The line spectrum of delta cephei

Walraven, Th.

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THE CURVE OF GROWTH

9. THE GENERAL CURVE OF GROWTH

For the determination of the physical parameters of the stellar atmosphere and of the concentration of the chemical elements from the line intensities the knowledge of the empirical curve of growth, i.e. the relation between log A and log Nf is necesssary. Defined in this way, each empirical curve of growth refers to one definite energy level of a particular atom or ion, while N is the number of atoms per cm^2 on this level and f and A the oscillator strengths and equivalent widths for the lines absorbed from this level. The catalogues of line intensities give values for log A, which may be used, without any further reduction, and plotted against log Nf values, in order to obtain the empirical curve of growth.

Several reasons make log Nf values, derived from solar line intensities, by means of the solar curve of growth, the most suitable for this purpose. These log Nf values differ from those of the star by a horizontal shift for each curve of growth, which does not influence the determination of Doppler width and damping constant by means of the theoretical curve of growth to be considered presently. We have made use of the Fraunhofer Intensity Tables of Allen 8), as the most general information at the time. As a rule the values of the equivalent width W in A.U., (column 8 of Allen's ,,General Intensity Table"), was taken; if a value of W was found directly from the contours and given in column 10, this was used. Sometimes no value of W was indicated, whereas in the stellar spectrum the line in question was quite well measurable. In such cases the equivalent width was derived from the Rowland intensity with the aid of the Mulders calibration.

For the reduction of W to log Nf Allen's Table 3 was used, interpolating between the values of λ and atomic weight of the table. Though better data have since become available, e.g. the excellent curve of growth determined by Wright 9) or by Menzel, Baker and Goldberg 10), it did not seem necessary to repeat the computations.

The measurements on the combined spectra in the first catalogue give the most reliable results and it seemed appropriate therefore to construct curves of growth for delta Cephei from these data. This was done by grouping the measurements according to Allen's log Nf values and computing the mean of the log A of each group. In this way curves of growth for delta Cephei were obtained, for both the maximum and minimum phases, for the neutral atoms, for the ionized atoms, and, in the case of the numerous Fe lines, for six separate wavelength regions. All these curves showed the same character, a large flat portion bending down into a Doppler portion; in some cases strong lines indicated a rise into a damping portion. Moreover it appeared that all neutral line curves were identical and could be brought to coincidence by a horizontal shift, i.e. in the log Nf direction. The direction of the shift is such, that the number of absorbing atoms is larger, i.e. the spectral lines are stronger, in the minimum phase. The dependence on wavelength is such that the lines become stronger towards the longer wavelengths. In the latter case the fact that the curves have an identical shape tests the reliability of the method of measuring, since corresponding parts of these curves are based on entirely different contours owing to the difference in dispersion; thus the objection that the results might be influenced by the choice of the parameters for the construction of the contours is refuted. In the case of the maximum and minimum phases the identity of figure is important, for it expresses that the change in equivalent width of the spectral lines in delta Cephei must be ascribed chiefly to changes in the numbers of atoms rather than to changing Doppler width.

The only exception of the uniformity of shape was given by the ion lines. The ion line curves were strongly shifted, indicating increased ionization in delta Cephei, relative to the sun. The upward damping portion was present, the Doppler portion, however, was steeper than in the case of the neutral atoms. Now this part of the curves is based on the weakest lines of the spectrum and its uncertainty is therefore increased. The curve of the neutral atoms can be falsified by the increased disturbance by unknown ion

