Models of interacting binary stars
Kool, M.

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MODELS OF INTERACTING BINARY STARS

M. de Kool
MODELS OF
INTERACTING BINARY STARS

ACADEMISCH PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE
WISKUNDE EN NATUURWETENSCHAPPEN AAN DE
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RECTOR MAGNIFICUS DR. S.K. THODEN VAN VELZEN,
HOGLERAAR IN DE FACULTEIT DER TANDHEELKUNDE, IN
HET OPENBAAR TE VERDEIDIGEN IN DE AULA DER
UNIVERSITEIT (OUDE LUTHERSE KERK, SINGEL 411,
HOEK SPUI) OP WOENSDAG 3 JUNI 1987 TE 15.00 UUR
PRECIES.

DOOR

MARTHIJN DE KOOL

GEBOREN TE ROTTERDAM
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De belangrijkste reden hiervoor is dat de studie van compacte objecten inzicht verschaf over het gedrag van materie onder extreme omstandigheden (hoge temperatuur en dichtheid, sterke zwaartekracht en een zeer sterk magneetveld), die niet in het laboratorium op aarde bereikt kunnen worden. Geïsoleerde compacte sterren zenden over het algemeen weinig straling uit, zodat ze moeilijk zijn waar te nemen. Als ze echter deel uitmaken van een dubbelster veroorzaakt de wisselwerking tussen het object en zijn begeleider een aantal opvallende verschijnselen. Zo zijn een aantal van deze systemen zeer heldere Rontgenbronnen, en vrijwel allemaal zijn ze in de meeste golflengtegebieden van het elektromagnetische spectrum variabel, op een zodanig korte tijdschaal dat relevante gegevens snel te verzamelen zijn. Deze eigenschappen houden beide verband met het feit dat de uitgezonden straling wordt opgewekt doordat er materie (afkomstig van de begeleidende ster) op het oppervlak van de compacte ster valt. Bij dit zogenaamde "accretieproces" komt zeer veel energie vrij omdat de zwaartekracht aan het oppervlak van een compacte ster erg sterk is. Doordat de afmeting van een compacte ster zeer klein is (ca. 10000 km voor een witte dwerg, 10 km voor een neutronenster of een zwart gat van stellaire massa) zijn de tijdschalen van het accretieproces zeer kort (milliseconden tot minuten). Verder moet in de meeste gevallen de afstand tussen de twee componenten van de dubbelster klein zijn, wil er materie van de begeleider op de compacte ster terecht kunnen komen, hetgeen inhoudt dat de baanperiode (waarin beide sterren om elkaar heen draaien) kort moet zijn (minuten tot dagen). Om deze redenen kan de waargenomen straling op tijdschalen van milliseconden tot dagen variëren. Voor de theoreticus is het een grote uitdaging om al deze waargenomen verschijnselen te verklaren en in een consistent model onder te brengen.

Dubbelsterren met compacte objecten worden gewoonlijk verdeeld in twee groepen. In de eerste groep accreteert de compacte ster materie uit de sterrenwind van een zware begeleidende ster; in de tweede groep wordt de massa naar de compacte ster overgedragen doordat de sterren zo dicht bij elkaar staan dat de compacte ster door zijn zwaartekracht materie van het oppervlak van de begeleidende ster kan trekken ("Roche lobe
overflow")) In dit laatste geval is de begeleider meestal een lichte ster. Een belangrijk verschil tussen deze twee groepen is dat in de tweede groep de materie nooit rechtstreeks op het oppervlak van de compacte ster kan vallen, maar er eerst in een schijf omheen komt te draaien. Deze zogenaamde "accretieschijf" is verantwoordelijk voor de meeste waargenomen eigenschappen van het systeem. De aanwezigheid van zo'n schijf in de systemen van de eerste groep is nog niet eenduidig bewezen, en zal ook de meeste waargenomen eigenschappen niet domineren.

Dit proefschrift gaat voornamelijk over systemen uit de tweede groep: dubbelstersystemen met een compacte ster en een lichte begeleider die door middel van Roche-lobe overflow massa verliest. Het werk kan verdeeld worden in drie delen die hierna apart zullen worden samengevat:

1) Een studie van de eigenschappen van de wijde lage-massa systemen (heldere galaktische bulge-bronnen, symbiotische sterren).
2) Het construeren van evolutie scenario's voor een aantal waargenomen systemen (A0620-00, 1E2259-586).
3) Een studie van de vorming en evolutie van nauwe systemen (nauwe Röntgendubbelsterren van lage massa, cataclysmische variabelen), in het bijzonder die evolutiefase waarin het systeem een gemeenschappelijk gasomhulsel heeft.

**Eigenschappen van wijde Röntgendubbelsterren van lage massa**

De galaktische bulge bevat een aantal zeer heldere Röntgenbronnen die vanwege de grote extinctie niet optisch kunnen worden geïdentificeerd en waarvan geen baanperiode bekend is. Webbink, Rappaport en Savonije (WRS) stelden voor dat deze bronnen bestaan uit een neutronenster met een lage massa reus (lichter dan de neutronenster) als begeleider. De massa-overdracht wordt aangedreven door de evolutionaire expansie van de reus, en omdat de massa verliezende ster lichter is dan de accreterende is de massa-overdracht stabiel. De evolutie van deze dubbelsterren kan op een simpele semi-analytische wijze worden beschreven, hetgeen WRS deden voor die systemen waarin de reus een lichte helium-kern heeft, met daaromheen een waterstof verbrandende schil.

In hoofdstuk II.1 wordt dit werk uitgebreid om te onderzoeken of ook andere typen dubbelsterren, met name die waarin de massaoverdracht sneller verloopt dan in de Röntgendubbelsterren, kunnen worden verklaard met een zelfde soort model waarin de reus verder geëvolueerd is.
tabel 1. De gemiddelde snelheid van massaoverdracht (\( M_{\odot}/\text{jaar} \)) en de duur van de massaoverdracht fase (in jaren) voor een dubbelster bestaande uit een reus van lage massa en een neutronenster, als functie van de initiële baanperiode (in dagen).

Naarmate een reus verder geëvolueerd is neemt de snelheid waarmee de evolutiefasen worden doorlopen toe, en daarmee ook de snelheid van massaoverdracht, die immers wordt aangedreven door de evolutionaire expansie. De massa-overdrachtsnelheid en levensduur van dergelijke systemen is samengevat in tabel 1, als functie van de baanperiode waarbij de massaoverdracht begint. Een duidelijk voorbeeld van een dubbelster bestaande uit een neutronenster en een ver geëvolueerde reus, waarvan het voornaamste kenmerk een zeer hoge (super Eddington) massa-overdrachtsnelheid zou zijn, lijkt op dit moment nog niet waargenomen te zijn. Als de compacte ster echter een zware witte dwerg is zal een dergelijke dubbelster vermoedelijk als een zogenaamde symbiotische ster worden waargenomen. Door de eigenschappen van onze modellen te vergelijken met die van een aantal waargenomen symbiotische sterren is het mogelijk een aantal systemen aan te wijzen waarvan de eigenschappen door dit model goed verklaard kunnen worden: AG Dra, V443 Her, RW Hyi en SY Mus.

Het model is door ons ook gebruikt om de eigenschappen te verklaren van een aantal axiaal symmetrische niet-thermische radiobronnen die in de buurt van het galactisch centrum werden waargenomen. Uit de energie-inhoud en de vorm van de radiobronnen valt af te leiden dat er een continue energie-injectie met een vermogen van ca. \( 10^{38} \) ergs/sec moet hebben plaatsgevonden gedurende een periode van ongeveer \( 10^5 \) jaar. In hoofdstuk II.2 wordt beargumenteerd dat het enige bekende soort astrofysisch object dat wij kennen dat aan deze voorwaarden kan voldoen het soort dubbelster-systeem is dat wij modelleren.

Tenslotte hebben we in hoofdstuk II.3 het model toegepast om de evolutie van de spinperiode van een accreterende, magnetische neutronenster in een wijde lage massa Röntgendiffibelster te berekenen. Als we een
figuur 1. De spinperiode van de neutronenster als functie van de baanperiode aan het einde van de massa-overdracht-fase, met en zonder een bodem in het magnetveld. De krommes met label 1, 2 en 3 geven de resultaten voor een vervaltijdsschaal van het magnetveld van $2 \times 10^6$, $5 \times 10^6$ en $10^7$ jaar.

Aantal veronderstellingen doen over de evolutie van de sterkte van het magnetveld van de neutronenster, dan voorspellen deze berekeningen een verband tussen de spinperiode van de neutronenster en de baanperiode van de dubbelster op het moment dat de massa-overdracht stopt (figuur 1). Aangezien de nakomelingen van deze systemen waargenomen worden als wijde dubbelster-radiopulsars, stelt een vergelijking tussen de waargenomen en theoretische relatie in het spinperiode/baanperiode diagram ons in staat limiten te stellen op het gedrag van het magnetveld als functie van de tijd. De positie van de pulsar PSR 1953+29 zet een bovenlimiet op de vervaltijd van het magnetveld vlak na het ontstaan van de neutronenster van $1.2 \times 10^7$ jaar. De positie van PSR 1855+09 geeft aan dat dit verval niet onbeperkt doorgaat, maar stopt als het veld een bodemwaarde van ca. $5 \times 10^8$ Gauss heeft bereikt.

Evolutie scenarios

1E2259+586

De dubbelster 1E2259+586 heeft een baanperiode van 38 minuten en bevat een 7 seconden Röntgenpulsar. Zij staat in het midden van een schil van diffuse Röntgenemissie die er uitziet als een ca. $10^4$ jaar oud supernova-restant, op een afstand van $3.6 \pm 0.4$ kpc. Dit betekent vermoedelijk dat de neutronenster recent is gevormd door het instorten van een witte dwerg.
Fig. 2 De baanperiode (in minuten) als functie van de massa van de He-ster waarvan de massa door accretie boven de Chandrasekhar limiet is gekomen. Uit nauwkeurige metingen van de modulatie van de baanperiode verkregen Gregory en Fahlman een massafunctie voor de dubbelster van $0.008 \pm 0.0002 \, M_\odot$, hetgeen voor een standaard pulsarmassa van $1 \, M_\odot$ inhoudt dat de massa van de begeleider groter moet zijn dan $0.2 \, M_\odot$. De korte baanperiode maakt het onmogelijk dat de begeleider een normale hoofdreeksster is, aangezien zulke sterren te groot zijn om in de baan te passen. Een ster bestaande uit gedegenereerde materie (witte dwerg) is daarentegen zo klein dat zij nooit massa via Roche-lobe overflow kan verliezen.

In hoofdstuk III.1 van dit proefschrift stellen wij voor dat de begeleider van de neutronenster een niet-gedegenereerde He-ster is. Om deze hypothese te testen hebben wij numerieke evolutie-berekeningen gedaan van een dubbelster die aanvankelijk bestaat uit een $1 \, M_\odot$ neutronenster en een $0.6 \, M_\odot$ He-ster begeleider, waarin de massa-overdracht wordt aangedreven door impulsommentoverliezen ten gevolge van gravitatiestraaling. Als voorbeeld van de resultaten van onze berekeningen staat in figuur 2 de relatie uitgezet tussen de massa van de begeleider en de baanperiode. Aangezien eenvoudig valt af te leiden dat de baanperiode van 1E2259+586 voor de supernovaexplosie ca. 22 minuten moet zijn geweest, kan uit de figuur worden afgelezen dat de huidige massa van de He-ster $0.37 \, M_\odot$ is. Met gebruik van de lichtkracht en effectieve temperatuur uit de berekeningen en de bekende afstand en extinctie van het systeem voorspelt het model een blauwe magnitude van 23.7, wat zeer goed overeenkomt met de waargenomen waarde van 23.5 voor de optische tegenhanger. Een ander interessant punt in figuur 2 is het bestaan van een minimum periode van 11 minuten voor een
Dubbelster waarin een He-ster via Roche-lobe overflow massa overdraagt. De minimumperiode wordt veroorzaakt doordat bij een massa van ca. 0.2 M\(_\odot\) de He-ster zal expanderen bij verder massaverlies, door de toenemende degeneratie van de elektronen. Het bestaan van een dergelijke minimumperiode voor waterstofrijke sterren (70 minuten) was al eerder aangetoond.

A0620-00

Dit systeem is een tijdelijke Rontgenbron die in 1975 uitbarstte en toen de grootste schijnbare helderheid bereikte die tot nu toe van een Rontgenbron is waargenomen. Tegelijkertijd nam de optische helderheid met ca. 7 magnituden toe. Studie van oude fotografische platen toonde aan dat deze uitbarstingen waarschijnlijk een recurrentie tijdschaal van ca 70 jaar hebben. Uit spectra genomen van het systeem nadat het weer tot rust was gekomen, blijkt dat de begeleider van het compacte object een K5 dwerg is (massa ongeveer 0.8 M\(_\odot\)), hetgeen een afstand inhoudt van ca. 850 pc. De baanperiode van het systeem is 7.75 uur. De radiële snelheid van de begeleider varieert met de zeer grote amplitude van 457 km/sec, wat een absolute onderlimiet op de massa van de begeleider oplevert van 3.2 M\(_\odot\).

Dit betekent vrijwel zeker dat het compacte object een zwart gat is.

Door de evolutie van het systeem zoals het nu wordt waargenomen terug in de tijd te volgen hebben wij in hoofdstuk III.2 een evolutie scenario voor deze dubbelster opgesteld, dat geïllustreerd is in figuur 3. De typische beginconfiguratie bestaat uit een 40 M\(_\odot\) hoofdreeksster met een begeleider van 1 M\(_\odot\), in een baan met een periode van ca. 500 dagen. Door gebruik te maken van de voorwaarden dat de dubbelster een zogenaamde "common envelope" fase heeft overleefd, gebonden bleef tijdens de supernova-explosie en dat de begeleider niet van de hoofdreeks af is geëvoveerd voordat zij massa begon over te dragen, konden wij

Fig. 3 Een schematische weergave van de evolutie van A0620-00
limieten stellen op de beginmassa's van de sterren. Voor de meest waarschijnlijke massa van het zwarte gat van 7 M☉ houden deze limieten in dat de beginmassa van de begeleider kleiner moet zijn geweest dan 2 M☉, en dat de zware ster een massa tussen de 27 en 46 M☉ moet hebben gehad, hetgeen vrij licht is gezien de huidige ideeën over de vorming van zwarte gaten.

Common Envelope evolutie en Bondi-Hoyle accretie.

Als we de evolutie van nauwe dubbelsterren met een compact object beschouwen is het direct duidelijk dat de dubbelster in het verleden veel wijder moet zijn geweest: wil een ster een gedegenereerde kern (witte dwerg) ontwikkelen of tot neutronenster evolueren dan moet zij eerst door het reuzenstadium gaan, en een dergelijke reus zou nooit in de huidige baan passen. Het algemeen geaccepteerde idee over de evolutie van deze systemen is dat als de voorloper van de compacte ster een reus wordt, deze op onstabiele wijze massa zal gaan overdragen aan zijn begeleider, zodat het oorspronkelijke reuzen-omhulsel een gemeenschappelijk gasomhulsel ("common envelope") zal vormen rondom een intern dubbelstersysteem dat bestaat uit de begeleider en de kern van de reus. Wrijvingsprocessen in dit omhulsel zijn dan de oorzaak van een geleidelijk naar elkaar toe spiraleren van de componenten van de interne dubbelster, waarbij de hierbij vrijkomende energie verantwoordelijk wordt geacht voor het afwerpen van het gasomhulsel zodat alleen de interne dubbelster overblijft.

Een goede beschrijving van dit ingewikkelde 3-dimensionale hydodynamische proces behoort op dit moment nog niet tot de mogelijkheden. Een kritisch overzicht van de ruwe pogingen hiertoe die in de literatuur zijn verschenen staat in hoofdstuk V van dit proefschrift, met als belangrijkste conclusie dat geen enkel model tot nu toe in staat is gebleken om op enigszins geloofwaardige wijze de uitkomst van een common envelope fase te voorspellen. Op grond van een aantal eenvoudige algemene beschouwingen wordt beargumenteerd dat het verliezen van de common envelope niet noodzakelijkerwijs het gevolg is van de interne wrijvingsprocessen.

In hoofdstuk IV wordt een studie beschreven naar de hoeveelheid energie die gedissipeerd wordt als een graviterend lichaam door een gasvormig medium beweegt, wat als deelprobleem van de common-envelope evolutie kan worden worden beschouwd. Om een betere schatting te verkrijgen dan de gewoonlijk gebruikte klassieke benadering werden er
**Table 2** De energie die in de omringende materie wordt gedeponeerd bij Bondi-Hoyle accretiestroming (zie tekst)

<table>
<thead>
<tr>
<th>Model</th>
<th>$E_s/E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M=1.5$, $\gamma=5/3$</td>
<td>2.07</td>
</tr>
<tr>
<td>$M=2.0$, $\gamma=5/3$</td>
<td>2.18</td>
</tr>
<tr>
<td>$M=3.0$, $\gamma=5/3$</td>
<td>2.36</td>
</tr>
<tr>
<td>$M=3.75$, $\gamma=5/3$</td>
<td>2.42</td>
</tr>
<tr>
<td>$M=1.5$, radiaatief</td>
<td>1.36</td>
</tr>
<tr>
<td>$M=2.0$, radiaatief</td>
<td>2.67</td>
</tr>
<tr>
<td>$M=3.0$, radiaatief</td>
<td>4.12</td>
</tr>
<tr>
<td>$M=3.75$, radiaatief</td>
<td>4.90</td>
</tr>
</tbody>
</table>

Numerieke simulaties uitgevoerd van het probleem (gewoonlijk aangeduid als Bondi-Hoyle accretiestroming), waarbij gebruik werd gemaakt van de zogenaamde Particle-In-Cell methode. Als voorbeeld van de resultaten uit hoofdstuk IV.1 wordt in tabel 2 de hoeveelheid energie gegeven die per seconde in het omringende medium wordt gedeponeerd, normaliseerd op de klassieke schatting $E_0 = \frac{2\pi G m^2 v^2}{ho}$ waarin $G$ de gravitatieconstante is, $m$ de massa van het graviterend object, $\rho$ de dichtheid van het medium en $v$ de snelheid van het object. De energie-opwekking wordt gegeven als functie van het Mach getal $M$, en zowel voor een polytrope toestandsvergelijking ($\gamma=5/3$) als voor een mengsel van een ideaal gas en straling. In het laatste geval is stralingstransport meegenomen in de berekeningen met behulp van de diffusiebenadering.

Tijdens het werk aan het Bondi-Hoyle probleem kwam een samenwerking tot stand met M. Livio en N. Soker (Haifa, Israel), om met een gemodificeerde versie van onze code 3-dimensionale berekeningen van het probleem uit te voeren. Dit is noodzakelijk als men een uitspraak wil doen over de hoeveelheid impulsmoment die door het graviterend object wordt ingevoerd als het medium niet homogeen is, bijvoorbeeld door de aanwezigheid van een dichtheidsgradient. Dit is een belangrijke vraagstelling in de context van accretie uit een sterrenwind, zoals die plaats vindt in de zware Röntgendubbelsterren. De resultaten van deze berekeningen (hoofdstukken IV.2, IV.3 en IV.4) bevestigen een al eerder uitgesproken vermoeden dat de schatting die gewoonlijk gebruikt wordt om de hoeveelheid geaccreteerd impulsmoment uit te rekenen een sterke overschatting is, met ten minste een factor 5. Vervolgens worden de gevolgen die dit heeft voor de vraag van het al of niet aanwezig zijn van accretieschijven in de zware Röntgendubbelsterren besproken.
CHAPTER I

INTRODUCTION AND SUMMARY
Binary stars containing a compact object (white dwarf, neutron star or black hole) have been intensively studied in recent years. The most important reason for this is that the study of compact objects can yield important information on the behaviour of matter under extreme conditions (high density and temperature, strong gravity and magnetic fields), that are not available in the laboratory on earth. Most of our knowledge of compact objects is derived from those that are in binary systems, since isolated compact stars emit almost no radiation, whereas the interaction between the compact star and its companion in a binary gives rise to a number of conspicuous phenomena. A number of these binaries are very bright X-ray sources, and they often exhibit considerable variability, on short timescales and at many different wavelengths. These properties are both related to the fact that most of the luminosity of these systems derives from accretion of matter (coming from the companion) onto the compact star. The brightness of the systems is caused by the effectiveness of accretion on a compact object as an energy generation mechanism, and the reasons for the short timescale variability are that i) most of the luminosity is generated in a small volume close to the compact star (millisecond to minute variability) and ii) in most cases the orbital separation of the binary (and hence the orbital period) has to be small for interaction to take place (minute to day variability).

Interacting binaries containing a compact star are usually divided into two groups. In the first group the compact star accretes matter from the stellar wind of a massive companion star, and in the second group matter is transferred to the compact star by Roche-lobe overflow from a low-mass companion. An important difference in the accretion process between these two groups is that in the case of Roche-lobe overflow an accretion disk will always be present in the system, which is responsible for most of the observable properties. The presence of a disk in the massive wind-accreting binaries is still controversial, and it is certainly not the dominant observational feature.

In our project we have been mainly concerned with obtaining a better understanding of the formation and evolution of the low-mass systems. The work can be divided into three parts:
1) studies of the properties of wide, low-mass interacting binaries (bright galactic bulge sources, symbiotic stars?)
2) the construction of evolutionary scenarios for observed binaries (A0620-00, 1E2259-586)
3) studies of the formation and evolution of close interacting binaries (short period, low-mass X-ray binaries, cataclysmic variables) through common envelope or spiral-in evolution.

**Properties of wide, low-mass X-ray binaries**

To account for the presence of some very bright X-ray sources in the galactic bulge, which have not been identified optically because of the large interstellar extinction and for which no orbital periods are known, Webbink, Rappaport and Savonije (WRS) proposed the following model. The sources are thought to consist of a neutron star with a low-mass giant companion (less massive than the neutron star). The mass transfer is driven by the evolutionary expansion of the giant, and because the mass losing star is less massive than the gaining star this mass transfer is stable. The evolution of these binaries can be described in a simple semi-analytical way, which WRS did for binaries containing normal giants (single H-burning shell).

We have extended this work to investigate whether other types of interacting binaries, especially those that require higher mass transfer rates than those of X-ray binaries, can be explained by a similar model, in which the giant is a more evolved double shell source giant. These giants evolve (and hence expand) much faster, and give rise to higher mass transfer rates than in the systems considered by WRS. The calculated mass transfer rates and lifetimes are summarized in table 1, as a function of the orbital period at which mass transfer starts. We

<table>
<thead>
<tr>
<th>$P_{\text{ini}}$</th>
<th>$\log(\dot{m}_{\text{i}})$</th>
<th>$\log(t_{\text{AL}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.99</td>
<td>-7.38</td>
<td>7.16</td>
</tr>
<tr>
<td>429.6</td>
<td>-6.63</td>
<td>6.25</td>
</tr>
<tr>
<td>812.4</td>
<td>-6.36</td>
<td>5.90</td>
</tr>
<tr>
<td>2016</td>
<td>-6.40</td>
<td>5.51</td>
</tr>
<tr>
<td>3738</td>
<td>-6.21</td>
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<td>5551</td>
<td>-6.08</td>
<td>4.66</td>
</tr>
<tr>
<td>7313</td>
<td>-5.95</td>
<td>4.30</td>
</tr>
</tbody>
</table>

Table 1. The average mass transfer rate ($M_\odot$/yr) and the duration of the mass transfer phase (in years) of low-mass giant neutron star binaries, as a function of initial orbital period, which is given in days.
have not been able to find clear examples of the neutron star - giant binaries we model, which would be accreting at highly super-Eddington rates. If the compact star is a white dwarf the system will probably be observed as a symbiotic star. By comparing the properties of the systems we model with the properties of a sample of observed symbiotic stars, we find that AG Dra, V443 Her, SY Mus and RW Hyl are likely candidates for our model.

We have also invoked this binary model to explain the properties of some axially symmetric non-thermal radio sources that have been observed near the galactic center. From the energy content and the shape of the radio sources it can be derived that an energy injection of about $10^{38}$ ergs/s must have been going on for a period of a few times $10^5$ yr. It is argued that the only known type of astrophysical object that can emit such a large amount of energy for such a long time is the type of binary we describe in our model.

Finally, we have employed the model to calculate the spin evolution of an accreting magnetized neutron star in a wide, low-mass X-ray binary. For a certain set of assumptions about the time evolution of the magnetic field strength of the neutron star, this calculation predicts a relation between the orbital period and the spin period of the neutron star when mass transfer stops (figure 1). Since the descendants of these systems are

Fig. 1 The neutron star spin period as a function of orbital period at the end of the mass transfer phase, with and without a bottom in the magnetic field. The curves labeled 1, 2 and 3 represent magnetic field decay time-scales of $2 \times 10^6$, $5 \times 10^6$ and $10^7$ year respectively.
observed as wide binary radio pulsars, a comparison of the observed and theoretical relation in the orbital period versus spin period diagram allows us to put constraints on the evolution of the magnetic field. The position of the binary pulsar PSR 1953+29 sets an upper limit on the initial decay timescale of the magnetic field of $1.2 \times 10^7$ yr and the position of PSR 1855+09 tells us that this decay does not continue indefinitely, but stops when the surface field strength has reached a value of about $5 \times 10^8$ Gauss.

**Evolutionary scenarios.**

**1E2259+586**

The binary 1E2259+586 has an orbital period of 38 minutes and contains a 7 second X-ray pulsar. It is located at the center of a shell of diffuse X-ray emission which is probably a $10^4$ yr old supernova remnant at a distance of $3.6 \pm 0.4$ kpc. This implies that the neutron star was recently formed by accretion induced collapse of a white dwarf. From pulsar delay timing Gregory and Fahlman obtained a mass function of $0.008 \pm 0.0002 M_\odot$, which for a standard pulsar mass of about $1 M_\odot$ implies that the mass of the secondary must be $> 0.2 M_\odot$. The short orbital period excludes a normal main sequence companion, since such a star would overfill its Roche-lobe considerably. A degenerate star would never fill its Roche-lobe, and hence could not transfer any mass.

![Graph](image)

**Fig. 2** The orbital period (in minutes) versus the mass of the He-star
We propose that the companion in this system is a non-degenerate helium burning He-star. To test this hypothesis we have performed detailed evolutionary calculations of a binary system which initially consists of a 1 $M_\odot$ neutron star and a 0.6 $M_\odot$ He-star companion, which is transferring mass due to angular momentum losses due to gravitational radiation. As an example of the results of our calculations we have plotted in fig. 2 the relation between the mass of the companion and the orbital period. Since the pre-supernova orbital period of 1E2259+586 must have been about 22 minutes, we find that the present mass of the He-star companion is 0.37 $M_\odot$. Using the luminosity and effective temperature from our calculations and the known distance and extinction to the system we estimate a blue magnitude of 23.7, in excellent agreement with the observed value of 23.5 for the optical counterpart. Another interesting point from fig. 2 is the existence of a minimum period of 11 minutes for a binary with a Roche-lobe filling He-star secondary. This minimum period occurs because at a mass of about 0.2 $M_\odot$ the increasing electron degeneracy in the secondary will make it expand in response to mass loss. For hydrogen rich secondaries a similar minimum period of about 70 minutes had already been obtained before.

A0620-00

This is a transient X-ray source which went in outburst in 1975, and reached an apparent X-ray brightness which is the highest so far observed. At the same time the optical counterpart brightened by 7 magnitudes. Study of old plates revealed that the recurrence timescale of these outbursts is about 70 yr. Spectra taken after the system had returned to quiescence indicated that the companion to the compact object is a K5 dwarf (0.8$M_\odot$) which allows a distance determination of about 800 pc. The orbital period is 7.75 hours. From a series of radial velocity measurements of the K dwarf spectrum Figure 3 A schematic illustration of the evolution of A620-00
very high radial velocity amplitude of 457 km/sec was obtained, which gives an absolute lower limit to the mass of the compact object of 3.2 M\(_\odot\). This almost certainly means that it is a black hole.

By following the evolution of the presently observed binary backward in time we have constructed an evolutionary scenario for this system, which is illustrated in fig. 3. The initial configuration typically consists of a 40 M\(_\odot\) star and a 1 M\(_\odot\) companion with an orbital period of about 500 days. By making use of the conditions that the binary survived a common envelope phase, remained bound during the supernova and that the companion did not evolve off the main sequence before it started to transfer mass to the black hole, we were able to put limits on the initial masses of the stars. For the most likely black hole mass of 7 M\(_\odot\) these limits are that the companion must have had an initial mass less than 2 M\(_\odot\), and that the massive star was between 27 and 46 M\(_\odot\), which according to present views is rather low for a black-hole progenitor.

**Common envelope evolution and Bondi-Hoyle accretion flow.**

When we consider the evolution of close binaries that contain a compact object it immediately becomes clear that they must have been much wider in the past: in order for a star to develop a white dwarf core or to become a neutron star it has to pass through a giant phase in its evolution, and such a giant would never fit inside the present orbit. The generally accepted idea about the evolution of these binaries is that when the progenitor of the compact star has evolved into the giant stage it will start to overflow its Roche-lobe in an unstable way, so that the initial giant envelope becomes a common envelope around an "internal" binary that consists of the companion and the core of the giant. Friction in this envelope will cause the components of the internal binary to spiral together, while the generated frictional energy is thought to be responsible for the ejection of the common envelope, leaving a "naked" internal binary.

A proper description of this complicated 3-dimensional hydrodynamical process is at present beyond the possibilities. A critical review of the crude attempts that have been made in the literature is given in Chapter V. The conclusion of this review is that none of the attempts made so far are able to predict the outcome of common envelope evolution with any degree of credibility.
We have restricted ourselves to a few subtopics, the main one being a study of the frictional energy generated when a gravitating body moves through a medium. We have performed numerical simulations of this problem (which is usually referred to as Bondi-Hoyle accretion flow) to obtain better estimates of the frictional luminosity, using the so-called particle-in-cell method. As an example of our results table 2 gives the rate at which the dissipated energy is deposited into the surrounding medium, normalized to the classical estimate \( E_0 = \frac{2}{3} \frac{m^2 G \nu}{\rho} \) (\( G \) is the gravitational constant, \( m \) the mass of the gravitating object, \( \rho \) the density of the medium and \( \nu \) the velocity of the object. The energy generation rate is given as a function of the Mach number \( M \), and both for a polytropic equation of state (\( \gamma=5/3 \)), and for a mixture of an ideal gas and radiation. In the last case radiation transport is included in the diffusion approximation.

While working on Bondi-Hoyle accretion flow, we started a collaboration with M. Livio and N.Soker from Haifa, Israel to perform 3-dimensional calculations of the problem. This is necessary if one wants to investigate the accretion of angular momentum in the case that the star is moving through a medium which is not homogeneous, for instance if a density gradient is present. The amount of angular momentum accreted is an important quantity in the context of accretion from a stellar wind, as is the case in the massive X-ray binaries. The calculations confirmed our suspicion (based on earlier 2-dimensional work) that the expression usually employed to calculate the accretion rate of angular momentum is a severe overestimate, by at least a factor of 5. This can have significant implications for the question whether a disk is present in massive X-ray binaries, and we discuss this question for several observed systems in the light of our findings.

<table>
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<tr>
<th>Model</th>
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<tr>
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</tr>
<tr>
<td>( M=2.0, \gamma=5/3 )</td>
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</tr>
<tr>
<td>( M=3.0, \gamma=5/3 )</td>
<td>2.36</td>
</tr>
<tr>
<td>( M=3.75, \gamma=5/3 )</td>
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</tr>
<tr>
<td>( M=1.5, \text{radiative} )</td>
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</tr>
<tr>
<td>( M=2.0, \text{radiative} )</td>
<td>2.67</td>
</tr>
<tr>
<td>( M=3.0, \text{radiative} )</td>
<td>4.12</td>
</tr>
<tr>
<td>( M=3.75, \text{radiative} )</td>
<td>4.90</td>
</tr>
</tbody>
</table>

Table 2 The energy deposited in the surrounding medium in Bondi-Hoyle accretion flow.
CHAPTER II

THE EVOLUTION OF WIDE, LOW-MASS INTERACTING BINARIES.

II.1) Very high mass transfer rates in binaries containing low mass giants

II.2) A wide low-mass binary model for the origin of axially symmetric non-thermal radio sources

II.3) Neutron star spin-up in wide, low-mass X-ray binaries
Very high mass transfer rates in binaries containing low mass giants

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Summary. We study the evolution of binaries in which a low-mass giant with a helium or carbon-oxygen core is transferring matter to a more massive compact companion. In the case of the He-core giants we find mass and transfer rates of $\sim 10^{-6} - 10^{-7} M_\odot$/yr, which persist for $\sim 10^6$ yr, while for the CO-core giants corresponding values are $\sim 10^{-6} M_\odot$/yr and $\sim 10^3$ yr respectively. The stellar wind from the giant may play an important role in the evolution of these binaries. When the compact companion to the giant is a white dwarf the systems we model might be observed as symbiotic stars.

Key words: binaries – symbiotic stars – novae – X-ray binaries

1. Introduction

Many models for interacting binary stars such as X-ray binaries, cataclysmic binaries and symbiotic stars require a certain rate of mass-transfer from one star to the other. In the case of the X-ray binaries a transfer rate of $10^{-9} - 10^{-8} M_\odot$/yr is sufficient to explain the observed properties, in other cases (symbiotics, SS 433, etc.) a much larger rate ($10^{-5} - 10^{-4} M_\odot$/yr) is necessary.

For the bright low-mass X-ray sources near the galactic center a very simple and plausible model was proposed by Webbink, Rappaport and Savonije (1983, hereafter WRS) and Taam (1983), wherein a low-mass giant transfers mass on a nuclear timescale to a more massive neutron star. In this paper we extend the model of WRS to include giants with higher core masses. Our main purpose is to investigate whether the larger transfer rates which are needed to explain other types of interacting binaries can be achieved in a similar way.

2. Method of calculation

In the basic model (WRS) a low-mass giant, in a wide binary system with a slightly more massive companion, is slowly ascending the giant branch. These stars have the property that, for a given chemical composition, their radius and luminosity are, to good approximation, functions of the core mass only (Rafsdal and Weigert, 1970; Paczynski, 1970; Kippenhahn, 1981). As the core mass of the giant increases due to nuclear evolution, the radius of the giant also increases until it fills its Roche-lobe and mass transfer to the compact companion commences. Since the mass transfer takes place from the less massive to the more massive star, it will cause the orbit to widen on a dynamical timescale until the radius dictated by the core mass of the giant can again be accommodated within the Roche-lobe of the giant.

Because this adjustment can take place on a much shorter timescale than that on which the radius of the giant changes, the mass-transfer rate will adjust itself to a value at which the rate of increase in radius of the Roche-lobe is just equal to the evolutionary expansion rate of the giant. The equation determining the mass-transfer rate can be written as:

$$\frac{\partial R_l}{\partial t} = \frac{\partial R_i}{\partial t} + \frac{\partial R_\star}{\partial t},$$

(1)

where $R_l$ is the giant's radius and $R_\star$ is the radius of its Roche-lobe.

