Advanced evolution of helium stars and massive close binaries
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IIIC. THE LOWER LIMIT TO THE MASSES OF PROGENITORS OF NEUTRON STARS.

SUMMARY.

In this section the various scenarios for the formation of neutron stars by the evolution of single hydrogen stars and by binary evolution are reviewed. Some of these scenarios are adjusted to the results which we have obtained for the evolution of helium stars in the mass range 2.0 to 4.0 $M_\odot$. The lower mass limit for a helium star (with $Z = 0.03$) for forming a core of mass larger than the Chandrasekhar limiting mass, $M_{\text{Ch}}$, is derived. This mass limit $- 2.20 \pm 0.05$ $M_\odot$ yields also a lower limit for the initial mass of the hydrogen star progenitors of neutron stars that were formed by core collapse triggered by photo-disintegrations (i.e. $\approx 9$ $M_\odot$). The upper mass limit for the progenitors of single C-O white dwarfs (i.e. $\approx 8$ $M_\odot$) and single O-Ne-Mg white dwarfs (i.e. $\approx 8$ $M_\odot$, but $9$ $M_\odot$ in case of substantial mass loss in the red-giant phases) are derived by using our results in combination with the stellar evolution calculations of Nomoto (1984a, e) for helium stars (with $Z = 0.02$) in the mass range 2.0 to 2.6 $M_\odot$.

1. INTRODUCTION, REVIEW OF PREVIOUS WORK.

a. Relation between supernova types and various types of core-evolution.

Trimble (1982b, 1983) and Van den Heuvel (1983) reviewed the various ways in which a neutron star can be formed by the evolution of a single hydrogen star, and by the evolution of a component in a binary, respectively. The present ideas on the possible identification of Type I and II supernovae are as follows. The Type I supernova is thought to be identified with carbon detonation or deflagration of the core of a low- to intermediate-mass star from which about $0.2 - 1$ $M_\odot$ $^{56}$Ni is ejected in the explosion (Arnett, 1979b, and Nomoto, 1981). Whether or not in this mass range a single star or a non-accreting star in a binary will leave a condensed remnant (e.g. a white dwarf) is not clear; most probably the core of a single star will be completely disrupted (Müller and Arnett, 1984). From a white dwarf in a mass-accreting binary, a white dwarf remnant will be left for a limited range of masses of
the C-O core (of the initial white dwarf) and for a limited range of mass accretion rates (Nomoto, 1984b). Type II supernovae are generally thought to be associated with the formation of a neutron star either by core collapse triggered by electron-capture or by photo-disintegration reactions in intermediate-mass to massive stars [in which the $^{56}$Ni (always produced in a shell) falls back onto the neutron star (Arnett, 1979b); see Trimble (1982b, 1983)]. However, there are exceptions to the general picture that the ejecta of Type II supernovae are hydrogen rich. A number of the observed supernova remnants are oxygen rich and do not show evidence of hydrogen, helium, and nitrogen [Hillebrandt (1984); Canizares (1984)]. Recent numerical calculations of the core collapse of hydrogen stars with masses ~12 - 100 $M_\odot$ fail to give explosions by the core bounce mechanism, which is possibly due to wrong initial models and which is suggested to be possibly realistic by Hillebrandt (1984), however, see also Woosley and Weaver (1984), Weaver et al. (1984), Woosley et al. (1984), Bodenheimer and Woosley (1983), and see section IIIB.

In Figures 23a and b and 24a and b, we present the theoretical and observational mass limits for single and binary stars for white dwarf and planetary nebula formation, for complete disruption, and for neutron star formation. These limits will be discussed below. Whenever a mass limit interval (indicated by a solid line) has an extra uncertainty on one or both sides of the interval, the uncertainty is shown with a dashed line. Solid lines with a thick dot in the middle indicate the adopted range of the mass limit (the uncertainty is half the length of the range of the line).

b. Observational upper mass limit for white dwarf formation and observational lower mass limit for various supernova types.

For single stars the latest observational and theoretical results indicate that most of the single hydrogen stars with masses in the range 6 to 8 $M_\odot$ produce Type I supernovae leaving no compact remnant. Reimers and Koester (1982, from small number statistics) and Weidemann and Koester (1983, using results from open clusters) find an observational upper mass limit of 6 to 9 $M_\odot$ and 8 $M_\odot$, respectively, for single hydrogen stars to become white dwarfs and planetary nebulae, whereas Müller and Arnett (1984) claim from two-dimensional core-collapse calculations that the C-O cores of 4 to 8 $M_\odot$ single hydrogen stars will be disrupted by a Type I supernova explosion (most likely initiated by C-deflagration). Similarly, the O-Ne-Mg cores of 8 to 10 $M_\odot$
single hydrogen stars are expected by some authors to be disrupted completely by a Type I supernova explosion (initiated by O-deflagration). The high observational upper mass limit for white dwarf progenitors may not be in contradiction with the low theoretical mass limit derived from core collapse calculations if effects of rotation, magnetic fields, and initial composition are taken into account (Trimble, 1982b). Another semi-empirical mass limit for supernova progenitors was derived by Wood et al. (1983) from observations of long-period variables in the Magellanic Clouds. These authors find from a
Fig. 23b.—The observational upper mass limits for white dwarf and planetary nebula formation and the lower mass limits for neutron star formation (see the text and Table 14), for single hydrogen stars.

combination of stellar evolution and pulsation theories that in the Magellanic Clouds single hydrogen stars with initial masses larger than \( \sim 5 \, M_\odot \) will undergo a supernova explosion and that less massive stars will form planetary nebulae. Moreover, Kennicutt [1984, using the H-\( \alpha \) derived star formation rate of a sample of nearby Sc-Sbc galaxies, the mean rate of Type II supernovae, and the correlation between these rates (although derived from small number statistics); also accounting for the low Galactic supernova rate and for the observed strong concentration of Type II supernovae towards spiral arms] deduced a range for the observational lower limiting initial mass of the progenitors of Type II supernovae of \( 8 \pm 3 \, M_\odot \), which overlaps with the
theoretical lower limits of the initial masses of progenitors of neutron stars. Similarly, from the space distribution of radio pulsars in the solar neighbourhood Blaauw (1984) derived that most pulsars originate from stars with initial masses in the range \(5 - 10 \, M_\odot\), with an uncertainty of a few solar masses. Lyne et al. (1984) derived from a statistical analysis of an approximately flux limited sample of radio pulsars in the entire sky and from an assumed life-time of the Galaxy of \(12 \times 10^{10}\) years a lower and an upper limit for the birthrate of pulsars of \(0.8 \times 10^{-11} \, \text{pc}^{-2} \, \text{yr}^{-1}\) and \(3 \times 10^{-11} \, \text{pc}^{-2} \, \text{yr}^{-1}\), respectively. If one further assumes that these pulsars originated from all stars more massive than a certain mass, \(M_{\text{pulsar}}\), or, alternatively, if one assumes that all pulsars descended from stars with a mass \(M_{\text{pulsar}}\) in a certain mass interval, the values derived from these lower and upper limits to the birthrate are: \(M_{\text{pulsar}} > 16 \, M_\odot\) or the interval \(8 \, M_\odot < M_{\text{pulsar}} < 10 \, M_\odot\), and \(M_{\text{pulsar}} > 9 \, M_\odot\) or the interval \(8 \, M_\odot < M_{\text{pulsar}} < 25 \, M_\odot\), respectively (Lyne et al., 1984); see Figure 23b.

c. Theoretical mass limit for neutron star formation and for complete disruption derived from numerical hydrodynamic collapse calculations.

