Models of interacting binary stars

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CHAPTER V

NOTES ON THE THEORY OF COMMON ENVELOPE EVOLUTION
V.1 INTRODUCTION

The evolution of binary stars through a common envelope phase, in which the binary is entirely surrounded by one gaseous envelope, is one of the least understood stages of binary evolution. This is an unsatisfactory situation since the formation of such a common envelope is believed to be a crucial stage in the evolution of all short-period binaries that contain at least one compact star. The best known examples of these are the short-period low-mass X-ray binaries and binary radio-pulsars, in which the compact star is a neutron star, and the cataclysmic variables in which the compact star is a white dwarf. The conclusion that a common envelope (CE) phase has occurred in the history of these binaries is unavoidable, since the progenitor of the compact star must have been a giant, with dimensions far greater than the present binary separation. During this phase the compact star or its progenitor (the core of the giant) and the companion must have reduced their separation, while the envelope of the giant was ejected from the system (Paczynski 1976, Webbink 1979). Direct evidence for the actual occurrence of this type of evolution in nature can be found in the short-period double cores of planetary nebulae (Bond, 1985).

To gain a better understanding of the evolutionary history of presently observed binary systems of the types mentioned above, we would like to be able to predict the final outcome of the CE phase from the parameters of the binary system just before the formation of the CE. In particular we want to know whether the giant core and the companion will eventually coalesce, or, if this is not the case, what the orbital period of the remaining binary will be. One simple approach to this question, that has been used widely in the literature (see eg. Chapter III.2), is to assume that all gravitational energy that is gained by the reduction of the orbital separation between companion and giant core is used to eject the envelope. Since the separation can not be reduced beyond the point at which the companion fills its Roche-lobe if the system is to survive as a binary, there is a maximum to the amount of energy to be gained in this way. By comparing this to the amount of energy necessary to eject the envelope, one can decide from the details of the initial configuration whether it is possible to eject the envelope before companion and core coalesce. If this is the case, the
method also yields an upper limit on the final separation. One of the purposes of a more detailed study of the CE-phase is to investigate whether the assumptions entering this estimate are justified.

In the literature there have been two ways of approaching the problem of CE evolution, which are probably applicable for distinct initial conditions. The first model (Meyer and Meyer-Hofmeister, 1978, hereafter referred to as MMH) applies when the giant star is corotates with the orbital motion (or at least very near) at the time it starts to transfer mass to its companion. In this case the Roche-geometry can be used to describe the mass transfer, which greatly simplifies the problem. A CE can form around the binary in not too violent a way, without significant mass loss from the system. The second approach (Taam, Bodenheimer and Ostriker, 1978; Livio and Soker 1984; Bodenheimer and Taam, 1984), which describes the start of the CE phase as a plunge of the companion star into the more or less stationary giant envelope, is appropriate when the giant is far from corotation. In this review I shall first discuss what determines the rotation rate of the giant at the time mass transfer commences, and then give a more detailed description of the two approaches mentioned above.

V.2 THE FORMATION OF A COMMON ENVELOPE

When a single star evolves off the main sequence towards the giant stage its rotation rate is expected to decrease significantly as the moment of inertia increases very rapidly, while the total amount of angular momentum available remains constant. If the star is in a binary tidal interaction will transfer orbital angular momentum into the spin angular momentum of the giant. When the stellar radius becomes comparable to the binary separation (or equivalently, the giant almost fills its Roche-lobe) this tidal interaction becomes very effective (see e.g. Zahn, 1978), and because of the high turbulent viscosity in the convective giant envelope the tidal spin-up timescale will become shorter than the evolutionary expansion timescale, on which the stellar moment of inertia is changing (Alexander 1973, Zahn 1978). This means that the giant will be nearly corotating with the orbital motion when it starts to transfer mass to its companion.

A necessary condition for this scenario is however that the binary is tidally stable (Counselman 1973), i.e. that the increase in orbital angular velocity ($\omega_{\text{orb}}$) due to orbital angular momentum lost to the
giant is not greater than the increase in rotational angular velocity of
the giant ($\omega_{\text{rot}}$) due to angular momentum gained from the orbit, i.e.

$$\left| \frac{\partial \omega_{\text{orb}}}{\partial J} \right| < \left| \frac{\partial \omega_{\text{rot}}}{\partial J} \right|$$

(1)

Using Kepler's laws it is easily shown that this is equivalent to the
condition that the moment of inertia of the giant has to be less than
1/3 of the orbital moment of inertia. If we take typical values for the
moment of inertia of a giant as found from evolutionary calculations,
and assume that the giant nearly fills its Roche lobe, the above
condition for stability translates to the more practical condition that
$q$, the ratio of the mass of the companion to that of the giant, must be
greater than 1/6 (Sparks and Stecher, 1974; MMH).

