CHAPTER 4

PULSATION AND ORBITAL PARAMETERS OF CENTAURUS X-3 AND VELA X-1

4.1 CHARACTERISTICS OF THE CEN X-3 NEUTRON STAR FROM CORRELATED SPIN-UP AND X-RAY LUMINOSITY MEASUREMENTS

4.2 THE ORBITAL PARAMETERS AND THE X-RAY PULSATION OF VELA X-1 (4U 0900-40)
Characteristics of the Cen X-3 Neutron Star from Correlated Spin-up and X-ray Luminosity Measurements

M. van der Klis1, J. M. Bonnet-Bidaud2, and N. R. Robba3

1 Cosmic Ray Working Group, Huygens Laboratory, Leiden, The Netherlands
2 Service d'Electronique Physique, Centre d'Etudes Nucléaires, Saclay, France
3 Istituto Fisica, Università di Palermo, Italy

Received June 18, 1979

Summary. Results are presented of one month of observations with the ESA COS-B satellite of the X-ray pulsation period of Cen X-3, together with X-ray luminosity measurements during this same time.

The data are shown to be compatible with an unbraked spin-up by accretion from a Keplerian disk onto a neutron star with $f = 4.10^{37} \text{erg/s} (M/M_\odot)^{-3} = 1.65 \pm 0.44$, and no braking occurring down to a luminosity of $L_x = 4.210^{37} \text{erg/s}$. The mean intrinsic luminosity during an observed turn-on episode is calculated from the high observed spin-up as $L_x = (9.0 \pm 2.6)10^{37} \text{erg/s}$, which together with the long term behaviour of the pulsation period shows that certain low states of the source are in fact periods of high accretion.

It is argued that some current neutron star models are reconcilable with the value of $f$ determined in this paper. The present $f$ determination is shown to imply a lower mass limit for the Cen X-3 neutron star of 1.1 $M_\odot$.

Key words: X-ray binaries - neutron stars - spin-up - COS-B

I. Introduction

The X-ray source Cen X-3, since its discovery in 1972 as an X-ray pulsar in a binary system (Giacconi et al., 1971; Schreier et al., 1972), has been relatively well studied both from the point of view of its luminosity (Pounds et al., 1975; Long et al., 1975; Schreier et al., 1976) and of its pulsation period behaviour (Tuohy, 1976; Fabbiano and Schreier, 1977).

In particular, it has been shown to exhibit OFF periods during which the source apparently decreases in intensity to a level comparable to the eclipses (Schreier et al., 1977). Several interpretations have been proposed to explain this behaviour in terms of a precessing disk (Holt et al., 1978), a change in the absorption by surrounding matter (Schreier et al., 1976) or an intrinsic change in the accretion rate.

The presence of an accretion disk was also suspected on the basis of the amount of angular momentum expected to be transmitted during the high states of the source and the observed changes in the X-ray luminosity (Bonnet-Bidaud and van der Klis, 1979, hereafter Paper 1).

On the other hand, the measured spin-up rate of the source (Fabbiano and Schreier, 1977) was found to be lower than the one expected from a standard accretion disk model with a neutron star (Pringle and Rees, 1972), and periods of slowing down were observed which require additional braking torques (Fabbiano and Schreier, 1977).

Since in X-ray binaries, the observed spin-up rate of the compact object, interpreted as a result of accretion is related to the angular momentum transmitted, a detailed study of the relation between period changes and luminosity of the X-ray source should allow us to put interesting constraints on both accretion models and compact object characteristics.

Some conclusions were already drawn from the measured spin-up rates for X-ray sources of different mean luminosities (Rappaport and Joss, 1977; Mason, 1977), but the large expected spread in the characteristics of these different systems has somewhat weakened the possible conclusions.

Here, we present data obtained with the high time resolution mode of the ESA COS-B satellite during a one-month continuous observation of Cen X-3 from December 24, 1975 to January 23, 1976. From these high time resolution data, we were able to study the pulsation period changes from one binary cycle to the next one. These results together with the overall monitoring of the source flux during the same period (see Paper 1), allow an interesting study of the correlation between pulsation period changes and the luminosity.

II. Analysis

a) Satellite

The COS-B satellite was pointed at the X-ray source Cen X-3 ($\alpha = 16^h7^m7^s$, $\delta = -60^\circ3^\prime$) continuously between December 24, 1975 and January 23, 1976.

The satellite payload includes an X-ray detector which has been described by Boella et al. (1974). It is a collimated proportional counter with an effective area of $80 \text{cm}^2$ at maximum response and an energy range from 2 to 12 keV with a peak value response around 6 keV, the collimated field of view is $10^\circ$ FWHM with a flat top of 1.1 wide. The X-ray photons detected are accumulated over 25.4 s intervals every 102.4 s for monitoring the source over the full month of observation. To allow a finer temporal analysis, the arrival times of the four (or eight, depending on telemetry constraints) first events in successive telemetric windows of 0.64 s are also recorded with an accuracy better than 0.25 ms.

This window sampling and the rejection of some part of the data because of contamination by charged particles, reduce the effective observing time in this high time resolution mode to about $10^3$ s. The total number of recorded arrival times is about $10^7$, but
the limited efficiency of the rejection of charged particles makes the background contribution more than 50%.

**b) Data Reduction**

The X-ray arrival times in the high time resolution mode were used to study the 4.8 s pulsation period of Cen X-3. The arrival times in the satellite frame were first corrected for the satellite and Earth motion and transported from the satellite to the Solar System Barycenter (SSB) using the Lincoln Laboratory Ephemeris and the precise satellite position. This procedure was essentially the same as the one applied to the data from radio pulsars from the same satellite (Bennett et al., 1976; Bonnet-Bidaud and d'Amico, 1979). The resulting absolute accuracy after these corrections is about 0.5 ms.

Because of the relatively high background level and the low photon sampling rate, the accumulation of X-ray arrival times over periods of a few hours was necessary to determine the pulsation period with a good accuracy. Therefore a first correction using the literature values for the orbital parameters (Tuohy, 1976; Fabbiano and Schreier, 1977) was applied to correct the arrival times for the motion of the X-ray source with respect to the binary system barycenter during the integration intervals.

Phase histograms were then built using the distribution of the time intervals between successive events in each phase bin, since the average mean interval for source photons is 30 ms, much smaller than the Cen X-3 pulsation period. As the number of time intervals in each phase bin is always large, the mean of the distribution was taken to be the arithmetical mean and the corresponding counting rate:

\[ n = N / \sum_{i=1}^{N} t_i \]

where \( N \) is the total number of time intervals in the phase bin, and \( t_i \) is the length of a time interval in the bin.

The light curves, resulting for different values of the period, were then tested against a curve of constant flux using a \( \chi^2 \) test. The best value for the period was taken to be the one for which the \( \chi^2 \) value was at its maximum, with an uncertainty corresponding to the width of the \( \chi^2 \) peak at a 3\( \sigma \) statistical standard deviation level.

**c) Orbital Parameters**

Using the method described above, a set of values for the pulsation period \( P \), was derived, combing several stretches of data to diminish the effect of orbital phase. The best fit to the points in the resulting plot of pulse period against time, representing the general trend in \( P \), was then subtracted from the individual points. The residuals still showed a slight dependence on orbital phase.

If we analyse the influence of the orbital period \( P_{\text{orb}} \), the mid-eclipse time \( \phi \), and the projected radius of the binary orbit \( A = a \sin i \) on the periodic change of the pulsation period due to a Doppler delay of the form:

\[ \Delta t = A \sin \left( \frac{2\pi}{P_{\text{orb}}} \left( t - \phi \right) + \frac{\pi}{2} \right) \equiv A \sin \phi. \]

we find to first order:

\[ \delta \left( \frac{\Delta P}{P} \right) = \frac{2\pi}{P_{\text{orb}}} \left( \delta A - \frac{A}{P_{\text{orb}}} \delta P_{\text{orb}} \right) \cos \phi \]

\[ + \frac{\pi A^2}{P_{\text{orb}}^2} \left( \delta \phi \right) \sin \phi, \]

where the eccentricity of the orbit was taken to be zero. The value of the factor \( A/P_{\text{orb}} \) is 2.2 \( \pm \) 0.4, \( (t - \phi) / P_{\text{orb}} \) is 7 at most in the present observation. The best determination of \( A \) is 39.792 \( \pm \) 0.005 light s (from 1972 Uhuru data, Fabbiano and Schreier, 1977), while the time of observation \( P_{\text{orb}} \) is 2.08712 \( \pm \) 0.00002 d and \( \phi \) is known to an accuracy of 3 \( \pm \) 3 d (Paper 1). Therefore \( \delta P_{\text{orb}} \) is small enough to exclude it from further analysis, the factors \( A/P_{\text{orb}}, \delta A/P_{\text{orb}}, \) and \((t - \phi) \delta P_{\text{orb}} / P_{\text{orb}}\) being an order of magnitude smaller than \( \delta A \) and \( \delta \phi \), respectively.

A sinusoidal curve with this period was fitted to the residuals using the method of van Paradijs et al. (1977). After correction for the fact that each pulse period determination represents a finite binary phase interval the best zero phase time was determined to be JD 2442787.1755 \( \pm \) 0.0007, the best projected orbital radius 39.71 \( \pm \) 0.04 light s.

As pulse period determinations with these orbital parameters do not show any significant residual correlation with orbital phase, these were used in the subsequent data analysis.

The value for \( \phi \), is in accordance with that of JD 2442787.177 \( \pm \) 0.003, independently determined from the COS-B X-ray eclipse observations (Paper 1), is in the range of the values deduced from Uhuru observations.

From the present determination of \( \phi \), the mean binary period between the Ariel-V and COS-B observations is calculated to be 2.08711 \( \pm \) 0.00002 d.

**III. Results**

**a) Pulsation Period**

After correction for the binary orbital motion in the above described way, the intrinsic pulsation period of Cen X-3 was determined to be (4.834477 \( \pm \) 0.0076) s at the middle of the observation (JD 2442785.3).

Figure 1 shows how this value fits in with earlier observations (Tuohy, 1976; Fabbiano and Schreier, 1977). The mean spin-up between the Ariel-V and COS-B observations was \(- \dot{P}/P = 5.52 \times 10^{-5} \) yr\(^{-1} \), resulting in a significant period change.
Fig. 2. The Cen X-3 pulsation period behaviour inside the COS-B observation. The pulsation period in each interval is determined as indicated in the text. One sigma error bars and the extension in time of each interval are shown. Both have been taken into account to determine the best linear fits (solid lines) through the three groups of points (see text). Values are quoted in Table 1. Also shown in the upper part of the figure, is the luminosity behaviour during the same time. Each luminosity point was determined from the detailed light curve (Paper I). Typical error bar is shown. Three mean pulse profiles (see text) are shown in the lower part of the figure.

\[ \pm 0.09_{10} - 4 \text{ yr}^{-1} \], considerably higher than it was during the years 1971–1974 (about $3\pm 0.4 \text{ yr}^{-1}$).

The COS-B data allow a much more detailed monitoring of the pulsation period during the time of observation. The results of this study are presented in Fig. 2, which also shows the X-ray luminosity of Cen X-3 during the same time. As can be seen from this figure, the mean spin-up is relatively high (8.6 ± 0.6_{10} - 4 \text{ yr}^{-1}), comparable to the 1971 August–September Uhuru observation (Fabiano and Schreier, 1977), and higher than the long term spin-up between the observations of Ariel-V and COS-B. Also, the spin-up rate is seen to be changing during the COS-B observation: it is higher near the start and lower at the end of the observation. Therefore the month of observation has been divided in three parts and straight lines have been fitted to the pulsation period points inside these parts to make it possible to relate the mean spin-up to the mean \( L_x \) during the same stretch of time (see Chap. IV).

The pulsation period determinations and the results of the three least square fits are given in numerical form in Table 1.

b) Pulse Shape

We also examined the Cen X-3 pulse shape during the COS-B observation. For that purpose mean light curves were produced for each binary cycle, using the best fit value of the period as indicated above. An averaged pulse shape was then built by summing these curves phased relatively to each other. This mean pulse shape is shown in Fig. 3. It is a one peak curve with a sharp increase and a smooth decrease.

The uncertainties in the background level do not allow one to derive an accurate pulsed fraction, however, a good estimate will be about 40\% with an equivalent duty cycle (total pulsed power divided by peak value) of 0.25.

A significant fraction of the pulsed counts is present in the second part of the cycle (phase 0.5 – 1.0), corresponding to about 30% of the total pulsed power.

