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CHAPTER 1

MASS-FLUX INDUCED ORBITAL-PERIOD CHANGES IN X-RAY BINARIES
1. MASS-FLUX INDUCED ORBITAL-PERIOD CHANGES IN X-RAY BINARIES

I. Introduction

X-ray binaries are good candidates for the detection of orbital-period changes, for two reasons. First of all, we know that mass transfer is taking place in these systems, and simple theoretical considerations lead us to expect an influence on the orbital period from this. Secondly, using the Doppler shifts of the pulsation which are often emitted by the accreting component of the binary, the position of this component in its orbit can be measured accurately enough to detect changes in the orbital period within a few years. In many cases, eclipses give us another handle on the orbital period.

The first report of a changing orbital period in an X-ray binary system, Cen X-3, was due to Schreier (1973). His original claim has essentially been confirmed by further observations (see papers 3.1 and 4.1 and Kelley et al. 1982).

Since, orbital period variations have been detected in a few other X-ray binaries: Cyg X-3 (Manzo et al. 1978 and paper 2.1), Vela X-1 (paper 4.2). A strict upper limit has been obtained for Her X-1 (Deeter et al. 1981). The situation is still unclear for SMC X-1 (paper 3.2) and a number of other sources for which orbits have been determined more recently. A more extensive review of the observational data is given in section IV of this paper.

Early theoretical work on the orbits of binaries with secularly changing component masses was carried out by Stromgren (1903) and Jeans (1924). For a recent discussion, including the effects of mass transfer, mass loss and asymmetric interaction between the stars and the gas streaming in the system, and for further references, see Kruszewski (1966).

In principle, the rate of change of the orbital period contains information on the way the mass fluxes occur in the system and on the magnitude of these fluxes. Apart from the mass streams, however, there are several other ways in which an X-ray binary system can change its orbital period. Gravitational radiation (Paczynski 1967) has to be considered only for very compact systems (such as, for example, the 41 min. orbital-period system 4U 1626-67), and is therefore negligible for the massive systems containing an X-ray pulsar considered here. Apsidal motion can cause apparent changes in the period of the eclipses (Batten 1973, see paper 2.2), but this does not influence the orbital period as measured from Doppler shifts (Deeter et al. 1981, see paper 4.2).

Tidal interaction, finally, could in principle be important in each of the systems mentioned. The time scale on which tidal interaction can change the orbital period, depends very sensitively on factors such as the stellar radius, the internal structure of the star (Zahn 1977, Savonije and Papaloizou 1982), and the interplay between tidal synchronization and circularization (see, e.g., Hut 1982). These factors cannot presently be determined with sufficient accuracy from observations, or cannot even be observed at all. Therefore, the effects of tidal interaction can in most systems not be predicted with much confidence.

For this reason, observed orbital parameter changes could in many cases in principle be due to either tidal interaction, or to mass streams in the system, or to a combination of both. Because the mass fluxes are usually easier to estimate than the parameters determining the tidal interaction, the hopes are to obtain information on both mechanisms by a careful study of the mass streams. If in a specific case it can be established that the mass streams are not sufficient to explain an observed effect, the conclusion that tidal interaction must be important in this system will place strong constraints on the stellar parameters.

In the present paper, attention will be restricted exclusively to the effects of the mass flows on the orbital period.

In section II, the basic equations are given which describe the changes of binary orbital parameters, and the procedure is outlined by which these equations will be used to estimate

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It is noted here, that also the position of the line of apsides can be influenced by the mass streams (Hadjimetriou 1967, Huang 1956): apsidal motion does not have to be exclusively tidal in origin.
the change of the orbital period.

Section III describes which changes in orbital period can be expected to occur for various assumptions about the nature of the mass loss and the mass transfer. As a part of this, in section III.3 a parametrized description is introduced of the changes of orbital angular momentum which can be expected as a consequence of the mass streams. This parametrization is meant to facilitate the evaluation of the effects of different model assumptions. Physics is kept to the level of conservation laws; no attempts are made to apply the perturbation techniques of celestial mechanics for specific mass-stream trajectories.

At the end of this paper, in section IV, the results of these considerations are compared to the available observations.

For a discussion of the observational considerations pertinent to the measurement of periods of binaries with eccentric orbits, the reader is referred to paper 4.2.

II. Theory

1. Basic equations

Consider a binary system consisting of two spherically symmetric stars with masses \( M_1 \) and \( M_2 \). The relation between the orbital period \( P \) and the semi-major axis \( a \) of the relative orbit is given by Kepler's third law as:

\[
\frac{P^2}{a^3} = \frac{4\pi^2}{GM}
\]

where \( M = M_1 + M_2 \) is the total mass of the system.