Hence, if $q > 1/6$ the giant is expected to be rotating almost
synchronously with the orbit at the onset of mass transfer. Following
the Roche-geometry, matter will flow through the L1 point into an
accretion disk around the companion. What happens then is again
determined by the value of $q$. If $q > q_{\text{crit}}$ (where $q_{\text{crit}}$ varies between
0.836 and 1.2, see below) the response of the binary to mass transfer is
an increase in orbital separation, and hence also in the size of the
Roche lobe of the giant (see eg. Webbink, Rappaport and Savonije, 1983).
In this case the mass transfer will remain stable, since an increase in
mass transfer will cause an extra increase in the size of the Roche-lobalobe, which in turn reduces the mass transfer rate. If however $q < q_{\text{crit}}$,
the response of the Roche-lobe of the giant is to shrink because of the
mass transfer, which causes the mass transfer rate to become even
greater, which is an obviously unstable situation. This instability is
aggravated when the mass transfer rate has become so high that the giant
can no longer maintain thermal equilibrium. The extended convective
envelope of the giant is to first approximation isentropic, and will
behave like an $n=1.5$ polytrope, i.e. it will expand when its mass is
reduced. If the mass loss is sufficiently slow, this behaviour is
suppressed because the entropy of the envelope will adjust itself (on a
thermal timescale) in such a way that the radius of the star remains
approximately constant. If the timescale for mass loss becomes shorter
than the thermal timescale the giant will start to expand adiabatically,
increasing the transfer rate even more. A star which becomes subject to
this instability will transfer a significant fraction of its envelope on
a dynamical timescale, which is of the same order as the orbital period.
(In fact, a giant with $q > q_{\text{crit}}$ is also potentially unstable to this last instability, but the stable transfer rates never become high enough to disturb the thermal equilibrium). The exact value of $q_{\text{crit}}$ depends on the details of the mass transfer process. If all mass transferred is accreted by the companion $q_{\text{crit}} = 1.2$, whereas if all mass transferred is lost from the system carrying the specific orbital angular momentum of the companion (as is probably the case in super-critical disk accretion with matter being carried away in jets) $q_{\text{crit}} = 0.836$ (de Kool, Rappaport and van den Heuvel, 1986). Summarizing, we have the following situation. In the case $q < 1/6$, i.e. a giant which is much more massive than its companion, the binary is tidally unstable and will not be corotating with the orbit when (unstable) mass transfer starts. If $1/6 < q < q_{\text{crit}}$ the mass transfer is still unstable, but can at least initially be described as Roche-lobe overflow. If $q > q_{\text{crit}}$ the giant will be corotating, and the mass transfer is stable. Typical mass transfer rates in this case are $10^{-6}$ to $10^{-9} \, M_\odot/\text{yr}$ (Webbink, Rappaport and Savonije 1983), far less than the rates needed to form a common envelope.

The actual formation of the common envelope is a 3-dimensional, time dependent hydrodynamical problem involving very different length and time scales and hence remains rather obscure, although there are a few model calculations available in the literature that relate to the problem. The models which are most applicable to the corotating case are probably those of Sawada et al. (1984). These are 2-dimensional hydrodynamic calculations using cylindrical coordinates $(r, \phi, z)$ in which the $z$-dependence ($\vec{e}_z$ parallel to the rotation axis) of the flow is neglected. This might yield an impression of the real flow pattern in the equatorial plane. The authors model a binary in which both stars are assumed to be corotating and exactly filling their Roche-lobe, and in which one star is losing mass over its entire surface at a constant rate and velocity. An example of the resulting flow is shown in fig. 1, where density contours and the velocity in the corotating frame are plotted for a binary with $q = 0.5$. Although the simplifications used in these calculations seem rather drastic, it is nevertheless interesting to see that a number of features that have been predicted on the basis of physical intuition (MMH) are found back in these results. The matter that leaves the primary slowly builds up a common envelope which seems to consist of an inner part which is almost corotating with the binary (low velocity in the corotating frame), and a differentially rotating outer envelope. As we shall see below, this is very similar to the
Fig. 1. Velocity distribution and density contours in an envelope around a binary with $q=0.5$, in which both stars are filling their Roche-lobe. The most massive star is losing mass over its entire surface with constant density and velocity. Velocities are represented in the frame corotating with the binary (from Sawada et al., 1984).

predictions of MMH.

