Resonant excess quantum noise in lasers with mixed guiding

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We show experimentally that the combination of soft-edged gain and index guiding can lead to resonant excess quantum noise. Resonances with excess noise factors close to 100 are observed in end-pumped Nd$^{3+}$:YVO$_4$ lasers for cavity lengths in which two modes experience similar gain. An associated increase in the relaxation oscillation damping rate demonstrates that the fluctuation enhancement is indeed caused by excess quantum noise and not by dynamic instabilities. © 2003 Optical Society of America

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Mixed guiding, the combination of gain and index guiding, is a commonly occuring feature of many laser systems. A typical example of mixed guiding is the end-pumped planar microchip cavity, in which a narrowly focused pump constitutes a gain guide and the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror. In experiments with the index guide is induced by a transverse thermal gradient or a curvedmirror. In experiments with the index guide is induced by a transverse thermal gradient or a curved mirror.

Excess quantum noise, which manifests itself as an enhancement of spontaneous emission into the lasing mode, is caused by the nonorthogonality of the laser eigenmodes. This mode nonorthogonality is often caused by the combination of gain and index guides, i.e., dissipative and conservative spatial guiding, respectively. Typical experiments with excess quantum noise have involved so-called unstable cavities with the light exiting the cavity by bypassing the smaller of two mirrors. This hard-edge configuration allows for large transverse variation in losses, because only light incident on the small mirror is reflected, whereas all other light is lost. Such experiments have led to the observation of record-high excess-noise factors ($\approx 330$). In this Letter we surprisingly reach excess-noise factors of similar magnitude in experiments with a stable, soft-edged cavity with smooth gain and index profiles, thereby confirming recent predictions. Because of the resonant nature of the excess noise, a direct quantitative comparison with the model in Ref. 5 (extended to include frequency detuning) would require the laser parameters to be determined to very high accuracy; this is beyond the scope of this Letter. The noise enhancements resulting from our experiments are observed as intensity fluctuations, unlike in typical excess quantum noise experiments in which increased phase fluctuations are measured. Such an approach is possible because of the intrinsic strength and the slowness of the intensity fluctuations in the Nd$^{3+}$:YVO$_4$ laser.

The inset in Fig. 1 shows our cavity; it is constructed from a 0.23(3)-mm Nd$^{3+}$:YVO$_4$ crystal with a coating that is highly reflective at 1064 nm and antireflective at 809 nm on one crystal side but antireflective at 1064 nm on the opposite side. A concave mirror with a curvature of 200 mm faces the second facet. Between the crystal and the mirror there is an air gap of $\approx 50 \mu m$. This configuration can be mapped on a planar cavity with a longitudinally homogeneous gain and index guides. For the above configuration and a crystal refractive index of $n_c = 2.165$ we calculate an optical path length $l_{opt} = 550 \mu m$, an effective length of $L = 290 \mu m$, and a base refractive index of $n_0 = 1.87$. The atomic-doping level is 1% of Nd$^{3+}$. The gain guide, which has an approximately Gaussian profile $[\exp(-2\sigma^2/w^2)]$, has a width of $w_g = 17(2) \mu m$; this is compared with the purely index-guided mode, which is calculated to have a width of $w_0 \approx 43 \mu m$. The latter estimate is in good agreement with the 37(2)-$\mu m$ width measured with a concave mirror with $R = 98\%$ reflectivity (eigenmodes in such a high-reflectivity cavity

![Figure 1](image_url)

Fig. 1. Average output intensity as a function of cavity detuning. Here $\Delta L$ is the change in cavity length and $\lambda_{em}$ is the spontaneous emission wavelength, while $\Delta L = 0$ is the point at which maximum output is observed. Tuning ranges $A_1$, $A_2$, $A_3$, $B$, and $C$ are discussed in the text. The top curve is measured at $M = 5.0$, and the bottom curve at $M = 3.0$. Inset, cavity setup. The Gaussian pump beam (dotted curve) is tightly focused compared with the purely index-guided mode (dashed curve).
undergo only weak gain guiding). For the actual experiments we used a $R = 85\%$ mirror. This provided a balance of similarly sized gain and index guiding that made the eigenmodes highly nonorthogonal.

To eliminate pump noise, we passed the output of our pump laser, a Coherent 899-01 Ti:sapphire ring laser ($\lambda = 809$ nm), through an electro-optic feedback loop. This brought the rms pump noise down to $\approx 0.1\%$. A straightforward estimate shows that the pump noise is negligible compared with the quantum noise of the Nd$^{3+}$:YVO$_4$ laser, especially since the pump noise has a very low frequency (typically $\approx 1$ kHz) compared with the quantum noise bandwidth, which extends well beyond the relaxation oscillation frequency (0.5–10 MHz, depending on the pump and detuning).