The orbital energy \( E \) and orbital angular momentum \( J \) (see, e.g., Kruszewski 1966) are:

\[
E = -\frac{GM_1 M_2}{2a}
\]

\[
J = 2\pi a^2 \frac{M_1 M_2}{M} \left(1-e^2\right)^{1/2}
\]

with \( G \) the gravitational constant and \( e \) the orbital eccentricity.

The structure of these equations is such, that if for each of the two stars the rate of change of the mass is known, the way in which perturbing forces, whatever their nature, influence \( a \), and thus \( P \), is completely determined by their effect on the orbital energy \( E \) only. The changes in both \( E \) and \( J \) have to be known, however, to make it possible to predict the effect on \( e \). Note that, mathematically, \( e \) and \( J \) are inseparable.

2. Procedures to estimate \( \dot{P} \)

Changes in the orbit due to interaction of the components of the binary system with the mass streams can occur in many ways: by mass loss (not necessarily isotropic) and mass transfer, both possibly variable over the orbit, and by various forms of friction. Calculations for a few idealized cases can be found in, e.g., Huang (1956), Kiang (1961), Chevalier (1975). These authors assume some specific perturbing force, find its effect on \( E \) and \( J \) as a function of orbital phase, and calculate the time-averaged effect on the orbital motion by integration.

In realistic cases, however, it is difficult, in the presence of dissipative mechanisms, to estimate directly the rate of change of the orbital energy \( \dot{E} \).

Another approach (Huang 1963, Plavec 1968, Paczynski 1971, van den Heuvel and De Loore 1973) has proven useful to estimate time scales for orbital-period variations. In this
procedure, a direct estimate of $\dot{E}$ is avoided by estimating instead the change of the orbital angular momentum $J$ and of the eccentricity. $\dot{J}$ is easier to estimate than $\dot{E}$, because, contrary to energy, angular momentum cannot be dissipated or generated within the system. The problems associated with estimating $\dot{e}$ are discussed below.

It is this approach which I will adopt in this paper. Rewriting equations (1)-(3) (in differential form) in a way adapted to the described procedure, we have the following general relations between the changes of the masses, orbital elements and orbital energy and angular momentum:

$$\frac{\dot{M}}{M} = -3 \left( \frac{M_1}{M_1} + \frac{M_2}{M_2} \right) + \frac{\dot{M}}{M} + \frac{3\dot{e}}{1-e^2} + 3 \frac{\dot{J}}{J}$$  \hspace{1cm} (4)

$$\frac{\dot{a}}{a} = \frac{2\dot{P}}{3P} + \frac{1}{3} \frac{\dot{M}}{M}$$  \hspace{1cm} (5)

$$\frac{\dot{E}}{E} = -\frac{\dot{a}}{a} + \left( \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} \right)$$  \hspace{1cm} (6)

Note, that though $\dot{E}$ is not directly estimated in our procedure, its value follows from the other estimates. Occasionally, it can be instructive, to check how a certain model requires $E$ to change.

a) Estimating $\dot{e}$

The price we pay to avoid estimating $\dot{E}$ directly, is that in addition to $\dot{J}$, we now have to estimate the contribution of a term containing $\dot{e}$. This task is alleviated by the fact, that $\dot{e}$ appears in the equations (see eq. (4)) multiplied by a factor $e$, which is itself in many cases a small quantity.

If the orbit has a finite eccentricity, quite undetectable changes in $\dot{e}$ can already be important for the overall picture: The typical order of magnitude of the mass-change terms in massive X-ray binaries with mass loss in the form of a stellar wind is known from observations to be $\sim 10^{-7}$ y$^{-1}$. For, say, $e \sim 0.1$ (as observed in Vela X-1, see paper 4.2), a change of $e$ on a time scale $e/\dot{e} \sim 10^5$ y or less would already be sufficient to put the $\dot{e}\dot{e}$-term in eq. (4) on the same order of magnitude level of $\sim 10^{-7}$ y$^{-1}$.

In practice, in an eccentric binary mass streams which can change the eccentricity, will also affect the orbital angular momentum; it will depend on the details of the mechanisms involved what will be the net effect of the last two terms in eq. (4) on the orbital period. (The same is true for the effect of tidal circularization and decircularization.)

For some modes of mass loss and transfer, at least, it has been established theoretically that $\dot{e} = 0$. This is true for isotropic mass loss or accretion (Jeans 1924) and for ejection of matter along the line joining the two stars (Huang 1956). For mass transfer which occurs primarily at periastron, Piotrowski (1964) finds for $e \ll 1$ that $\dot{e} \sim -2K/M$ (Lea and Margon 1973), which would make the $\dot{e}\dot{e}$-term in eq. (4) a factor $\sim e$ smaller than the mass flux terms.