When the giant is far from corotation the Roche-geometry can not be employed, and as far as I am aware there are no numerical hydrodynamical calculations of this problem. The first to consider this case were Sparks and Stecher (1973), who wanted to explain some supernovae as a result of the spiral-in of a white dwarf into a giant. They regarded the giant envelope as nearly stationary, the white dwarf moving over the surface with its orbital velocity. The acceleration and resulting velocity of a matter element in the giant envelope due to the gravity of the white dwarf is then calculated using an impulse approximation, i.e. the displacement of the element while the white dwarf passes is neglected. Depending on the distance to the white dwarf some matter will attain escape speed from the red giant, another part of the envelope will just be lifted and fall back, radiating away its excess energy. Under the assumption that the kinetic energy put into the envelope matter derives from the orbital energy (an 'error of a factor of 2 is made here) it is then possible to follow the decay of the orbit. The radius of the giant is simply assumed to decrease proportional to the
orbital separation. However, the assumptions and simplifications in this model are not really acceptable. The neglect of all hydrodynamical effects is at least questionable, and use of the impulse approximation is not justified since the duration of the passage of the white dwarf is not short compared to the dynamical time of the envelope. Also the assumption that the giant is simply "peeled", i.e. that its radius decreases when mass is ejected from the envelope, is contrary to what we might expect in the light of the discussion above on the reaction of the radius of a giant to mass loss.

Other relevant calculations are those of Morris (1981), who attempted to model mass transfer in a non-corotating binary using particle trajectories and a crude form of hydrodynamical interaction when these particles collide. The giant is simulated by a sphere covered with particles, that are attracted to the giant by its gravity, and repelled by some artificial force which prevents them from entering the surface. They can however be pulled off by the gravity of a companion. This giant is then placed in a binary, and the trajectories of the particles are followed to see how much mass escapes, how much is accreted by the companion, and how much falls back to the giant after being pulled off. A number of models with different orbital separations, mass ratios and rotation rates of the giant are constructed in this way. It is found that the resulting flow is indeed strongly dependent on the rotation rate of the giant, in the sense that the more the giant deviates from corotation, the more violent the interaction. As a result of this a large fraction of the particles is ejected or forms an extended cloud about the binary. In the case that the giant is nearly corotating most particles are simply accreted by the companion. This strong dependence confirms that common envelope evolution proceeds differently in the corotating and non-corotating case.

All these calculations are however unsatisfactory because they do not include the response of the giant to the mass loss. Sparks and Stecher (1974) introduced an (unrealistic) ad hoc assumption to describe this response, and Sawada et al. (1984) and Morris (1981) only considered the case where the mass transfer rates are small, i.e. where the mass of the stars does not change appreciably during the computed time interval of several orbital periods. A proper description of the hydrodynamical reaction of the giant to mass loss would seem to be an essential ingredient to understanding the formation of a common envelope.
To investigate what kind of dynamical effects can be expected during the formation of a common envelope when the response of the giant is in some way taken into account, and if no a priori assumptions about the symmetry are made (except the symmetry with respect to the orbital plane), we have performed a grossly simplified simulation of this process using a method called Smooth Particle Hydrodynamics (SPH). This method, first used by Lucy (1978) and further developed by Gingold and Monaghan (1980, and references therein), represents matter by a number of particles, which move according to an equation of motion in which the acceleration due to gravitation and pressure gradients is calculated using particle-particle interactions. For details of the method we refer to the papers mentioned above, and we only remark here that the method can be reasonably well tested by modelling radial oscillations of polytropes.

