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Hommes, C.H.

Publication date
1999

[Link to publication](#)

Citation for published version (APA):

Hommes, C. H. (1999). *Cobweb dynamics under bounded rationality*. EPRINTS.

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Cobweb Dynamics under Bounded Rationality

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September 1999

Abstract This paper discusses recent work on expectation formation in the familiar cobweb 'hog cycle' model. A convenient feature of the model is the fact that it has a unique rational expectations equilibrium (REE). Attention will be focused on whether this unique REE can be learned under bounded rationality, with agents using simple habitual rule of thumb forecasting rules, or whether other 'approximate rational expectations equilibria' can arise, where forecasts are, although not perfect, approximately correct. In particular, the possibility of 'chaotic equilibria' under bounded rationality will be discussed. Both the homogeneous agent case and the heterogeneous case, with evolutionary competition between different forecasting rules, will be discussed. Recent experimental work on expectations formation in the cobweb economy suggests excess volatility and endogenous, possibly chaotic price fluctuations driven by the interaction of competing boundedly rational agents.

Keywords: expectations, adaptive learning, heterogeneity, nonlinear dynamics.

Acknowledgement. Financial support by the Netherlands Organization for Scientific Research (NWO) under Pionier grant 400-41-002 is gratefully acknowledged. I would also like to thank Joep Sonnemans, Henk van de Velden and Roy van der Weide for their help with some of the numerical simulations. Special thanks are due to Jan Tuinstra for his comments on an earlier draft.

1. Introduction

The main theme of my contribution will be the expectations hypothesis in economics and finance. How should we model expectations? Should we assume full rationality with agents deriving optimal forecasts from economic theory using underlying market equilibrium equations? Or is 'bounded rationality', with agents only observing time series and using simple habitual rule of thumb predictors, a more accurate description of expectation formation in a dynamic environment? Incomplete information about the model is only one aspect of bounded rationality. Another important issue is the possibility of heterogeneity. Can we use a *representative agent* model, or does *heterogeneity* in expectations contribute to excess price volatility? I will use the familiar demand-supply cobweb model to discuss these issues, because this model has the convenient feature of having a unique rational expectations equilibrium (REE). The main questions are then whether a representative agent could learn the REE from time series observations and whether in a heterogeneous world agents will be able to learn and coordinate on the unique REE.

In addressing these fundamental issues, I will discuss work that goes back to my thesis, as well as more recent work on cobweb dynamics that I have been doing over the years jointly with Buz Brock, with Gerhard Sorger, and more recently also some experimental work with Joep Sonnemans, Jan Tuinstra and Henk van de Velden, my co-workers from CeNDEF at the University of Amsterdam. In doing so, I will try not to repeat exactly the results from published papers, but instead focus on some relevant departures.

The paper is organized as follows. Section 2 discusses cobweb dynamics under various expectations rules, such as adaptive expectations. Section 3 discusses the notion of consistent expectations in the cobweb model, where forecasting rules are, although not perfect, correct in terms of sample averages and sample autocorrelations. Section 4 focuses on heterogeneity and evolutionary competition between different forecasting rules. Section 5 discusses recent experimental work on expectation formation in the cobweb model and finally section 6 concludes.

2. The Cobweb Model

The cobweb model describes fluctuations of equilibrium prices in an independent market for a non-storable good, that takes one time period to produce. Producers form price expectations one period ahead and derive their optimal supply from expected profit maximization:

$$S(p_t^e) = \operatorname{argmax}_{q_t} \{p_t^e q_t - c(q_t)\} = (c')^{-1}(p_t^e). \quad (2.1)$$

The cost function $c(\cdot)$ is assumed to be strictly convex so that the second order condition for profit maximization is satisfied. The marginal cost function is then invertible and supply strictly increasing in expected price. The simplest case arises when the cost function is quadratic, $c(q) = q^2/(2b)$, yielding a linear supply curve

$$S(p^e) = bp^e. \quad (2.2)$$

Here, we focus on a nonlinear, increasing, S-shaped supply curve derived from expected profit maximization with an increasing and convex cost function given by a third or higher order polynomial¹

$$c(q) = \frac{1}{d+1}(q-1)^{d+1} + q, \quad (2.3)$$

where d is an odd integer. Optimal supply then becomes

$$q = S(p^e) = (p^e - 1)^{\frac{1}{d}} + 1. \quad (2.4)$$

Consumer demand D depends upon the current market price p_t . The demand curve D can be derived from consumer utility maximization, but