As to the theoretical lower mass limit for neutron star formation (see Figure 23a) the current ideas are as follows. Hillebrandt (1982b) argued that single stars more massive than \(8 \, M_\odot\) will undergo core collapse and leave a neutron star remnant after a Type II supernova explosion. He argued that in single stars more massive than \(\sim 10 - 12 \, M_\odot\) this is the result of the formation of collapsing Fe cores (see e.g. Nomoto, 1984a, e). In the mass range \(8 \, (\pm 1)\) to \(10 \, (\pm 1) \, M_\odot\) the core collapse is thought to be triggered by electron captures on O-Ne-Mg cores [Ikeuchi et al. (1972), Miyaji et al. (1980), Nomoto (1981), Hillebrandt et al. (1984), Nomoto (1984a, e)]; the explosion and ejection of material in these stars are due to the development of a steep density gradient in the pre-collapse phase (Hillebrandt, 1984) and a neutron star remnant with mass \(\sim 1.2 \, M_\odot\) is found to be left (Hillebrandt et al., 1984). The alternative opinions are that single stars in the mass range \(8 - 10 \, M_\odot\) either undergo a detonation or a deflagration and disrupt (most likely completely) or that a neutron star may be formed if the collapse is initiated by core-silicon burning after the removal of the outer layers due to neon flashes, see Woosley et al., 1980). If this is true, 8 to 10 \(M_\odot\) single hydrogen stars can also be candidates for Type I supernovae (Woosley et al.,
1980; Weaver et al., 1980) depending on the validity of certain assumptions (see Nomoto, 1984a, c, e) and on the restricted parameter space for a detonation or deflagration to occur.

A weak lower mass limit is given by the mass limit for single stars which become C-O white dwarfs, because there is a higher mass limit for stars to become O-Ne-Mg white dwarfs. Iben and Renzini (1983) found a maximum initial mass for a single star to end as a C-O white dwarf of 8 $M_\odot$ assuming a very high stellar wind mass-loss rate in the asymptotic giant branch phase.

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**Fig. 24a.**—The theoretical mass ranges for various types of final evolution of primaries in a binary: the mass ranges for the formation of various types of white dwarfs, the lower mass limit for electron-capture induced core collapse, and the lower mass limit for photo-disintegration induced Fe-core collapse. These limits are dependent on the precise evolutionary phases in which the primary star begins to lose mass to the companion (i.e., cases B, B8, and C), which in turn depend on the initial orbital period and the mass ratio of the binary system.
Lower mass limit for the direct formation of neutron stars in binaries, as estimated so far.

Among the formation mechanisms of neutron stars in binaries three are simply variants of single star evolution, viz.: evolution in very wide orbits (which is not dominated by mass transfer), or capture of a neutron star by a single star, or exchange of a neutron star with a companion in an existing binary (the latter two mechanisms can occur in globular clusters, but capture may also occur in the galactic disk, see e.g. Taam and Fryxell, 1984). The other ways to form a neutron star in a close binary are: mass-transfer dominated evolution in (massive) close binaries and accretion of matter onto a white dwarf in a (close) cataclysmic-variable type or (wide) symbiotic type, low-mass, binary. These two cases are considered lateron, in Figures 24a and 24b, and in the following paragraph.

In a close binary, case B (or case BB and late case A) of mass transfer leads to a higher value of the lower mass limit for formation of neutron stars than case C of mass transfer does: depending on the orbital separation the value of the limit ranges from $8 \pm 1$ to $14 \pm 2 \, M_\odot$ for case B and $8 \pm 1$ to $10 \pm 2 \, M_\odot$ for case C [see Van den Heuvel (1983), the difference between cases B and C is due to the larger helium core mass formed in case C evolution). These results are, among others, based on stellar evolution computations of Nomoto (1980 - 1982) and Arnett (1978b). The Be/X-ray binaries are suggested to have been formed by mass-transfer dominated evolution in binaries with primaries less massive than $\sim 16 - 20 \, M_\odot$ (i.e. with helium core masses $< 4 - 5 \, M_\odot$), see Rappaport and Van den Heuvel (1982). Binaries with more massive primaries are thought to yield the standard massive X-ray binaries [Rappaport and Van den Heuvel and Van den Heuvel (1983)]. The Be/X-ray binaries are expected to evolve lateron into binary radio pulsars with short-period orbits in which the companion is a neutron star or a massive white dwarf, or into two runaway pulsars (Van den Heuvel and Taam, 1984).

Recently Iben and Tutukov (1984c) studied numerically the evolution of hydrogen star primaries with masses in the range $3 - 12 \, M_\odot$ and with $Z = 0.02$ (population I). Assuming non-conservative mass loss and mass transfer, which may be generalized to conservative cases B and C, they find that hydrogen primaries in the mass range $3 - 10.3 \, M_\odot$ may leave C-O electron degenerate white dwarfs if the initial conditions (initial orbital period and mass ratio of the binary system) are chosen appropriately. Furthermore, they find that
primaries in the mass range $8.8 - 11 M_\odot$ may leave O-Ne-Mg white dwarfs (again with appropriate initial conditions). Finally, they derive by extrapolation that the O-Ne-Mg cores of primaries with a mass of $10.6 M_\odot$ and larger (at least up to $12 M_\odot$) may undergo electron captures. The limiting values $10.3 M_\odot$ and $10.6 M_\odot$ are upper and lower mass limits for primaries to form of C-O white dwarfs and to undergo electron captures, respectively. Thus, the mass interval for primaries to form O-Ne-Mg white dwarfs may be very small ($10.3 - 10.6 M_\odot$) for an appropriate choice of the initial conditions of the binary system.

e. Formation of neutron stars in binaries by accretion-induced collapse:

formation of neutron stars in an old stellar population.

(i) Introduction.

In low-mass (close) cataclysmic variable systems or (wide) symbiotic systems containing a massive white dwarf accretion of mass onto the white dwarf can lead to the formation of a neutron star (see Whelan and Iben, 1973; Van den Heuvel (1977); Canal et al., 1980; Van den Heuvel, 1983; Nomoto, 1984a). The end-product of such an evolution may be a low-mass X-ray binary and, finally, a binary radio pulsar (Van den Heuvel and Taam, 1984).

The precise outcome of accretion onto a white dwarf depends strongly on the interior state (either fluid or solid), on the interior composition of the white dwarf (either He, C-O, or O-Ne-Mg), on the mass of the core of the white dwarf, on the mass accretion rate, and only slightly on the composition of the accreted material (either H, He, C-O, or O-Ne-Mg). Mass accretion onto a helium white dwarf gives always complete disruption, see e.g. Nomoto and Sugimoto (1977).

(ii) Results of mass accretion onto relatively young C-O white dwarfs.