We started our calculations by constructing a simplified giant model, taking one very massive particle (3 $M_\odot$) as the giant core, and adding a large number (600) particles to represent the envelope ($M_{\text{env}} = 8 M_\odot$). The gas of the envelope was taken to obey a polytropic equation of state with index $n=1.5$. We relaxed this model until a hydrostatic structure was reached, and then transferred it to a rotating coordinate system revolving around the center of gravity of the giant and a companion star, which is also represented as a massive particle (mass 4 $M_\odot$). The dimensions of this binary were chosen in such a way that the giant would not overfill its Roche-lobe. We then relaxed the system again until the giant was hydrostatic in this new configuration. The separation of the binary was then slowly reduced until Roche-lobe overflow started, and from this point on the system was left to evolve hydrodynamically. In figures 2a,b,c we show the situation at the onset of mass transfer. The position of the particles is indicated in the initially corotating coordinate system system $(x,y,z)$. The $z$-direction is perpendicular to the orbital plane, and the $x$-direction points from the giant core to the companion. Figure 2a gives the projections of the particle positions on the orbital $(x,y)$ plane, fig. 2b on the $(x,z)$ plane and fig. 2c on the $(y,z)$ plane. The positions of the core and the companion are indicated by the large solid dots, and the cross represents the center of mass of the entire system. The typical form of the Roche-lobe is easily recognized. In figures 2d,e,f the situation is shown at approximately 15 times the initial orbital period after mass transfer started, and the companion has made slightly more than one
Fig. 2. A simulation of the formation of a common envelope. Figures 2a, b, c represent the situation at the start of unstable mass transfer, figures 2d, e, f after approximately 15 times the initial orbital period. For a full explanation: see text.
revolution about the giant core in the initially corotating frame.

Since the method used is so crude (no energy equation, very simple equation of state, poor description of hydrodynamics in regions of low particle density, lack of spatial resolution) the details of these results should not be taken seriously. They do however illustrate the following points:

1) one has to be very cautious before assuming any symmetry (spherical or cylindrical).

2) one has to take account of the fact that the giant core will start to move through the envelope, which presumably has important consequences for the nuclear burning.

V.3 COMMON ENVELOPE EVOLUTION IN THE CASE OF INITIAL COROTATION:
THE MODEL OF MEYER AND MEYER-HOFMEISTER

If the mass ratio of a binary lies in the range $1/6 < q < q_{\text{crit}}$ the common envelope is expected to form in a relatively quiet way. According to MMH this CE will consist of two distinct parts: i) an inner region corotating with the "internal" binary formed by the star spiraling in and the dense core of the giant, and ii) an outer region which can rotate differentially. The situation is schematically shown in Fig. 3. The inner region is forced to corotate by viscosity caused by tidally induced turbulence, which resists differential rotation. The strength of this tidally induced turbulence determines the size of the corotating region. This strength is estimated by calculating the acceleration of a non-corotating mass element due to the part of the gravitational potential that varies during the orbital period, neglecting the displacement during the acceleration. From this periodic acceleration a typical turbulent velocity is deduced. Estimating the size of the largest turbulent eddies ($l_t$) to be the distance $r$ of the matter element to the center of gravity then yields an effective viscosity coefficient of

$$\eta = 0.5 \rho v l_t \quad (2)$$

where $\rho$ is the density and $v$ the typical turbulent velocity. By using a multipole expansion of the time-varying potential MMH show that the induced turbulent velocity scales with $(a/r)^4$, which makes $\eta$ strongly dependent on $r$. (In fact the typical size of the turbulent eddies would
be better estimated by the integral of the turbulent velocity over one half of the orbital period, which yields \( I_C = 0.19 a (a/r)^4 \), causing \( \eta \) to be even more strongly dependent on \( r \). The rapid decrease of \( \eta \) with radius implies that the radius of the corotating region will always be similar to the orbital separation, rather independent of the details of the calculations.

Since the time scale for convective transport in the envelope is shorter than the timescale on which the angular velocity of the corotating core is changing, the envelope is expected to be in a quasi-stationary state, which is described by the equation:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( \eta r^4 \frac{\partial \omega}{\partial r} \right) = 0
\]

Assuming \( \eta \) to be constant this equation has two solutions, one which has \( \omega \) proportional to \( r^{-3} \) and another with \( \omega \)-constant. MMH assume that the second solution applies in the outer part of the envelope, and the first solution describes the angular velocity distribution in a narrow region just outside the corotating inner region, in which the angular velocity changes from that of the core to that of the outer envelope. As pointed out by MMH, the angular velocity distribution derived in this way remains subject to some doubt, since the viscosity in the envelope is the result of turbulence driven by convection, which is non-isotropic, whereas equation 3 is only correct for isotropic viscosity (Biermann, 1951)
The high viscosity in the inner envelope, caused by turbulence driven by convection and by the Rayleigh-unstable angular velocity distribution (specific angular momentum increasing inwards), effectively transports angular momentum from the corotating region to the much more slowly rotating outer envelope. Since this angular momentum derives from the internal binary, the orbit of this binary will shrink at a rate determined by the magnitude of the angular momentum loss, which in turn is determined by the viscosity coefficient in the inner envelope and the surface area of the interface between corotating region and envelope.