¹ For simplicity we have chosen a specification such that the point $(p, q) = (1, 1)$ becomes the unique inflection point of the corresponding supply function.

for our purposes it is not necessary to specify these preferences explicitly. Throughout the paper we will simply work with a linear demand curve

$$D(p_t) = A - Bp_t + \epsilon_t, \quad (2.5)$$

where ϵ_t is an independently and identically distributed (IID) stochastic series representing an exogenous random demand shock. If beliefs are homogeneous, i.e., all producers have identical price expectations p_t^e , market equilibrium becomes

$$D(p_t) = S(p_t^e) \quad (2.6)$$

yielding the realized market price

$$p_t = D^{-1}(S(p_t^e)) = \frac{A + \epsilon_t - S(p_t^e)}{B}. \quad (2.7)$$

In order to complete the model, we have to specify how producers form price expectations. If producers have naive expectations, i.e. their prediction equals the latest observed price $p_t^e = p_{t-1}$, price dynamics becomes

$$p_t = D^{-1}(S(p_{t-1})). \quad (2.8)$$

According to the well known cobweb theorem (see e.g. Ezekiel (1938)), there are essentially two possibilities for the price dynamics under naive expectations, depending upon the ratio of marginal supply and marginal demand at the steady state p^* . When $-1 < S'(p^*)/D'(p^*) < 0$ the RE steady state p^* is (locally) stable, and prices converge to the REE. If on the other hand $S'(p^*)/D'(p^*) < -1$ the RE steady state p^* is (locally) unstable, and prices diverge from the REE. In the case of a nonlinear supply curve as in (2.4), if the REE is unstable, prices will converge to a stable 2-cycle.

It has been argued however, that simple forecasting rules such as naive expectations, lead to systematic forecasting errors. In particular, this argument seems to be valid, at least in theory, in the case the model generates a 2-cycle, even in the presence of exogenous shocks. When producers expect a high (low) price, they will supply a high (low) quantity and consequently the realized market price will be low (high). Along a ‘hog cycle’, expectations are thus systematically wrong, and forecasting errors are correlated. Rational agents would learn from these systematic forecasting errors and revise expectations accordingly, so the argument goes. Such considerations lead Muth (1961), using the cobweb model, to the introduction of rational expectations, where producers’ subjective price expectations equal the objective mathematical expected value of the market price. Using market equilibrium (2.6) with the linear demand curve (2.5) rational expectations yields

$$p_t^e = E_t[p_t] = p^*, \quad (2.9)$$

where p^* is the unique price corresponding to the intersection of demand and supply. Given producers’ rational forecast $p_t^e = p^*$, the actual law of motion (2.7) becomes

$$p_t = p^* + \frac{\epsilon_t}{B}. \quad (2.10)$$

The cobweb model therefore has a unique REE, given by an IID process with mean p^* . Along a REE expectations are self fulfilling and producers make no systematic forecasting errors; forecasting errors are uncorrelated. It is important to note that, in order to form rational expectations, perfect knowledge of underlying market equilibrium equations is required.

It is worthwhile to reconsider the issue of ‘systematic forecasting errors’ in the light of the recent discovery of chaotic dynamics in simple nonlinear deterministic systems and under the more plausible assumption of *bounded rationality*, where agents do not know underlying market equilibrium equations, but only use time series observations to forecast². As an example, consider the cobweb model with *adaptive expectations*, i.e.,

$$p_t^e = (1 - w)p_{t-1}^e + wp_{t-1}, \quad 0 \leq w \leq 1, \quad (2.11)$$

where w is the expectations weight factor. The expected price is a weighted average of yesterday’s expected and realized prices, or equivalently, the expected price is adapted by a factor w in the direction of the most recent realization. Adaptive expectations may thus be seen as ‘error learning’ with a constant factor. Under adaptive expectations and, given the linear demand curve (2.5), the dynamics of expected prices in the cobweb model becomes

$$p_t^e = (1 - w)p_{t-1}^e + w \frac{A + \epsilon_t - S(p_{t-1}^e)}{B}. \quad (2.12)$$

Chiarella (1988) and Hommes (1991,1994) have shown that, without any random shocks ϵ_t , for *nonlinear*, but monotonic, demand and/or supply curves, this nonlinear deterministic difference equation can easily generate chaotic fluctuations in expected prices, and therefore also in prices, quantities and forecasting errors. Fig. 2.1 shows a bifurcation diagram w.r.t. the expectations weight factor w , with the nonlinear, S-shaped supply curve (2.4). For high values of w prices converge to a 2-cycle, whereas for small values of w prices converge to the RE steady state. For intermediate w -values however, chaotic price oscillations arise.