If mass is accreted onto a white dwarf with a mass smaller than $M_{\text{Ch}}$ and with a fluid, electron degenerate C-O core [i.e. its cooling age since core-helium burning is less than a few times $10^8$ years, cf. Nomoto (1982a) and Paczynski (1971b)], the white dwarf may be disrupted or not [see Nomoto (1980a, b) and revised in Nomoto et al. (1984b) and in Nomoto (1984d) for the case of hydrogen accretion; Iben and Tutukov (1984a) do not exclude the possibility that the remnant will be a neutron star]. The precise way of
Fig. 24b.—The theoretical core-mass ranges for accretion-induced complete disruption, for accretion-induced white dwarf formation, and for accretion-induced core collapse (which may occur in various ways, see the text) in a binary. For the case in which a white dwarf remains after the supernova explosion the mass accretion rate and the composition of the accreted fuel is given.

disruption [which is actually occurring in a more complex way of fuel combustion than described by one- or two-dimensional detonation or deflagration models, see Müller and Arnett (1982), and Nomoto (1984c)] depends on the mass accretion rate and even more heavily on the input physics and computational method (see Müller and Arnett, 1984); however, for a limited mass range of its C-O core the white dwarf will not be disrupted if the mass
accretion rate is chosen properly [see Nomoto et al. (1984b) for H-, He-, and C-O accretion, Miyaji et al. (1980) for O-Ne-Mg accretion, Woosley et al. (1984) and references therein for He-accretion, and Fujimoto and Taam (1982) for H-accretion; see also Figure 24b]. A neutron star may be formed if C-O material is accreted onto a C-O white dwarf [e.g. in a double C-O white dwarf system, see Webbink (1979, 1984) and Iben and Tutukov (1984a, c)]. In that case the C-O white dwarf may turn into an O-Ne-Mg white dwarf for an appropriate choice of the accretion rate [see Nomoto (1984d) and references therein] and lateron collapse to a neutron star [Nomoto et al. (1979)]. It should be noticed, that these results are obtained with input physics (e.g. convection theory) which is sometimes used outside the regime for which it is has been developed. Furthermore, low rates for H-accretion do not lead to the growth of the mass of the C-O core, but instead hydrogen shell flashes are triggered which give rise to mass ejection from the accreting star in nova outbursts [the details are reviewed by Nomoto (1984b), by MacDonald (1984) and by Iben and Tutukov (1984a)].

(iii) Results of mass accretion onto old C-O white dwarfs.

On the other hand, accretion of mass onto a white dwarf with a massive, solid C-O core (and with a total mass > 1.2 M⊙, but < MCh, cf. Labay et al., 1983; Isern et al., 1983) is suggested to lead to the collapse of the white dwarf and to the formation of a neutron star in a Type I supernova event (Isern et al., 1984). Notice, that solid, electron degenerate C-O cores are expected to be formed in a cooling time longer than ~ 109 - 4 × 109 years after core-helium burning for C-O stars of about 0.6 - 1.0 M⊙ [cf. Labay et al. (1983), Isern et al. (1983), Lamb and Van Horn (1975), and Iben and Tutukov (1984b)]. In such cores a separation between C and O is expected to occur (Stevenson, 1980), the O being concentrated to the inner core; this makes the situation more favourable for electron captures on O to occur in the centre or, in case of partial separation of C and O in the core, for off-centre C-ignition or central ignition of 12C and 16O [the latter occurs if the 12C (α, γ) 16O reaction is as fast as suggested by Kettner et al. (1982)].

Mass-accreting white dwarfs with solid, electron degenerate C-O cores in which element separation has occurred, are expected to undergo a collapse initiated by electron captures, by off-centre carbon ignition (see Canal et al., 1980, and Isern et al., 1983), or by central ignition of 12C + 16O (Isern et al.,
either leaving a condensed remnant (e.g. a neutron star, Isern et al., 1983, 1984) or leading to total disruption of the white dwarf in some cases of off-centre C-ignition (Canal et al., 1982, Labay et al., 1983, Isern et al., 1984). Nomoto (1984b) and Nomoto et al. (1984b) do not rule out that separation of carbon and oxygen in the electron degenerate C-O core during the cooling may just prevent the formation of a neutron star [rather than be favourable to it— as suggested by Isern et al. (1983, 1984)] after the supernova explosion: as a result of both mass accretion onto the white dwarf and separation of carbon and oxygen during crystallization of the interior of the white dwarf, off-centre ignition of carbon may still occur, which can lead to total disruption of the white dwarf (Nomoto, 1984b). However, Isern et al. (1984) have argued that at least partial collapse certainly occurs in accreting solid, C-O white dwarfs due to the solidification of the core and independent of the currently favoured $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rates and of whether element separation does occur partially, totally, or not at all. Isern et al. (1984) suggest that all the observed kinds of Type I supernovae may be formed by mass accretion onto solid C-O white dwarfs (in some cases leaving a neutron star or white dwarf remnant, in other cases leaving no remnant).

(iv) Results of mass accretion onto O-Ne-Mg white dwarfs.

On the other hand, accreting white dwarfs with fluid, electron degenerate O-Ne-Mg cores and with masses less than $M_{\text{Ch}}$ (~ 1.2 – 1.37 $M_\odot$, Nomoto, 1984a) are expected to collapse and form a neutron star: when the mass of the white dwarf becomes larger than $M_{\text{Ch}}$ (by accretion), hydrogen burning in a shell increases the core mass and the collapse is triggered by electron-capture reactions, see Sugimoto and Nomoto (1980), Van den Heuvel (1981) and Nomoto (1984a, b, e).

The fate of these white dwarfs with fluid O-Ne-Mg cores has not yet been fully studied for various rates of mass accretion; but the outcome will be most likely a neutron star remnant formed by core collapse triggered by electron captures (also if instead of H, He or C and O is accreted onto a stripped white dwarf without outer hydrogen or helium layers, respectively), see Nomoto (1984b, 1980a, and Figure 24b). Mass accretion onto a white dwarf with a solid O-Ne-Mg core has never been studied; however, it is expected to yield a supernova explosion and to leave a neutron star remnant.

In order to form a white dwarf with an electron degenerate, fluid C-O or
O-Ne-Mg core a case BB mass exchange was invoked by Nomoto (1980a and 1984a, see also section IIIA). Law and Ritter (1983) and Ritter (1982) have pointed out that a much larger fraction of the massive white dwarfs is produced by case C mass transfer in binaries with 3 to 6 (-8) M☉ hydrogen primaries than by case BB mass transfer in binaries with primaries in this mass range. However, the case C mechanism is likely to produce more C-O than O-Ne-Mg white dwarfs: the O-Ne-Mg white dwarfs have descended only from the most massive of such primaries (~7–8 M☉), as only these are able to ignite carbon (see Chapter IV); due to the form of the initial mass function the C-O white dwarfs are expected to be far more abundant than O-Ne-Mg white dwarfs.

In the calculations of Miyaji et al. (1980) on the collapse of an O-Ne-Mg core triggered by electron captures on 24Mg and 24Na, and subsequently on 20Ne, the growth of the mass of the fluid, degenerate O-Ne-Mg core is 10 times as fast as in their progenitor hydrogen stars as derived by Paczynski (1970a, where the growth of the core mass is determined by hydrogen shell-burning), which is due to the helium-shell burning induced by mass accretion. Such a growth of the core mass is possible in the case in which the hydrogen-burning shell is extinguished (see Barkat et al., 1974) and can be mimicked by accretion of mass (of the same composition as the core) onto an O-Ne-Mg core, see Miyaji et al. (1980) and see Nomoto et al. (1979). If the mass coordinate of the carbon-burning shell above the fluid O-Ne-Mg core of the white dwarf becomes larger than 1.37 M☉, either an electron-capture or a photodisintegration supernova may occur, leaving most likely a neutron star remnant in both cases.

(v) Collapse or solidification of non-accreting cooling O-Ne-Mg white dwarfs.