The laser operates in a single transverse and longitudinal mode that is selected by piezoadjustment of the cavity length.\(^6\) The two curves in Fig. 1 show the output of the laser as a function of cavity length, with each protuberance representing a separate laser mode. The dimensionless pump strength $M = P_{\text{out}}/P_{\text{thr}}$ for the two curves is defined according to the threshold at the point of maximum gain. The fundamental lasing mode is labeled $A_1$, $A_2$, and $A_3$, depending on the detuning. $A_2$ corresponds to maximum output (gain), whereas $A_1$ and $A_3$ are measurements for shorter and longer cavities, respectively. Two higher-order modes, with ring-shaped far-field patterns, are labeled B and C. The measurements were made with a comparatively slow detector with a bandwidth of only $\approx 50$ kHz. Since this detector could not follow the relatively fast relaxation oscillations (0.5 MHz and above), a time-averaged signal was obtained.

When the laser is tuned from $A_1$ to $A_3$ (via $A_2$) in Fig. 1, the laser mode changes shape and widens in the near field. This and the dependency of the laser frequency on cavity length lead to a change in the spatial and spectral overlap (given by width $w$ and detuning $\Delta \omega$) of the lasing mode with the gain, which in turn affects the fraction of spontaneous emission into the lasing mode, $\beta(w, \Delta \omega)$. In Fig. 1 this is observed as a change in output power, but variations in $\beta(w, \Delta \omega)$ also affect the noise. Since it is difficult to find $\beta(w, \Delta \omega)$ by estimating the spectral and spatial overlap of the mode with the gain, the best indication for $\beta(w, \Delta \omega)$ is given by the output power, $P_{\text{out}}$, since $P_{\text{out}} \approx \Gamma_c \hbar \omega n_0 = [(\omega/\omega_p)P_{\text{in}} - \hbar \omega \Gamma_c/\beta(w, \Delta \omega)]$ above threshold. The cavity decay rate $\Gamma_c$, the pump frequency $\omega_p$, and the laser’s output frequency $\omega$ can be assumed to be constant here.

The effect of excess quantum noise is easily incorporated into the laser rate equations, where it acts as a multiplicative factor $K$ to the spontaneous emission. The effect of the $K$ factor was determined in Ref. 8 by use of the rate equations given in Ref. 7. The result is as follows: Small Nd$^{3+}$:YVO$_4$ lasers are said to operate in the microscopic regime, where the relaxation oscillation damping is relative weak compared with the oscillation driving force (quantum noise), potentially giving rise to very strong anharmonic oscillations.\(^7\) Such lasers have relative intensity fluctuations given by the reduced factorial moment

$$Q_2(0) = \frac{\gamma_n}{\gamma_n + \gamma_{\text{NL}}} = \left[1 + \frac{\beta(w, \Delta \omega)}{K} \frac{\gamma_\parallel n_0^2}{\gamma_{\text{LL}}} \right]^{-1},$$

where $Q_2(0) = g_2(0)/1 = \delta I(t)^2/I^2$ in the classical limit, with $I(t) = I + \delta I(t)$ being the output intensity of the laser and $g_2(0)$ being the second-order coherence function. The damping rates $\gamma_n = K\Gamma_c/n_0$ and $\gamma_{\text{NL}} = \Gamma_c \gamma_\parallel (\beta(w, \Delta \omega)) n_0/\gamma_{\text{LL}}$ are denoted as the photonic and the nonlinear damping, being the result of uncompensated cavity loss and a finite lower-level decay rate $\gamma_{\text{LL}}$, respectively; $\gamma_\parallel$ is the upper-level decay rate. The approximation sign in Eq. (1) indicates that the inversion damping, $\gamma_n$, was neglected; this is allowed because $\gamma_{\text{NL}} > \gamma_n$ (see Ref. 7).

One of the implications of Eq. (1) is that, for a specific photon number and thus also output intensity $P_{\text{out}} = \hbar \nu \Gamma_c/n_0$, $Q_2$ depends on only the ratio $\beta(w, \Delta \omega)/K$ and the cavity-independent ratio $\gamma_\parallel/\gamma_{\text{LL}}$. Hence, by comparing the $A_1$ and $A_3$ for the same output power, much can be learned about the excess-noise factor $K$, especially since the same output power indicates the same $\beta(w, \Delta \omega)$ (see above discussion).

In Fig. 2, $Q_2$ is plotted in terms of the average output intensity for two different pump values, $M = 3$ and $M = 5$. These values were obtained from the time traces, which were measured with a dc-coupled detector with a bandwidth greater than 10 MHz. To vary the output intensity, the cavity length was changed while the pump was kept constant, moving through all three segments, $A_1$, $A_2$, and $A_3$, as shown in Fig. 1. Errors in $Q_2$, estimated to be $0.05–0.1$ rms, are caused mainly by piezo-related cavity length drift and difficulties in estimating dc contributions to the time trace as a result of detector offset, oscilloscope resolution, and background light. Note the difference in fluctuations between segments $A_1$ and $A_3$, both for a pump of $M = 3$ and $M = 5$; the $A_3$ segments lie much higher than the $A_1$ segments, suggesting enhanced