When prices fluctuate chaotically, the corresponding forecasting errors will be highly unpredictable and the question arises whether boundedly rational agents would be able to detect structure in these chaotic forecasting errors and improve upon their simple adaptive forecasts. If patterns are indeed hard to discover, then such a chaotic situation might be a satisfactory (long run) equilibrium.

² At the time of the introduction of rational expectations by Muth (1961) and its introduction into macroeconomics by Lucas (1971) and others, the phenomenon of deterministic chaos was still unknown.

Fig. 2.1. Bifurcation diagram w.r.t. the expectations weight factor w , $0.45 \leq w \leq 0.65$, with the other parameters fixed at $A = 2$, $B = 0.25$ and $d = 5$.

3. Consistent Expectations Equilibria

In order to detect structure in a chaotic time series generated by an unknown model, it seems natural that agents start to use linear time series techniques to detect patterns. Hommes (1998) investigated for various simple forecasting rules, including adaptive expectations and linear AR(p) forecasting rules, whether chaotic forecasting errors in the cobweb model are uncorrelated. In all chaotic examples the chaotic forecasting errors had strongly significant negative autocorrelation coefficient at the first lag. This suggests, that even in a nonlinear, chaotic cobweb economy, boundedly rational agents using simple linear time series techniques would be able to improve their forecasts. In related work, Sorger (1998) presented an example of an overlapping generations model with production, where agents believe that interest rates follow a stochastic IID process, and given this belief the actual interest rate follows a chaotic process without any autocorrelations. This is an example where beliefs, although not perfect, are self fulfilling in terms of mean and autocorrelations. Subsequently, Hommes and Sorger (1998) generalized this example and developed the notion of *consistent expectations equilibrium (CEE)*, which will be discussed below.

Assume that all agents believe that prices are generated by a stochastic AR(1) process³. Given this perceived law of motion and prices known up to p_{t-1} , the predictor for p_t minimizing the mean squared prediction error is

$$p_t^e = \alpha + \beta(p_{t-1} - \alpha), \quad (3.1)$$

³ Hommes and Sorger (1998) emphasize that the definition of CEE can be generalized to higher order belief processes, e. g., AR(k) processes with $k \geq 2$.

where α and β are real numbers, $\beta \in [-1, 1]$. Given that agents use the linear predictor (3.1), the *implied actual law of motion* becomes

$$p_t = F_{\alpha, \beta}(p_{t-1}) := D^{-1}S(\alpha + \beta(p_{t-1} - \alpha)). \quad (3.2)$$

The empirical or sample average of a time series $(p_t)_{t=0}^\infty$ is defined as

$$\bar{p} = \lim_{T \rightarrow \infty} \frac{1}{T+1} \sum_{t=0}^T p_t \quad (3.3)$$

and the empirical or sample autocorrelation coefficients are given by

$$\rho_j = \lim_{T \rightarrow \infty} \frac{c_{j,T}}{c_{0,T}}, \quad j \geq 1, \quad (3.4)$$

where

$$c_{j,T} = \frac{1}{T+1} \sum_{t=0}^{T-j} (p_t - \bar{p})(p_{t+j} - \bar{p}), \quad j \geq 0. \quad (3.5)$$

A CEE is now defined as

Definition 3.1. A triple $\{(p_t)_{t=0}^\infty; \alpha, \beta\}$, where $(p_t)_{t=0}^\infty$ is a sequence of prices and α and β are real numbers, $\beta \in [-1, 1]$, is called a *consistent expectations equilibrium* (CEE) if

1. the sequence $(p_t)_{t=0}^\infty$ satisfies the implied actual law of motion (3.2),
2. the sample average $\bar{p}$ in (3.3) exists and is equal to α , and
3. the sample autocorrelation coefficients ρ_j , $j \geq 1$, in (3.4) exist and the following is true:
 - a. if $(p_t)_{t=0}^\infty$ is a convergent sequence, then $\text{sgn}(\rho_j) = \text{sgn}(\beta^j)$, $j \geq 1$;
 - b. if $(p_t)_{t=0}^\infty$ is not convergent, then $\rho_j = \beta^j$, $j \geq 1$.