The pulse shape reported here is found to be similar to the Uhuru and the OSO-7 results (Ulmer et al., 1974; Ulmer, 1976). In particular there is no evidence of a double peak structure as reported for this source from other observations (Tuohy, 1976; Long et al., 1975). As the energy ranges were different for those experiments, no conclusion can be drawn about spectral or long term variability of the pulse structure.

Short term time variability of the pulsation curve during the month of observation has been investigated. The pulse shape was found to vary appreciably during the turn-on, showing an increasing pulsed fraction during the first orbits (orbit 1-2-3) and a wider peak in orbit 4-5-6. In the rest of the observation, no significant changes were found. However, a tendency to produce a weak
second peak around phase 0.75 was noted in the beginning of the observation (orbit 1-9) as compared to the last orbits (10-14). The features mentioned are reflected in the averaged pulse shapes presented in Fig. 2 for the three different parts of the observation.

IV. Discussion

Having presented the observational results concerning the pulsation of Cen X-3, we will now discuss these results in terms of the relation between the observed spin-up rate and the observed luminosity of the source, leading to constraints on the neutron star parameters and providing a test for neutron star models.

a) Theories on Spin-up

Only two theoretical models to our knowledge give quantitative predictions about the relation between $L_x$ and $\dot{P}$: the simple model (Pringle and Rees, 1972; Lamb et al., 1973), in which it is assumed that the only torque working on the neutron star is that of the accreting mass, which transfers the angular momentum corresponding to a Keplerian orbit at the magnetospheric radius, and the model of Ghosh and Lamb (1978), who except for this accretion torque take into account the force of the matter in the disk on the magnetic field of the neutron star.

In the simple model it is possible to derive a straightforward relation between the spin-up rate and the X-ray luminosity.

The matter circulating in the disk at Kepler speed has a specific angular momentum $L = (GM/R)^{1/2}$, where $M$ is the mass of the neutron star and $R$ the distance to its center. At the Alfvén radius, which we take to be

$$r_A = 3.210^{-3} \mu_{30}^{1/2} M_7^{1/2} \left( M/M_\odot \right)^{-1/2}$$

(Lamb 1977)

where $\mu_{30}$ is the magnetic moment of the neutron star (units 10^{30} Gauss cm^3), and $M_7$, is the accretion rate (units 10^{17} g s^{-1}), the matter transfers its angular momentum to the neutron star and its magnetosphere.

Estimating the X-ray luminosity produced in the accretion process to be equal to the gravitational energy, i.e., to about $10^{36}$ of the rest mass of the accreting matter: $L_x = (GM/R)M$, where $R$ is the radius of the neutron star (Lamb et al., 1973), it is found that

$$-\dot{P}; P = 2.10 - 5 f L_x^{5/7} \text{yr}^{-1}$$

with $I_{45}$ the moment of inertia of the neutron star (units 10^{45} g cm^2). Where the factor containing the neutron star parameters is

$$f = 4\mu_{30}^{7/3} R_8^{5/3} I_{45}^{-1} (M/M_\odot)^{-1/7}$$

For both the simple model and the idea of Ghosh and Lamb, curves are given in Fig. 4 for several values of the neutron star parameters. As can be seen from the figure, for Cen X-3 these curves are nearly identical, though for somewhat different values of the neutron star parameters.

![Fig. 4. Comparison of theoretical relations between $-\dot{P}/P$ and $L_x$ with the results of the present observation of Cen X-3. Dashed lines are for simple unbraked accretion spin-up; the value of $f$ is shown. Dashed lines represent the model of Ghosh and Lamb (1978) for neutron stars of $\mu_{30} = 1$, $R_8 = 1$, $M = 1.3M_\odot$ and $I_{45}$ as shown.](image-url)
Braking Torques

As already noted by several authors, the simple model cannot explain all characteristics of the Cen X-3 pulsation behaviour: the spin-up is often lower than would be expected from the observed $L_n$, and also occasional spin-down episodes have been reported (Fabbiano and Schreier, 1977). Davidson and Ostriker (1973) pointed out that for relatively low accretion rates the Alfven radius could become larger than the corotation radius, causing matter to be ejected, thereby braking the rotation of the neutron star. In this way the spin period tends to keep an equilibrium value $P_{\text{eq}}$, at which the Alfven radius is equal to the corotation radius.

Adopting the Alfven radius of formula (3), this equilibrium period is calculated to be

$$P_{\text{eq}} = 3.1 \mu_0^{5/3} M_1^{1/3}(M/M_\odot)^{3/5} s$$

(6)

Schreier (1977) further developed the braking idea, showing that reasonable amounts of ejected matter could explain the observed long term spin-up as well as occasional spin-down episodes. However, this model does not give a quantitative prediction for the behaviour of the source in the $L_n \leftrightarrow P$ diagram during the time that the Alfven radius is large enough for braking to occur.

**Long Term Spin-up**

Savonije (1978) proposed an explanation for secular spin-up in X-ray binaries by showing that the mean accretion rate is expected to increase slowly because of the evolution of the primary star overflowning its Roche lobe. This causes the equilibrium period to decrease at a rate comparable to the spin-up rate observed in Cen X-3. Thus the counterring forces of accretion and ejection of matter ensure that the rotation period of the neutron star stays near $P_{\text{eq}}$, but this equilibrium point is slowly shifting for reasons of stellar evolution.

From the long term spin-up seen in Cen X-3, one can, taking the assumption of Savonije that this is due to a secular change in $P_{\text{eq}}$, deduce the change in accretion rate $M/M$ necessary to explain this $P_{\text{eq}}/P_{\text{eq}}$. The model for the evolution of the star then gives an indication for the mean $M$ and thereby for the mean $L_n$ at the present stage of evolution. This mean $L_n$ is then the $L_n$ at which the neutron star is spinning exactly at its equilibrium rotation period.

For a higher $L_n$, we expect unbraked accretion spin-up, when $L_n$ gets lower some braking should start to occur.

For spin-up rates between 2.8 and $5.5 \times 10^{-4} \text{ yr}^{-1}$ we find this critical $L_n$ to be about $4.1 \times 10^{37} \text{ erg/s} (M = 4.4 \times 10^{17} \text{ g s}^{-1})$. This number, derived from evolutionary calculations, should be considered as a good order of magnitude estimate, not as an exact value (Savonije, 1978).

In the next two paragraphs, the theoretical ideas described above will be compared to the results of the present observation.

**b) Long Term Pulsation Period Change**

The most striking feature of Fig. 1 is the high mean spin-up rate between the Ariel V and COS-B observations (5.52 \pm 0.09) \times 10^{-4} \text{ yr}^{-1}, as compared to the one determined between Uhuru and Ariel V (3.23 \pm 0.04) \times 10^{-4} \text{ yr}^{-1} (Fig. 1). Fortunately, during the year 1975 the source was well monitored by the Ariel V All Sky Monitor (ASM) (Holt et al., 1978), so that the resulting mean luminosity observed can be fully checked against the observed spin-up rate during this period.

Assuming the simple model, we can rewrite formula (4) as:

$$L_{37} = 3.01 \times 10^{-5} f^{-7/6} \times (1 - P/P_{\text{yr}^{-1}})^{7/6} p^{-7/6}$$

(7)

where $f$ can be assumed to be 1.68 \pm 0.74 for Cen X-3 (see next paragraph). This luminosity is actually a lower limit since braking torques are known to occur and to reduce the transmitted angular momentum and the observed long term spin-up rate.

For the measured slope in Fig. 1 between Ariel V and COS-B observations, the mean luminosity required for the source is then at least $L_n = (4.2 \pm 1.6) \times 10^{37} \text{ erg/s}$. This is significantly higher than the mean luminosity observed by Ariel V during this interval. In fact, from the overall light curve reported (Holt et al., 1978) the mean flux is about 0.12 ph cm$^{-2}$ s$^{-1}$, corresponding to a luminosity of $L_n = 1.87 \pm 0.25 \times 10^{37} \text{ erg/s}$, as deduced from a calibration on the Crab source (Kaluzienski, 1976). Both luminosities quoted are for a distance of 8 kpc.

This clearly shows, that the intrinsic luminosity of the source is affected by absorption during part of this stretch of data. It is stressed that this conclusion is independent of the distance assumed for the source since any correction to this distance will be reflected in the $f$ value deduced from the source flux seen by COS-B. As during 20% of the time of the ASM observation the source can be considered to be in a low state compatible with the eclipse level (Holt et al., 1978), there is a strong indication that the low periods of the source correspond to periods of high accretion rate with high spin-up rate, and that the low luminosity at that time is caused by absorption. (See also next paragraph for the spin-up during the turn-on.)

Unfortunately the same coverage of the source flux is not available during the Uhuru-Ariel V observations interval, where the mean spin-up rate is found to be appreciably lower (3.23 \pm 0.04 \times 10^{-4} \text{ yr}^{-1}). Since the braking torques are expected to be efficient at luminosities lower than $4.1 \times 10^{37} \text{ erg/s}$, such a change could be explained by a gradual increase in the accretion rate from Uhuru to COS-B observations. This is compatible with both the larger luminosity found in the COS-B observation, and the strong braking effect seen by Fabbiano and Schreier (1977).

c) Short Term Pulsation Period Variation

Turning to the detailed pulsation period behaviour during the month of observations (Fig. 2), the first feature to remark is the high mean spin-up of about $10^{-3} \text{ yr}^{-1}$. Assuming that the angular momentum required for this spin-up is transferred to the neutron star by accreting matter, taking $L_n = 0.1 c^2M = 4.1 \times 10^{37} \text{ erg/s}$ and the moment of inertia of the neutron star $I = 1.0 \times 10^{45} \text{ g cm}^2$, the specific angular momentum of the matter is calculated to be $9.2 \times 10^{16} \text{ cm}^2 \text{s}^{-1} \text{g}^{-1}$, in good accordance with the specific angular momentum expected in case of disk accretion (Petterson, 1978).

From the existence of a disk in Cen X-3 we may not conclude, however, what the mode of mass transfer is, because at accretion rates of this magnitude the matter has a specific angular momentum high enough to form a disk, not only if the mass transfer is by Roche lobe overflow, but also if it is by stellar wind (Paper 1).

In Paper 1 it was concluded from the luminosity behaviour of the source that the COS-B observation of Cen X-3 essentially consists of three parts:

a) the turn on, during which the absorption by circumstellar matter is high and decreases quickly,

b) the high luminosity orbits, where absorption is unimportant, but the accretion rate high, and

c) the low luminosity orbits, which show a more or less normal accretion rate together with low absorption.
The mean \( \bar{P} \) and also the pulse shape in each of these three parts of the observation confirm the general picture given above and lead to the interesting possibility of comparing these data to predictions by theories on spin-up.

The spin-up during the turn-on is not significantly different from that during the high orbits, confirming the assertion that the accretion rate during the turn-on is high and that the low \( L_x \) there is caused by absorption. This is also confirmed by the relatively flat pulse shape during the turn-on, attributed to scattering of the X-rays in the absorbing circumstellar matter. During the low luminosity orbits – \( \bar{P}/P \) is significantly lower than before, which shows that accretion was really less at that time. Because the high and low orbits show little absorption, these two points can be compared to the theoretical predictions of the correlation between \( \bar{P} \) and \( L_x \) (Fig. 4). It is seen that both the high and the low luminosity point are compatible with the simple spin-up model (dotted lines).

The factor \( f \) containing the neutron star parameters is determined to be \( 1.68 \pm 0.74 \) for the high luminosity point and \( 1.64 \pm 0.47 \) for the low luminosity point. These values are not significantly different, so there is no evidence from these data of braking occurring at luminosities above \( L_x = 4.2 \times 10^{37} \text{ erg/s} \), in accordance with the suggestion from evolutionary arguments above. The mean value of \( f \) is \( 1.65 \pm 0.44 \).

Assuming the dependence of intrinsic luminosity on \( \bar{P} \) to be given by the same relation (formula (4)) during the turn-on as during the high and low luminosity orbits, the unabsorbed \( L_x \) there can be calculated to be \( (9.0 \pm 2.6) \times 10^{37} \text{ erg/s} \).

Behind this determination of \( f \) lie the assumptions of the simple model (paragraph a), which could lead to an underestimate of \( f \) (Lamb, 1977), and the observational value of \( L_x \), which could be influenced by the assumed distance (8 kpc) and spectral shape (Paper I), as well as by a possible correction for beaming of the radiation (Henry and Schreier, 1977). It can be seen from Fig. 4, however, that \( f \) does not depend very sensitively on \( L_x \); the dependence on \( P/\bar{P} \) is much more important, and this quantity is measured directly.