![Fig. 2. Reduced factorial moment $Q_2$ versus output power $P_{\text{out}}$ for a pump power (a) $M = 5$ and (b) $M = 3$. The points $A_1$, $A_2$, and $A_3$ correspond to the same regimes as outlined in Fig. 1. The curves represent Eq. (1) with different $K$ factors: (i) $K = 1$, (ii) $K = 4$, (iii) $K = 16$, (iv) $K = 64$.](image-url)
excess noise. Curves (i), (ii), (iii), and (iv) are defined by Eq. (1) but have different values for $K$. Curves (i), for both $M = 5$ and $M = 3$, is based on a fit to the $A_1$ segment in Fig. 2(b). The fit yields a value of $\beta(w, \Delta \omega)\gamma_{\parallel}/\gamma_{\perp}(\hbar \nu G_0)^{-2} = 0.45 \text{ mW}^{-1}$. With reasonable estimates of $\Gamma_c = -(c/2L_{\text{opt}}) \ln R = 4.4 \times 10^{10} \text{ s}^{-1}$, $\gamma_{\parallel} = 0.61(5) \times 10^6 \text{ s}^{-1}$, $\gamma_{\perp} = 1.6(1) \times 10^9 \text{ s}^{-1}$, and $K = 1$, we found that $\beta(w, \Delta \omega) = 3 \times 10^{-6}$, which is in excellent agreement with previous observations at zero detuning. Unlike the $A_1$ segment, which has $K \approx 1$ and approximately follows curve (i), the $A_3$ segment deviates from this curve because of the excess noise enhancement. For a certain detuning the proximity of the segment $A_3$ to one of the other curves instead provides an indication of the excess-noise factor at that point. This comparison indicates that $K$ factors may come close to 100, with the largest noise enhancements close to the point at which the laser switches to another mode (a mode crossing). No measurements were conducted at the mode crossing, since the laser starts to mode switch and pulse for this detuning. We note that some of the fluctuation enhancement in Fig. 2 may be caused by a decrease in $\beta(w, \Delta \omega)$. We expect this contribution to be almost negligible compared with excess quantum noise. A small decrease in $\beta(w, \Delta \omega)$ is, however, observed as an increase in fluctuations for large detuning in the $A_1$ segment of Fig. 2(a); for very large detuning and a much diminished $\beta(w, \Delta \omega)$ the laser eventually switches off.

On the basis of measurements of $Q_2$ alone we cannot be sure that the increase in intensity fluctuations is really caused by an increase in (spontaneous emission) noise and not by a decrease in damping. The latter suggestion could seem plausible because of the proximity to a second mode, where this nonlinear system might become dynamically unstable. To distinguish between the two explanations, we note that reduced damping is expected from the competing nonlinear theory, whereas one distinctive feature of excess quantum noise is the fact that the fluctuation increase is accompanied by increased damping of the relaxation oscillations. This is because the relaxation oscillation damping rate $\gamma_{\text{ro}} = (\gamma_n + \gamma_{\text{NL}})/2$ increases with enhanced excess quantum noise as $\gamma_n = K\Gamma_c/n_0$.

The role of damping is demonstrated in Fig. 3, where the rf spectra are compared for the same output power [6.7 mW in Fig. 2(a)] in two different segments. From the width of the main harmonics in this figure we find that the relaxation oscillation damping has increased from $8.4(9) \times 10^5 \text{ s}^{-1}$ in Fig. 3(a) to $3.3(3) \times 10^6 \text{ s}^{-1}$ in Fig. 3(b), an increase in damping by a factor 4. This comparison was made with the same detector used for the $Q_2$ measurements.

This enhanced damping can be directly related to excess quantum noise. For $M = 5$ in Fig. 2 the $A_1$ segment has a reduced factorial moment of $Q_2 = \gamma_n/(\gamma_n + \gamma_{\text{NL}}) = 0.047$ at $P_{\text{out}} = 6.7 \text{ mW}$. In combination with the value for $\gamma_{\text{ro}} = (\gamma_n + \gamma_{\text{NL}})/2$ we have two equations and two unknowns, which allows us to deduce $\gamma_{\text{NL}} = 1.60 \times 10^9 \text{ s}^{-1}$ and $\gamma_n = 7.9 \times 10^4 \text{ s}^{-1}$. For the $A_3$ segment a similar comparison of the measured $Q_2 = 0.75$ at $P_{\text{out}} = 6.7 \text{ mW}$ and the $\gamma_{\text{ro}}$ yields $\gamma_{\text{NL}} = 0.33\gamma_n = 1.65 \times 10^8 \text{ s}^{-1}$. The nonlinear damping, $\gamma_{\text{NL}}$, is thus nearly identical in both segments, as it should be, whereas the photonic damping, $\gamma_n = K\Gamma_c/n_0$, differs by more than a factor of 60, which is a clear indication of enhanced excess quantum noise.

In this Letter we have shown that stable soft-edged cavities can have noise resonance with excess-noise factors close to 100. Strong evidence that the fluctuation enhancement is caused by excess quantum noise and not by dynamic instabilities was provided by the fact that the relaxation oscillation damping increased with increased fluctuations. Similar enhancements (although less drastic) may be expected for intensity fluctuations in electrically pumped semiconductor lasers.

References