A CEE is a price sequence together with an AR(1) belief such that expectations are self-fulfilling in terms of the *observable* sample average and sample autocorrelations. Along a CEE expectations are thus correct in a linear statistical sense. Given an AR(1) belief, there are at least three possible types of CEE:

- a *steady state CEE* in which the price sequence $(p_t)_{t=0}^\infty$ converges to a steady state p^* , with $\alpha = p^*$ and $\beta = 0$;
- a *2-cycle CEE* in which the price sequence $(p_t)_{t=0}^\infty$ converges to a period two cycle $\{p_1^*, p_2^*\}$, $p_1^* \neq p_2^*$, with $\alpha = (p_1^* + p_2^*)/2$;
- a *chaotic CEE* in which the price sequence $(p_t)_{t=0}^\infty$ is chaotic, with sample average α and autocorrelations β^j .

Which of these cases occurs in the cobweb model depends on the composite mapping $D^{-1}S$ in (3.2), determined by demand and supply curves, and will be discussed below.

So far, the notion of CEE involves a given AR(1) belief, with fixed parameters α and β . Now consider the more flexible situation of *adaptive learning* with agents changing their forecasting function over time within the class of AR(1) beliefs, and updating their belief parameters α_t and β_t , as additional observations become available. A natural learning scheme fitting the framework of CEE is based upon sample average and sample autocorrelation coefficients.

For any finite set of observations $\{p_0, p_1, \dots, p_t\}$ the sample average is

$$\alpha_t = \frac{1}{t+1} \sum_{i=0}^t p_i, \quad t \geq 1 \quad (3.6)$$

and the first order sample autocorrelation coefficient is

$$\beta_t = \frac{\sum_{i=0}^{t-1} (p_i - \alpha_t)(p_{i+1} - \alpha_t)}{\sum_{i=0}^t (p_i - \alpha_t)^2}, \quad t \geq 1. \quad (3.7)$$

When, in each period, the belief parameters are updated according to (3.6) and (3.7) the (temporary) law of motion (3.2) becomes

$$p_{t+1} = F_{\alpha_t, \beta_t}(p_t) = D^{-1}S(\alpha_t + \beta_t(p_t - \alpha_t)), \quad t \geq 0. \quad (3.8)$$

We call the dynamical system (3.6) - (3.8) the actual dynamics with *sample autocorrelation learning* (SAC-learning)⁴. The initial state for the system (3.6) - (3.8) can be any triple (p_0, α_0, β_0) with $\beta_0 \in [-1, 1]$.

Which type of CEE exist in the cobweb model, and to which of them will the SAC-learning dynamics converge? Hommes and Sorger (1998) show that in the most relevant case, when demand is decreasing and supply is increasing, the *only* CEE is the RE steady state price p^* . This means that, even when underlying market equilibrium equations are *not* known, agents should be able to learn and coordinate on the REE price simply by looking at sample averages and sample autocorrelations. Although other simple forecasting rules, such as adaptive expectations, might lead to chaotic price fluctuations, these forecasting rules are *inconsistent* in terms of sample autocorrelations. Hence, in a nonlinear cobweb economy with monotonic demand and supply, boundedly rational agents should, at least in theory, be able to learn the unique REE from time series observations⁵.

⁴ Another adaptive learning process that has received much attention in the bounded rationality literature is ordinary least squares (OLS) learning, see Sargent (1993) for a survey and many references. Hommes and Sorger (1998) argue that the SAC- and the OLS-learning schemes are in fact closely related.

⁵ Bray and Savin (1986) show that OLS learning also converges to the REE steady state in the cobweb model. Arifovic (1994) shows that agents using genetic algorithms can learn the REE steady state. See also Böhm and Wenzelburger (1999) for 'perfect prediction rules' in the cobweb and other models.

