Models of interacting binary stars

Kool, M.

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CHAPTER IV

BONDI-HOYLE ACCRETION FLOW

IV.1) A numerical study of cylindrically symmetric accretion flow

IV.2) On the accretion of angular momentum from an inhomogeneous medium

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Abstract

We present model calculations of cylindrically symmetric accretion flow to a gravitational point source. The flow is calculated using a form of the particle-in-cell method. We consider two polytropic equations of state (\( \gamma=1.0 \) and \( \gamma=5/3 \)), and one in which the effects of radiation pressure are taken into account. In the latter case radiation transport is included in the diffusion approximation. We confirm the result of previous studies that the mass accretion rate is well approximated by the classical Hoyle-Lyttleton estimate. The kinetic energy dissipation rate is found to be several times larger than the classical estimate, the exact value depending on the equation of state and the Mach number of the flow. It is stressed that although the mass accretion rate is necessarily limited to the Eddington rate, the kinetic energy dissipation rate is not limited in this way as long as the surroundings of the accreting object are optically thick.

I. Introduction.

The gravitational capture of matter by a body moving relative to its surroundings is a process which occurs in several astrophysical problems. The first attempts at solving this problem (Hoyle and Lyttleton 1939, Bondi and Hoyle 1944) were instigated by a study of accretion from an intergalactic medium by galaxies and the possibility of stars gaining a non-negligible amount of mass from the interstellar medium during their lifetimes. A more recent application is the capture of matter from a stellar wind by a compact star, which is thought to occur in some X-ray binaries. Our own interest in the problem derives from the study of common envelope evolution of binaries, during which a compact or dwarf star is moving through the envelope of a giant companion.

In the general problem matter is flowing from infinity towards a gravitational point source. Far away from the object forces caused by pressure gradients and viscosity will be small and the matter will follow a free Keplerian orbit. Behind the object there will be a line, the so-called "accretion-line", where the orbits of matter coming from different sides of the object intersect. Assuming that the matter follows a free orbit leads to an infinite density on this line, so obviously that approximation breaks down here. On the accretion line pressure and viscous forces will become important and, because of the
cylindrical symmetry of the problem, will cause a complete cancellation of momentum transverse to the accretion line. Assuming that the only interaction with other matter takes place when the accretion line is crossed, Hoyle and Lyttleton (1939) were able to give a first estimate of the rate at which matter is accreted:

$$A_{HL} = \pi R_{HL}^2 \rho_\infty v_\infty$$

(1)

where $\rho_\infty$ and $v_\infty$ are the density and velocity of the gas at infinity and $R_{HL}$ is the accretion radius, defined by

$$R_{HL} = \frac{2GM}{v_\infty^2}$$

(2)

with $G$ the gravitational constant and $M$ the mass of the gravitational point source. This result can be derived by assuming that all matter which does not have sufficient kinetic energy left to escape from the gravitational field after the cancellation of transverse momentum at the accretion line is accreted. When using the simple picture of a very thin accretion line it should always be kept in mind that Cowie (1977) showed that this assumption leads to an instability in the accretion flow that precludes any time-independent solutions for the accretion rate.

It was shown by Bondi (1951) that in the case of stationary accretion (i.e. $v_\infty=0$) the accretion rate can be estimated by an expression similar to equation 1, with $v_\infty$ replaced by $c_\infty$, the sound speed at infinity. Since in the accretion line picture, where the pressure forces are neglected ($c_\infty<<v_\infty$), the accretion rate is given by eq. 1, and in the stationary case ($v_\infty<<c_\infty$) by the same expression with $v_\infty$ replaced by $c_\infty$ it was proposed (Bondi and Hoyle 1944) that in the intermediate case the accretion radius would be given by

$$R_A = \frac{2GM}{v_\infty^2 + c_\infty^2}$$

(3)

To get beyond the many simplifications used to derive the results above and to obtain more details of the actual flow pattern it seems necessary to perform detailed numerical simulations of the problem. This was first done by Hunt (1971) who gave a full 2-dimensional treatment of the gas flow around a gravitating object of very small geometrical radius ($R<<R_A$). These calculations were done using an adiabatic equation of state for an ideal gas. In the case that $v_\infty < c_\infty$ (subsonic) it was found
that the flow closely resembles that of the case \( v_\infty = 0 \). When \( v_\infty > c_\infty \) (supersonic) the matter colliding on the accretion line behind the compact object will shock, and this shock moves out to the sides to form a bow shock. For subsonic flow the accretion rate was approximately that given by the Bondi-rate (eq. 3), and for large Mach numbers the accretion rate tends to the Hoyle-Lyttleton value (eq. 1). In a later paper (Hunt 1979) these calculations were repeated for a gas with a polytropic equation of state with \( \gamma = 4/3 \). Other model calculations covering more or less the same topic are those by Livio, Shara and Shaviv (1979) and Shima et al. (1985). Also related to this work are the calculations of Da Costa and Fryxell (1981) who considered the flow around a star in a moving medium in the case that its geometrical radius is much larger than the gravitational accretion radius, in which case gravity has only a minor influence on the flow. A study of the 3-dimensional aspects of accretion flow was recently started by Livio et al. (1986), using a simplified version of the method used in this paper. These 3-dimensional aspects become important when the upstream boundary conditions are not constant in space, which destroys the cylindrical symmetry. An example of this is the presence of a density gradient in the undisturbed gas.

II. The numerical method.

The way in which we simulate the gas flow is based on the particle in cell method (PIC) (see e.g. Potter 1973, Hensler 1982a,b). The gas is represented by a large number of particles, which move according to an equation of motion in which the effects of gravity, pressure forces and viscosity are present. The pressure and viscous forces are calculated from the particle distribution on a 2-dimensional grid of cells in space, and are then interpolated to the particle positions.

To describe the method in more detail we shall use the following notational convention: particle properties like mass, thermal energy, velocity, momentum, position are given the index \( n \) (\( m_n, e_n, \vec{v}_n, \vec{P}_n, \vec{x}_n \)) to indicate the particle to which they refer. Grid properties like density, thermal energy, pressure, velocity and temperature are given the double index \( i,j \) (\( \rho_{i,j}, e_{i,j}, \vec{p}_{i,j}, \vec{v}_{i,j}, T_{i,j} \)) to denote the gridcell to which they refer (one for each dimension). We employ a 2-dimensional grid of cylindrical coordinates \((x,r)\) where \( r \) is the coordinate perpendicular to \( \vec{v}_\infty \) and \( x \) is parallel to it. The flow is
Fig. 1. A schematic illustration of the way the particle properties are distributed over the grid cells. For explanation see text.

assumed to be rotationally symmetric around the x-axis, and the azimuthal velocity (perpendicular to both x and r) is assumed to be zero.

To calculate the grid properties density, thermal energy density and momentum density from the particle properties we use a standard PIC method, slightly modified because of the non-cartesian coordinates. We shall only describe the calculation of mass density; the others are calculated in the same way. The mass of a particle is distributed over the four cells closest to the particle position. In standard PIC the weights for this distribution are taken to be proportional to the area of overlap between a square the size of a gridcell centered on the particle position and the fixed gridcells (see fig. 1). We take the weights to be proportional to the volume of the tori around the x-axis whose cross-section with the x,r plane is given by the areas indicated in fig 1. If we define

\[ g_n = \frac{x_{n+1} - x_i}{\delta} \]

\[ h_n = \frac{r_{n+1} - r_i}{\delta} \left( \frac{\delta + r_{n+1}}{2r_n} \right) \]

where \( \delta \) is the grid cell size then the weights are given by
After the mass of the particles is distributed over the cells using the above weights, the density $\rho_{i,j}$ is calculated by dividing the total mass in a cell $m_{i,j}$ by its volume $V_{i,j}$, which is again a torus around the x-axis. In the same way we obtain the momentum $p_{i,j}$ and thermal energy $e_{i,j}$. These grid properties are subsequently used to calculate the rate of change of the particle velocity and thermal energy.

The velocity of a particle (or gas element) can change due to gravitational forces, pressure forces and viscous forces. To simulate the viscous forces we first calculate the average velocity in each cell

$$ v_{i,j} = \frac{p_{i,j}}{m_{i,j}} $$

The next step is to give the particles a new velocity which is interpolated from the grid values using the same weight factors as were used for calculating the averages. For example, the particle illustrated in fig 1 gets the velocity

$$ v_n = w_{11}v_{i-1,j} + w_{12}v_{i-1,j+1} + w_{21}v_{i,j} + w_{22}v_{i,j+1} $$

In this way momentum is exactly conserved. Test calculations which were made to estimate the diffusion of momentum associated with this alternation between grid and particle velocities showed that it is equivalent to a viscous force with kinematic viscosity $v = 0.16 \delta^2/\Delta t$, with $\delta$ the size of a grid cell and $\Delta t$ the timestep.

The pressure is calculated on the grid from the density and thermal energy density, using a choice of equations of state. In the case of an isothermal gas the pressure is proportional to the density, and in an ideal gas with $\gamma = 5/3$ the pressure is two-thirds of the thermal energy density. For the more complicated case of a mixture of an ideal gas and radiation in thermal equilibrium an iterative procedure is used to calculate the temperature and pressure from the density and thermal energy density. Once the pressure in all grid cells is known the acceleration that a gas element in the center of a grid cell would undergo is calculated, according to
These accelerations are then interpolated from the grid to the particle positions to yield \( \ddot{z}_n \), the time rate of change of the velocity due to pressure forces.

Since we neglect the self-gravity of the gas (the total mass of gas in our grid is always less than \( 10^{-4} \) times the mass of the moving object) the gravitational acceleration of a particle is simply

\[
\ddot{z}_n = - \frac{GM(\dot{x}_n - x_{co})}{|\dot{x}_n - x_{co}|^3}
\]

where \( G \) is the gravitational constant, \( M \) the mass of the gravitational source and \( x_{co} \) its position.

The velocity of a particle at time \( t+\Delta t \) can now be expressed as

\[
\tilde{v}_n(t+\Delta t) = \tilde{v}_n + (\ddot{z}_n + \dot{z}_n) \Delta t
\]

The thermal energy of a particle can change due to three causes: i) the dissipation of kinetic energy by the velocity averaging procedure, ii) pressure work, and in the models that take radiation effects into account by iii) radiation diffusion. In contrast to the evolution of velocity, which is directly applied to the particle properties, it is more convenient in this case to calculate the time rate of change of the thermal energy in a grid cell, and redistribute the new thermal energy over the particles. The time rate of change of the total thermal energy \( e_{i,j} \) in a gridcell due to viscous interactions is calculated by taking the difference in kinetic energy of each particle before and after the velocity averaging procedure and distributing this over the gridcells using the weights \( w_{11}, w_{12} \) etc.: 

\[
(\Delta E_{\text{kin}})_n = \frac{1}{2} m_n (|\tilde{v}_n|^2 - |\dot{v}_n|^2)
\]

\[
\Delta e_{i,j} = E_w (\Delta E_{\text{kin}})_n
\]

where the sum goes over all particles contributing to cell \((i,j)\).

To find the change in thermal energy due to pressure work we first calculate the change in thermal energy per gram.
\[ \Delta e_{i,j} = \frac{p_{i,j}}{\rho_{i,j}^2} \Delta \rho_{i,j} \]  

(13)

The change in density is found by taking the difference between the old and the new density \( \Delta \rho = \rho_{\text{new}} - \rho_{\text{old}} \). The change in total thermal energy in a cell due to pressure work then follows by multiplying with the total mass in a cell

\[ \Delta e_{i,j} = m_{i,j} \Delta e_{i,j} \]  

(14)

The third way in which \( e_{i,j} \) can change is by radiation diffusion. This can be described by the equation

\[ \frac{1}{V_{i,j}} \frac{\partial \delta e_{i,j}}{\partial t} = \nabla \left( \frac{c}{3\rho\kappa} \nabla (aT^4) \right) \]  

(15)

in which \( V_{i,j} \) is again the volume of cell \((i,j)\), \( c \) the velocity of light, \( \kappa \) the opacity (electron scattering) and \( a \) the radiation constant. This equation can be solved directly on the grid. A complication is that when radiation diffusion is important the time step required for stable integration of eq. (15) is smaller than the timestep required in the dynamical part of the calculations. We therefore integrate eq. (15) separately within each dynamical timestep, keeping the density constant. Equations (12), (14) and (15) yield the non-advective part of the change in thermal energy of a gridcell in a timestep. The new energy is distributed over the particles in the normal way.