**d) Neutron Star Parameters**

Having shown that the present observation is consistent with the theoretical picture sketched in paragraph a), we now proceed, within this theoretical framework, to take a closer look at the neutron star itself.

It can be seen from expression (5) that the factor \( f \) appearing in the formula for classical accretion spin-up is only weakly dependent on \( \mu \). For radiopulsars, Ruderman (1972) derives a range of magnetic moments of \( \mu_{30} = (0.26 - 4.4) \times 10^{12} \) from observations of spin-down by assuming the rotational energy loss of the pulsar to be equal to one caused by radiation of a nonaligned rotating magnetic dipole in vacuum. Assuming the \( \mu \) of Cen X-3 to be somewhere in this range, and using the present determination of \( f = 1.68 \pm 0.74 \) (excluding the low luminosity orbits to be sure no braking is affecting the value of \( f \)), it is found, that

\[
R_n/I_{45} = (0.11 - 0.87)(M/M_\odot)^{1/2}.
\]

This result is compared for a large range of possible masses to predictions which can be made from theoretical models of neutron stars in Fig. 5. The models A–N appear with the same designations in the paper by Arnett and Bowers (1977). Not all models given there are included in the picture to avoid cluttering, but the full range of \( R_n/I_{45} \) values is covered by the curves shown. The models T1 and T11 were taken from Pandharipande et al. (1976); these are models with a tensor interaction equation of state in which a solid crust is believed to enclose a liquid interior. We assumed the crust to be totally decoupled from the interior and took only the moment of inertia of the former to compute \( R_n/I_{45} \).

The assumption of total decoupling clearly bends the curves for these models out of the range allowed by the present observation, while without this assumption the tensor interaction model (curve \( M \)) is quite within this range. Also the model B (Pandharipande, 1971), \( F \) (Arponen, 1972) and \( G \) (Canuto and Chitre, 1974, not shown in the figure), are made improbable by this observation. It is remarkable that following the classification by Arnett and Bowers (1977) only the models based on the stiffer equations of state can account for the observed ratio of \( R \) and \( I \).

Under the assumption that a neutron star has a structure which is somewhere in the range of models proposed to this date, it is clear from Fig. 5 that we can, independent from other determinations, derive a lower limit to the mass of the Cen X-3 neutron star of \( 1.1 M_\odot \).

Another constraint on the neutron star parameters can be obtained from the assumption that Cen X-3 is spinning at the equilibrium period (formula (6)). The neutron star is expected to rotate at the period corresponding to the mean accretion rate \( \dot{M} \). A good order of magnitude estimate of this number was already

![Fig. 5. The ratio \( R_n/I_{45} \) as a function of mass for different neutron star models. The hatched area represents the range on \( R_n/I_{45} \) allowed for Cen X-3 as derived from the present determination of \( f(1.7 \pm 0.7) \). The models A–N were taken from Arnett and Bowers (1977) and span the full range of models published for the models T10 and T110 (Pandharipande et al., 1976) only the moment of inertia of the neutron star solid crust was taken into account.](image-url)
given in paragraph a): \( \dot{M} / \dot{L} = 4.4 \). The range of masses of Cen X-3 from radial velocity curves and eclipse durations is often quoted in recent publications as \( M = (0.6 - 1.8) M_\odot \) (Avni and Bahcall 1976, Joss and Rappaport 1976), but see also Lamb (1977).

Combining these data it is found, that the assumption \( P = P_{o} \) implies: \( \mu_{30} = 2.3 - 5.7 \), consistent with the range assumed for \( \mu \) above.

V. Conclusion

Correlated \( P - L \), measurements of neutron stars in binary X-ray systems provide an important test of theories both on accretion and on neutron star structure.

The present observation provides a determination of the properties of an individual neutron star rather than of the class as a whole.

From the results emerges the picture of a neutron star heavier than \( 1.1 M_\odot \) which is accelerating its rotation by accretion from a Keplerian disk, with no significant braking forces acting at least at luminosities over \( 4.2 \times 10^{37} \) erg/s. Sometimes the X-rays it emits are obscured by absorbing matter, but its accumulation rate and thereby the intrinsic luminosity during these low states is even higher than otherwise, as is betrayed by its high spin-up in the December 1975 turn-on; there its calculated \( L_x \) was \( \sim 9 \times 10^{37} \) erg/s.

We found some conditions constraining the possible equations of state governing the interior of this neutron star (the stiffer ones seem to be favored), but most possibilities are still admitted; future observations should make it possible further to tighten these constraints.

References

Bonnet-Bidaud, J.M., d'Amico, N.: 1979, preprint
4.2 The Orbital Parameters and the X-ray Pulsation of Vela X-1 (4U 0900-40)

M. van der Klis$^{1,2}$ and J.M. Bonnet-Bidaud$^3$

$^1$ Astronomical Institute 'Anton Pannekoek', University of Amsterdam, Roetersstraat 15, 1018 WB Amsterdam, The Netherlands.


$^3$ Section d'Astrophysique, CEN-Saclay, DPh-EP/Sp, F-91191 Gif-sur-Yvette, France.

A shorter version of this paper will be submitted to Astronomy and Astrophysics, Main Journal.

SUMMARY

Results are presented of an analysis of 83 days of 2-12 keV X-ray observations of Vela X-1 (4U 0900-40) obtained during three separate pointings with the ESA COS-B satellite. For each of the three observations, a full set of orbital and pulsation parameters is derived. Combining the present determinations of the orbital epoch from pulse timing with those previously obtained with other satellites, the data are found to be consistent with an increase of the orbital period at a rate $(\dot{P}/P)_{\text{orb}} = (1.2 \pm 0.4) 10^{-4}$ y$^{-1}$ during the interval 1975-1980. Comparison with observed epochs of mid-eclipse shows that this effect cannot have been present since the time that an eclipse was first measured in Vela X-1, in 1971. These results are discussed in terms of models for non-secular orbital-period changes. The pulsation period is shown to undergo very rapid intrinsic changes, at a rate of up to $\dot{P}/P \sim 10^{-1}$ y$^{-1}$ during intervals of a few days. The lower values of $\dot{P}/P \sim 10^{-3}$ y$^{-1}$ which were previously observed over longer intervals, appear to result from an averaging out of these rapid changes. Transfer of angular momentum to the neutron star by accreting and non-accreting matter is considered in an attempt to explain these pulsation-period changes.

Key words: X-ray binaries - X-ray pulsations - Vela X-1 - COS-B

I. Introduction

Vela X-1 (4U 0900-40) was discovered to be an eclipsing X-ray binary with a period of $\sim 8.9$ d in 1972 (Ulmer et al. 1972). Its 283 s pulsation, not discovered until 1976 (McClintock et al. 1976), is characterized by a complex, energy-dependent pulse profile. Measurements of the Doppler delays of the pulsation show that the binary orbit is slightly eccentric ($e \sim 0.1$, Rappaport et al. 1980). The determination of the radial velocity curve of the B0.5Ib optical companion to the X-ray source, HD 77581, made it possible to determine the masses of the two stars as $\sim 21$ M$_{\odot}$ and $\sim 1.7$ M$_{\odot}$, respectively (van Paradijs et al. 1977). HD 77581 shows mass loss in a strong stellar wind at a rate of $\sim 10^{-6}$ y$^{-1}$ (Dupree et al. 1980), and the relatively low X-ray luminosity of the source ($\lesssim 1.4 \times 10^{36}$ erg s$^{-1}$, Bradt et al. 1979), is consistent with accretion of matter from this stellar wind onto a neutron star (Lamers et al. 1976, see also Henrichs 1980).

The pulsation period has been decreasing at an average rate of $1.5 \times 10^{-4}$ y$^{-1}$ between 1975 and 1978, consistent with the effect expected from accretion from a spherically symmetric wind, but since 1978, it is increasing, at a rate of $3 \times 10^{-4}$ y$^{-1}$ (Nagas e et al. 1982a). During shorter intervals of time, both slow-downs and speed-ups of the pulsation period have been reported (Gelman et al 1977, Nagase et al. 1982a), at rates of up to $4 \times 10^{-3}$ y$^{-1}$.

The possibility to detect changes in the orbital parameters, either due to mass streams in the system or to tidal effects, has aroused some interest. So far, only upper limits were reported for changes of the periastron angle (Rappaport et al. 1980) and the orbital period (Nagase et al. 1982b).
Vela X-1 was observed with the COS-B X-ray detector on three occasions, in 1975 (during 19 d), in 1976 (29 d), and finally in 1979 (35 d). Previous studies, dealing with the X-ray intensity and pulse period during these observations and with the orbital motion during those in 1975 and 1976 (Ögelman et al. 1977, Molteni et al. 1982), were based on the 100 s resolution 'ratemeter' data mode of the instrument.

In this paper, we present an analysis of the pulsational and orbital behaviour of Vela X-1 which makes use of both the ratemeter and the 0.25 ms resolution 'high time resolution' data modes. The analysis covers all three observations and provides a uniform treatment of all available COS-B X-ray data on Vela X-1.

After giving the observational details in Section II, we describe in Section III the determination of the pulsational and orbital parameters, and in Section IV the correlation of these measurements, extending over 4 years, with the results obtained with other experiments. The presented data are discussed in Section V.

II. Observations

The COS-B X-ray observations of Vela X-1 were performed in 1975 from November 9 to 28, in 1976 from July 24 to August 22 and in 1979 from October 11 to November 15. In all three cases, the source was kept within 0.8 of the center of the 10° FHWM field of the 80 cm² 2-12 keV detector (Boella et al. 1974). The collimator transmission was close to 100%, small variations (<1%) due to pointing drift were corrected.

The data were collected simultaneously in two modes: 'ratemeter' data, consisting of 25 s of X-ray flux measurements separated by 75 s intervals of charged-particle background monitoring, and 'high time resolution' (HTR) data, consisting of arrival time measurements, accurate to 0.25 ms, of the first few photons arriving in consecutive 0.64 s intervals. The HTR mode is telemetry limited: arrival times of either 8, 4 or 0 photons are transmitted every 0.64 s, depending on the demands of the other COS-B experiments.

The typical charged-particle background in the X-ray detector was ~100 c/s; this measured background was subtracted from the ratemeter data before further analysis. During the 1979 observation, changes in the background on time scales down to several hundred seconds occurred frequently, and subtraction could not always be satisfactorily performed. We found, that this did not interfere noticeably with the detection of the pulsation (see Section III.2 below).

The contribution to the total detected flux by other sources than Vela X-1 within the field of the collimator was assessed from a compilation of X-ray catalogues by Amnuel et al. (1982). We estimate the counting rate from these sources to less than 1.5 c/s, assuming that the transient source 0836-42, which has not been seen since 1973, was still in its low state during the observations. At its previously observed peak level (Cominsky et al. 1978), this source could have contributed ~3 c/s.

III. Data analysis and results

1. Binary light curve

In each of the observations, the eclipses of the source are clearly visible; the flux outside eclipse relative to the eclipse level typically varies between 5 and 10 c/s. The COS-B X-ray experiment does not give spectral information; for typically observed spectra (Forman et al. 1973, Becker et al. 1978, Kallman and White 1982), this count rate corresponds to a 2-12 keV luminosity of 3 to 6 \(10^{35}\) erg s⁻¹ at a distance of 1.4 kpc (Bradt et al. 1979). Many flares with typical durations of a few hours occur, especially in the 1979 observation. They reach levels of up to ten times the usual source flux. Low-flux episodes with a duration of ~1 d are occasionally seen at phase 0.5; there are also indications for the occurrence of occasional intensity dips with durations of less than an hour at various orbital phases. Similar features have been reported previously (Eadie et al. 1975, Charles et al. 1978,
Kallman and White 1982).

To investigate the global properties of the binary light curve, the ratemeter data of each observation were folded with a period \( P_{\text{orb}} = 8.9643 \) d, phase zero corresponding to JD 2442728.43 + n\( P_{\text{orb}} \) (Ögelman et al. 1977).

The results are presented in Figure 1. The flaring behaviour is evident; it is also clear that in all three cases the mean out-of-eclipse flux after phase 0.5 is somewhat attenuated with respect to the first half of the light curve. This is in accordance with previous observations (Watson and Griffiths 1977, Nagase et al. 1982b). In the 1976 observation, there is a clear low-flux episode between phases 0.4 and 0.6.