The first partial derivative on the right-hand side gives the rate of change of the radius of the Roche-lobe due to mass transfer through the $L_\star$ point, the second allows for changes in the radius of the Roche-lobe due to a stellar wind. An analytic expression for the evolutionary change in the radius of the giant can be obtained from fits to the core mass-luminosity [$L = L(m_c)$] and core mass-radius [$R_c = R_c(m_c)$] relations mentioned above. Since the energy in these giants is generated in a shell source around the core, the increase of core mass with time is given by the luminosity of the star divided by the amount of energy generated by burning one gram of matter in the shell:

$$m_c = L/\epsilon.$$

(2)

The left-hand side of Eq. (1) can therefore be written as:

$$\frac{\partial R_l}{\partial t} = \frac{\partial R_i}{\partial m_c} m_c = \frac{\partial R_\star}{\partial m_c} L_c,$$

(3)

To derive an expression for the right-hand side of Eq. (1) we begin by writing the radius of the critical Roche-lobe as:

$$R_c = f(q) a,$$

(4)

where $a$ is the binary separation, $q$ the ratio of the mass of the compact star to that of the giant $m_i/m_c$, and $f(q)$ an analytic fit to numerically calculated values of $R_c/a$. We shall employ here the expression by Paczynski (1971):

$$f(q) = \frac{2}{3^{1/3}} \left(\frac{1}{1 + q}\right)^{1/3}$$

(5)

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Differentiating (4) with respect to time yields

$$R_{\perp} = a \left( \frac{df}{dq} \right) \dot{q} + f(q) \dot{a}. \quad (6)$$

To obtain expressions for $\dot{q}$ and $\dot{a}$ as a function of the transferred mass we have to make some assumptions about the way mass is lost from the giant. WRS assumed that all matter lost from the giant is transferred to the companion, implying conservation of total system mass and orbital angular momentum. Since we want to consider cases where the total system mass (and hence also angular momentum) is not conserved, we shall use more general expressions. We have simply

$$\dot{q} = \frac{m_s}{m_a} \frac{dE}{dq},$$

$$\dot{a} = \frac{J}{M_1 \mu} \frac{dE}{dq}. \quad (7)$$

With Kepler's law we can write $a$ as

$$a = \frac{J^2}{GM_1 \mu^2}, \quad (8)$$

with $J$ the orbital angular momentum, $M_1$ the total system mass and $\mu$ the reduced mass. Differentiating with respect to time gives us an expression for $\dot{a}$:

$$\dot{a} = \frac{2J}{M_1} \dot{M}_1 \frac{2\mu}{\dot{\mu}}.$$  \quad (9)

To be able to calculate $\dot{q}$ and $\dot{a}$ from (7) and (9) we make the following assumptions regarding the mass loss:

1) There is a wind emanating from the giant, which is totally lost from the system taking with it the specific angular momentum (orbital plus rotation) of matter at the surface of the giant.

2) Mass is transferred to the compact star through the $L_1$-point. When the mass transfer rate necessary to keep the giant within its Roche-lobe is lower than that required to yield the Eddington luminosity from the compact star ($\dot{m}_e$), all transferred matter is accreted.

3) If, however, the mass transfer rate is in excess of $\dot{m}_e$, the accretion rate is limited to $\dot{m}_e$ and the rest is lost from the system carrying the specific orbital angular momentum of the compact star. By making this assumption about the angular momentum loss we implicitly assume that the matter which is lost first passes through an accretion disk before it is ejected from the vicinity of the compact star (possibly in the form of beams or jets).

Using these assumptions in Eqs. (9), (7) and (6) one obtains the change in size of the Roche-lobe of the giant as a function of the mass-transfer rate and the wind mass-loss rate. Equating these results to the evolutionary change in the radius of the giant obtained in Eq. (3), yields an expression for the mass-transfer rate:

$$|\dot{m}_e| = \left| \frac{\alpha R_1}{3M_a} \frac{m_e}{m_a} \frac{\dot{m}_w}{F(q)} \right| G(q), \quad (10)$$

with

$$F(q) = \frac{2}{3} + \frac{6q}{3(1 + q)} + \left( \frac{2}{3!} \right)^{1/3} \left( 1 + q \right)^{1/3} q^{-1},$$

and

$$G(q) = \frac{2}{3} - \frac{2}{3} \left( 3 + q + 2\beta q \right).$$

Here $m_e$ is the stellar wind mass loss rate of the giant and $\beta$ is the fraction of the transferred mass that is actually accreted. By studying the behaviour of the functions $F$ and $G$ some general conclusions can be derived. The function $F$ determines the effect of the wind mass loss: when $F > 0$ the wind mass loss will increase the transfer rate. The last term in the expression for $F$ represents the rotational angular momentum of matter at the surface of the giant which is carried out of the system by the stellar wind. This term is always positive and therefore tends to increase the mass transfer rate. The sum of the first two terms is $< 0$ for $q < 3$. Hence in the absence of spin angular momentum losses the mass transfer rate will be reduced by the stellar wind for $q < 3$ and increased for $q > 3$. Stable solutions require that mass flow from the giant to the compact star leads to an increase in the radius of the Roche-lobe of the giant. This translates to the requirement that $G > 0$, which for $\beta = 1$ implies $q > 1.2$. This result was also found by WRS. Note that for $\beta = 0$ (i.e., all matter transferred through the $L_1$-point is ejected) the requirement is only $q > 0.836$.

3. Properties of the giant

To carry out the calculation outlined above we have to know the following properties of the mass-losing giant:

1) The core mass-radius and core mass-luminosity relations. In their paper, WRS considered giants with a degenerate He-core and a hydrogen burning shell, with core masses in the range $0.15 - 0.35 M_\odot$. For these stars the fits to the core mass-luminosity and core mass-radius relations for a Population I composition ($Y = 0.28, Z = 0.02$) as derived by WRS are:

$$\ln (L/L_\odot) = 3.50 + 8.11 z - 0.61 z^2 - 2.13 z^3,$$

$$\ln (R/R_\odot) = 5.38 + 5.10 z - 0.05 z^2 - 1.71 z^3, \quad (11)$$

where $z = \ln (m_e/(0.25 M_\odot))$. These fits can be used for core masses up to 0.45 $M_\odot$.

Since we are interested in cases of high mass-transfer rates, we have made similar fits to the numerical models of Paczynski (1970) for stars on the asymptotic giant branch with a degenerate CO-core and two burning shells, which have core masses in the range $0.6 - 0.9 M_\odot$. We find the following relations:

$$\ln (L/L_\odot) = 10.227 + 2.057 y - 0.687 y^2 + 2.282 y^3,$$

$$\ln (R/R_\odot) = 6.771 + 1.299 y - 0.332 y^2 + 1.920 y^3, \quad (12)$$

where $y = \ln (\dot{m}_e/M_\odot)$.

2) A value for the efficiency of energy generation per unit mass in the burning shell $\varepsilon_\text{H}$ in Eq. (3). For the He-core giants we have used the same burning efficiency as WRS, $\varepsilon_\text{H} = 5.89 \times 10^{18}$ erg g$^{-1}$. For the giants with two burning shells we used the result found by Kippenhahn (1981) that in these cases typically about $13\%$ of the luminosity comes from the He-burning shell, and simply raised $\varepsilon$ accordingly. In this simple model we neglect the possible effects of thermal pulsations (Iben and Truran, 1978).

3) The wind mass-loss rate. For the wind mass-loss rate we used the expression by Kudritzki and Reimers (1978):

$$\dot{m}_w = 5.5 \times 10^{-3} \left( R/R_\odot \right) (L/L_\odot) (M/M_\odot)^{-1} M_\odot/yr. \quad (14)$$

In the case of giants with high core masses these wind losses become very large (larger than the transfer rate through the Roche-lobe) and the giant envelope is lost mostly through the wind. Because expression (14) is still rather uncertain we repeated the calculations with the wind mass-loss rate reduced by a factor of...
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Table 1. Low mass giant – neutron star binaries

<table>
<thead>
<tr>
<th>( m_2 )</th>
<th>( P_{\text{init}} )</th>
<th>Full wind mass loss</th>
<th>Reduced wind mass loss</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( \log (m_2) )</td>
<td>( \log t_{\text{RL}} )</td>
<td>( m_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( m_1 )</td>
</tr>
<tr>
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<tr>
<td>0.40</td>
<td>429.6</td>
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<td>6.25</td>
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<td>812.4</td>
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</tr>
<tr>
<td>0.60</td>
<td>2016</td>
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<td>5.51</td>
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<tr>
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<tr>
<td>0.90</td>
<td>7313</td>
<td>-5.95</td>
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</tr>
</tbody>
</table>

In these models \( m_2 = 1.1 M_\odot \), \( m_1 = 1.4 M_\odot \) and all matter transferred in excess of the Eddington limit (\( 1.5 \times 10^{-8} M_\odot/\text{yr} \)) is ejected from the system. \( P_{\text{init}} \) is the initial orbital period of the binary in days, \( \langle m_1 \rangle \) the time average of the mass transfer rate in \( M_\odot/\text{yr} \), \( t_{\text{RL}} \) the duration of the mass transfer phase in years, and \( P_{\text{fin}} \) the orbital period when mass transfer stops.

Table 2. Low mass giant – white dwarf binaries

<table>
<thead>
<tr>
<th>( m_2 )</th>
<th>( P_{\text{init}} )</th>
<th>Full wind mass loss</th>
<th>Reduced wind mass loss</th>
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<td>( \log (m_2) )</td>
<td>( \log t_{\text{RL}} )</td>
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<td>0.90</td>
<td>7670</td>
<td>-5.40</td>
<td>3.87</td>
</tr>
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</table>

In these models \( m_2 = 1.0 M_\odot \) and \( m_2 = 1.3 M_\odot \). Symbols have the same meaning as in Table 1.

100. If \( m_2 \) were significantly larger than the value given by (14), as has been suggested for giants with massive degenerate cores (the so-called "super-wind," see e.g. Iben, 1981), the evolution would be completely dominated by stellar wind losses. The evolutionary timescale would then become so short that our model would no longer be applicable. We therefore do not consider this case any further.

4. Results

We considered giants with a Population I composition that start to transfer mass with either a He-core in the range 0.3–0.45 \( M_\odot \) or a CO-core in the range 0.6–0.9 \( M_\odot \). Two types of binary systems were investigated. The first consisted of a 1.1 \( M_\odot \) giant and a 1.4 \( M_\odot \) neutron star. The maximum accretion rate of the neutron star, \( m_2 \), was taken to be \( 1.5 \times 10^{-8} M_\odot/\text{yr} \). The second type of system consisted of a 1.0 \( M_\odot \) giant and a 1.3 \( M_\odot \) white dwarf. In this case the maximum accretion rate of the companion is much larger, with \( m_2 = 1.5 \times 10^{-5} M_\odot/\text{yr} \). The main results are presented in Tables 1 and 2, in which we list the average mass-transfer rate, the duration of the mass-transfer phase and the initial and final orbital periods of the binary. Some general conclusions can be drawn from Tables 1 and 2. By comparing the transfer rates with high and low wind mass-loss rates we find that the presence of a wind from the giant tends to increase the mass transfer rate. This increase, which may seem contrary to the usual result that mass loss from a star in a binary always increases the separation, is caused by the inclusion of rotational angular momentum in the giant wind [see Eq. (10)].

The mass-transfer rate does not change drastically when comparing the models of the most massive He-core giants with those of the least massive CO-core giants, and becomes at most a few times \( 10^{-8} M_\odot/\text{yr} \). On the other hand the duration of the mass transfer is much shorter in the case of the CO-core giants (10–100 times). This effect is partly caused by the small envelope masses allowed in this case, and partly by the high stellar wind mass loss rate, which in the most extreme cases can become four times as large as the transfer rate.

5. Discussion

We now turn to the question of which types of observed interacting binaries might possibly be explained by our model. The only system which has been identified with a wide binary consisting of a neutron star and a giant which fills or nearly fills its critical potential lobe is GX 1 + 4. Our model is probably not applicable here because of the relatively low inferred mass transfer rate of \( 5 \times 10^{-9} M_\odot/\text{yr} \). All of our models with neutron star primaries yield
super-Eddington transfer rates, so the obvious features to look for are a combination of X-ray emission and mass ejection from the system, possibly in the form of jets. The two objects that fulfill these criteria, ScoX-1 and SS 433, do not seem to fit in the model because of the relatively short observed orbital periods of 0.794 and 13.14, respectively. In SS 433 a giant of low core mass (0.25 \(M_\odot\)) might fit into the orbit, but it would not provide the mass estimated to be leaving the system in the jets (\(10^{-1} - 10^{-2} \dot{M}_\odot/yr\); Begelman et al., 1980). Apart from this, there are also other arguments against a low-mass companion to the compact object in SS 433 (Crampton and Hutchings, 1981). Another case in which supercritical accretion onto a neutron star has been invoked is in the explanation of the recently observed axially symmetric radio-remnants (Shaver et al., 1985; Becker and Helfand, 1985). In these objects we see evidence for highly energetic mass ejection that has been going on for a few times \(10^5\) yr, a time interval very similar to the lifetimes found in our calculations. For more details about this model see de Kool and van den Heuvel (1985). When the accreting star in the binary is a white dwarf more promising candidates for our model can be found. To recognize what these systems would look like we employ the work of Nomoto (1981) concerning the accretion of hydrogen-rich matter onto white dwarfs. It is found that for \(\log m \simeq -6.6\) the accretion matter burns steadily on the surface of the white dwarf. For \(\log m \leq -6.6\) the matter will burn in flashes, which may or may not eject some of the accreted matter. For \(\log m \geq -6.0\) a giant envelope will form around the white dwarf. In Table 2 we find that for a large range in parameters, \(<m>\) falls in the interval apparently required for steady H-burning. These binaries would display many of the properties normally associated with symbiotic stars. Such objects are defined (see Friedjung and Viotti, 1981) as stars which (i) show both absorption features typical of late-type giants and high excitation emission lines in their spectrum, and (ii) are quasi-periodically variable. These systems are now widely interpreted as binaries consisting of a giant with a strong stellar wind and a very hot dwarf or sub-dwarf companion. The high excitation emission lines are caused by photoionization of the giant’s wind by the radiation field of the hot dwarf. Accreting white dwarfs have already been proposed as candidates for the hot companion (Tutukov and Yungelson, 1976, 1981; Paczynski and Rudak, 1980) since their properties explain many observed characteristics of symbiotic stars.

Kenyon and Webbink (1984) have made an extensive study of several well known symbiotic stars by trying to fit their observed spectra with different theoretical models. Some systems are best explained by a binary consisting of a giant and an accreting main sequence star, in which case the mass transfer mechanism we discuss is not applicable since the giant has to be less massive than its companion. Better candidates are those systems for which a hot subdwarf companion, with a radius intermediate between that of a white dwarf and a main sequence star, would best explain the observed spectra. Model calculations of accreting white dwarfs (Paczynski and Zytkowski, 1979) indicate that the photospheric radius of such stars can indeed become an order of magnitude larger than the white dwarf itself. Of the systems considered by Kenyon and Webbink the best candidates for our model are AG Draconis, RW Hydrae, V443 Herculis and SY Muscae. These are the systems in which (i) the hot component of the binary is very compact and has a high temperature (\(\sim 10^4\) K) and (ii) the giant can be filling its Roche lobe. It is interesting that AG Dra and V 443 Her are the only two systems out of the 19 considered by Kenyon and Webbink that lie far from the galactic plane, which is consistent with the age expected for the binaries we model. Other symbiotics with a hot compact source are AG Peg and BF Cyg, but in these systems the giant does not seem to be filling its Roche lobe (Kenyon and Gallagher, 1983). In these cases accretion is thought to take place from the wind of the giant, although if these giants underfill their Roche lobe by a factor of 2 or more, very large wind mass loss rates (\(<\sim\ 10^{-6} \dot{M}_\odot/yr\)) are necessary to explain the inferred accretion rate. In fact, this is a more general problem of the wind accretion model, since even if the giants do nearly fill their Roche lobes a substantial increase in mass loss rate over the value given by the Reimers and Kudritzki expression is still necessary.

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A wide low-mass binary model for the origin of axially symmetric non-thermal radio sources

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The extended non-thermal radio sources G357.7 - 0.1 and G5.3 - 1.0 in the galactic bulge have about the same energy content in the form of relativistic particles and magnetic fields as the brightest ordinary supernova remnants (original energy input $\sim 10^{50}$ erg), but differ from the latter in having a marked axial symmetry. Their surface brightness gradually fades away towards one end of the symmetry axis, and at the opposite end one finds a compact radio source. The recently discovered radio source G18.95 - 1.1 (ref. 3) may belong to the same category. Helfand and Becker have argued that these structural properties can be explained most plausibly if these 'remnants' were produced by the ejection of matter with high kinetic energy by accreting binary systems at a rate $\gg 10^{51}$ erg s$^{-1}$, the ejection lasting for several times 10$^5$ yr while the systems were travelling through the interstellar medium with a speed of $\sim 100$ km s$^{-1}$. We show here that the only type of neutron star binary that can fulfil both the condition of longevity and of a continuous high-mass transfer rate is a relatively wide binary in which the companion of the neutron star is a low-mass giant, with an orbital period of the order of weeks to months. Binaries of this type are expected to resemble closely the eight brightest galactic bulge X-ray sources, as well as the progenitors of the two wide radio pulsar binaries.

The arguments for an accreting binary model are:

1. The only known types of sources that can inject large amounts of energy into non-thermal radio shells for an extended period of time are young pulsars and accreting neutron star binaries.

2. The linear extent of the radio remnants measured along the axis are $\sim 20 - 30$ pc. At a plausible speed of $\sim 100$ km s$^{-1}$ this requires $\geq 2 - 3 \times 10^5$ yr travel time. (3) The same amount of time is required to obtain their $\sim 10^{50}$ erg energy content from a young Crab-like pulsar or a neutron star accreting at the Eddington limit (both having $dE/dt = 10^{39}$ erg s$^{-1}$). As the high energy-loss phase of a young pulsar ceases within $\sim 10^5$ yr after its birth, an accreting neutron star (or, possibly, a black hole) seems the only suitable candidate for providing the energy. (4) Those neutron-star binaries around which we infer the presence of relativistic particles (such as SS 433 (refs. 10), Sco X-1 (refs 11, 12), Cyg X-3 (ref. 13) and Cir X-1 (ref. 14)) have rates of mass transfer near or above the Eddington limit ($m_{\nu} = 1.5 \times 10^{-3} M_{\odot}$ yr$^{-1}$). (5) The presence of the compact radio source (of finite angular size) on the symmetry axis, which at least in the case of G35.3 - 1.0 is obviously connected to the radio remnant, suggests that continuous injection of energy and relativistic particles may still be taking place (see also ref. 15).

The possible absence of a strong X-ray point-source in or near the shells is not necessarily worrying, because in the case that the mass transfer rate exceeds the Eddington value, most or all of the X-ray emission may be blocked on its way towards us, by the excess matter which is piling up around the compact object, or in a thick accretion disk around this object. The energy generated by the accretion will then come out in other forms, presumably mostly as kinetic energy. Our example here is SS 433, where the kinetic energy in the mass outflow at least equals the Eddington luminosity, while on the other hand the X-ray point source of SS 433 has an X-ray luminosity of only $10^{36}$ erg s$^{-1}$, more than a factor of 10 below the Eddington value.

In view of the arguments above, we adopt the accreting binary model and now examine which of the known types of accreting-neutron-star binary systems can achieve a more or less continuous near or super-Eddington-limit mass transfer rate for several hundreds of thousands of years. The three main types of accreting neutron star binaries known in the Galaxy are: (1) the massive X-ray binaries, in which the companion to the neutron star has a mass $\geq 8 - 10 M_{\odot}$; (2) the close low-mass X-ray binaries (orbital period $\lesssim 0.5$ days), in which the companion to the neutron star is presumably an unevolved red dwarf; (3) the wider low-mass X-ray binaries (orbital period $\geq 0.5$ days), in which the companion to the neutron star is an evolved low-mass giant ($< 1.2 M_{\odot}$) and the mass transfer is driven by the internal evolution of this star. Examples of the last category are Cyg X-2 ($P = 9.8$ days), 2S 0921 - 63 ($P = 8.99$ days) and GX 1 + 4 ($P$ at least several months, as the optical star is a low-mass M6 IIIe red giant).

The first two categories of systems are unable to maintain a high mass transfer rate for several times $10^5$ yr, for the following reasons. The massive systems can reach a near- or super-Eddington rate only by means of Roche lobe overflow, which is, however, highly unstable as the mass transfer takes place from the much more massive to the less massive component, which causes the orbit to shrink rapidly. In these systems, the Eddington limit is reached in $\sim 5 \times 10^5$ yr on average (at most $10^6$ yr) beyond which the transfer rate will increase rapidly ($10^8$ yr) to $\sim 10^{-4} - 10^{-3} M_{\odot}$ yr$^{-1}$ (see ref. 17) and a common envelope will form, presumably leading to rapid spiral-in of the neutron star and ejection of this envelope on a timescale $\sim 10^5 - 10^6$ yr (refs 16, 18 - 20). In the close low-mass systems, where gravitational radiation losses and/or magnetic braking are driving the mass transfer, it is very difficult to obtain mass transfer rates $\gg m_{\nu}$ without coming into serious conflict with the observed luminosity distribution of X-ray sources. This makes these systems improbable candidates for having produced the radio shells.

On the other hand, the third category of systems, consisting of a neutron star and a less massive giant, seems ideally suited for fulfilling the required conditions: here almost any rate of mass transfer $m_{\nu}$, below or above the Eddington limit, can be achieved for long periods of time (ranging from $10^5$ to $10^7$ yr, depending on the value of $m_{\nu}$). The transfer rate depends mainly
on the initial orbital period and to a lesser extent on the initial mass and metal content of the companion\(^2\).\(^3\). Moreover, the mass transfer, even if it is super-Eddington, never becomes unstable as the transfer takes place from the less massive to the more massive component, which causes the orbit to widen. An additional merit of this type of binary model is that the location of the radio sources considered (near the galactic centre) is very similar to that of the bright galactic bulge X-ray sources, for which the model was originally devised and seems well established.

For conservative mass transfer (with conservation of total mass and total orbital angular momentum) from a giant with a degenerate He core, the mean mass transfer rate \((\dot{m})\) is in lowest order approximation given by (see ref. 5):

\[
(\dot{m}) = 5.3 \times 10^{-10} P \left[ M_\odot \text{yr}^{-1} \right]
\]

where \(P\) is the initial orbital period in days. Equation (1) shows that to reach \(\dot{m} \sim 3 \times 10^{-8} M_\odot \text{yr}^{-1} (= 2m_{\text{Edd}})\), the required initial orbital period is \(\sim 60\) days. The duration of the mass transfer phase is roughly \(M_{\text{init}}/\dot{m}\), where \(M_{\text{init}} \approx 0.7 M_\odot\) is the envelope mass of the giant, which for \(\dot{m} = 2m_{\text{Edd}}\) yields \(\sim 2 \times 10^7\) yr. Giants with a degenerate He core yield a maximum \((\dot{m})\) of \(\sim 10m_{\text{Edd}}\), just before the He flash. A higher \(\dot{m} (10-100m_{\text{Edd}})\) can be reached by giants without a degenerate CO core.

The conservative assumptions used in ref. 5 are probably not applicable to the case of very high mass transfer rates considered here, as a considerable amount of mass (and angular momentum with it) is ejected from the system. We have therefore carried out calculations of orbital evolution and mass transfer rates similar to those in refs 5-7 for binaries initially consisting of a \(1 M_\odot\) giant, either with a He core of mass \(0.3 M_\odot < M_\text{He} < 0.45 M_\odot\), or with a CO core of mass \(0.6 M_\odot < M_\text{He} < 0.9 M_\odot\), in which we did not assume total conservation as in ref. 5, but allowed for loss of mass and angular momentum from the system. We also included the effects of the mass loss due to the strong stellar wind expected from the luminous giants considered here. As we assume a jet-like ejection mechanism from the inner parts of an accretion disk, the mass which is transferred by Roche lobe overflow in excess of \(m_{\text{Edd}}\) leaves the system with the specific orbital angular momentum of the neutron star. The stellar wind carries the specific orbital and spin angular momentum of the giant. The method of calculation is based on the assumption that the orbit evolves in such a way that the giant's equivalent Roche radius remains equal to the radius dictated by its core mass. Together with our assumptions on the way mass and angular momentum are lost from the system, this uniquely determines the mass transfer rate as a function of time. The method is discussed in detail in ref. 5, so we shall not give detailed equations here but only list the \(M_\odot - R\) and \(M_\odot - L\) relations used, where \(R\) is the giant radius and \(L\) its luminosity.

\begin{table}[h]
\centering
\caption{Mean mass transfer rate \((\dot{m})\), duration \(\tau_{\text{obs}}\), and final orbital period \(P_{\text{fin}}\) as a function of initial (degenerate) core mass \(M_1\) for wide binary systems initially consisting of a \(1.4 M_\odot\) neutron star and a \(1 M_\odot\) giant companion.}
\begin{tabular}{|c|c|c|c|c|}
\hline
\(M_1\) & \(\dot{m}\) & \(\tau_{\text{obs}}\) & \(P_{\text{fin}}\) & \(\tau_{\text{fin}}\) \\
\hline
0.30 & 65.01 & 2.4(-8) & 18.1(6) & 272 & 2.4(-8) & 23.1(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(405\) \\
0.40 & 450.6 & 1.3(-7) & 2.05(6) & 982 & 1.3(-7) & 3.63(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(1355\) \\
0.45 & 852.0 & 2.3(-7) & 0.86(6) & 1524 & 2.1(-7) & 1.84(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(1397\) \\
0.60 & 2,114 & 2.5(-7) & 0.28(6) & 2,994 & 2.0(-7) & 1.30(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(4282\) \\
0.80 & 3,921 & 4.0(-7) & 0.08(6) & 4,844 & 3.9(-7) & 0.66(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(6053\) \\
0.90 & 5,622 & 5.5(-7) & 0.03(6) & 6,588 & 3.1(-7) & 0.34(6) \(\tau_{\text{obs}} = \tau_{\text{fin}}\) \(7432\) \\
\hline
\end{tabular}
\end{table}

\(\dot{m}\) is uniquely determined by \(M_1\). The degenerate cores with \(M_1 < 0.45 M_\odot\) consist of helium, with \(M_1 = 0.60 M_\odot\) of C and O. The matter transferred in excess of the Eddington limit is assumed to leave the system with the specific orbital angular momentum of the neutron star. Mass transfer rates are in \(M_\odot \text{yr}^{-1}\), \(\tau_{\text{obs}}\) in yr and \(\tau_{\text{fin}}\) in days. Numbers in parentheses indicate powers of 10.

For the He-core giants these were (with \(z = \ln (M_1/(0.25 M_\odot))\)):

\[
\ln (L/L_0) = 3.50 + 8.11z \ln 10 - 0.61z^2 - 2.13z^3
\]

and for the CO-core giants (with \(y = \ln (M_1/M_0)\)):

\[
\ln (L/L_0) = 10.227 + 2.057z - 0.687z^2 + 2.282z^3
\]

Equations (2) are from ref. 5 assuming a Population I composition, since galactic bulge stars are known to have a rather high metal content, and equations (3) (S. A. Rappaport, M. de K. and E.I.P. van den H.) represent a fit to the curves calculated in refs 23 and 24. For the mass loss in the giant wind we used the expression by Kudritzki and Reimers\(^2\):\

\[
\dot{m}_b = 5.5 \times 10^{-13} (R/R_\odot) (L/L_\odot) (M_1/M_\odot)^{-3} M_\odot \text{yr}^{-1}
\]

We also did calculations with \(\dot{m}_b\) given by \(\dot{m}_b/100\); an example of the results is given in Fig. 1, and all results are summarized in Table 1. Although the lifetimes of the systems considered here are shorter than those in ref. 5, these low-mass giant plus neutron star binaries are still able to provide the high (super-Eddington) mass transfer rates for periods \(> 10^6\) yr, and thus can easily produce the radio sources.

We conclude that the wide binary model gives a good agreement with the present observations. Further work, especially a careful search for X-ray emission or detailed structure in the compact sources, could yield more evidence in this direction.

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Neutron star spin evolution in wide low-mass X-ray binaries

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Summary. We present the results of calculations of the spin evolution of a magnetized neutron star in a wide low-mass X-ray binary, with an evolved secondary star. In these calculations the history of the mass-accretion rate is obtained from a semi-analytical description of the evolution of the low-mass X-ray binary, and the magnetic field of the neutron star is assumed to decay on a variable timescale. The torques associated with the accretion have been derived from Ghosh and Lamb's (1979) model of disk accretion. Adopting the idea that these LMXRB's are the progenitors of the wide low-mass binary radio pulsars we find that the relation between spin and orbital period of the binary radio pulsars PSR 1855+09 and PSR 1953+29 is in good agreement with our models if we assume that after the formation of the neutron star its magnetic field decayed exponentially on a timescale of a few times $10^6$ yr to a bottom value of $\sim 5 \times 10^9$ Gauss. From the orbital and spin period of the binary radio pulsar PSR 1953+29 we derive an upper limit to the initial decay timescale of the magnetic field of the neutron star of $\sim 1.2 \times 10^7$ yr.

Key words: X-ray: binaries – stars: neutron – pulsars: general

1. Introduction

The distribution of binary radio pulsars in the spin-period versus magnetic-field diagram differs markedly from that of the single radio pulsars (for a review see e.g. van den Heuvel, 1984). This has led to the idea that radio pulsars in binary systems have been "recycled", i.e. they have been spun up due to mass accretion during a previous episode of mass exchange of the binary (Smarr and Blandford, 1976; Radhakrishnan and Srinivasan, 1982; Alpar et al. 1982).

The properties of the binary radio pulsars in wide circular orbits can be understood in a natural way, if they are the descendants of low-mass X-ray binaries with an evolved companion star (Savonije, 1983; Paczynski, 1983; Joss and Rappaport, 1983). In the latter systems the transfer of mass, which occurs by Roche-lobe overflow, is driven by the nuclear evolution of the secondary, whose expansion leads to a secular increase of the orbital separation (Webbink et al. 1983; Taam, 1983). After the envelope of the secondary has been completely stripped the system consists of a neutron star, and the remaining core of the secondary (a low-mass helium white dwarf), i.e. a wide binary radio pulsar. The orbital separation of these two compact stars depends mainly on the initial secondary mass, and the orbital period at the beginning of the mass-transfer stage of the low-mass X-ray binary.

Due to the accretion of mass the neutron star in the low-mass X-ray binary will undergo a torque and its rotation period will change; the resulting spin-rate derivative depends on the neutron-star spin, its magnetic field, and the accretion rate. As a consequence, the final spin rate of the neutron star, when the low-mass X-ray binary is transformed into a binary radio pulsar, is determined by the histories of the latter two quantities.

The time dependence of the accretion rate can be obtained from the reasonably well understood evolution of low-mass X-ray binaries with an evolved companion (Webbink et al. 1983; Taam, 1983).

For the history of the magnetic field of the neutron star the situation is far less clear. We will take the generally accepted point of view that when a neutron star is formed it has a surface magnetic field of a few times $10^{12}$ G, which decays on a timescale of less than $10^7$ years (see e.g. Lyne et al., 1985).

It is generally assumed that neutron stars in low-mass X-ray binaries are formed by the accretion-induced collapse of a white dwarf, during the same stage of mass transfer which gives rise to the X-ray emission. This idea is, in particular, based on the difficulty to explain otherwise how the systems managed to remain bound during the supernova explosion in which the neutron star was formed (Helfand, Ruderman and Shaham, 1983). The problem lies in the fact that the neutron star progenitor must have been more massive than $\sim 8 M_{\odot}$ and the remaining neutron star is only $\sim 1.4 M_{\odot}$, which means that several $M_{\odot}$ are ejected from the system. Since the system can only remain bound if less than half of its mass is ejected, this implies that the companion must have been sufficiently massive (at least a few $M_{\odot}$), which is not the case in the low-mass X-ray binaries. Since this consideration argues strongly against the idea that the neutron star is a very old object, we shall use the scenario of accretion induced collapse throughout this work.

In this paper we calculate the spin-up history of neutron stars in low-mass X-ray binaries with an evolved secondary star, and the resulting spin periods of wide binary radio pulsars, for a range of assumed initial values of the relevant system parameters. These results are compared with the observed spin and orbital periods, and magnetic fields of wide binary radio pulsars. We discuss possible implications for the formation of neutron stars in low-mass X-ray binaries, and for the decay of neutron-star magnetic fields.
2. Evolution of the neutron-star spin rate

To calculate the change in rotation period of an accreting magnetized neutron star, we use the model proposed by Ghosh and Lamb (1979) (hereafter GL) for the interaction between an accretion disk and the magnetosphere of a neutron star. In this model, the torque exerted on the neutron star by the accreting matter can be conveniently expressed as a function of a "fastness parameter" $\omega$, which is the ratio of $\Omega_0$, the angular velocity of the star, to $\Omega_k$, the Keplerian angular velocity at the boundary of the magnetosphere, and is given by

$$\omega = \frac{\Omega_0}{\Omega_k} = 1.44 \times 10^{-3} B_8^{6/7} P^{-1} m_8^{-3/7} M^{-5/7}$$  \hspace{1cm} (1)

Here $B_8$ is the surface magnetic field in units of $10^8$ Gauss, $P$ the spin period in seconds, $m_8$ the accretion rate in $10^{-8} M_{\odot}$/yr and $M$ the mass of the neutron star in $M_{\odot}$. The net accretion torque can then be expressed as

$$N = N_0 n(\omega)$$  \hspace{1cm} (2)

where $N_0$ is the angular-momentum flux carried by matter flowing through the inner boundary of the disk, at the magnetospheric radius $R_m$:

$$N_0 = j_k(R_m) \sin \theta$$  \hspace{1cm} (3)

Here $j_k(R_m)$ is the specific angular momentum of matter in a Keplerian orbit at the magnetospheric boundary $R_m$. The dimensionless torque $n(\omega)$ is approximately given by (GL):

$$n(\omega) = 1.39(1 - \omega_d(4.03(1 - \omega_d)^{0.173} - 0.878)(1 - \omega_d)^{-1}$$  \hspace{1cm} (4)

Combining Eqs. (1,2,3,4), and using the result that the moment of inertia of a neutron star does not change significantly by mass accretion (see e.g. Canuto, 1977), and is well approximated by a constant value of $10^{45}$ gcm$^2$ then yields an expression for the rate of period change $\dot{P}$:

$$\dot{P} = -1.054 \times 10^{-10} B_8^{6/7} n(\omega_d) \omega_d^{-2} M^{-1} \text{ (syr} \text{-}^{-1} \text{)}$$  \hspace{1cm} (5)

Note from Eq. 4 that for $\omega_d = 0.35$ the total torque on the star is zero, in which case the star is said to rotate at its equilibrium period. For later reference we introduce here the spin-up timescale $t_{sp}$ defined as the time it would take to spin up a neutron star which is initially at rest ($\omega = 0$) to the equilibrium period at which $n(\omega) = 0$:

$$t_{sp} = \frac{\Omega_{eq}}{\dot{P}(\omega = 0)} = 3.43 \times 10^6 B_8^{8/7} m_8^{-3/7} M^{-5/7} \text{ (yr)}$$  \hspace{1cm} (6)

The model of Ghosh and Lamb (1979) is only valid as long as the magnetospheric boundary lies outside the stellar surface ($R_\ast = 10^6$ cm):

$$R_\ast < 1.91 \times 10^8 B_8^{-5/7} m_8^{-3/7} M^{-1/7} \text{ (cm)}$$  \hspace{1cm} (7)

When the magnetic field is very weak this condition can be violated. In that case we use a simple boundary layer model in which the torque is given by

$$N = j_k(R_\ast) \sin \theta$$  \hspace{1cm} (8)

where $R_\ast$ is the stellar radius. This yields

$$\dot{P} = -4.350 \times 10^{-5} P^2 m_8 \text{ (syr} \text{-}^{-1} \text{)}$$  \hspace{1cm} (9)

We impose a lower limit of 1.5 ms on the spin period, following the work of Papaloizou and Pringle (1978), Harding (1982), and Shapiro et al. (1982), who showed that at this period a neutron star becomes unstable to non-radial oscillations which effectively radiate away spin angular momentum in the form of gravitational waves.

