Chandrasekhar upper mass limit for a white dwarf (Hamada and Salpeter 1961). This shows that the compact object is almost certainly a neutron star.

For an estimate of \( \sin i \) we use the results obtained by Avni and Bahcall (1976) and Zuiderwijk et al. (1977), which show that the inclination should certainly be larger than 74°, in order to be consistent with the optical light curve of HD 77581.

It is not easy to combine this estimate of the lower limit of \( i \) in a formal way with the errors found for \( m_x \sin^3 i \) and \( m_{opt} \sin^3 i \). If we assume either a uniform probability distribution for \( i \) in the interval from 74° to 90°, or one which varies as \( \sin^2 i \) (all directions of the system's angular momentum vector equally probable), we find in both cases an average value for \( \sin^3 i \) of 0.96. Taking this as the "best value" for...
\[ \sin^3 i \text{ we obtain as "best values" for the masses: } m_x = 1.74 \ m_\odot \text{ and } m_{\text{opt}} = 21.3 \ m_\odot. \]

The allowed range in inclination angle gives a pseudo 1σ range for \( m_x \) between 1.55 and 1.87 \( m_\odot \), and for \( m_{\text{opt}} \) between 19.5 and 22.4 \( m_\odot \).

The errors given here have been formally derived from a least squares analysis of observational data. The results of the discussion of the orbital parameters show that there are no strong indications that the present determination of \( m_x \) contains a systematic error larger than 10 percent upward or downward. The effect on \( m_{\text{opt}} \) of possible systematic errors in the optical data is, of course, much smaller.

The present result for \( m_x \) is consistent with theoretically predicted values of the upper mass limit for neutron stars, and a stiff equation of state for high-density matter is favoured (Bowers \textit{et al.} 1976, Malone \textit{et al.} 1976, Pandharipande \textit{et al.} 1976). The mass of the early-type component is rather low, as compared with the value expected from stellar evolutionary tracks, and from the luminosity of the star. For the bolometric magnitude of HD 77581 a value \( -9.00 \pm 0.75 \) has been derived (van Paradijs \textit{et al.} 1976). According to recent evolutionary tracks by Stothers and Chin (1977) for blue supergiants this corresponds to masses between 30 and 60 \( m_\odot \).

A similar type of undermassiveness has also been found for Sk 160, the optical counterpart of SMC X-1 (Hutchings \textit{et al.} 1977), and for Krzminski's star, the optical counterpart of Cen X-3 (Conti 1977). A reasonable explanation for this phenomenon is a previous mass loss, probably by stellar wind (Hutchings 1976).

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REFERENCES


APPENDIX

A number of measurements \( x_i \) and \( y_i \) are given, with weight \( g_i \). We need a least squares solution for \( \alpha, \beta \) and \( \gamma \), according to the relation

\[
y = \alpha + \beta \sin(x - \gamma)
\]

Define the following quantities:

\[
A = \frac{\sum y_i}{\sum g_i} \\
B = \frac{\sum g_i y_i \sin x_i}{\sum g_i} \\
C = \frac{\sum g_i \sin 2x_i}{\sum g_i} \\
D = \frac{\sum g_i \cos 2x_i}{\sum g_i} \\
E = \frac{\sum g_i \sin x_i}{\sum g_i} \\
F = \frac{\sum g_i \cos x_i}{\sum g_i} \\
Y = \frac{\sum g_i y_i}{\sum g_i} \\
Z = \frac{\sum g_i \gamma_i}{\sum g_i}
\]

The solution is in terms of these sums:

\[
tg \gamma = \frac{2E(AF - BE) - [AC - B(1 - D)] + Y[CE - F(1 - D)]}{2F(AF - BE) + [BC - A(1 + D)] - Y[CF - E(1 + D)]}
\]

\[
\beta = \frac{2(AF - BE)}{[E(1 + D) - FC] \sin \gamma + [F(1 - D) - EC] \cos \gamma}
\]

\[
\alpha = Y + \beta(F \sin \gamma - E \cos \gamma)
\]

The simplest way to derive these equations is to transform equation (A1) into a linear equation in \( x, \beta \sin \gamma \) and \( \beta \cos \gamma \). The ratio of \( \beta \sin \gamma \) and \( \beta \cos \gamma \) gives equation (A3). Then equations (A4) and (A5) are simply derived.

The value of \( L_0 \) for the solution is:

\[
L_0 = \frac{[\sum g_i (O - C)^2 / \sum g_i]}{\sum g_i} = Z - \alpha Y + \beta(B \sin \gamma - A \cos \gamma)
\]

In order to find confidence regions for this solution we use:

\[
L = [\sum g_i (y_i - \alpha - \beta(x_i - \gamma))^2] / \sum g_i
\]