Observationally, for nearly circular orbits, it is possible by repeated orbital determinations to set useful upper limits to the $\dot{e}\dot{e}$-term: Careful analysis of Doppler-delay curves of X-ray pulsars can put upper limits to $e$ of $\sim 10^{-8}$ (Deeter et al. 1981, Primini et al. 1977). If such an observation is repeated after one year, we already have $e\dot{e} < 10^{-8}$ y$^{-1}$.

Throughout most of this paper, I will assume the $\dot{e}\dot{e}$-term to be zero, but it is good to remember that it exists, and could play an important role in some cases.

b) Estimating $\dot{J}$

If we consider only the effects of the mass streams, and neglect tidal effects, as I intend to do in this paper for reasons stated in section I, the only way in which $\dot{J}$ can change is by the ejection from the system of matter that carries off orbital angular momentum.

One of the main objectives of the different models discussed in the next section is to provide a way to estimate the amount of orbital angular momentum taken away by the matter that
is leaving the system. It will be shown that plausible assumptions about the loss of orbital angular momentum usually imply \( J \) to be of order \( jM \), where \( j \) is the specific orbital angular momentum of the binary system, defined as \( J/M \). This means that \( J/J \) will be of order \( \dot{M}/M \). It is clear then from equation (4), that apart from the effect of eccentricity changes, the relative changes in orbital period are in general expected to be of the order of the relative mass changes.

III. Models for loss of mass and angular momentum from the binary system

In this section I first examine two simple models for the mass streams frequently found in literature, and then proceed to rewrite equations (4)-(6) in a way designed to facilitate investigation of the effects of different model assumptions.

Throughout this paper the index 1 will refer to the mass-losing, the index 2 to the mass-gaining star, so that \( \dot{M}_1 < 0 \), \( \dot{M}_2 > 0 \), and, in the absence of matter inflow into the system, \( \dot{M} < 0 \). The two stars will be referred to as 'primary' and 'secondary', respectively, regardless of their relative mass.

1. No mass loss from system

The model in which no mass loss from the system is allowed (Plavec 1968, Paczynski 1971), i.e., \( \dot{M} = 0 \) and \( \dot{J} = 0 \) is sometimes referred to as the 'conservative' case, although the orbital energy is in general not conserved in this model.

![Diagram](image)

It is easily found, that

\[
\frac{\dot{R}}{R} = -3 \frac{M_2}{M_1} \left[ 1 - \frac{M_2}{M_1} \right] + \frac{3e\dot{e}}{1-e^2} \tag{7}
\]

\[
\frac{\dot{a}}{a} = \frac{2}{3} \frac{\dot{P}}{P} \tag{8}
\]

\[
\frac{\dot{E}}{E} = 3 \frac{M_2}{M_1} \left[ 1 - \frac{M_2}{M_1} \right] - \frac{2e\dot{e}}{1-e^2} = -\frac{\dot{P}}{P} + \frac{\dot{e}}{1-e^2} \tag{9}
\]

Assuming \( e\dot{e} = 0 \), we have (remember that \( E < 0 \)):

\[
\begin{align*}
M_1 > M_2 & \quad \dot{P} < 0 \quad \dot{e} < 0 \quad E < 0 \\
M_1 < M_2 & \quad \dot{P} > 0 \quad \dot{e} > 0 \quad E > 0
\end{align*}
\tag{10}
\]

By specifying \( e\dot{e} \), the effects of the mass transfer have become completely determined. For \( M_1 < M_2 \), for example, the orbit will, in the absence of other mechanisms than mass transfer, expand, and the orbital energy must increase. Table 1 shows what conclusions can still be drawn when the condition \( e\dot{e} = 0 \) is relaxed.
Table 1. Effects of mass transfer (eccentric orbit).