From the above it can be seen that the evolution of the neutron-star spin period is completely determined once the mass-accretion rate and the magnetic field of the neutron star are known as a function of time.

The mass-transfer rate in a binary system can, in general, only be obtained from detailed evolutionary calculations involving the response of the secondary star to mass loss. However, for the wide low-mass X-ray binaries the structure of the mass-losing star (which slowly ascends the giant branch) can easily be parametrized, and the evolution of these binaries followed semi-analytically. These stars have the property that, for a given chemical composition, their radius and luminosity are, to good approximation, functions of the core mass only. As the core mass of the giant increases due to nuclear evolution, the radius of the giant also increases until it fills its Roche-lobe and mass transfer to the compact companion commences. Since the mass transfer takes place from the less massive to the more massive star, it will cause the orbit to widen on a dynamical timescale until the radius dictated by the core mass of the giant can again be accommodated within the Roche-lobe of the giant. Because this adjustment can take place on a much shorter timescale than that on which the radius of the giant changes, the mass-transfer rate will adjust itself to a value at which the rate of increase in radius of the Roche-lobe is just equal to the evolutionary expansion rate of the giant. Hence, from the equations describing the radius change of the giant and the changes of the Roche lobe due to mass transfer the transfer rate can be solved. This method was first proposed by Webbink et al. (1983); the details of the method used in the present paper have been described by de Kool et al. (1986).

The time evolution of the magnetic field of a neutron star is not yet well understood, and one of the purposes of this work is to make an attempt to put constraints on this evolution. From observations of single radio pulsars it appears that neutron stars are formed with a magnetic field of $\sim 10^{12}$ to $10^{13}$ Gauss, which initially decays on a timescale of $\sim 5 \times 10^5$ yr (Gunn and Ostriker 1970; Lyne et al., 1985). However, there is recent evidence, that this decay does not continue indefinitely but stops (or proceeds at a much longer time scale) at a value of order $10^6$ Gauss (Kulkarni, 1986; Van den Heuvel et al., 1986; Bhattacharya and Sinha, 1986). To parametrize this behaviour we have assumed that a neutron star forms with a magnetic field of $10^{12}$ Gauss, which decays exponentially on a timescale $\tau_B$, between $10^6$ and $10^7$ yr until it reaches a transition field strength $B_T (10^5$ to $10^6$ Gauss). After $B_T$ is reached the field only decays on a much longer timescale $\tau_{B2}$ ($10^8$ to $10^{10}$ yr).

3. Description of the models

The starting point of the evolutionary history of the wide low-mass X-ray binary is a system consisting of a low-mass star ($\sim 1 M_{\odot}$) and a massive white dwarf ($1.2$ to $1.4 M_{\odot}$) companion, with an orbital period between 0.5 and 50 days. As the secondary leaves the main sequence its radius will increase until it fills the critical potential lobe and mass transfer starts. While the system is slowly widening due to the mass transfer the mass of the white dwarf increases until it reaches the Chandrasekhar limit and col-
lapse to a neutron star follows. During this collapse at least 0.1 \( M_\odot \) is lost from the system (the change in binding energy of the primary), and the binary will temporarily detach. The nuclear evolution of the secondary will continue and after some time it will again fill its Roche lobe and resume transfer of mass. It is at this point that our calculations start.

Our models begin with a 1.3 \( M_\odot \) neutron star and a 0.8 \( M_\odot \) companion with heavy element abundance \( Z = 0.02 \) (see discussion). Through the condition that the secondary fills its Roche-lobe the orbital period is determined by the radius of the secondary, i.e. by how far it has evolved up the giant branch. It is assumed that the surface magnetic field when the neutron star formed \( (B_0) \) was \( 10^{12} \) Gauss, and that it decayed (according to the above description) during the detached phase. The length of this phase is calculated under the assumption that 0.1 \( M_\odot \) is lost from the system during the supernova explosion. The initial neutron star spin period was taken to be 1 second, because young pulsars which are born with a much shorter period are probably spun down on a very short timescale (a few times \( 10^3 \) yr) after their formation. The exact value of this initial spin period has very little influence on the results since as soon as mass transfer starts the period quickly evolves towards the equilibrium period.

The calculations were done for a range of initial orbital periods and for different parameters describing the magnetic field decay.

The results of the calculations are best understood by considering the relative magnitude of the following timescales:

\[ \tau_{\text{in}}, \tau_{\text{sp}}, \tau_{\text{d}}, \tau_{\text{t}}, \tau_{\text{a}}, \tau_{\text{b}}, \tau_{\text{c}} \]  

- \( \tau_{\text{in}} \) - the duration of the mass transfer phase;
- \( \tau_{\text{sp}} \) - the spin-down time scale, \( \tau_{\text{sp}}/\dot{Q} \) for \( \dot{Q} = 0 \) (see Eq. 6);
- \( \tau_{\text{d}} \) - the initial \( B \)-field decay timescale;
- \( \tau_{\text{b}} \) - the second \( B \)-field decay timescale;
- \( \tau_{\text{t}} \) - the time needed to reach \( B_r \).

The magnitudes of \( \tau_{\text{in}} \) and \( \tau_{\text{sp}} \) are determined by the initial orbital period chosen: a shorter period implies a less massive core, and hence much more slowly evolving giant. The magnitude of \( \tau_{\text{sp}} \) is determined by the mass accretion rate and the magnetic field strength; the other time scales are essentially free parameters.

We shall discuss one model in detail to illustrate the types of spin-up behaviour that can be encountered. The system considered started to transfer mass again after the supernova at an orbital period of 9.7 days, which yields a \( \tau_{\text{in}} = 1.0 \times 10^4 \) yr and a \( \tau_{\text{sp}} = 7.0 \times 10^3 \) yr. The average mass transfer rate in such a system is \( 4.5 \times 10^{-9} M_\odot/\text{yr} \). The magnetic field decay parameters used are \( \tau_{\text{b}} = 2.1 \times 10^7 \) yr, \( B_0 = 5 \times 10^8 \) Gauss and \( \tau_{\text{in}} = 10^{10} \) yr. In Fig. 1 the spin period and the fastness parameter \( \dot{Q} \) are given as a function of time elapsed since the supernova explosion. Because \( \tau_{\text{b}} \) is the same order as \( \tau_{\text{in}} \), the magnetic field has not yet decayed very much during the detached phase, and when mass transfer starts the neutron star is spun up to its equilibrium period of \( \sim 60 \) milliseconds on the spin-up time scale \( \tau_{\text{in}} \), which is then of order \( 10^4 \) yr. For some time the star remains near its equilibrium period, which is decreasing on a timescale \( \tau_{\text{sp}} \) due to the decay of the magnetic field (the mass accretion rate is almost constant). At \( 8 \times 10^6 \) yr the period goes out of equilibrium again. This is caused by the fact that the very small accretion torque on the star when it is close to equilibrium is no longer able to spin it up sufficiently to follow the change in the equilibrium period due to the magnetic field decay. In terms of time scales this means that \( \tau_{\text{in}} \) has become comparable to, or larger than, \( \tau_{\text{sp}} \). The fastness parameter deviates more and more from its equilibrium value, and if the rapid field decay would not stop this trend would continue indefinitely since \( \tau_{\text{sp}} \) only increases (Eq. 6). The further evolution in this case is shown by the dotted lines in Fig. 1. The dynamical importance of the magnetic field is quickly reduced, and the magnetospheric boundary is pushed to the stellar surface, at which point boundary layer accretion determines the spin-up rate. In this way the minimum period of 1.5 ms is reached in about \( 3 \times 10^7 \) yr. If however the magnetic field decay time scale changes from \( \tau_{\text{b}} \) to the much longer \( \tau_{\text{in}} \) at \( \tau_t \) we again enter the regime where \( \tau_{\text{sp}} < \tau_{\text{b}} \), the timescale on which the equilibrium period is changing. The fastness parameter increases again and has reached the equilibrium value at \( 3 \times 10^7 \) yr. When mass transfer stops the spin period of the neutron star is 2.21 ms, slightly above the minimum period.

4. Discussion

If a prescription is given for the evolution of the magnetic field of the neutron star our calculations yield a definite prediction of the relation between orbital period and spin period of the neutron star when mass transfer stops. It is very plausible (Savanije, 1983; Paczynski, 1983; Joss and Rappaport, 1983) that the end products of this type of evolution are observed as wide binary radio pulsars. A comparison of the theoretical relation between spin period and orbital period at the end of the mass transfer phase with the observed relation for binary radio pulsars may therefore provide constraints on the magnetic field decay. In Fig. 2 we have plotted the predicted relations for two different values of \( B_0 \), and for three values of \( \tau_{\text{b}} \). The dashed lines represent the results in the case where it was assumed that the rapid field decay continues indefinitely (\( \tau_{\text{sp}} > \tau_{\text{b}} \)). It is seen that in this case all neutron stars in binaries that end with orbital periods <63 days are spun up to the minimum period of 1.5 ms, mainly by boundary layer accretion. Only for systems that end with orbital periods

![Fig. 1. The neutron star spin period and the fastness as a function of time elapsed since the formation of the neutron star. The parameters describing the magnetic field decay in this particular model were \( B_0 = 10^{12} \) Gauss, \( B_0 = 5 \times 10^8 \) Gauss, \( \tau_{\text{in}} = 2 \times 10^6 \) yr and \( \tau_{\text{b}} = 10^{10} \) yr. The initial values of the orbital period, neutron star mass and giant mass were 9.7 days, 1.3 \( M_\odot \) and 0.8 \( M_\odot \) respectively.](image-url)
corresponding to a field of 5/10^8 Gauss is equal to the equilibrium period at this moment. The equal mass system thus does not affect the outcome, since for these change since we have R(t) < R. For the shorter period systems the situation now changes since we have R(t) > R, and R(t) < R, which means that they will be able to remain spinning at the equilibrium period corresponding to a field of 5/10^8 Gauss until the mass transfer stops. The increase in spin period to shorter orbital periods reflects the difference in accretion rates for these systems.

The sensitivity of these results to the input parameters is illustrated in Fig. 3. Here we have plotted the results for a standard case (B_0 = 10^12 Gauss, R(t) = 5/10^9 yr, R(t) = 5/10^9 yr, B_0 = 5/10^9 Gauss, M_(giant) = 0.8 M_☉), and also the results if either B_0, B_0, M_(giant) or R(t), which is a measure of the amount of mass lost in the supernova explosion, is varied. It is clear that the results of the calculations are not very sensitive to variations of M_(giant) or R(t) within a reasonable range. If a larger initial value of the magnetic field at the formation of the neutron star is chosen the expected spin period for the long period systems increases, since the magnetic field, and hence the equilibrium period, are larger when mass transfer stops. The short period systems are affected in the same way by an increase in the value of B_0. Also plotted is one example in which the giant has an extreme Population II composition. These giants have smaller radii than Population I giants and hence the mass transfer stops at a shorter orbital period (for the same core mass). For shorter orbital periods the relation between spin and orbital period is hardly affected by the composition.

There are three wide binary radio pulsars with a low mass companion that might have followed the evolutionary scenario we model. These are (see e.g. Taylor and Stinebring, 1986):

>63 days is R(t) short enough that there still can be a noticeable field left when the mass transfer stops, in which case the spin period is equal to the equilibrium period at this moment. The solid lines in Fig. 2 represent the results if we assume that B_0 equals 5/10^8 Gauss, and R(t) = 10^9 yr, which is almost identical to assuming a bottom value of the field at B_0. For the longer period system this does not affect the outcome, since for these change since we have R(t) < R. For the shorter period systems the situation now changes since we have R(t) > R, and R(t) < R, which means that they will be able to remain spinning at the equilibrium period corresponding to a field of 5/10^8 Gauss until the mass transfer stops. The increase in spin period to shorter orbital periods reflects the difference in accretion rates for these systems.

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There are three wide binary radio pulsars with a low mass companion that might have followed the evolutionary scenario we model. These are (see e.g. Taylor and Stinebring, 1986):

PSR 1855+09 (P_{spin} = 5.3 ms, P_{orb} = 12.3 days, B_0 = 4/10^8 G (Taylor, private comm.)), PSR 1953+29 (P_{spin} = 0.6 ms, P_{orb} = 117 days, B_0 = 4/10^8 G) and PSR 0820+02 (P_{spin} = 865 ms, P_{orb} = 1232 days, B_0 = 3.3/10^11 G).

In our calculations we did not consider systems with orbital periods like in PSR 0820+02, because in these long period binaries the giant starts to transfer mass when it is already very evolved, in which case very high (10^{-6} to 10^{-7} M_☉/yr) mass transfer rates are obtained. These rates are highly super-Eddington, and we can not guess how this will affect the spin-up mechanism. Nevertheless, the evolutionary scenario is very well possible, and the fast evolution expected in these systems is in good agreement with both the high magnetic field strength and the small age of the white dwarf companion observed in this system (Kulkarni 1986).

The position of PSR 1855+09 in the diagram is in reasonable agreement with the model that has a bottom in the magnetic field decay. The spin period is very close to the equilibrium period given its observed magnetic field and the mass transfer rate from our calculations. Since the spin period will decrease by the normal pulsar spin-down mechanism after the mass transfer stops, the fact that the spin period is so close to the expected value at the end of the mass transfer phase implies that this phase must have ended less than one spin-down timescale (210^9 yr) ago. The duration of the mass transfer phase for a system at this orbital period (t_{trans} ~ 10^9 yr) is long compared to the time the magnetic field needed to decay to its present value (<10^9 yr). This means that the assumption that the magnetic field decay slows down at about 10^9 G can only be avoided if the neutron star formed during the last few percent of the mass transfer phase, and we are now observing it not more than a few times t_{trans} after mass transfer stopped. This problem becomes even worse when it is taken into account that after the collapse of the white dwarf there must have been a detached time which lasted for a few t_{trans}. Although this special set of circumstances can not be definitely

![Fig. 2. The neutron star spin period as a function of orbital period at the end of the mass transfer phase, with and without a bottom in the magnetic field decay. Three models with t_{trans} = 210^9 yr (curve labeled 1), 510^9 yr (2) and 10^9 yr (3) are shown. All three have t_{trans} = 10^9 yr, B_0 = 10^12 Gauss, M_(giant) = 510^9 G and initial values of the giant and neutron star mass of 0.8 and 1.3 M_☉, respectively.](image1)

![Fig. 3. The dependence of our results on the model parameters. Plotted are the results for a standard case (the same as in Fig. 2 with t_{trans} = 510^9 yr), and the results if one of the following parameters is set to a different value: B_0 = 10^13 Gauss, B_0 = 10^9 Gauss, an initial giant mass of 0.9 M_☉, and t_{trans} twice the standard value. Also shown is the relation in the case that the giant has an extreme Population II composition (Z = 0.0001) with a bottom in the magnetic field.](image2)
excluded, we prefer the explanation that the rapid magnetic field decay slows down at about $10^9$ G. The binary radio pulsar PSR 1935+29 cannot tell us anything about the existence of a bottom in the magnetic field because it is at an orbital period where $\tau_{\nu} < \tau_{\nu}$. It can however yield information about the initial field decay timescale $\tau_{B1}$. Comparing the position of this pulsar in the diagram with the predictions for different values of $\tau_{B1}$ it seems that $6 \times 10^6 \text{yr}$ is a good estimate for the parameters we have assumed. Any value derived for $\tau_{B1}$ in this way is only an upper limit since the pulsar may already have been spinning down. To estimate the best upper limit for $\tau_{B1}$ we have tried to fit the position of PSR 1935+29 with a model in which the other parameters are taken to have extreme values, in such a way that a large value for $\tau_{B1}$ results. These parameters were the lowest initial B-field inferred from radio pulsar observations ($5 \times 10^{14}$ G, see e.g. van den Heuvel 1984), a high initial giant mass (1.0 $M_\odot$) and a long detached time (twice the normal value). To account for the uncertainty in the GL-theory we used a slightly different form for the dimensionless torque $n(\omega)$ which yields an $\omega_{\text{eq}}$ of 0.5 (see GL Fig. 3). The upper limit derived in this way is $1.2 \times 10^7 \text{yr}$.

We conclude that our detailed calculations of neutron star spin-up in LMXRB's offer strong support to the idea that these are the progenitors of the wide low-mass binary millisecond pulsars. If, in turn, we accept this evolutionary scenario, calculations of this type may be a promising way of acquiring more knowledge about the decay of the magnetic field in neutron stars.

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CHAPTER III

EVOLUTIONARY SCENARIOS FOR OBSERVED INTERACTING BINARIES

III.1) The minimum orbital period for ultra-compact binaries with Helium burning secondaries

III.2) An evolutionary scenario for the black-hole binary A0620-00
The minimum orbital period for ultra-compact binaries with helium burning secondaries

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Summary. We present evolutionary calculations of an ultra-compact binary system undergoing mass transfer, consisting initially of a 0.6 $M_\odot$ nondegenerate helium-star accompanied by a 1.3 $M_\odot$ compact star in a tight orbit of 37 minutes. The binary parameters were chosen to model the ultra-compact binary 1E 2259 + 586 which is currently thought to have an orbital period of 38 min. The total system mass and the orbital angular momentum is assumed to be constant during the evolution apart from the (substantial) losses of angular momentum caused by the emission of gravitational waves. These losses cause the orbit to decay until a minimum orbital period of 10.6 min is reached. The thermal structure of the secondary, which by then has a mass of 0.2 $M_\odot$, becomes severely perturbed and causes the star to remain semi-degenerate for a significant fraction of the further evolution the binary system, during which the binary separation increases again. We comment on possible nonconservative effects during the approach of the minimum orbital period when the mass transfer rate becomes super-Eddington.

Key words: close binaries — X-ray binaries — millisecond radio pulsars

1. Introduction

The evolution of binary systems with orbital periods less than about 10 hours is substantially influenced by loss of orbital angular momentum due to the emission of gravitational waves (cf. Kraft et al., 1962; Faulkner, 1971; Taam et al., 1980). The substantial loss of angular momentum in these compact binaries causes an orbital decay that enhances the mass transfer significantly. It has been shown that compact binary systems with mass transfer from a hydrogen burning low-mass main-sequence star towards a collapsed companion of $\sim 1 M_\odot$ cannot reach orbital periods below 70 min for initial compositions with $X = 0.7$, or 50 min when $X \approx 0.2$ (Paczynski and Sienkiewicz, 1981; Rappaport et al., 1981). The minimum orbital period is reached when the mass transfer has reduced the mass of the main sequence star, and thereby the temperature at its centre, to such a degree that the fusion of hydrogen at its centre is extinguished. The secondary then cools down to a degenerate configuration whereby its radius starts to increase with further mass loss. Sufficiently high mass transfer to the more massive primary causes the binary orbit to expand as well, so that the expanding secondary can still be accommodated inside its critical lobe.

A similar phenomenon occurs with low-mass core helium burning helium stars that transfer matter to a compact companion. Because the radius of such helium stars is only roughly 0.7 of that of a core hydrogen burning star of the same mass, the Roche-lobe, and thus the orbital separation, must be correspondingly smaller. This means that binaries of this type undergo much more severe angular momentum losses by gravitational radiation and consequently sustain much higher mass transfer rates. When the compact companion is a neutron star systems like this will appear as extremely bright X-ray sources (Savonije, 1983), unless self-absorption of the (super-Eddington) sources becomes important.

We have studied the evolution of a binary consisting initially of a 0.6 $M_\odot$ helium-star in the phase of core helium burning transferring mass to a companion of 1.3 $M_\odot$. We followed the evolution of the binary through the minimum orbital period, assuming all the transferred mass to be accreted by the primary.

In Sect. 2.1, we present simple analytical expressions for the mass transfer rate due to gravitational radiation and for the resulting orbital changes. The stellar evolution code and input physics is briefly described in Sect. 2.2, followed by a brief summary of the general features of the evolution of low-mass helium-stars. The results of the detailed evolutionary calculations and comparison with observations is presented in Sect. 3. In Sect. 4 we speculate on the possible formation of ultra-compact binary systems, followed by a discussion of our results in Sect. 5.

2. Method of calculation

2.1. Analytical expressions for the mass transfer rate and the orbital evolution

Since we do not know the detailed hydrodynamics of tidal mass transfer in close binaries we will simply assume that all the mass lost by the secondary is captured by the (more massive) primary. We also assume that the secondary rotates synchronously with the (circular) orbital motion and that the spin angular momenta of the stars (including the accretion disc) can be neglected. The only loss of angular momentum is due to the emission of gravitational waves at a rate (Landau and Lifshitz, 1959):

$$j = \frac{32}{5} \frac{G^3}{c^5} M_1 M_2 M a^{-3}$$

(1)
where $M_1$ and $M_2$ are the two stellar masses, $M = M_1 + M_2$, and $a$ is the orbital separation of the binary system. Differentiating with respect to time the expression for the binary angular momentum yields:

$$\frac{\dot{a}}{a} = 2\left(\frac{M_2}{M_1} - 1\right)\frac{M_1}{M_2} + 2\frac{\dot{J}}{J}$$

(2)

where $M_2$ is the mass of the mass transferring secondary. Note that in the absence of angular momentum losses ($\dot{J} = 0$) mass transfer from the secondary would increase the orbital separation. In order to derive a simple analytical expression for the rate of mass transfer in the binary we will in this section approximate the secondary by a simple mass-radius relation of the form:

$$R_2/R_\odot = \alpha(M_2/M_\odot)^\beta$$

(3)

where $\alpha$ and $\beta$ are to be considered as constants derived from a local fit to the mass-radius relation. We assume that the secondary's radius remains exactly equal to the volume averaged radius of the Roche lobe, which for $M_2/M_1 < 0.8$ can be expressed as (Paczynski, 1971a):

$$R_c = \left(\frac{2}{3\sqrt{3}}\right)^{1/3} \left(\frac{M_2}{M}\right)^{1/3}$$

(4)

If we require $R_2 = R_c$ we obtain

$$\frac{\dot{a}}{a} = (\beta - 3) \frac{M_2}{M_1}$$

(5)

Eliminating $\dot{a}/a$ by substituting Eq. (5) and Eq. (2) yields an expression for the mass transfer rate:

$$-\dot{M}_2 = (\beta + 2 - M_2/M_1)^{-1}(M_2/\tau_a)$$

(6)

where $\tau_a = -(\dot{J}/\dot{J})^{-1}$ is the timescale on which the orbit decays by the emission of gravitational waves (Eq. 1). We infer from Eq. (5) that a minimum orbital separation of the binary is reached when $\beta = \frac{5}{3}$. For smaller values of $\beta$ the mass losing secondary can no longer be accommodated in a binary with a decaying orbit. The mass transfer to the more massive companion will then counteract the gravitational radiation induced shrinking of the orbit (Eq. 2) and the binary orbit starts to expand.

### 2.2. Short description of input physics and the starting model

The evolution code originally developed by Eggleton (1971, 1972), was modified to calculate tidal mass transfer and orbital angular momentum loss due to gravitational radiation. The standard set of Cox-Stewart radiative opacities (Cox and Stewart, 1970) was used. The equation of state is described in Eggleton et al. (1979). In the evolutionary calculations it was not simply assumed that the radius of the secondary exactly equals the equivalent Roche-radius during mass transfer, as we did for simplicity in Sect. 2.1. Instead the mass transfer was calculated from a simplified hydrodynamic treatment of the flow through the region about the inner Langrangian point (Savonije, 1979). The Roche-radius of the secondary was calculated from the approximation by Eggleton (1983), which gives a continuous fit better than 1% over the whole range of mass ratios.

In constructing an evolutionary sequence an initial orbital separation has to be chosen, which determines the time (and hence the evolutionary status of the helium-star) when mass transfer starts. At this point it is useful to briefly summarize the evolution of single low-mass helium-stars. During the first phase of core helium burning, helium stars expand slightly (e.g. Paczynski, 1971b). Low-mass helium stars start to contract when the core helium content approaches a value of about $\sim 0.2$. After the end of core helium burning the stars re-expand, but for stars less massive than about $0.8M_\odot$ the re-expansion is only moderate. For these low-mass stars the stellar radius does not, or only marginally, surpass the maximum radius attained during core helium burning. As soon as the stars have reached a maximum size a rapid shrinking sets in towards a compact degenerate configuration. Binaries in which the low-mass helium star secondary does not come into contact during core helium burning can therefore only begin a (long lasting) phase of mass transfer when, after a long period of time, the gravitational radiation losses have reduced the orbital separation sufficiently for the degenerate helium-star to fill its Roche lobe. Mass transfer from a degenerate star can be calculated fairly well from Eq. (6) with appropriate values for $\beta$ and $\alpha$ (e.g. Li et al., 1980). Here we will study the case where contact is reached during the core helium burning phase, and a simple analytical calculation is not possible.

To this end a $0.6M_\odot$ helium-star with initially a homogeneous composition of $Y = 0.98$, $Z = 0.02$ and a radius of 0.124$R_\odot$ was evolved until the helium abundance in the core was reduced to 0.26. At that time the star had a convective core of $0.117M_\odot$ and an adjacent semi-convective region with reduced convective mixing of 0.06$M_\odot$. By adopting an orbital period of 36.6 min the star (with a radius of $R = 0.142R_\odot$) suddenly commenced Roche-lobe overflow whereby its evolutionary expansion came to a halt.

### 3. Results

#### 3.1. Results of the evolutionary calculations

The evolution of the compact semi-detached binary under study can be qualitatively described in terms of three timescales:

(i) the evolutionary timescale $\tau_\text{ev}$ defined here as the time elapsed since the beginning of helium burning in the core of the secondary;

(ii) the Kelvin-Helmholtz or thermal timescale $\tau_\text{KH}$ of the secondary defined as $\tau_\text{KH} = (GM^2/R_\odot)^{1/2}$;

(iii) the timescale for decay of the orbit by gravitational radiation $\tau_\text{gr}$, which can be expressed as (Eq. 1): $\tau_\text{gr} = 1.2 \times 10^{10} (M_1M_2/M)^{1/2}$ where the time is in years and the binary parameters are in solar units.

Figure 1 shows these three timescales as a function of decreasing secondary mass during the evolution. The first phase of binary evolution is characterized by $\tau_\text{KH} < \tau_\text{KH} < \tau_\text{ev}$. This means that the orbit initially decays on a timescale $\tau_\text{KH}$ (much) longer than the timescale on which the secondary relaxes from the thermal perturbations caused by the mass transfer. The secondary remains thus in thermal equilibrium, although it loses mass at a considerable rate. We can estimate the rate of mass transfer from Eq. (6) by taking $\beta = 1$ and $M_2/M_1 = \frac{1}{3}$ as $\dot{M} \sim M_2/\tau_\text{gr}$, which amounts to $\sim 3 \times 10^{-8}$ $M_\odot$/yr, consistent with the results of the detailed evolutionary calculations shown in Fig. 2. Because $\tau_\text{gr} > \tau_\text{ev}$ nuclear evolution is only marginally important. Figure 1 shows that $\tau_\text{ev}$ drops rapidly as a result of the rapid decay of the orbit caused by the emission of gravitational radiation. As
the secondary becomes less massive its luminosity drops and \( \tau_{KH} \) increases steadily. When \( M_2 \sim 0.36 \, M_\odot \), the gravitational decay timescale \( \tau_g \) and the thermal timescale \( \tau_{KH} \) have become comparable \( \tau_g \approx \tau_{KH} \). This occurs \( 12 \times 10^6 \, \text{yr} \) after the onset of mass transfer. It is roughly from this moment on that the mass transfer begins to drive the secondary out of thermal equilibrium. The mass-loss induced motions in the sub-adiabatic envelope cause a substantial absorption of energy so that the luminosity of the secondary drops quickly and the star departs more and more from thermal equilibrium. When the secondary's mass is reduced to \( 0.35 \, M_\odot \), only 70\% of the core luminosity (3.23 \( L_\odot \)) reaches the surface. When the mass of the secondary drops below 0.28 \( M_\odot \) (13 \( 10^6 \) yr after the beginning of mass transfer) helium burning begins to fade. The convective core disappears and is replaced by an outer convection zone that penetrates inwards. At this stage the rate of mass transfer shows a little dip (Fig. 2). The reduced stellar luminosity (Fig. 3) causes \( \tau_{KH} \) to rise even faster and the thermal structure of the secondary is severely perturbed. The core luminosity is by now reduced to 1.58 \( L_\odot \), of which only 0.14 \( L_\odot \) can escape from the star.

From Fig. 4, which shows the secondary's evolution in the central density/central temperature plane, it can be inferred that the electron gas in the stellar interior becomes mildly degenerate when the secondary's mass decreases below 0.3 \( M_\odot \). After the nuclear energy source has disappeared the central temperature drops faster. But, because the mass transfer dominates the thermal evolution of the star \( (\tau_g < \tau_{KH}) \), the central density decreases as well, causing the interior of the secondary to remain only mildly degenerate during the further evolution. Figure 5 depicts the evolution of the secondary's radius as a function of its diminishing mass. We have indicated 4 points on the curve that correspond with special features in the other figures.

In point 1 the thermal and gravitational timescales become equal \( (\tau_g = \tau_{KH}) \). This point roughly marks the beginning of thermal imbalance and underluminosity of the mass losing secondary.

Point 2 corresponds to the little dip in the rate of mass transfer just before the steep rise to maximum (Fig. 2). Near this point the convective core disappears and is replaced by an inward penetrating convective envelope. The pronounced rise of the mass transfer rate corresponds to the steepening of the \( \tau_{g} \)-curve (Fig. 1) for \( M_2 < 0.3 \, M_\odot \). The secondary can only adhere to the shrinking

![Fig. 3](https://example.com/fig3.png)

**Fig. 3.** The secondary's evolutionary path in the HR-diagram. The diminishing stellar mass is indicated in solar units.

![Fig. 4](https://example.com/fig4.png)

**Fig. 4.** The secondary's evolutionary path in the central density versus central temperature plane. The diminishing stellar mass is again indicated in solar units. The straight line indicates where the fully degenerate electron-gas pressure equals the extrapolated Maxwell-Boltzmann pressure.

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Roche limit by strongly augmenting its mass loss. This results in a significantly steeper shrinking of the radius with decreasing mass (between points 2 and 3 in Fig. 5) caused by the development of a severe thermal imbalance and underluminosity of the secondary which has a predominantly sub-adiabatic temperature structure. The heavy mass transfer to the more massive primary opposes the orbital decay and shrinking of the Roche lobe. As a result of the flattening of the secondary's mass-radius relation near point 3 \( (\tau_2 \propto R_2^2 M_2^{-0.5} \text{ when } M_2 < M) \) the orbital angular momentum losses start to decrease even before the turning point for the orbital separation is reached.

At point 3 in Fig. 5 \( \tau_2 \) reaches a minimum value of \( 3 \times 10^9 \) yr. By now the interior of the secondary has become mildly degenerate (Fig. 4) and the secondary's radius appears insensitive to mass loss.

Therefore, although the orbital decay is already abating, the secondary continues to increase its mass loss rate in order to remain within the limits of its Roche lobe. But gradually the widening effect of the mass transfer becomes preponderant, and the orbital decay decelerates ever more (Fig. 6).

At point 4, corresponding to \( M_2 = 0.21 \, M_\odot \), the dominating effect of gravitational radiation losses are finally overcome and the orbit begins to expand. Thereby the rate of mass transfer passes a peak value of \( 9 \times 10^{-8} \, M_\odot/\text{yr} \). Simultaneously the orbital period attains a minimum value of \( 10.6 \, \text{min} \) (Fig. 6).

After this point the increasing orbital separation causes \( \tau_2 \propto a^3 \) to increase even faster so that the mass transfer abates strongly, as can be observed in Fig. 2. When ultimately, for \( M_2 = 0.16 \, M_\odot \), the nuclear energy source at the centre is completely extinguished, the stellar radius reaches a minimum value of \( 0.0422 \, R_\odot \). Further mass loss from the semi-degenerate secondary leads to a larger stellar radius. The stellar luminosity has become extremely small: \( L \approx 6 \times 10^{-8} \, L_\odot \).

We followed the evolution of the binary until the secondary's mass was \( 0.11 \, M_\odot \), approximately the convective core mass at the beginning of mass transfer. This implies that the surface abundance of the remnant has become similar to that of the initial convective core, i.e., \( Y_c = 0.26 \). Nuclear burning during the mass transfer phase resulted in a central helium abundance of \( Y_c = 0.15 \). The remnant consists therefore primarily of carbon with some admixture of oxygen and helium.

The coefficients of the mass-radius relation (Eq. 3) of the semi-degenerate remnant are \( \alpha = 0.029 \) and \( \beta = -0.19 \) which differs substantially from the fully-degenerate values \( \alpha = 0.013 \) and \( \beta = -1.5 \). Because of the extreme under-luminosity of the remnant (caused by the severe mass loss) its thermal timescale has meanwhile become much longer than a Hubble-time. The remnant is thus expected to remain semi-degenerate for a substantial part of its further lifetime. One can easily verify that for \( M_2 < M \) the rate of gravitational radiation driven tidal transfer in a binary with a given orbital period obeys a proportionality relation of the form:

\[
\frac{\dot{M}}{M} \propto M^{2/3} \left( \frac{M_1^2}{M_2^2} \right)
\]

For a given total system mass and orbital period the rate of mass transfer from the semi-degenerate remnant is therefore a factor 0.6 smaller than the rate from a fully degenerate star of the same mass and composition.

### 3.2. Schematic description of the further evolution and a possible explanation for the single millisecond radio-pulsars

We have for simplicity neglected mass loss and losses of orbital angular momentum other than by gravitational radiation. However, these “conservative” assumptions may become invalid approximations when the rate of mass transfer rises significantly above the Eddington limit, as happens when the secondary's mass decreases below 0.25 \( M_\odot \).

Unfortunately, the non-conservative processes referred to are very difficult to analyse and we can do no better here than discuss two extreme possibilities for the further evolution of the binary system. Let us first discuss the “conservative” approximation, and consider the fate of the secondary after the binary has passed the minimum orbital period. As long as the secondary expands in reaction to mass loss the orbit must expand as well in order to accommodate the contact star inside its Roche lobe (Eq. 4). The gravitational radiation timescale \( \tau_2 \) increases steadily, with a corresponding slow-down of the binary evolution and mass transfer rate.

After a prolonged phase of slow mass transfer the secondary will eventually reach a critical mass below which electrostatic forces can no longer balance the gravitational forces and the secondary will collapse into a neutron star.
forces between the atomic particles become larger than the bulk gravitational forces, and matter is transformed to a solid state. The mass density then becomes independent of the remnant mass, so that \( R \sim \frac{M}{M_{\odot}}^{1/3} \). This implies that the remnant must attain a maximum size, after which it will again shrink in reaction to further mass loss. The maximum radius of spherical zero temperature structures was calculated by Zapolsky and Salpeter (1969) for various chemical compositions. They found for pure carbon configurations a maximum radius of \( R \approx 0.039 R_{\odot} \) attained when \( M \approx 0.0022 M_{\odot} \). Although tidal interaction becomes ineffective for extreme mass ratios, let us for simplicity adopt a simple Roche geometry to estimate the orbital dimensions of the binary at the time the secondary reaches this maximum radius. We obtain an orbital separation \( \approx 0.8 R_{\odot} \) corresponding to a binary period of \( \sim 100 \) minutes and a gravitational radiation timescale \( \tau_g = 6 \times 10^{10} \) yr. This estimate seems a lower limit, since it takes much more time (Sect. 3.1) for the secondary to cool down, so that it is expected to become larger than the zero temperature estimate of Zapolsky and Salpeter. We conclude that binary systems of the type discussed cannot evolve to this stage within the estimated lifetime of our galaxy.