Accreting or non-accreting white dwarfs with fluid O-Ne-Mg cores of 1.2 to 1.37 M☉ may become solid O-Ne-Mg white dwarfs if the mass of the O-Ne-Mg core does not increase above 1.37 M☉ in the course of the evolution, see Nomoto (1984a). Possibly these white dwarfs may O-deflagrate and disrupt completely before ever becoming solid. [Once the core of the white dwarf is solid, mass accretion may still lead to one of the various types of supernova explosions, see before].
2. THE LOWER MASS LIMIT FOR NEUTRON STAR FORMATION AS DERIVED FROM OUR COMPUTATIONS.

a. Evolution of helium stars in the mass range 2.0 - 4.0 $M_\odot$.

(1) Introduction

In this paragraph we deduce the lower mass limits for a helium star, for a single hydrogen star, and for a hydrogen primary in a close binary, for forming a neutron star by core collapse (triggered by photo-disintegration reactions in a core with a mass larger than the Chandrasekhar limiting mass), or triggered by electron capture reactions [the precise trigger mechanism for a core mass just above the limiting mass still being a matter of debate, see Nomoto et al. (1984a)]. To deduce these limits we use our calculations of sections IIIA and IIIB. To be safe, we will assume that $0$-$Ne$-$Mg$ white dwarfs and hydrogen stars with core masses larger than $M_{Ch}$ at neon ignition will become dynamically unstable and leave a neutron star remnant after a supernova explosion. Also, we combine the calculations of Nomoto (see above) and of other authors in order to summarize the evolution of helium stars with masses in the range 0.85 - 4.0 $M_\odot$ as descendents from single hydrogen stars or from hydrogen primaries in a close binary.

(ii) The lower mass limit for neutron star formation derived from stellar evolution calculations.

a. this work and calculations of Delgado and Thomas (1981).

Figure 25 depicts the evolution of the radius against core mass, $M_{core}$, from core-helium burning up to (or very close to) neon ignition, for helium stars in the mass range 2.2 to 4.0 $M_\odot$ and for a 2.0 $M_\odot$ helium star up to the time when the convective carbon-burning shell reaches the centre (peacefully). During central helium burning (i.e. from letter A to B in Figure 25) the core mass is defined as the mass of the convective helium-burning core (including adjacent semi-convective regions); after core-helium burning the core mass is defined as the mass of the core (consisting of $^{12}$C, $^{16}$O, and possibly $^{20}$Ne, $^{24}$Mg, and some $^{4}$He at the boundary of the core), which is limited by the helium shell with maximum energy generation.
A figure similar to Figure 25 was constructed by Law and Ritter (1983, figure 1) for helium stars of 1 to 4 $M_\odot$ using evolutionary calculations of Law (1982, for a 1 $M_\odot$ helium star) and of Delgado and Thomas (1981, for 2 to 4 $M_\odot$ helium stars).

b. the lower mass limit for neutron star formation.

The results in Figure 25 differ considerably from those of figure 1 of Law and Ritter: the resulting lower mass limits for formation of a neutron star (most likely due to core collapse initiated by photo-disintegrations and probably not initiated by electron captures) from a helium star (with a helium envelope) is 2.20 (± 0.05) $M_\odot$ in our work and 3.2 (± 0.1) $M_\odot$ as deduced from the calculations of Delgado and Thomas. The range of uncertainty of 0.05 $M_\odot$ which we set to our limiting mass value accounts for the effect of the decrease of the core mass just before neon ignites. [The lower mass limit is given by the criterion that the mass of the core of the helium star becomes larger than $M_{Ch}$ in the course of the evolution. If this criterion is fulfilled, our calculations show that the mass of the O-Ne-Mg core (which is limited by the carbon-burning shell with maximum net energy generation and which is constituted mainly of $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, and some $^{12}\text{C}$ at the boundary

![Fig. 25.—The core mass versus radius relation for 2.0 to 4.0 $M_\odot$ helium stars undergoing helium and carbon burning and neon ignition (in the case of the 2.9 $M_\odot$ helium star till very close to neon ignition; the evolution of the 2.0 $M_\odot$ helium star was followed only up to the time at which the convective carbon-burning shell reaches the centre). During convective core-helium burning (from letter A to B) the core mass is defined as the mass of the convective helium-burning core including adjacent semi-convective regions. After convective core-helium burning the core mass is defined as the mass of the core (consisting of $^{12}\text{C}$, $^{16}\text{O}$, and possibly $^{20}\text{Ne}$ and $^{24}\text{Mg}$, and some $^{4}\text{He}$ at the outer boundary of the core) which is limited by the helium-burning shell with maximum net energy generation. The open circles mark the onset of either central or off-centre convective core-carbon burning (at letter C). The dash-dotted curve has been drawn through these circles. The horizontal marking between B and C indicates the point where carbon burning begins in the radiative core. The letters along the curves for the various masses have the same meaning as in Figure 12a (see section IIIB).]
of the core) will become larger than 1.37 \( M_\odot \), which is the limiting mass for an O-Ne-Mg core to ignite neon (either in the centre or off-centre; also for pure neon cores) as deduced by Nomoto (1984a); The dotted line in Figure 18 (see section IIIB) demarcates the boundary of the O-Ne-Mg core of the 2.2 \( M_\odot \) helium star. Figure 18 shows that the mass of the O-Ne-Mg core becomes larger than 1.37 \( M_\odot \). The Chandrasekhar limiting mass used here is based on the Newtonian theory of gravity; the limiting mass based purely on the general relativistic theory of gravity (Einsteinian) is called the critical mass which is even smaller than \( M_{Ch} \) for a given chemical composition (see Ibanez Cabanell, 1984). This critical mass has to be corrected for some effects (e.g. neutronization by electron captures). 

\[ \text{c. calculations of Nomoto (1984a, e) and collaborators.} \]

From the results for helium stars in the mass range 2.2 to 2.8 \( M_\odot \) [see Nomoto (1984a, e)] Nomoto (1984a) derived a lower mass limit of 2.5 - 2.8 \( M_\odot \) for the formation of a neutron star from a helium star (with a hydrogen envelope) by core collapse triggered by photo-disintegrations or electron captures (or due to neon flashes and silicon burning in the core, see Woosley et al. (1980) and see also Nomoto (1984a)]. Hillebrandt et al. (1984) derived for helium stars a lower limiting mass of 2.0 to 2.5 \( M_\odot \) for formation of neutron stars by core collapse (initiated by electron captures only).

\[ \text{d. causes of the differences.} \]

In all these determinations of the limiting mass the neutrino loss rates were corrected for neutral current effects. The neutrino-Bremsstrahlung may not have been included by Delgado and Thomas (1981) and Arnett (1978a, who did not correct for neutral current effects). Arnett (1978a) found about the same limiting helium star mass (> 3 \( M_\odot \)) as Delgado and Thomas. However, Arnett, Nomoto, and Delgado and Thomas did not allow the convective helium core to grow in mass by convective overshoot (see Chapter IV). Actually, the lower mass limit derived from figure 1 of Law and Ritter should be lower than the 3.2 \( M_\odot \) which they derived, for the following reason. Delgado and Thomas calculated the evolution of a 3.33 \( M_\odot \) helium star somewhat beyond central carbon ignition in the core and the evolution of a 2.7 \( M_\odot \) helium star up to off-centre ignition of carbon. Since the mass of the core grows during as well
as after core-carbon burning (see Figures 17 - 22, and 25), we estimate that also in the calculations of Delgado and Thomas a 3 \( M_\odot \) helium star might well have reached a core mass \( > M_{ch} \) after core-carbon burning.