<table>
<thead>
<tr>
<th>Assumptions +</th>
<th>( \dot{e} &gt; 0 )</th>
<th>( \dot{e} &lt; 0 )</th>
<th>( \dot{P} &gt; 0 )</th>
<th>( \dot{P} &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conclusions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_1 &gt; M_2 )</td>
<td>*</td>
<td>( \dot{P}, \dot{E} &lt; 0 )</td>
<td>( \dot{e} &gt; 0 )</td>
<td>( \dot{E} &lt; 0 )</td>
</tr>
<tr>
<td>( M_1 &lt; M_2 )</td>
<td>( \dot{P}, \dot{E} &gt; 0 )</td>
<td>*</td>
<td>( \dot{E} &gt; 0 )</td>
<td>( \dot{e} &lt; 0 )</td>
</tr>
<tr>
<td>( \dot{P} &gt; 0 )</td>
<td>*</td>
<td></td>
<td>( M_1 &lt; M_2 )</td>
<td>( \dot{E} &gt; 0 )</td>
</tr>
<tr>
<td>( \dot{P} &lt; 0 )</td>
<td></td>
<td></td>
<td>( M_1 &gt; M_2 )</td>
<td>*</td>
</tr>
<tr>
<td>( \dot{E} &lt; 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(\( \dot{a} \) always has the same sign as \( \dot{P} \))
* Assumptions not sufficient to constrain other variables: no conclusion.

2. Mass loss in a stellar wind

Matter is leaving the system in this model at a rate \( \dot{M} = \dot{M}_1 + \dot{M}_2 \). The effect on the orbit depends on the amount of orbital angular momentum which this matter carries away. I will first discuss the standard case of a high-velocity wind and then consider two modifications to this model.

![Diagram](image)

a) High-velocity wind

In this case, the matter lost by the primary is supposed to leave the system so quickly that it does not interact with the secondary. As it is assumed to be ejected by the primary in an isotropic way, the specific orbital angular momentum carried away is equal to that of the primary: \( J = (M_2/M_1)J \). For the present purpose, this mode of mass loss is equivalent to the mass being annihilated in the star at a rate \( \dot{M} \), and it was with this in mind that Jeans (1924) first derived the mass-loss term below.

With \( J = j \dot{M} \), we find after some rearranging:

\[
\frac{\dot{P}}{P} = -3 \left[ 1 - \frac{M_2}{M_1} \right] \frac{M_2}{M_2} - 2 \frac{\dot{M}}{M} + \frac{3 \dot{e} \dot{e}}{1-e^2} \tag{11}
\]

For \( M_1 > M_2 \) the mass-transfer term (first right-hand side term) and the mass-loss term (second) have opposite signs, and it depends on the ratio of the accretion rate \( \dot{M}_2 \) and the net mass-loss rate \( \dot{M} \) how the period will change. Specifically, in this case, \( \dot{P} > 0 \) implies (for \( \dot{e} \dot{e} = 0 \)): \( |\dot{M}_2/\dot{M}| < (2/3)[M_1 M_2/(M_1^2 - M_2^2)] \), and vice versa.

In a typical massive X-ray binary powered by accretion from a stellar wind, the accretion
efficiency $|\dot{M}/\dot{M}|$ is typically of the order of $10^{-3}$ to $10^{-5}$ (Davidson and Ostriker 1973, see Ch. 3), and $M_1 M_2/(M_1^2 - M_2^2) = M_2/M_1 \sim 0.05$ to 0.10, so that we can expect $\dot{P}$ to be positive.

b) Low-velocity wind

It can be expected, that due to the interaction of the outflowing matter with the secondary in its orbit (orbital 'stirring' of the wind), the flow will be slightly disturbed. The specific orbital angular momentum of the matter which finally leaves the system will thus be somewhat larger than $\tilde{j}_1$. Assuming $\tilde{j} = a_1 \tilde{j}_1 \dot{M}$, we have (for $\varepsilon = 0$):

$$\dot{P}/P = -3 \left[ \frac{M_2}{M_1} \right] \frac{M_2}{M_2} + \left[ 3(a_1 - 1) \frac{M_2}{M_1} - 2 \right] \frac{M}{M}$$

(12)

The contribution of the net mass loss to $\dot{P}$, and hence, for $M_1 > M_2$, the entire expression, becomes negative only for $a_1 > 1 + (2/3)(M_1/M_2)$, which in the case of a massive binary is an appreciable factor. Consequently, some orbital 'stirring' is not likely to invert the lengthening effect of the stellar wind on the orbital period.

c) Tidal coupling

The outflowing matter will in most cases also have some extra angular momentum which originates in the rotation of the primary. This will only indirectly influence the orbit, by means of tidal coupling.

An upper limit to the effect of this additional angular-momentum loss can be obtained by assuming the tidal-synchronization time scale to be zero (Fabbiano and Schreier 1977). Let the matter leaving the system carry a specific rotational angular momentum $j_{\text{rot}}$ corresponding to a point comoving with the stellar rotation at a distance $r$ from the center of the primary. $r$ can be equal to the stellar radius, or it can be larger, in the case that magnetic acceleration of the outflowing matter takes place (Huang 1966, Verbunt and Zwaan 1981). Then $j_{\text{rot}} = qr^2$, where $q$ is the stellar angular velocity.