Just as the velocity averaging procedure causes a diffusion of momentum (viscosity), so the alternation between grid and particle thermal energy causes a diffusion of energy with a diffusion coefficient \( 0.16 \delta^2 / \Delta t \), which in case of high densities or low temperatures can become larger than the energy diffusion by radiation, which has a diffusion coefficient \( c / \rho \kappa \) (eq.15).

Given an initial distribution and a set of boundary conditions on the grid the method above can describe the time evolution of a system of particles.

III. Boundary conditions.

A complication of a (semi) Lagrangian method like PIC in the case of accretion flow calculations is that we can not consider a fixed amount of mass. Matter is continuously entering the grid fixed to the compact
object from the upstream side and leaving it either on the downstream side or by being captured. Hence in every timestep new particles must be created to represent the matter entering, while particles which have left the grid need no longer be considered. Since it is most convenient to use a fixed number of particles this is handled as follows. After the particle positions are advanced one timestep it is checked which particles are outside the grid or have been accreted. (A particle is assumed to be accreted when it has entered one of the two gridcells adjacent to the gravitational source, which is considered to be very compact.) These particles are subsequently injected at the upstream edge of the grid, representing the matter which must have entered the grid in the last timestep. The position, velocity and mass of the particles is chosen in such a way that they represent matter which has come from a homogeneous density distribution far upstream \((x=-\infty)\) and has moved from there in a free Keplerian orbit. Because of the gravitational focusing particles enter the grid at both \(x=x_{\text{min}}\) and \(r=r_{\text{max}}\). Because of this the grid edge \(r=r_{\text{max}}\) is handled as an upstream boundary, and we have to take care that the bow shock of the compact object does not pass through it.

The boundary conditions on the axis \((r=0)\) are imposed by the condition of cylindrical symmetry, i.e. the radial velocity is zero and the derivatives in the \(r\)-direction are zero. Pressure gradients (and temperature gradients in the case that radiation diffusion is taken into account) on the sides of the grid are calculated by assuming an extra gridcell just outside the grid with a pressure (or temperature) equal to the adjacent gridcell. The more exact method of extrapolation of the outer two gridcells can give rise to an instability when a shock crosses the boundary (Roache, 1976).

The pressure in the cells next to the compact object is calculated by taking the average of the two values one obtains by extrapolating the pressure in the cells adjacent to it in the \(x\)- and \(r\)-direction. The exact choice of this inner boundary condition hardly influences the results, as was verified with a test calculation in which the pressure in these cells was simply set to zero.

We use a grid of \(64\times64\) cells, which extends upstream from the compact object for one accretion radius \((R_{\text{HL}})\) and downstream for two accretion radii. All calculations were performed with 50,000 particles, giving an average number of particles per cell of \(~12\). We start the calculations with the grid filled with particles with velocity \(\mathbf{v}_0\) and let the system evolve until a steady state is reached.
IV. Results.

a) Isothermal equation of state.
We calculated models using an isothermal equation of state \( P = \rho RT \) for Mach numbers 1.5, 2.0, 3.0 and 3.75, where the Mach number is calculated relative to the isothermal sound speed \( c_\infty^2 = \frac{p_\infty}{\rho_\infty} \). In fig 2a and b the density and velocity distribution for the case \( M=1.5 \) is shown. Typical for an isothermal shock are the large density and velocity jumps across it. Since the information about the presence of an obstacle in the flow can never propagate upstream, the shock will always be attached to the inner boundary, however small this is taken. For every Mach number the stagnation point of the flow on the accretion axis behind the compact object lies near \( R_{HL} \).

b) Adiabatic ideal gas.
For this case models of Mach number 1.5, 2.0, 3.0 and 3.75 were constructed. Density and velocity plots are shown in fig. 3 a and b for the case \( M=3 \). The general features of the flow pattern are similar to those of Hunt(1971) and Shima et. al. (1985). There is some difference in the location of the stagnation point in the flow on the accretion axis, in the sense that in our calculations this point is situated further away from the compact object. The deviation is largest for small Mach numbers. A consequence is that the mass accretion rate we find is also larger (see V.). A possible cause for these differences might be the rather high diffusivity in our method, which would explain why agreement is better for the more supersonic cases.

c) Mixture of gas and radiation
In this case the Mach number alone does not completely determine the flow, since the relative importance of radiation pressure increases with temperature. We chose an upstream density \( \rho_\infty \) of \( 10^{-8} \) g/cm\(^3\) and a temperature \( T_\infty \) of \( 10^5 \) K. This corresponds to a value for \( \beta \left( \equiv \frac{P_{gas}}{P_{gas} + P_{rad}} \right) \) in the undisturbed gas of 0.35. In the region close to the compact object the temperature will be higher and \( \beta \) will decrease even further, and the results should become comparable to those of other authors for \( \gamma = 4/3 \). For a mixture of gas and radiation the adiabatic sound speed is given by

\[
c_\infty^2 = \left( \frac{4 a T_\infty^4 + \rho_\infty RT_\infty}{\rho_\infty (4 a T_\infty^4 + 3 \rho_\infty RT_\infty)} \right)^2 + RT_\infty
\] (16)
Fig. 2a. The density distribution in the $\gamma=1.0$, $M=1.5$ model. The mass density is proportional to the density of points in the plot, the unit of distance along the axis is $k_{HL}$. The maximum density is $42.9 \rho_\infty$.

Fig. 2b. The velocity distribution in the central part of the grid. The maximum velocity $3.0 v_\infty$. 

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Fig. 3a. The density distribution in the $\gamma=5/3$, $M=3.0$ model. The maximum density is $25.8 \rho_\infty$.

Fig. 3b. The velocity distribution in the central part of the grid. The maximum velocity $2.3 v_\infty$. 
Fig. 4a. The density distribution in the model with a mixture of gas and radiation, M=3.0. The maximum density is 42.4 $\rho_\infty$.

Fig. 4b. The velocity distribution in the central part of the grid. The maximum velocity 3.0 $v_\infty$. 
and the Mach number is determined relative to this sound speed. We calculated models for Mach numbers 1.5 and 3.0, and for the latter case density and velocity distribution plots are presented in figures 4 a and b. Generally the shock has a smaller opening angle than in case of an ideal gas because the dissipated kinetic energy can build up less pressure for an effective $\gamma$ less than 5/3.

V. The accretion rate.

In fig. 5 the average mass accretion rate by the compact object is shown as a function of Mach number for the different equations of state, and the typical short-time variability of the accretion is indicated. The values are reasonably consistent with the results of Shima et al, the greatest difference being the somewhat higher rates found for the $\gamma=5/3$ case. In the isothermal models the accretion rate remains more variable in time than in the $\gamma=5/3$ models, which is probably a consequence of the fact that the accretion line instability described by Cowie (1977) is less suppressed by pressure effects. Note that in our models in which radiation pressure dominates we do not find the substantial increase in accretion rate over the Hoyle-Lyttleton value that was reported by Hunt (1979) for the $\gamma=4/3$ case. This is in agreement with the results of Shima et al.

An interesting phenomenon is the rise of the accretion rate over the classical Hoyle-Lyttleton value for Mach numbers just larger than 1 in the isothermal case (or near isothermal, $\gamma=1.1$ in the work of Shima et al.). This can be understood by a slight modification of the accretion line model. In this model it is assumed that all matter is accreted which does not have sufficient kinetic energy left to escape from the gravitational field after the cancellation of transverse momentum on the accretion line. When gas pressure effects are taken into account the cancellation of momentum and the associated kinetic energy dissipation take place in the shock. When the equation of state is isothermal the velocity jump across the shock is very large, and the kinetic energy associated with the velocity perpendicular to the shock is completely dissipated. Now for Mach numbers close to one, when the opening angle of the shock cone is $\sim 45^\circ$, matter coming from upstream in an almost free Keplerian orbit will pass the shock at a point which lies deeper inside the potential well than the intersection point of the orbit with the accretion line, and at which a larger component of the
Fig. 5. The mass accretion rate as a function of Mach number for the different equations of state. The accretion rate is normalized to the Hoyle-Lyttleton rate ($A_{HL}$). The typical short-timescale variability is indicated by the bars.
total velocity is perpendicular to the shock. These two factors tend to increase the mass accretion rate. As the Mach number increases, the opening angle of the shock decreases, and the accretion rate approaches the Hoyle-Lyttleton value.

IV. The kinetic energy dissipation rate

Interesting parameters of the accretion flow problem in the context of common envelope evolution of binaries are the total amount and the distribution of frictional energy liberated in the surrounding medium by the passage of the compact object. A first estimate of the kinetic energy dissipation rate as derived from the Hoyle-Lyttleton picture is

\[ E_0 = \frac{1}{2} \pi R_{HL}^2 \rho v_f^3 = 1.12 \times 10^{38} \left( \frac{M/M_\odot}{10^{-8}} \right)^2 \left( \frac{v_f}{10^7} \right)^{-1} \text{erg/sec} \quad (17) \]

We have made an estimate of the total amount of thermal energy which is deposited in the surrounding medium \( E_s \) in our calculations by determining the total kinetic energy dissipation rate \( E_{\text{dis}} \), and subtracting the thermal energy which is carried by the matter being accreted (see table 1). We find that the total amount of dissipated kinetic energy is between 7 and 10 times the estimate in equation (17). In the \( \gamma=5/3 \) case the larger part of this thermal energy disappears with the accreted matter, and approximately \( 2E_0 \) is deposited in the surroundings. In the radiative case both \( E_{\text{dis}} \) and \( E_s \) increase with Mach

<table>
<thead>
<tr>
<th>model</th>
<th>( E_{\text{dis}}/E_0 )</th>
<th>( E_s/E_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M=1.5, ( \gamma=5/3 )</td>
<td>7.61</td>
<td>2.07</td>
</tr>
<tr>
<td>M=2.0, ( \gamma=5/3 )</td>
<td>7.30</td>
<td>2.18</td>
</tr>
<tr>
<td>M=3.0, ( \gamma=5/3 )</td>
<td>7.30</td>
<td>2.36</td>
</tr>
<tr>
<td>M=3.75, ( \gamma=5/3 )</td>
<td>6.98</td>
<td>2.42</td>
</tr>
<tr>
<td>M=1.5, radiative</td>
<td>7.54</td>
<td>1.36</td>
</tr>
<tr>
<td>M=2.0, radiative</td>
<td>8.41</td>
<td>2.67</td>
</tr>
<tr>
<td>M=3.0, radiative</td>
<td>9.07</td>
<td>4.12</td>
</tr>
<tr>
<td>M=3.75, radiative</td>
<td>9.17</td>
<td>4.90</td>
</tr>
</tbody>
</table>

Table 1. The total energy dissipation rate \( E_{\text{dis}} \) and the rate \( E_s \) at which thermal energy is deposited in the surrounding medium, as a function of Mach number and equation of state. Both are normalized to the first-order estimate in equation 17.
number. The large values of $E_a$ are caused by the fact that the hot regions near the gravitating object lose energy to their surroundings by radiation. The reason that these numbers are generally larger than the first order estimate above is probably that much more matter is shocked than is accreted, since the bow shock extends to distances further than $R_{HL}$. From our calculations it is difficult to determine the final distribution of this energy, since we only consider the region within a few $R_{HL}$ from the compact object. We can however make a qualitative estimate from the width of the bow shock, which would yield a distribution with a width of $\sim 1 R_{HL}$ for the higher Mach numbers increasing to about $2 R_{HL}$ for a Mach number of about 1.5.