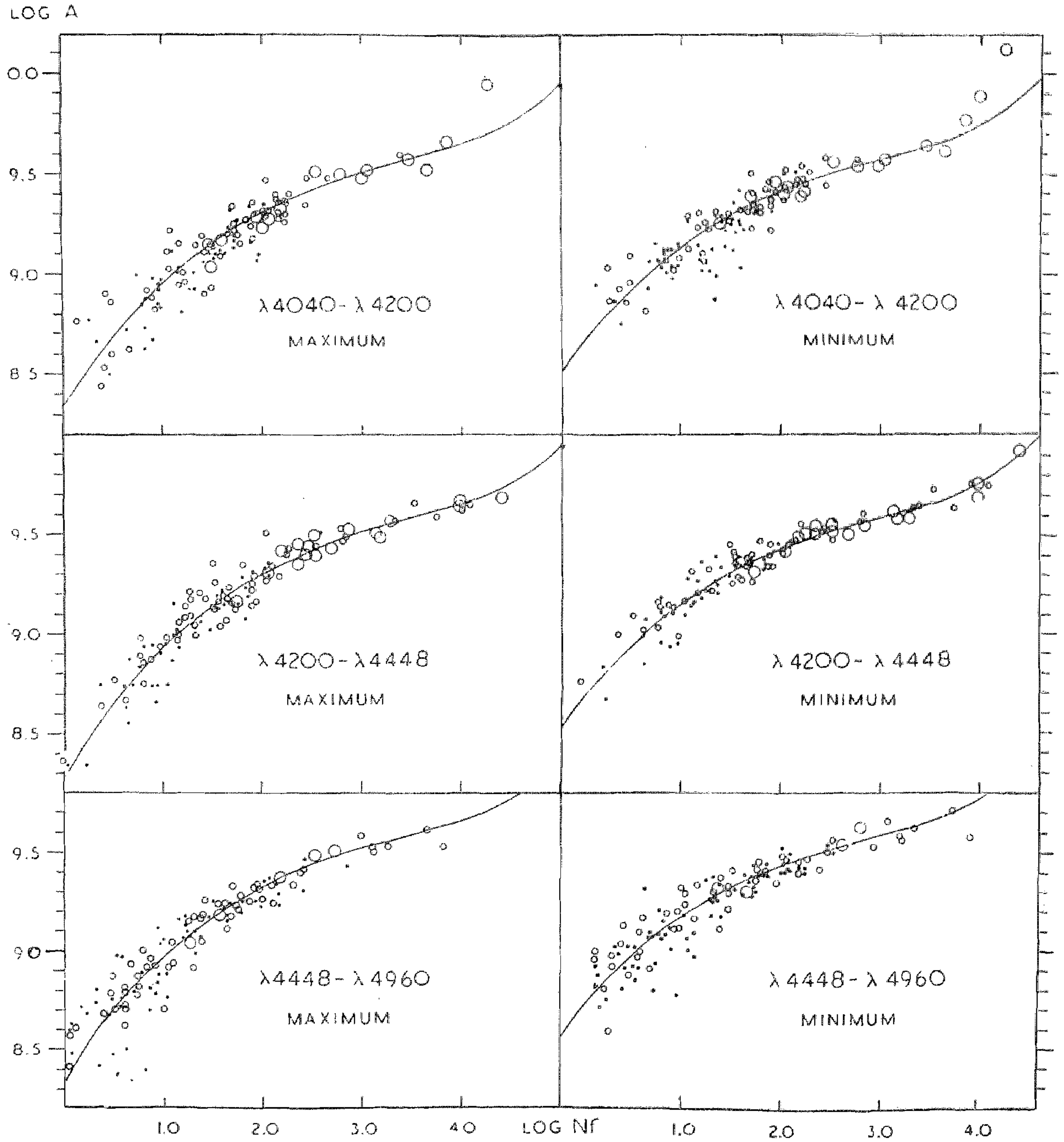

Fig. 6. Curves of growth from combined spectra. Fe lines only. Diameter indicates weight. Full lines represent adopted general curve of growth.

lines in the spectrum of delta Cephei; also by the method of taking the mean of log A, whereas no account was taken of the errors in log Nf, and finally by a selection of stronger lines. The curve of the ionized atoms, on the other hand, can be falsified by the large uncertainty in the solar log Nf of such ion lines as are extremely weak in the sun, but well observable in delta Cephei. Some of these log Nf values may be too large by a disturbing neutral component in the solar spectrum. It was believed, that the above mentioned influences are entirely responsible for the observed differences in the curves of growth.

A final determination of the curve of growth was made, now graphically, and only well selected Fe lines were used, which are so numerous, that a good representation of the whole curve is given by them. The other atoms were not included, as they might show relative shifts. By dividing each of the combined spectra in three wavelength parts, in order to prevent too dense crowding of points in the plot, six curves were obtained, from which, after mutual shifts in horizontal direction a mean curve was drawn and this was assumed to be the general curve of growth of delta Cephei.

Table 8. Coordinates of general curve of growth for delta Cephei.

log A	log Nf	log A	log Nf
8.20	0.64	9.30	2.72
8.30	0.76	9.35	2.90
8.40	0.88	9.40	3.11
8.50	1.01	9.45	3.34
8.60	1.15	9.50	3.62
8.70	1.31	9.55	3.96
8.80	1.47	9.60	4.37
8.90	1.65	9.65	4.71
9.00	1.87	9.70	4.99
9.05	1.98	9.75	5.21
9.10	2.11	9.80	5.38
9.15	2.25	9.85	5.51
9.20	2.39	9.90	5.64
9.25	2.55	9.95	5.76
9.30	2.72	0.00	5.87

It is given in Table 8. In the plots, shown in Figure 6, the general curve is drawn in as a full line.