Let us now turn to a discussion of possible non-conservative effects on the evolution of ultra-compact binaries. Ruderman and Shaham (1983) have argued that a tidal instability with runaway mass transfer sets in before the secondary can actually reach a solid state. An argument for this is that the interaction between the secondary and the outer edge of the accretion disc centered on the primary is rendered inefficient when the mass ratio becomes extremely small. Tidal conversion of disc angular momentum into orbital angular momentum is thought crucial for limiting the size of the accretion disc (Papaloizou and Pringle, 1977). When the tidal coupling between the outer layers of the accretion disc and the secondary becomes weak the disc will expand until the disc matter becomes unbound at the edge and is ejected (Lin and Papaloizou, 1979). Ruderman and Shaham (1983) now assume that this ejected matter carries off so much angular momentum from the binary that a tidal runaway mass transfer, whereby the secondary is completely disrupted, becomes unavoidable. The possibility of such instability was further studied by Hut and Paczynski (1984), Ruderman and Shaham (1985), by Bonsema and van den Heuvel (1985) and by Taam and Wade (1985), who conclude that the instability requires such extreme mass ratios that it is unlikely to occur within a Hubble time.

The matter is of interest in view of the existence of single millisecond radio-pulsars (cf. Backer et al., 1982). These rapidly rotating radio-pulsars are thought to have been spun-up by accretion from a binary companion which has meanwhile disappeared as a result of tidal break-up of the degenerate low-mass secondary as discussed above. However, a more promising possibility to dispose of the secondary seems a tidal break-up triggered by non-conservative effects during the steep rise of the mass transfer from a helium-star secondary near minimum orbital period, as discussed in the previous section. During this phase the secondary hardly shrinks in reaction to mass loss as it turns to become semi-degenerate. The secondary can only be kept inside its Roche lobe by the stabilizing effect of the heavy mass transfer to the primary which opposes the rapid decay of the orbit caused by angular momentum losses (\( \tau_g \approx 3 \times 10^4 \) yr) during this very compact binary phase. When a sufficiently large fraction of the transferred mass cannot be captured by the neutron star and is either ejected from the binary (taking away orbital angular momentum) or re-captured by the secondary, the stabilizing effect of mass transfer may vanish and give way to a tidal break-up of the secondary. The tidal dispersion of the secondary would presumably result in a large (not tidally truncated) accretion disc of remnant material around the neutron star that may continue to spin the neutron star up to the required millisecond rotation period. If this extreme non-conservative scenario is real the evolution of low-mass helium star secondaries could produce a class of very bright X-ray sources (compared with Webbink et al., 1983) that evolve into single millisecond radio pulsars.

### 3.3. Comparison with observations

At present five systems are thought to be ultra-compact binaries with orbital periods less than an hour. These systems are respectively the X-ray burster 4U1916-05 with \( P_s = 50 \) min (Walter et al., 1982 and White and Swank, 1982), the cataclysmic variable GP Com with \( P_s = 46 \) minutes (Nather et al., 1981), the 7.7 second X-ray pulsar 4U1626-67 with \( P_s = 41 \) min (Middleditch et al., 1981), the 7 second X-ray pulsar 1E 2259 + 586 with an orbital period of 38 min (Gregory and Fahlgren, 1984) and, finally, PG 1346 + 082 a faint blue object which shows rapid flickering and regular optical variations (\( P \approx 25^\circ \)) (Nather et al., 1983). The existence of a companion to the compact star in these systems is inferred from the presence of accretion. Since this is only indirect evidence, we always have to keep in mind that the interpretation of observed periodicities as orbital is not unique.

The system 1E 2259 + 586 seems very interesting in that it is situated at the geometric centre of curvature of a semi-circular shell of diffuse X-ray emission (Gregory and Fahlgren, 1980, 1981) which has been identified as a \( \sim 10^4 \) yr old supernova remnant at an estimated distance of 3.6 \( \pm \) 0.4 kpc. The compactness of the binary system strongly suggests that the 7s pulsar was created by a mass accretion induced collapse of a heavy white dwarf (Canal et al., 1980; Miyaji et al., 1980; Nomoto, 1980; Labay et al., 1983 and Lipunov and Postnov, 1985). The X-ray source has a very faint optical counterpart (\( B \approx 23.5 \)) that certainly rules out a massive star companion. From pulsar delay timing Gregory and Fahlgren (1984) (GF) determined a best Keplerian orbit fit, represented by the mass function

\[
\frac{M_s^2 \sin^2 i}{(M_1 + M_2)^2} = 0.008 \pm 0.0002 \, M_{\odot}
\]

with \( e \approx 0.3 \). Adoption of these parameters implies that the mass of the secondary \( M_2 > 0.2 \, M_{\odot} \) for a standard pulsar mass of about \( 1 \, M_{\odot} \). As GF mentioned, the short orbital period (\( P_s = 38^\circ \)) excludes a normal main-sequence companion to the pulsar since such a star would be too large and severely overfill its critical lobe (as far as this can be defined for \( e \neq 0 \)). A degenerate star would, even for the minimum mass of \( 0.2 \, M_{\odot} \), underfill its critical lobe by a large factor and presumably transfer no matter to the pulsar at all, even at periastron passage. The pre-supernova system, containing a heavy white dwarf, must have been even more massive and compact than the currently observed system. GF estimated the original orbit to be some 30\(^\circ\), smaller than observed now, which corresponds to an orbital period of \( \sim 22 \) min. However, as observed in Fig. 6, a non-degenerate helium-star secondary of mass \( M_2 \approx 0.37 \, M_{\odot} \) would fit very well into this binary system.
Our calculations show (Fig. 3) that a helium-star secondary of 0.37 $M_\odot$ has a luminosity $L = 4.4 L_\odot$ and $T_{\text{eff}} = 2.9 \times 10^4$ K. Adopting the distance of 3.6 kpc to the SN-remnant and an absorption in the blue $A_{\text{B}} \sim 5.0$ (Fahlman et al., 1982) we obtain by applying a bolometric correction corresponding to $T_{\text{eff}} = 29000$ K an apparent magnitude $B \sim 23.7$ consistent with the observed optical counterpart. At present, after the supernova explosion, the secondary can probably only transfer mass to the pulsar when it is close to periastron. This would be consistent with the relatively low X-ray luminosity $L_x \sim 2.10^{33}$ erg/s (in the band 0.5–4.0 keV) currently observed. For the model parameters adopted the present timescale for orbital decay would be $t_\text{r} \sim 6 \times 10^7$ yr, so that a 30% shrinkage of the orbit would roughly take some $10^7$ yr. This is shorter than the time needed to finish helium burning in the core, so that the further evolution of the system is expected to be qualitatively similar to the results of our calculations presented in Sect. 3.1. Apart from PG 1346 + 082 (which together with GP Com presumably has a white dwarf primary) the other ultra-compact systems have orbital periods that are longer, so that a model similar to 1E 2259 + 586 would require more massive and hence more luminous helium burning secondaries. This can be ruled out, however, because the optical counterparts all appear very inconspicuous. Possibly the secondaries in the other ultra-compact systems are (semi-) degenerate and situated in binaries that have already passed the minimum orbital period, without breaking up by tidal effects. In Sect. 3.2 we discussed that such systems evolve to a period maximum during a long phase of weak mass transfer and slow orbital expansion. The potentially attained maximum orbital period ($> 100$ min) is indeed larger than the derived orbital periods of these three ultra-compact binaries. We estimate by extrapolating our mass-radius relation for semi-degenerate stars that it takes the systems less than $10^5$ yr to re-expand to an orbital period of 50 min. For a discussion of this possibility we refer to Rappaport and Joss (1984). We only note here that the observed spectra of GP Com, 4U 1626-67 and PG 1346 + 082 indicate that these systems contain very little or no hydrogen. However, in a discussion of the recurrence properties of the X-ray burster 4U 1916-05 Swank et al. (1984) argue that the secondary in this system cannot be hydrogen-poor.

4. The evolutionary history of compact binary systems

A priori it does not seem very plausible that a white dwarf or neutron star can be accompanied by a mass transferring helium burning secondary because it requires a rather complicated evolutionary history for the binary. Although helium-stars in binaries are the natural result of case B mass transfer they are expected to be fully detached stars because of their small size compared to their immediate progenitors. These progenitors with a hydrogen burning shell source at the base of the hydrogen-rich envelope must have had stellar radii more than an order of magnitude larger.

It is commonly believed that white dwarfs or neutron stars can have low-mass hydrogen burning secondaries and manifest themselves respectively as catalysmic variables or as a special type of low-mass X-ray binaries (e.g.see Van Paradisj, 1983). It is obvious that all these systems must have evolved from originally much wider binaries in order to accommodate the relatively huge progenitor of the degenerate component. The current compact state of the binaries was probably achieved by expelling the envelope of the compact star's progenitor during a tidally induced spiral-in of its main-sequence companion (cf. Paczynski, 1976; Meyer and Meyer-Hofmeister, 1979 and Van den Heuvel, 1983). Helium-star companions to white dwarfs or neutron-stars in ultra-compact binaries require a more complicated interplay between nuclear and spiral-in evolution. We shall try to outline below a possible way of forming compact binaries, in terms of a double spiral-in scenario (e.g. see Webbink 1979, 1984 and Iben and Tutukov, 1984), basing ourselves on the starting model of our calculations and the case of 1E 2259 + 56.

Since the neutron star in 1E 2259 + 56 is thought to be formed in the recent supernova-explosion initiated by accretion-induced collapse of a white dwarf, we assume the pre-supernova configuration consisted of a helium-star and a white dwarf. A helium-star of $\sim 0.6 M_\odot$ is formed as the core of a main sequence star of mass $\sim 5 M_\odot$. The immediate predecessor of the He-star/WD binary was therefore presumably a $5 M_\odot$ main sequence plus WD binary with an orbital separation of $10-100 R_\odot$. Such a binary will start transferring mass while the $5 M_\odot$ star is on its way from the MS to the base of the giant branch (case B mass transfer). Mass is transferred on a thermal time scale, thus at a high transfer rate, which is further increased by the shrinking of the orbit caused by mass transfer from the heavier to the lighter component of the binary. The white dwarf will not be able to accommodate this mass and will try to form a red giant envelope again. Since this envelope obviously will not fit into the white dwarf's Roche-lobe a common envelope will be formed, from which we may expect the He-star/WD binary to emerge. This spiral-in scenario is similar to the one suggested for the formation of cataclysmic variables. During this spiral-in phase the orbital separation has to be sufficiently reduced for the gravitational decay timescale to become comparable to the evolutionary timescale of the He-star. Gravitational radiation losses will then cause the He-star to fill its Roche-lobe before helium depletion in the core.

According to present views the accretion of matter on a white dwarf only produces a neutron star when the white dwarf was already massive from the beginning, being formed as the remaining (O-Ne-mg) core of a star that was just not sufficiently massive to reach the supernova-phase by itself (e.g. Nomoto, 1980). This massive white dwarf thus has to be the product of a star with a MS-mass of $\sim 8 M_\odot$. This consideration suggests that the initial configuration consisted of a $5 M_\odot$ plus $8 M_\odot$ binary in a wide orbit (orbital separation $\sim 10^4$ R_\odot). In the case of a large orbital separation mass transfer will start when the $8 M_\odot$ star is a giant or supergiant with a completely convective envelope. This mass transfer is dynamically unstable and will lead to a common envelope. Because the mass ratio is near 1 and the amount of mass in the envelope is not more than that of the "inner" binary, consisting of the $5 M_\odot$ star and the giant core, the orbit of this "inner" binary will not have to shrink very drastically in order to provide sufficient energy and angular momentum to expel the envelope. After this expulsion we would be left with the $5 M_\odot$ star plus WD binary required to precede the He-star plus WD system.

5. Conclusions

We have shown that ultra-compact binaries with helium burning secondaries and neutron star primaries undergo heavy mass
transfer that peaks at a value of about $10^{-7} M_{\odot}$/yr when a minimum orbital period of about 11 min is approached. At this phase helium burning in the secondary is extinguished and the star reaches a state of severe thermal imbalance where almost all luminosity is absorbed by the mass spilling envelope. For simplicity we have assumed the total system mass to be constant, and the total orbital angular momentum to be conserved apart from gravitational radiation losses. These approximations may not be accurate, particularly when the minimum orbital period is reached where the gravitational radiation losses become substantial. Tidal stability requires that a substantial part of the mass transferred at a super-Eddington rate is actually captured and accreted by the more heavy neutron star. This implies also a sufficiently effective tidal interaction between the accretion disc and the secondary in order to remove the excess Keplerian disc angular momentum. It is quite possible that these conditions are not fulfilled near the period minimum and that the secondary, which by this time hardly shrinks in response to mass loss, cannot be kept inside its critical lobe and begins a catastrophic tidal mass loss. This could destroy the binary system and eventually yield a single millisecond radio pulsar regenerated by accretion induced spin-up.

If, however, the orbital angular momentum of the binary is sufficiently conserved to prevent this catastrophic run of events, the secondary will become semi-degenerate. Because it is very under-luminous it remains far from thermal equilibrium and substantially oversized (by a factor of $\approx 2$ compared to a fully degenerate star of the same mass and composition) for a large fraction of its lifetime. This implies that the use of a simple degenerate mass-radius relation for secondaries in ultra-compact binaries can be misleading.

We note that it seems important to affirm the orbital solution of 1E 2259 + 586 by Gregory and Fahlman as this would strongly suggest that white dwarfs or neutron stars can indeed be accompanied by nondegenerate low-mass helium stars.

References

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AN EVOLUTIONARY SCENARIO FOR THE BLACK-HOLE BINARY A0620-00

Abstract
We present an evolutionary scenario for the black hole binary A0620-00, which starts from an initial configuration consisting of a massive star and a low-mass companion in a very wide orbit. By using the presently observed system parameters and following the evolution backward in time we derive an upper limit to the initial mass of the companion of 2\(M_\odot\), a relation between the present mass of the black hole and the mass of its main-sequence progenitor which for the most likely black hole mass of 7 M_\odot yields a progenitor mass between 27 and 46 M_\odot, and a lower limit on the initial orbital period of 240 days.

INTRODUCTION

The large radial velocity amplitude of 457 ± 8 km/s of the K-dwarf component of the transient binary X-ray source A0620-00 (\(P_{\text{orb}} = 7.75\) hours) indicates an absolute lower mass limit of 3.20 M_\odot for the compact star in this system, strongly suggesting that it is a black hole (McClintock and Remillard 1986, hereafter referred to as MR).

We investigate how this binary may have originated. We show that it is excluded that the black hole was formed by gradual accretion of matter onto a neutron star. Therefore, the only possibility, aside from triple star models (Fabian et al. 1986; Eggleton and Verbunt, 1986) is that the black hole in this system was formed by core collapse of the remnant of an originally very massive star. McClintock and Remillard have suggested that the model which can explain the peculiar combination of a short orbital period and a low companion mass is similar to the one proposed by Van den Heuvel and Habets (1984) (hereafter referred to as HH) for the formation of the black hole X-ray binary LMC X-3. In such a model, which also seems most likely to us, the original binary system started out with a fairly wide orbit and a very large mass ratio, of order 15 or more. When the massive component of the system evolved to the red supergiant stage, it engulfed its low-mass companion which then spiralled in and removed the envelope of the supergiant. The outcome of this process is a very close binary system consisting of the core of the supergiant (composed of helium and possibly heavier elements) together with the low-mass companion. The final collapse of the burned-out core then produced a close binary consisting of a black hole and a dwarf star.
Figure 1 A schematic outline of the evolution of A0620-00.
which, after tidal circularization of the orbit, and orbital decay (for example by gravitational radiation losses) turned into the present system configuration. This evolutionary scenario is illustrated schematically in fig. 1.

We study here, in terms of this model, the constraints which the presently observed system parameters set to the initial system parameters, by taking into account that: (i) the system was not disrupted in the supernova explosion, (ii) the K-dwarf has only slightly evolved away from the Zero Age Main Sequence (MR). We show that these conditions allow one to constrain the original mass of the low-mass star to \(< 2.0 \, M_\odot\)\), the original mass of the massive star to \(> 20 \, M_\odot\) and the original orbital period to \(> 240\) days. Assuming that, in order to produce a black hole, the original mass of the massive star should have been \(> 40 \, M_\odot\) (HH; Schild and Maeder 1985), the original orbital period is constrained to \(> 450\) days.

**THE EVOLUTION AFTER THE SUPERNOVA**

**An upper limit to the initial mass of the secondary star.**

The optical star, which at present is very close to filling its Roche lobe, has a spectrum showing the typical features of a mid-K-dwarf (~K5, not earlier than K2). The short orbital period indeed only allows a K-dwarf, i.e. a main-sequence star which has slightly evolved away from the Zero Age Main Sequence (ZAMS). The latter is required since a star on the ZAMS with a mass of a K2 to K7 dwarf (i.e.: \(< 0.8 \, M_\odot\) underfills its Roche-lobe by ~20% in a binary with \(P = 7.75^h\). The most likely present mass of the K-companion is ~0.7 \(M_\odot\), independent of the mass of the compact star, which, dependent on the orbital inclination, is most likely to range between 4 \(M_\odot\) (3σ lower limit) and 13 \(M_\odot\) (cf. MR).

As pointed out by McClintock and Remillard, the mass of the companion might possibly be as low as 0.25 \(M_\odot\), which is the measured mass of the K2 IV secondary in the old nova GK Per (Watson, King and Osborne 1985). An upper limit on the initial mass of the secondary, i.e. the mass before it started to transfer mass, can be derived from evolutionary considerations. If this initial mass was considerably larger than the present mass, the orbital separation also must have been
larger at that time to accommodate the companion within its Roche lobe, and the binary subsequently evolved to its present state by shrinking of the orbit and transferring mass. This implies that orbital angular momentum must have been lost since, if the mass-losing star is the less massive one, conservation of orbital angular momentum dictates that the orbital separation should increase due to mass transfer. (Even if the companion was initially the more massive one, a short phase of very high mass transfer rates would have followed the onset of mass transfer until the mass ratio was reversed). A reduction in orbital separation accompanied by mass transfer can therefore only be achieved if angular momentum is being removed from the orbit. By analogy to the situation in e.g. the cataclysmic variables the two processes that could be responsible for these angular momentum losses are gravitational radiation (GR) or magnetic braking. If the angular momentum losses are caused by GR an upper limit to the initial companion mass can be derived semi-analytically by comparing $\tau_{\text{ms}}$, the main sequence lifetime of the companion with $\tau_\gamma$, the timescale on which the orbit shrinks due to GR. As long as $\tau_{\text{ms}} > \tau_\gamma$, the evolution can proceed as is required for the system considered: the binary is slowly shrinking and transferring mass due to angular momentum losses by GR until the secondary has nearly disappeared. If however $\tau_{\text{ms}}$ becomes $< \tau_\gamma$ the companion will start to leave the main sequence before the orbit has shrunk appreciably. The star will subsequently expand on a timescale $<< \tau_\gamma$ and the mass transfer rate will increase drastically. The widening of the orbit caused by this sharp increase in mass transfer can not be compensated by GR angular momentum losses, the orbital separation and period will increase again and a wide low-mass giant black hole binary will form (Webbink, Rappaport and Savonije, 1983, Taam 1983). Since this scenario has to be avoided to arrive at the present system, the criterion $\tau_{\text{ms}} > \tau_\gamma$ can be used to derive an upper limit on the initial mass of the companion $M_{\text{IC}}$. To determine a numerical value for this upper limit we have used the fact that if the mass losing star has a simple mass-radius relation of the form

$$R_\star = \alpha M_\star^\beta$$

(for stars in the mass range we consider $\alpha = 1.0 \, R_\odot$, $\beta = 0.6769$, van der Linden, 1982) then the mass transfer rate due to angular momentum losses by GR is given by (Ritter 1980, Savonije 1984):
Fig. 2 The mass of the companion as a function of time for $M_{BH}=7 \, M_{\odot}$ and $M_{IC}=1.5, 1.86$ and $2.0 \, M_{\odot}$ respectively, as calculated by equation (2). When the elapsed time becomes longer than the main sequence lifetime $\tau_{MS}$ of the companion it will evolve to the giant stage in a widening orbit.

\[
\frac{\dot{M}_C}{M_C} = -(5/6 + \beta/2 - M_C/M_{BH})^{-1} \tau_Y^{-1}
\]  

(2)

where $\tau_Y$ is the orbital decay timescale

\[
\tau_Y \equiv \frac{J_{orb}}{J_{GR}} = 1.24 \times 10^9 \left( \frac{M_C M_{BH} (M_C + M_{BH})}{M_{\odot}} \right)^{-1} \left( \frac{a}{R_{\odot}} \right)^4 \text{ (yr)}
\]  

(3)

where $J_{orb}$ is the orbital angular momentum, $J_{GR}$ the angular momentum losses and a the orbital separation. Equation 2 can be integrated numerically for different values of $M_{IC}$ and $M_{BH}$. In figure 2 we show the mass of the companion as a function of time for $M_{BH}=7 \, M_{\odot}$ and $M_{IC}$ equal to $1.5, 1.86$ and $2.0 \, M_{\odot}$ respectively. In the same diagram we have drawn a line which gives the relation between the mass of a star and its main sequence lifetime. The evolutionary track for $M_{IC}=2.0$ crosses the $\tau_{MS}$ curve at $t=10^9 \, \text{yr}$, after which the star will evolve to the giant stage while the orbit is widening. For $M_{IC}=1.5 \, M_{\odot}$ the evolutionary track never crosses the $\tau_{MS}$ curve: the companion never leaves the main sequence and
will transfer practically all of its mass while the orbit is shrinking. From fig.2 it can be seen that $M_{IC}=1.86\,M_\odot$ is the critical mass which separates these two cases. For $M_{BH}=4\,M_\odot$ and $M_{BH}=13\,M_\odot$ the critical initial companion masses are 1.75 and 2.01 $M_\odot$ respectively. These masses are of course strictly upper limits since the main sequence lifetime of a star which was originally more massive and has arrived at its present mass by mass transfer is always shorter than that of a star which evolves at constant mass.

As mentioned above, in making this estimate we did not take into account the possible effects of angular momentum losses by magnetic braking, as these do not lend themselves to analytical estimates. We are presently engaged in doing detailed numerical calculations where these effects are included. The first results indicate that our conclusions remain valid, since the dynamo mechanism responsible for the magnetic activity is not expected to operate in stars which are more massive than the upper limit derived above.

A closer consideration of the evolutionary scenario outlined above also rules out the possibility that the black hole in A0620-00 was formed by accretion induced collapse of a neutron star. If the initial configuration consisted of a main sequence star (of about 2 $M_\odot$) and a 1.4 $M_\odot$ neutron star, the main sequence star would start to transfer mass on a thermal timescale until the mass ratio reverses. In this case the mass transfer rate is strongly super-Eddington, and practically all transferred mass must be lost from the system. After this mass ejection there would be less mass left in the system than is presently observed.

The maximum post-supernova orbital period, as a function of the initial companion mass.

The above considerations show that the present system configuration, with a K-dwarf-like Roche-lobe filling companion, can only be obtained if the following conditions are fulfilled:

(i) the initial companion mass - just after spiral-in - was $< 1.4 - 2.0$ $M_\odot$ (for $M_{BH}$ between 4 $M_\odot$ and 13 $M_\odot$);

(ii) Gravitational radiation already started driving the mass transfer when the companion was still in the core-hydrogen burning phase.

The second condition follows from the fact that if the system immediately after the supernova was so wide that $\tau_Y > \tau_{ev}$, then the
companion would have evolved away from the main sequence to overflow its Roche lobe before gravitational radiation could become effective. In this case the mass transfer would presently be driven by interior nuclear evolution, which will cause a gradually widening of the system. The companion would in that case at present be a post-main-sequence star, a giant or a subgiant - which disagrees with the observations.

Thus, an upper limit to \( P_{\text{post-SN}} \) is set, for any value of \( M_{\text{IC}} \), by the condition that this period must be such that \( \tau_{\gamma} (P_{\text{post-SN}}) \leq \tau_{\text{ev}} \), for \( \tau_{\text{ev}} \leq \tau_{\text{Hubble}} \) and \( \tau_{\gamma} (P_{\text{post-SN}}) \leq \tau_{\text{Hubble}} \) for \( \tau_{\text{ev}} > \tau_{\text{Hubble}} \), respectively. In figure 3 the curve \( P_{\text{contact}}(M_{\text{IC}}) \) indicates the orbital period at which the companion is filling its Roche-lobe on the ZAMS. Notice that this period is independent of \( M_{\text{BH}} \) (see Faulkner 1971). To obtain from the \( P_{\text{contact}} \) values the maximum possible initial orbital periods \( P_{\text{max}}(M_{\text{IC}}) \), we computed the orbital decay by GR-losses backward in time over a time interval \( \text{Min}(\tau_{\text{ev}}, \tau_{\text{Hubble}}) \). This yielded the three \( P_{\text{max}} \) - curves in the figure, for the three values of \( M_{\text{BH}} \). The figure shows that the orbital period of the system, just after the formation of the black hole, cannot have been larger than about one day.

Fig. 3 The orbital period when the companion exactly fills its Roche lobe on the main sequence (\( P_{\text{contact}} \)) and the maximum post supernova orbital period (\( P_{\text{max}} \)) as a function of companion mass. \( P_{\text{max}} \) is calculated by following the decay of the orbit by gravitational radiation backward in time for one main sequence lifetime.
THE SYSTEM CONFIGURATION PRIOR TO THE FORMATION OF THE BLACK HOLE

The evolution after spiral-in: constraints to the Zero Age mass of the progenitor of the black hole.

In the wide system with very small mass ratio which we adopt as initial configuration, spiral-in of the low-mass star into the envelope of the massive star is unavoidable if the massive star becomes a red supergiant. After the Common-Envelope (CE) phase and spiral-in, the system will consist of the helium core of the initially more massive star, together with the low-mass companion, in a very narrow orbit. The lower limit to the post-spiral-in orbital period is set by the condition that the low-mass star filled its Roche lobe in the new system (helium stars have much smaller radii than hydrogen stars, and will therefore not fill their Roche lobe, cf. Arnett 1978).

Massive helium stars ($M > 5 M_\odot$) are thought to be identified with Wolf-Rayet stars (Paczynski 1967; Kippenhahn 1969; Willis 1985). Such stars have very strong stellar winds, with $v = 2000$ km/s and a mass loss rate $> 10^{-5} M_\odot$/yr (cf. Willis 1985). This type of gradual spherically symmetric mass loss will make the orbital period increase, but does not affect the circularity of the orbit (cf. van der Klis 1983). The subsequent supernova explosion of the helium star - leaving the black hole as a remnant - will also make the orbital period increase but makes the orbit eccentric. After tidal circularization of the orbit (assuming conservation of orbital angular momentum, i.e. neglecting the rotational angular momenta of the components), the orbital period will, - relative to the pre-explosion period - have changed by just the same amount as in the case that the explosively ejected mass had been lost gradually by means of a spherically symmetric wind.

Thus, for the change in orbital period from $M_{\text{He}} (P_1)$ down to $M_{\text{BH}} (P_f)$, one can use the same formula (cf. van der Klis 1983):

$$\frac{P_f}{P_1} = \left( \frac{M_{\text{IC}} + M_{\text{He}}}{M_{\text{IC}} + M_{\text{BH}}} \right)^2$$

with the added constraint that no more than $M_{\text{IC}} + M_{\text{BH}}$ can have been lost during the supernova to avoid disruption of the binary.

For a given combination of values of $M_{\text{IC}}$, $M_{\text{He}}$ and $M_{\text{BH}}$ one can derive constraints to the post-SN orbital period from the following considerations:
(i) a lower limit to $P_{\text{post-CE}}$ is set by the constraint that after CE evolution, the low mass star filled its Roche lobe;
(ii) the increase in $P$ from $P_{\text{post-CE}}$ to $P_{\text{Post-SN}}$ is determined by the amount of mass $\Delta M = (M_{\text{He}} - M_{\text{BH}})$ lost by the massive star while evolving from a He-star to a black hole.
(iii) the thus obtained period $P_{\text{post-SN}} (M_{\text{IC}}, M_{\text{BH}}, M_{\text{He}})$ should be lower than the upper limit $P_{\text{max}} (M_{\text{IC}}, M_{\text{BH}})$ to the post-SN orbital period, required for gravitational-radiation trapping, as represented in figure 3.

From these conditions, one derives, for each combination ($M_{\text{IC}}, M_{\text{BH}}$) an upper limit to $M_{\text{He}}$. These upper limits are given by the dashed lines in figure 4. The next step is to convert these $M_{\text{He}}$ values into initial masses of the massive stars when they were on the ZAMS. Taking stellar wind mass loss during the hydrogen-burning stages of these stars into account in the way described by HH, the relation between the main-sequence progenitor mass $M_{\odot}$ and the helium core mass $M_{\text{He}}$, for $X = 0.70$, $Z = 0.02$ is given by

$$M_{\text{He}} = 0.1015 M_{\odot}^{1.285}$$  \hspace{1cm} (5)

(For other chemical compositions slightly different relations are

![Fig. 4](image-url)  

Fig. 4 Upper limits to the masses of the WR and main sequence progenitors of the black hole, as a function of companion mass and for different black hole masses.
obtained, cf. Tutukov et al. 1973). Using relation (5) to derive the maximum initial ZAMS progenitor masses from the maximum He-core masses, the $M_1$-curves in figure 4 are obtained.

The figure shows that for $M_{BH} = 4 M_\odot$, the maximum progenitor mass is rather small, ranging from $< 20 M_\odot$ for $M_{IC} = 2 M_\odot$ to $< 30 M_\odot$ for $M_{IC} = 0.7 M_\odot$. For $M_{BH} = 13 M_\odot$ these ranges change into: $< 50 M_\odot$ for $M_{IC} = 2 M_\odot$, to $< 80 M_\odot$ for $M_{IC} = 0.7 M_\odot$.

Of course, the lower mass limits of $M_{He}$ are equal to $M_{BH}$ in each case. Converting these values into $M_1$-values, one obtains a lower limit to $M_1$ of $17.44 M_\odot$ for $M_{BH} = 4 M_\odot$, $27 M_\odot$ for $M_{BH} = 7 M_\odot$ and $43.7 M_\odot$ for $M_{BH} = 13 M_\odot$.

One thus observes, for example, that for $M_{BH} = 7 M_\odot$, the allowed range of $M_1$-values is $27 M_\odot - 45.7 M_\odot$ for $M_{IC} = 0.7 M_\odot$, and $27 M_\odot - 30 M_\odot$ for $M_{IC} = 2.0 M_\odot$.

Taking a possible range of 4-13 $M_\odot$ for $M_{BH}$, the total allowed range for $M_1$ becomes $17 M_\odot - 80 M_\odot$. These absolute lower and upper limits do not really contribute an interesting result. If, however, a better estimate of $M_{BH}$ could be made (for instance by the study of ellipsoidal variations in an improved light curve, which yields a better estimate of the inclination), the possible values for $M_1$ would be much more restricted.

Further narrowing down of the Zero Age mass range of the progenitor.

The lower mass limits derived in the foregoing section are absolute lower limits derived from the present system parameters of AO620-00. However, stronger lower mass limits for black hole formation have been derived by HH from the presence of neutron stars in some very massive X-ray binaries, and by Schild and Maeder (1985) from the presence of neutron stars in some very young OB associations. The thus derived lower (Zero Age) mass limit for black hole formation is $40 \pm 5 M_\odot$.

Lower limit to the initial orbital period before Common-Envelope evolution.

We use the formulation of the spiral-in evolution as given by Webbink (1984). In this formulation it is assumed that the change in orbital binding energy during the CE-evolution is entirely used for removing the envelope, i.e. equals the binding energy of this envelope. The latter
binding energy can be expressed as

\[ E_{\text{env}} = \frac{G (M_{2f} + M_{2e}) M_{2e}}{\lambda_1 a_1 R_L} \]  \hspace{1cm} (6)

where \( M_{2f} \) and \( M_{2e} \) denote the masses of the core and envelope of the giant, respectively, \( a_1 R_L \) is its radius (= Roche-lobe radius) at the onset of the spiral in, \( a_1 \) is the orbital separation and \( \lambda \) is a factor of order unity, which depends on the stellar density distribution.

Setting expression (6) equal to the change in orbital energy

\[ \frac{GM_1 (M_{2e} + M_{2f})}{2a_1} + \frac{GM_1 M_{2f}}{2a_2} \]  \hspace{1cm} (7)

one obtains for the ratio of final and initial orbital separation:

\[ \frac{a_2}{a_1} = \frac{M_{2f} M_1}{(M_{2f} + M_{2e})} / \left( \frac{2 M_{2e}}{M_1 + \frac{2 M_{2e}}{\lambda_1 R_L}} \right) \]  \hspace{1cm} (8)

where \( M_1 \) is the mass of the low-mass companion.

For a given value of \( M_1 \), one can calculate \( a_2/a_1 \) as a function of the mass \( M_2 \) of the red giant, taking the core mass of the giant (eq. 4) as \( M_{2f} \) and the mass of the envelope at the time of central H-exhaustion

![Graph](image)

**Fig. 5** The minimum initial orbital period, as a function of companion mass and for several choices of the main sequence mass of the black hole progenitor.
as the envelope mass $M_{2e}$ (Chiosi et al., 1978). For $R_L$, we take the value 1, since the giant is much more massive than its companion and is not expected to be corotating, and for $\lambda_1$ we take 0.5, which is a reasonable value if we compare expression (6) with the results of detailed numerical stellar models. In this way, we can derive the pre-spiral-in orbital period. To find the initial orbital period, we correct this for the wind mass loss during the main-sequence evolution of the massive star, and thus arrive at the periods plotted in fig. 5. As the figure shows, this orbital period is not very dependent on the initial mass of the BH-progenitor. For an initial mass of $40 \, M_\odot$, the shortest initial orbital periods are obtained, which range between 240 and 660 days depending on the mass of the companion.

**DISCUSSION.**

The model which we adopt for the formation of the system is, as mentioned above, similar to the one suggested by HH for the origin of LMC X-3, i.e.: a wide and massive binary with a mass ratio very different from unity. Adopting the semi-empirically derived result that, in order to terminate with a black hole, the initial hydrogen-rich stellar mass must have been $> 40 \, M_\odot$, and combining this with our result that $M_{IC} < 2 \, M_\odot$, implies an initial mass ratio of the system $< 0.05$. Observationally, unevolved O-type binaries with such a low mass ratio are not known (Garmany 1979, Garmany et al. 1980). However, the orbital period range investigated observationally so far is rather short (a few months). Moreover, the lower detection limit of the radial velocity variations (15 km/s, due to the large width and variability of line profiles in O-type stars) limited the detection to companions with mass ratios larger than about 0.1. Therefore, no observational indications concerning the presence or absence of companions with mass ratios $< 0.05$ and orbital periods larger than a few months are available. Among the lower main-sequence stars, including the sun, many systems are present with dark companions having orbital periods of several years and mass ratios of $< 0.05$, so we find no a priori reasons to assume that similar companions could not be present among the O-type stars.

Eggleton and Verbunt (1986) have argued against an evolutionary scenario along the lines of HH because i) they could not see how the system managed to remain bound during the supernova, and ii) the spiral-in in the model of HH took place before the massive star becomes
a red supergiant, which makes it difficult to eject the envelope with a companion as light as in A0620-00.