VII. Possible effects of an accretion luminosity

An assumption made in most models up to date is that all matter which flows through the inner boundary near the accreting object simply disappears, together with the thermal and kinetic energy it carries. In some applications the accretion rate can become much larger than the Eddington accretion rate at which the luminosity, originating in the region close to the accreting object where the inflowing matter is stopped and accumulates, becomes so great that the outward radiation pressure overcomes the gravitational attraction, and accretion stops. The effect this will have on the flow pattern depends on the optical depth of the surroundings. When these surroundings are optically thin ($R_{HL} \rho_0 < 1$) the effects of radiation pressure from the accretion luminosity can be described as an effective reduction of the gravitational force. A simple analytical estimate of the accretion rate based on the Hoyle-Lyttleton expression then yields (for $A_{Edd} < A_{HL}$)

$$A = A_{Edd} \left[ 1 - \left( \frac{A_{Edd}}{2A_{HL}} \right)^{1/2} \right]$$

where $A_{Edd}$ is the Eddington accretion rate, at which the radiation force exactly equals gravitation. In this way the accretion rate can be reduced significantly. Because the effective gravity is reduced at all distances to the compact object the flow pattern will simply scale to the new effective accretion radius. This means that the energy dissipation rate will also be reduced.

The limit opposite to the optically thin case is when the
surroundings are sufficiently optically thick that the photon diffusion time becomes larger than the dynamical time

$$\frac{R_{HL}^2 \rho K}{c} > \frac{R_{HL}}{v_\infty}$$

(19)

In this case the flow far upstream can not be affected by the accretion luminosity since the photons which diffuse out of the region close to the compact object are swept away with the matter flowing along it. In this case the accretion rate will be limited to a value near the Eddington limit, but the energy dissipation rate will remain of the same order since the typical size of the bow shock is not affected. We have tried to simulate this behaviour by letting the energy which is carried by the accreting matter accumulate in the cells next to the compact object, so that it could only be removed from there by radiation diffusion. We find that the accretion rate decreases as expected. The width of the bow shock however remains unaffected. The bow shock develops an extra bulge on the front edge, very similar to the flow pattern found by Shima et al for a hard non-absorbing sphere.

VIII. Conclusions

These model calculations of accretion flow, and others from the literature, have shown us that although the actual flow patterns can be very different from the simple accretion line picture, the mass accretion rate is very well approximated by the simple Hoyle-Lyttleton estimate. The total amount of kinetic energy dissipation seems to be underestimated in the classical estimate.

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On accretion of angular momentum from an inhomogeneous medium

M. Livio and N. Soker  Department of Physics, Technion-Israel Institute of Technology, Technion City, 32000 Haifa, Israel
M. de Kool and G. J. Savonije  Astronomical Institute, University of Amsterdam, 1018WB Amsterdam, The Netherlands

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Summary. The problem of a compact object accreting from an inhomogeneous medium has been studied, using a three-dimensional numerical scheme. When pressure effects are neglected, it has been shown that the mass accretion rate is given by the Bondi–Hoyle value. Not more than a few per cent of the angular momentum deposited into the accretion cylinder are accreted by the compact object. Some possible consequences for the case of neutron stars and white dwarfs accreting from a stellar wind are discussed.

1 Introduction

The problem of axisymmetric accretion from an infinite medium, by a gravitating point mass, has many astrophysical applications. It is not surprising, therefore, that many workers have treated various aspects of the problem both analytically (e.g. Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Spiegel 1970; Ruderman & Spiegel 1971; Lyttleton 1972; Wolfson 1977a; Rephaeli & Salpeter 1980) and numerically (e.g. Hunt 1971; Wolfson 1977b; Livio, Shara & Shaviv 1979; Okuda 1983; de Kool & Savonije 1985, in preparation).

In the non-axisymmetric case (e.g. a medium containing a density gradient), progress has been impeded by the lack of a fundamental theory. Some aspects of the problem were pointed out in the early works of Getting (1951) and Dodd & McCrea (1952), which used the classical Bondi & Hoyle (1944) approach.

The non-axisymmetric case has gained renewed interest in the context of compact objects accreting from a stellar wind, the question of spin-up and the possibility of forming a disc from wind accretion. Most workers have used a direct application of the Bondi–Hoyle result, noting that a density or velocity gradient in the flow results in a net deposition of angular momentum into the accretion cylinder (e.g. Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981) and assuming all that angular momentum to be accreted. Davies & Pringle (1980) pointed out, using a simplified two-dimensional picture, that the conditions required for matter to be accreted at all (in the Hoyle–Lyttleton picture) conflict with the possibility of accretion of angular
momentum. They concluded that to first order in $R_{\text{acc}}/H$ ($R_{\text{acc}}$ the accretion radius, $H$ the density gradient scale) no angular momentum will be accreted in their case but admitted that the situation can be more complex in the realistic, three-dimensional case. Their approach has been criticized by Wang (1981), who claimed that their result was merely a direct consequence of the particular simplification of using an ‘accretion line’ going into a point mass.

In a recent paper, Soker & Livio (1984) have attempted to examine the problem of accretion from a medium containing a density gradient, in the three-dimensional case, when the interactions were assumed to take place only on the accretion axis. Using a perturbative, analytical, Hoyle–Lyttleton-type approach, they have first shown that even in the three-dimensional case, the matter becomes confined to a thin ‘accretion layer’, after encountering the accretion axis. They have then attempted to calculate (under the above-mentioned assumptions) the first-order corrections to the specific angular momentum, due to pressure in the ‘accretion layer’.

In the present work, we present a three-dimensional numerical study of accretion from an inhomogeneous medium. Our basic assumptions and method of calculation are described in Section 2. The results are presented in Section 3 and discussed in Section 4.

2 Assumptions and method of calculation

We have used a pseudo-particle method to describe the hydrodynamics. The gas is treated as being divided into individual particles with given masses. Similar calculations have been used by Lucy (1977, 1980), Gingold & Monaghan (1977, 1978), Lin & Pringle (1976) and Hensler (1982a, b). Our method can be described as follows:

2.1 EQUATION OF MOTION

In the present, still preliminary calculation, we have neglected pressure gradients (the flow can be considered, therefore, as hypersonic). Most of the existing calculations which include pressure gradients do not conserve angular momentum and thus are not suitable, at least a priori for our present purpose, which is to study the accretion of angular momentum (e.g. Hensler 1982a). The equation of motion for the particles is thus in general (in dimensionless form)

$$\frac{d^2r}{dt^2} = -\frac{1}{2} \frac{r}{r^3} + a_i$$

where we have used as our unit length

$$R_{\text{BH}} = \frac{2GM}{V_0^2}$$

where $M$ is the mass of the accreting object and $V_0$ the flow velocity at infinity. Our unit time was chosen as $R_{\text{BH}}/V_0$. The term $a_i$ in equation (1) describes the effect of inter-particle interaction, to be described later.

2.2 THE GRID

We have used a three-dimensional, rectangular block shaped grid, $-1.5 < x < 3.9$, $-1.5 < y < 1.5$, $0 < z < 1.5$. The flow direction was taken as the $x$-axis. In the calculations with an inhomogeneous medium, the density gradient was taken in the $y$ direction as (achieved by
changing the masses of the particles)

\[ \rho = \rho_0 \left(1 + \frac{y}{H}\right). \]  

(3)

Since the problem is symmetric about the \(xy\) plane, the calculation was performed for the \(z>0\) half space (for every particle crossing with \(V_z\) to the \(z<0\) half space, we inject one at \(-z>0\) with \(-V_z\)). The compact object was taken as a cube at the origin of size 0.30. Our standard grid has been divided into equal cubic cells of size 0.15. The number of cells used was 36x20x10 (which means an effective number of 14400 because of the use of the symmetry plane). We have also performed calculations using other cell sizes (and compact object sizes). We have used an average of six particles per cell (a total of 43200), which is more than the number required for standard PIC techniques (e.g. Potter 1973). The number of cells and particles that have been used was in fact determined by the limitations imposed by the maximally allowed memory requirements in the IBM 3081D.

2.3 INTERACTIONS AMONG PARTICLES IN THE SAME CELL

We have taken the particles in each cell \(j\), to interact in the following way:

First, the centre of mass and velocity of each cell are calculated

\[ V_{c,j} = \frac{\sum m_i V_i}{\sum m_i} \]

\[ r_{cm,j} = \frac{\sum m_i r_i}{\sum m_i} \]  

(4)

the summations being on the particles in the \(j\)th cell. Then, an angular velocity can be defined by

\[ L_j^k = -I_j^{kl} \Omega_j^l \]  

(5)

where \(\bar{R}_i\) is the particle's coordinate in the centre of mass (of the cell) frame, \(L_j^k\) is the angular momentum component and \(I_j^{kl}\) is the moment of inertia component. We then find the new velocity of the particle by (see also Hensler 1982a; Lin & Pringle 1976).

\[ V_{new,i} = V_i (1 - \alpha) + \alpha U_i \]  

(6)

where

\[ U_i = V_{c,j} + \bar{R}_i \times \Omega_j \]  

(7)

and \(\alpha\) is a parameter determining the strength of the interaction in the cell (\(\alpha = 0\) means no interaction, \(\alpha = 1\) full interaction).

2.4 INTER-CELL INTERACTION

In order to prevent jumps in the fluid velocity in neighbouring cells and the development of instabilities, we have introduced a smearing of velocities over adjacent grid cells by the following
procedure. At even time-steps the vertices of the cells were taken at the coordinates

\[
\begin{align*}
N_x &= -N_{x_{\text{min}}}, \ldots, N_{x_{\text{max}}} \\
N_y &= -N_{y_{\text{max}}}, \ldots, N_{y_{\text{max}}} \\
N_z &= 0, \ldots, N_{z_{\text{max}}}
\end{align*}
\]

where \( \Delta R \) is the cell size. At odd time-steps the vertices were taken at

\[
(x, y, z) = \Delta R(N_x, N_y, N_z)
\]

for \( N_x = -N_{x_{\text{min}}} + \frac{1}{2}, N_y = \frac{1}{2}, N_z = \frac{1}{2} \).

The scheme is shown symbolically in Fig. 1 for a two-dimensional grid. The scheme has the following advantages: (i) It conserves angular momentum explicitly. This is to be compared to some of the methods for inter-cell interaction which use a cell around each particle, overlapping with neighbouring cells and do not conserve angular momentum (e.g. Hensler 1982a). (ii) The scheme allows, in principle at least, the freedom of choosing different interaction strengths \( \alpha_A, \alpha_B \) for the two grids A and B (Fig. 1) and checking the effects of different choices. Obviously a numerical viscosity \( \nu \) which is proportional to \( \Delta R^2/\Delta r \) is introduced.

2.5 CRITERIA USED IN NUMERICAL CALCULATIONS

All models were started with the particles randomly distributed and with a velocity \( V = (1, 0, 0) \). The time-step was always chosen to obey the Courant–Friedrich–Levy condition. New particles were injected into the grid for particles escaping from the grid or accreted.

The procedure of injection has been the following: An impact parameter \( b \) and an angle with the \( y \)-axis \( \phi \), were chosen randomly the particle was assumed to travel on an unperturbed hyperbolic orbit corresponding to \( b \) and \( \phi \) outside the grid and was injected into the grid along that orbit (its mass has been determined according to the \( y \) coordinate corresponding to \( b \) and \( \phi \)). Use was made of the \( z = 0 \) symmetry plane. In order to determine whether the system could be considered in a steady state (which actually served as an initial state for the real calculation), we have used several criteria:

(i) The number of particles injected (or escaping). The calculation has been carried out until the number of particles we had to inject converged to a limiting value (apart from obvious fluctuations, see Fig. 2).

\[ \text{GRID A} \]

\[ \text{GRID B} \]

\[ V_x \]

\[ \text{Figure 1. A schematic representation in two dimensions of the two grids on which the calculation has been performed at alternative time-steps, producing inter-cell interaction (see text).} \]
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Figure 2. The number of particles injected every 10 time-steps as a function of the number of time-steps. The arrows at the top indicate crossing times of the grid.