It is useful to discuss the possibility of more complicated, 2-cycle and chaotic CEE in the cobweb model. Hommes and Sorger (1998) present an example where demand is linear and supply is non-monotonic and piecewise linear. We slightly extend this example by adding IID demand shocks ϵ_t , uniformly distributed over the interval $[-0.25, 0.25]$. Let demand be given by

$$D(p) = \begin{cases} \max\{9 - p + \epsilon_t, 0\} & \text{if } 0 \leq p \leq 9, \\ \max\{0, \epsilon_t\} & \text{if } p > 9 \end{cases}$$

and supply by the piecewise linear curve⁶.

$$S(p) = \begin{cases} (25/2)p & \text{if } 0 \leq p \leq 18/25, \\ 10 - (25/18)p & \text{if } 18/25 < p \leq 18/5, \\ 5 + \epsilon[p - (18/5)] & \text{if } p > 18/5. \end{cases}$$

In the deterministic case without exogenous shocks ϵ_t , this example has a chaotic CEE with belief parameters $\alpha = 2$ and $\beta = 4/5$, as well as three steady state CEE (corresponding to the three intersection points of D and S , p_1^* , p_2^* , and p_3^*), and three 2-cycle CEE (corresponding to $\{p_1^*, p_2^*\}$, $\{p_1^*, p_3^*\}$, and $\{p_2^*, p_3^*\}$). To which of these CEE the SAC-learning process will converge depends upon the initial state of the learning process.

Fig. 3.1 shows an example of *learning to believe in chaos*, where the SAC-learning dynamics converges to a chaotic CEE. The key feature is that learning parameters converge to constants whereas prices do not converge but fluctuate chaotically on a strange attractor, with the correct sample average and sample autocorrelations⁷. A chaotic CEE may be seen as an example of what Sargent (1998) calls an *approximate rational expectations equilibrium*, with optimal misspecified forecasts. A chaotic CEE is not a REE, because expectations do not coincide with the conditional mathematical expectations which could only be derived if underlying market equilibrium equations would be known. Along a CEE agents use time series observations only and arrive at an optimal linear predictor to forecast an unknown nonlinear actual law of motion. Along a chaotic CEE forecasting errors are uncorrelated.

⁶ At first sight, this non-monotonic supply curve may seem odd. However, Hommes and Rosser (1999) have recently shown that overfishing and logistic growth in an optimal fishery management model leads to a smooth, non-monotonic supply curve similar to the piecewise linear curve. Chaotic CEE arise in this renewable resource model and they are robust w.r.t. noise.

⁷ The notion *learning to believe in chaos* has been introduced in Hommes (1998, p.360), and the first examples have been given by Sorger (1998) and Hommes and Sorger (1998). For related work on the instability of OLS learning, see e.g. Bullard (1994) and Grandmont (1998). Schönhofer (1999) has recently employed the notion of learning to believe in chaos in a somewhat different context, namely when the entire OLS-learning process fluctuates chaotically. In Schönhofer's examples belief parameters of the OLS-learning scheme do *not* converge but keep fluctuating chaotically, and at the same time, due to inflation, prices diverge to infinity, so that agents are in fact running an OLS-regression on a non-stationary time series.

Fig. 3.1. Learning to believe in chaos. Belief parameters converge to constant values, i.e. $(\alpha_t, \beta_t) \rightarrow (\alpha^*, \beta^*) \approx (2.22, -0.89)$, but at the same time prices keep fluctuating chaotically with sample mean α^* and sample autocorrelations as for the corresponding AR(1) model (bottom left). Forecasting errors are chaotic and uncorrelated (bottom right). Statistical hypothesis testing would *not* reject the null hypothesis that prices follow a stochastic AR(1) process.

4. Heterogeneous Expectations

So far we have focused on a representative agent cobweb model, where all producers have identical expectations. In this section we discuss *heterogeneous* expectations, with evolutionary competition between different forecasting rules, as introduced in Brock and Hommes (1997). Producers can choose between different forecasting rules H_j . The fractions $n_{j,t}$ of agents using predictor H_j at date t , will be updated over time based upon a publically available evolutionary fitness measure associated to each predictor.