Regarding the second point, indeed we find (§3.3) that the massive star has to become a red supergiant before the common envelope phase. It is still an open question whether very massive stars become red supergiants, or proceed directly to the Wolf-Rayet (WR) phase by losing their entire envelope through a massive stellar wind while they are blue supergiants. In this respect it may be significant that A0620-00 is located in the direction of the galactic anticenter. Maeder et al. (1980) noticed that the ratio $N_R/N_{WR}$ of the numbers of red supergiants (R) and Wolf-Rayet (WR) stars as a function of galactic longitude is sharply peaked into that direction, whereas it reaches a minimum into the direction of the galactic center. This longitude effect is presumably directly related to the heavy-element abundance $Z$, which decreases as a function of galactocentric distance. The Magellanic Clouds which have a lower $Z$-value than the anti-center region, have an even higher ratio $N_R/N_{WR}$. This suggests (Maeder et al.) that at high $Z$, the loss of the stellar envelope during the blue supergiant phase may occur already for stars of relatively low mass, i.e. upwards from 35 $M_\odot$ (Pylyser et al. 1985), upon which they become WR stars – without passing through the red supergiant phase. On the other hand, for lower $Z$-values the same will happen only if the stars are more massive, say $> 45 M_\odot$. This means that our scenario may only be possible in regions that have a small metal abundance, which is exactly as observed.

Discussing their first objection, Eggleton and Verbunt argue that it is unlikely that the system remains bound during the supernova, since more than half of the system mass will probably be lost during the supernova when the progenitor WR star is sufficiently massive to produce a black hole. In our opinion exactly the reverse is true: the presence of the black hole makes it easier to understand that the system remained bound. The essential difference between neutron star and black hole formation is whether the shock that moves outward after the collapse of the 1.4 $M_\odot$ iron core is able to expel the outer layers. If it succeeds in doing this only the remnant of the iron core remains as a neutron star. If on the other hand the shock stalls, the outer layers start to fall in again, and the whole star collapses. In this case very little mass could be lost from the system, and the companion can easily remain bound.
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LITERATURE
van der Klis, M., 1983, thesis at the University of Amsterdam.
CHAPTER IV

BONDI-HOYLE ACCRETION FLOW

IV.1) A numerical study of cylindrically symmetric accretion flow

IV.2) On the accretion of angular momentum from an inhomogeneous medium

IV.3) On the accretion of angular momentum from an inhomogeneous medium II: Isothermal flow

IV.4) On the accretion of angular momentum from an inhomogeneous medium III: General case and observational consequences
Abstract
We present model calculations of cylindrically symmetric accretion flow to a gravitational point source. The flow is calculated using a form of the particle-in-cell method. We consider two polytropic equations of state ($\gamma=1.0$ and $\gamma=5/3$), and one in which the effects of radiation pressure are taken into account. In the latter case radiation transport is included in the diffusion approximation. We confirm the result of previous studies that the mass accretion rate is well approximated by the classical Hoyle-Lyttleton estimate. The kinetic energy dissipation rate is found to be several times larger than the classical estimate, the exact value depending on the equation of state and the Mach number of the flow. It is stressed that although the mass accretion rate is necessarily limited to the Eddington rate, the kinetic energy dissipation rate is not limited in this way as long as the surroundings of the accreting object are optically thick.

I. Introduction.
The gravitational capture of matter by a body moving relative to its surroundings is a process which occurs in several astrophysical problems. The first attempts at solving this problem (Hoyle and Lyttleton 1939, Bondi and Hoyle 1944) were instigated by a study of accretion from an intergalactic medium by galaxies and the possibility of stars gaining a non-negligible amount of mass from the interstellar medium during their lifetimes. A more recent application is the capture of matter from a stellar wind by a compact star, which is thought to occur in some X-ray binaries. Our own interest in the problem derives from the study of common envelope evolution of binaries, during which a compact or dwarf star is moving through the envelope of a giant companion.

In the general problem matter is flowing from infinity towards a gravitational point source. Far away from the object forces caused by pressure gradients and viscosity will be small and the matter will follow a free Keplerian orbit. Behind the object there will be a line, the so-called "accretion-line", where the orbits of matter coming from different sides of the object intersect. Assuming that the matter follows a free orbit leads to an infinite density on this line, so obviously that approximation breaks down here. On the accretion line pressure and viscous forces will become important and, because of the
cylindrical symmetry of the problem, will cause a complete cancellation of momentum transverse to the accretion line. Assuming that the only interaction with other matter takes place when the accretion line is crossed, Hoyle and Lyttleton (1939) were able to give a first estimate of the rate at which matter is accreted:

\[ A_{HL} = \pi R_{HL}^2 \rho_\infty v_\infty \]  

where \( \rho_\infty \) and \( v_\infty \) are the density and velocity of the gas at infinity and \( R_{HL} \) is the accretion radius, defined by

\[ R_{HL} = \frac{2GM}{v_\infty^2} \]  

with \( G \) the gravitational constant and \( M \) the mass of the gravitational point source. This result can be derived by assuming that all matter which does not have sufficient kinetic energy left to escape from the gravitational field after the cancellation of transverse momentum at the accretion line is accreted. When using the simple picture of a very thin accretion line it should always be kept in mind that Cowie (1977) showed that this assumption leads to an instability in the accretion flow that precludes any time-independent solutions for the accretion rate.

It was shown by Bondi (1951) that in the case of stationary accretion (i.e. \( v_\infty = 0 \)) the accretion rate can be estimated by an expression similar to equation 1, with \( v_\infty \) replaced by \( c_\infty \), the sound speed at infinity. Since in the accretion line picture, where the pressure forces are neglected (\( c_\infty \ll v_\infty \)), the accretion rate is given by eq. 1, and in the stationary case (\( v_\infty \ll c_\infty \)) by the same expression with \( v_\infty \) replaced by \( c_\infty \) it was proposed (Bondi and Hoyle 1944) that in the intermediate case the accretion radius would be given by

\[ R_A = \frac{2GM}{\frac{v_\infty^2}{2} + c_\infty^2} \]  

To get beyond the many simplifications used to derive the results above and to obtain more details of the actual flow pattern it seems necessary to perform detailed numerical simulations of the problem. This was first done by Hunt (1971) who gave a full 2-dimensional treatment of the gas flow around a gravitating object of very small geometrical radius (\( R \ll c_R A \)). These calculations were done using an adiabatic equation of state for an ideal gas. In the case that \( v_\infty \ll c_\infty \) (subsonic) it was found
that the flow closely resembles that of the case $v_\infty=0$. When $v_\infty>c_\infty$ (supersonic) the matter colliding on the accretion line behind the compact object will shock, and this shock moves out to the sides to form a bow shock. For subsonic flow the accretion rate was approximately that given by the Bondi-rate (eq. 3), and for large Mach numbers the accretion rate tends to the Hoyle-Lyttleton value (eq. 1). In a later paper (Hunt 1979) these calculations were repeated for a gas with a polytropic equation of state with $\gamma=4/3$. Other model calculations covering more or less the same topic are those by Livio, Shara and Shaviv (1979) and Shima et al. (1985). Also related to this work are the calculations of Da Costa and Fryxell (1981) who considered the flow around a star in a moving medium in the case that its geometrical radius is much larger than the gravitational accretion radius, in which case gravity has only a minor influence on the flow. A study of the 3-dimensional aspects of accretion flow was recently started by Livio et al. (1986), using a simplified version of the method used in this paper. These 3-dimensional aspects become important when the upstream boundary conditions are not constant in space, which destroys the cylindrical symmetry. An example of this is the presence of a density gradient in the undisturbed gas.

II. The numerical method.

The way in which we simulate the gas flow is based on the particle in cell method (PIC) (see e.g. Potter 1973, Hensler 1982a,b). The gas is represented by a large number of particles, which move according to an equation of motion in which the effects of gravity, pressure forces and viscosity are present. The pressure and viscous forces are calculated from the particle distribution on a 2-dimensional grid of cells in space, and are then interpolated to the particle positions.

To describe the method in more detail we shall use the following notational convention: particle properties like mass, thermal energy, velocity, momentum, position are given the index \(n\) (\(m_n\), \(e_n\), \(\vec{v}_n\), \(\vec{p}_n\), \(\vec{x}_n\)) to indicate the particle to which they refer. Grid properties like density, thermal energy, pressure, velocity and temperature are given the double index \(i,j\) (\(\rho_{i,j}\), \(e_{i,j}\), \(\vec{p}_{i,j}\), \(\vec{v}_{i,j}\), \(T_{i,j}\)) to denote the gridcell to which they refer (one for each dimension). We employ a 2-dimensional grid of cylindrical coordinates \((x,r)\) where \(r\) is the coordinate perpendicular to \(\vec{v}_\infty\) and \(x\) is parallel to it. The flow is
Fig. 1. A schematic illustration of the way the particle properties are distributed over the grid cells. For explanation see text.

assumed to be rotationally symmetric around the x-axis, and the azimuthal velocity (perpendicular to both x and r) is assumed to be zero.

To calculate the grid properties density, thermal energy density and momentum density from the particle properties we use a standard PIC method, slightly modified because of the non-cartesian coordinates. We shall only describe the calculation of mass density; the others are calculated in the same way. The mass of a particle is distributed over the four cells closest to the particle position. In standard PIC the weights for this distribution are taken to be proportional to the area of overlap between a square the size of a gridcell centered on the particle position and the fixed gridcells (see fig. 1). We take the weights to be proportional to the volume of the tori around the x-axis whose cross-section with the x,r plane is given by the areas indicated in fig 1. If we define

\[ g_n = \frac{x_{n+1} - x_n}{\delta} \]

\[ h_n = \frac{r_{n+1} - r_n}{\delta} \left( \frac{\delta + r_n + r_{n+1}}{2r_n} \right) \]  

(4)

where \( \delta \) is the grid cell size then the weights are given by
\[ w_{11} = (1-g_n) (1-h_n) \]
\[ w_{12} = (1-g_n) h_n \]
\[ w_{21} = g_n (1-h_n) \]
\[ w_{22} = g_n h_n \]

After the mass of the particles is distributed over the cells using the above weights, the density \( \rho_{i,j} \) is calculated by dividing the total mass in a cell \( m_{i,j} \) by its volume \( V_{i,j} \), which is again a torus around the \( x \)-axis. In the same way we obtain the momentum \( \vec{p}_{i,j} \) and thermal energy \( e_{i,j} \). These grid properties are subsequently used to calculate the rate of change of the particle velocity and thermal energy.

The velocity of a particle (or gas element) can change due to gravitational forces, pressure forces and viscous forces. To simulate the viscous forces we first calculate the average velocity in each cell

\[ \vec{v}_{i,j} = \vec{p}_{i,j} / m_{i,j} \]  

The next step is to give the particles a new velocity which is interpolated from the grid values using the same weight factors as were used for calculating the averages. For example, the particle illustrated in fig 1 gets the velocity

\[ \vec{v}_n = w_{11} \vec{v}_{i-1,j} + w_{12} \vec{v}_{i-1,j+1} + w_{21} \vec{v}_{i,j+1} + w_{22} \vec{v}_{i,j+1} \]  

In this way momentum is exactly conserved. Test calculations which were made to estimate the diffusion of momentum associated with this alternation between grid and particle velocities showed that it is equivalent to a viscous force with kinematic viscosity \( v = 0.16 \delta^2/\Delta t \), with \( \delta \) the size of a grid cell and \( \Delta t \) the timestep.

The pressure is calculated on the grid from the density and thermal energy density, using a choice of equations of state. In the case of an isothermal gas the pressure is proportional to the density, and in an ideal gas with \( \gamma = 5/3 \) the pressure is two-thirds of the thermal energy density. For the more complicated case of a mixture of an ideal gas and radiation in thermal equilibrium an iterative procedure is used to calculate the temperature and pressure from the density and thermal energy density. Once the pressure in all grid cells is known the acceleration that a gas element in the center of a grid cell would undergo is calculated, according to
\( \mathbf{a}_{i,j} \) = \(-\frac{1}{\rho_{i,j}} \mathbf{(VP)}_{i,j} \) \hfill (8)

These accelerations are then interpolated from the grid to the particle positions to yield \( \mathbf{a}_n \), the time rate of change of the velocity due to pressure forces.

Since we neglect the self-gravity of the gas (the total mass of gas in our grid is always less than \( 10^{-4} \) times the mass of the moving object) the gravitational acceleration of a particle is simply

\[
\mathbf{a}_n = - \frac{G(M) (\mathbf{x}_n - \mathbf{x}_{co})}{|\mathbf{x}_n - \mathbf{x}_{co}|^3}
\]

where \( G \) is the gravitational constant, \( M \) the mass of the gravitational source and \( \mathbf{x}_{co} \) its position.

The velocity of a particle at time \( t+\Delta t \) can now be expressed as

\[
\mathbf{v}_n(t+\Delta t) = \mathbf{v}_n + (\mathbf{a}_n^p + \mathbf{a}_n^g) \Delta t
\]

The thermal energy of a particle can change due to three causes: i) the dissipation of kinetic energy by the velocity averaging procedure, ii) pressure work, and in the models that take radiation effects into account by iii) radiation diffusion. In contrast to the evolution of velocity, which is directly applied to the particle properties, it is more convenient in this case to calculate the time rate of change of the thermal energy in a grid cell, and redistribute the new thermal energy over the particles. The time rate of change of the total thermal energy \( E_{i,j} \) in a gridcell due to viscous interactions is calculated by taking the difference in kinetic energy of each particle before and after the velocity averaging procedure and distributing this over the gridcells using the weights \( w_{11}, w_{12} \) etc.:

\[
(\Delta E_{\text{kin}})_n = \frac{1}{2} m_n (|\mathbf{v}_n|^2 - |\mathbf{v}_{\text{in}}|^2) \hfill (11)
\]

\[
\Delta e_{i,j} = \mathbf{E}_n w_{i,j} (\Delta E_{\text{kin}})_n \hfill (12)
\]

where the sum goes over all particles contributing to cell \((i,j)\).

To find the change in thermal energy due to pressure work we first calculate the change in thermal energy per gram.
\[ \Delta \varepsilon_{i,j} = \frac{p_{i,j}}{\rho_{i,j}^2} \Delta \rho_{i,j} \] (13)

The change in density is found by taking the difference between the old and the new density \( \Delta \rho = \rho_{\text{new}} - \rho_{\text{old}} \). The change in total thermal energy in a cell due to pressure work then follows by multiplying with the total mass in a cell

\[ \Delta \varepsilon_{i,j} = m_{i,j} \Delta \varepsilon_{i,j} \] (14)

The third way in which \( \varepsilon_{i,j} \) can change is by radiation diffusion. This can be described by the equation

\[ \frac{1}{V_{i,j}} \frac{\partial \varepsilon_{i,j}}{\partial t} = \nabla \left( \frac{c}{3 \kappa \alpha} \nabla (aT^4) \right) \] (15)

in which \( V_{i,j} \) is again the volume of cell \((i,j)\), \( c \) the velocity of light, \( \kappa \) the opacity (electron scattering) and \( \alpha \) the radiation constant. This equation can be solved directly on the grid. A complication is that when radiation diffusion is important the time step required for stable integration of eq. (15) is smaller than the timestep required in the dynamical part of the calculations. We therefore integrate eq. (15) separately within each dynamical timestep, keeping the density constant. Equations (12), (14) and (15) yield the non-advective part of the change in thermal energy of a gridcell in a timestep. The new energy is distributed over the particles in the normal way.

Just as the velocity averaging procedure causes a diffusion of momentum (viscosity), so the alternation between grid and particle thermal energy causes a diffusion of energy with a diffusion coefficient \( 0.16 \delta^2/\Delta t \), which in case of high densities or low temperatures can become larger than the energy diffusion by radiation, which has a diffusion coefficient \( c/\kappa \alpha \) (eq.15).

Given an initial distribution and a set of boundary conditions on the grid the method above can describe the time evolution of a system of particles.

III. Boundary conditions.

A complication of a (semi) Lagrangian method like PIC in the case of accretion flow calculations is that we can not consider a fixed amount of mass. Matter is continuously entering the grid fixed to the compact
object from the upstream side and leaving it either on the downstream side or by being captured. Hence in every timestep new particles must be created to represent the matter entering, while particles which have left the grid need no longer be considered. Since it is most convenient to use a fixed number of particles this is handled as follows. After the particle positions are advanced one timestep it is checked which particles are outside the grid or have been accreted. (A particle is assumed to be accreted when it has entered one of the two gridcells adjacent to the gravitational source, which is considered to be very compact.) These particles are subsequently injected at the upstream edge of the grid, representing the matter which must have entered the grid in the last timestep. The position, velocity and mass of the particles is chosen in such a way that they represent matter which has come from a homogeneous density distribution far upstream \((x=-\infty)\) and has moved from there in a free Keplerian orbit. Because of the gravitational focusing particles enter the grid at both \(x=x_{\text{min}}\) and \(r=r_{\text{max}}\). Because of this the grid edge \(r=r_{\text{max}}\) is handled as an upstream boundary, and we have to take care that the bow shock of the compact object does not pass through it. The boundary conditions on the axis \((r=0)\) are imposed by the condition of cylindrical symmetry, i.e. the radial velocity is zero and the derivatives in the \(r\)-direction are zero. Pressure gradients (and temperature gradients in the case that radiation diffusion is taken into account) on the sides of the grid are calculated by assuming an extra gridcell just outside the grid with a pressure (or temperature) equal to the adjacent gridcell. The more exact method of extrapolation of the outer two gridcells can give rise to an instability when a shock crosses the boundary (Roache, 1976).

The pressure in the cells next to the compact object is calculated by taking the average of the two values one obtains by extrapolating the pressure in the cells adjacent to it in the \(x\)- and \(r\)-direction. The exact choice of this inner boundary condition hardly influences the results, as was verified with a test calculation in which the pressure in these cells was simply set to zero.

We use a grid of 64x64 cells, which extends upstream from the compact object for one accretion radius \((R_{\text{HL}})\) and downstream for two accretion radii. All calculations were performed with 50,000 particles, giving an average number of particles per cell of \(~12\). We start the calculations with the grid filled with particles with velocity \(\mathbf{v}_0\) and let the system evolve until a steady state is reached.
IV. Results.

a) Isothermal equation of state.
We calculated models using an isothermal equation of state \( P=\rho RT_\infty \) for Mach numbers 1.5, 2.0, 3.0 and 3.75, where the Mach number is calculated relative to the isothermal sound speed \( c_\infty^2=\frac{P_\infty}{\rho_\infty} \). In fig 2a and b the density and velocity distribution for the case \( M=1.5 \) is shown. Typical for an isothermal shock are the large density and velocity jumps across it. Since the information about the presence of an obstacle in the flow can never propagate upstream, the shock will always be attached to the inner boundary, however small this is taken. For every Mach number the stagnation point of the flow on the accretion axis behind the compact object lies near \( R_{HL} \).

b) Adiabatic ideal gas.
For this case models of Mach number 1.5, 2.0, 3.0 and 3.75 were constructed. Density and velocity plots are shown in fig. 3 a and b for the case \( M=3 \). The general features of the flow pattern are similar to those of Hunt (1971) and Shima et. al. (1985). There is some difference in the location of the stagnation point in the flow on the accretion axis, in the sense that in our calculations this point is situated further away from the compact object. The deviation is largest for small Mach numbers. A consequence is that the mass accretion rate we find is also larger (see V.). A possible cause for these differences might be the rather high diffusivity in our method, which would explain why agreement is better for the more supersonic cases.

c) Mixture of gas and radiation
In this case the Mach number alone does not completely determine the flow, since the relative importance of radiation pressure increases with temperature. We chose an upstream density \( \rho_\infty \) of \( 10^{-8} \) g/cm\(^3\) and a temperature \( T_\infty \) of \( 10^5 \) K. This corresponds to a value for \( \beta \) (\( \equiv \frac{P_{\text{gas}}}{(P_{\text{gas}}+P_{\text{rad}})} \)) in the undisturbed gas of 0.35. In the region close to the compact object the temperature will be higher and \( \beta \) will decrease even further, and the results should become comparable to those of other authors for \( \gamma=4/3 \). For a mixture of gas and radiation the adiabatic sound speed is given by

\[
c_\infty^2 = \frac{\left( \frac{4}{3} T_\infty + \frac{4}{3} \rho_\infty RT_\infty \right)^2}{\rho_\infty \left( \frac{4}{3} T_\infty + \frac{4}{3} \rho_\infty RT_\infty \right)} + RT_\infty
\]  

(16)
Fig. 2a. The density distribution in the $\gamma=1.0$, $M=1.5$ model. The mass density is proportional to the density of points in the plot, the unit of distance along the axis is $k_{HL}$. The maximum density is $42.9 \rho_\infty$.

Fig. 2b. The velocity distribution in the central part of the grid. The maximum velocity $3.0 v_\infty$. 
Fig. 3a. The density distribution in the $\gamma=5/3$, $M=3.0$ model. The maximum density is $25.8 \rho_\infty$.

Fig. 3b. The velocity distribution in the central part of the grid. The maximum velocity $2.3 \, v_\infty$. 
Fig. 4a. The density distribution in the model with a mixture of gas and radiation, $M=3.0$. The maximum density is $42.4 \rho_\infty$.

Fig. 4b. The velocity distribution in the central part of the grid. The maximum velocity $3.0 \, v_\infty$. 
and the Mach number is determined relative to this sound speed. We calculated models for Mach numbers 1.5 and 3.0, and for the latter case density and velocity distribution plots are presented in figures 4 a and b. Generally the shock has a smaller opening angle than in case of an ideal gas because the dissipated kinetic energy can build up less pressure for an effective γ less than 5/3.

V. The accretion rate.

In fig. 5 the average mass accretion rate by the compact object is shown as a function of Mach number for the different equations of state, and the typical short-time variability of the accretion is indicated. The values are reasonably consistent with the results of Shima et al, the greatest difference being the somewhat higher rates found for the γ=5/3 case. In the isothermal models the accretion rate remains more variable in time than in the γ=5/3 models, which is probably a consequence of the fact that the accretion line instability described by Cowie (1977) is less suppressed by pressure effects. Note that in our models in which radiation pressure dominates we do not find the substantial increase in accretion rate over the Hoyle-Lyttleton value that was reported by Hunt (1979) for the γ=4/3 case. This is in agreement with the results of Shima et al.

An interesting phenomenon is the rise of the accretion rate over the classical Hoyle-Lyttleton value for Mach numbers just larger than 1 in the isothermal case (or near isothermal, γ=1.1 in the work of Shima et al.). This can be understood by a slight modification of the accretion line model. In this model it is assumed that all matter is accreted which does not have sufficient kinetic energy left to escape from the gravitational field after the cancellation of transverse momentum on the accretion line. When gas pressure effects are taken into account the cancellation of momentum and the associated kinetic energy dissipation take place in the shock. When the equation of state is isothermal the velocity jump across the shock is very large, and the kinetic energy associated with the velocity perpendicular to the shock is completely dissipated. Now for Mach numbers close to one, when the opening angle of the shock cone is ~45°, matter coming from upstream in an almost free Keplerian orbit will pass the shock at a point which lies deeper inside the potential well than the intersection point of the orbit with the accretion line, and at which a larger component of the

75
Fig. 5. The mass accretion rate as a function of Mach number for the different equations of state. The accretion rate is normalized to the Hoyle-Lyttleton rate ($A_{HL}$). The typical short-timescale variability is indicated by the bars.
total velocity is perpendicular to the shock. These two factors tend to increase the mass accretion rate. As the Mach number increases, the opening angle of the shock decreases, and the accretion rate approaches the Hoyle-Lyttleton value.

IV. The kinetic energy dissipation rate

Interesting parameters of the accretion flow problem in the context of common envelope evolution of binaries are the total amount and the distribution of frictional energy liberated in the surrounding medium by the passage of the compact object. A first estimate of the kinetic energy dissipation rate as derived from the Hoyle-Lyttleton picture is

$$E_0 = \frac{1}{2} \pi R^2_{HL} v^3 = 1.12 \times 10^{38} \left(\frac{\rho_\omega}{M_{\odot}}\right)^2 \left(\frac{\rho_\omega}{10^{-8}}\right) \left(\frac{v_\omega}{10^7}\right)^{-1} \text{erg/sec} \quad (17)$$

We have made an estimate of the total amount of thermal energy which is deposited in the surrounding medium ($E_S$) in our calculations by determining the total kinetic energy dissipation rate ($E_{\text{dis}}$), and subtracting the thermal energy which is carried by the matter being accreted (see table 1). We find that the total amount of dissipated kinetic energy is between 7 and 10 times the estimate in equation (17). In the $\gamma=5/3$ case the larger part of this thermal energy disappears with the accreted matter, and approximately $2E_0$ is deposited in the surroundings. In the radiative case both $E_{\text{dis}}$ and $E_S$ increase with Mach

<table>
<thead>
<tr>
<th>model</th>
<th>$E_{\text{dis}}/E_0$</th>
<th>$E_S/E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M=1.5, \gamma=5/3$</td>
<td>7.61</td>
<td>2.07</td>
</tr>
<tr>
<td>$M=2.0, \gamma=5/3$</td>
<td>7.30</td>
<td>2.18</td>
</tr>
<tr>
<td>$M=3.0, \gamma=5/3$</td>
<td>7.30</td>
<td>2.36</td>
</tr>
<tr>
<td>$M=3.75, \gamma=5/3$</td>
<td>6.98</td>
<td>2.42</td>
</tr>
<tr>
<td>$M=1.5,$ radiative</td>
<td>7.54</td>
<td>1.36</td>
</tr>
<tr>
<td>$M=2.0,$ radiative</td>
<td>8.41</td>
<td>2.67</td>
</tr>
<tr>
<td>$M=3.0,$ radiative</td>
<td>9.07</td>
<td>4.12</td>
</tr>
<tr>
<td>$M=3.75,$radiative</td>
<td>9.17</td>
<td>4.90</td>
</tr>
</tbody>
</table>

Table 1. The total energy dissipation rate $E_{\text{dis}}$ and the rate $E_S$ at which thermal energy is deposited in the surrounding medium, as a function of Mach number and equation of state. Both are normalized to the first-order estimate in equation 17.
number. The large values of $E_\text{s}$ are caused by the fact that the hot regions near the gravitating object lose energy to their surroundings by radiation. The reason that these numbers are generally larger than the first order estimate above is probably that much more matter is shocked than is accreted, since the bow shock extends to distances further than $R_{\text{HL}}$. From our calculations it is difficult to determine the final distribution of this energy, since we only consider the region within a few $R_{\text{HL}}$ from the compact object. We can however make a qualitative estimate from the width of the bow shock, which would yield a distribution with a width of $\sim 1$ $R_{\text{HL}}$ for the higher Mach numbers increasing to about $2$ $R_{\text{HL}}$ for a Mach number of about 1.5.

VII. Possible effects of an accretion luminosity

An assumption made in most models up to date is that all matter which flows through the inner boundary near the accreting object simply disappears, together with the thermal and kinetic energy it carries. In some applications the accretion rate can become much larger than the Eddington accretion rate at which the luminosity, originating in the region close to the accreting object where the inflowing matter is stopped and accumulates, becomes so great that the outward radiation pressure overcomes the gravitational attraction, and accretion stops. The effect this will have on the flow pattern depends on the optical depth of the surroundings. When these surroundings are optically thin ($R_{\text{HL}} \alpha < 1$) the effects of radiation pressure from the accretion luminosity can be described as an effective reduction of the gravitational force. A simple analytical estimate of the accretion rate based on the Hoyle-Lyttleton expression then yields (for $A_{\text{Edd}} < A_{\text{HL}}$)

$$A = A_{\text{Edd}} \left( 1 - \left( \frac{A_{\text{Edd}}}{2A_{\text{HL}}} \right)^{1/2} \right)$$

(18)

where $A_{\text{Edd}}$ is the Eddington accretion rate, at which the radiation force exactly equals gravitation. In this way the accretion rate can be reduced significantly. Because the effective gravity is reduced at all distances to the compact object the flow pattern will simply scale to the new effective accretion radius. This means that the energy dissipation rate will also be reduced.

The limit opposite to the optically thin case is when the
surroundings are sufficiently optically thick that the photon diffusion time becomes larger than the dynamical time

\[ \frac{R_{HL}^2 \rho \, c}{c} > \frac{R_{HL}}{v_\infty} \]  

(19)

In this case the flow far upstream can not be affected by the accretion luminosity since the photons which diffuse out of the region close to the compact object are swept away with the matter flowing along it. In this case the accretion rate will be limited to a value near the Eddington limit, but the energy dissipation rate will remain of the same order since the typical size of the bow shock is not affected. We have tried to simulate this behaviour by letting the energy which is carried by the accreting matter accumulate in the cells next to the compact object, so that it could only be removed from there by radiation diffusion. We find that the accretion rate decreases as expected. The width of the bow shock however remains unaffected. The bow shock develops an extra bulge on the front edge, very similar to the flow pattern found by Shima et al for a hard non-absorbing sphere.

VIII. Conclusions
These model calculations of accretion flow, and others from the literature, have shown us that although the actual flow patterns can be very different from the simple accretion line picture, the mass accretion rate is very well approximated by the simple Hoyle-Lyttleton estimate. The total amount of kinetic energy dissipation seems to be underestimated in the classical estimate.

Acknowledgements.
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On accretion of angular momentum from an inhomogeneous medium

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Summary. The problem of a compact object accreting from an inhomogeneous medium has been studied, using a three-dimensional numerical scheme. When pressure effects are neglected, it has been shown that the mass accretion rate is given by the Bondi-Hoyle value. Not more than a few per cent of the angular momentum deposited into the accretion cylinder are accreted by the compact object. Some possible consequences for the case of neutron stars and white dwarfs accreting from a stellar wind are discussed.

1 Introduction

The problem of axisymmetric accretion from an infinite medium, by a gravitating point mass, has many astrophysical applications. It is not surprising, therefore, that many workers have treated various aspects of the problem both analytically (e.g. Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Spiegel 1970; Ruderman & Spiegel 1971; Lyttleton 1972; Wolfson 1977a; Rephaeli & Salpeter 1980) and numerically (e.g. Hunt 1971; Wolfson 1977b; Livio, Shara & Shaviv 1979; Okuda 1983; de Kool & Savonije 1985, in preparation).

In the non-axisymmetric case (e.g. a medium containing a density gradient), progress has been impeded by the lack of a fundamental theory. Some aspects of the problem were pointed out in the early works of Gethin (1951) and Dodd & McCrea (1952), which used the classical Bondi & Hoyle (1944) approach.

The non-axisymmetric case has gained renewed interest in the context of compact objects accreting from a stellar wind, the question of spin-up and the possibility of forming a disc from wind accretion. Most workers have used a direct application of the Bondi–Hoyle result, noting that a density or velocity gradient in the flow results in a net deposition of angular momentum into the accretion cylinder (e.g. Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981) and assuming all that angular momentum to be accreted. Davies & Pringle (1980) pointed out, using a simplified two-dimensional picture, that the conditions required for matter to be accreted at all (in the Hoyle–Lyttleton picture) conflict with the possibility of accretion of angular
momentum. They concluded that to first order in $R_{\text{acc}}/H$ ($R_{\text{acc}}$ the accretion radius, $H$ the density gradient scale) no angular momentum will be accreted in their case but admitted that the situation can be more complex in the realistic, three-dimensional case. Their approach has been criticized by Wang (1981), who claimed that their result was merely a direct consequence of the particular simplification of using an 'accretion line' going into a point mass.

In a recent paper, Soker & Livio (1984) have attempted to examine the problem of accretion from a medium containing a density gradient, in the three-dimensional case, when the interactions were assumed to take place only on the accretion axis. Using a perturbative, analytical, Hoyle–Lyttleton-type approach, they have first shown that even in the three-dimensional case, the matter becomes confined to a thin 'accretion layer', after encountering the accretion axis. They have then attempted to calculate (under the above-mentioned assumptions) the first-order corrections to the specific angular momentum, due to pressure in the 'accretion layer'.

In the present work, we present a three-dimensional numerical study of accretion from an inhomogeneous medium. Our basic assumptions and method of calculation are described in Section 2. The results are presented in Section 3 and discussed in Section 4.

2 Assumptions and method of calculation

We have used a pseudo-particle method to describe the hydrodynamics. The gas is treated as being divided into individual particles with given masses. Similar calculations have been used by Lucy (1977, 1980), Gingold & Monaghan (1977, 1978), Lin & Pringle (1976) and Hensler (1982a, b). Our method can be described as follows:

2.1 EQUATION OF MOTION

In the present, still preliminary calculation, we have neglected pressure gradients (the flow can be considered, therefore, as hypersonic). Most of the existing calculations which include pressure gradients do not conserve angular momentum and thus are not suitable, at least a priori for our present purpose, which is to study the accretion of angular momentum (e.g. Hensler 1982a). The equation of motion for the particles is thus in general (in dimensionless form)

$$\frac{d^2r}{dt^2} = - \frac{1}{2} \frac{r}{r^3} + a_i$$

(1)

where we have used as our unit length

$$R_{\text{BH}} = \frac{2GM}{V_0^2}$$

(2)

where $M$ is the mass of the accreting object and $V_0$ the flow velocity at infinity. Our unit time was chosen as $R_{\text{BH}}/V_0$. The term $a_i$ in equation (1) describes the effect of inter-particle interaction, to be described later.

2.2 THE GRID

We have used a three-dimensional, rectangular block shaped grid, $-1.5 < x < 3.9$, $-1.5 < y < 1.5$, $0 < z < 1.5$. The flow direction was taken as the $x$-axis. In the calculations with an inhomogeneous medium, the density gradient was taken in the $y$ direction as (achieved by
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changing the masses of the particles)

\[ \rho = \rho_0 \left(1 + \frac{y}{H}\right). \]  

(3)

Since the problem is symmetric about the xy plane, the calculation was performed for the \( z > 0 \) half space (for every particle crossing with \( V_z \) to the \( z < 0 \) half space, we inject one at \( -z > 0 \) with \( -V_z \)). The compact object was taken as a cube at the origin of size 0.30. Our standard grid has been divided into equal cubic cells of size 0.15. The number of cells used was \( 36 \times 20 \times 10 \) (which means an effective number of 14400 because of the use of the symmetry plane). We have also performed calculations using other cell sizes (and compact object sizes). We have used an average of six particles per cell (a total of 43200), which is more than the number required for standard PIC techniques (e.g. Potter 1973). The number of cells and particles that have been used was in fact determined by the limitations imposed by the maximally allowed memory requirements in the IBM 3081D.

2.3 INTERACTIONS AMONG PARTICLES IN THE SAME CELL

We have taken the particles in each cell \( j \), to interact in the following way:

First, the centre of mass and velocity of each cell are calculated

\[ V_{c,j} = \frac{\sum m_i V_i}{\sum m_i} \]

\[ r_{cm,j} = \frac{\sum m_i r_i}{\sum m_i} \]  

(4)

the summations being on the particles in the \( j \)th cell. Then, an angular velocity can be defined by

\[ L_j^k = -I_j^{kl} \Omega_j^l \]  

(5)

where \( \vec{R}_j \) is the particle's coordinate in the centre of mass (of the cell) frame, \( L_j^k \) is the angular momentum component and \( I_j^{kl} \) is the moment of inertia component. We then find the new velocity of the particle by (see also Hensler 1982a; Lin & Pringle 1976).

\[ V_{new,i} = V_i (1 - \alpha) + \alpha U_i \]  

(6)

where

\[ U_i = V_{c,j} + \vec{R}_j \times \Omega_j \]  

(7)

and \( \alpha \) is a parameter determining the strength of the interaction in the cell (\( \alpha = 0 \) means no interaction, \( \alpha = 1 \) full interaction).