(ii) **Velocity criteria on the accretion axis.** We have checked for the sums of the $V_x$ component of the velocity in three regions of 16 cells each, located as shown in Fig. 3 (dashed areas) one cell above the $z=0$ plane. Again the calculation has been carried out until a limiting value has been approached.

(iii) **The number of particles at the accretion axis’ ‘tail’.** The total number of particles in 40 cells in the $z=0$ plane, at the edge of the grid around the accretion axis (see Fig. 3 marked by heavy line) was counted and followed till a limiting value was approximately reached.

Following the establishment of a steady state according to the above criteria, each run was

Figure 3. Regions used for velocity criteria (dashed areas) and number of particles criterion (marked by heavy line), for the establishment of a steady state (see text).
3 Results

3.1 Symmetric Case

In order to test the numerical code and investigate the effect of various factors, we have first run an axisymmetric case in which no density gradient existed. The standard run assumed $\alpha_A = \alpha_B = 1$. The results for the velocity profile in the $z = 0$ plane are shown in Fig. 5 and the density profile in Fig. 6. The stagnation point is clearly seen. Matter is seen to accumulate along the accretion axis in an ‘accretion cone’ the width of which is largely determined by the viscosity. The accretion radius obtained was $R_{\text{acc}} = 1.0 (2GM/V_0^3)$, in very close agreement to the Bondi & Hoyle (1944) and Hunt (1971) results. In Fig. 7 we present $V_x$ on the axis, as a function of $X$. Any solution along the largely dashed line that passes through the shaded region is a Bondi–Hoyle solution with the same stagnation point as ours (e.g. Lyttleton 1972). Our numerical solution clearly satisfies these conditions (the small deviation very close to the accreting body results from numerical viscosity).

An extensive study of axisymmetric accretion in two dimensions, using the PIC method is presently carried out by de Kool & Savonije (1985, in preparation). The results of the present work, which necessarily uses a more coarse grid, are consistent with theirs.

3.2 Accretion from a Medium with a Density Gradient

We have assumed the existence of a density gradient as described by equation (3) with $H = 4$. The main results are the following.
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Figure 5. The velocity profile in the \( z = 0 \) plane for the symmetric case.

As predicted by Soker & Livio (1984), the gas is strongly concentrated towards the \( z = 0 \) plane. This can be clearly seen in Fig. 8, where the mass in the downstream side of successive planes is plotted as a function of \( z \).

In the \( z = 0 \) plane, the matter forms a displaced 'accretion cone', very similar to the one described by Davies & Pringle (1980) in their two-dimensional example. The 'accretion cone' is shown in Fig. 9 which represents an instantaneous picture of the location of all particles. The velocity and density profiles in the \( z = 0 \) plane are presented in Figs 10 and 11 respectively and again exhibit the clear formation of the displaced 'accretion cone'. The accretion rate obtained (for \( a_A = a_B = 1 \)) is of the order \( M_{\text{acc}} \approx 1.0 M_{\text{BH}} \), where \( M_{\text{BH}} \) is the Bondi–Hoyle accretion rate in the symmetrical

Figure 6. The density profile (represented by the areas of the squares) in the \( z = 0 \) plane for the symmetric case.
case (for $R_{\text{acc}}=2GM/V_0^2$). As an additional check we tried decreasing $\alpha_B$ which resulted in lower accretion rates as expected (the dependence being almost linear). It should be remembered that the results of Bondi & Hoyle (1944) for the symmetrical case actually only state that the accretion radius is between $GM/V_0^2$ and $2GM/V_0^2$.

Our most important new result concerns the accretion of angular momentum. Our results indicate that the rate of accretion of angular momentum is very low and certainly not more than a few per cent of

$$L_{\text{BH}} = \frac{1}{4H} M_{\text{BH}} \frac{(2GM)^2}{V_0^3}$$

which was the rate assumed by Illarionov & Sunyaev (1975) and Shapiro & Lightman (1976), based on the accretion of all the angular momentum entering the accretion cylinder. The results are in fact consistent with almost no accretion of angular momentum (other than that of matter hitting the accreting object directly from the upstream side and within the accuracy of the calculation). Most of the angular momentum accretion rate obtained, $L_{\text{acc}} \approx 0.08 L_{\text{BH}}$, results from the fact that our accreting body is relatively large, so that matter coming from the upstream side can be accreted, with its angular momentum, prior to reaching the accretion cone. This fact can be realized by noting that even just the free orbits of particles hitting the accreting object from the upstream side would lead to an accretion rate of $L_{\text{acc}} \approx 0.06 L_{\text{BH}}$ and the interaction effectively increases the size of the body. Calculations performed with different sizes of the accreting body have indeed shown that a significant fraction of the obtained angular momentum accretion comes from upstream.

Figure 7. The velocity component $V_x$ on the accretion axis, as a function of $x$, for the symmetric case. Our numerical results are represented by the full line, the dashed line and the shaded region represent a Bondi–Hoyle solution.
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Figure 8. The mass in successive planes in the downstream side as a function of $z$ (for two grid cell sizes).

Figure 9. An instantaneous picture of the location of all particles in the grid, in the inhomogeneous case. The accreting object is marked by a cross.
The fact that very little angular momentum is accreted can be traced to two causes:

(i) A displacement of the accretion cylinder (or its cross-section facing the flow) towards lower densities (which is related to the displacement of the accretion cone).

(ii) Cancellation of transverse momentum at the displaced accretion cone as in the Bondi–Hoyle (1944) picture. With respect to point (i), it can be shown that if the displacement is small and the cross-section of the accretion cylinder is still roughly circular (which is actually usually not the case), then the decrease in the rate of angular momentum accretion resulting from the displacement alone can be roughly estimated as

\[
\frac{L_{\text{dis}}}{L_{\text{sym}}} = 1 - \frac{4Hd}{M_{\text{acc}}/M_{\text{BH}}}
\]

(11)
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where \(d\) is the displacement of the cross-section (in dimensionless units), \(\dot{L}_{\text{dis}}\) is the rate of angular momentum deposition into the displaced cross-section and \(\dot{L}_{\text{sym}}\) is the rate of deposition if the same mass had gone into the symmetric cross-section. Typically in the numerical calculation a reduction by a factor of \(\sim 4\) in the rate of angular momentum deposition resulted from this displacement.

Point (ii) above is in certain respects a manifestation of the point raised by Davies & Pringle (1980) in the two-dimensional case, that a cancellation of transverse momentum is required for accretion to take place. It should be noted that in our case \(R_{\text{acc}}/H = 1/4\) and the result is no longer a first-order approximation in the \(R_{\text{acc}}/H < 1\) case.

Keeping \(\alpha_{\text{as}} = \alpha_{\text{as}} = 1\) and changing other numerical parameters such as the grid size and the time-step gave always results of the order \(M_{\text{acc}} = M_{\text{BH}}\), \(\dot{L}_{\text{acc}} = 0.07 - 0.1 \dot{L}_{\text{BH}}\). Similar results were obtained when a different density gradient was used \((H = 16\), equation 3\).

4 Discussion

The indications of the preliminary results of the present study (which neglects pressure) can be summarized as follows:

(i) The mass accretion rate, on to a compact object moving through a medium containing a density gradient, is not very different from the Bondi–Hoyle value, obtained in the axisymmetric case.

(ii) The rate of accretion of angular momentum is not more than a few per cent of the rate at which angular momentum enters the Bondi–Hoyle (symmetric) accretion cylinder.

The present calculation does not include pressure effects, a calculation with pressure gradients is now in progress. Because of the preliminary nature of the results we do not want at this stage to speculate on all their possible consequences. We would like, however, to point out certain topics which may be significantly influenced.

Accretion of angular momentum from a stellar wind has been invoked to explain spin-up (and spin-down under certain circumstances) of some neutron stars (e.g. Vela X-1, 4U 1538-52; Wang 1981). The time-scale for the spin-up was taken as

\[
- \frac{P}{P} \approx 0.8 \frac{1}{\eta} \left( \frac{M_{\text{ns}}}{M_{\text{acc}}} \right) \left( \frac{R_{\text{ns}}}{R_{\text{acc}}} \right)^2 \left( \frac{P_{\text{orb}}}{P_{\text{ns}}} \right)
\]

(12)

where \(M_{\text{ns}}, R_{\text{ns}}\) and \(P_{\text{ns}}\) are the neutron star’s mass, radius and spin period respectively, \(P_{\text{orb}}\) is the binary orbital period and \(\eta\) is a parameter depending on the velocity and density gradients in the wind.

If the results obtained in the present work are confirmed by our more realistic calculations (including e.g. pressure), then the time-scale for spin-up should be about \(10 - 100\) time longer than the one expressed by equation (12). Furthermore, the formation of an accretion disc around neutron stars accreting from the companion’s wind is marginal even in the existing ‘theory’, since it requires very low wind velocities (e.g. Wang 1981). Such a disc formation becomes virtually impossible when our present results are considered since a relative velocity of less than \(\sim 100\, \text{km s}^{-1}\) is required between the neutron star and the wind.

The possibility of forming a disc from wind accretion would have been more favourable (if equation 10 is used) in the case of a white dwarf accreting from the wind of a cool giant, as pointed out by Livio & Warner (1984) for Mira, SY For and 56 Peg. However, again if our present results are confirmed, a disc cannot form in this case either.
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It should be pointed out, however, that if an accretion disc starts to form, even temporarily, it will probably grow due to the viscous interaction. This adds further weight to the point made by Livio & Warner (1984), that the observational establishment of the existence or non-existence of discs in these systems and in similar ones such as HR 3080, ν Her, HR 8157, can contribute significantly to the understanding of the accretion process.

References

Accretion of angular momentum from an inhomogeneous medium – II. Isothermal flow

N. Soker and M. Livio* Department of Physics, Technion, Haifa 32000, Israel
M. de Kool and G. J. Savonije Astronomical Institute, University of Amsterdam, 1018WB Amsterdam, The Netherlands

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Summary. We have studied the problem of accretion (by a compact object) from an inhomogeneous medium, for the case of an isothermal flow.

Using a three-dimensional numerical scheme, we found the mass and angular-momentum accretion rates. The mass accretion rate agrees well with the Bondi–Hoyle theory. The rate of accretion of angular momentum is only a small fraction of the rate at which angular momentum is deposited into the accretion cylinder. This confirms our previous results which were obtained without the inclusion of pressure effects.

1 Introduction

Accretion from an inhomogeneous medium, by a compact object, is an important process for two classes of objects:

(i) Neutron stars accreting from the wind of early-type companions and
(ii) white dwarfs accretion from the winds of cool giants.

Because of the lack of a basic theory in the non-axisymmetric case, progress has been rather limited. Most workers have simply tried to make use of the Bondi & Hoyle (1944) results which were obtained for the axisymmetric case (e.g. Dodd & McCrea 1952; Illarionov & Sunyaev 1975; Shapiro & Lightman 1976).

A fundamental question in the case of accretion from an inhomogeneous medium is whether the accreting object can accrete angular momentum. A simple inspection of the Bondi–Hoyle picture, reveals, that if the accretion cylinder (of radius $R_{\text{acc}}$) remains unchanged, then the existence of a density (or velocity) gradient in the medium results in a net deposition of angular momentum into the cylinder. Several authors have assumed that all the angular momentum entering the accretion cylinder is actually accreted (Illarionov & Sunyaev 1975; Shapiro &

*Present address: Department of Astronomy, University of Illinois, Urbana, IL61801, USA.
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Lightman 1976; Wang 1981). An objection to this assumption has been raised by Davies & Pringle (1980), who pointed out that the condition imposed on the matter to be accreted (a cancellation of the momentum transverse to the accretion line), conflicts with the idea that this matter can still possess angular momentum. Davies & Pringle (1980) have indeed shown, in a highly simplified two-dimensional case, that no angular momentum is accreted.

Two important questions that thus emerged were:

(i) Is angular momentum accreted in the three-dimensional case? Wang (1981) argued that the result obtained by Davies & Pringle (1980) was a consequence of the use of a restricted two-dimensional model and the concept of an 'accretion line' [still in the Hoyle-Lyttleton (1939) approach neglecting pressure].