We find for a circular orbit the following expression for the maximum contribution to the orbital period derivative of this indirect orbital angular momentum loss:

$$[\dot{P}/P]_{\text{rot}} = 3 \left[ \frac{\omega}{\omega} \right] \left( \frac{r}{a} \right)^2 \frac{M_1}{M_2} + \frac{M_2}{M_1} + 2 \frac{M}{M}$$

(13)

where $\omega = 2\pi/P$, by assumption equal to $\omega$.

It can be seen that, if the tidal-synchronization time scale is really very short, as assumed, this mechanism could have a considerable shortening effect on the orbital period. As noted in Section I, the actual time scale for the tidal interaction is difficult to estimate, however.

3. General case: arbitrary mass-loss mechanism

Although the expression for the orbital-period change (4) does not distinguish between the different possible ways in which the streams of matter can leave the system, it is convenient for the sake of model discussions to split $\dot{M}$ into two components, $\dot{M}_{1\lambda}$ and $\dot{M}_{2\lambda}$ (both $< 0$). $\dot{M}_{1\lambda}$ represents matter ejected from the system by direct loss from the primary (e.g., in a stellar wind) and carries a specific orbital angular momentum $\alpha_1 j_1$, where $\alpha_1$ is an adjustable dimensionless parameter. $\dot{M}_{2\lambda}$ is the mass lost from the system via the critical lobe of the accreting star (e.g., ejected by an accretion disk or via $L_2$), and has a specific orbital angular momentum $\alpha_2 j_2$, with $j_2$ the specific orbital angular momentum of the secondary. We thus have the situation that a particle lost by the primary can end up in one of three 'output channels', $\dot{M}_{1\lambda}$, $\dot{M}_{2\lambda}$, or $\dot{M}_{2\lambda}$. 

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This approach is similar to that of van den Heuvel and De Loore (1973), who, however, assume \( a_1 = a_2 = 1 \) and neglect \( M_2 \).

With
\[
\frac{J}{M} = \alpha_1 \frac{M_2}{M} \frac{dM_1}{dt} + \alpha_2 \frac{M_1}{M} \frac{dM_2}{dt} \tag{14}
\]
and
\[
\dot{M} = \dot{M}_1 + \dot{M}_2 = \dot{M}_{1e} + \dot{M}_{2e} \tag{15}
\]
we have
\[
\frac{\dot{P}}{P} = 3[q-q^{-1}] \frac{M_2}{M} + [3(a_1-1)q - 2] \frac{M_{1e}}{M} + [3a_2q^{-1} - 3q - 2] \frac{M_{2e}}{M} \tag{16}
\]
\[
\frac{\dot{a}}{a} = 2[q-q^{-1}] \frac{M_2}{M} + [2(a_1-1)q - 1] \frac{M_{1e}}{M} + [2a_2q^{-1} - 2q - 1] \frac{M_{2e}}{M} \tag{17}
\]
\[
\frac{\dot{E}}{E} = -3[q-q^{-1}] \frac{M_2}{M} + [(3-2a_1)q + 2] \frac{M_{1e}}{M} + [-2a_2q^{-1} + 3q + 2] \frac{M_{2e}}{M} \tag{18}
\]

where the mass ratio \( q = M_2/M_1 \), and again \( \dot{e} = 0 \) was assumed to be zero.

a) Effects of individual mass streams

From eq. (16), we are now able to answer the following questions:

- What is the total effect on the orbital period of matter ending up in one of the three output channels, measured over its full trajectory starting with ejection by the primary?
- How much should the specific orbital angular momentum of this matter deviate from the canonical \( (\alpha = 1) \) value in order to change this effect by a certain amount, and, in particular, to change its sign?

In both cases, the answers will, in general, depend on the mass ratio \( q \). Figure 1 (upper half) shows in which regions in the \( \alpha,q \)-plane the contributions of the three mass fluxes to the orbital period derivative are positive (+) or negative (−). Table 2 lists the equations of the curves of \( \dot{P} = 0 \) (heavy curves in Figure 1), and the critical value of \( q \) at which \( \dot{P} \) changes sign if \( \alpha = 1 \). It can be noted immediately, that (i) the effect of the accretion onto the secondary only depends on the mass ratio and cannot be changed by changes in the specific orbital angular momentum of the matter involved, and (ii) mass loss in a stellar wind for \( a_1 < 1 \) always leads to a longer orbital period, independent of the mass ratio.