2.4 INTER-CELL INTERACTION

In order to prevent jumps in the fluid velocity in neighbouring cells and the development of instabilities, we have introduced a smearing of velocities over adjacent grid cells by the following
procedure. At even time-steps the vertices of the cells were taken at the coordinates

\[ \begin{align*}
N_x &= -N_{x_{\text{min}}} \ldots N_{x_{\text{max}}} \\
N_y &= -N_{y_{\text{max}}} \ldots N_{y_{\text{max}}} \\
N_z &= 0, \ldots, N_{z_{\text{max}}}
\end{align*} \]

where \( \Delta R \) is the cell size. At odd time-steps the vertices were taken at

\[ (x, y, z) = \Delta R(N_x + \frac{1}{2}, N_y + \frac{1}{2}, N_z + \frac{1}{2}) \]  \quad \text{(9)}

The scheme is shown symbolically in Fig. 1 for a two-dimensional grid. The scheme has the following advantages: (i) It conserves angular momentum explicitly. This is to be compared to some of the methods for inter-cell interaction which use a cell around each particle, overlapping with neighbouring cells and do not conserve angular momentum (e.g. Hensler 1982a). (ii) The scheme allows, in principle at least, the freedom of choosing different interaction strengths \( \alpha_A, \alpha_B \) for the two grids A and B (Fig. 1) and checking the effects of different choices. Obviously a numerical viscosity \( \nu \) which is proportional to \( \Delta R^2/\Delta r \) is introduced.

2.5 CRITERIA USED IN NUMERICAL CALCULATIONS

All models were started with the particles randomly distributed and with a velocity \( V = (1, 0, 0) \). The time-step was always chosen to obey the Courant–Friedrich–Levy condition. New particles were injected into the grid for particles escaping from the grid or accreted.

The procedure of injection has been the following: An impact parameter \( b \) and an angle with the y-axis \( \phi \), were chosen randomly the particle was assumed to travel on an unperturbed hyperbolic orbit corresponding to \( b \) and \( \phi \) outside the grid and was injected into the grid along that orbit (its mass has been determined according to the y coordinate corresponding to \( b \) and \( \phi \)). Use was made of the \( z = 0 \) symmetry plane. In order to determine whether the system could be considered in a steady state (which actually served as an initial state for the real calculation), we have used several criteria:

(i) The number of particles injected (or escaping). The calculation has been carried out until the number of particles we had to inject converged to a limiting value (apart from obvious fluctuations, see Fig. 2).

Figure 1. A schematic representation in two dimensions of the two grids on which the calculation has been performed at alternative time-steps, producing inter-cell interaction (see text).
Figure 2. The number of particles injected every 10 time-steps as a function of the number of time-steps. The arrows at the top indicate crossing times of the grid.

(ii) Velocity criteria on the accretion axis. We have checked for the sums of the $V_x$ component of the velocity in three regions of 16 cells each, located as shown in Fig. 3 (dashed areas) one cell above the $z=0$ plane. Again the calculation has been carried out until a limiting value has been approached.

(iii) The number of particles at the accretion axis' 'tail'. The total number of particles in 40 cells in the $z=0$ plane, at the edge of the grid around the accretion axis (see Fig. 3 marked by heavy line) was counted and followed till a limiting value was approximately reached.

Following the establishment of a steady state according to the above criteria, each run was

Figure 3. Regions used for velocity criteria (dashed areas) and number of particles criterion (marked by heavy line), for the establishment of a steady state (see text).
Figure 4. The velocity component $V_x$ on the accretion axis as a function of $x$, in the symmetric case. The standard case (six particles per cell and $\Delta R = 0.15$) is represented by the long dashes. The full line represents four particles per cell and the short dashes four particles per cell and $\Delta R = 0.1$.

performed for at least 15 crossing times of the grid and average values were calculated for the relevant quantities. As we have already mentioned, our standard calculations were performed with an average of six particles per cell and a cell size of $\Delta R = 0.15$. In Fig. 4 we present the results for $V_x$ on the accretion axis, from calculations performed with an average of four particles per cell and a cell size of $\Delta R = 0.1$. As can be seen, no significant changes are introduced.

3 Results

3.1 Symmetric Case

In order to test the numerical code and investigate the effect of various factors, we have first run an axisymmetric case in which no density gradient existed. The standard run assumed $\alpha_A = \alpha_B = 1$. The results for the velocity profile in the $z = 0$ plane are shown in Fig. 5 and the density profile in Fig. 6. The stagnation point is clearly seen. Matter is seen to accumulate along the accretion axis in an 'accretion cone' the width of which is largely determined by the viscosity. The accretion radius obtained was $R_{\text{acc}} = 1.0(2GM/V_0^2)$, in very close agreement to the Bondi & Hoyle (1944) and Hunt (1971) results. In Fig. 7 we present $V_x$ on the axis, as a function of $x$. Any solution along the largely dashed line that passes through the shaded region is a Bondi–Hoyle solution with the same stagnation point as ours (e.g. Lyttleton 1972). Our numerical solution clearly satisfies these conditions (the small deviation very close to the accreting body results from numerical viscosity).

An extensive study of axisymmetric accretion in two dimensions, using the PIC method is presently carried out by de Kool & Savonije (1985, in preparation). The results of the present work, which necessarily uses a more coarse grid, are consistent with theirs.

3.2 Accretion from a Medium with a Density Gradient

We have assumed the existence of a density gradient as described by equation (3) with $H = 4$. The main results are the following.
As predicted by Soker & Livio (1984), the gas is strongly concentrated towards the $z=0$ plane. This can be clearly seen in Fig. 8, where the mass in the downstream side of successive planes is plotted as a function of $z$.

In the $z=0$ plane, the matter forms a displaced 'accretion cone', very similar to the one described by Davies & Pringle (1980) in their two-dimensional example. The 'accretion cone' is shown in Fig. 9 which represents an instantaneous picture of the location of all particles. The velocity and density profiles in the $z=0$ plane are presented in Figs 10 and 11 respectively and again exhibit the clear formation of the displaced 'accretion cone'. The accretion rate obtained (for $a_A=a_B=1$) is of the order $M_{\text{acc}} \approx 1.0 M_{\text{BH}}$, where $M_{\text{BH}}$ is the Bondi–Hoyle accretion rate in the symmetrical
Figure 7. The velocity component $V_x$ on the accretion axis, as a function of $x$, for the symmetric case. Our numerical results are represented by the full line, the dashed line and the shaded region represent a Bondi–Hoyle solution.

case (for $R_{\text{acc}}=2GM/V_0^2$). As an additional check we tried decreasing $\alpha_B$ which resulted in lower accretion rates as expected (the dependence being almost linear). It should be remembered that the results of Bondi & Hoyle (1944) for the symmetrical case actually only state that the accretion radius is between $GM/V_0^2$ and $2GM/V_0^2$.

Our most important new result concerns the accretion of angular momentum. Our results indicate that the rate of accretion of angular momentum is very low and certainly not more than a few per cent of

$$L_{\text{BH}}\approx \frac{1}{4H} \dot{M}_{\text{BH}} \left(\frac{2GM}{V_0^2}\right)^2$$

which was the rate assumed by Illarionov & Sunyaev (1975) and Shapiro & Lightman (1976), based on the accretion of all the angular momentum entering the accretion cylinder. The results are in fact consistent with almost no accretion of angular momentum (other than that of matter hitting the accreting object directly from the upstream side and within the accuracy of the calculation). Most of the angular momentum accretion rate obtained, $L_{\text{acc}}=0.08 L_{\text{BH}}$, results from the fact that our accreting body is relatively large, so that matter coming from the upstream side can be accreted, with its angular momentum, prior to reaching the accretion cone. This fact can be realized by noting that even just the free orbits of particles hitting the accreting object from the upstream side would lead to an accretion rate of $L_{\text{acc}} \approx 0.06 L_{\text{BH}}$ and the interaction effectively increases the size of the body. Calculations performed with different sizes of the accreting body have indeed shown that a significant fraction of the obtained angular momentum accretion comes from upstream.
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Figure 8. The mass in successive planes in the downstream side as a function of z (for two grid cell sizes).

Figure 9. An instantaneous picture of the location of all particles in the grid, in the inhomogeneous case. The accreting object is marked by a cross.
The fact that very little angular momentum is accreted can be traced to two causes:

(i) A displacement of the accretion cylinder (or its cross-section facing the flow) towards lower densities (which is related to the displacement of the accretion cone).

(ii) Cancellation of transverse momentum at the displaced accretion cone as in the Bondi–Hoyle (1944) picture. With respect to point (i), it can be shown that if the displacement is small and the cross-section of the accretion cylinder is still roughly circular (which is actually usually not the case), then the decrease in the rate of angular momentum accretion resulting from the displacement alone can be roughly estimated as

\[
\frac{\dot{L}_{\text{dis}}}{\dot{L}_{\text{sym}}} = 1 - \frac{4Hd}{M_{\text{acc}}/M_{\text{BH}}} \quad (11)
\]
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where \( d \) is the displacement of the cross-section (in dimensionless units), \( \dot{L}_{\text{dis}} \) is the rate of angular momentum deposition into the displaced cross-section and \( \dot{L}_{\text{sym}} \) is the rate of deposition if the same mass had gone into the symmetric cross-section. Typically in the numerical calculation a reduction by a factor of \( -4 \) in the rate of angular momentum deposition resulted from this displacement.

Point (ii) above is in certain respects a manifestation of the point raised by Davies & Pringle (1980) in the two-dimensional case, that a cancellation of transverse momentum is required for accretion to take place. It should be noted that in our case \( R_{\text{ac}}/H = \frac{1}{4} \) and the result is no longer a first-order approximation in the \( R_{\text{ac}}/H < 1 \) case.

Keeping \( \alpha_s = \alpha_B = 1 \) and changing other numerical parameters such as the grid size and the time-step gave always results of the order \( \dot{M}_{\text{acc}} = \dot{M}_{\text{BH}}, \); \( \dot{L}_{\text{acc}} = 0.07 - 0.1 \dot{L}_{\text{BH}} \). Similar results were obtained when a different density gradient was used (\( H = 16 \), equation 3).

4 Discussion

The indications of the preliminary results of the present study (which neglects pressure) can be summarized as follows:

(i) The mass accretion rate, on to a compact object moving through a medium containing a density gradient, is not very different from the Bondi–Hoyle value, obtained in the axisymmetric case.

(ii) The rate of accretion of angular momentum is not more than a few per cent of the rate at which angular momentum enters the Bondi–Hoyle (symmetric) accretion cylinder.

The present calculation does not include pressure effects, a calculation with pressure gradients is now in progress. Because of the preliminary nature of the results we do not want at this stage to speculate on all their possible consequences. We would like, however, to point out certain topics which may be significantly influenced.

Accretion of angular momentum from a stellar wind has been invoked to explain spin-up (and spin-down under certain circumstances) of some neutron stars (e.g. Vela X-1, 4U 1538-52; Wang 1981). The time-scale for the spin-up was taken as

\[ \frac{P}{\dot{P}} \sim 0.8 \frac{1}{\eta} \left( \frac{M_{\text{ns}}}{\dot{M}_{\text{acc}}} \right) \left( \frac{R_{\text{ns}}}{R_{\text{ac}}} \right)^2 \left( \frac{P_{\text{orb}}}{P_{\text{ns}}} \right) \]  

(12)

where \( M_{\text{ns}}, R_{\text{ns}} \) and \( P_{\text{ns}} \) are the neutron star's mass, radius and spin period respectively, \( P_{\text{orb}} \) is the binary orbital period and \( \eta \) is a parameter depending on the velocity and density gradients in the wind.

If the results obtained in the present work are confirmed by our more realistic calculations (including e.g. pressure), then the time-scale for spin-up should be about 10–100 time longer than the one expressed by equation (12). Furthermore, the formation of an accretion disc around neutron stars accreting from the companion's wind is marginal even in the existing 'theory', since it requires very low wind velocities (e.g. Wang 1981). Such a disc formation becomes virtually impossible when our present results are considered since a relative velocity of less than \( \sim 100 \text{ km s}^{-1} \) is required between the neutron star and the wind.

The possibility of forming a disc from wind accretion would have been more favourable (if equation 10 is used) in the case of a white dwarf accreting from the wind of a cool giant, as pointed out by Livio & Warner (1984) for Mira, SY For and 56 Peg. However, again if our present results are confirmed, a disc cannot form in this case either.
It should be pointed out, however, that if an accretion disc starts to form, even temporarily, it will probably grow due to the viscous interaction. This adds further weight to the point made by Livio & Warner (1984), that the observational establishment of the existence or non-existence of discs in these systems and in similar ones such as HR 3080, $\nu$ Her, HR 8157, can contribute significantly to the understanding of the accretion process.

References

Summary. We have studied the problem of accretion (by a compact object) from an inhomogeneous medium, for the case of an isothermal flow.

Using a three-dimensional numerical scheme, we found the mass and angular-momentum accretion rates. The mass accretion rate agrees well with the Bondi–Hoyle theory. The rate of accretion of angular momentum is only a small fraction of the rate at which angular momentum is deposited into the accretion cylinder. This confirms our previous results which were obtained without the inclusion of pressure effects.

1 Introduction

Accretion from an inhomogeneous medium, by a compact object, is an important process for two classes of objects:

(i) Neutron stars accreting from the wind of early-type companions and
(ii) white dwarfs accretion from the winds of cool giants.

Because of the lack of a basic theory in the non-axisymmetric case, progress has been rather limited. Most workers have simply tried to make use of the Bondi & Hoyle (1944) results which were obtained for the axisymmetric case (e.g. Dodd & McCrea 1952; Illarionov & Sunyaev 1975; Shapiro & Lightman 1976).

A fundamental question in the case of accretion from an inhomogeneous medium is whether the accreting object can accrete angular momentum. A simple inspection of the Bondi–Hoyle picture, reveals, that if the accretion cylinder (of radius $R_{\text{acc}}$) remains unchanged, then the existence of a density (or velocity) gradient in the medium results in a net deposition of angular momentum into the cylinder. Several authors have assumed that all the angular momentum entering the accretion cylinder is actually accreted (Illarionov & Sunyaev 1975; Shapiro &

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Lightman 1976; Wang 1981). An objection to this assumption has been raised by Davies & Pringle (1980), who pointed out that the condition imposed on the matter to be accreted (a cancellation of the momentum transverse to the accretion line), conflicts with the idea that this matter can still possess angular momentum. Davies & Pringle (1980) have indeed shown, in a highly simplified two-dimensional case, that no angular momentum is accreted.

Two important questions that thus emerged were:

(i) Is angular momentum accreted in the three-dimensional case? Wang (1981) argued that the result obtained by Davies & Pringle (1980) was a consequence of the use of a restricted two-dimensional model and the concept of an 'accretion line' [still in the Hoyle-Lyttleton (1939) approach neglecting pressure].

(ii) What are the effects of pressure on the rate of accretion of angular momentum?

In an attempt to answer the first question Livio et al. (1986, hereafter Paper I) have performed a three-dimensional numerical calculation of accretion onto a compact object, from a medium containing a density gradient. The calculation was performed neglecting pressure gradients (in the Hoyle-Lyttleton approach) and using the PIC method. The results of Paper I have shown that the accretion rate (of mass) in the non-axisymmetric case, was very similar to the one obtained in the case of accretion from an homogeneous medium (e.g. Bondi & Hoyle 1944; Hunt 1971). The rate of accretion of angular momentum obtained, was very low and amounted to not more than a few per cent of the angular momentum flowing into the symmetrical accretion cylinder.

In the present work we make a first step towards answering the second question above, by including pressure effects in the calculation of an isothermal flow.

The assumptions made, the numerical scheme and the results are given in Section 2 and the results are summarized and discussed in Section 3.

2 The isothermal case, numerical scheme and results

The pseudo-particle scheme that was used to treat the three-dimensional hydrodynamics has been fully described in Paper I, thus we shall not repeat this description here. The new element that was introduced was the inclusion of pressure effects: (i) a calculation of the pressure in each cell, which in our velocity units ($V_m=V_0=1$) reads

$$ P = \frac{1}{\mathcal{U}^2} \varrho $$

where $\mathcal{U}$ is the Mach number for the flow. (ii) Pressure gradients in the equation of motion that were calculated in the following way (see Hensler 1982): If we look at three adjacent grid cells $j-1$, $j$, $j+1$ [say in the $x$ (flow) direction] and a particle, $i$, that is located in the $j$th cell then the $x$ component of the pressure gradient is

$$ \frac{1}{\varrho} \frac{dP}{dX} = \frac{1}{\varrho} \left[ \frac{(P_{j+1}-P_j)(x_i-x_{j+1})+(P_j-P_{j-1})(x_{j+1}-x_i)}{\Delta R^2} \right] $$

where $\Delta R$ is the cell size and $x_j$, $x_{j+1}$ are the boundaries of the respective cells.

The following requirements were fulfilled by all runs: (i) in all the calculations we required that the shock will be 'contained' in the grid, namely, that the shock will not cross the grid edges that are parallel to the flow. (ii) The average number of particles per cell was larger than 4.5 (e.g. Potter 1973).

We have used the same criteria as described in Paper I, to test the stability of the flow.
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The flow direction was taken as the $x$ axis and again, a density gradient was assumed, of the form

$$\rho = \rho_0 \left( 1 + \frac{y}{H} \right).$$  \hspace{1cm} (3)

The number of cells used was $24 \times 28 \times 14$.

We have performed calculations with a Mach number of 4 and $H = 5$ (equation 3) and with a Mach number of 2 and $H = 16$. The density gradients corresponding to each Mach number were chosen in such a way that requirements (i) and (ii) above were fulfilled.

The accretion rates obtained were $M_{\text{acc}} = 0.98 \dot{M}_{\text{HL}}$ for the Mach 4 case and $M_{\text{acc}} = 0.89 \dot{M}_{\text{HL}}$ for the Mach 2 case, where the Hoyle–Lyttleton accretion rate $\dot{M}_{\text{HL}}$ is (see also Bondi & Hoyle 1944)

$$\dot{M}_{\text{HL}} = \frac{4\pi (GM)^2 \rho_0}{V_0^3}. \hspace{1cm} (4)$$

The density profile in the $z = 0$ plane for the Mach 4 case is shown in Fig. 1, and for the Mach 2 case in Fig. 2. We also give the density contours for the symmetrical case in Fig. 3. The shock is very clear and it exhibits the typical broadening as one goes to lower Mach numbers. Another feature that is demonstrated in the figures is the displacement of the accretion cone towards the lower density. The displaced accretion cone is more clearly visible in Fig. 4 which represents the instantaneous location of all particles for the Mach 4 case. The velocity profiles in the $z = 0$ plane are shown in Figs 5 and 6 for the Mach 4 and Mach 2 cases respectively.

Our main interest has been in the accretion of angular momentum. We found an accretion rate of $L_{\text{acc}} = 0.1 L_{BH}$ in the Mach 4 case and $L_{\text{acc}} = 0.14 L_{BH}$ in the Mach 2 case where

$$L_{BH} = \frac{1}{4H} \dot{M}_{\text{HL}} \frac{(2GM)^2}{V_0^3}.$$  \hspace{1cm} (5)

Figure 1. The density profile (represented by the areas of the squares) in the $z = 0$ plane for the Mach 4 isothermal case ($H = 5$, equation 3).
is the rate at which angular momentum is deposited into the (symmetric) Bondi–Hoyle accretion cylinder (see Dodd & McCrea 1952). We therefore find, as in Paper I, where pressure effects were not included, that only a small fraction of the angular momentum assumed to be accreted in previous works (e.g. Shapiro & Lightman 1976; Wang 1981) is actually accreted.

In an attempt to follow the process of depletion of angular momentum for accreted matter (by interactions and angular-momentum transfer), we have followed the mass and angular momentum of a ring about the accretion axis (for simplicity, in a calculation neglecting pressure). The entire ring was contained also in the actual displaced, accretion cylinder cross-section. The results are presented in Fig. 7. The small increase of the angular momentum between points $t'$ and $t''$ (marked only for discussion purposes), is a result of the fact, that due to the displacement of the accretion cone towards lower densities, all parts of the ring do not enter the interaction region simultaneously. We then observe the steep decrease in the angular momentum (as the matter of

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**Figure 2.** Same as Fig. 1 for Mach 2 ($H=16$) case.

**Figure 3.** Density contours the $z=0$ plane, for the homogeneous, isothermal case, Mach=2.
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Figure 4. The instantaneous location of all particles in the inhomogeneous, isothermal, Mach=4 (H=5) case.

the ring collides with other matter) which precedes the decrease in the mass of the ring, as matter starts to be accreted. It is thus demonstrated that the various parts of the mass that is accreted, are depleted of their angular momentum in the accretion cone region, as required in the Bondi–Hoyle (1944) picture and consistently with the point raised by Davies & Pringle (1980).

A different exploratory calculation is described in Fig. 8. In this calculation we do not look for the steady state, but rather observe dynamical effects as accretion is initiated. We start with a cloud of matter (with a density gradient) at some distance from the accreting body and follow it as it hits the compact object. From the figure we see that the accretion rate increases first at $t_0$, as matter encounters the accreting body. It then stays at a constant value, as accretion takes place only from the upstream side (for $t_0<r<t_1$) and then it increases abruptly (at $t_1$) to roughly its final value, as matter starts to accrete from the accretion cone downstream. This demonstrates clearly

Figure 5. The velocity profile in the $z=0$ plane for the Mach 4 (H=5) case.
that most of the accretion takes place via the accretion cone. The angular momentum accreted from upstream (denoted by triangles), stays more or less constant (apart from fluctuations) from $t_0$ onwards. The total rate of accretion of angular momentum (empty circles) increases temporarily as matter starts to accrete from downstream, but then settles to a value only slightly

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**Figure 6.** The velocity profile in the $z=0$ plane for the Mach 2 ($H=16$) case.

**Figure 7.** The mass and angular momentum (relative units) of a ring about the accretion axis. $t'$ and $t''$ are chosen arbitrarily around the increase of angular momentum (see text). The centre of mass of the ring $X_{cm}$ (in units of $2GM/V_0^2$) is shown on the right.
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Figure 8. The accretion rate, from an impinging cloud, the rate of accretion of angular momentum and the rate of accretion of angular momentum from the upstream side $[\dot{L}(V_x>0)]$, in relative units, as a function of time (see text).

above the rate of accretion from upstream. This demonstrates the point noted in Paper I, that much of the accreted angular momentum comes from upstream, from matter hitting the accreting object directly, without passing through the accretion cone.

We should mention that due to the presence of the pressure gradient imposed on our grid (caused by the assumed density gradient), the total angular momentum is not exactly conserved (e.g. it increases by 8.6 per cent in the Mach 4, $H=5$ case). This, however, does not affect our conclusion that only a small fraction of the angular momentum deposited into the accretion cylinder is accreted. This was confirmed by a number of different runs with different conditions (e.g. different sizes of accreting bodies). We also performed one run of Mach 2 ($H=16$) in which we have intentionally taken a narrower grid (which caused the shock to 'escape' through the sides of the grid, a situation not allowed normally, as explained at the beginning of this section), this resulted in a decrease of total angular momentum by ~20 per cent, but nevertheless ~13.1 per cent of the angular momentum entering the (symmetric) accretion cylinder was accreted, in very good agreement with the standard case.

3 Discussion

The mass and angular-momentum accretion rates have been obtained, for accretion from an inhomogeneous medium, in the isothermal case. Bondi (1952) suggested an interpolation formula for the accretion rate between the velocity-dominated and pressure-dominated regimes. This formula can be expressed as

$$\dot{M}_{\text{acc}} = M_{\text{HL}} \frac{\mathcal{H}^3}{(1+\mathcal{H}^2)^{3/2}}$$

where $M_{\text{HL}}$ is given by equation (4) and $\mathcal{H}$ is the Mach number. For the cases calculated in the present work (in all of which $\gamma=1$) this would give $\dot{M}_{\text{acc}}=0.91 M_{\text{HL}}$ for the Mach 4 case and
$\dot{M}_{ac} = 0.72 \dot{M}_{HL}$ for the Mach 2 case. While the numerical values do not agree exactly with the numbers obtained (0.98 and 0.89, respectively), the qualitative trend does agree (the same trend is obeyed by the calculation without pressure, corresponding to hypersonic flow). Two things should be remembered here; (i) equation 6 does not represent an exact solution and (ii) numbers differing by a few per cent only, in the present calculation (having a relatively coarse grid), should be treated with caution.

It has been confirmed (at least in the isothermal case) that the rate of accretion of angular momentum represents only a small fraction of the net angular momentum deposited into the Bondi–Hoyle (symmetrical) accretion cylinder. It should be noted, however, that the rate of accretion of angular momentum when pressure effects are included is somewhat larger than in the Hoyle–Lyttleton picture (neglecting pressure). It can be therefore expected that a somewhat larger fraction of angular momentum will be accreted for $\gamma > 1$. A calculation with $\gamma = 4/3$ is presently being carried out. Also, for any given value of $\gamma$, the rate of accretion of angular momentum can be expected to be somewhat larger for lower Mach numbers.

As already mentioned in paper I, the fact that the rate of accretion of angular momentum is lower than it has been previously assumed, (at least when pressure is neglected and in the isothermal case), can have important consequences for two physical processes: (i) Spin-up (and spin-down) of neutron stars accreting from the winds of early-type companions, and (ii) the possible formation of accretion discs around white dwarfs accreting from the winds of cool giants.

We shall postpone a detailed discussion of these issues, as well as a discussion of individual systems to future work, when we shall have a complete picture of the accretion process for different values of $\gamma$.

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References


Accretion from an inhomogeneous medium – III. General case and observational consequences

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Summary. We study the problem of accretion by a compact object from an inhomogeneous medium, in the general $\gamma \neq 1$ case. The mass accretion rate is found to decrease with increasing $\gamma$. The rate of accretion of angular momentum is found to be significantly lower than the rate at which angular momentum is deposited into the Bondi–Hoyle, symmetrical, accretion cylinder. We discuss the consequences of our results for the cases of neutron stars accreting from the winds of early-type companions and white dwarfs and main-sequence stars accreting from winds of cool giants.

1 Introduction

The classical problem of accretion by a gravitating object, moving through an infinite medium (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952) has regained new interest through the use of multi-dimensional hydrodynamic calculations (e.g. Hunt 1975, 1979; Livio, Shara & Shaviv 1979; Okuda 1983; Shima et al. 1985; Takeda et al. 1985). The problem of accretion from an inhomogeneous medium, however, suffered from both the lack of a basic theory (although see the works of Gething 1951 and Dodd & McCrean 1952) and the need to perform three-dimensional calculations. At the same time, it has been realized that accretion from an inhomogeneous medium has important consequences for such processes as spin-up and disc formation, in the case of compact objects accreting from stellar winds. In an attempt to produce results that can be related to observations, several authors have therefore used the Bondi–Hoyle (1944) picture to argue that all the angular momentum deposited into the symmetrical Bondi–Hoyle accretion cylinder is actually accreted (Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). Davies & Pringle (1980) were the first to point out that in the Bondi–Hoyle picture, for matter to be accreted at all, a cancellation of the momentum transverse to the accretion line is required and thus no angular momentum can be accreted. It was not clear, however, whether this conclusion

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remains valid in a realistic three-dimensional case, in which the 'accretion line' broadens into a column or a cone (as argued by Wang 1981).

In an attempt to resolve the question of accretion from an inhomogeneous medium, we have performed a three-dimensional calculation, first neglecting pressure (Livio et al. 1986, hereafter LSKS) and then for an isothermal flow (Soker et al. 1986, hereafter SLKS). We found that for those cases, while the mass accretion rate was very close to the one predicted by the Bondi–Hoyle theory (for the homogeneous case), the rate of accretion of angular momentum was very much lower than the rate assumed by previous authors (Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). In that present work we expand upon our previous work and calculate the general case of $\gamma\neq1$ ($\gamma$ – the specific heats ratio). The equations and method of calculation are described in Section 2, our results are presented in Section 3 and discussed in Section 4.

2 Equations and method of calculation

The method of calculation used is the same as that described by LSKS and SLKS (apart from the treatment of the energy equation); we shall thus describe it only briefly for completeness.

2.1 EQUATION OF MOTION (FOR PARTICLES)

$$\frac{d^2r}{dt^2} = -\frac{1}{r} \frac{1}{2r^3} \nabla P + a_i$$

(1)

where the unit length was chosen as $R_{HL} = 2GM/V_0^2$ and the unit time as $R_{HL}/V_0 (V_0=1)$. The inter-particle interaction is represented by $a_i$. The pressure gradient term was calculated as in SLKS.

2.2 INTER-PARTICLE INTERACTION

The velocity of each particle following the interaction is given by (see also Lin & Pringle 1976; Hensler 1982)

$$v_{\text{new},i} = v_i (1-\alpha) + \alpha u_i$$

(2)

where $\alpha$ is a parameter defining the strength of the interaction (typically taken as 1) and

$$u_i = v_{i,j} + \vec{R}_i \times \Omega_j$$

(3)

where $v_{i,j}$ is the centre of mass velocity of the $j$th cell and $\vec{R}_i$ is the particle’s coordinate in the centre of mass (of the cell) frame. The angular velocity $\Omega$ is defined by

$$L_i^j = -I_i^k \Omega_j^k$$

(4)

where $L_j$ is the angular momentum of the cell and $I_i^k$ is the moment of inertia tensor components. The inter-cell interaction is treated by the two-grid method described by LSKS.

2.3 THE ENERGY EQUATION

The energy equation was written in general as

$$\frac{\partial}{\partial t} \left[ \rho \left( \frac{1}{2} V^2 + \epsilon \right) \right] + \nabla \cdot \left[ \rho V \left( \frac{1}{2} V^2 + \epsilon + \frac{p}{\rho} \right) + F_i + T_j \right] = \rho V \cdot \mathbf{g}$$

(5)
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where $\epsilon$ is the internal energy, $F_j$ represents the energy dissipation rate in the $j$th cell due to inter-particle interaction, $T_j$ represents the effective rate of heat transport (by inter-particle interactions) and $g$ is the gravitational acceleration. We have calculated the change in energy in two steps; in the first step we calculated the change due to interactions alone (no acceleration due to gravity). We have assumed that the dissipation in kinetic energy is transformed into internal energy and thus the enthalpy $E_j$ of each cell is given by

$$E_j^{\text{new}} = E_j^{\text{old}} + \frac{1}{2} \sum m_i [(V_i^{\text{old}})^2 - (V_i^{\text{new}})^2],$$

where $V_i^{\text{old}}, V_i^{\text{new}}$ represent the particle's velocities before and after the interaction respectively. The specific energy per particle $\epsilon_i$ is related to $E_j$ through

$$E_j = \gamma \sum m_i \epsilon_i.$$  

In the second, acceleration step, we have

$$e_i^{\text{new}} = e_i^{\text{old}} + \frac{1}{\gamma} \left[ \frac{1}{2} (V_i^{\text{old}})^2 - \frac{1}{2} (V_i^{\text{new}})^2 + \frac{1}{2} \left( \frac{1}{r_i^{\text{new}}} - \frac{1}{r_i^{\text{old}}} \right) \right]$$

where old and new in this case refer to the stages before and after the acceleration has taken place. The pressure in the $j$th cell is calculated by (cell size normalized)

$$P_j = (\gamma - 1) \sum m_i \epsilon_i.$$  

For the particles that are injected into the grid (see LSKS) we have

$$\epsilon_{in} = [M^2 \gamma(\gamma - 1)]^{-1}$$

where $M$ is the Mach number.

We have used the same criteria as described by LSDS for the establishment of a (quasi) steady state. Following that, we have carried out the different runs for 35 crossing times of the grid and then average values of the physical quantities were calculated. The grid in all runs (apart from a few test runs to be described shortly) contained $24 \times 28 \times 14$ cells (use was made of the $z=0$ symmetry plane). The accreting body was represented by a cube of size 0.15 (in our unit of length). The average number of particles per cell was four. These numbers were chosen based on trial runs and the constraints imposed by the maximum allowable memory on the IBM 3081D. A number of tests with different grid sizes (e.g. $32 \times 24 \times 12$, $32 \times 20 \times 10$) and different average numbers of particles per cell (e.g. 6.37, 3.2) were performed and we shall discuss the effects of such changes in the next section, when we present the results.

3 Results

In all calculations we have used a density profile at infinity of the form

$$\rho = \rho_0 \left( 1 + \frac{y}{H} \right),$$

where the flow direction was taken as the $x$ axis. In the present work we have used $H=16$ (other values of $H$ have been used in LSKS and SLKS). We have performed calculations with $\gamma=7/6$ at Mach numbers $M=3, 16, \gamma=4/3, 3/2$, and $5/3$ at Mach number 16. The velocity and density profiles that were obtained in the $z=0$ plane are presented in Figs 1-4. As can be seen in the
Figure 1. (a) The velocity profile in the \( z = 0 \) plane for \( \gamma = 7/6, \text{Mach} = 3, H = 16 \). (b) The density profile (represented by the areas of the squares) in the \( z = 0 \) plane for \( \gamma = 7/6, \text{Mach} = 3, H = 16 \).
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(a)

\[ \Gamma = \frac{7}{6}, \quad \text{Mach} = 16, \quad H = 16 \]

(b)

\[ \Gamma = \frac{7}{6}, \quad \text{Mach} = 16, \quad H = 16 \]

Figure 2. (a) The velocity profile in the \( z = 0 \) plane for \( \gamma = 7/6, \) Mach = 16, \( H = 16 \). (b) The density profile (represented by the areas of the squares) in the \( z = 0 \) plane for \( \gamma = 7/6, \) Mach = 16, \( H = 16 \).
Figure 3. (a) The same as Fig. 2(a) for $\gamma = 4/3$. (b) The same as Fig. 2(b) for $\gamma = 4/3$. 

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Figure 4. (a) The same as Fig. 2(a) for $\gamma=5/3$. (b) The same as Fig. 2(b) for $\gamma=5/3$. 
figures, the shock slightly 'escapes' from the grid in the \( \gamma = 5/3 \) case (also for \( \gamma = 3/2 \)) so we should therefore treat the numerical values obtained in these runs with caution. The results can be summarized as follows:

(i) For a given value of \( \gamma \), the shock angle is larger for a smaller Mach number [e.g. Figs 1(a), 2(a) and figs 1–2 of SLKS]. This is of course a known result from flows past non-gravitating bodies, where the cone angle is \( \text{arcsin}(1/M) \). However, it should be remembered that the shock in the case of a gravitating body is not produced by the fact that the flow directly impinges on the body, but rather by the dense region generated through the gravitational influence.

(ii) For a given Mach number the shock angle is larger for a larger value of \( \gamma \) (Figs 1–4). The same result was found by Shima et al. (1985) in their two-dimensional hydrodynamic study. This can be expected from the fact that, as \( \gamma \) is reduced (towards the isothermal, \( \gamma = 1 \) case), less pressure support is available for the shock. In a realistic flow, the situation with \( \gamma = 1 \) would correspond to a cooling time for the gas that is short compared to the flow time-scale, while \( \gamma = 5/3 \) would correspond to a radiationless case.

The increase of the shock angle with \( \gamma \) was obtained also in the self-similar solutions of Bisnovatyi–Kogan et al. (1979) and Wolfson (1977), corresponding essentially to an infinite accretion radius.

We find (as did Shima et al. 1985) that in the \( \gamma = 5/3 \) case, an 'accretion cone' rather than an 'accretion column' is formed, namely, the density in this case is highest behind the shock and not along an accretion line. Our resolution is not good enough (because of the memory constraints imposed on a three-dimensional calculation) to be able to detect the formation of a bow shock rather than a shock attached to the accreting body.