(ii) What are the effects of pressure on the rate of accretion of angular momentum?

In an attempt to answer the first question Livio et al. (1986, hereafter Paper I) have performed a three-dimensional numerical calculation of accretion onto a compact object, from a medium containing a density gradient. The calculation was performed neglecting pressure gradients (in the Hoyle-Lyttleton approach) and using the PIC method. The results of Paper I have shown that the accretion rate (of mass) in the non-axisymmetric case, was very similar to the one obtained in the case of accretion from an homogeneous medium (e.g. Bondi & Hoyle 1944; Hunt 1971). The rate of accretion of angular momentum obtained, was very low and amounted to not more than a few per cent of the angular momentum flowing into the symmetrical accretion cylinder.

In the present work we make a first step towards answering the second question above, by including pressure effects in the calculation of an isothermal flow.

The assumptions made, the numerical scheme and the results are given in Section 2 and the results are summarized and discussed in Section 3.

2 The isothermal case, numerical scheme and results

The pseudo-particle scheme that was used to treat the three-dimensional hydrodynamics has been fully described in Paper I, thus we shall not repeat this description here. The new element that was introduced was the inclusion of pressure effects: (i) a calculation of the pressure in each cell, which in our velocity units \( V_m = V_0 = 1 \) reads

\[
P = \frac{1}{\mathcal{A}^2 Q} \tag{1}
\]

where \( \mathcal{A} \) is the Mach number for the flow. (ii) Pressure gradients in the equation of motion that were calculated in the following way (see Hensler 1982): If we look at three adjacent grid cells \( j-1, j, j+1 \) [say in the \( x \) (flow) direction] and a particle, \( i \), that is located in the \( j \)th cell then the \( x \) component of the pressure gradient is

\[
\frac{1}{Q} \frac{dP}{dX} = \frac{1}{Q_j} \left[ \frac{(P_{j+1}-P_j)(x_j-x_i)+(P_j-P_{j-1})(x_{j+1}-x_i)}{\Delta R^2} \right] \tag{2}
\]

where \( \Delta R \) is the cell size and \( x_j, x_{j+1} \) are the boundaries of the respective cells.

The following requirements were fulfilled by all runs: (i) in all the calculations we required that the shock will be 'contained' in the grid, namely, that the shock will not cross the grid edges that are parallel to the flow. (ii) The average number of particles per cell was larger than 4.5 (e.g. Potter 1973).

We have used the same criteria as described in Paper I, to test the stability of the flow.
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The flow direction was taken as the $x$ axis and again, a density gradient was assumed, of the form

$$
\rho = \rho_0 \left(1 + \frac{y}{H}\right).
$$

(3)

The number of cells used was $24 \times 28 \times 14$.

We have performed calculations with a Mach number of 4 and $H = 5$ (equation 3) and with a Mach number of 2 and $H = 16$. The density gradients corresponding to each Mach number were chosen in such a way that requirements (i) and (ii) above were fulfilled.

The accretion rates obtained were $\dot{M}_{\text{acc}} = 0.98 \dot{M}_{\text{HL}}$ for the Mach 4 case and $\dot{M}_{\text{acc}} = 0.89 \dot{M}_{\text{HL}}$ for the Mach 2 case, where the Hoyle–Lyttleton accretion rate $\dot{M}_{\text{HL}}$ is (see also Bondi & Hoyle 1944)

$$
\dot{M}_{\text{HL}} = \frac{4\pi (GM)^2 \rho_0}{V_0^3}.
$$

(4)

The density profile in the $z = 0$ plane for the Mach 4 case is shown in Fig. 1, and for the Mach 2 case in Fig. 2. We also give the density contours for the symmetrical case in Fig. 3. The shock is very clear and it exhibits the typical broadening as one goes to lower Mach numbers. Another feature that is demonstrated in the figures is the displacement of the accretion cone towards the lower density. The displaced accretion cone is more clearly visible in Fig. 4 which represents the instantaneous location of all particles for the Mach 4 case. The velocity profiles in the $z = 0$ plane are shown in Figs 5 and 6 for the Mach 4 and Mach 2 cases respectively.

Our main interest has been in the accretion of angular momentum. We found an accretion rate of $\dot{L}_{\text{acc}} = 0.1 \dot{L}_{\text{BH}}$ in the Mach 4 case and $\dot{L}_{\text{acc}} = 0.14 \dot{L}_{\text{BH}}$ in the Mach 2 case where

$$
\dot{L}_{\text{BH}} = \frac{1}{4H} \dot{M}_{\text{HL}} \frac{(2GM)^2}{V_0^3}.
$$

(5)

Figure 1. The density profile (represented by the areas of the squares) in the $z = 0$ plane for the Mach 4 isothermal case ($H = 5$, equation 3).
is the rate at which angular momentum is deposited into the (symmetric) Bondi–Hoyle accretion cylinder (see Dodd & McCrea 1952). We therefore find, as in Paper I, where pressure effects were not included, that only a small fraction of the angular momentum assumed to be accreted in previous works (e.g. Shapiro & Lightman 1976; Wang 1981) is actually accreted.

In an attempt to follow the process of depletion of angular momentum for accreted matter (by interactions and angular-momentum transfer), we have followed the mass and angular momentum of a ring about the accretion axis (for simplicity, in a calculation neglecting pressure). The entire ring was contained also in the actual displaced, accretion cylinder cross-section. The results are presented in Fig. 7. The small increase of the angular momentum between points \( t' \) and \( t'' \) (marked only for discussion purposes), is a result of the fact, that due to the displacement of the accretion cone towards lower densities, all parts of the ring do not enter the interaction region simultaneously. We then observe the steep decrease in the angular momentum (as the matter of

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**Figure 2.** Same as Fig. 1 for Mach 2 (\( H=16 \)) case.

**Figure 3.** Density contours the \( z=0 \) plane, for the homogeneous, isothermal case, Mach=2.
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Figure 4. The instantaneous location of all particles in the inhomogeneous, isothermal, Mach = 4 (H = 5) case.

the ring collides with other matter) which precedes the decrease in the mass of the ring, as matter starts to be accreted. It is thus demonstrated that the various parts of the mass that is accreted, are depleted of their angular momentum in the accretion cone region, as required in the Bondi–Hoyle (1944) picture and consistently with the point raised by Davies & Pringle (1980).

A different exploratory calculation is described in Fig. 8. In this calculation we do not look for the steady state, but rather observe dynamical effects as accretion is initiated. We start with a cloud of matter (with a density gradient) at some distance from the accreting body and follow it as it hits the compact object. From the figure we see that the accretion rate increases first at t₀, as matter encounters the accreting body. It then stays at a constant value, as accretion takes place only from the upstream side (for t₀<r<t₁) and then it increases abruptly (at t₁) to roughly its final value, as matter starts to accrete from the accretion cone downstream. This demonstrates clearly

Figure 5. The velocity profile in the z=0 plane for the Mach 4 (H = 5) case.
that most of the accretion takes place via the accretion cone. The angular momentum accreted from upstream (denoted by triangles), stays more or less constant (apart from fluctuations) from $t_0$ onwards. The total rate of accretion of angular momentum (empty circles) increases temporarily as matter starts to accrete from downstream, but then settles to a value only slightly

![Figure 6. The velocity profile in the $z=0$ plane for the Mach 2 ($H=16$) case.](image)

![Figure 7. The mass and angular momentum (relative units) of a ring about the accretion axis. $t'$ and $t''$ are chosen arbitrarily around the increase of angular momentum (see text). The centre of mass of the ring $X_{cm}$ (in units of $2GM/V_0^2$) is shown on the right.](image)
Figure 8. The accretion rate, from an impinging cloud, the rate of accretion of angular momentum and the rate of accretion of angular momentum from the upstream side $[\dot{L}(V_x>0)]$, in relative units, as a function of time (see text).

above the rate of accretion from upstream. This demonstrates the point noted in Paper I, that much of the accreted angular momentum comes from upstream, from matter hitting the accreting object directly, without passing through the accretion cone.

We should mention that due to the presence of the pressure gradient imposed on our grid (caused by the assumed density gradient), the total angular momentum is not exactly conserved (e.g. it increases by 8.6 per cent in the Mach 4, $H=5$ case). This, however, does not affect our conclusion that only a small fraction of the angular momentum deposited into the accretion cylinder is accreted. This was confirmed by a number of different runs with different conditions (e.g. different sizes of accreting bodies). We also performed one run of Mach 2 ($H=16$) in which we have intentionally taken a narrower grid (which caused the shock to ‘escape’ through the sides of the grid, a situation not allowed normally, as explained at the beginning of this section), this resulted in a decrease of total angular momentum by ~20 per cent, but nevertheless ~13.1 per cent of the angular momentum entering the (symmetric) accretion cylinder was accreted, in very good agreement with the standard case.

3 Discussion

The mass and angular-momentum accretion rates have been obtained, for accretion from an inhomogeneous medium, in the isothermal case. Bondi (1952) suggested an interpolation formula for the accretion rate between the velocity-dominated and pressure-dominated regimes. This formula can be expressed as

$$\dot{M}_{\text{acc}} = \dot{M}_{\text{HL}} \frac{\mathcal{H}^3}{(1+\mathcal{H}^2)^{3/2}}$$

where $\dot{M}_{\text{HL}}$ is given by equation (4) and $\mathcal{H}$ is the Mach number. For the cases calculated in the present work (in all of which $\gamma=1$) this would give $\dot{M}_{\text{acc}} = 0.91 \dot{M}_{\text{HL}}$ for the Mach 4 case and
\( \dot{M}_{\text{acc}} \approx 0.72 \dot{M}_{\text{H}} \) for the Mach 2 case. While the numerical values do not agree exactly with the numbers obtained (0.98 and 0.89, respectively), the qualitative trend does agree (the same trend is obeyed by the calculation without pressure, corresponding to hypersonic flow). Two things should be remembered here; (i) equation 6 does not represent an exact solution and (ii) numbers differing by a few per cent only, in the present calculation (having a relatively coarse grid), should be treated with caution.

It has been confirmed (at least in the isothermal case) that the rate of accretion of angular momentum represents only a small fraction of the net angular momentum deposited into the Bondi–Hoyle (symmetrical) accretion cylinder. It should be noted, however, that the rate of accretion of angular momentum when pressure effects are included is somewhat larger than in the Hoyle–Lyttleton picture (neglecting pressure). It can be therefore expected that a somewhat larger fraction of angular momentum will be accreted for \( \gamma > 1 \). A calculation with \( \gamma = 4/3 \) is presently being carried out. Also, for any given value of \( \gamma \), the rate of accretion of angular momentum can be expected to be somewhat larger for lower Mach numbers.

As already mentioned in paper I, the fact that the rate of accretion of angular momentum is lower than it has been previously assumed, (at least when pressure is neglected and in the isothermal case), can have important consequences for two physical processes: (i) Spin-up (and spin-down) of neutron stars accreting from the winds of early-type companions, and (ii) the possible formation of accretion discs around white dwarfs accreting from the winds of cool giants.

We shall postpone a detailed discussion of these issues, as well as a discussion of individual systems to future work, when we shall have a complete picture of the accretion process for different values of \( \gamma \).

Acknowledgments

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References

Accretion from an inhomogeneous medium – III. 
General case and observational consequences

M. Livio* Department of Astronomy, University of Illinois, Urbana, IL 61801, USA
N. Soker Department of Physics, Technion, Haifa 32000, Israel
M. de Kool and G. J. Savonije Astronomical Institute, University of Amsterdam, 1018WB Amsterdam, Netherlands

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Summary. We study the problem of accretion by a compact object from an inhomogeneous medium, in the general γ≠1 case. The mass accretion rate is found to decrease with increasing γ. The rate of accretion of angular momentum is found to be significantly lower than the rate at which angular momentum is deposited into the Bondi–Hoyle, symmetrical, accretion cylinder. We discuss the consequences of our results for the cases of neutron stars accreting from the winds of early-type companions and white dwarfs and main-sequence stars accreting from winds of cool giants.