The discrepancy between Nomoto's work and our work can be explained completely by the growth in mass of our convective helium-burning cores and by the almost zero growth of these cores in the calculations of Nomoto. Nevertheless, the differences between the work of Delgado and Thomas and Arnett on the one hand and that of ourselves and of Nomoto on the other are too large to be solely explained by growing or non-growing masses of convective helium-burning cores. Hence, the lower mass limit is considerably affected by the input physics in the evolutionary code employed.

In Figure 25 the dash-dotted curve through the open circles (at the letter C) indicates the core mass versus radius relation at the beginning of convective-carbon burning in the centre (or off-centre in case of the 2.0 \( M_\odot \) helium star). The radii which we obtained at the onset of convective carbon-core burning are systematically smaller for a given total stellar helium mass in the range 2 to 4 \( M_\odot \) than those calculated by Delgado and Thomas. This is a consequence of the larger convective helium-burning core at that mass in our computations. Hence, the core masses which we calculated correspond to smaller total stellar helium masses than those of Delgado and Thomas. This effect of the increased core mass disappears for helium stars more massive than \( \sim 4 \ M_\odot \).

b. The occurrence of a second mass-transfer phase for a helium star in a binary.

(1) The fate of helium stars more massive than 2.20 (± 0.05) \( M_\odot \)

a. summary.

From inspection of Figure 25 we can draw several conclusions regarding a second mass-transfer phase in a close binary. For example, a 2.5 \( M_\odot \) helium star attains a core mass larger than \( M_{ch} \) before carbon ignites in the centre; the bare (i.e. without most of its helium layers) core remnant after a possible second mass-transfer phase of the 2.2 \( M_\odot \) helium star is \( > 1.12 \ M_\odot \) at the end of convective core-helium burning. Therefore this bare core can ignite carbon off-centre [to do so the core has to have a mass \( > 1.06 \ M_\odot \) (the maximum mass for a Z of about 0.02 and is lower for all other values of Z), see
Chapter IV]; thus an O-Ne-Mg white dwarf will result from the second phase of mass transfer from the original primary onto the original secondary. The helium stars with masses in the range 2.7 - 4.0 \(M_\odot\) do not develop red-giant envelopes after central helium burning. These stars can lose their helium envelopes only by a second mass-transfer phase in a post-common-envelope binary [i.e. non-conservative case BB mass exchange, see Law and Ritter (1983) and Paczynski (1971c)] or by extensive stellar wind mass loss after the first case B or case C mass-transfer phase. The helium stars with masses in the range \(\sim 2.15\) to 2.7 \(M_\odot\) develop red-giant envelopes (after central helium or carbon burning, see Figure 25) and may lose almost all of their helium layers either by a (conservative or non-conservative) second mass-transfer phase or by extensive stellar wind mass loss [as is suggested to occur for intermediate-mass stars in order to form white dwarfs, cf. Weidemann (1984a), Wood et al. (1983), and Iben and Renzini (1983)].

The second mass-transfer phase sets in after ignition of carbon in the core (see Figure 25), and therefore always O-Ne-Mg white dwarfs will be formed by the mass exchange. The fate of such a case BB or stellar wind mass-loss remnant (i.e. O-Ne-Mg cores surrounded by a thin carbon-burning shell), is somewhat uncertain: if the mass coordinate of the C-O shell is larger than \(1.37\ M_\odot\) or if it will reach this value in the course of the cooling in the subsequent evolution, the core may undergo a supernova explosion triggered by electron captures, see Nomoto (1984a) and Ikeuchi et al. (1972); however, the supernova explosion may also be triggered by photo-disintegrations or if both electron captures and photo-disintegrations do not occur the remnant will cool down (over more than 10 years) and just leave a white dwarf with a solid O-Ne-Mg core, or, alternatively, the O-Ne-Mg core will subsequently O-"deflagrate".

b. one special case: the evolution of an O-Ne-Mg core which undergoes O-
deflagration.

In case the O-Ne-Mg core O-deflagrates (which is rather unlikely), the O-Ne-Mg core may be completely disrupted (Müller and Arnett, 1984). This type of deflagration evolution is expected to occur for O-Ne-Mg cores surrounded by a (not too thin) carbon burning C-O shell with mass coordinate smaller than \(1.37\ M_\odot\) (but larger than \(1.005\) to \(1.06\ M_\odot\), see Chapter IV), see Nomoto (1984a). Notice that the helium and hydrogen shell sources are absent in these stripped
cores. After carbon burning in the core the bare O-Ne-Mg cores will not produce a core-mass luminosity relation such as that of Paczynski (1970a and 1971b) for non-stripped electron degenerate (C-O) cores with masses in the range 0.57 to 1.39 M\(_\odot\) [or the relation deduced by Kippenhahn (1981, who gives a unique relation which extends also to lower C-O core masses, see also Refsdal and Weigert, 1970)] nor that of Havazelet and Barkat (1979). The stripped O-Ne-Mg cores will not follow the same track in the \(\rho_C - T_C\) diagram, because these bare cores will not grow in mass like the non-stripped C-O cores do by nuclear burning in the helium and hydrogen layers. After central or off-centre carbon burning the O-Ne-Mg cores will grow in mass by the rate at which carbon burns in a shell; the growth will be halted if the carbon-burning shell is convective; eventually, the growth will be controlled by the remnant helium-burning shell as soon as the carbon-burning shell is very thin; however, in our helium stars the growth is halted, because the helium-burning shells are convective, cf. Barkat et al. (1974) and Miyaji et al. (1980).

(ii) The fate of helium stars in the mass range 1.9 (± 0.1) - 2.20 (± 0.05) M\(_\odot\) (without loss of the envelope)

If a second mass-transfer phase or extensive stellar wind mass loss does not occur before central carbon burning, the 1.9 (± 0.1) to 2.20 (± 0.05) M\(_\odot\) helium stars may develop fluid electron degenerate O-Ne-Mg cores of which the less massive ones will either finally be disrupted or will form solid O-Ne-Mg white dwarfs and of which the more massive ones will become unstable to electron captures or even to photo-disintegrations. Our 2.0 M\(_\odot\) helium star has a core mass of \(< 1.22 M_\odot\) in the last computed model, but it may become as large as 1.28 M\(_\odot\) at the end of the carbon shell-burning phase. Stellar evolution computations of Woosley et al. (1984) and Nomoto (1982d) for a 2 M\(_\odot\) helium star yield smaller convective helium-burning cores than our calculations do (see Chapter IV). Therefore, we expect that our 2.0 M\(_\odot\) helium star is likely to undergo an electron-capture supernova explosion leaving a neutron star remnant (> 1.2 M\(_\odot\)), as is suggested to occur in 2 to 2.5 M\(_\odot\) helium stars with hydrogen envelopes by Hillebrandt et al. (1984) and Nomoto (1984a, e). We thus do not expect it to be disrupted by deflagrative burning (and leaving no compact remnant), see Müller and Arnett (1984), Woosley et al. (1984), and Nomoto (1982d), or to form a white dwarf with a solid O-Ne-Mg core [as suggested by Nomoto (1980a, 1984a) and see above].
(iii) The fate of helium stars which attain core masses too low to leave a neutron star remnant in the course of their evolution

a. with and without loss of the helium envelope.