BH focus on the special but important case of rational expectations, which can be obtained at costs C per time period, and naive expectations, which is freely available. Rational expectations represents a sophisticated forecasting rule, whereas naive expectations represents a rule of thumb forecasting rule. BH show the occurrence of a *rational route to randomness*, i.e. a bifurcation route to strange attractors and chaos as traders become more rational in the sense of using more frequently forecasting rules that performed well in the recent past.

Rational agents have perfect knowledge about market equilibrium equations and are aware of the fact that the market equilibrium price is affected by the presence of naive traders. Hence, rational agents have perfect knowledge about prices and quantities, but also about *beliefs* of *all* other traders. In a real market, this seems highly unrealistic. Therefore we will focus here on another two type case, with two simple, competing linear predictors. As a special case, the case of fundamentalists versus naive expectations is obtained. In contrast to rational agents, fundamentalists do not take into account the presence of other trader types, but instead predict that the price will always be equal to the REE steady state price p^* .

Consider the two linear AR(1) prediction rules

$$H_j(p_{t-1}) = \alpha_j + \beta_j p_{t-1}, \quad j = 1, 2. \quad (4.1)$$

These predictors specialize to the case with fundamentalists versus naive expectations when $\alpha_1 = p^*$, $\beta_1 = 0$, $\alpha_2 = 0$ and $\beta_2 = 1$. Throughout this section we focus on the case where the supply curve is linear as in (2.2), with corresponding cost function $c(q) = q^2/(2b)$.

Market equilibrium prices in the cobweb model with two trader types, with linear predictors as in (4.1), is determined by⁸

$$A - Bp_t = n_{1,t-1}b(\alpha_1 + \beta_1 p_{t-1}) + n_{2,t-1}b(\alpha_2 + \beta_2 p_{t-1}), \quad (4.2)$$

where $n_{1,t-1}$ and $n_{2,t-1}$ denote the fractions of agents using respectively H_1 and H_2 , at the beginning of period t . These fractions will be updated

⁸ In our simulations we will work in deviations $x_t = p_t - p^*$ from the fundamental RE steady state price p^* . This is equivalent to setting the parameter $A = 0$, so that the RE steady state $p^* = 0$.

according to past realized profits. Net realized profit in period t for traders using predictor H_j is given by

$$\pi_{j,t} = bp_t H_j(p_{t-1}) - \frac{b}{2}(H_j(p_{t-1}))^2 - C_j, \quad (4.3)$$

where C_j represents the average costs per time period for obtaining predictor H_j . For a simple habitual rule of thumb predictor, such as naive or adaptive expectations, these costs C_j will be zero, whereas for more sophisticated predictors such as fundamentalists beliefs, based on fundamental analysis, costs C_j may be positive.

The fraction of agents using predictor H_j in period t are given by the discrete choice ‘probabilities’

$$n_{j,t} = \frac{\exp(\beta\pi_{j,t})}{Z_t}, \quad Z_t = \sum_{j=1}^H \exp(\beta\pi_{j,t}), \quad (4.4)$$

where Z_t is a normalization factor so that all fractions add up to one.

A key feature of this evolutionary predictor selection is that agents are *boundedly rational*, in the sense that predictors with higher evolutionary fitness attract more followers. The parameter β is called the *intensity of choice*, measuring how fast producers switch between different prediction strategies. For $\beta = 0$, all fractions are fixed over time and equal to $1/H$, whereas for the other extreme $\beta = \infty$, in each period *all* producers choose the optimal predictor. Hence, the higher the intensity of choice, the more rational agents are in choosing prediction strategies. Equivalently, the inverse $1/\beta$ is proportionally to the noise level in the fitness measure.

The timing of predictor selection in (4.4) is important. In (4.2) the old fractions $n_{t-1,1}$ and $n_{t-1,2}$ determine the new equilibrium price p_t . This new equilibrium price p_t is used in the fitness measure for predictor choice and the new fractions $n_{t,1}$ and $n_{t,2}$ are updated according to (4.4). These new fractions are then used in determining the next equilibrium price p_{t+1} , etc..

Fig. 4.1 shows the strange attractor and corresponding chaotic price fluctuations in the case of costly fundamentalists versus naive expectations. It is remarkable that both the attractor and the price time series are very similar to the case of rational versus naive expectations studied in BH⁹.