1 Introduction

The classical problem of accretion by a gravitating object, moving through an infinite medium (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952) has gained new interest through the use of multi-dimensional hydrodynamic calculations (e.g. Hunt 1975, 1979; Livio, Shara & Shaviv 1979; Okuda 1983; Shima et al. 1985; Takeda et al. 1985). The problem of accretion from an inhomogeneous medium, however, suffered from both the lack of a basic theory (although see the works of Gething 1951 and Dodd & McCrea 1952) and the need to perform three-dimensional calculations. At the same time, it has been realized that accretion from an inhomogeneous medium has important consequences for such processes as spin-up and disc formation, in the case of compact objects accreting from stellar winds. In an attempt to produce results that can be related to observations, several authors have therefore used the Bondi–Hoyle (1944) picture to argue that all the angular momentum deposited into the symmetrical Bondi–Hoyle accretion cylinder is actually accreted (Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). Davies & Pringle (1980) were the first to point out that in the Bondi–Hoyle picture, for matter to be accreted at all, a cancellation of the momentum transverse to the accretion line is required and thus no angular momentum can be accreted. It was not clear, however, whether this conclusion

*On leave from: Department of Physics, Technion, Haifa, Israel.
remains valid in a realistic three-dimensional case, in which the ‘accretion line’ broadens into a column or a cone (as argued by Wang 1981).

In an attempt to resolve the question of accretion from an inhomogeneous medium, we have performed a three-dimensional calculation, first neglecting pressure (Livio et al. 1986, hereafter LSKS) and then for an isothermal flow (Soker et al. 1986, hereafter SLKS). We found that for those cases, while the mass accretion rate was very close to the one predicted by the Bondi–Hoyle theory (for the homogeneous case), the rate of accretion of angular momentum was very much lower than the rate assumed by previous authors (Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). In that present work we expand upon our previous work and calculate the general case of γ≠1 (γ = the specific heats ratio). The equations and method of calculation are described in Section 2, our results are presented in Section 3 and discussed in Section 4.

2 Equations and method of calculation

The method of calculation used is the same as that described by LSKS and SLKS (apart from the treatment of the energy equation); we shall thus describe it only briefly for completeness.

2.1 EQUATION OF MOTION (FOR PARTICLES)

\[
\frac{d^2 \mathbf{r}}{dt^2} = -\frac{1}{2} \frac{\mathbf{r}}{r^3} + \nabla P + \mathbf{a}_i \tag{1}
\]

where the unit length was chosen as \( R_{HL} = 2GM/V_0^2 \) and the unit time as \( R_{HL}/V_0 \) (\( V_0 = 1 \)). The inter-particle interaction is represented by \( \mathbf{a}_i \). The pressure gradient term was calculated as in SLKS.

2.2 INTER-PARTICLE INTERACTION

The velocity of each particle following the interaction is given by (see also Lin & Pringle 1976; Hensler 1982)

\[
\mathbf{V}_{\text{new},i} = \mathbf{V}_i (1-\alpha) + \alpha \mathbf{U}; \tag{2}
\]

where \( \alpha \) is a parameter defining the strength of the interaction (typically taken as 1) and

\[
\mathbf{U}_i = \mathbf{V}_{i,j} + \mathbf{R}_i \times \mathbf{\Omega}_j \tag{3}
\]

where \( \mathbf{V}_{i,j} \) is the centre of mass velocity of the \( j \)th cell and \( \mathbf{R}_i \) is the particle’s coordinate in the centre of mass (of the cell) frame. The angular velocity \( \mathbf{\Omega} \) is defined by

\[
\mathbf{L}_j = -\mathbf{I}_j^{ik} \mathbf{\Omega}_j^k \tag{4}
\]

where \( \mathbf{L}_j \) is the angular momentum of the cell and \( \mathbf{I}_j \) is the moment of inertia tensor components. The inter-cell interaction is treated by the two-grid method described by LSKS.

2.3 THE ENERGY EQUATION

The energy equation was written in general as

\[
\frac{\partial}{\partial t} \left[ \rho \left( \frac{1}{2} V^2 + \epsilon \right) \right] + \nabla \cdot \left[ \rho \mathbf{V} \left( \frac{1}{2} V^2 + \epsilon + \frac{P}{\rho} \right) + F_j + T_j \right] = \rho \mathbf{V} \cdot \mathbf{g} \tag{5}
\]

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where $\varepsilon$ is the internal energy, $F_j$ represents the energy dissipation rate in the $j$th cell due to inter-particle interaction, $T_j$ represents the effective rate of heat transport (by inter-particle interactions) and $g$ is the gravitational acceleration. We have calculated the change in energy in two steps; in the first step we calculated the change due to interactions alone (no acceleration due to gravity). We have assumed that the dissipation in kinetic energy is transformed into internal energy and thus the enthalpy $E_j$ of each cell is given by

$$E_j^{\text{new}} = E_j^{\text{old}} + \frac{1}{2} \sum m_i [ (V_i^{\text{old}})^2 - (V_i^{\text{new}})^2 ] ,$$  \hspace{1cm} (6)$$

where $V_i^{\text{old}}$, $V_i^{\text{new}}$ represent the particle’s velocities before and after the interaction respectively. The specific energy per particle $e_i$ is related to $E_j$ through

$$E_j = \gamma \sum m_i e_i .$$  \hspace{1cm} (7)$$

In the second, acceleration step, we have

$$e_i^{\text{new}} = e_i^{\text{old}} + \frac{1}{\gamma} \left[ \frac{1}{2} (V_i^{\text{old}})^2 - \frac{1}{2} (V_i^{\text{new}})^2 + \frac{1}{2} \left( \frac{1}{r_i^{\text{new}}} - \frac{1}{r_i^{\text{old}}} \right) \right]$$  \hspace{1cm} (8)$$

where old and new in this case refer to the stages before and after the acceleration has taken place. The pressure in the $j$th cell is calculated by (cell size normalized)

$$P_j = (\gamma - 1) \sum m_i e_i .$$  \hspace{1cm} (9)$$

For the particles that are injected into the grid (see LSKS) we have

$$e_{i0} = [M^2 \gamma (\gamma - 1)]^{-1}$$  \hspace{1cm} (10)$$

where $M$ is the Mach number.

We have used the same criteria as described by LSDS for the establishment of a (quasi) steady state. Following that, we have carried out the different runs for 35 crossing times of the grid and then average values of the physical quantities were calculated. The grid in all runs (apart from a few test runs to be described shortly) contained $24 \times 28 \times 14$ cells (use was made of the $z=0$ symmetry plane). The accreting body was represented by a cube of size 0.15 (in our unit of length). The average number of particles per cell was four. These numbers were chosen based on trial runs and the constraints imposed by the maximum allowable memory on the IBM 3081D. A number of tests with different grid sizes (e.g. $32 \times 24 \times 12$, $32 \times 20 \times 10$) and different average numbers of particles per cell (e.g. 6.37, 3.2) were performed and we shall discuss the effects of such changes in the next section, when we present the results.

3 Results

In all calculations we have used a density profile at infinity of the form

$$\varrho = \varrho_0 \left( 1 + \frac{y}{H} \right) ,$$  \hspace{1cm} (11)$$

where the flow direction was taken as the $x$ axis. In the present work we have used $H=16$ (other values of $H$ have been used in LSKS and SLKS). We have performed calculations with $\gamma=7/6$ at Mach numbers $M=3$, 16, $\gamma=4/3$, 3/2, and 5/3 at Mach number 16. The velocity and density profiles that were obtained in the $z=0$ plane are presented in Figs 1–4. As can be seen in the
Figure 1. (a) The velocity profile in the $z=0$ plane for $\gamma = 7/6$, Mach = 3, $H = 16$. (b) The density profile (represented by the areas of the squares) in the $z=0$ plane for $\gamma = 7/6$, Mach = 3, $H = 16$. 

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**Figure 2.** (a) The velocity profile in the $z=0$ plane for $y=7/6$, Mach = 16, $H = 16$. (b) The density profile (represented by the areas of the squares) in the $z=0$ plane for $y=7/6$, Mach = 16, $H = 16$. 

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Figure 3. (a) The same as Fig. 2(a) for $\gamma=4/3$. (b) The same as Fig. 2(b) for $\gamma=4/3$. 
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\[ \Gamma = \frac{5}{3}, \quad M = 16, \quad H = 16 \]

Figure 4. (a) The same as Fig. 2(a) for \( \gamma = 5/3 \). (b) The same as Fig. 2(b) for \( \gamma = 5/3 \).
figures, the shock slightly 'escapes' from the grid in the \( \gamma = 5/3 \) case (also for \( \gamma = 3/2 \)) so we should therefore treat the numerical values obtained in these runs with caution. The results can be summarized as follows:

(i) For a given value of \( \gamma \), the shock angle is larger for a smaller Mach number [e.g. Figs 1(a), 2(a) and figs 1–2 of SLKS]. This is of course a known result from flows past non-gravitating bodies, where the cone angle is \( \arcsin(1/M) \). However, it should be remembered that the shock in the case of a gravitating body is not produced by the fact that the flow directly impinges on the body, but rather by the dense region generated through the gravitational influence.

(ii) For a given (large) Mach number the shock angle is larger for a larger value of \( \gamma \) (Figs 1–4). The same result was found by Shima et al. (1985) in their two-dimensional hydrodynamic study. This can be expected from the fact that, as \( \gamma \) is reduced (towards the isothermal, \( \gamma = 1 \) case), less pressure support is available for the shock. In a realistic flow, the situation with \( \gamma = 1 \) would correspond to a cooling time for the gas that is short compared to the flow time-scale, while \( \gamma = 5/3 \) would correspond to a radiationless case.

The increase of the shock angle with \( \gamma \) was obtained also in the self-similar solutions of Bisnovatyi-Kogan et al. (1979) and Wolfson (1977), corresponding essentially to an infinite accretion radius.

We find (as did Shima et al. 1985) that in the \( \gamma = 5/3 \) case, an 'accretion cone' rather than an 'accretion column' is formed, namely, the density in this case is highest behind the shock and not along an accretion line. Our resolution is not good enough (because of the memory constraints imposed on a three-dimensional calculation) to be able to detect the formation of a bow shock rather than a shock attached to the accreting body.

A very crude estimate of the shock angle (at distances larger than the accretion radius) can be obtained by noting that the post-shock flow is more or less parallel to the accretion axis [e.g. Figs 1(a), 2(a) and 3(a); figs 4 and 5 in Hunt 1971, figs 2 and 3 in Shima et al. 1985]. We then obtain from the shock conditions (see Fig. 5 for the definition of the angles)

\[
\tan \alpha_2 = \tan \alpha_1 \frac{(\gamma-1)M^2+2}{(\gamma+1)M^2}.
\]

From equation (12) it can be seen that for a given (large) \( M \), \( \alpha_2 \) increases for increasing \( \gamma \) and for a given \( \gamma \), \( \alpha_2 \) is a decreasing function of \( M \), as was found in the calculation.

(iii) The cross-section of the accretion cylinder is displaced towards the lower density and so is the accretion column or cone behind the accreting body. This effect is not so pronounced in the present calculation because of the relatively large value of \( H \), but is very pronounced in the larger density gradient calculations of LSKS and SLKS.

(iv) For a given (large) Mach number, the accretion rate decreases with increasing \( \gamma \) (see Table 1). The same result was found by Shima et al. (1985, their fig. 9). It is interesting to note that the

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**Figure 5.** A schematic representation of the pre-shock and post-shock velocities (at distances larger than the accretion radius, see text).