Finally we examine the fate of those helium stars which do not ignite carbon just after core-helium burning (i.e., in case that a second mass-transfer or a stellar wind mass-loss phase occurs: for helium stars with masses < 2.15 M_☉ as estimated from Figure 25, and with masses < 1.9 ± 0.1 M_☉ if such phases do not occur). If after a first mass-exchange phase a helium star less massive than ∼ 1.4 M_☉ is left behind, this helium star will never ignite carbon. The C-O core of such a star keeps growing in mass up to its outer boundary, but the core mass is always too small to ignite carbon (see e.g., Becker and Iben, 1980). Such a helium star ends as a solid C-O white dwarf or as a planetary nebula.

If after a second mass-transfer or stellar wind mass-loss phase the helium layers are lost and if the C-O core is less massive than ∼ 1.005 to 1.06 M_☉ (see Chapter IV) the core fails to ignite carbon. These stripped stars will end their lives as solid C-O white dwarfs too. The progenitor helium star of such a stripped C-O core was < 2.15 M_☉, but > 0.85 M_☉ (this is required in order to undergo a second mass-transfer phase, see section IIIIB).

The helium stars in the mass range ∼ 1.4 to 1.9 (± 0.1) M_☉ can only ignite carbon explosively under very degenerate conditions at the time when the mass of the C-O core has grown towards ∼ 1.4 M_☉. In case these stars have undergone an additional mass-transfer phase or extensive stellar wind mass loss phase by which it is no more possible to develop a C-O core of 1.4 M_☉, these stars will end as C-O white dwarfs. Iben and Renzini (1983, and references therein) find evidence that these C-O white dwarfs have a maximum mass of 0.9 to 1 M_☉. At the time the C-O core of the helium stars in this mass range, with hydrogen envelopes, is about 1.4 M_☉, these stars may eventually deflagrate and produce Type I supernovae, see Nomoto (1982d). The same will probably happen to helium stars with helium envelopes. Present numerical calculations of the collapse phase indicate a total disruption of the star at the explosion. Alternatively, if one considers the still rather poor numerical treatment and approximations made in the input physics, it may not be excluded that these stars will not be completely disrupted in the explosion and that they leave O-Ne-Mg white dwarfs.
b. the evolution of the core.

Single hydrogen stars of ~ 3 to 7 \( M_\odot \) when obtaining a mass of the helium core of ~ 0.57 to ~ 1.45 \( M_\odot \) will after core-helium burning follow a core-mass luminosity relation (cf. Paczynski, 1970a and 1971b; Kippenhahn, 1981) and the C-O cores will evolve along a common cooling track in the \( p_C - T_C \) diagram up to the point were carbon ignites explosively (here the effects of neutronization and crystallization are neglected; the more massive ones of these stars will follow these common tracks due to the reduction of the C-O core mass by the penetration of the convective hydrogen envelope into the helium core). For case B, case C, and case BB evolution in binaries the helium star remnants do not follow a common track (see Figure 16 of section IIIB. Furthermore, a common core-mass luminosity relation will not exist for mass-exchange remnants.

(iv) **Summary of the expected final evolution of helium and hydrogen stars**

In Table 14 we summarize the expected various types of final evolution of helium and hydrogen stars as described above. The table was adapted from table 8.4 of Van den Heuvel (1983). We include the fact that hydrogen stars more massive than about 3 \( M_\odot \) do ignite helium [Paczynski (1970b, for \( X = 0.7 \) and \( Z = 0.03 \); however, Havazelet and Barkat (1979) found that a 2 \( M_\odot \) hydrogen star with \( X = 0.7 \) and \( Z = 0.01 \) also ignited helium], and that helium stars in the mass range ~ 0.85 ~ 2.9 \( M_\odot \) become (almost) red giant stars after convective core-helium burning (see section IIIB). We assumed for the construction of Table 14 that the mass of the initial hydrogen star is four times that of the helium core.

In Figures 26a and 26b we show the theoretical mass limits for single helium stars (with and without wind mass loss) and for binary helium stars with masses in the range 0.85 - 4.0 \( M_\odot \). In the following we briefly explain the causes for the different outcomes of single star and binary evolution as displayed in these Figures. First, observations of young Galactic clusters (see before) suggest that C- or O-ignition and the thereby-induced deflagration (or, less likely, detonation) in the fluid, electron degenerate cores of (single) young white dwarfs may not always occur due to e.g. substantial stellar wind mass loss. Instead, these young white dwarfs become old white dwarfs with solid cores. Second, if a binary star deflagrates due to
mass accretion, the outcome will be complete disruption in most cases, but in a few cases a remnant white dwarf is found to be left. Furthermore, there is an overlap between the range of core masses for which O-Ne-Mg white dwarfs are formed and the range of core masses for which electron-capture or even photodisintegration triggered supernova explosions occur. Both these mass ranges appear to be very sensitive to the input physics. In the case of binary evolution the precise outcome is additionally dependent on the orbital period and mass ratio of the binary system.

Following Nomoto (1984a) and Weidemann (1979) we introduce a limiting initial mass $M_{\text{wd}, \text{single}}$ for a single hydrogen star defined as follows: $M_{\text{wd}, \text{single}}$ is the maximum mass of a single hydrogen star that develops a degenerate O-Ne-Mg core that terminates its life as a solid O-Ne-Mg white
Fig. 26b.—The theoretical mass ranges for the various types of final evolution of helium stars in binaries as derived in this paper (see the text for an explanation). The helium stars are assumed to be formed by evolution dominated by cases B and C of mass transfer. In cases in which a second mass-transfer (case BB) phase occurs, the remnant mass of the C-O or O-Ne-Mg white dwarf is not shown. The C- and O-deflagrations may occur only for those primaries more massive than 1.9 (± 0.1) M\(_\odot\) which lose a substantial fraction of their mass in a second mass-transfer phase or in a stellar wind mass-loss phase. The precise outcome of the evolution depends also on the initial orbital period and mass ratio of the binary.

...
an electron-capture supernova explosion, respectively. In the latter case they leave a neutron star remnant. However, above another critical initial mass for single hydrogen stars, M_e.c., single; electron captures do not occur and the O-Ne-Mg cores of these stars will undergo collapse triggered by photo-disintegrations (this is definitely the case for helium stars more massive than 2.20 ± 0.05 M_☉). Observations (see paragraph 1, above) and theory (see Nomoto (1984a, b) and Hillebrandt et al. (1984), see paragraph 1, above) indicate that M_wd, single is in the mass range 6 to 9 M_☉; whereas M_e.c., single is in the mass range ~7 to ~9 M_☉ according to our results. The upper limits to M_wd, single and M_e.c., single following from our calculations can be the same (~9 M_☉, if one assumes that the high upper mass limit for formation of O-Ne-Mg white dwarfs is attained due to, for example, extensive stellar wind mass loss), and they are in the middle of the range for these limits derived by Nomoto (1984a) and Hillebrandt et al. (1984). However, in the following part of this section we will adopt that M_wd, single is 8 M_☉, which does not include, for example, the effect of stellar wind mass loss after the ignition of carbon in the centre of the star. Similarly we define the limiting initial masses M_wd, binary and M_e.c., binary for hydrogen star primaries in a binary system.

The following differences between the evolution of single hydrogen stars and of hydrogen primaries with mass M_1 in close binaries (which is dominated by mass transfer by cases B, BB, and C) are found (assuming no total loss of the hydrogen envelope by stellar wind and no other means to prevent carbon or oxygen ignition or electron captures to occur in low- to intermediate-mass single hydrogen stars):

(a) for ~ 3 - 4 M_☉ < M_1 < ~ M_2 (case BB), where M_2 is in the range 5.6 M_☉ - M_wd, binary (and has a value which depends on the binary period and mass ratio), and

for ~ 3 - 4 M_☉ < M_1 < ~ 5.6 M_☉ (cases B and C):
the primaries terminate as solid C-O white dwarfs (or solid O-Ne-Mg white dwarfs in case BB evolution) instead of evolving towards C- or O-deflagration as in single stars. If a second mass-transfer phase does not occur, primaries with masses in the range ~ 5.6 M_☉ to M_wd, binary (< ~ 8 M_☉, for case C evolution) will undergo C- or O-deflagration.