By comparing equations (16) and (17) it is clear that there are narrow regions in the \( \alpha,q \)-plane where \( \dot{a} < 0 \) and \( \dot{P} > 0 \): the system is shrinking while at the same time the period is...
Table 2. Properties of individual mass streams.

<table>
<thead>
<tr>
<th>Output channel</th>
<th>Critical value of $q$ if $\alpha = \alpha_0 = 1$</th>
<th>Curve of $\dot{P} = 0$</th>
<th>Physical picture (example)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>1</td>
<td>$q = 1$</td>
<td>accretion</td>
</tr>
<tr>
<td>$M_{1L}$</td>
<td>$\infty$</td>
<td>$\alpha_1 = (2/3)q^{-1} + 1$</td>
<td>wind mass loss</td>
</tr>
<tr>
<td>$M_{2L}$</td>
<td>$(1/3)(\sqrt{10} - 1) \sim 0.72$</td>
<td>$\alpha_2 = q^2 + (2/3)q$</td>
<td>mass loss from disk or through $L_2$</td>
</tr>
</tbody>
</table>

Increasing. The curves of $\dot{a} = 0$ are indicated in Figure 1 by the lightly drawn curves alongside the heavy $\dot{P} = 0$ curves.

The dotted line in the third frame in Figure 1 represents the $a_2$ of a particle which moves along with the external Lagrange point on the side of the accreting star, calculated according to the tabulated values for the position of $L_2$ in Kopal (1978). For $q < 1$, this $a_2$ of $L_2$ is given to within 3% by:

$$a_{L_2} = [1 + 0.69(q^{1/3} + q)]^2$$

Clearly, from Figure 1, matter leaving the system with this orbital angular momentum will always shrink the system and will decrease the orbital period. The same is true for matter which has $a_2 = 1$ (for example, matter spinning off a disk at high velocity in all directions) in massive binaries. Only if $q > 0.72$ will the system slow down under these conditions, if $q > 0.78$ it will also expand.

b) Effects of mass flux shifts

Another type of questions one frequently wants to answer in model discussions is:

- What would be the effect of shifting some of the mass flux from one output channel to another?

In answering questions of this type, one should be particularly careful to take into account the full effect of such a shift. For example, the effect of losing more mass in $M_{2L}$ instead of accreting it ($M_2$), will for $a_2 = 1$ be an increase of the orbital period, independent of the mass ratio, despite the fact that the amount of angular momentum lost from the system increases considerably due to this mass flux shift: the reduction of the accretion rate dominates the other effects (see below).

One of the ways to see the effects of this kind of mass flux shifts is to rewrite eq. (16) in terms of $M_1$ and two of the 'output' mass fluxes, using eq. (15):

$$\frac{\dot{P}}{P} = [3(a_1-1)q - 2] \frac{\dot{M}_1}{M} + [3a_1q -3q^{-1} -2] \frac{\dot{M}_2}{M} + 3[a_2q^{-1} -a_1q] \frac{\dot{M}_{2L}}{M}$$  \hspace{1cm} (20a)

$$\frac{\dot{P}}{P} = [3a_2q^{-1} -3q -2] \frac{\dot{M}_1}{M} + [3(a_2-1)q^{-1} -2] \frac{\dot{M}_2}{M} + 3[a_1q-a_2q^{-1}] \frac{\dot{M}_{1L}}{M}$$  \hspace{1cm} (20b)

As we are now only considering the effect of a shift in mass flux between two of the output channels, the flux in the third channel and in $\dot{M}_1$ will be constant. The result we are looking for can thus be obtained by considering the effect of a change in only one type of mass flux, keeping $\dot{M}_1$ and one other output flux constant in the appropriate formula (20a or b). Just as before, we can investigate the effect of varying the $\alpha$'s on the value of $\dot{P}$.
The regions in the \((a,q)\)-plane where \(\dot{P} > 0\) (+), \(0\) (−) and \(0\) (heavy curves). \(a_{1,2}\) is the ratio of the specific orbital angular momentum of the matter lost from the system to the specific orbital angular momentum of star 1,2. The abscissa, \(q\), is the mass ratio \(M_2/M_1\). Upper three frames: the individual effect of matter accreted (\(\dot{M}_1\)) lost from the primary (\(\dot{M}_1\)) and lost from the secondary (\(\dot{M}_2\)). Lower three frames: the effect of shifting mass flux from one output channel to another (see text). Lightly drawn curves correspond to the locus of \(a = 0\). Dashed curves indicate asymptotic values of \(P = 0\) curves. Dotted curves represent the value of \(a_2\) for comoving matter in \(L_2\). Arrows indicate the 'canonical' \(a = 1\) level.