The damping portion caused some trouble. It depends on rather few lines, which are but little stronger than the bulk of lines in the flat portion of the curve. Of these Fe 4045 and Sr$^+$ 4077 deviated, being too strong. The reason for this is not understood, it may be not real, as both lines lie at the very end of the spectrum. In constructing the damping portion, these lines were excluded.

In the theoretical curve of growth given by Unsöld 13) (p. 268) the quantity log $(A_\lambda/\Delta\lambda_D)$ is plotted vertically. After the observed curve, where log A is plotted vertically, A being defined as $A_\lambda\ 4300/\lambda$, is brought into coincidence with this theoretical curve, the constant difference in vertical scale yields therefore log $\Delta\lambda_D$, referring to λ 4300. Figure 7 shows the comparison of the observed general curve of growth with Unsöld's theoretical curve. An exact fit is not possible, but the parameters, Doppler width $\Delta\lambda_D$ and damping constant a (in units of $\Delta\omega_D$) of the best fitting curve shown in the figure are given by log $\Delta\lambda_D$ = 0.86 — 2 and log a = 0.59 — 3. The position of the curve on the log Nf scale can be characterized by the straight line, also shown in the figure, into which the Doppler portion merges, which may be called the Doppler asymptote. It is given by the equation log A = log Nf — 2.53.

Strömgren's more rigorous theoretical curve of growth based on the M.E. model gave log $\Delta\lambda_D$ =

0.90 — 2, log a = 0.45 — 3 and a Doppler asymptote log A = log Nf — 2.50. The Doppler asymptote fitting to the observed curve itself is log A = log Nf — 2.41. Taking the mean of these values, the Doppler width at λ 4300 becomes:

$$\Delta \lambda_D = 0.076 \text{ A.U.} \pm 0.004 \tag{14}$$

agreeing approximately with the value of 0.10 A.U. estimated from the shape of the profiles and the damping constant:

$$\gamma = 2.1\,\gamma_{cl.} \tag{14a}$$

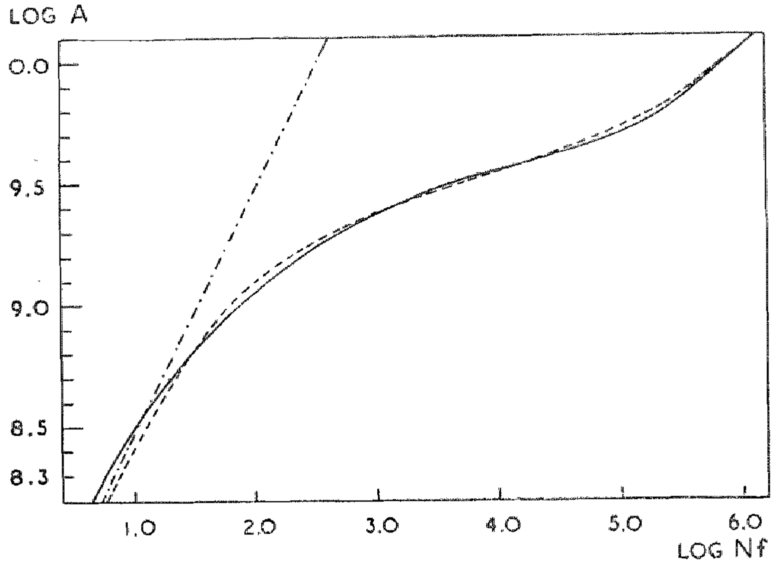

Fig. 7. General curve of growth of delta Cephei (————), compared with theoretical curve (— — — —).

The observed Doppler width corresponds to a turbulent velocity of 5.0 km/sec. If considered as kinetical temperature of Fe atoms it would correspond to a temperature of 100.000°.

The main discrepancy between theory and observation is the weak curvature of the observed curve in the part, where the Doppler portion merges into the flat portion. In this respect the agreement is better with the Milne—Eddington curves. Now, where the agreement between theoretical and observed curves of growth even in the case of the sun is not perfect, as it appears from the curve by Wright 9), one should not expect it to be in the case of a variable atmosphere like that of delta Cephei. The effect might easily be explained by a non-Maxwellian velocity distribution. If, for example, preference exists for vertical up and downward velocities, the curve of growth may assume the observed shape. Although therefore the discrepancy might be explained by errors in measurement, there are also arguments for its reality and it seemed not necessary, therefore, to insist on the agreement with the theoretical curve of growth; and the curve of growth may be used as it is found.