The data obtained in 1975 and 1976 were used to estimate the eclipse width. Due to background problems, no eclipse duration could be derived from the 1979 observation. Following the method outlined by Watson and Griffiths (1977), we estimate, that the duration of the flat bottom of the eclipse is \( 1.66 \pm 0.20 \) d and \( 1.95 \pm 0.20 \) d for 1975 and 1976, respectively, where the errors reflect both the statistical errors in the data and the uncertainties inherent in the method we used. The fact that the coverage of eclipse entrance in 1976 was not complete was also taken into account in the error estimates.

Due to the different definition of the eclipse width adopted here, the 1975 value measured for this quantity is 0.24 d less than the value quoted for the same eclipse by Ögelman et al. (1977). The present result is close to the combined duration of a number of eclipses between 1974 and 1976 obtained by Watson and Griffiths (1977), who used the same definition.

2. Pulsation parameters

By folding with the 283 s period, the pulsation is detectable during every 1.5 d satellite orbit in each of the three observations, both in the ratemeter and in the high-time resolution data. The phase shifts due to the orbital motion are readily evident. There is a clear correlation between the flux level of the source and the amplitude of the pulsation, which indicates that the pulsed fraction is approximately constant at \( \sim 30\% \) of the total flux.

The X-ray pulse profile of Vela X-1 at the energies seen by the COS-B X-ray detector is known to present repeatable structures on time scales down to 15 s (McClintock et al. 1976, Nagase et al. 1982b). Rappaport et al. (1980) noted, that the use of these high-frequency components in a cross-correlation analysis makes the phase determination more accurate and reduces the probability of spurious effects. We therefore made an analysis of the pulsations of Vela X-1 observed in 1975, 1976 and 1979 using the millisecond resolution data.

The X-ray arrival times were first corrected for the satellite and the seasonal earth motion. All arrival times were reduced to the solar system barycenter reference frame. The accuracy of the correction used is better than 1 ms, which was checked by observations of fast pulsars (Wills et al. 1982).

We split up the three observations into intervals ranging from 0.2 to 1 d, depending on what was found necessary to obtain a good signal-to-noise ratio. Using a trial pulsation period \( P_{0} \), the data in each interval were then folded to derive an average pulse profile (see van der Klis et al. 1980).

The magnitude of the Doppler shifts induced by the orbital motion of the neutron star can be as large as 70 s in one day. To preserve the high frequency structure of the pulses, we therefore applied, prior to folding, a first-order orbital correction of the form:

\[
\Delta t = \frac{a \sin i}{c} \left[ \sin(M+\omega) + \frac{e}{2} \sin(2M+\omega) - \frac{3}{2} e \sin \omega \right]
\]

where \( a \sin i \) is the projected semi-major axis, \( c \) the velocity of light, \( e \) the orbital eccentricity, \( \omega \) the periastron angle and \( M \) the mean anomaly. Second and higher order terms of \( \Delta t \) were neglected at this stage. For the calculation of this correction, we assumed the orbital parameters given by Ögelman (1977) in the 1975 observation, and those of Rappaport (1980) in the other two observations.

In each of the three observations, a set of 40 to 50 average pulse profiles was obtained. The highest-quality profile was used as a master template and was cross-correlated to the
other ones to determine time delays of the pulses across the observation. The typical accuracy
which was achieved, mainly determined by statistical fluctuations, was 2 to 5 s.

The measured time delays can be attributed to three effects:

1) a residual orbital effect due to the approximate (first-order) nature of the
correction and/or to different values of the orbital parameters from the trial
ones,

2) a non-constant pulsation period across the observation, and/or one different
from the trial period $P_0$,

3) short-term changes in the pulsating clock.

The effect of 1) is expected to be less than 5 s (see below). Due to 2), variations of up to
40 s are present in the data. We determined the correction to the pulsation period, and its
derivative, $\dot{P}$, from a second order polynomial fitted to the delay times $t_n$:

$$t_n = t_0 + n \Delta P_0 + (1/2)n^2 P_0 \dot{P},$$

where $n$ is the pulse number, and $\Delta P_0$ is the difference between
the best-fit period at $n = 0$ and the trial period.

Figure 2 shows the time delays with the best fitting parabola superimposed. The points in
the figure were already corrected for the residual orbital effect (see Section 3). There is
clear evidence for pulse-period variations on shorter time scales, particularly during the
first observation. Due to this, the formal $\chi^2$-values of the fits are very high. We will here
quote the results of these fits in order to compare our data to previous measurements of $\dot{P}$
(which refer to intervals of similar length), and analyze the short time scale variations in
Section 4.

The results of the fits are given in Table 1. Significant quadratic terms are present in
1975 and 1979, corresponding, in both cases, to a continuous period increase over the
observation of $\dot{P} \sim 10^{-7}$, while for 1976 a value consistent with zero is derived. The 2σ upper
limit to $|\dot{P}|$ in this observation is 1.0 $10^{-9}$, an order of magnitude below the other two. The
values in Table 1 are not significantly different from, but much more accurate than the values
derived from ratemeter-data analysis (Ögelman et al. 1977, Molteni et al. 1982).

The pulsation period derivatives we find here for observation intervals of 20 to 40 d
appear to be intermediate (in absolute value) between the values found over ~ 10 d intervals
($|\dot{P}|$ up to 4 $10^{-8}$, Nagase et al. 1982a), and the long-term values observed over intervals of
years.

To fit the observed $\dot{P}$ values into the long-term picture, we note that the 1975 and 1976
observations took place when, considered on a time scale of years, Vela X-1 was spinning up.
The 1975 observation is situated, however, inside an ~ 200 d interval during which this trend
was apparently temporarily reversed (Becker et al. 1978). The 1979 observation occurred
probably less than 100 d after the reversal of the long-term period trend from spin-up to
spin-down, in the rising branch of the long-term pulsation-period plot.

3. Orbital parameters

The residuals of the pulse-arrival times obtained from the analysis described in the last
section were used to determine the best-fit orbital parameters for each of the three
observations.

As described above, the arrival times of the individual photons were corrected
 provisionally for an assumed binary orbital delay curve in order to be able to derive well-
de fined pulse profiles. Therefore, the residual phase shifts of the pulse profiles do not
reflect the full, ~ 100 s amplitude effect of the orbital motion, but only the $< 5$ s
difference between the assumed delay curve and the real one.

The function to fit to these residual delays was derived in the following way: (i) First
the usual expression for the delay $z(t,x_i)$, where $x_i$ symbolizes the orbital parameters, was
expanded into a Fourier series in terms of the mean longitude $z = M + \omega$ (Deeter et al. 1981,
Russell 1902), retaining all terms up to order $e^2$ (see Appendix I). In closed form, $z$ is
implicit in the time $t$; by expanding, it becomes explicit in this variable. (ii) We then
applied the method of variation of the orbital parameters: the residual delay function was
written in terms of the differential corrections $\delta x_i$ to the orbital parameters as
\[
\delta z = \sum_{i} \frac{\partial^2 z}{\partial x_i \partial z} \delta x_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 z}{\partial x_i \partial x_j} \delta x_i \delta x_j.
\]

(iii) The \(e^2\) terms of \(z\), which were neglected in the provisional correction to the individual photon arrival times (see Section 2), were added to \(\delta z\) to complete the fitting function. (iv) Finally, we dropped all terms in this fitting function which could be estimated from the known values and uncertainties of the assumed orbital parameters to have an amplitude of less than 0.5 s. Among the terms dropped were all those containing corrections to the orbital period, which was thus kept fixed at its assumed value, 8.9643 d (Ögelman et al. 1977). We also checked that no terms of order higher than \(e^2\) or \(\delta x_i \delta x_j\) could reach an amplitude in excess of 0.5 s.

The fitting function obtained by this approach is given by:

\[
\delta z = A + B \sin \alpha + C \cos \alpha + D \sin 2\alpha + E \cos 2\alpha + \frac{3}{8} e^2 a \sin \sin(3\alpha - 2\omega)
\]

where \(a\) is the projected semi-major axis of the orbit. This function is linear in its coefficients \(A\) to \(E\), so that the fit can be made by straightforward matrix inversion instead of by scanning the parameter space for the lowest \(\chi^2\) value. The calculations are further facilitated by the fact that the function is explicit in time.

The corrections to the orbital parameters are obtained as functions of the coefficients \(B\) to \(E\) and the assumed orbital parameters. To first approximation, \(a\) is determined by the amplitude of the fundamental frequency sinusoidal; \(T_0\), the time of \(x = 0\), by its phase; \(e\) by the amplitude of the first harmonic and \(\omega\) by its phase. The higher order terms carried in the derivation of \(\delta z\) provide corrections to these simple relations.

The corrected orbital parameters (\(a\), \(T_0\), \(e\) and \(\omega\)) were used, in place of those originally assumed, to derive new residual phase delays, and the procedure was iterated a few times until the coefficients \(B\) to \(E\) were all less than 0.1 s, the approximate value of the neglected terms in \(e^3\). Tests on synthetic and on existing real data of Vela X-1 verified the correctness and rapid convergence of this procedure. The errors introduced by the approximations of the method are an order of magnitude less than the statistical errors.

The results of the fits are given in Table 2. During the 1975 observation, the phase coverage of the high time resolution data close to eclipse was not sufficient (due to telemetry constraints) to accurately determine the amplitude of the fundamental frequency component of the delay curve, i.e., \(a\). It was decided in this case to adopt the value of \(a = 113.0\) s previously determined from the ratemeter data. The effect of fixing this parameter on the best-fit values of the other parameters was found to be negligible. No such problems occurred for the other observations. All results are in the range of values determined with other experiments, and are consistent with the earlier determinations by Ögelman et al. (1977) and Molteni et al. (1982), at somewhat improved accuracy.

We stress, that the definition of orbital epoch used here is different from that of these authors. They used the center of eclipse as determined from intensity measurements, while in the present analysis the orbital epoch \(T_0\) is determined in a self-consistent way, simultaneously with the other parameters directly from the delay curve.

4. Short-term intrinsic pulse-period variations

We tested the residuals of the final orbital fits for the presence of a linear or parabolic trend, which would indicate a deviation from the mean value of the pulse period and its derivative as determined prior to the orbital analysis.

No such deviations were found, confirming the validity of our approach to determine pulsational and orbital parameters separately. As noted in Section 2, however, there is evidence for variations of the pulse period on time scales of days: particularly in the 1975 observation, the time delays (corrected for orbital motion) show clear deviations from the best fit parabola (see Figure 2).

Typical values of these deviations are 5-10 s, corresponding to up to 6 times the statistical uncertainty of individual points. The fact that the deviations sometimes occur in a systematic way increases their significance. As they occur on a time scale of days, the
effect corresponds to changes of the period of the order of $10^{-2}$ s ($\Delta P/P \sim 10^{-4}$).

This cannot be due to residual uncertainties in the orbital parameters, as the effect of these is estimated to be less than 0.1 s (see Section 3). Moreover, the residuals do not show any correlation with orbital phase, as shown in Figure 2 (right-hand side frames). The deviations are not connected to any major changes of the shape of the pulse profile. Therefore, they have probably to be attributed to intrinsic changes of the period of the pulsation.

Using the data obtained in 1975, we computed $P$ and $\dot{P}$ for different parts of the observation. The results are given in Table 3. The values of the short-term $\dot{P}$ are all larger than the monthly averaged value of $1.4 \times 10^{-8}$ (see Table 1), and fluctuate from $(-0.5$ to $+0.0) \times 10^{-7}$.

We note that these $\dot{P}$ values are actually lower limits, as we cannot from our data determine whether the pulsation-period changes result from a gradual change taking place during an interval of a few days (as assumed by fitting a constant $\dot{P}$), or by even more rapid changes separated by intervals with an approximately constant pulsation period. The division of the short-term $\dot{P}$ values quoted in Table 3 into very rapid spin-down and somewhat slower spin-up episodes is determined to some extent by the way we split up the observation.

The highest spin-down value occurs during an interval of 2 days around JD 2442733.0. This is clearly visible in Figure 2, as well as the other spin-down event around JD 2442741.0 during an interval of 3 days. In both cases, the rate of change of the period derived here is an order of magnitude larger than previously detected in Vela X-1 (Rappaport et al. 1976, Nagase et al. 1982a). This is probably explained by the fact, that the previous measurements were all based on intervals comparable to the orbital period (8-10 d), which tends to smooth out the shorter time scale variations discussed here. As a check, we took an interval of 9 d centered around JD 2442734.0. The spin-down value in this interval is as low as $(3.5 \pm 1.2) \times 10^{-8}$, similar to the values found by Nagase et al. (1982a).