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Table 1. Results of numerical calculations for mass and angular momentum accretion rates.
Numbers appearing in parentheses should be viewed with caution (see text). The rate of accretion of angular momentum from upstream is denoted by $L_+$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Mach=2</th>
<th>Mach=4</th>
<th>7/6</th>
<th>4/3</th>
<th>3/2</th>
<th>5/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{M}/\dot{M}_{HL}$</td>
<td>0.89</td>
<td>0.98</td>
<td>0.88</td>
<td>0.72</td>
<td>(0.58)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>$L/L_{BH}$</td>
<td>0.14</td>
<td>0.10</td>
<td>0.17</td>
<td>0.23</td>
<td>(0.18)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$L_+/L$</td>
<td>0.34</td>
<td>0.23</td>
<td>0.37</td>
<td>0.43</td>
<td>(0.49)</td>
<td>(0.50)</td>
</tr>
</tbody>
</table>

(maximal) accretion rate obtained in the case of spherically symmetrical accretion from a stationary cloud (Bondi 1952) behaves similarly. This of course reflects the effect of the pressure that builds up, in the dense region, on the accretion rate. With respect to the dependence on the Mach number, the isothermal calculation of SLKS has shown the dependence of the accretion rate on the Mach number to agree qualitatively with the Bondi (1952) interpolation formula (with an additional factor of 2, see also Shima et al. 1985; Livio 1986). We can, therefore, write the accretion rate as $(M_{CO} -$ the mass of the compact object)

$$\dot{M}_{acc} = \alpha(\gamma) \frac{M^3}{(1+M^2)^{3/2}}$$

with $\alpha(\gamma)$ an almost linearly decreasing function of $\gamma$, the values of which are approximately given by $\dot{M}/\dot{M}_{HL}$ in Table 1 (at least for $\gamma \leq 4/3$), $M_{HL}$ being the Hoyle–Lyttleton value. In the results of Bondi (1952) also a close to linear relation appears.

(v) The rate of accretion of angular momentum (see Table 1) is in all cases significantly less than the rate at which angular momentum is deposited into the symmetrical Bondi–Hoyle accretion cylinder (which has been assumed to be the rate of accretion of angular momentum by Illarionov & Sunyaev 1975; Shapiro & Lightman 1976; Wang 1981). The rate of accretion of angular momentum is smaller ($L/L_{BH} \approx 0.1$) in the isothermal and hypersonic cases than in $\gamma \neq 1$ cases (when $L/L_{BH} \approx 0.2$), here

$$L_{BH} = \frac{\dot{M}_{HL}}{H} \frac{(GM_{CO})^2}{V_0^3}.$$  

Furthermore, of the accreted angular momentum a significant part comes from upstream (denoted by $L_+/L$ in Table 1), from matter that hits the (relatively large) accreting body directly without passing through the interaction region downstream.

The fact that the rate of accretion of angular momentum is much lower than that expected naively, by calculating the rate at which angular momentum enters the symmetrical Bondi–Hoyle cylinder, is in fact consistent with the Bondi–Hoyle picture, in which the matter that is actually accreted cannot have high specific angular momentum. This has been confirmed by following the mass and angular momentum of an accreted ring of mass (see SLKS). Our calculation thus supports the suspicion, first raised by Davies & Pringle (1980), that relatively very little angular momentum can be accreted from an inhomogeneous medium.

Test runs performed with other grid sizes and average numbers of particles per cell have
shown that: (a) The results do not change when an average number of 6.37 particles per cell is used (instead of 4); however, the calculation tends to become unstable when the average number is smaller than 3.2. (b) When different grid sizes were used (e.g. $32 \times 24 \times 12$) differences of at most 8 per cent in the accretion rate (but smaller in the angular momentum accretion rate) were found. These could usually be attributed to either a reduction in the average number of particles per cell in the downstream side, or the shock slightly 'escaping' through the sides of the grid. Nevertheless, possible errors in the quoted values of up to a few per cent have probably to be realistically assumed, due to the relatively coarse grid.

In Section 4 we shall discuss some of the possible implications that our results may have for compact objects accreting from a stellar wind.

4 Discussion

Accretion by a compact object, from an inhomogeneous medium, occurs in the case of a neutron star accreting from the stellar wind of an early-type companion and in the case of a white dwarf (or a main-sequence star) accreting from the wind of a cool giant. We shall discuss each of these classes separately in the context of the results of the present work (see also the discussions by White 1985; Henrichs 1983; Livio & Warner 1984; Livio 1986).

In Table 2 we present the parameters for some of the better studied X-ray binaries (taken from Wang 1981; White 1985; Eisner et al. 1985, and references therein). We would like to discuss the implications of our results for three properties of these binaries: (i) the X-ray luminosity, (ii) the spin-up (or spin-down) rate, and (iii) the possibility of forming an accretion disc.

(i) The luminosity. The accretion rate can be expressed as

$$\dot{M}_{\text{acc}} = \frac{\delta^4 \pi G^2 M^2 \rho}{V_{\text{rel}}^3},$$

where we have neglected the speed of sound compared to the relative velocity (between the neutron star and the wind) and $\delta = \dot{M}_{\text{acc}}/\dot{M}_{\text{HL}}$ represents the deviation from the Hoyle–Lyttleton (1939) value (as found in Table 1). We shall now assume a spherically symmetrical wind from the giant, with $V_w = V_{\text{rel}}$ (actually a questionable assumption, as will be discussed later). We adopt an average value of $\delta = 0.8$ (see Table 1 and Shima et al. 1985) and for the neutron star we take $M_x = 1 M_\odot$, $R_x = 10^6$ cm. Equation (15) can then be expressed as

$$V_{\text{rel}} = 4.1 \times 10^7 \left(\frac{\delta}{0.8}\right)^{1/4} \left(\frac{M_x}{M_\odot}\right)^{3/4} \left(\frac{R_x}{10^6 \text{ cm}}\right)^{-1/4} \left(\frac{\dot{M}_w}{10^{-6} M_\odot \text{ yr}^{-1}}\right)^{1/4}$$

$$\times \left(\frac{L_x}{10^{37} \text{ erg s}^{-1}}\right)^{-1/4} \left(\frac{a}{30 R_\odot}\right)^{-3/2} \text{ cm s}^{-1},$$

where $L_x$ is the X-ray luminosity, $\dot{M}_w$ is the rate of mass loss from the giant and $a$ is the separation. The resulting relative velocities are listed in Table 2 under the column labelled $V_{\text{rel}}$ (luminosity). We shall discuss the values that have been obtained after studying the implications of the spin-up and the possible existence of a disc ($\dot{M}_w$ has been taken from White 1985, and references therein).

(ii) Spin-up (or spin-down) rates. We shall ignore for the moment the question of whether spin-up (which occurs most of the time) or spin-down is actually observed, and treat average values of $P/P_* (P_*$, the spin period) observed over relatively long time-scales (short time-scale variations will be mentioned later). The observed $P/P_*$ can be directly related to an implied rate
Table 2. Parameters of X-ray binaries and required relative velocities (see text).

<table>
<thead>
<tr>
<th>Object</th>
<th>$P_{\text{orb}}$ (d)</th>
<th>$P_a$ (sec)</th>
<th>$L_x (10^{37} \text{ erg s}^{-1})$</th>
<th>$\dot{P}/P_a$ (sec$^{-1}$)</th>
<th>$V_{\text{rel}}$ (luminosity) cm s$^{-1}$</th>
<th>$V_{\text{rel}}$ (spin-up) cm s$^{-1}$</th>
<th>$V_{\text{rel}}$ (disk) cm s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cen X-3</td>
<td>2.09</td>
<td>4.84</td>
<td>5</td>
<td>$9.3 \times 10^{-12}$</td>
<td>$3.4 \times 10^7$</td>
<td>$5.3 \times 10^7$</td>
<td>$3.5 \times 10^7$</td>
</tr>
<tr>
<td>Vela X-1</td>
<td>8.96</td>
<td>283</td>
<td>0.14</td>
<td>$3.0 \times 10^{-12}$</td>
<td>$5.9 \times 10^7$</td>
<td>$5.5 \times 10^7$</td>
<td>$2.2 \times 10^7$</td>
</tr>
<tr>
<td>1538-52</td>
<td>3.73</td>
<td>529</td>
<td>0.4</td>
<td>$&lt; 3.2 \times 10^{-11}$</td>
<td>$4.7 \times 10^7$</td>
<td>$&gt; 5.8 \times 10^7$</td>
<td>$4.7 \times 10^7$</td>
</tr>
<tr>
<td>GX 301-2</td>
<td>41.5</td>
<td>699</td>
<td>0.3</td>
<td>$10^{-9}$</td>
<td>$3.3 \times 10^7$</td>
<td>$1.3 \times 10^7$</td>
<td>$1.5 \times 10^7$</td>
</tr>
<tr>
<td>SMC X-1</td>
<td>3.89</td>
<td>0.714</td>
<td>50</td>
<td>$2.0 \times 10^{-11}$</td>
<td>$1.4 \times 10^7$</td>
<td>$4.1 \times 10^7$</td>
<td>$3.3 \times 10^7$</td>
</tr>
<tr>
<td>0352+31</td>
<td>581?</td>
<td>835</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$5.3 \times 10^{-12}$</td>
<td>$3.8 \times 10^7$</td>
<td>$0.7 \times 10^7$</td>
<td>$0.6 \times 10^7$</td>
</tr>
</tbody>
</table>
| 0535+26  | $> 20$d              | 104         | 2                                | $3.2 \times 10^{-10}$    | $5.1 \times 10^7$  
(for $a=30 R_0$) | $2.1 \times 10^7$  
(for $P_{\text{orb}}=20$d) | $1.9 \times 10^7$                  |
| 1700-33  | 3.4                  | ---         | 0.04                             | ---                      | $1.6 \times 10^8$                  | ---                               | $2.6 \times 10^7$                  |
| Cyg X-1  | 5.6                  | ---         | 0.6                              | ---                      | $2.1 \times 10^8$                  | ---                               | $2.6 \times 10^8$                  |
| GX 1+4   | ---                  | 122         | 4                                | $7.0 \times 10^{-10}$    | ---                               | $4.8 \times 10^7$  
($P_{\text{orb}}^{-1/4}$) | $4.2 \times 10^7$  
($P_{\text{orb}}^{-1/4}$) |

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of accretion of angular momentum by

\[ L_{\text{obs}} = \left| \frac{\dot{P}}{P_{\text{obs}}} \right| \left( \frac{2\pi}{P_\ast} \right) I_x \]

\[ = 6.28 \times 10^{32} \left( \frac{\dot{P}/P_\ast}{10^{-11} \text{s}^{-1}} \right) \left( \frac{P_\ast}{100 \text{s}} \right)^{-1} \left( \frac{I_x}{10^{45} \text{g cm}^2} \right) \text{dyne cm,} \]

(17)

where \( I_x \) is the moment of inertia of the neutron star (e.g. Lamb, Pethick & Pines 1973). The predicted rate of accretion of angular momentum based on the present study is

\[ L_{\text{pred}} = 3.89 \times 10^{32} \left( \frac{\xi}{0.2} \right) \left( \frac{L_x}{10^{37} \text{erg s}^{-1}} \right) \left( \frac{M_x}{M_\odot} \right) \left( \frac{P_{\text{orb}}}{\text{1 day}} \right)^{-1} \]

\[ \times \left( \frac{R_x}{10^6 \text{cm}} \right) \left( \frac{V_{\text{rel}}}{10^8 \text{cm s}^{-1}} \right)^{-4} \text{dyne cm,} \]

(18)

where \( \xi \equiv l/l_{BH} \) is the ratio of the accreted specific angular momentum (according to our calculations, Table 1) to the specific angular momentum of matter that is deposited into the Bondi–Hoyle, symmetrical, accretion cylinder. Equating the rates in equations (17) and (18), gives us for the relative velocity the values listed in Table 2 under \( V_{\text{rel}} \) (spin-up). We have adopted \( M_x = M_\odot, R_x = 10^6 \text{cm}, \xi = 0.2, \) and \( I_x = 10^{45} \text{g cm}^2 \).