Using the model outlined above to describe the hydrodynamical processes MMH subsequently use a normal stellar evolution code to calculate the evolution of such a common envelope formed by a $5 M_\odot$ giant and a $1 M_\odot$ main sequence companion. The central luminosity is given by the luminosity of the giant core (which is determined only by its mass, see MMH) plus an accretion luminosity from the companion star. In addition there are energy source terms that describe the viscous energy dissipation in the envelope. It is found that the frictional luminosity generated in the inner envelope dominates the evolution. The calculations show that this luminosity evolves to a constant value due to a feed-back mechanism in the inner envelope region: as the frictional luminosity increases the radiation pressure in this region also increases, which causes the density to decrease. This density decrease in turn decreases the viscous dissipation (see eq. 2), so that the luminosity decreases again. This mechanism causes the luminosity of the star to remain at a nearly constant value during the entire evolution. Since the frictional energy derives from the binding energy of the internal binary, the evolution of the orbital separation is well approximated by

$$\frac{d}{dt} \left( \frac{1}{a} \right) = \text{constant.} \quad (4)$$

From this expression it can be seen that initially the orbit shrinks very fast, but as it gets smaller the rate of shrinking also decreases. MMH conjecture that the evolution continues in this way until the star which is spiralling in fills its Roche-lobe in the internal binary and suddenly starts to release a large amount of mass in the inner envelope. This increases the coefficient of viscosity in this region and causes a sudden increase in the frictional luminosity that could drive off the entire envelope. It is, however, questionable whether sufficient energy
is available at this point, since most of the gravitational energy gained by the closing in of the giant core and companion has been radiated away, without causing a significant reduction in envelope mass. Another way of removing the envelope may be in the form of a massive stellar wind. The giant has a very high luminosity throughout the evolution ($10^5 L_\odot$), and some observations of very evolved red supergiants (which are thought to be stars that reach a luminosity of similar magnitude by double-shell burning) indicate that they can have very large wind mass loss rates (the so-called super wind, Iben 1981).

Apart from the unsatisfactory description of viscosity and turbulence in the MMH model, the major uncertainty lies in the assumption of spherical symmetry, and the associated neglect of dynamical effects. From the fact that the radius of the corotating region is about equal to the orbital separation it immediately follows that (to first order) the equator of this region is rotating at Keplerian speed, and that the material at this position will be forced to flow outwards by the pressure gradient. This will lead to a circulation pattern in the envelope as sketched in fig. 4, which transports angular momentum far more efficiently even than turbulent viscosity, and will cause a major deviation from the evolution as calculated by MMH.
V.4 COMMON ENVELOPE EVOLUTION WITHOUT INITIAL COROTATION

V.4.1 Spiral-in timescales

If the pre-common envelope binary has a very small mass ratio \( q < 1/6 \), the giant will not be rotating synchronously with the orbit when it starts to transfer mass to its companion. After a short initial period of violent interaction, in which the giant expands due to mass loss and the orbit shrinks due to the frictional interaction between the companion and the transferred mass, the orbit of the companion will lie inside the giant envelope. The star will move through this envelope with a relative velocity equal to the orbital speed minus the local rotational speed of the giant. The accretion flow near the star which is spiralling in has a scale which is generally much smaller than the size of the envelope as a whole (see below), and causes a disturbance in the envelope which destroys any symmetries that could be used to simplify the problem. This makes a simultaneous solution of the response of the giant envelope and the accretion flow very difficult. The calculations that have been done so far have concentrated on the envelope structure, while using the so-called Bondi-Hoyle approximation to describe the interaction between star and envelope. Since the Bondi-Hoyle problem is discussed elaborately elsewhere in this thesis I will only give a summary of the pertinent results here, and discuss their implications in the context of spiral-in evolution.