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(b) for $M_{e.c.}$, single $< M_1 < M_3$, where $M_3$ is in the range $M_{e.c.}$, single - 14 $M_\odot$ (and has a value which depends on orbital period and mass ratio):

1) the primaries of close binaries evolve after a short or long cooling time ($< \text{a few times } 10^6 - 10^7 \text{ years and }> \text{a few times } 10^8 \text{ years, respectively}$) into fluid or solid $0$-$Ne$-$Mg$ white dwarfs, respectively, whereas single hydrogen stars almost immediately evolve into a photo-disintegration-induced collapse (within $< 3 \times 10^5 \text{ years after the end of core-helium burning}$).

2) in a later stage the primaries of close binaries either will be disrupted by deflagrative carbon- or oxygen-burning or they will undergo an electron-capture supernova (cases B and C) or leave solid $0$-$Ne$-$Mg$ white dwarfs (in the lower part of the mass range for case BB evolution), instead of evolving into a photo-disintegration supernova like single hydrogen stars in that mass range do.

3) if reversed mass transfer (from the original secondary to the original primary) takes place, this mass accretion onto the original primary leads to complete disruption or, alternatively, to the formation of a solid C-O or $0$-$Ne$-$Mg$ white dwarf (in case of a low mass-accretion rate and for the lowest part of the mass range for case BB evolution), and for all other cases to an electron-capture supernova (leaving a neutron star remnant).

(c) for case B and case BB evolution the upper limit to $M_{e.c.}$, binary is 12 $M_\odot$ and 14 $M_\odot$, respectively, and the lower limit to $M_{wd}$, binary is $< 8$ and $< 10 M_\odot$, respectively; for case C evolution the upper limits to $M_{wd}$, binary and $M_{e.c.}$, binary are about the same as for single hydrogen stars, i.e. about 8 and 9 $M_\odot$, respectively.

3. SUMMARY OF OBSERVABLE SUPERNOVA EFFECTS EXPECTED FOR THE VARIOUS TYPES OF EVOLUTION

Here we summarize the possible observational appearances of the various types of final evolution of binaries and single stars as summarized in Table 14.

1) helium cores of $1.4 - 1.9 (\pm 0.1) M_\odot$

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If in a binary a second mass-transfer phase is avoided a deflagrative supernova explosion may occur for helium stars in this mass range (i.e. \( \sim 5.6 \) to \( \sim M_{\odot} \), binary for hydrogen star primaries undergoing case C mass exchange). The optical lightcurve is expected to resemble that of a Type I supernova [i.e. hydrogen poor - lack of hydrogenic lines, Woosley et al. (1984)], because sufficient \(^{56}\)Ni will be formed by the explosion in order to produce such a lightcurve, see e.g. Nomoto (1984d), and also due to the lack of a possibly hydrogen-rich envelope the light curve will resemble that of a Type I supernova (Falk and Arnett, 1973, and Weaver and Woosley, 1980); the low masses of the helium stars are consistent with the general opinion that Type I supernovae are produced by low-mass stars [which is supported by observations and theory, see Woosley et al. (1984) and see the references therein].

If after a second mass-transfer or stellar wind mass-loss phase so much mass of the helium layers is lost that the remnant C-O core has a mass less than \( M_{\text{Ch}} \), then the primary will end as a solid, electron degenerate C-O white dwarf. In case that not too much of the helium layers is lost the mass of the fluid C-O core can grow beyond \( M_{\text{Ch}} \) and possibly a Type I supernova occurs, because of lack of hydrogen in the envelope [the explosion will be dim if the helium envelope is not of red-giant size, cf. Woosley et al. (1984)].

In single hydrogen stars less massive than the mass limit for single stars to form C-O white dwarfs, \( M_{\odot} \), single C-O (\( \sim 8 M_{\odot} \)) but more massive than \( 5.6 M_{\odot} \), just like in case C binaries, carbon deflagration of the C-O core leads to a Type I supernova (cf. Nomoto, 1982a, b, c, d) if no extensive stellar wind mass loss takes place (Nomoto, 1984a) or other means to prevent carbon ignition in these stars do not work (cf. Trimble, 1982b). [Heavy mass loss during the asymptotic giant branch phase is suggested by the observations, cf. Weidemann (1984a), Weidemann and Koester (1983), and Reimers and Koester (1982). Therefore, the final product of the evolution of these single stars is not always carbon deflagration, but may be a rather massive white dwarf with a C-O core.].

\( (11) \) Helium cores of mass 1.9 (\( \pm 0.1 \)) - 2.20 (\( \pm 0.05 \)) \( M_{\odot} \)

Possibly also helium primaries in this mass range give rise to Type I supernovae (if a second mass-transfer or extensive stellar wind mass-loss phase does not occur in the helium red-giant phase). While the helium primaries in the mass range 1.4 to 1.9 (\( \pm 0.1 \)) \( M_{\odot} \) most likely do not leave any
remnant after the supernova explosion, the helium stars in the mass range 1.9
(± 0.1) to 2.20 (± 0.05) M⊙ do leave a neutron star remnant if an electron-
capture supernova occurs. It is not clear whether such a supernova will be of
Type I or II. Alternatively, stars in this mass range may not explode at all
and leave a white-dwarf remnant with a solid O-Ne-Mg core, instead of
undergoing a (C- or) O-deflagration supernova explosion (of Type I ?) and
leaving no remnant.

For single stars in the mass range M^singl e- M^singl e, single, (i.e.
with helium core masses of 1.9 (± 0.1) to 2.20 (± 0.05) M⊙ and corresponding
to hydrogen stellar masses of < 7.6 to 9 M⊙) an electron-capture supernova of
Type II may (or may not, see before) occur leaving a neutron star remnant with
mass > 1.2 M⊙ (or an O-Ne-Mg white dwarf), cf. Hillebrandt et al. (1984). In
case the hydrogen envelope is not lost by stellar wind, if other means to
prevent O-ignition do not work (but which prevent electron-capture reactions
to dominate) O-deflagration of the core occurs leading to supernova presumably
looking like a Type II (with possibly no compact remnant).

(iii) Helium cores in the mass range 2.20 (± 0.05) - 5 M⊙

A photo-disintegration supernova of Type II is expected to occur for
single hydrogen stars of ~ 9 to ~ 20 M⊙ (i.e. with helium cores of about 2.20
(± 0.05) to 5.0 M⊙) leaving either a neutron star or a black hole. (Most of
these supernovae have spectra with hydrogen, helium and nitrogen lines from
the ejectae, but some do not show these lines and have oxygen-rich ejectae).
On the other hand, in helium stars with helium envelopes and with masses in
the range 2.20 (± 0.05) to 5.0 M⊙ the photo-disintegration-induced supernova
event will be very dim and brief (due to the small extent of the helium
envelope, see e.g. Woosley et al., 1984). If the pre-supernova star was
rapidly rotating, the supernova shell is expected to have a ring structure
[which is seen in oxygen-rich supernova remnants, Canizares (1984)]. A black
hole may be formed if no bounce occurs during the core collapse, cf. Arnett
(1979a), Rees et al. (1974), and Woosley and Weaver (1982a, b) which is,
however, not very likely (Woosley and Weaver, 1984). On the other hand,
Bodenheimer and Woosley (1983) find evidence from two-dimensional stellar
evolution collapse calculations that a 25 M⊙ hydrogen star with a rapidly
rotating core will evolve into a supernova explosion and that the supernova
remnant will become so massive (~ 3.8 M⊙) that it ends as a black hole. The
supernova event for such a star is expected to resemble one of Type II, but to be less luminous and to last longer.