We also looked for a possible correlation of the period changes with the X-ray flux, by plotting the mean period changes against the mean counting rate. No correlation was found in any of the three observations. (See also Table 3, where both the rate of change of the period and the mean counting rate are given.) In particular, we do not confirm the suggestion by Makishima and Nagase (1980) of a coincidence of spin-down and X-ray flaring activity.

5. Pulse profiles

In the course of the pulsation-parameter analysis, pulse profiles averaged over 0.2 to 1 d intervals were produced by folding the HTR data modulo the pulsation period into 56-bin histograms. The time delays determined from a cross-correlation of each pulse with a template curve (see Section 2), were then used to align the individual profiles. In each observation, the aligned pulses were summed to derive a grand-average pulse profile. Figure 3 shows the resulting 2-12 keV profiles. The profiles were normalized by subtracting the mean flux level; the flux scale is arbitrary, but identical in each frame.

The pulse profiles in Figure 3 are similar to those observed in the same energy range by McClintock et al. (1976) and Nagase et al. (1982b). A double peaked structure is clearly visible. The major pulse (between phase 0.45 and 0.80) contains about 2/3 of the pulsed flux. Stable structures of shorter time extension (from 15 to 50 s) are present in each of the three observations, of which the most stable are the deep minimum near phase 0.4, and the depression in the middle of the major pulse near phase 0.65.

No clear variations of the pulse shape from one observation to the other are present, except possibly for the suppression of the first part of the major pulse in 1975. We also looked for changes of the pulse period across the orbital cycle, by summing up the individual profiles within 4 orbital-phase intervals outside of eclipse. The mean pulse profiles in the phase interval $\varphi_{orb} = 0.7 - 0.9$ were found to be somewhat smoothened out, but they still clearly show the short time scale structures. This is different from the results of Nagase et al. (1982b), who, in the same orbital-phase interval, found a smooth pulse profile with only two peaks, similar to the pulse shape seen at higher energies. Although during this orbital-phase interval (0.7 - 0.9) the X-ray flux is on the average somewhat lower than during the
rest of the un eclipsed part of the orbital cycle, there is no strict correlation of pulse shape with X-ray intensity, since the low state observed in 1976 in the interval $\phi_{\text{orb}} = 0.4 - 0.6$ (see Figure 1) does not cause any observable changes in pulse shape.

IV. Long-term Changes of the Orbital Parameters

1. Orbital epoch

The three $T_0$ values listed in Table 2 bring the total number of published orbital epochs measured from the pulse-delay curve to seven. To be able to use these data to investigate the evolution of the orbital period, however, the epochs should all refer to the same event in the orbital cycle.

As pointed out by Deeter et al. (1981), the accuracy of epochs defined by the periastron passage suffers greatly when the periastron angle cannot be measured accurately, as is the case in Vela X-1. They also note that epochs in true longitude are influenced by the possible presence of apsidal motion, and in addition can be determined less accurately by pulse timing than epochs in mean longitude. They conclude that the best choice for the fiducial event is the time when the mean longitude $\chi = M + \omega$ reaches a certain fixed value. We will use the point $\chi = \pi/2$ for this purpose, and refer to the corresponding time (close, but not identical to mid-eclipse time) as $T_{\pi/2}$.

The results of the conversion to $T_{\pi/2}$ of the seven orbital epochs are listed in Table 4 (see Appendix II for the conversion formulae used). The errors given in Table 4 are those originally quoted in literature, reduced to the 1σ-level. Because of the reasons stated above, these errors are expected to overestimate the statistical uncertainties in the case of the four $T_{\pi/2}$-values which were converted from orbital epochs defined in terms of true longitude or periastron passage.

On the other hand, systematic errors in $T_{\pi/2}$ due to differences in phase coverage between the seven observations cannot be excluded. From tests on our own data, we estimate these to be less than the statistical errors, however. This also applies to possible systematic shifts of $T_{\pi/2}$ due to the errors in the other fitting parameters: we find that the best-fit value of $T_{\pi/2}$ is only weakly dependent on the values of the pulse period and its derivative, on the values of the other orbital parameters and on the precise shape of the pulse-profile master template.

The shift of the $T_{\pi/2}$ values with respect to the constant period ephemeris given by Ögelman et al. (1977), $T_{\pi/2} = 8.9643 + 2442728.43$, is plotted in Figure 4 (upper frame). Clearly, the best fit parabola (drawn curve), corresponding to a $(P/P_{\text{orb}} - 1)$ of $(1.2 \pm 0.4) 10^{-4} y^{-1}$ is a closer representation of the data than the linear best fit (dashed line), representing a constant period of $P_{\text{orb}} = 8.964 22 \pm 0.000 11$ d. The reduced chi-squared of the parabolic fit has the very small value of $x^2_v = 0.08$ at $v = 4$, which is understandable if the accuracy of the converted $T_{\pi/2}$ values was indeed underestimated by our procedure (by a factor of ~3).

We note that the parabolic effect found here is not an artifact of the very different statistical uncertainties of the points which enter into the fit. When we assign identical error bars to all points, we again find a parabola to fit considerably better to the data than a line, and even obtain a more significant (though somewhat lower) value of the orbital period derivative: $(0.84 \pm 0.15) 10^{-4} y^{-1}$.

Further discussion of this possible very rapid orbital-period slow-down is deferred to section V.

2. Periastron angle

The comparison of the different values of $\omega$ observed in Vela X-1 up to now is more straightforward than that of the orbital epochs. The results of this comparison are listed in Table 4 and plotted in Figure 4 (lower frame) as a function of time. There is no conclusive detection of apsidal motion. The formal value of $\omega$, $(2.1 \pm 1.6) y^{-1}$, is consistent with the
upper limits quoted by Rappaport et al. (1980) and Nagase et al. (1982a).

3. **Eclipse timings.**

In the absence of disturbing effects caused by absorption of X-rays in the atmosphere of HD 77581 and variations of its tidal distortion, measurements of the times of eclipse immersion and emersion would in principle yield information on the combined effects of orbital period changes and apsidal motion (see also Appendix II). The effect of apsidal motion is small in the case of Vela X-1. For observations with a duration much shorter than the apsidal motion period $P_{\text{aps}}$, the spurious orbital period derivative $\dot{P}_{\text{conj}}$ (in fact the derivative of the period of the X-ray source superior conjunction) introduced by the apsidal motion is given to first order in $e$ by $\dot{P}_{\text{conj}} = 4\pi e (P_{\text{orb}}/P_{\text{aps}})^2 \cos \omega$. For Vela X-1, at a rate of apsidal advance of $< 3.8 \, \text{yr}^{-1}$ (Rappaport et al. 1980), this comes to $|\dot{P}/P_{\text{conj}}| < 3 \times 10^{-6} \, \text{y}^{-1} (\dot{P} < 0)$, two orders of magnitude below the observed intrinsic period derivative.

Table 5 lists the mid-eclipse timings available in literature. In some cases the quoted value was estimated from published intensity plots; this was done according to the method outlined by Watson and Griffiths (1977). A few closely spaced eclipse measurements made with the same experiments were combined into one value by shifting them by an integer number of $8.96433$ periods.

The eclipse transitions of Vela X-1 are known to show considerable phase jitter, even from one eclipse to the next. This effect sets an upper limit to the accuracy of the period as determined from eclipse timing. From the list of eclipse transitions published by Watson and Griffiths (1977), the effect of the jitter on mid-eclipse determinations is estimated to have an amplitude of $0.1 \, \text{d}$ (sd). We have attempted to take this into account by quadratically adding $0.1 \, \text{d}$ to the errors quoted by the different authors for the mid-eclipse times.

A linear fit to the data gives a period quite consistent with, but four times less accurate than, the value determined from the pulsation measurements. The best-fit value of the epoch of mid-eclipse is $T_{\text{ec1}} = JD 2442728.40 \pm 0.04$. The results obtained are very close to those quoted by Nagase et al. (1982b), who, however, mix in some epochs determined from pulse timing with the mid-eclipse times.

The epoch of mid-eclipse occurs $\sim 0.1 \, \text{d}$ earlier than theoretically expected (see Appendix II), which could be the result of greater absorption of X-rays at eclipse immersion than at emersion. This idea is supported by the fact that eclipse immersion is more gradual than emersion (Watson and Griffiths 1977). The general depletion of the 2-12 keV flux in the second half of the binary light curve is perhaps related to this.

Inclusion of a quadratic term in the fitting function does not improve the fit to the eclipse-center data, nor does it change the value of $T_{\text{ec1}}$. The formal value for the period derivative is $(0.15 \pm 0.33) \times 10^{-4} \, \text{y}^{-1}$, consistent with zero and different (by 2s) from the value of $(1.2 \pm 0.4) \times 10^{-4} \, \text{y}^{-1}$ inferred from the time dependence of $T_{\text{e}2}$. It can be concluded, therefore, that from eclipse measurements there is no evidence for a persistent non-zero $P_{\text{orb}}$ during the interval from December, 1971 to March, 1980.

Restricting the fit to the interval from July, 1975 to March, 1980 for which reliable orbital pulse-delay curves are available, the $T_{\text{ec1}}$ period derivative is $(0.5 \pm 1.4) \times 10^{-4} \, \text{y}^{-1}$, consistent with both the $T_{\text{e}2}$ value and with zero.

Summarizing, eclipse measurements do not mark orbital phase accurately enough to confirm or reject the orbital period change detectable by pulsation measurements over a five-year interval. They disconfirm the persistence of this $P_{\text{orb}}$ during the full eight-year interval in which eclipses have been observed in Vela X-1.

We stress, that these conclusions are subject to the condition, that no systematic changes occurred in the atmospheric effects which influenced the epoch of mid-eclipse.
V. Discussion

1. Orbital period change

The orbital-period change inferred from the pulse-timing data (Table 4) is a 3a-effect, and the reduced $\chi^2$-value of the linear fit to these data, 2.0, implies that a constant period can not strictly be excluded. Assuming the period change to be real, it is interesting to consider the possible causes of such a period change.

A secular increase of the orbital period is expected in systems losing mass by means of a stellar wind. The eclipse data show, however, that the $\frac{\dot{P}}{P}$ derived from the pulse-timing data is probably not a secular effect. Its value, $\sim 10^{-4}$ $y^{-1}$, is astoundingly high. The only time a similar $\dot{P}/P$ was observed in an X-ray binary, was during August-October 1972 in Cen X-3 (Fabbiano and Schreier 1977). This was only a three-month fluctuation, however, in a general speed-up of the orbital motion. From the expression of orbital angular momentum we have, with Kepler's third law:

$$\left(\frac{\dot{P}}{P}\right)_{\text{orb}} = \frac{3}{1} \left(\frac{M_1 + M_2}{M}\right) + \frac{3}{1} \left(\frac{3e^2}{M}\right) + \frac{3}{1} \left(\frac{J}{M}\right)$$

where $M_1$ and $M_2$ are the masses of the mass-losing and gaining star, respectively, $M = M_1 + M_2$ is the total mass of the system, and $J$ is the orbital angular momentum. The mass-loss rate in the stellar wind of HD 77581 has been reliably estimated from UV studies as $\sim 10^{-6}$ $M_\odot$ $y^{-1}$ (Dupree et al. 1980). The accretion rate, $\dot{M}_2 \sim L_\gamma/0.1 c^2$ is $\sim 10^{-10}$ $M_\odot$ $y^{-1}$, negligible for the present purpose. Assuming that the wind carries off a specific orbital angular momentum equal to that of the primary, and that the orbital eccentricity is constant, we find for $M_1 = 21 M_\odot$, $M_2 = 1.7 M_\odot$ (van Paradijs et al. 1977), that $\dot{P}/P_{\text{orb}} \sim 10^{-7}$ $y^{-1}$. So, the wind mass loss is three orders of magnitude below the level required to explain the observed orbital-period increase.

a) Tidal interaction

Without invoking extra mass fluxes in the system, we would have to explain the observed orbital-period change by an increase of the orbital energy at a rate of $E/E \sim -\dot{E}/a \sim -2/3 \left(\frac{\dot{P}}{P}\right)_{\text{orb}} \sim -0.8 \ 10^{-4}$ $y^{-1}$, implying $\dot{E} \sim 3 \times 10^{36}$ erg $s^{-1}$. This could be done without changing the orbital angular momentum if the orbit would at the same time decircularize at a rate $e \sim 3 \times 10^{-4}$ $y^{-1}$. At constant eccentricity, the expansion of the orbit would imply $\dot{J}/J \sim 1/2 \dot{a}/a \sim 1/3 \left(\frac{\dot{P}}{P}\right)_{\text{orb}} \sim 0.4 \ 10^{-4}$ $y^{-1}$, or $\dot{J} \sim 3 \times 10^{41}$ g cm$^2$ $s^{-2}$.