In our applications \( \alpha \) is always an "uninteresting" parameter, so in determining the confidence region we maintain \( \partial L / \partial \alpha = 0 \) by eliminating \( \alpha \) between equations (A5) and (A7). The result is:

\[
\mu = \frac{L - L_0}{L_0} = -1 + \frac{1}{L_0} (Z - Y^2) + \frac{2B}{L_0} [(B - FY) \sin \gamma - (A - EY) \cos \gamma] + \frac{B^2}{2L_0} [(1 - E^2 - F^2) + (2EF - C) \sin 2\gamma] + (F^2 - E^2 - D) \cos 2\gamma
\]

or:

\[
\mu = a_1 + \beta(a_2 \sin \gamma + a_3 \cos \gamma) + \beta^2(a_4 + a_5 \sin 2\gamma + a_6 \cos 2\gamma)
\]

\( \mu \) can be identified directly with \( p^2/(n - m) \) in equation (6).
The limiting values of $\gamma$ in the confidence region follow from:
\begin{equation}
\tan \gamma = \frac{4a_2(a_1 - \mu) - a_2a_3 \pm 4 \sqrt{(a_2^2 - a_3^2 - a_6^2)(a_1 - \mu)^2}}{a_2^2 + 4(a_6 - a_4)(a_1 - \mu)}
\end{equation}
(A10)

The surprising fact that this solution contains $a_1$ and $\mu$ in other combinations than $(a_1 - \mu)$ only, is a consequence of a relation between the coefficients of equation (A9), viz.:
\begin{equation}
2a_2a_3a_5 + a_3a_6 - a_2a_6 - a_3a_4 - a_2a_4 + 4a_1(a_2^2 - a_3^2 - a_6^2) = 0
\end{equation}
(A11)

The extreme values of $\beta$ and other functions of $\beta$ and $\gamma$ in the confidence region are determined through an iterative interpolation procedure.

Figure 2 A problem determined by two parameters is considered. If one is interested only in the limits on the value of $a$, one needs the single-parameter confidence limit. This is the coarsely hatched area: $b$ can have any value. If one is interested in $a$ and $b$ simultaneously, the extreme values of $a$ and $b$ are determined by a confidence region, bounded in the direction of both $a$ and $b$, but with the same total probability inside. This gives the finely hatched area. Now the confidence limit on $a$ has increased and the value of $L(p)$ increases with the number of interesting parameters.
Table 1 Observational data

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Table 2 L.s used for radial velocity measurements

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Table 3 Adopted accuracy p of a radial velocity measurement of a single spectral line

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<th>Spectral line</th>
<th>p (km/sec)</th>
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<tr>
<td>all lines (except H)</td>
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Table 5 Orbital parameters for 3U0900-40*

| P | 282.89138 |
| P / P | 1500 ± 750 years |
| e | 0.096 ± 0.019 |
| w | 161° ± 15° |
| a sin i | (32.83 ± 0.45) 10 km |
| i | 0.111 ± 0.048 |

* Constraints: P=8.9681±0.0016 (this paper) mid-eclipse at JD 2442611.89 ± 0.057 (Rappaport et al. 1976). Errors given are 1σ.

Table 6 Orbital parameters for HD 77581*

<table>
<thead>
<tr>
<th>O II</th>
<th>Si III</th>
<th>Si IV</th>
<th>He I</th>
<th>all lines</th>
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<tbody>
<tr>
<td>period (days)</td>
<td>8.9682 ± 0.0022</td>
<td>8.9699 ± 0.0027</td>
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<tr>
<td>eccentricity</td>
<td>&lt; 0.105</td>
<td>0.171 ± 0.055</td>
<td>0.145 ± 0.037</td>
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<tr>
<td>K (km/s)</td>
<td>22.81 ± 1.75</td>
<td>26.22 ± 1.92</td>
<td>22.35 ± 0.85</td>
<td>20.77 ± 1.35</td>
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<td>g</td>
<td>undefined</td>
<td>310° ± 26°</td>
<td>21° ± 15°</td>
<td>16° ± 20°</td>
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</table>

* Errors given are 1σ.

Notes to table 1:
* plate number F series: 20 Â/mm
G series: 12 Â/mm
* phase calculated according to (JD-2441446.54)/8.9681

H: G. Hammerschlag
VDH: E.P.J. van den Heuvel
L: C. de Loore
P: J. van Paradijs

The Orbit and Masses of 3U0900-40/HD 77581

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The Orbit and Masses of 3U0900-40/HD 77581

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**Notes:**

- The table continues with identical data entries, indicating a pattern or repeated measurements.
- The entries are likely related to physical measurements, such as masses or orbital parameters, in a scientific context.
- The documentation suggests these are precise measurements, possibly for astronomical purposes, given the context of 3U0900-40/HD 77581.
Figure 1a  Variation of the heliocentric radial velocity of HD 77581 with X-ray phase (phase 0.0 corresponding to X-ray eclipse), as obtained from the HeI lines. Each point represents the average radial velocity for one plate. Only 12 Å/mm results have been plotted in this figure. The error bars denote the mean error of the radial velocity determination, as discussed in the text.

Figure 1b  Same as figure 1a for lines of OII.

Figure 1c  Same as figure 1a for lines of SiIII.

Figure 1d  Same as figure 1a for lines of SiIV.
Figure 1e  Same as figure 1a for all lines together (except HI lines). Each point gives the average for all plates, obtained during one night. The curve drawn through the points represents the best-fit solution to the data points.

Figure 1f  The same as figure 1a, for lines of HI.