It is found that when a point-like source of gravitation moves relative to a gaseous medium surrounding it, the typical size of the region affected by the gravitational field is given by the accretion radius \( R_a \):

\[
R_a = \frac{2GM}{v^2 + c^2}
\]

(5)

where \( M \) is the mass of the gravitating object, \( v \) the relative velocity and \( c \) the sound speed in the gas far away from the object. Physically this can be interpreted as the distance to the object at which the gravitational binding energy of a matter element is of the same order as its original total energy at infinity. If the motion is supersonic, the matter which is deflected by gravitation will collide and shock behind the star and its kinetic energy is largely converted to thermal energy. This kinetic energy dissipation rate is typically
where $\rho$ is the density in the gas at infinity. If we consider a star moving through a stationary gas we see that the gas moving through the accretion radius is i) heated in the shock and ii) accelerated to the velocity of the star. Hence the total energy loss from the motion of the star is approximately 2 times $E_{\text{dis}}$. If the motion of the star is subsonic there is much less dissipation of kinetic energy, and the entropy of matter passing close to the compact star which is not accreted is only slightly increased by turbulent dissipation. In this case the drag exerted on the compact star is mainly caused by the gravitational force due to the higher density behind the star, and can become smaller than the drag found from a straight application of eq. (6) (Shima et al, 1985). Assuming that the star moving through the envelope remains in an approximately Keplerian orbit, and that the loss of kinetic energy given by eq. (6) derives from the change in binding energy of the system, one finds that the orbital separation $a$ decays at a rate given by

$$-\frac{GmM(a)}{2a^2} \frac{da}{dt} = 2E_{\text{dis}}$$

where $m$ is the mass of the star that is spiralling in and $M(a)$ the mass of the giant interior to the orbit. These expressions form the basis of all calculations of the evolution of a non-corotating common envelope that have been done so far.

Before describing in more detail the models in the literature which employ the Bondi-Hoyle approximation, we shall investigate what can be inferred from a simple application of the above equations to the spiral-in process. To this end we have taken three giant models (kindly provided by dr. G.J. Savonije) which represent different evolutionary phases:

a) a 1 M$_\odot$ giant with a 0.48 M$_\odot$ He-core, and a radius $R = 209$ R$_\odot$

b) a 5 M$_\odot$ giant with a 0.90 M$_\odot$ CO-core, and a radius $R = 104$ R$_\odot$

c) a 3 M$_\odot$ giant with a 1.39 M$_\odot$ ONeMg-core, and a radius $R = 860$ R$_\odot$

As a first order approximation we calculate the parameters of the accretion flow as a function of the distance to the giant center, assuming that the envelope structure is undisturbed. These parameters are of course also dependent on the mass of the accreting object, and in
our examples we will assume this to be 0.1 times the mass of the giant. In figure 5 the orbital decay time, defined as

\[ \tau_d = -\frac{a}{\frac{da}{dt}} \quad (8) \]

is plotted as a function of radius. The behaviour of model a and b is similar: The decay time is longest in the outer regions of the envelope, has a plateau in the inner part and decreases strongly again as the core is entered. In the much more evolved model c this plateau has developed into a broad maximum in the orbital decay time at a radius between 1 and 10 R_\odot. This maximum is caused by the fact that the envelope of such very evolved giants have a nearly constant density, whereas the orbital velocity of the star spiralling in increases with decreasing radius. From eq. (5) and (6) we see that (if the relative velocity has become subsonic) this implies a reduction in the energy dissipation rate, and hence a lengthening of the orbital decay timescale. It is interesting to note that the maximum occurs at a radius that is comparable to the observed orbital separation in several

![Graph](image)

**Fig. 5.** The orbital decay time scale during spiral-in (\( \tau_d \)), as a function of radius in our three giant models (see text).
types of post common envelope binaries. The presence of the maximum certainly favors the possibility of ejecting the giant envelope before the star collides with the core, but it can not be a necessary condition. This is because it only develops in giants with a sharply defined massive core (near the Chandrasekhar limit), while many cataclysmic variables appear to have white dwarf masses well below 1 M\(_\odot\). There are some indications that white dwarfs in cataclysmic variables are on average more massive than the general population (Warner, 1976) but this is very difficult to establish because of selection effects (Livio and Soker 1984b, Ritter 1986).