### Table 3. Properties of shifts between mass streams.

<table>
<thead>
<tr>
<th>Mass flux shift</th>
<th>Critical value of (q) if (a_1 = a_2 = 1)</th>
<th>Curve of (\dot{P} = 0)</th>
<th>Physical picture (example)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{M}_1 + \dot{M}_2)</td>
<td>1</td>
<td>(a_2 = a_1 q^2)</td>
<td>more mass captured from wind but not accreted and finally lost from secondary critical lobe</td>
</tr>
<tr>
<td>(\dot{M}_1 + \dot{M}_2)</td>
<td>((1/3)(\sqrt{10} + 1) \approx 1.39)</td>
<td>(a_1 = q^2 + (2/3)q^{-1})</td>
<td>more mass captured from wind and accreted</td>
</tr>
<tr>
<td>(\dot{M}_2 + \dot{M}_2)</td>
<td>0</td>
<td>(a_2 = (2/3)q + 1)</td>
<td>same amount of mass transferred to secondary critical lobe, but more of it is rejected and finally lost from lobe</td>
</tr>
</tbody>
</table>
The results are listed in Table 3 and pictured in Figure 1 (lower half). For massive binaries, the effect on $\dot{P}$ of a higher capture rate from a stellar wind is in general negative, both if the mass is finally accreted and if it is ejected from the secondary critical lobe. Rejection instead of accretion of mass at the secondary $(M_2 + M_{2*})$ will tend to increase $\dot{P}$ for all mass ratio's if $a_2 = 1$, but for massive binaries the specific angular momentum of the matter lost from the secondary critical lobe needs to increase by only a small amount in order to invert this effect; for $q = 0.1$, $a_2 > 1.067$ is sufficient to make the effect on $\dot{P}$ negative. Matter lost with an angular momentum corresponding to $L_2$ (dotted curves) will in all cases decrease the period.

IV. Comparison with the observations

The most promising systems in which to detect variations of the orbital period are those containing an X-ray pulsar. The reason for this is that accurate orbital epoch determinations are possible in these systems using the Doppler delays of the pulsation.

Table 4 lists the information available on the orbital-period derivative of the X-ray binaries of which the orbit is, with the exception of Cyg X-3, known from pulse timing. A few sources (LMC X-4, 1538-52, O115+63, GX301-2), of which the orbit has only been measured once or twice, have been omitted from the table.

<table>
<thead>
<tr>
<th>Source</th>
<th>P (days)</th>
<th>$\dot{P}/P$ (y$^{-1}$)</th>
<th>References and remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyg X-3</td>
<td>0.2</td>
<td>$(2.1 \pm 0.3) \times 10^{-6}$</td>
<td>Van der Klis and Bonnet-Bidaud 1981 (paper 2.1)</td>
</tr>
<tr>
<td>Her X-1</td>
<td>1.4</td>
<td>$</td>
<td>\dot{P}/P</td>
</tr>
<tr>
<td>Cen X-3</td>
<td>2.1</td>
<td>$(-2.8 \pm 0.2) \times 10^{-6}$</td>
<td>Kelley et al. 1982</td>
</tr>
<tr>
<td>SMC X-1</td>
<td>3.9</td>
<td>$(1.2 \pm 1.5) \times 10^{-5}$</td>
<td>Few measurements. Prinini et al. 1977, Davison 1977</td>
</tr>
<tr>
<td>Vela X-1</td>
<td>9.0</td>
<td>$(1.0 \pm 0.4) \times 10^{-4}$</td>
<td>Van der Klis and Bonnet-Bidaud 1983 (paper 4.2)</td>
</tr>
</tbody>
</table>

*from optical, X-ray eclipse and pulsation measurements (Bonnet-Bidaud and Van der Klis 1981).

In the X-ray binary Cygnus X-3, an orbital period variation has been detected without the help of pulsations. The nature of the components of this system is unknown, so that it is impossible to compare the observed value of $\dot{P}$ with theoretical expectations. The reader is referred to Chapter 2 for a discussion of the constraints placed on the nature of this system by the observed period variations. The $\dot{P}$ of Cyg X-3 is consistent with mass loss from the system at a rate of a few $10^{-6} M_\odot$ y$^{-1}$, depending on the component masses. This is in accordance with the observation that the eclipses in the system are partial, which is probably due to scattering of the X-rays in circumstellar matter.