(iv) Supernovae produced by accretion onto white dwarfs

Mass accretion onto massive C-O or O-Ne-Mg white dwarfs in close binaries is expected to give rise to a Type I supernova [because sufficient $^{56}$Ni is produced by carbon detonation or deflagration of the core, see e.g. Nomoto (1980b), Isern et al. (1984)] or to a "silent" electron-capture triggered supernova event, respectively. [The explosion will be faint due to the lack of an extended envelope around the star (see e.g. Woosley et al., 1984) and due to the small amount of $^{56}$Ni produced (see e.g. Arnett, 1982, and Nomoto, 1984b, and references therein). In the O-Ne-Mg white dwarf most of the energy released in the explosion and related to the shock will turn into kinetic energy of the explosion (Nomoto, 1984b) and only a small amount of mass will be ejected by the explosion (Nomoto, 1980a). Hence, the optical event will be dim, which is called a "silent supernova" (Nomoto, 1984b) or "quiet supernova" (Nomoto, 1980a)]. In case that the accreting star is an O-Ne-Mg white dwarf a neutron star will be formed (Nomoto et al., 1984b) and in the case of C-O white dwarfs it is a matter of debate whether or not there will be a stellar supernova remnant (for the more massive C-O cores either a white dwarf or a neutron star): depending on the accretion rate the less massive C-O white dwarfs will either collapse and being disrupted completely, or still leave a white dwarf remnant, see Nomoto et al. (1984b), Isern et al. (1984), and Sutherland and Wheeler (1984).

4. CONCLUSIONS.

Our calculations of the evolution of helium stars with growing, convective helium-burning core masses lead to the following lower mass limits for progenitors of neutron stars (we assume the mass of the initial hydrogen star to be four times that of the helium core):

If electron-capture supernovae do occur single hydrogen stars of 7.6 ($\pm$ 0.4) - 8.8 ($\pm$ 0.2) $M_\odot$ are expected to yield Type II supernovae leaving a neutron star of mass $> 1.2 M_\odot$. Hydrogen primaries in binary systems may undergo the same fate if they have about the same mass, in case C mass-transfer dominated evolution and if they have masses in the range 7.6 ($\pm$ 0.4)
- 12 $M_\odot$ in case B mass-transfer dominated evolution; in these cases a second mass-transfer phase must be avoided. In case B the precise value of the upper mass limit for this type of evolution depends on the initial orbital period and mass ratio. Furthermore, if electron captures do occur in single hydrogen stars down to the low masses as indicated by Hillebrandt et al. (1984), the upper limit for formation of O-Ne-Mg white dwarfs will be 8 $M_\odot$. However, the core of a single hydrogen star with a mass larger than 9 $M_\odot$ will collapse, because it attains a mass larger than $M_{\text{Ch}}$ during carbon burning. Accordingly, the mass range to produce O-Ne-Mg white dwarfs from single stars might be very small if one considers the high upper mass limit of 8 $M_\odot$ for formation of C-O white dwarfs as obtained by Iben and Renzini (1983) and in this work.

Single hydrogen stars more massive than 8.8 ($\pm$ 0.2) $M_\odot$ are expected to evolve into a Type II supernova, most likely initiated by photo-disintegration reactions leaving a neutron star remnant of $\sim 1.4 M_\odot$ [if the core bounce mechanism for the explosion works, Woosley and Weaver (1984), however, see Hillebrandt (1984)]. Hydrogen star primaries of about the same mass in case C, with masses in the range 8.8 ($\pm$ 0.2) to 12 $M_\odot$, and in any case with masses larger than 12 $M_\odot$ in case B; and with masses in the range 8.8 $M_\odot$ ($\pm$ 0.2) to 14 $M_\odot$, and in any case with masses larger than 14 $M_\odot$ in case BB mass-transfer dominated evolution are expected to have the same fate. The precise value of the lower mass limit in case B and case BB again depends on the initial orbital period and mass ratio.

Furthermore, from these types of evolution one infers that: hydrogen primaries in close binaries with masses in the range 7.6 ($\pm$ 0.4) - 14 $M_\odot$ can yield white dwarfs with C-O or O-Ne-Mg cores in (close) low-mass cataclysmic variable systems and in (wide) symbiotic systems, which may end as neutron stars upon mass accretion from the companion. To achieve this accretion, a reverse mass-transfer phase (either after a case B or C mass transfer together with substantial stellar wind mass loss after core-helium burning or after a second mass transfer (case BB)) has to be invoked. However, in case of the O-Ne-Mg white dwarfs the corresponding supernovae will eject only little mass and are called quiet or silent supernovae (not of Type II and, most likely, also not of Type I).

The theoretical lower mass limits which we estimated for hydrogen stars (single as well as in close binaries) to form neutron stars, are consistent with the lower mass limits derived observationally for progenitors of pulsars in the solar neighbourhood [Blaauw (1984) and Lyne et al. (1984)] and for Type
II supernova progenitors (Kennicutt, 1984), as well as with the observational upper mass limit to progenitors of single white dwarfs. They are also consistent with the weak observational upper mass limit to the progenitor of white dwarfs in close binaries which yield Type I supernovae upon mass accretion from the companion (see e.g. Weidemann and Koester, 1983, who give an upper limit of $8 \, M_\odot$ to the initial mass of the progenitor of a single white dwarf, which is also a weak upper mass limit to the progenitor of a white dwarf in a close binary; see also Reimers and Koester (1982) who give an upper limit of $8 \pm (3, -2) \, M_\odot$).

The Type I supernovae are thought to be either formed by carbon or oxygen deflagration in the C-O or O-Ne-Mg cores of single hydrogen stars less massive than $\sim 8 \, M_\odot$ and $9 \, M_\odot$, respectively. Or, alternatively, induced by mass accretion onto not too massive white dwarfs with C-O or O-Ne-Mg cores in binaries which have descended from hydrogen primaries of $\sim 8 - 12 \, M_\odot$ or $\sim 10 - 14 \, M_\odot$ (roughly) after case B or case BB evolution dominated by mass-transfer and/or stellar wind mass loss. However, the Type I supernovae do probably not result from mass accretion onto white dwarfs with O-Ne-Mg cores, and with masses larger than $\sim 1.37 \, M_\odot$. Moreover, the carbon and oxygen deflagration of the cores of single hydrogen stars with masses less than $\sim 9 \, M_\odot$ may not have occurred in the young Galactic clusters studied by Weidemann and Koester (1983) and Reimers and Koester (1982). In these clusters hydrogen stars with masses less than 8 or 9 $M_\odot$ appear to have ended their lives as white dwarfs. Hence, it is expected that the Galactic Type I supernovae result almost exclusively from mass accretion onto not too massive white dwarfs with C-O or O-Ne-Mg cores (with masses smaller than $M_{\text{Ch}}$ or smaller than 1.37 $M_\odot$, respectively).