It would be conceivable that tidal interaction with a primary star rotating faster than synchronously with the orbit could supply the required energy and, if necessary, angular momentum to the orbit; already for synchronous rotation, the energy and angular momentum of the rotation of the primary each are only $\sim 4$ times less than the orbital values. If the rotation of the primary were the energy source for the orbital expansion, this would imply time scales of $\sim 10^4$ $y$ for rotational slow-down.

Time scales of tidal interaction are critically dependent on the internal structure of the star (Zahn, 1977). For an evolved star like HD 77581, Zahn's tidal coefficient $E_2$ becomes very small, and time scales much longer than $10^4$ $y$ are expected. This result depends critically, however, on the viscosity of the outer layers of the star.

On evolutionary grounds, Suntanto (1974) assumes that tidal circularization has been operating in Vela X-1 for about $5 \times 10^4$ $y$, and concludes from the finite eccentricity that the tidal-circularization time scale is of this order or larger. Applying the same kind of reasoning to tidal synchronization, it is clear that tidal interaction operating on an evolutionary time scale at an approximately constant level can never explain the observed orbital period slow-down: Either the synchronization time scale is short, and the system should be synchronized by now, or it is long, with the result that it does not sufficiently affect the orbital period for the effect to be observable.

The results obtained by Savonije and Papaloizou (1982) by numerical calculations of the effects of dynamical tides, suggest that episodes of enhanced tidal interaction can occur when the orbital frequency becomes equal to a resonant frequency of the star. The calculations of
these authors are for main-sequence stars, i.e., their results do not apply directly to Vela X-1. Possibly this mechanism could provide a way to temporarily produce tidal interaction with a sufficiently short characteristic time scale to explain the observed $P_{\text{orb}}$. Because the long-term average of the tidal-interaction time scale would be, in this picture, much longer than the current value, the system could have maintained, during the course of its evolution, a finite eccentricity and faster-than-synchronous rotation.

However, HD 77581 is a supergiant, which must have expanded by a factor of $\sim 5$ since it left the main sequence. Any rotation the star had originally must have slowed down considerably by this process. This makes somewhat improbable the super-synchronous rotation required to enable the star to feed energy into the orbit. Indeed, there is spectroscopic evidence that the rotation of HD 77581 is slower, rather than faster than synchronous (Conti 1978; Zuiderwijk 1982, private communication).

So, it is very unlikely that the observed orbital expansion could be explained by tidal interaction. The only way out would be to assume differential rotation in the star, the atmosphere rotating much slower than the region below it.

b) Rapid mass loss

Alternatively, the observed orbital-period increase could be interpreted as an indication of an episode of rapid mass loss from the system. To have a positive effect on the orbital-period derivative, this mass should not carry away too much orbital angular momentum. For example, assume that the mass has a specific angular momentum equal to $J = (1-\alpha) j_1 + \alpha j_2$, $0 < \alpha < 1$, where $j_1$ and $j_2$ are the specific orbital angular momenta of the mass-losing and gaining star, respectively (compare van den Heuvel and de Loore, 1973). We then find that if this mass loss is to cause an increase of $P_{\text{orb}}$, this implies that $\alpha < (2/3) M_1 M_2/(M_1^2-M_2^2)$. In our case $\alpha < 0.05$. This means that the matter lost by the primary should quickly travel through the system, without acquiring too much transverse velocity from the compact star.

Except with a constant $P_{\text{orb}}$ during five years, the orbital epoch data presented in Section IV are also consistent with a rapid increase of the period by $(dP/P)_{\text{orb}} = (2.4 \pm 0.8) \times 10^{-4}$, which occurred between JD 2443000 and JD 2443800. (Note that the sign reversal of the long-term pulse period derivative as discovered by the Hakuro team (Nagase et al. 1981) must have taken place during the same time.)

The observed effect could therefore be the result of either a relatively gradual mass-loss process operating during the interval 1975-1980, or of 'instantaneous' matter ejection somewhere halfway this interval. In both cases, a few times $10^{-3} M_\odot$ of gas must have been lost from the system.

As spherically symmetric outflow, either gradual, like in a stellar wind, or more abruptly, by ejection of a massive ($\sim 10^{-3} M_\odot$) shell, would contradict the optical and UV data, this gas must have been ejected in an asymmetric way.

As mentioned above, HD 77581 is in the evolutionary stage of shell hydrogen burning. It must be expanding on a time scale of only a few $10^3$ yr. Perhaps the star is losing mass on this time scale by, possibly episodic, overspillting of its critical lobe. If the stellar rotation is non-synchronous, it is possible that most of the matter leaves the system directly, without ever coming very near to the neutron star (e.g., Hettersen, 1978). The asymmetry of the binary X-ray light curve (see Section III.1) could be caused by this.

Alternatively, one may speculate that the mass loss occurs by ejection by the primary of high-velocity clouds of matter. When such a cloud hits the neutron star, an X-ray flare would ensue; when it passes through the line of sight to the X-ray source, this would be seen as an absorption dip.

Assume, that the clouds move at a velocity $v_c$ and that the source is in a flaring state (identified with the passage of the neutron star through a cloud) during a fraction $f$ of the time. It is easily found, that then, in order to lose $10^3$ times more mass in the clouds than via a normal stellar wind, the density of the gas within the clouds should be $\rho_{\text{cloud}} = 10^3 f^{-2} (v_c/v_w)$, where $\rho_w$ and $v_w$ are the density and velocity of the wind at the neutron star orbit. This result is independent of the number and size of the clouds, but assumes them to move much faster than the neutron star in its orbit. Assuming that the accretion efficiency from the clouds is proportional to $\rho_c v_c^{-3}$, like in the case of accretion from a stellar wind,
we derive \( v_c/v_w = (10^3/f \xi)^{1/4} \), where \( \xi \) is the factor by which \( L_x \) increases during a flare.

For \( f = 0.05 \) (corresponding to, on the average, 10 hours of flaring per binary cycle) and \( \xi = 10 \), we have \( v_c/v_w = 6.7 \) and \( p_c/p_w = 3 \times 10^3 \).

The duration of a flare is typically between several minutes and a few hours (Charles et al. 1978, Kallman and White 1982). At \( v_w \sim 860 \text{ km s}^{-1} \) (Dupree et al. 1980), this implies cloud sizes which for the longest flares are of the order of the binary separation. This problem could be removed by assuming that the interaction time between the neutron star and the cloud is much shorter than the typical flare duration (which could be connected to, e.g., temporary storage of matter in a disk with subsequent disk decay). If we require the same mass-loss rate in the form of clouds, however, this implies an even higher cloud density than \( 3 \times 10^3 \rho_\odot \). In this picture, the cloud would be well removed from the neutron star during most of the flare, which would be in accordance with the observational evidence that the column density does not change appreciably when a flare occurs (Charles et al. 1978, Kallman and White 1982).

Interception of the X-rays by a high-velocity cloud passing through the line of sight could cause absorption dips with a duration of only a few minutes, such as observed by Kallman and White (1982), and could not explain the more extended absorption events observed by Becker et al. (1978).

Which physical mechanism could produce these clouds is unclear. In any case, the energy requirements would be very large: about \( 10^{50} \text{ erg s}^{-1} \), \( \sim 5 \) times the optical luminosity of the star.

Inhomogeneities in the stellar wind, connected to the observed X-ray dips and flares, do almost certainly occur, but it is not clear whether they could carry away \( 10^{-3} M_\odot \text{ y}^{-1} \).

c) Third body

A periodic effect on the orbital period could be expected from the influence of a hypothetical third body in the system.

The approach of periastron by a third body in an eccentric orbit around a close binary will cause an expansion of its orbit due to tidal effects by \( a/a_0 = (M_3/M) (a/a_0)^3 (1-e_3)^{-3} \) (Fabbiano and Schreier 1977), where \( M = M_1 + M_2 \) as before, and where the index 3 refers to the relative motion of the third body. It is clear from Figure 4, that to explain the orbital period change by a periodic effect, the associated period should be \( \geq 2500 \text{ d} \). Assuming \( P_3 = 2500 \text{ d} \), it is found that in order to explain the observed \( \Delta P/P \) it is sufficient that \( e_3 > 1-0.43 (1+M/M_3)^{-1/3} \). For a wide range on \( M_3 \)-values, \( a_3 \sim 10^{14} \text{ cm} \) and even for \( M_3 \) as low as \( 1 M_\odot \), \( e_3 \sim 0.85 \), \( a_3 \sim 1.5 \times 10^{14} \text{ cm} \), the required point of closest approach \( (a_3(1-e_3)) \) lies well away from the close binary orbit, which has a semi-major axis of 3.4 \( 10^{12} \text{ cm} \). The duration of the phase of closest approach, defined as the time when \( -\pi/2 < \text{true anomaly} < \pi/2 \) however, is relatively short: \( \sim 300 \text{ d} \) for \( M_3 = M_1 \), and only \( \sim 100 \text{ d} \) for \( M_3 = 1 M_\odot \), which is difficult to reconcile with the long time scale of the observed orbital-period change.

Another effect of a third body would be the apparent change of \( P_{\text{orb}} \) due to the Doppler effect caused by the motion of the close binary in the triple system. For \( P_3 = 2500 \text{ d} \), we have \( v_{\text{max}}/c = 5.2 \times 10^{-5} (M_3/M_\odot)^{1/3} (1+M_3/M)^{-2/3} (1-e_3^2)^{-1/2} \), where \( v_{\text{max}} \) is the maximum velocity of the close binary. As an example, for \( e_3 = 0.5 \) and \( M_3 = 27 M_\odot \), the full amplitude of the Doppler shift would become approximately equal to the observed \( \Delta P/P \) of 2.4 \( 10^{-4} \).

The Doppler shift due to the third body would not only affect the binary period, but the pulsation period as well. It would be difficult to detect this effect, however, because the observed, probably intrinsic, variation of the pulsation period is much larger: \( (\Delta P/P)_{\text{pulsar}} \sim 8 \times 10^{-4} \) in \( 10^3 \text{ days} \) (Nagase et al. 1982a). To explain this observed \( \Delta P/P \) by third body effects would require even more extreme assumptions.

In conclusion, if the orbital period increase deduced from the data is real, it is probably not part of a trend which extends over an evolutionary time scale. Its cause could be the, presumably asymmetric, ejection of several times \( 10^{-3} M_\odot \) of matter from the system during the interval 1975-1980, or the presence of a massive (\( > 25 M_\odot \)) third body in the system. In the latter case one would expect to see long-periodic variations of the orbital period. However, it would be expected that a star of this mass would easily be observable in the UV or optical part of the spectrum. As it is not observed, we consider this possibility unlikely. As
also the ejection of a large amount of mass as required by the first-mentioned possibility (Section b) seems hardly possible without contradicting the observations, we are essentially left without an explanation for the change in orbital period.

2. Intrinsic pulse period changes

The rate of change of the pulsation period in Vela X-1 seems to depend strongly on the considered time scales. The observed values of $\dot{P}/P$ range from $+3$ to $-1.5 \times 10^{-4} \text{ yr}^{-1}$ on a time scale of years (Nagase et al. 1982a), to $\dot{P}/P = (+9$ to $-1) \times 10^{-2} \text{ yr}^{-1}$ on a time scale of days (this paper).

While for rapid (< 10 s) X-ray pulsars the observed spin-up rates are commonly in the range $10^{-3}$ to $10^{-4} \text{ yr}^{-1}$ (e.g., Rappaport and Joss 1982), spin-up rates of up to $2 \times 10^{-2} \text{ yr}^{-1}$ have been observed in several long-period (> 100 s) pulsars (Doty et al. 1981, Kelley et al. 1980). This could be a consequence of the lower total angular momentum of these neutron stars, which have a long rotation period. Rapid spin-down episodes were also observed in the long-period sources, with values of up to $2 \times 10^{-3} \text{ yr}^{-1}$ in a one year interval for GX 301+2 ( Kelley et al. 1980), and $4 \times 10^{-3} \text{ yr}^{-1}$ in 10 day interval for Vela X-1 (Nagase et al. 1982a).