The tightest upper limit has been put on the $\dot{P}$ of Hercules X-1. The component masses of this system are $M_1 \sim 2.3 M_\odot$ and $M_2 \sim 1.4 M_\odot$ (Rappaport and Joss 1983). The X-ray luminosity is $L_x < 10^{37}$ erg s$^{-1}$ (Bradt and McClintock 1983), implying an accretion rate of $M_2 < 2 \times 10^{-9} M_\odot$ y$^{-1}$. 26
The orbital period derivative expected from this accretion is estimated from eq. (16) as $|\dot{P}/P| < 2 \times 10^{-9} \, y^{-1}$ (with $\dot{P} < 0$), well below the quoted upper limit. An upper limit of $4 \times 10^{-8} \, M_\odot \, y^{-1}$ can be set on isotropic mass loss from the system, and with eq. (19) we find that mass loss along $L_2$ should be less than $5 \times 10^{-9} \, M_\odot \, y^{-1}$. The two last-mentioned mass fluxes have opposite effects on the orbital period, and if they occur simultaneously, their effects could cancel.

The other systems listed in Table 4 are massive binaries, with $M = 20 - 30 \, M_\odot$ and $q = 0.05 - 0.10$. The primaries in these systems lose mass in a stellar wind at a rate of $10^{-5} \, M_\odot \, y^{-1}$ at most (Conti 1978). In addition to this, some of these stars are believed to be in a state of incipient or atmospheric Roche-lobe overflow (Savonije 1978, 1979).

With eq. (16) we find, that the effect of the stellar wind is expected to be $\dot{P}/P = +10^{-6} \, y^{-1}$ or less. This clearly can not be the explanation of the negative $\dot{P}$ found in Centaurus X-3.

The long-term average of the intrinsic X-ray luminosity of this system is difficult to estimate because of the occurrence of low states probably caused by the absorption of the X-ray flux, during which the luminosity can not be measured. It is probably of the order of $4 \times 10^{-9} \, \text{erg s}^{-1}$ (paper 4.1), which at $M_1 \sim 19 \, M_\odot$, $M_2 \sim 1.1 \, M_\odot$ (Rappaport and Joss 1983) would be consistent with an accretion rate corresponding to a $P/P$ of only $\sim 2 \times 10^{-6} \, y^{-1}$.

Tidal interaction has been proposed by several authors as an explanation of the observed $\dot{P}$ (Wheeler et al. 1974, Chevalier 1975, Fabbiano and Schreiber 1977). However, the variations which are seen in $\dot{P}$ itself are difficult to understand in the tidal picture (Kelley et al. 1982).

It is likely that during the low states more matter is transferred through $L_1$ to the neutron star's critical lobe than it can accrete (see papers 3.1 and 4.1). If this matter has sufficient velocity, it could leave the system along $L_2$. Loss of mass with an angular momentum corresponding to $L_2$, at an average rate of $\sim 6 \times 10^{-7} \, M_\odot \, y^{-1}$, would be sufficient to explain the observed effect in the orbital period. The observed changes in the orbital period derivative would then be explained by the known irregularities in the occurrence of this mass loss and the low states they cause.

The very high orbital-period increase observed during 5 years in Vela X-1 is discussed at length in paper 4.2. It probably is a non-secular effect. To explain the $P$ in terms of the mechanisms discussed in this paper, one would require large amounts of low-angular-momentum mass to be ejected from the system.

The orbit of SMC X-1 is one of the best known stellar orbits in astronomy. Surprisingly, however, it has only been measured twice (see Table 4). No accurate $\dot{P}$ can therefore be determined. The range quoted in Table 4 was derived from a combination of optical, X-ray intensity and X-ray pulsation measurements. Its limits are one or two orders of magnitude wider than the values expected from the accretion and the possible stellar wind mass loss (paper 3.2 and Conti 1978). Judging from the accuracy of the orbital epoch quoted by Primini et al. (1977), it would be possible to detect a $P/P$ of the order of $10^{-6} \, y^{-1}$, as observed in the similar system of Cen X-3, within 1 to 2 years.

In conclusion, in two systems, Cyg X-3 and Cen X-3, mass fluxes are a likely explanation for the observed orbital period change, although in both cases the mass stream causing the $\dot{P}$ has not been unambiguously observed. The orbital period change observed in Vela X-1 is difficult to explain on all accounts. The limits on $\dot{P}$ obtained for Her X-1 and SMC X-1 include the values that would be expected from the known mass fluxes in these systems.

For SMC X-1 it could prove possible to measure the orbital period derivative in the near future. In several other systems the $\dot{P}$ is potentially detectable. These systems span a wide range of orbital parameters and X-ray emission characteristics which will hopefully facilitate interpretation of the observations.

Continued monitoring of the orbits of X-ray binaries by observation of the pulsations is necessary to gain a better understanding of the effects of the mass fluxes, and thus, by elimination, of the tidal interaction in these systems.
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