(iii) The possibility of forming a disc from wind accretion. An important question in the case of a compact object accreting from a stellar wind is whether an accretion disc can be formed. The radius at which a disc can start forming can be obtained by equating the specific angular momentum of the accreted matter to that in a Keplerian disc \( l = (GM_x/r_D)^{1/2} \). In the case in which the compact object is a magnetized neutron star, it is necessary, for a disc to form, that the resultant \( r_D \) will be larger than the magnetospheric radius \( R_M \). This imposes the following condition on the relative velocity (Shapiro & Lightman 1976; Wang 1981)

\[ V_{\text{rel}} \leq 4.0 \times 10^7 \left( \frac{\xi}{0.2} \right)^{1/4} \mu_{30}^{-1/14} \left( \frac{M_x}{M_\odot} \right)^{5/14} \left( \frac{P_{\text{orb}}}{\text{1 day}} \right)^{-1/4} \left( \frac{R_x}{10^6 \text{cm}} \right)^{1/28} \]

\[ \times \left( \frac{L_x}{10^{37} \text{erg s}^{-1}} \right)^{1/28} \text{cm s}^{-1} \]

(19)

where \( \mu_{30} \) is the neutron star's magnetic moment (in units of \( 10^{30} \text{erg g}^{-1} \)). The upper limits on the relative velocity for disc formation are listed in Table 2 under \( V_{\text{rel}} \) (disc). In the case of Cyg X-1, the upper limit is derived by requiring the disc radius to be larger than the innermost stable orbit around the (possible) black hole (we have adopted \( M_x = 10M_\odot \)).

Let us now discuss the implications of the relative velocities obtained in Table 2. The most striking general property, that is revealed by examining Table 2, is the fact that almost all the required relative velocities are much smaller than those that could be expected for radiatively driven winds, typically of order \( V_w \sim 1000–2000 \text{km s}^{-1} \). In fact, column density estimates, obtained from the attenuation of the X-ray spectrum by photoelectric absorption, also seem to indicate low velocities (e.g. White 1985). Even from this result alone, we can therefore immediately conclude that the simple picture of a smooth, spherically symmetrical, radiatively driven wind, is in general not applicable. The two major factors that can both change the wind-flow picture and produce significantly smaller wind velocities (Roche lobe overflow will be discussed separately) are: (i) ionization by the X-ray source, which can decrease the radiatively
driven, UV line accelerations (e.g. Hatchett & McCray 1977; MacGregor & Vitello 1982; Dupree et al. 1980) and (ii) a wind flow concentrated towards the compact object (and in fact resembling Roche lobe overflow), caused by the primary being close to filling its lobe (Friend & Castor 1982).

We shall now look at some of the individual systems and examine what can be learned about each of them. For Cen X-3, the relative velocity required to explain the luminosity and for disc formation is in fact lower than the orbital velocity (see also Conti 1978; Petterson 1978). The velocity derived from $P$ is only slightly larger than the orbital velocity. Taking into account the facts that the spin-up appears quite smooth (Rappaport & Joss 1983) and that there is additional evidence suggesting the presence of a disc (lack of flaring due possibly to smoothing of fluctuations, evidence for scattering from a possible disc in the spectrum), we have to conclude that Roche lobe overflow must occur in this system, at least occasionally. The situation is almost identical in the case of SMCX-1; we conclude therefore that Roche lobe overflow occurs in that system too. An additional system, on which there is less information available, but for which the smooth spin-up rate (and the short spin-up time-scale, Elsner et al. 1985) would require unreasonably low relative velocities is GX 1 + 4; we therefore predict that Roche lobe overflow occurs in that system. The situation is somewhat less clear regarding 1538−52, where the velocity required to explain the luminosity (and for the possibility of forming a disc) appears to be smaller than the orbital velocity but there are no good spin-up data to support this conclusion. More observations related to spin-up or to the possible existence of a disc are required to establish whether Roche lobe overflow is expected in this case too.

It should be pointed out that if an accretion disc is formed, even temporarily, it can then spread by viscous angular momentum transport (see Livio 1986, for a discussion).

The velocities required to explain the luminosity and average spin-up (and spin-down) rates of Vela X-1 are consistent with a stellar wind. However, the wind velocity should be significantly reduced with respect to the unperturbed, radiatively driven wind. In addition, a considerable amount of inhomogeneities in the wind, on several scales, is required to explain $P/P$ as high as $1.8 \times 10^{-10}$ s$^{-1}$ (Boynton et al. 1984). Under such circumstances, the spin-up and spin-down behaviour is consistent with a random noise process (Boynton et al. 1984). Less information is available on 1700−33, but accretion from a stellar wind appears consistent with the observations existing so far on this object. In the case of Cyg X-1, it appears possible in fact for an accretion disc to form from wind accretion.

The situation is quite complicated concerning GX 301-2. The wind there is clearly variable due to the fact that the orbit is elliptical $e=0.47$. The velocities (Table 2) that are necessary in order to explain the luminosity and the spin-up are extremely low. The absence of a smooth spin-up behaviour, together with the extremely low velocity required to form a disc, argue against the existence of a disc in this system. This is consistent with the fact that WRA 977 is not close to filling its Roche lobe and with the absence of any lag between the 41.5 day period outbursts and the times of periastron passage (White & Swank 1984).

The second class of objects for which accretion from an inhomogeneous medium is applicable involves white dwarfs (or main-sequence stars) accreting from the winds of cool giants. This class includes such objects as (Livio & Warner 1984; Livio 1986): Mira AB, SY For, $\xi$ Cap, $\xi$ Cyg, S6 Peg, $\xi$ Aur, 32 Cyg, 31 Cyg, 22 Vul, and possible $\nu$ Her, HR 3080, and HR 8157. The condition on the relative (and wind) velocity in this case can be written as

$$V_{\text{rel}} = \left( \frac{V_w}{V_{\text{rel}}} \right)^{1/4} = 1.3 \times 10^6 \left( \frac{\delta}{0.8} \right)^{1/4} \left( \frac{M_{\text{wd}}}{0.6 M_\odot} \right)^{3/4} \left( \frac{\dot{M}_w}{10^{-7} M_\odot \text{yr}^{-1}} \right)^{1/4} \left( \frac{R_{\text{wd}}}{9.5 \times 10^8 \text{cm}} \right)^{-1/4} \times \left( \frac{a}{10^{15} \text{cm}} \right)^{-1/2} \left( \frac{L_{\text{acc}}}{10^{33} \text{erg s}^{-1}} \right)^{-1/4} \text{cm s}^{-1},$$

(20)
where we have used the average mass of single white dwarfs (Koester et al. 1979) due to the large separation (and the appropriate white dwarf radius). For an accretion disc to form, the radius of the disc must be larger than the radius of the white dwarf (for non-magnetic white dwarfs); this implies the condition

$$V_{\text{rel}} \leq 3.7 \times 10^6 \left( \frac{\xi}{0.2} \right)^{1/4} \left( \frac{M_{\text{WD}}}{0.6 M_\odot} \right)^{3/8} \left( \frac{P_{\text{orb}}}{10 \text{ yr}} \right)^{-1/4} \left( \frac{R_{\text{WD}}}{9.5 \times 10^8 \text{ cm}} \right)^{-1/8} \text{ cm s}^{-1}. \quad (21)$$

Recent IUE observations of Mira B have claimed the existence of an accretion disc around the white dwarf (Reimers & Cassatella 1985; Cassatella et al. 1985, and see also the description of the optical spectrum of Yamashita & Maehara 1977). In trying to establish the possibility of forming a disc from wind accretion in this system, we face the unpleasant situation of no known orbital period. Fernie & Brooker (1961) found the possible solutions of 59, 169 and 261 yr of which the last one was considered the most plausible. Hopmann (1964) found 139 and 842 yr. Baize (1980) found 400 yr (quoted as private communication from P. Couteau by Reimers & Cassatella 1985). Walker (1985) found all existing orbits to be bad. Since the period is likely to be larger than 100 yr (van Biesbroeck 1959) and the average value of all the estimated periods above 100 yr is 362 yr, we shall adopt 400 yr as the period; it should be remembered, however, that this should not be regarded as an accurate determination. Using $M_w=10^{-7} M_\odot \text{ yr}^{-1}$ (Reimers & Cassatella 1985), $a=9.8 \times 10^{14} \text{ cm}$ (Jenkins 1952) and $J_{\text{acc}}=10^{33} \text{ erg s}^{-1}$ (a lower limit of $3.3 \times 10^{32} \text{ erg s}^{-1}$ is indicated, Reimers & Cassatella 1985) we find $V_{\text{rel}}(V_w/V_{\text{rel}})^{1/4}=1.31 \times 10^6 \text{ cm s}^{-1}$. Now $V_w=5.6 \times 10^6 \text{ cm s}^{-1}$ (Wannier et al. 1980) giving $V_{\text{rel}}=1.7 \times 10^6 \text{ cm s}^{-1}$ which is in reasonable agreement with the assumed orbit (giving $V_{\text{rel}} \approx 1.1 \times 10^6 \text{ cm s}^{-1}$). Now the condition for disc formation (equation 20) reads $V_{\text{rel}} \leq 1.5 \times 10^6 \text{ cm s}^{-1}$ which indicates, considering the uncertainties, that disc formation is indeed possible in this system. The initial disc radius that is obtained if we take $V_{\text{rel}}=1.1 \times 10^6 \text{ cm s}^{-1}$ is $R_d=10^{10} \text{ cm}$ and thus much smaller than the one obtained by Reimers & Cassatella who used $\xi=1$. However, once a disc forms, it spreads due to viscous transport of angular momentum and thus the observational determination of $R_d=10^{11} \text{ cm}$ by Reimers & Cassatella may be correct.

A different system which quite clearly contains an accretion disc around the mass-gainer star is RZ Oph (Olson & Hickey 1983; Baldwin 1978). While an inclination of $i=76^\circ$ which would have enabled the K5 mass-losing star to fill its Roche lobe has been suggested by Smak (1981), it has been argued by Olson & Hickey (1983) that $80^\circ \leq i \leq 88^\circ$.

If we adopt the parameters of Olson & Hickey (1983) for the mass-gainer F star, $M=3 M_\odot$, $R=3.8 R_\odot$, we find that for a disc to form ($P=262$ day) we must have $V_{\text{rel}} \leq 6.8 \times 10^6 \text{ cm s}^{-1}$. This would require a wind velocity $V_w \leq 3.8 \times 10^6 \text{ cm s}^{-1}$. We cannot entirely exclude, therefore, the possibility that an accretion disc does form from wind accretion. However, the large dimensions of the disc would suggest to us that Roche lobe overflow (or at least a wind concentrated towards the accreting star, Friend & Castor 1982) does occur in this system.

A different group of objects for which the presence of a disc generated by wind accretion has been suggested (at least for \zeta Aur and \delta Sge) are the \zeta Aur binaries (Che, Hempke & Reimers 1983; Che-Bohnenstengel & Reimers 1985). Using the same parameters as Che-Bohnenstengel & Reimers (1985), but introducing $\xi=0.2$ (equation 20) in the rate of accretion of angular momentum, as indicated by our results, makes disc formation from wind impossible in the case of \zeta Aur and only marginally possible for \delta Sge. Indeed the extremely high temperatures ($\approx 70000 \text{ K}$) quoted for the disc in \zeta Aur cannot occur in a steady disc model around a star with a radius of $R=3.6 \times 10^{11} \text{ cm}$ (which would rather give temperatures of order $\sim 400 \text{ K}$). More observations of these systems and a possible re-interpretation of the observations (very probably in terms of a shocked region) are thus strongly recommended.
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A re-examination of the systems 56 Peg and ζ Cap (discussed by Livio & Warner 1984) reveals that the formation of an accretion disc from wind accretion becomes only marginally possible in the case of 56 Peg ($R_D \sim 8 \times 10^8 \text{cm}$) while it becomes impossible for the assumed parameters of ζ Cap. This may explain the appearance of only very narrow (FWHM $\sim 114 \text{km s}^{-1}$) UV emission lines in 56 Peg (Schindler et al. 1982). More observations of these systems are encouraged.

To conclude, we have established the dependence of the accretion rate on the specific heat ratio. The rate of accretion of angular momentum from an inhomogeneous medium is significantly lower than has been previously assumed. The results on the rate of accretion of angular momentum of the present study can be used to place severe constraints on models for systems involving a compact object accreting from the stellar wind of its companion. More observations of such systems, in particular in the case of white dwarfs and main-sequence stars accreting from the winds of cool giants, are extremely important for a better understanding of the accretion process.

References

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