Figure 4.2 illustrates an example without any information costs, with two competing linear forecasting rules. The first type is slightly downward biased, and predicts a constant price just below the (unknown) RE steady state, whereas the second type has again naive expectations. This example shows that, without any information costs, the evolutionary competition between simple, linear forecasting rules can generate chaotic price fluctuations.

⁹ See the original working paper Brock and Hommes (1995) for more details concerning the case of fundamentalists versus naive expectations.

Fig. 4.1. Parameters: $A = 0$, $B = 0.5$, $b = 1.35$, $\beta = 3$, $\alpha_1 = 0$, $\beta_1 = 0$, $C_1 = 1$, $\alpha_2 = 0$, $\beta_2 = 1$ and $C_2 = 0$. Costly fundamentalists versus naive expectations. Strange attractor in the $(x_t, n_{1,t})$ phase space, where $x_t = p_t - p^*$ is the deviation from the fundamental steady state price, and chaotic time series x_t .

Fig. 4.2. Parameters: $A = 0$, $B = 0.5$, $b = 1.35$, $\beta = 18$, $\alpha_1 = -0.1$, $\beta_1 = 0$, $C_1 = 0$, $\alpha_2 = 0$, $\beta_2 = 1$ and $C_2 = 0$. Strange attractor and chaotic fluctuations due to evolutionary competition between downward biased forecast versus naive expectations, without any information costs.

5. Laboratory Experiments

In joint work with my colleagues Joep Sonnemans, Jan Tuinstra and Henk van de Velden from CeNDEF, we have run individual as well as group experiments on expectation formation in the cobweb model. The results of these experiments are described in detail in Hommes, Sonnemans and van de Velden (1999a,1999b) and Sonnemans et al. (1999), and will be discussed here briefly.

The participants in the experiments were asked to predict next periods price of a certain, unspecified, good. The realized price p_t in the experiment was determined by the cobweb market equilibrium equation

$$D(p_t) = \frac{1}{K} \sum_{i=1}^K S(p_{i,t}^e), \quad (5.1)$$

where $D(p_t)$ is the demand for the good at price p_t , K is the size of the group, $p_{i,t}^e$ is the price forecast by participant i and $S(p_{i,t}^e)$ is the supply of producer i depending upon the forecast by participant i . Demand and supply curves D and S were fixed during all experiments (except for the random shocks to the demand curve) and unknown to the participants. In the group experiments $K = 6$, whereas in the individual experiments $K = 1$. Notice that on the righthand side of the market equilibrium equation (5.1) we take the average supplied quantity, in order to be able to compare the group experiments directly to the individual experiments. Solving (5.1) for the market equilibrium price, for the linear demand curve (2.5), yields

$$p_t = \frac{A_t + \epsilon_t - \frac{1}{K} \sum_{i=1}^K S(p_{i,t}^e)}{B}, \quad (5.2)$$

where ϵ_t are IID demand shocks, which are drawn from a normal distribution $N(0, 0.5)$ in the experiments discussed here. In the experiments we used the S-shaped supply curve

$$S(p_{i,t}^e) = \tanh(\lambda(p_{i,t}^e - 6)) + 1, \quad (5.3)$$

which is geometrically equivalent to the S-shaped supply curve (2.4).

Expectation formation of the producers is the only part of the model that is affected by the participants in the experiments. Participants thus did not know underlying market equilibrium equations, nor were they informed about the distribution of any exogenous shocks to demand and/or supply. The participants were told that they were advisors of producers of an unspecified good and that the price was determined by market clearing. Based upon this information we asked the participants to predict next periods price. The predicted price had to be between 0 and 10 and the realized price was also always between 0 and 10. Participants' earnings in each period were a quadratic function of their squared forecasting error. The better their

forecast, the higher their earnings. After every period the participants were informed about the realized price in the experiment. Also a time series of the participant's own prediction and a time series of the realized price in the experiment was shown on each computer screen.

In real markets, individual expectations usually only have a small effect upon realized market prices. Therefore, in the individual experiments, participants were not informed that the realized prices only depended upon their own expected price (and some small noise), and in the group experiments participants were not informed that the realized price depended upon their own price forecast and the forecasts of (five) other participants.