The maximum observed values of the short-term spin-up and spin-down rates presented here (Table 3) are at least one order of magnitude higher. They indicate that strong accelerating and braking torques must be acting on the neutron star. We will now discuss in what way these torques could arise.

a) Wind accretion

First, we consider the torque transmitted to the neutron star by matter accreted from a stellar wind. The spin-period change resulting from this process is estimated to be: $|\dot{P}/P| = \frac{\alpha \Omega_a Q x}{2(a \Omega_a)}$ $(M/\Omega a)^2$, where the dimensionless factor $\alpha$ is discussed below, $\Omega_a$ and $Q_x$ are the angular velocity corresponding to the orbital motion and the neutron-star rotation, respectively, $M$ is the accretion rate, $I$ the moment of inertia of the neutron star, and $r_a$ the accretion radius. An estimate of $r_a$ is given by $r_a = 2G M/v_{\text{rel}}^2$, where $M$ is the mass of the neutron star and $v_{\text{rel}}$ is the velocity of the wind relative to the neutron star (Shapiro and Lightman 1976).

The value and sign of the factor $\alpha$ in the expression for $\dot{P}/P$ will depend on the assumed wind characteristics: $\alpha$ is usually assumed to be equal to 1 in the case of a spherically symmetric non-rotating wind with a constant velocity near the compact star (Shapiro and Lightman 1976), but it could be as low as zero according to the analytical particle treatment of Davies and Pringle (1980). On the other hand, if the radial and azimuthal gradients in the density and velocity of the wind are considered which can be expected as a consequence of, e.g., primary rotation and radial acceleration of the wind, $\alpha$ could vary between $\sim -5$ and +5 (see Wang 1981). For Vela X-1, the magnitudes of these changes are computed to be of the order (for $\alpha = \pm 5$): $|\dot{P}/P| = 2.8 \times 10^{-5} \text{ yr}^{-1} \times 16 I_{45}^{-1} (r_a/10^{10} \text{ cm})^2$, where $M$ and $I$ are expressed in units of $10^{16} \text{ g cm}^2$ and $10^{45} \text{ g cm}^2$, respectively. The luminosity during our observations is probably $\sim 0.5 \times 10^{36} \text{ erg s}^{-1}$ (see Section III.1). To take into account the possible effects of absorption, we will here assume the maximum steady X-ray luminosity of $1.4 \times 10^{36} \text{ erg s}^{-1}$ quoted for Vela X-1 by Bradt et al. (1978). For this $L_x$, $M$ (estimated as $\sim L_x/0.1c^2$) is about $1.5 \times 10^{16} \text{ g s}^{-1}$. Assuming that the wind velocity near the neutron star is 860 km s$^{-1}$ (Dupree et al. 1980), we find $r_a \sim 3.3 \times 10^{10} \text{ cm}$, so that for the standard value $I_{45} = 1$ the maximum ($\alpha = \pm 5$) value of $|\dot{P}/P|$ is only $\sim 4.5 \times 10^{-4} \text{ yr}^{-1}$, two orders of magnitude below that observed.

If we would require the specific angular momentum of the matter which is accreted from the wind to be sufficiently high to match the maximum observed spin-down rate, the accretion radius would need to be very large: $r_a \sim 4.7 \times 10^{11} \text{ cm}$. This would imply a capture fraction $(r_a/2a)^2$ of matter from the wind of $\sim 0.5\%$, which is much too high to be consistent with the X-ray intensity and the mass-loss rate in the wind. In addition to this, to allow the accretion radius to become this large, the relative wind velocity should be $\sim 240$ km s$^{-1}$, lower than the orbital velocity, and much below the observed wind velocities (Dupree et al. 1980).

In conclusion, angular momentum transferred by matter accreted from a stellar wind which
has only small gradients in velocity and density, cannot explain the observed \( \dot{P} \) values.

b) Disk accretion

We now assume that the matter in some way acquires a sufficient specific angular momentum to form an accretion disk. The maximum accreted torque can be derived in this case as \( N = \dot{M}Q \), where \( Q = (GM_m)^{1/2} \) is the specific angular momentum of matter in a Kepler orbit at the inner edge of the disk, assumed to be located at the magnetospheric radius \( r_m \). Such a torque would result in the case of Vela X-1 in a rate of change of the spin period of \( |\dot{P}/P| = \pm 1.6 \times 10^{-7} \) s\(^{-1} \) \( M_{16}^{-1} \), where the sign depends on whether the disk is prograde or retrograde.

If we assume standard values for the neutron star magnetic moment: \( \mu_{30} (= \mu \text{ in units of } 10^{30} \text{ Gauss cm}^3) = 1 \), which for a neutron star with a radius of \( 10^6 \text{ cm} \) corresponds to a surface magnetic field strength of \( B_{12} (= B \text{ in units of } 10^{12} \text{ G}) = 1 \), and adopt the expression:

\[
r_m = 6.2 \times 10^8 \mu_{30}^{-2/7} M_{16}^{-1/7} \text{ cm}
\]

(see van der Klis et al. 1980), we find for \( M_{16} = 1.5 \) that \( r_m \sim 5.5 \times 10^5 \text{ cm} \). The maximum spin-up or spin-down rate derived from this is \( |\dot{P}/P| \sim 5.6 \times 10^{-3} \) s\(^{-1} \), still a factor of 16 below the maximum observed rates. To explain these, the tangential velocity of the matter entering the magnetosphere would have to be \( 8 \times 10^8 \text{ cm s}^{-1} \), well above the escape velocity. Furthermore, the rapid alternation which we observe between spin-up and spin-down would mean that the disk is inverting its sense of rotation several times during the interval of a few days.

It might be argued that the intrinsic X-ray luminosity, and thus the accretion rate, were underestimated in this discussion, because of absorption effects. However, column densities and X-ray 'bolometric corrections' derived from X-ray spectra of Vela X-1 (Kallman and White 1982, Dolan et al. 1981, Becker et al. 1978) show that absorption could not consistently affect the 2-12 keV luminosity sufficiently to explain the discrepancy between \( L_X \) and \( \dot{P} \) discussed here. In fact, the factor of about three by which we raised our observed \( L_X \) at the outset of this discussion to take these effects into account is already a generous estimate. In addition to this, no trace is observed during the spin-down episodes of the X-ray flares which would, even in the presence of some absorption, be the consequence of the high accretion rates needed to transfer the angular momentum (Nagase et al. 1982a; this paper, Sect. III.4).

We therefore conclude, that it is unlikely that the observed rapid spin-down episodes are caused by angular-momentum transfer by the accreting matter. If the torque is related at all to matter moving in the surroundings of the neutron star, other mechanisms, which do not involve accretion, have to be invoked.

c) Propeller effect

One possibility could be the 'propeller effect' (Illarionov and Sunyaev 1975, Davidson and Ostriker 1973), which starts to act when the magnetospheric radius becomes larger than the corotation radius. If we require the magnetosphere to accelerate the matter, originally moving in Kepler orbits, to the escape velocity, we find that for Vela X-1 \( r_m \sim 9 \times 10^8 \text{ cm} \). Assuming the matter is spun away in the form of free particles, the approximation which provides the most efficient braking (Davies et al. 1979), we find that \( \mu_{30} \) should be as large as \( 3.5 \times 10^6 \) and that \( \sim 10^{17} \text{ g s}^{-1} \) should be expelled to produce the observed spin-down episodes.

d) High-velocity clouds

If, on the other hand, the magnetic moment is assumed to have a 'normal' value (\( \mu_{30} = 1 \)), then, in order to be able to escape from the neutron star the matter must, after having interacted with the magnetosphere, move at a characteristic velocity which is much higher than the corotation velocity at \( r_m \). This would be possible if the matter already has a large velocity when it approaches the neutron star. If we take the picture (see Section 1b) of dense clouds of matter which are ejected by the primary with high velocity, we find that an encounter of the magnetosphere with a cloud with a mass of \( \sim 10^{23} \text{ g} \) which changes the velocity of the cloud by \( \sim 10^4 \text{ km s}^{-1} \) ('windmill effect'), could explain the magnitude of the observed pulsation period changes (\( \Delta P/P \sim 10^{-4} \)). The same caveats as mentioned in Section 1b apply, however: the density of the clouds would be relatively high, and a large amount of energy would be needed to accelerate them.
Neutron star interior

In a very different approach Lamb et al. (1978), proposed to explain pulsation-period changes in X-ray pulsars by the accumulated effect of random torque fluctuations filtered by the response of the interior of the neutron star. In particular, adopting a two-component model of the neutron star, these authors are able, for a certain choice of parameters describing the accretion torques and the neutron star response, to reproduce the 1972 spin-down episode in Cen X-3 which involved a relative period change of \( \Delta P/P \approx 5 \times 10^{-4} \) in 20 d. The maximum change of the pulse period in the present observations is of the same order, although it takes place in a much shorter interval of time. In the absence of sufficiently quantitative knowledge of the crust-core coupling times and of other parameters describing the torque fluctuations (Lamb et al. 1978), we can not, at this point, either confirm or exclude the possibility that torques originating in the interior of the neutron star cause the observed spin-down episodes.

VI. Conclusion

Further determinations of the binary orbit of Vela X-1 by pulse timing analysis will show whether the trend towards a longer orbital period which can be deduced from the pulse delay data up to now, continues. For the evaluation of the significance of such a trend it will be advantageous to use orbital epochs defined in such a way, that they do not depend strongly on the periastron angle. Changes of the orbital period occurring at a rate of \( \approx 10^{-4} \text{ yr}^{-1} \) are difficult to explain; if they would be confirmed, rather drastic conclusions about the system seem unavoidable.

The very rapid pulsation period changes we observe, up to \( 10^{-1} \text{ yr}^{-1} \), are inconsistent with the standard assumptions of the stellar-wind accretion model. If the angular momentum required for these changes is transferred to the neutron star by external matter, considerable inhomogeneities in the density and/or velocity of this matter must be present. Perhaps these inhomogeneities could strongly increase the mass-loss rate of the system as a whole over that observed in the stellar wind of HD 77581, and thus be responsible for changes in the orbital period as well.

The torques responsible for the rapid pulse-period changes are evidently variable on time scales of a few days or less. Possibly, the changes of the pulsation period occurring on longer time scales are determined by other, less rapidly variable torquing processes. It is conceivable, however, that the long-term \( \dot{P} \) changes are nothing but the effect of the way that the short time-scale torque fluctuations average out.

Acknowledgements

The authors gratefully acknowledge useful comments and discussion by J. van Paradijs, E.P.J. van den Heuvel, J.A.M. Bleeiker and G.J. Savonije. MvdK acknowledges financial support by ZWO (ASTRON).

Appendix I. The expansion of \( z \) in powers of \( e \)

While the first-order terms of the expansion of \( z \) in powers of \( e \) are easily found in literature (Fabbiano and Schreiber 1977, Deeter et al. 1981), this is not the case for the higher order terms. In the expansion below, terms up to the second order were computed directly from the expression \( z = r \sin i \sin (\nu + \omega) \), where \( \nu \) is the true anomaly. Third order terms were converted from Russell (1902), who gives an expansion of the radial velocity in terms of the mean anomaly.
\[
\frac{z}{(a \sin i)} = z_0 + z_1 e + z_2 e^2 + \ldots, \text{ where}
\]

\[
z_0 = \sin \xi
\]
\[
z_1 = (1/2) \sin (2 \xi - \omega) - (3/2) \sin \omega
\]
\[
z_2 = -(1/2) \sin \xi - (1/8) \sin (\xi - 2\omega) + (3/8) \sin (3\xi - 2\omega)
\]
\[
z_3 = -(3/8) \sin (2\xi - \omega) - (1/24) \sin (2\xi - 3\omega) + (1/3) \sin (4\xi - 3\omega),
\]

where \( z \) = line-of-sight displacement with respect to the center of mass of the binary
\( a \sin i \) = projected semi-major axis
\( e \) = eccentricity
\( \omega \) = periastron angle
\( \xi \) = mean longitude \( = M + \omega \)
\( M \) = mean anomaly \( = 2\pi (t - T_\omega)/P_{\text{orb}} \)
\( T_\omega \) = time of periastron passage.