10. CURVES OF GROWTH OF THE INDIVIDUAL SPECTRA

While discussing the curve of growth of the combined spectra it was remarked, that no perceptible difference existed between the curves of the maximum and minimum phases, whereas the curves of different wavelengths could be brought to coincidence by horizontal displacement.

In the case of the curves of growth of the individual spectra, corresponding to various phases, the numbers of lines available was not sufficient to study simultaneously the shape and the wavelength dependence of the curves. The procedure followed was therefore a different one. For each line of the individual spectra the log Nf corresponding to the measured A was read off from the general curve of growth; Table 8. Then this log Nf shows a dependence on wavelength as well as on phase. The dependence on wavelength for each phase turned out to be linear, but the slope of this linear relation varied greatly for different phases, and thus seems to be related to the mean photographic density of the spectrum.

It was eliminated empirically by applying the correction:

$$-\,w\,(\lambda - 4500)$$

to log Nf, where the factor w ranges from 0.14 to 0.45 for the various plates.

Then curves of growth were found for each plate, by plotting the log A of the Fe lines against log Nf, corrected for excitation and wavelength. The excitation temperatures used, to be discussed later on, are given in Table 10. Within the accuracy, limited by the scatter of the points, the curves appeared to be identical with the general curve, which therefore is used in all further computations.

An exception was made for plates 17920 and 17765, at phases 0.065 and 0.261. The curves of growth, obtained from these plates, differed markedly in slope and could be brought to coincidence with the general curve only after a vertical displacement, downward for 17920 and upward for the other plate. A reason for this could not be found and it was ascribed to the use of an erronous characteristic curve, because it afterwards appeared, that the deviation from the run with phase of the log Nf values shown by these plates, disappeared after a correction was applied to the observed log A, equal to the vertical displacement of the curve of growth. The applied corrections were — 0.06 for 17920 and + 0.08 for 17765. In the second catalogue the log A given is the uncorrected value.

11. VARIATION OF THE DOPPLER WIDTH

The use of the general curve of growth for all phases is based on the obvious similarity of the curves, obtained for the individual plates, with that for the combined spectra. Van Albada 12), however, introducing the Doppler width as an unknown parameter in a least squares solution, found different values of it at different phases. This was a reason to try a numerical determination of the Doppler width from the individual curves of growth, to see, whether a variation with phase was indicated. As only relative variations are of interest, this was done by determining the horizontal displacement m (log Nf direction), and the vertical displacement n (log A direction), necessary to obtain best fit with the general curve. Now n is small, and it is sufficiently accurate to use in a least squares solution the equations:

$$\log Nf = m + n\,q,$$

where log Nf for the various A is read off the general curve of growth (Table 8), and q is the slope of the curve of growth at the observed log A. The values found for n, which represents the variation of log $\Delta\lambda_D$, with respect to the value given on p. 21, are given in Table 9.

Table 9. Variation of Doppler width.

Phase	n	m.e.
0.01	—0.02	±0.02
0.06	+0.06	0.02
0.10	+0.02	0.02
0.14	0.00	0.02
0.21	+0.01	0.02
0.26	—0.05	0.02
0.38	+0.04	0.02
0.46	+0.06	0.02
0.56	+0.04	0.02
0.84	+0.04	0.03
0.90	+0.06	0.03

A variation with phase seems to exist. Its value, however, is not much in excess of the mean error. Deviations of the general run with phase are shown for the plates 17920 and 17765, in accordance with the remark concerning these plates in the previous section. The table confirms the results of Van Albada, who found a difference of 0.08 between the values of log $\Delta\lambda_D$ at the phases of minimum volume (0.88), and of maximum brightness (0.07), of the star.

This variation of the Doppler width, if allowed for by a vertical displacement of the curve of growth, has a corresponding effect on the horizontal displacement, and therefore on the log Nf. But, by doing so, the log Nf is made to depend, besides on the horizontal shift of the curve of growth, which is a well determinable quantity, also on the slope of the curve of growth, which is very uncertain.

Now, instead of trying to find a theoretically better founded quantity, but which is undeterminable from our spectra, we preferred to have a well determined value and to keep in mind that its run with phase may be slightly altered by an eventual variation of Doppler width. Therefore in the further discussion the Doppler width will be considered as constant.