Participants in the experiments thus had very little information about the price generating process and had to rely mainly upon time series observations of past prices and predictions. The information in the experiment was thus similar to the information assumption underlying much of the bounded rationality literature, where agents form expectations based upon time series observations. Our setup thus enables us to test the expectations hypothesis in a controlled dynamic environment. The main question was whether agents can learn and coordinate on the unique REE, in a world where consumers and producers act as if they were maximizing utility and profits, but where they do *not* know underlying market equilibrium equations and only observe time series of prices and expected prices. Our choice for a nonlinear, S-shaped supply curve enables us to investigate whether agents can learn a REE steady state in a world where simple forecasting rules can lead to periodic cycles or even chaotic price fluctuations.

In the individual experiments, only about 35% of the participants were able to learn the unique REE steady state; in the remaining 65% prices kept fluctuating with large amplitude around the unstable REE steady state. In the group experiments, forecasting errors were significantly lower than in the individual experiments. Fig. 5.1 shows time series of the realized prices in two group experiments and Table 5.1 shows the mean and standard deviations over different subsamples in the experiments. Under RE the mean price would be 5.91 and the standard deviation (SD) would be 0.71 (equal to the SD of the demand shocks). For both time series, the sample mean over the full sample is fairly close to the RE mean, so that on average the price level is predicted correctly. In the first time series the amplitude of price fluctuations clearly decreases over time. Table 5.1 shows that the standard deviation in the last 25 time periods is close to the RE standard deviation. One might thus say that this group experiment converges to RE. In the second time series the amplitude of price fluctuations does *not* decrease much over time. The standard deviation in the last 25 periods is in fact close to the standard deviation in the first 25 periods, and about twice the SD under rational expectations. One might thus say that this group experiment exhibits excess price volatility due to endogenous expectations.

Fig. 5.1. Realized market prices in two different cobweb group experiments with exogenous demand shocks $\epsilon_t \sim N(0, 0.5)$. The first market settles down closely to REE, whereas the second one exhibits expectations driven excess price volatility.

Table 5.1. Statistics of realized market prices

statistic sample	mean 1-50	mean 1-25	mean 26-50	SD 1-50	SD 1-25	SD 26-50
rational expectations	5.91	5.91	5.91	0.71	0.71	0.71
group 31	5.78	5.92	5.64	1.41	1.83	0.81
group 42	5.95	6.12	5.77	1.52	1.61	1.43

Fig. 5.2. Distribution of long run dynamics based upon 620 strategy simulations. Only 10% of all simulations converges to the RE steady state. In the final round, more than 60% appears to be chaotic.

Sonnemans et al. (1999) conducted a four round strategy experiment on expectation formation in the cobweb economy. Participants submitted a forecasting strategy and could revise their strategy after each round. Earnings were based upon the relative performance of the strategy. Over the four rounds, forecasting errors decreased, but at the same time the complexity of the price fluctuations also increased. Figure 5.2 shows the distribution of the long run dynamics, based upon 620 simulations, in each of the four rounds. Convergence to the unique REE steady state occurred in less than 10% of all cases. In the final round, more than 60% of the simulations appears to be chaotic, with a positive Lyapunov exponent. Strategy simulations with homogeneous agents typically show regular behaviour, with convergence to the REE steady state or to a stable periodic cycle. Heterogeneous interaction of simple prediction strategies thus seems to be the main source of the endogenous, possibly chaotic price fluctuations.

6. Concluding Remarks

In a cobweb economy with nonlinear, but monotonic demand and supply curves, it should be possible to learn the unique REE steady state price from time series observations. The steady state price forecast is the only forecast where sample averages and sample autocorrelations are correct. In theory, simply by looking at sample averages and sample autocorrelations, in particular trying to learn the negative first order autocorrelation so typical for the ‘hog cycle’, boundedly rational agents should be able to learn the unique REE. Experimental evidence shows however that this is not as easy as theory suggests. Even in individual experiments more than 65% is *not* able to learn the REE steady state. In heterogeneous agents experiments, although forecasting errors are significantly lower, typically prices do not settle down to the REE steady state. Interaction of simple, heterogeneous forecasting rules seems to generate complicated, possibly chaotic price fluctuations. Even in the simplest of all dynamic economic models, heterogeneity may thus cause expectations driven excess price volatility.

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