A very crude estimate of the shock angle (at distances larger than the accretion radius) can be obtained by noting that the post-shock flow is more or less parallel to the accretion axis [e.g. Figs 1(a), 2(a) and 3(a); figs 4 and 5 in Hunt 1971, figs 2 and 3 in Shima et al. 1985]. We then obtain from the shock conditions (see Fig. 5 for the definition of the angles)

\[
\tan \alpha_2 = \tan \alpha_1 \frac{(\gamma-1) M^2 + 2}{2 \gamma M^2}.
\]

From equation (12) it can be seen that for a given (large) \( M \), \( \alpha_2 \) increases for increasing \( \gamma \) and for a given \( \gamma \), \( \alpha_2 \) is a decreasing function of \( M \), as was found in the calculation.

(iii) The cross-section of the accretion cylinder is displaced towards the lower density and so is the accretion column or cone behind the accreting body. This effect is not so pronounced in the present calculation because of the relatively large value of \( H \), but is very pronounced in the larger density gradient calculations of LSKS and SLKS.

(iv) For a given (large) Mach number, the accretion rate decreases with increasing \( \gamma \) (see Table 1). The same result was found by Shima et al. (1985, their fig. 9). It is interesting to note that the

Figure 5. A schematic representation of the pre-shock and post-shock velocities (at distances larger than the accretion radius, see text).
Accretion from an inhomogeneous medium

Table 1. Results of numerical calculations for mass and angular momentum accretion rates. Numbers appearing in parentheses should be viewed with caution (see text). The rate of accretion of angular momentum from upstream is denoted by \( \dot{L}_+ \).

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Mach=2</th>
<th>Mach=4</th>
<th>7/6</th>
<th>4/3</th>
<th>3/2</th>
<th>5/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{M}/\dot{M}_{HL} )</td>
<td>0.89</td>
<td>0.98</td>
<td>0.88</td>
<td>0.72</td>
<td>(0.58)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>( \dot{L}/\dot{L}_{BH} )</td>
<td>0.14</td>
<td>0.10</td>
<td>0.17</td>
<td>0.23</td>
<td>(0.18)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>( \dot{L}_+ /\dot{L} )</td>
<td>0.34</td>
<td>0.23</td>
<td>0.37</td>
<td>0.43</td>
<td>(0.49)</td>
<td>(0.50)</td>
</tr>
</tbody>
</table>

The (maximal) accretion rate obtained in the case of spherically symmetrical accretion from a stationary cloud (Bondi 1952) behaves similarly. This of course reflects the effect of the pressure that builds up, in the dense region, on the accretion rate. With respect to the dependence on the Mach number, the isothermal calculation of SLKS has shown the dependence of the accretion rate on the Mach number to agree qualitatively with the Bondi (1952) interpolation formula (with an additional factor of 2, see also Shima et al. 1985; Livio 1986). We can, therefore, write the accretion rate as \( (M_{CO} - \text{the mass of the compact object}) \)

\[
\dot{M}_{acc} = \alpha(\gamma) \frac{M^3}{(1 + M^2)^{3/2}} \dot{M}_{HL} = \alpha(\gamma) \frac{M^3}{(1 + M^2)^{3/2}} \frac{4\pi(GM_{CO})^2}{V_0^3},
\]

with \( \alpha(\gamma) \) an almost linearly decreasing function of \( \gamma \), the values of which are approximately given by \( \dot{M}/\dot{M}_{HL} \) in Table 1 (at least for \( \gamma \leq 4/3 \), \( \dot{M}_{HL} \) being the Hoyle–Lyttleton value. In the results of Bondi (1952) also a close to linear relation appears.

(v) The rate of accretion of angular momentum (see Table 1) is in all cases significantly less than the rate at which angular momentum is deposited into the symmetrical Bondi–Hoyle accretion cylinder (which has been assumed to be the rate of accretion of angular momentum by Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). The rate of accretion of angular momentum is smaller \( (L/L_{BH} \approx 0.1) \) in the isothermal and hypersonic cases than in \( \gamma \neq 1 \) cases (when \( L/L_{BH} \approx 0.2 \)), here

\[
L_{BH} = \dot{M}_{HL} \frac{1}{H} \frac{(GM_{CO})^2}{V_0^3}.
\]

Furthermore, of the accreted angular momentum a significant part comes from upstream (denoted by \( \dot{L}_+ /\dot{L} \) in Table 1), from matter that hits the (relatively large) accreting body directly without passing through the interaction region downstream.

The fact that the rate of accretion of angular momentum is much lower than that expected naively, by calculating the rate at which angular momentum enters the symmetrical Bondi–Hoyle cylinder, is in fact consistent with the Bondi–Hoyle picture, in which the matter that is actually accreted cannot have high specific angular momentum. This has been confirmed by following the mass and angular momentum of an accreted ring of mass (see SLKS). Our calculation thus supports the suspicion, first raised by Davies & Pringle (1980), that relatively very little angular momentum can be accreted from an inhomogeneous medium.

Test runs performed with other grid sizes and average numbers of particles per cell have
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shown that: (a) The results do not change when an average number of 6.37 particles per cell is used (instead of 4); however, the calculation tends to become unstable when the average number is smaller than 3.2. (b) When different grid sizes were used (e.g. $32 \times 24 \times 12$) differences of at most 8 per cent in the accretion rate (but smaller in the angular momentum accretion rate) were found. These could usually be attributed to either a reduction in the average number of particles per cell in the downstream side, or the shock slightly 'escaping' through the sides of the grid. Nevertheless, possible errors in the quoted values of up to a few per cent have probably to be realistically assumed, due to the relatively coarse grid.

In Section 4 we shall discuss some of the possible implications that our results may have for compact objects accreting from a stellar wind.

4 Discussion

Accretion by a compact object, from an inhomogeneous medium, occurs in the case of a neutron star accreting from the stellar wind of an early-type companion and in the case of a white dwarf (or a main-sequence star) accreting from the wind of a cool giant. We shall discuss each of these classes separately in the context of the results of the present work (see also the discussions by White 1985; Henrichs 1983; Livio & Warner 1984; Livio 1986).

In Table 2 we present the parameters for some of the better studied X-ray binaries (taken from Wang 1981; White 1985; Eisner et al. 1985, and references therein). We would like to discuss the implications of our results for three properties of these binaries: (i) the X-ray luminosity, (ii) the spin-up (or spin-down) rate, and (iii) the possibility of forming an accretion disc.

(i) The luminosity. The accretion rate can be expressed as

$$\dot{M}_{\text{acc}} = \frac{\delta 4\pi G^2 M_x^2 \rho}{V_{\text{rel}}^3},$$

where we have neglected the speed of sound compared to the relative velocity (between the neutron star and the wind) and $\delta = \dot{M}_{\text{acc}}/\dot{M}_{\text{HL}}$ represents the deviation from the Hoyle–Lyttleton (1939) value (as found in Table 1). We shall now assume a spherically symmetrical wind from the giant, with $V_w = V_{\text{rel}}$ (actually a questionable assumption, as will be discussed later). We adopt an average value of $\delta = 0.8$ (see Table 1 and Shima et al. 1985) and for the neutron star we take $M_x = 1 M_\odot$, $R_x = 10^6$ cm. Equation (15) can then be expressed as

$$V_{\text{rel}} = 4.1 \times 10^7 \left(\frac{\delta}{0.8}\right)^{1/4} \left(\frac{M_x}{M_\odot}\right)^{3/4} \left(\frac{R_x}{10^6 \text{ cm}}\right)^{-1/4} \left(\frac{\dot{M}_w}{10^{-6} M_\odot \text{ yr}^{-1}}\right)^{1/4} \times \left(\frac{L_x}{10^{37} \text{ erg s}^{-1}}\right)^{-1/4} \left(\frac{a}{30 R_\odot}\right)^{-1/2} \text{ cm s}^{-1},$$

where $L_x$ is the X-ray luminosity, $\dot{M}_w$ is the rate of mass loss from the giant and $a$ is the separation. The resulting relative velocities are listed in Table 2 under the column labelled $V_{\text{rel}}$ (luminosity). We shall discuss the values that have been obtained after studying the implications of the spin-up and the possible existence of a disc ($\dot{M}_w$ has been taken from White 1985, and references therein).

(ii) Spin-up (or spin-down) rates. We shall ignore for the moment the question of whether spin-up (which occurs most of the time) or spin-down is actually observed, and treat average values of $P/P_s$ ($P_s$, the spin period) observed over relatively long time-scales (short time-scale variations will be mentioned later). The observed $P/P_s$ can be directly related to an implied rate
Table 2. Parameters of X-ray binaries and required relative velocities (see text).

<table>
<thead>
<tr>
<th>Object</th>
<th>$P_{\text{orb}}$ (d)</th>
<th>$P_e$ (sec)</th>
<th>$L_x$ ($10^{37}$ ergs s$^{-1}$)</th>
<th>$\dot{m}$/$P_e$ (sec$^{-1}$)</th>
<th>$V_{\text{rel}}$ (luminosity) cm s$^{-1}$</th>
<th>$V_{\text{rel}}$ (spin-up) cm s$^{-1}$</th>
<th>$V_{\text{rel}}$ (disk) cm s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cen X-3</td>
<td>2.09</td>
<td>4.84</td>
<td>5</td>
<td>9.3 x $10^{-12}$</td>
<td>3.4 x $10^7$</td>
<td>5.3 x $10^7$</td>
<td>3.5 x $10^7$</td>
</tr>
<tr>
<td>Vela X-1</td>
<td>8.96</td>
<td>283</td>
<td>0.14</td>
<td>3.0 x $10^{-12}$</td>
<td>5.9 x $10^7$</td>
<td>5.5 x $10^7$</td>
<td>2.2 x $10^7$</td>
</tr>
<tr>
<td>1538-52</td>
<td>3.73</td>
<td>529</td>
<td>0.4</td>
<td>&lt; 3.2 x $10^{-11}$</td>
<td>4.7 x $10^7$</td>
<td>&gt; 5.8 x $10^7$</td>
<td>4.7 x $10^7$</td>
</tr>
<tr>
<td>GX 301-2</td>
<td>41.5</td>
<td>699</td>
<td>0.3</td>
<td>10^{-9}</td>
<td>3.3 x $10^7$</td>
<td>1.3 x $10^7$</td>
<td>1.5 x $10^7$</td>
</tr>
<tr>
<td>SMC X-1</td>
<td>3.89</td>
<td>0.714</td>
<td>50</td>
<td>2.0 x $10^{-11}$</td>
<td>1.4 x $10^7$</td>
<td>4.1 x $10^7$</td>
<td>3.3 x $10^7$</td>
</tr>
<tr>
<td>0352+31</td>
<td>581</td>
<td>835</td>
<td>1.2 x $10^{-3}$</td>
<td>5.3 x $10^{-12}$</td>
<td>3.8 x $10^7$</td>
<td>0.7 x $10^7$</td>
<td>0.6 x $10^7$</td>
</tr>
<tr>
<td>0535+26</td>
<td>&gt; 20d</td>
<td>104</td>
<td>2</td>
<td>3.2 x $10^{-10}$</td>
<td>5.1 x $10^7$ (for $a=30R_0$)</td>
<td>2.1 x $10^7$ (for $P_{\text{orb}}=20d$)</td>
<td>1.9 x $10^7$</td>
</tr>
<tr>
<td>1700-33</td>
<td>3.4</td>
<td>--</td>
<td>0.04</td>
<td>--</td>
<td>1.6 x $10^8$</td>
<td>--</td>
<td>2.6 x $10^7$</td>
</tr>
<tr>
<td>Cyg X-1</td>
<td>5.6</td>
<td>--</td>
<td>0.6</td>
<td>--</td>
<td>2.1 x $10^8$</td>
<td>--</td>
<td>2.6 x $10^8$</td>
</tr>
<tr>
<td>GX 1+4</td>
<td>--</td>
<td>122</td>
<td>4</td>
<td>7.0 x $10^{-10}$</td>
<td>--</td>
<td>4.8 x $10^7$ $(P_{\text{orb}}^{-1/4})$</td>
<td>4.2 x $10^7$ $(P_{\text{orb}}^{-1/4})$</td>
</tr>
</tbody>
</table>
of accretion of angular momentum by

\[ L_{\text{obs}} = \left| \frac{\dot{P}}{P_{\text{obs}}} \right| \left( \frac{2\pi}{P_{\text{obs}}} \right) I_x \]

\[ = 6.28 \times 10^{32} \left[ \frac{\dot{P}/P_{\text{obs}}}{10^{-11} \text{s}^{-1}} \right] \left( \frac{P_s}{100 \text{s}} \right)^{-1} \left( \frac{I_x}{10^{45} \text{g cm}^2} \right) \text{ dyn cm}, \]

(17)

where \( I_x \) is the moment of inertia of the neutron star (e.g. Lamb, Pethick & Pines 1973). The predicted rate of accretion of angular momentum based on the present study is

\[ L_{\text{pred}} = 3.89 \times 10^{32} \left[ \frac{\xi}{0.2} \right] \left( \frac{L_x}{10^{37} \text{erg s}^{-1}} \right) \left( \frac{M_x}{M_\odot} \right) \left( \frac{P_{\text{orb}}}{1 \text{ day}} \right)^{-1} \]

\[ \times \left( \frac{R_x}{10^6 \text{cm}} \right) \left( \frac{V_{\text{rel}}}{10^8 \text{cm s}^{-1}} \right)^{-4} \text{ dyne cm}, \]

(18)

where \( \xi = l/l_{\text{BH}} \) is the ratio of the accreted specific angular momentum (according to our calculations, Table 1) to the specific angular momentum of matter that is deposited into the Bondi–Hoyle, symmetrical, accretion cylinder. Equating the rates in equations (17) and (18), gives us for the relative velocity the values listed in Table 2 under \( V_{\text{rel}} \) (spin-up). We have adopted \( M_x = M_\odot, R_x = 10^6 \text{cm}, \xi = 0.2, \) and \( I_x = 10^{45} \text{ g cm}^2 \).

(iii) The possibility of forming a disc from wind accretion. An important question in the case of a compact object accreting from a stellar wind is whether an accretion disc can be formed. The radius at which a disc can start forming can be obtained by equating the specific angular momentum of the accreted matter to that in a Keplerian disc \( l = (GM_xr_D)^{1/2} \). In the case in which the compact object is a magnetized neutron star, it is necessary, for a disc to form, that the resultant \( r_D \) will be larger than the magnetospheric radius \( R_M \). This imposes the following condition on the relative velocity (Shapiro & Lightman 1976; Wang 1981)

\[ V_{\text{rel}} \leq 4.0 \times 10^7 \left( \frac{\xi}{0.2} \right)^{1/4} \mu_30^{-1/14} \left( \frac{M_x}{M_\odot} \right)^{5/14} \left( \frac{P_{\text{orb}}}{1 \text{ day}} \right)^{-1/4} \left( \frac{R_x}{10^6 \text{cm}} \right)^{1/28} \]

\[ \times \left( \frac{L_x}{10^{37} \text{erg s}^{-1}} \right)^{1/28} \text{ cm s}^{-1} \]

(19)

where \( \mu_30 \) is the neutron star's magnetic moment (in units of \( 10^{30} \text{erg g}^{-1} \)). The upper limits on the relative velocity for disc formation are listed in Table 2 under \( V_{\text{rel}} \) (disc). In the case of Cyg X-1, the upper limit is derived by requiring the disc radius to be larger than the innermost stable orbit around the (possible) black hole (we have adopted \( M_x = 10 M_\odot \)).

Let us now discuss the implications of the relative velocities obtained in Table 2. The most striking general property, that is revealed by examining Table 2, is the fact that almost all the required relative velocities are much smaller than those that could be expected for radiatively driven winds, typically of order \( V_w \sim 1000-2000 \text{km s}^{-1} \). In fact, column density estimates, obtained from the attenuation of the X-ray spectrum by photoelectric absorption, also seem to indicate low velocities (e.g. White 1985). Even from this result alone, we can therefore immediately conclude that the simple picture of a smooth, spherically symmetrical, radiatively driven wind, is in general not applicable. The two major factors that can both change the wind-flow picture and produce significantly smaller wind velocities (Roche lobe overflow will be discussed separately) are: (i) ionization by the X-ray source, which can decrease the radiatively
driven, UV line accelerations (e.g. Hatchett & McCray 1977; MacGregor & Vitello 1982; Dupree et al. 1980) and (ii) a wind flow concentrated towards the compact object (and in fact resembling Roche lobe overflow), caused by the primary being close to filling its lobe (Friend & Castor 1982).

We shall now look at some of the individual systems and examine what can be learned about each of them. For Cen X-3, the relative velocity required to explain the luminosity and for disc formation is in fact lower than the orbital velocity (see also Conti 1978; Petterson 1978). The velocity derived from $\dot{P}$ is only slightly larger than the orbital velocity. Taking into account the facts that the spin-up appears quite smooth (Rappaport & Joss 1983) and that there is additional evidence suggesting the presence of a disc (lack of flaring due possibly to smoothing of fluctuations, evidence for scattering from a possible disc in the spectrum), we have to conclude that Roche lobe overflow must occur in this system, at least occasionally. The situation is almost identical in the case of SMCX-1; we conclude therefore that Roche lobe overflow occurs in that system too. An additional system, on which there is less information available, but for which the smooth spin-up rate (and the short spin-up time-scale, Elsner et al. 1985) would require unreasonably low relative velocities is GX1+4; we therefore predict that Roche lobe overflow occurs in that system. The situation is somewhat less clear regarding 1538–52, where the velocity required to explain the luminosity (and for the possibility of forming a disc) appears to be smaller than the orbital velocity but there are no good spin-up data to support this conclusion. More observations related to spin-up or to the possible existence of a disc are required to establish whether Roche lobe overflow is expected in this case too.

It should be pointed out that if an accretion disc is formed, even temporarily, it can then spread by viscous angular momentum transport (see Livio 1986, for a discussion).

The velocities required to explain the luminosity and average spin-up (and spin-down) rates of Vela X-1 are consistent with a stellar wind. However, the wind velocity should be significantly reduced with respect to the unperturbed, radiatively driven wind. In addition, a considerable amount of inhomogeneities in the wind, on several scales, is required to explain $\dot{P}/P$ as high as $1.8 \times 10^{-10} \text{s}^{-1}$ (Boynton et al. 1984). Under such circumstances, the spin-up and spin-down behaviour is consistent with a random noise process (Boynton et al. 1984). Less information is available on 1700–33, but accretion from a stellar wind appears consistent with the observations existing so far on this object. In the case of Cyg X-1, it appears possible in fact for an accretion disc to form from wind accretion.

The situation is quite complicated concerning GX301-2. The wind there is clearly variable due to the fact that the orbit is elliptical $e=0.47$. The velocities (Table 2) that are necessary in order to explain the luminosity and the spin-up are extremely low. The absence of a smooth spin-up behaviour, together with the extremely low velocity required to form a disc, argue against the existence of a disc in this system. This is consistent with the fact that WRA 977 is not close to filling its Roche lobe and with the absence of any lag between the 41.5 day period outbursts and the times of periastron passage (White & Swank 1984).

The second class of objects for which accretion from an inhomogeneous medium is applicable involves white dwarfs (or main-sequence stars) accreting from the winds of cool giants. This class includes such objects as (Livio & Warner 1984; Livio 1986): Mira AB, SY For, $\zeta$ Cap, $\xi$ Cyg, S6 Peg, $\xi$ Aur, 32 Cyg, 31 Cyg, 22 Vul, and possible $\nu$ Her, HR 3080, and HR 8157. The condition on the relative (and wind) velocity in this case can be written as

$$V_{\text{rel}} = \left( \frac{V_w}{V_{\text{rel}}} \right)^{1/4} \approx 1.3 \times 10^6 \left( \frac{d}{0.8} \right)^{1/4} \left( \frac{M_{\text{WD}}}{0.6 M_\odot} \right)^{3/4} \left( \frac{M_w}{10^{-7} M_\odot \text{ yr}^{-1}} \right)^{1/4} \left( \frac{R_{\text{WD}}}{9.5 \times 10^8 \text{ cm}} \right)^{-1/4} \times \left( \frac{a}{10^{15} \text{ cm}} \right)^{-1/2} \left( \frac{L_{\text{acc}}}{10^{33} \text{ erg s}^{-1}} \right)^{-1/4} \text{ cm s}^{-1},$$

(20)
where we have used the average mass of single white dwarfs (Koester et al. 1979) due to the large separation (and the appropriate white dwarf radius). For an accretion disc to form, the radius of the disc must be larger than the radius of the white dwarf (for non-magnetic white dwarfs); this implies the condition

$$V_{\text{rel}} \leq 3.7 \times 10^6 \left( \frac{\xi}{0.2} \right)^{1/4} \left( \frac{M_{\text{WD}}}{0.6 M_\odot} \right)^{3/8} \left( \frac{P_{\text{orb}}}{10 \text{ yr}} \right)^{-1/4} \left( \frac{R_{\text{WD}}}{9.5 \times 10^8 \text{ cm}} \right)^{-1/8} \text{ cm s}^{-1}. \quad (21)$$

Recent IUE observations of Mira B have claimed the existence of an accretion disc around the white dwarf (Reimers & Cassatella 1985; Cassatella et al. 1985, and see also the description of the optical spectrum of Yamashita & Maehara 1977). In trying to establish the possibility of forming a disc from wind accretion in this system, we face the unpleasant situation of no known orbital period. Fernie & Brooker (1961) found the possible solutions of 59, 169 and 261 yr of which the last one was considered the most plausible. Hopmann (1964) found 139 and 842 yr. Baize (1980) found 400 yr (quoted as private communication from P. Couteau by Reimers & Cassatella 1985). Walker (1985) found all existing orbits to be bad. Since the period is likely to be larger than 100 yr (van Biesbroeck 1959) and the average value of all the estimated periods above 100 yr is 362 yr, we shall adopt 400 yr as the period; it should be remembered, however, that this should not be regarded as an accurate determination. Using $M_w = 10^{-7} M_\odot \text{ yr}^{-1}$ (Reimers & Cassatella 1985), $a = 9.8 \times 10^4 \text{ cm}$ (Jenkins 1952) and $L_{\text{acc}} = 10^{33} \text{ erg s}^{-1}$ (a lower limit of $3.3 \times 10^{32} \text{ erg s}^{-1}$ is indicated, Reimers & Cassatella 1985) we find $V_{\text{rel}} (V_u/V_{\text{rel}})^{1/4} \approx 1.31 \times 10^6 \text{ cm s}^{-1}$. Now $V_u = 5.6 \times 10^6 \text{ cm s}^{-1}$ (Wannier et al. 1980) giving $V_{\text{rel}} = 1.7 \times 10^6 \text{ cm s}^{-1}$ which is in reasonable agreement with the assumed orbit (giving $V_{\text{rel}} \approx 1.1 \times 10^6 \text{ cm s}^{-1}$). Now the condition for disc formation (equation 20) reads $V_{\text{rel}} \approx 1.5 \times 10^6 \text{ cm s}^{-1}$ which indicates, considering the uncertainties, that disc formation is indeed possible in this system. The initial disc radius that is obtained if we take $V_{\text{rel}} = 1.1 \times 10^6 \text{ cm s}^{-1}$ is $R_\text{d} = 10^{10} \text{ cm}$ and thus much smaller than the one obtained by Reimers & Cassatella who used $\xi = 1$. However, once a disc forms, it spreads due to viscous transport of angular momentum and thus the observational determination of $R_\text{d} = 10^{11} \text{ cm}$ by Reimers & Cassatella may be correct.

A different system which quite clearly contains an accretion disc around the mass-gainer star is RZ Oph (Olson & Hickey 1983; Baldwin 1978). While an inclination of $i = 76^\circ$ which would have enabled the K5 mass-losing star to fill its Roche lobe has been suggested by Smak (1981), it has been argued by Olson & Hickey (1983) that $80^\circ \leq i \leq 88^\circ$.

If we adopt the parameters of Olson & Hickey (1983) for the mass-gainer F star, $M = 3 M_\odot$, $R = 3.8 R_\odot$, we find that for a disc to form ($P = 262$ day) we must have $V_{\text{rel}} \leq 6.8 \times 10^6 \text{ cm s}^{-1}$. This would require a wind velocity $V_w \approx 3.8 \times 10^6 \text{ cm s}^{-1}$. We cannot entirely exclude, therefore, the possibility that an accretion disc does form from wind accretion. However, the large dimensions of the disc would suggest to us that Roche lobe overflow (or at least a wind concentrated towards the accreting star, Friend & Castor 1982) does occur in this system.

A different group of objects for which the presence of a disc generated by wind accretion has been suggested (at least for $\xi$ Aur and $\delta$ Sge) are the $\xi$ Aur binaries (Che, Hempke & Reimers 1983; Che-Bohnenstengel & Reimers 1985). Using the same parameters as Che-Bohnenstengel & Reimers (1985), but introducing $\xi = 0.2$ (equation 20) in the rate of accretion of angular momentum, as indicated by our results, makes disc formation from a wind impossible in the case of $\xi$ Aur and only marginally possible for $\delta$ Sge. Indeed the extremely high temperatures ($\approx 70,000 \text{ K}$) quoted for the disc in $\xi$ Aur cannot occur in a steady disc model around a star with a radius of $R = 3.6 \times 10^{11} \text{ cm}$ (which would rather give temperatures of order $\sim 400 \text{ K}$). More observations of these systems and a possible re-interpretation of the observations (very probably in terms of a shocked region) are thus strongly recommended.
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A re-examination of the systems 56 Peg and ζ Cap (discussed by Livio & Warner 1984) reveals that the formation of an accretion disc from wind accretion becomes only marginally possible in the case of 56 Peg (R_D~8×10^8 cm) while it becomes impossible for the assumed parameters of ζ Cap. This may explain the appearance of only very narrow (FWHM~114 km s^{-1}) UV emission lines in 56 Peg (Schindler et al. 1982). More observations of these systems are encouraged.

To conclude, we have established the dependence of the accretion rate on the specific heat ratio. The rate of accretion of angular momentum from an inhomogeneous medium is significantly lower than has been previously assumed. The results on the rate of accretion of angular momentum of the present study can be used to place severe constraints on models for systems involving a compact object accreting from the stellar wind of its companion. More observations of such systems, in particular in the case of white dwarfs and main-sequence stars accreting from the winds of cool giants, are extremely important for a better understanding of the accretion process.

References

CHAPTER V

NOTES ON THE THEORY OF COMMON ENVELOPE EVOLUTION
The evolution of binary stars through a common envelope phase, in which the binary is entirely surrounded by one gaseous envelope, is one of the least understood stages of binary evolution. This is an unsatisfactory situation since the formation of such a common envelope is believed to be a crucial stage in the evolution of all short-period binaries that contain at least one compact star. The best known examples of these are the short-period low-mass X-ray binaries and binary radio-pulsars, in which the compact star is a neutron star, and the cataclysmic variables in which the compact star is a white dwarf. The conclusion that a common envelope (CE) phase has occurred in the history of these binaries is unavoidable, since the progenitor of the compact star must have been a giant, with dimensions far greater than the present binary separation. During this phase the compact star or its progenitor (the core of the giant) and the companion must have reduced their separation, while the envelope of the giant was ejected from the system (Paczynski 1976, Webbink 1979). Direct evidence for the actual occurrence of this type of evolution in nature can be found in the short-period double cores of planetary nebulae (Bond, 1985).

To gain a better understanding of the evolutionary history of presently observed binary systems of the types mentioned above, we would like to be able to predict the final outcome of the CE phase from the parameters of the binary system just before the formation of the CE. In particular we want to know whether the giant core and the companion will eventually coalesce, or, if this is not the case, what the orbital period of the remaining binary will be. One simple approach to this question, that has been used widely in the literature (see eg. Chapter III.2), is to assume that all gravitational energy that is gained by the reduction of the orbital separation between companion and giant core is used to eject the envelope. Since the separation can not be reduced beyond the point at which the companion fills its Roche-lobe if the system is to survive as a binary, there is a maximum to the amount of energy to be gained in this way. By comparing this to the amount of energy necessary to eject the envelope, one can decide from the details of the initial configuration whether it is possible to eject the envelope before companion and core coalesce. If this is the case, the
method also yields an upper limit on the final separation. One of the purposes of a more detailed study of the CE-phase is to investigate whether the assumptions entering this estimate are justified.

In the literature there have been two ways of approaching the problem of CE evolution, which are probably applicable for distinct initial conditions. The first model (Meyer and Meyer-Hofmeister, 1978, hereafter referred to as MMH) applies when the giant star is corotates with the orbital motion (or at least very near) at the time it starts to transfer mass to its companion. In this case the Roche-geometry can be used to describe the mass transfer, which greatly simplifies the problem. A CE can form around the binary in not too violent a way, without significant mass loss from the system. The second approach (Taam, Bodenheimer and Ostriker, 1978; Livio and Soker 1984; Bodenheimer and Taam, 1984), which describes the start of the CE phase as a plunge of the companion star into the more or less stationary giant envelope, is appropriate when the giant is far from corotation. In this review I shall first discuss what determines the rotation rate of the giant at the time mass transfer commences, and then give a more detailed description of the two approaches mentioned above.

V.2 THE FORMATION OF A COMMON ENVELOPE

When a single star evolves off the main sequence towards the giant stage its rotation rate is expected to decrease significantly as the moment of inertia increases very rapidly, while the total amount of angular momentum available remains constant. If the star is in a binary tidal interaction will transfer orbital angular momentum into the spin angular momentum of the giant. When the stellar radius becomes comparable to the binary separation (or equivalently, the giant almost fills its Roche-lobe) this tidal interaction becomes very effective (see e.g. Zahn, 1978), and because of the high turbulent viscosity in the convective giant envelope the tidal spin-up timescale will become shorter than the evolutionary expansion timescale, on which the stellar moment of inertia is changing (Alexander 1973, Zahn 1978). This means that the giant will be nearly corotating with the orbital motion when it starts to transfer mass to its companion.

A necessary condition for this scenario is however that the binary is tidally stable (Counselman 1973), i.e. that the increase in orbital angular velocity ($\omega_{\text{orb}}$) due to orbital angular momentum lost to the
The giant is not greater than the increase in rotational angular velocity of
the giant \((\omega_{\text{rot}})\) due to angular momentum gained from the orbit, i.e.
\[
\left| \left( \frac{\partial \omega_{\text{orb}}}{\partial J} \right) \right| < \left| \left( \frac{\partial \omega_{\text{rot}}}{\partial J} \right) \right|
\]  
(1)

Using Kepler's laws it is easily shown that this is equivalent to the
condition that the moment of inertia of the giant has to be less than
1/3 of the orbital moment of inertia. If we take typical values for the
moment of inertia of a giant as found from evolutionary calculations,
and assume that the giant nearly fills its Roche lobe, the above
condition for stability translates to the more practical condition that
\(q\), the ratio of the mass of the companion to that of the giant, must be
greater than 1/6 (Sparks and Stecher, 1974; MMH).

Hence, if \(q>1/6\) the giant is expected to be rotating almost
synchronously with the orbit at the onset of mass transfer. Following
the Roche-geometry, matter will flow through the L1 point into an
accretion disk around the companion. What happens then is again
determined by the value of \(q\). If \(q>q_{\text{crit}}\) (where \(q_{\text{crit}}\) varies between
0.836 and 1.2, see below) the response of the binary to mass transfer is
an increase in orbital separation, and hence also in the size of the
Roche lobe of the giant (see eg. Webbink, Rappaport and Savonije,
1983). In this case the mass transfer will remain stable, since an increase in
mass transfer will cause an extra increase in the size of the Roche-
lobe, which in turn reduces the mass transfer rate. If however \(q<q_{\text{crit}}\),
the response of the Roche-lobe of the giant is to shrink because of the
mass transfer, which causes the mass transfer rate to become even
greater, which is an obviously unstable situation. This instability is
aggravated when the mass transfer rate has become so high that the giant
can no longer maintain thermal equilibrium. The extended convective
envelope of the giant is to first approximation isentropic, and will
behave like an \(n=1.5\) polytrope, i.e. it will expand when its mass is
reduced. If the mass loss is sufficiently slow, this behaviour is
suppressed because the entropy of the envelope will adjust itself (on a
thermal timescale) in such a way that the radius of the star remains
approximately constant. If the timescale for mass loss becomes shorter
than the thermal timescale the giant will start to expand adiabatically,
increasing the transfer rate even more. A star which becomes subject to
this instability will transfer a significant fraction of its envelope on
a dynamical timescale, which is of the same order as the orbital period.

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(In fact, a giant with \( q > q_{\text{crit}} \) is also potentially unstable to this last instability, but the stable transfer rates never become high enough to disturb the thermal equilibrium). The exact value of \( q_{\text{crit}} \) depends on the details of the mass transfer process. If all mass transferred is accreted by the companion \( q_{\text{crit}} = 1.2 \), whereas if all mass transferred is lost from the system carrying the specific orbital angular momentum of the companion (as is probably the case in super-critical disk accretion with matter being carried away in jets) \( q_{\text{crit}} = 0.836 \) (de Kool, Rappaport and van den Heuvel, 1986). Summarizing, we have the following situation.

In the case \( q < 1/6 \), i.e. a giant which is much more massive than its companion, the binary is tidally unstable and will not be corotating with the orbit when (unstable) mass transfer starts. If \( 1/6 < q < q_{\text{crit}} \) the mass transfer is still unstable, but can at least initially be described as Roche-lobe overflow. If \( q > q_{\text{crit}} \) the giant will be corotating, and the mass transfer is stable. Typical mass transfer rates in this case are \( 10^{-6} \) to \( 10^{-9} \) M\(_{\odot} \)/yr (Webbink, Rappaport and Savonije 1983), far less than the rates needed to form a common envelope.

The actual formation of the common envelope is a 3-dimensional, time dependent hydrodynamical problem involving very different length and time scales and hence remains rather obscure, although there are a few model calculations available in the literature that relate to the problem. The models which are most applicable to the corotating case are probably those of Sawada et al. (1984). These are 2-dimensional hydrodynamic calculations using cylindrical coordinates \((r, \phi, z)\) in which the z-dependence \( (z_2 \parallel \text{rotation axis}) \) of the flow is neglected. This might yield an impression of the real flow pattern in the equatorial plane. The authors model a binary in which both stars are assumed to be corotating and exactly filling their Roche-lobe, and in which one star is losing mass over its entire surface at a constant rate and velocity. An example of the resulting flow is shown in fig. 1., where density contours and the velocity in the corotating frame are plotted for a binary with \( q = 0.5 \). Although the simplifications used in these calculations seem rather drastic, it is nevertheless interesting to see that a number of features that have been predicted on the basis of physical intuition (MMH) are found back in these results. The matter that leaves the primary slowly builds up a common envelope which seems to consist of an inner part which is almost corotating with the binary (low velocity in the corotating frame), and a differentially rotating outer envelope. As we shall see below, this is very similar to the
Fig. 1. Velocity distribution and density contours in an envelope around a binary with $q=0.5$, in which both stars are filling their Roche-lobe. The most massive star is losing mass over its entire surface with constant density and velocity. Velocities are represented in the frame corotating with the binary (from Sawada et al., 1984).

predictions of MMH.