Appendix II. Orbital epochs

Of the several different orbital epochs used in literature, those which refer to a fixed value of the mean longitude \( \xi \) are the most useful to measure the (sidereal) orbital period (Deeter et al. 1981). Examples are \( T_0 \) and \( T_{\pi/2} \), the times when \( \xi = 0 \) and \( \xi = \pi/2 \), respectively, which are simply related by

\[
T_{\pi/2} = T_0 + \frac{1}{4} P_{\text{orb}} \tag{1}
\]

The periastron passage time \( T_\omega \), the time when \( M = v = 0 \) and \( \xi = \omega \) could be used, in principle, to measure the (anomalistic) orbital period, but is in practice difficult to observe when \( e \) is small; it is related to \( T_{\pi/2} \) by

\[
T_{\pi/2} = T_\omega + \frac{P_{\text{orb}}}{2\pi} (\frac{\pi}{2} - \omega) \tag{2}
\]

The time of superior conjunction of the X-ray source \( T_{\text{conj}} \) is an epoch defined in true anomaly: \( v_{\text{conj}} \equiv (\pi/2) - \omega \), and thus is in the presence of apsidal motion subject to a sinusoidal variation with a period equal to the apsidal motion period and an amplitude \( e P_{\text{orb}}/\pi \) (Batten 1973). To first order in \( e \):

\[
T_{\pi/2} = T_{\text{conj}} + \frac{e P_{\text{orb}}}{\pi} \cos \omega \tag{3}
\]

For Vela X-1 (\( e \sim 0.1 \), \( \omega \sim 160^\circ \)) \( T_{\text{conj}} \) will lag \( T_{\pi/2} \) by \( \sim 0.3 \) d.

It is pointed out that due to variations in transverse orbital velocity during eclipse, even in the absence of atmospheric and distortion effects, \( T_{\text{conj}} \) is not identical to the mid-eclipse time as defined by the average of eclipse immersion and emersion times. From the results given by Kopal (1959) it is derived that for a spherical primary with a radius \( R \), to first order in \( e \) the mid-eclipse time \( T_{\text{mid}} \) would be

\[
T_{\text{mid}} = T_{\text{conj}} - \frac{e P_{\text{orb}}}{\pi} \cos \omega \frac{(\sin i - \beta) (1 - \beta \sin i)}{\beta \sin^2 i} \tag{4}
\]

Where \( i \) is the inclination and \( \beta \equiv (1 - (R/a)^2 (1 - e^2)^{-1})^{1/2} \).

Assuming \( i = 90^\circ \), and adopting \( R/a = 0.6 \) (Rappaport et al. 1980) we have \( \beta = 0.8 \) and find that \( T_{\text{conj}} \) precedes \( T_{\text{mid}} \) in Vela X-1 by 20 min. \( (1.4 \times 10^{-2} \) d). In practice, naturally, in Vela X-1 this shift is dominated by other effects (see Avni 1976 for a discussion of the effects of tidal distortion).
### Table 1. Average pulsation parameters

<table>
<thead>
<tr>
<th>Observation</th>
<th>( t_0 ) (JD-2440000)</th>
<th>( P ) (s)</th>
<th>( \dot{P} ) ( (10^{-9}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>2728.430</td>
<td>282.9196 ± 0.0008</td>
<td>14.9 ± 1.1</td>
</tr>
<tr>
<td>1976</td>
<td>2997.121</td>
<td>282.8185 ± 0.0008</td>
<td>-0.4 ± 0.4</td>
</tr>
<tr>
<td>1979</td>
<td>4171.448</td>
<td>282.7791 ± 0.0002</td>
<td>9.1 ± 0.3</td>
</tr>
</tbody>
</table>

1σ single-parameter errors are quoted.

### Table 2. Orbital parameters

<table>
<thead>
<tr>
<th>Observation</th>
<th>1975</th>
<th>1976</th>
<th>1979</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \sin i ) (s)</td>
<td>113.0 ± 4.0*</td>
<td>112.1 ± 2.4</td>
<td>113.5 ± 0.2</td>
</tr>
<tr>
<td>( T_0 ) (JD - 2440000)</td>
<td>2726.013 ± 0.017</td>
<td>2994.878 ± 0.013</td>
<td>4169.224 ± 0.013</td>
</tr>
<tr>
<td>( e )</td>
<td>0.137 ± 0.022</td>
<td>0.077 ± 0.024</td>
<td>0.092 ± 0.021</td>
</tr>
<tr>
<td>( \omega (°) )</td>
<td>147 ± 13</td>
<td>132 ± 17</td>
<td>161 ± 13</td>
</tr>
</tbody>
</table>

\( P_{\text{orb}} = 8.9643 \) d (fixed)

1σ uncertainties were determined according to the method of Lampton et al. (1977).

* Ögelman et al. (1977)

$ The orbital epoch \( T_0 \) refers to the time when \( i = 0 \).

### Table 3. Short-term pulsation period changes (1975 observation)

<table>
<thead>
<tr>
<th>Interval</th>
<th>( t_0 ) (JD - 2440000)</th>
<th>( P ) (s)</th>
<th>( \dot{P} ) ( (10^{-7}) )</th>
<th>Mean Flux (c/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2724.87 - 2731.31</td>
<td>2728.090</td>
<td>282.9112 ± .0012</td>
<td>- 0.6 ± 0.2</td>
<td>95</td>
</tr>
<tr>
<td>2731.62 - 2733.97</td>
<td>2733.795</td>
<td>282.9127 ± .0009</td>
<td>+ 8.1 ± 1.4</td>
<td>96</td>
</tr>
<tr>
<td>2733.97 - 2739.65</td>
<td>2736.810</td>
<td>282.9181 ± .0010</td>
<td>- 1.2 ± 0.4</td>
<td>112</td>
</tr>
<tr>
<td>2739.67 - 2743.48</td>
<td>2741.575</td>
<td>282.9394 ± .0039</td>
<td>+ 3.5 ± 0.7</td>
<td>115</td>
</tr>
</tbody>
</table>

1σ single-parameter errors are quoted.
### Table 4. Orbital parameter changes

<table>
<thead>
<tr>
<th>Reference</th>
<th>Orbital epoch quoted by authors *</th>
<th>Cycle number</th>
<th>$T_{\pi/2}$ (JD-2440000)</th>
<th>$\omega$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charles et al. 1978</td>
<td>$T_\omega$</td>
<td>-31</td>
<td>2450.4 ± 1.6</td>
<td>125 ± 52</td>
</tr>
<tr>
<td>Rappaport et al. 1976</td>
<td>$T_{\text{conj}}$**</td>
<td>-13</td>
<td>2611.73 ± 0.05</td>
<td>146 ± 9</td>
</tr>
<tr>
<td>This paper</td>
<td>$T_0$</td>
<td>0</td>
<td>2728.254 ± 0.017</td>
<td>147 ± 13</td>
</tr>
<tr>
<td>&quot;</td>
<td>$T_0$</td>
<td>30</td>
<td>2997.119 ± 0.013</td>
<td>132 ± 17</td>
</tr>
<tr>
<td>Rappaport et al. 1980</td>
<td>$T_\omega$</td>
<td>122</td>
<td>3821.84 ± 0.13</td>
<td>158 ± 5</td>
</tr>
<tr>
<td>This paper</td>
<td>$T_0$</td>
<td>161</td>
<td>4171.465 ± 0.013</td>
<td>161 ± 13</td>
</tr>
<tr>
<td>Nagase et al. 1981</td>
<td>$T_\omega$</td>
<td>176</td>
<td>4305.94 ± 0.070</td>
<td>155.3 ± 2.5</td>
</tr>
</tbody>
</table>

Errors quoted are those given by the authors reduced to the 1σ level.

* See App. II for definition of the symbols

** Rappaport 1982 (private communication)

---

**Linear fit to $T_{\pi/2}$ values**

- $T_{\pi/2,0} = 2728.220 ± 0.011$
- $P_{\text{orb}}^0 = 8.96422 ± 0.00011$
- $x_\nu^2 = 2.0$ at $\nu = 5$

**Parabolic fit to $T_{\pi/2}$ values**

- $T_{\pi/2,0} = 2728.250 ± 0.015$
- $P_{\text{orb},0}^0 = 8.96196 ± 0.00075$
- $(P/P)_{\text{orb}} = (1.2 ± 0.4) \times 10^{-4} \text{ y}^{-1}$
- $x_\nu^2 = 0.08$ at $\nu = 4$

**Linear fit to $\omega$ values**

- $\omega^0 = 147 ± 6$
- $\omega = (2.1 ± 1.6) \times 10^{-1} \text{ y}^{-1}$
- $x_\nu^2 = 0.5$ at $\nu = 5$

**1σ single parameter errors are quoted for the fits.**

The suffix 0 in $T_{\pi/2,0}$, $P_{\text{orb},0}$ and $\omega^0$ refers to the best fit value of this parameter at cycle number 0.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Cycle number</th>
<th>( T_{\text{ecl}} ) (JD-2440000)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ulmer et al. 1972</td>
<td>- 158</td>
<td>1312.20 ± 0.22</td>
<td>own estimate</td>
</tr>
<tr>
<td>Forman et al. 1973</td>
<td>- 143</td>
<td>1446.54 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>Watson and Griffiths 1977</td>
<td>- 40</td>
<td>2369.78 ± 0.39</td>
<td></td>
</tr>
<tr>
<td>Charles et al. 1978</td>
<td>- 31</td>
<td>2450.47 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>Watson and Griffiths 1977</td>
<td>- 11</td>
<td>2629.74 ± 0.14</td>
<td>3 eclipses combined</td>
</tr>
<tr>
<td>Ögelman et al. 1977</td>
<td>0</td>
<td>2728.43 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>Watson and Griffiths 1977</td>
<td>13</td>
<td>2844.91 ± 0.20</td>
<td>2 eclipses combined</td>
</tr>
<tr>
<td>Molteni et al. 1982</td>
<td>29</td>
<td>2988.37 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>Dupree et al. 1980</td>
<td>100</td>
<td>3624.76 ± 0.17</td>
<td>own estimate</td>
</tr>
<tr>
<td>Dolan et al. 1981</td>
<td>102</td>
<td>3642.65 ± 0.19</td>
<td>own estimate; 2 eclipses combined</td>
</tr>
<tr>
<td>Nagase et al. 1982b</td>
<td>177</td>
<td>4315.07 ± 0.11</td>
<td></td>
</tr>
</tbody>
</table>

Errors were computed according to \((\text{error quoted by author})^2 + (0.1 \text{ d})^2 \)\(^{1/2}\) (see text).
References

Becker, R.H., Rotschild, R.E., Boldt, E.A., Holt, S.S., Pravdo, S.H., Serlemitsos, P.J.,
ESRO SP-106, 345.
Bradt, H.V., Doysey, R.E., Jernigan, J.G., 1979, Adv. in Space Expl. 3.
MNRA S 183, 813.
Davies, R.E., Fabian, A.C., and Pringle, J.E., 1979, MNRA S 186, 779.
McClintock, J.E., Rappaport, S., Joss, P.C., Bradt, H., Buff, J., Clark, G.W., Hearn, D.,
Ogelman, H., Beuermann, K.P., Kanbach, G., Mayer-Hasselwander, H.A., Capozzi, D.,
Van Paradijs, J., Zuidervijk, E.J., Takens, R.J., Hammerschlag-Hensberge, G.,
Figure 1. The intensity of Vela X-1 during each of the three observations, folded modulo 8.9643 $\alpha$. The bin width is 0.01 in phase; in a few cases, when insufficient phase coverage was available, it was extended to $\sim 0.04$. Intrinsic source variability dominates the dispersion of the points collected in one bin ($\sim 3.5$ c/s, standard deviation); the typical error per bin from counting statistics is $\sim 0.5$ c/s (1$\sigma$). Some of the structure of the 1979 light curve only may be due to residual background contamination.
Figure 2. Variations of the barycentric pulse arrival times in the three COS-B observations (left hand frames). The error bars were computed according to the method of Lampton et al. (1977). Linear trend and the effect of the binary orbit were subtracted: a zero time delay is expected for a constant pulsation period. A clear parabolic term is present in 1975 and 1979. Note the significant deviations from the parabolic trend in the 1975 observation around JD 2442733.0 and JD 2442741.0. Also shown (right-hand frames) are the residuals with respect to the fitted parabolas, plotted against orbital phase.
Figure 3. Mean pulse profiles. 2-12 keV pulse profiles were obtained in each observation by summing phase-aligned individual pulses. The profiles were normalized by subtracting the mean values in each case and are repeated once for clarity. Vertical scale is arbitrary, but identical in each frame. Bin width is ~ 5 s, statistical error per bin is ~ 0.8 units.
Figure 4. The evolution of orbital epoch (see text for definition) and periastron angle as determined from pulsation measurements. Error bars are 1σ. In the case of the orbital epochs, errors are probably overestimated in several cases (see text). Drawn curve is best fit parabola, dashed curves are best linear fits. A significant variation of the orbital period is deduced from the data presented in the upper frame.