When the giant is far from corotation the Roche-geometry can not be employed, and as far as I am aware there are no numerical hydrodynamical calculations of this problem. The first to consider this case were Sparks and Stecher (1973), who wanted to explain some supernovae as a result of the spiral-in of a white dwarf into a giant. They regarded the giant envelope as nearly stationary, the white dwarf moving over the surface with its orbital velocity. The acceleration and resulting velocity of a matter element in the giant envelope due to the gravity of the white dwarf is then calculated using an impulse approximation, i.e. the displacement of the element while the white dwarf passes is neglected. Depending on the distance to the white dwarf some matter will attain escape speed from the red giant, another part of the envelope will just be lifted and fall back, radiating away its excess energy. Under the assumption that the kinetic energy put into the envelope matter derives from the orbital energy (an 'error of a factor of 2 is made here) it is then possible to follow the decay of the orbit. The radius of the giant is simply assumed to decrease proportional to the
orbital separation. However, the assumptions and simplifications in this model are not really acceptable. The neglect of all hydrodynamical effects is at least questionable, and use of the impulse approximation is not justified since the duration of the passage of the white dwarf is not short compared to the dynamical time of the envelope. Also the assumption that the giant is simply "peeled", i.e. that its radius decreases when mass is ejected from the envelope, is contrary to what we might expect in the light of the discussion above on the reaction of the radius of a giant to mass loss.

Other relevant calculations are those of Morris (1981), who attempted to model mass transfer in a non-corotating binary using particle trajectories and a crude form of hydrodynamical interaction when these particles collide. The giant is simulated by a sphere covered with particles, that are attracted to the giant by its gravity, and repelled by some artificial force which prevents them from entering the surface. They can however be pulled off by the gravity of a companion. This giant is then placed in a binary, and the trajectories of the particles are followed to see how much mass escapes, how much is accreted by the companion, and how much falls back to the giant after being pulled off. A number of models with different orbital separations, mass ratios and rotation rates of the giant are constructed in this way. It is found that the resulting flow is indeed strongly dependent on the rotation rate of the giant, in the sense that the more the giant deviates from corotation, the more violent the interaction. As a result of this a large fraction of the particles is ejected or forms an extended cloud about the binary. In the case that the giant is nearly corotating most particles are simply accreted by the companion. This strong dependence confirms that common envelope evolution proceeds differently in the corotating and non-corotating case.

All these calculations are however unsatisfactory because they do not include the response of the giant to the mass loss. Sparks and Stecher (1974) introduced an (unrealistic) ad hoc assumption to describe this response, and Sawada et al. (1984) and Morris (1981) only considered the case where the mass transfer rates are small, i.e. where the mass of the stars does not change appreciably during the computed time interval of several orbital periods. A proper description of the hydrodynamical reaction of the giant to mass loss would seem to be an essential ingredient to understanding the formation of a common envelope.
To investigate what kind of dynamical effects can be expected during the formation of a common envelope when the response of the giant is in some way taken into account, and if no a priori assumptions about the symmetry are made (except the symmetry with respect to the orbital plane), we have performed a grossly simplified simulation of this process using a method called Smooth Particle Hydrodynamics (SPH). This method, first used by Lucy (1978) and further developed by Gingold and Monaghan (1980, and references therein), represents matter by a number of particles, which move according to an equation of motion in which the acceleration due to gravitation and pressure gradients is calculated using particle-particle interactions. For details of the method we refer to the papers mentioned above, and we only remark here that the method can be reasonably well tested by modelling radial oscillations of polytropes.

We started our calculations by constructing a simplified giant model, taking one very massive particle (3 $M_\odot$) as the giant core, and adding a large number (600) particles to represent the envelope ($M_{\text{env}} = 8 M_\odot$). The gas of the envelope was taken to obey a polytropic equation of state with index $n=1.5$. We relaxed this model until a hydrostatic structure was reached, and then transferred it to a rotating coordinate system revolving around the center of gravity of the giant and a companion star, which is also represented as a massive particle (mass 4 $M_\odot$). The dimensions of this binary were chosen in such a way that the giant would not overfill its Roche-lobe. We then relaxed the system again until the giant was hydrostatic in this new configuration. The separation of the binary was then slowly reduced until Roche-lobe overflow started, and from this point on the system was left to evolve hydrodynamically. In figures 2a,b,c we show the situation at the onset of mass transfer. The position of the particles is indicated in the initially corotating coordinate system system $(x,y,z)$. The $z$-direction is perpendicular to the orbital plane, and the $x$-direction points from the giant core to the companion. Figure 2a gives the projections of the particle positions on the orbital $(x,y)$ plane, fig. 2b on the $(x,z)$ plane and fig. 2c on the $(y,z)$ plane. The positions of the core and the companion are indicated by the large solid dots, and the cross represents the center of mass of the entire system. The typical form of the Roche-lobe is easily recognized. In figures 2d,e,f the situation is shown at approximately 15 times the initial orbital period after mass transfer started, and the companion has made slightly more than one
Fig. 2. A simulation of the formation of a common envelope. Figures 2a, b, c represent the situation at the start of unstable mass transfer, figures 2d, e, f after approximately 15 times the initial orbital period. For a full explanation: see text.
revolution about the giant core in the initially corotating frame.

Since the method used is so crude (no energy equation, very simple equation of state, poor description of hydrodynamics in regions of low particle density, lack of spatial resolution) the details of these results should not be taken seriously. They do however illustrate the following points:
1) one has to be very cautious before assuming any symmetry (spherical or cylindrical).
2) one has to take account of the fact that the giant core will start to move through the envelope, which presumably has important consequences for the nuclear burning.

V.3 COMMON ENVELOPE EVOLUTION IN THE CASE OF INITIAL COROTATION:
THE MODEL OF MEYER AND MEYER-HOFMEISTER

If the mass ratio of a binary lies in the range $1/6 < q < q_{\text{crit}}$ the common envelope is expected to form in a relatively quiet way. According to MMH this CE will consist of two distinct parts: i) an inner region corotating with the "internal" binary formed by the star spiraling in and the dense core of the giant, and ii) an outer region which can rotate differentially. The situation is schematically shown in Fig. 3. The inner region is forced to corotate by viscosity caused by tidally induced turbulence, which resists differential rotation. The strength of this tidally induced turbulence determines the size of the corotating region. This strength is estimated by calculating the acceleration of a non-corotating mass element due to the part of the gravitational potential that varies during the orbital period, neglecting the displacement during the acceleration. From this periodic acceleration a typical turbulent velocity is deduced. Estimating the size of the largest turbulent eddies ($l_{\text{t}}$) to be the distance $r$ of the matter element to the center of gravity then yields an effective viscosity coefficient of

$$\eta = 0.5 \rho v l_{\text{t}}$$

(2)

where $\rho$ is the density and $v$ the typical turbulent velocity. By using a multipole expansion of the time-varying potential MMH show that the induced turbulent velocity scales with $(a/r)^4$, which makes $\eta$ strongly dependent on $r$. (In fact the typical size of the turbulent eddies would
be better estimated by the integral of the turbulent velocity over one half of the orbital period, which yields $l_c = 0.19 \, a \, (a/r)^4$, causing $\eta$ to be even more strongly dependent on $r$. The rapid decrease of $\eta$ with radius implies that the radius of the corotating region will always be similar to the orbital separation, rather independent of the details of the calculations.

Since the time scale for convective transport in the envelope is shorter than the timescale on which the angular velocity of the corotating core is changing, the envelope is expected to be in a quasi-stationary state, which is described by the equation:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ \eta \, r^4 \frac{\partial \omega}{\partial r} \right] = 0 \tag{3}$$

Assuming $\eta$ to be constant this equation has two solutions, one which has $\omega$ proportional to $r^{-3}$ and another with $\omega=\text{constant}$. MMH assume that the second solution applies in the outer part of the envelope, and the first solution describes the angular velocity distribution in a narrow region just outside the corotating inner region, in which the angular velocity changes from that of the core to that of the outer envelope. As pointed out by MMH, the angular velocity distribution derived in this way remains subject to some doubt, since the viscosity in the envelope is the result of turbulence driven by convection, which is non-isotropic, whereas equation 3 is only correct for isotropic viscosity (Biermann, 1951).
The high viscosity in the inner envelope, caused by turbulence driven by convection and by the Rayleigh-unstable angular velocity distribution (specific angular momentum increasing inwards), effectively transports angular momentum from the corotating region to the much more slowly rotating outer envelope. Since this angular momentum derives from the internal binary, the orbit of this binary will shrink at a rate determined by the magnitude of the angular momentum loss, which in turn is determined by the viscosity coefficient in the inner envelope and the surface area of the interface between corotating region and envelope.

Using the model outlined above to describe the hydrodynamical processes MMH subsequently use a normal stellar evolution code to calculate the evolution of such a common envelope formed by a $5 M_\odot$ giant and a $1 M_\odot$ main sequence companion. The central luminosity is given by the luminosity of the giant core (which is determined only by its mass, see MMH) plus an accretion luminosity from the companion star. In addition there are energy source terms that describe the viscous energy dissipation in the envelope. It is found that the frictional luminosity generated in the inner envelope dominates the evolution. The calculations show that this luminosity evolves to a constant value due to a feed-back mechanism in the inner envelope region: as the frictional luminosity increases the radiation pressure in this region also increases, which causes the density to decrease. This density decrease in turn decreases the viscous dissipation (see eq. 2), so that the luminosity decreases again. This mechanism causes the luminosity of the star to remain at a nearly constant value during the entire evolution. Since the frictional energy derives from the binding energy of the internal binary, the evolution of the orbital separation is well approximated by

$$\frac{d}{dt} \left( \frac{1}{a} \right) = \text{constant.}$$  \hspace{1cm} (4)

From this expression it can be seen that initially the orbit shrinks very fast, but as it gets smaller the rate of shrinking also decreases. MMH conjecture that the evolution continues in this way until the star which is spiralling in fills its Roche-lobe in the internal binary and suddenly starts to release a large amount of mass in the inner envelope. This increases the coefficient of viscosity in this region and causes a sudden increase in the frictional luminosity that could drive off the entire envelope. It is, however, questionable whether sufficient energy
is available at this point, since most of the gravitational energy gained by the closing in of the giant core and companion has been radiated away, without causing a significant reduction in envelope mass. Another way of removing the envelope may be in the form of a massive stellar wind. The giant has a very high luminosity throughout the evolution (\(10^5 L_\odot\)), and some observations of very evolved red supergiants (which are thought to be stars that reach a luminosity of similar magnitude by double-shell burning) indicate that they can have very large wind mass loss rates (the so-called super wind, Iben 1981).

Apart from the unsatisfactory description of viscosity and turbulence in the MMH model, the major uncertainty lies in the assumption of spherical symmetry, and the associated neglect of dynamical effects. From the fact that the radius of the corotating region is about equal to the orbital separation it immediately follows that (to first order) the equator of this region is rotating at Keplerian speed, and that the material at this position will be forced to flow outwards by the pressure gradient. This will lead to a circulation pattern in the envelope as sketched in fig. 4, which transports angular momentum far more efficiently even than turbulent viscosity, and will cause a major deviation from the evolution as calculated by MMH.
V.4 COMMON ENVELOPE EVOLUTION WITHOUT INITIAL COROTATION

V.4.1 Spiral-in timescales

If the pre-common envelope binary has a very small mass ratio \( q<1/6 \), the giant will not be rotating synchronously with the orbit when it starts to transfer mass to its companion. After a short initial period of violent interaction, in which the giant expands due to mass loss and the orbit shrinks due to the frictional interaction between the companion and the transferred mass, the orbit of the companion will lie inside the giant envelope. The star will move through this envelope with a relative velocity equal to the orbital speed minus the local rotational speed of the giant. The accretion flow near the star which is spiralling in has a scale which is generally much smaller than the size of the envelope as a whole (see below), and causes a disturbance in the envelope which destroys any symmetries that could be used to simplify the problem. This makes a simultaneous solution of the response of the giant envelope and the accretion flow very difficult. The calculations that have been done so far have concentrated on the envelope structure, while using the so-called Bondi-Hoyle approximation to describe the interaction between star and envelope. Since the Bondi-Hoyle problem is discussed elaborately elsewhere in this thesis I will only give a summary of the pertinent results here, and discuss their implications in the context of spiral-in evolution.

It is found that when a point-like source of gravitation moves relative to a gaseous medium surrounding it, the typical size of the region affected by the gravitational field is given by the accretion radius \( R_a \):

\[
R_a = \frac{2GM}{v^2 + c^2}
\]  

(5)

where \( M \) is the mass of the gravitating object, \( v \) the relative velocity and \( c \) the sound speed in the gas far away from the object. Physically this can be interpreted as the distance to the object at which the gravitational binding energy of a matter element is of the same order as its original total energy at infinity. If the motion is supersonic, the matter which is deflected by gravitation will collide and shock behind the star and its kinetic energy is largely converted to thermal energy. This kinetic energy dissipation rate is typically

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\[ E_{\text{dis}} = \frac{1}{2} \pi R_a^2 \rho v^3 \] (6)

where \( \rho \) is the density in the gas at infinity. If we consider a star moving through a stationary gas we see that the gas moving through the accretion radius is i) heated in the shock and ii) accelerated to the velocity of the star. Hence the total energy loss from the motion of the star is approximately 2 times \( E_{\text{dis}} \). If the motion of the star is subsonic there is much less dissipation of kinetic energy, and the entropy of matter passing close to the compact star which is not accreted is only slightly increased by turbulent dissipation. In this case the drag exerted on the compact star is mainly caused by the gravitational force due to the higher density behind the star, and can become smaller than the drag found from a straight application of eq. (6) (Shima et al., 1985). Assuming that the star moving through the envelope remains in an approximately Keplerian orbit, and that the loss of kinetic energy given by eq. (6) derives from the change in binding energy of the system, one finds that the orbital separation \( a \) decays at a rate given by

\[ -\frac{G m M(a)}{2a^2} \frac{da}{dt} = 2 E_{\text{dis}} \] (7)

where \( m \) is the mass of the star that is spiralling in and \( M(a) \) the mass of the giant interior to the orbit. These expressions form the basis of all calculations of the evolution of a non-corotating common envelope that have been done so far.

Before describing in more detail the models in the literature which employ the Bondi-Hoyle approximation, we shall investigate what can be inferred from a simple application of the above equations to the spiral-in process. To this end we have taken three giant models (kindly provided by dr. G.J. Savonije) which represent different evolutionary phases:

a) a 1 \( M_\odot \) giant with a 0.48 \( M_\odot \) He-core, and a radius \( R = 209 R_\odot \)
b) a 5 \( M_\odot \) giant with a 0.90 \( M_\odot \) CO-core, and a radius \( R = 104 R_\odot \)
c) a 3 \( M_\odot \) giant with a 1.39 \( M_\odot \) ONeMg-core, and a radius \( R = 860 R_\odot \)

As a first order approximation we calculate the parameters of the accretion flow as a function of the distance to the giant center, assuming that the envelope structure is undisturbed. These parameters are of course also dependent on the mass of the accreting object, and in
our examples we will assume this to be 0.1 times the mass of the giant. In figure 5 the orbital decay time, defined as

\[ \tau_d = -\frac{a}{\frac{da}{dt}} \]  

is plotted as a function of radius. The behaviour of model a and b is similar: The decay time is longest in the outer regions of the envelope, has a plateau in the inner part and decreases strongly again as the core is entered. In the much more evolved model c this plateau has developed into a broad maximum in the orbital decay time at a radius between 1 and 10 $R_\odot$. This maximum is caused by the fact that the envelope of such very evolved giants have a nearly constant density, whereas the orbital velocity of the star spiralling in increases with decreasing radius. From eq. (5) and (6) we see that (if the relative velocity has become subsonic) this implies a reduction in the energy dissipation rate, and hence a lengthening of the orbital decay timescale. It is interesting to note that the maximum occurs at a radius that is comparable to the observed orbital separation in several

![Diagram](image)

**Fig. 5.** The orbital decay time scale during spiral-in ($\tau_d$), as a function of radius in our three giant models (see text).
Fig. 6. The Mach number of the orbital velocity as a function of radius in our three giant models (see text).

types of post common envelope binaries. The presence of the maximum certainly favors the possibility of ejecting the giant envelope before the star collides with the core, but it can not be a necessary condition. This is because it only develops in giants with a sharply defined massive core (near the Chandrasekhar limit), while many cataclysmic variables appear to have white dwarf masses well below 1 M\(_\odot\). There are some indications that white dwarfs in cataclysmic variables are on average more massive than the general population (Warner, 1976) but this is very difficult to establish because of selection effects (Livio and Soker 1984b, Ritter 1986).

The spiral-in times as given in figure 5 can change if the energy dissipation rate is not exactly given by equation 6. Following the results of Shima et al (1985) and de Kool and Savonije (Chapter IV.1) the energy dissipation rate can be substantially larger than the classical estimate for supersonic relative speeds, and smaller for subsonic speeds. To investigate the possible importance of this effect we have plotted in fig. 6 the Mach number of the orbital speed as a function of radius. The behaviour is very similar for the different models: the Mach number is never very high, and varies between 1 and 2 over the largest part of the envelope. Especially in model c the Mach number is very close to one. This implies that if the envelope is rotating by itself, either due to evolution prior to the spiral-in or
due to angular momentum transfer from orbit to envelope during the spiral-in, the relative velocity between star and envelope could easily become subsonic, with an accompanying reduction in the dissipation rate and increase in orbital decay time.

Since the relative velocity between star and envelope plays an important role in spiral-in evolution, we would like to know whether the angular momentum deposited in the envelope by friction is able to force the surroundings of the star to corotation. We therefore introduce the spin-up timescale \( \tau_{sp} \), defined as

\[
\tau_{sp} = I_e(a) \omega_{orb} \left( \frac{dJ}{dt} \right)^{-1}
\]  

(9)

in which \( \omega_{orb} \) is the orbital angular velocity, \( I_e(a) \) the moment of inertia of the part of envelope with radius \( a \), and \( dJ/dt \) is defined by

\[
\frac{dJ}{dt} = a \pi R_a^2 \rho \omega_{orb}^2
\]  

(10)

This very rough estimate may be as good as some more complicated estimates based on tidal interaction between star and envelope, since these depend on a very uncertain viscosity coefficient and employ a description of tidal interaction which is derived for situations in which the orbital separation is larger than the radius of the giant.

In figures 7a,b,c we compare the spin-up timescale with the orbital decay timescale. In model c the star can already spin up the inner part of the envelope to corotation when it has penetrated to a radius of 350 \( R_\odot \), and in models a and b these radii are 30 and 10 \( R_\odot \) respectively. The fact that \( \tau_{sp} \) can become shorter than \( \tau_d \) is caused by the presence of the plateau in the \( \tau_d \) curves. In figure 7d the two timescales are compared for a 16 \( M_\odot \) giant with a radius of 200 \( R_\odot \), which has a much much less sharply defined core, and we can see that in this case the spin-up time becomes smaller than the orbital decay time for very small radii (ca. 1 \( R_\odot \)), and at a radius where the density is about \( 10^6 \) times greater than in the other models. In this case coalescence will be difficult to avoid. How these results change for different initial mass ratios can be inferred from the fact that \( \tau_{sp} \) scales approximately with \( m^{-2} \), and \( \tau_d \) with \( m^{-1} \). From the form of the curves in fig. 6 it can be seen that this implies that for smaller initial mass ratios these timescales become equal at smaller radii.
Fig. 7 A comparison of the orbital decay time scale $\tau_d$ (solid) and the spin-up time scale $\tau_{sp}$ (dashed) of the surroundings of the star that spirals in, for the three models from the text (figures 7a,b,c) and for a less centrally condensed 16 $M_\odot$ giant (figure 7d).
To follow the evolution of a massive (16 $M_\odot$) giant with a 1 $M_\odot$ neutron star in its envelope Taam, Bodenheimer and Ostriker (1978) (hereafter TBO) used a normal one dimensional stellar evolution code, slightly modified to be able to follow hydrodynamical expansions or contractions. Other dynamical effects were neglected. First they constructed a model of a single 16 $M_\odot$ giant, and then introduced an extra energy source term given by eq. (6). The extra energy was deposited in a spherical shell of thickness $R_a$ and radius equal to the orbital separation. A first (and very fundamental) difficulty is immediately encountered: to calculate the velocity of the star relative to the envelope it is necessary to assume an angular velocity distribution in the giant. TBO make the assumption that outside the orbit of the star the angular velocity $v_e(r)$ varies with $r^{-2}$, so that the specific angular momentum is constant. This is based on the idea that convection in this region effectively redistributes angular momentum. (Note the difference with the MMH model in which convection is assumed to lead to solid body rotation). Inside the orbit the rotational velocity is assumed to be given by

$$v_e(r) = v_e(a) \exp \left(-\left(\frac{r-a}{R_a}\right)^2\right)$$

(11)

to mimick the effects of a diffusion process. In this way the angular momentum of the entire envelope can be derived from $v_e(a)$, and the value of this quantity is always chosen in such a way that the total angular momentum of star and envelope remains constant. Using eq. (7) the evolution of the orbit can be followed simultaneously with that of the star. (Energy source terms such as accretion onto the neutron star or turbulent viscous dissipation due to tides or differential rotation are also considered by TBO but are generally found to be less important than the frictional luminosity $E_{\text{dis}}$. ) TBO consider two cases: case i), in which the common envelope forms while the 16 $M_\odot$ star is a yellow giant, at the onset of He-burning. In this evolutionary phase the envelope is radiative, and not very extended. In case ii) the spiral in starts when the massive star has evolved to a red giant with an extended convective envelope ($R=535 R_\odot$)

In the first case the neutron star spends most of the time in the outer envelope because the density is very low there, which makes the
frictional energy losses very small. After about $3 \times 10^3$ yr the star starts to enter denser layers, and the orbital decay time is drastically reduced. Because of the very large frictional luminosity ($10^{42}$ ergs/sec) the temperature gradient in the envelope becomes super-adiabatic and convection sets in. It is found that this convection is able to transport all heat deposited in the envelope to the surface in a sufficiently short time, and so avoid a build up of thermal energy (and pressure) around the position of the neutron star that could drive off the envelope. The spiral-in continues until the neutron star is very close to the He-core of the giant, and the layers around the neutron star have been spun up to the velocity of the neutron star. When the relative velocity disappears the frictional luminosity drops to zero, and further evolution proceeds on the timescale of the tidal and viscous dissipation. In fact the situation becomes very similar to that in the MMH model. This is where TBO stopped the calculations.

In case ii) the neutron star passes through the outer layers of the giant much more quickly because the density is higher than in case i), and the entire spiral in process takes only about 20 yrs. Similarly to case i) the frictional luminosity increases as the star enters denser layers, but now the low densities and temperatures in the extended red giant envelope cause $\tau_c$, the timescale for convective energy transport to be much longer than in case i) ($10^{-1}$ versus $10^{-3}$ yr). When the orbital separation is reduced to only $3.5 R_\odot$ the orbital decay time becomes shorter than $\tau_c$. The energy deposited can no longer be transported to the surface quickly enough, and a pressure build-up at the radius of the neutron star then causes the envelope to acquire large outward velocities. When the calculations were stopped the layers just outside the orbit had velocities in excess of the escape velocity.

From these calculations it would appear that a simple recipe exists to predict the outcome of spiral-in evolution, based only on the total available energy and the efficiency of the convective energy transport. We will see below, however, that this result is severely dependent on the assumption of spherical symmetry that TBO had to use.

Delgado (1980) performed calculations very similar to those of TBO, but now considering a binary consisting of a $25 M_\odot$ blue supergiant and a $1 M_\odot$ neutron star. In this work it is argued that if the total luminosity of the giant exceeds the Eddington luminosity very large wind mass loss rates might occur that modify the further evolution of the common envelope. To test this he calculated the evolution using the
expression for wind mass loss due to Chiosi et al (1978), modified to make the mass loss increase by a factor $10^3$ when the luminosity becomes equal to $L_{\text{edd}}$. Because of numerical difficulties the evolution could only be followed to the time at which the neutron star had crossed 3.5 percent of the mass of the giant, which at that time had expanded to about 12 times its original radius. The frictional luminosity at this point exceeded the luminosity at the surface of the star, which led Delgado to the conclusion that the outer part of the envelope would be ejected, and that further evolution would proceed in similar steps, the neutron star entering the giant by a small mass fraction and subsequently blowing this off. However, this is only conjecture which is not directly supported by the results of the calculations.

V.4.3 The model of Livio and Soker

A different, semi-analytical approach (but also based on the Bondi-Hoyle approximation to describe the interaction between star and envelope) was taken by Livio and Soker (1984a,b). They modelled the common envelope evolution of a binary initially consisting of a very evolved low-mass (0.88 $M_\odot$) giant and a companion of planetary mass (0.001-0.025 $M_\odot$). An advantage of starting with such a low-mass companion is that the use of the Bondi-Hoyle approximation is very well justified in this case because the accretion radius is much smaller than any of the length scales associated with the giant structure. Neglecting all structural changes in the giant envelope (which again can be justified by the small mass of the companion) they calculate the decay of the orbit of the planet due to friction and tidal dissipation, and at the same time follow the change in mass of the planet due to accretion and thermal evaporation. Cases with and without angular momentum transfer from planet to envelope are considered. It is found that planets below a certain critical mass (the exact value of which is determined by the assumptions used) evaporate completely. Above this critical mass accretion dominates, and the mass of the planet increases while it spirals in. If the planet is sufficiently massive to start with, it is able to accrete the entire envelope (0.16 $M_\odot$) before colliding with the core, and a binary consisting of a white dwarf and a main sequence dwarf remains. Such a binary is an excellent candidate for a cataclysmic variable progenitor. The lower mass planets will collide with the giant core, presumably being disrupted in the process. Although
some aspects of this model may be far from reality, such as the assumption that the planet can accrete all matter falling on to it and continues to obey a simple mass radius relation in spite of the high temperature of the accreting gas, this scenario is a very interesting one in view of the probable ubiquity of the progenitor binaries.

Livio and Soker also calculated a few cases in which the planet was not circling inside the giant envelope but in the giant wind, to investigate whether spiral-in evolution can be induced by strong wind mass losses. It was found that the planet always spiralled out, because the widening of the orbit caused by the reduction in mass of the giant was stronger than the decay due to friction. By comparing the timescale for spiral-out (which is equal to the wind mass loss timescale $m_W/\dot{m}_W$, $m_W$ being the mass of the giant and $\dot{m}_W$ the wind mass loss rate) to the orbital decay time scale we find that spiral out results from wind mass loss as long as

$$\frac{\alpha \beta \frac{m_p}{\dot{m}_W} (\frac{a}{R})^{1/2}}{\frac{\dot{m}_W}{R}} > 1$$

(12)

Here $m_p$ is the mass of the planet, $\alpha$ the ratio of the velocity of the star relative to the wind to the orbital velocity ($\alpha > 1$), and $\beta$ the ratio of the wind velocity to the escape velocity at the surface of the giant. We can see that this expression is independent of the actual wind mass loss rate. Since in most cases $m_W$ will be much larger than $m_p$ the condition is generally well satisfied unless $\beta < 1$, i.e. the wind is extremely slow.

V.4.4 The model of Tutukov and Yungelson

For completeness we should also mention the paper by Tutukov and Yungelson (1979), who also attempt a semianalytical approach to the spiral-in problem. Unfortunately, the physical basis used in this work is incorrect, since the authors assume that the frictional force between star and envelope scales with the relative velocity squared, which is correct for a solid non-gravitating body moving through a gas, but not for gravitational accretion flow, in which this force scales with approximately the inverse of the relative velocity squared (see eq. 5 and 6). For this reason we shall not consider this model any further.
The most recent development in the study of common envelope or spiral-in evolution has been the application of two-dimensional numerical hydrodynamics (Bodenheimer and Taam, 1984, hereafter BT). The great difficulty in this type of calculations is the variety of length and time scales that is involved. To describe the structure of the giant properly, very small zones have to be taken near the core of the giant, where density and temperature change on a small length scale. The Courant-Friedrichs condition for stability of any explicit method for the solution of the hydrodynamical equations requires that the time step be smaller than the sound crossing time of a grid zone. Since in the zones closest to the core the temperature (and hence also the sound speed) is largest, the stability of the calculations in these inner regions require such small timesteps that following the entire common envelope phase becomes impossible in terms of calculation time. To investigate the possible effects of dropping the assumption of spherical symmetry anyhow, BT therefore considered the following limited problem. They started with a one-dimensional giant model from the calculations of TBO, taken at a time when the neutron star had already spiralled in very close to the core, and had an orbital period sufficiently short that the computation could cover at least one orbital period. This giant model was then transferred to a two-dimensional grid in the (r,z) coordinates of a cylindrical (r,φ,z) coordinate system. (In this way rotational symmetry about the orbital angular momentum vector is assumed) To avoid extremely small timesteps the core of the giant was then replaced by a point mass and a solid inner boundary. Further reduction of the computational effort was reached by not including the entire giant envelope in most calculations, but only the inner part. This does not influence the results, since even with these simplifications the time it took for the induced motions to reach the edge of the grid (1-2 orbital periods) was about all that could be calculated. One model was constructed on a larger grid in which the entire envelope was included, to check if any unexpected events occurred in the time that the motions needed to reach the surface of the star. The energy and angular momentum transferred from orbit to envelope by friction, which are calculated in the same way as by TBO, is distributed over an annular region consisting of the four zones closest to the position of the neutron star. The gravitational potential is calculated by solving the Poisson equation.
simultaneously with the hydrodynamical equations. An example of the
results is illustrated in figs 8a,b where the velocity and angular
momentum distribution in the red supergiant envelope after slightly more
than one orbital period is shown. It is found that an equatorial outflow
develops, in which the velocities after some time exceed the local
escape velocity. Physically, this result can be understood as follows.
Material close to the position of the neutron star first receives a kick
in the direction of the orbital motion, which will make it move
outwards. The effect of this initial kick soon becomes unimportant
relative to the velocity gained by the buoyancy force acting on the
material, because it has also been heated by dissipation, and has gained
an entropy excess relative to its surroundings. Since the Mach number of
the relative motion between star and envelope is not very high (see
fig. 6) the thermal energy gained by the material is of the same order
as the original thermal energy, and hence the buoyancy force is of the
same order as the local gravitational force, but working in the opposite
direction. Hence the material is accelerated radially outward with an
acceleration comparable to the local gravitational acceleration. Since
the outer layers are already convectively unstable, this acceleration
can continue over a long path. This description in terms of buoyancy
forces (which is not used by BT) gives an explanation of the gradual
acceleration of material from the vicinity of the neutron star to
velocities exceeding the local escape velocity further out in the
envelope. It is interesting to note that the buoyancy mechanism does not
operate when the relative motion is subsonic, because in this case the
entropy of the matter passing close to the star does not increase very
much.

In the work of TBO it was found that the spin-up of the layers in
the envelope around the neutron star reduced the relative velocity, and
hence the rate of orbital decay. In the two-dimensional models the
material that receives the angular momentum immediately moves outwards
and is replaced by low angular momentum material. This means the spiral-
in will proceed even more rapidly than in the spherically symmetric
case. The simplifications made by BT that will probably affect the
outcome of their calculations most severely are:

1) The symmetric, hydrostatic starting model. Since the structure of the
envelope changes completely in one orbital period, this can not be
consistent with the earlier evolution. Especially he effectiveness of
the buoyancy forces may depend on the initial hydrostatic structure.
Fig. 8a. The velocity field in the giant envelope as calculated by Bodenheimer and Taam (1984), with a two-dimensional hydrodynamics code. The largest velocities exceed the local escape velocity.

Fig. 8b. A contour plot of the angular momentum distribution in the same model as fig. 8a. Since the initial angular momentum distribution was independent of z, the height above the equatorial plane, the circulation pattern from 8a can also be traced in the deformation of the contours.
2) **The short computed time interval.** Since this interval is even shorter than one dynamical time of the envelope, one can not really conclude from the present results what the flow pattern developing on a longer timescale will be like.

3) **The rotational symmetry.** Even if the initial evolution could be properly described by a cylindrically symmetric model which is followed by a very sudden increase in frictional luminosity as the star hits denser layers near the giant core, the change in envelope structure in one orbital period is not consistent with rotational symmetry.

4) **The size of the region in which the energy and angular momentum is deposited.** For unclear reasons BT chose this region to be much smaller than the accretion radius of the neutron star. If the energy is distributed over a much larger amount of matter smaller pressure gradients and hence smaller outflow velocities might result.

In view of these difficulties the results of the two-dimensional calculations should not be used quantitatively. The conclusion that one-dimensional models can not describe the dominant physical processes is however very firm.

V.5 CONCLUSIONS

We conclude that the detailed studies of common envelope evolution made so far have yielded no more than an inventory of the physical processes that may be involved. The main difficulty is the complexity of the three-dimensional hydrodynamics with very different length- and time scales that is involved. This can probably only be resolved by using numerical techniques, on a scale which is presently unattainable.

Returning to the question posed in the introduction regarding the outcome of common envelope evolution, we can now conclude that there is no justification whatsoever for the assumption that the gravitational energy liberated by the spiral-in is efficiently used to eject the envelope, which is the basis of most predictions of the outcome of the CE phase (Chapter III.2, Iben and Tutukov 1984). In the 1-D models it is found that most of this energy is radiated away, in the 2-D models (at least as far as can be judged from the present results) it is not efficiently used, since a small fraction of the mass carries away a large excess of energy.
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Stellingen behorende bij het proefschrift
"Models of Interacting Binary Stars"

1. Het door Sawada et al. in hun numerieke berekeningen gevonden effect
dat de hoge effectieve viscositeit in accretieschijven in dubbelsterren
het gevolg is van door de begeleider geïnduceerde spiraalschokken
verdient meer aandacht dan het tot nu toe heeft gekregen.

   Sawada et al. 1985, MNRAS 219, 75

2. Een niet verwaarloosbare fractie van het aantal planetaire nevels is
bij het ontstaan beïnvloed door het feit dat de reus die zijn omhulsel
verliest, onderdeel uitmaakt van een dubbelster.

3. De conclusie van Matsuda et al. dat accretie van impulsmoment door
een neutronenster uit de sterrenwind van een begeleider zowel spin-up
als spin-down kan veroorzaken, zelfs als de sterrenwind azimuthaal
symmetrisch is, is gebaseerd op een onjuiste veronderstelling.

   Matsuda et al., preprint Kyoto University

4. Het aanduiden van met elkaar in contact gekomen supernovarestanten
als "tunnels" gevuld met heet gas, heeft bij veel mensen aanleiding
gegeven tot een verkeerd beeld van de structuur van het interstellaire
medium.


5. Een analytische oplossing van een fysisch probleem verschaft niet
altijd meer inzicht dan een numerieke oplossing.

6. Sterrenkunde en bergbeklimmen hebben met elkaar gemeen dat zij wel
degelijk een bepaalde romantiek in zich dragen, maar dat deze in de
dagelijkse praktijk ver te zoeken valt.

7. Het opleggen van de feitelijke dienstplicht aan één bepaalde groep
Nederlanders is een historisch gegroeide, maar naar huidige maatstaven
onacceptabele vorm van rechtsongelijkheid.
8. Christelijke politiek is principieel ondemocratisch, aangezien zij, als zij haar grondslagen serieus neemt, nooit de volkswil zal kunnen aanvaarden als deze in tegenspraak is met in de Bijbel vastgelegde regels.

9. Het is didactisch niet verantwoord om studenten bij het vak practische numerieke wiskunde uitsluitend gebruik te laten maken van kant-en-klare routines uit een numerieke bibliotheek.

10. Een weerskaart van het gehele noordelijk halfrond verschaf veel meer inzicht dan een kaart die slechts Europa of zelfs alleen de nabije omgeving van Nederland weergeeft.

H.H. Lamb, "Climate, Present, Past and Future", Methuen Press

11. Gezien de mate waarin de sociaal-economische voorspellingen van het Centraal Planbureau in het verleden zijn uitgekomen is het verbazingwekkend hoeveel aandacht er in de media aan besteed wordt.

Marthijn de Kool

Amsterdam, 3 juni 1987
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