The spiral-in times as given in figure 5 can change if the energy dissipation rate is not exactly given by equation 6. Following the results of Shima et al (1985) and de Kool and Savonije (Chapter IV.1) the energy dissipation rate can be substantially larger than the classical estimate for supersonic relative speeds, and smaller for subsonic speeds. To investigate the possible importance of this effect we have plotted in fig. 6 the Mach number of the orbital speed as a function of radius. The behaviour is very similar for the different models: the Mach number is never very high, and varies between 1 and 2 over the largest part of the envelope. Especially in model c the Mach number is very close to one. This implies that if the envelope is rotating by itself, either due to evolution prior to the spiral-in or

Fig. 6. The Mach number of the orbital velocity as a function of radius in our three giant models (see text).
due to angular momentum transfer from orbit to envelope during the spiral-in, the relative velocity between star and envelope could easily become subsonic, with an accompanying reduction in the dissipation rate and increase in orbital decay time.

Since the relative velocity between star and envelope plays an important role in spiral-in evolution, we would like to know whether the angular momentum deposited in the envelope by friction is able to force the surroundings of the star to corotation. We therefore introduce the spin-up timescale $\tau_{sp}$, defined as

$$\tau_{sp} = I_e(a) \omega_{orb} \left( \frac{dJ}{dt} \right)^{-1}$$

in which $\omega_{orb}$ is the orbital angular velocity, $I_e(a)$ the moment of inertia of the part of envelope with radius $a$, and $dJ/dt$ is defined by

$$\frac{dJ}{dt} = 2 \pi R_a^2 \rho v_{orb}^2$$

This very rough estimate may be as good as some more complicated estimates based on tidal interaction between star and envelope, since these depend on a very uncertain viscosity coefficient and employ a description of tidal interaction which is derived for situations in which the orbital separation is larger than the radius of the giant.

In figures 7a,b,c we compare the spin-up timescale with the orbital decay timescale. In model c the star can already spin up the inner part of the envelope to corotation when it has penetrated to a radius of 350 $R_\odot$, and in models a and b these radii are 30 and 10 $R_\odot$ respectively. The fact that $\tau_{sp}$ can become shorter than $\tau_d$ is caused by the presence of the plateau in the $\tau_d$ curves. In figure 7d the two timescales are compared for a 16 $M_\odot$ giant with a radius of 200 $R_\odot$, which has a much much less sharply defined core, and we can see that in this case the spin-up time becomes smaller than the orbital decay time for very small radii (ca. 1 $R_\odot$), and at a radius where the density is about 10$^6$ times greater than in the other models. In this case coalescence will be difficult to avoid. How these results change for different initial mass ratios can be inferred from the fact that $\tau_{sp}$ scales approximately with $\propto m^{-2}$, and $\tau_d$ with $\propto m^{-1}$. From the form of the curves in fig. 6 it can be seen that this implies that for smaller initial mass ratios these timescales become equal at smaller radii.
Fig. 7 A comparison of the orbital decay time scale $\tau_d$ (solid) and the spin-up time scale $\tau_{sp}$ (dashed) of the surroundings of the star that spirals in, for the three models from the text (figures 7a,b,c) and for a less centrally condensed 16 $M_\odot$ giant (figure 7d).
V.4.2 The spiral-in models of Taam, Bodenheimer and Ostriker and Delgado

To follow the evolution of a massive (16 \( M_\odot \)) giant with a 1 \( M_\odot \) neutron star in its envelope Taam, Bodenheimer and Ostriker (1978) (hereafter TBO) used a normal one dimensional stellar evolution code, slightly modified to be able to follow hydrodynamical expansions or contractions. Other dynamical effects were neglected. First they constructed a model of a single 16 \( M_\odot \) giant, and then introduced an extra energy source term given by eq. (6). The extra energy was deposited in a spherical shell of thickness \( R_a \) and radius equal to the orbital separation. A first (and very fundamental) difficulty is immediately encountered: to calculate the velocity of the star relative to the envelope it is necessary to assume an angular velocity distribution in the giant. TBO make the assumption that outside the orbit of the star the angular velocity \( v_e(r) \) varies with \( r^{-2} \), so that the specific angular momentum is constant. This is based on the idea that convection in this region effectively redistributes angular momentum. (Note the difference with the MEE model in which convection is assumed to lead to solid body rotation). Inside the orbit the rotational velocity is assumed to be given by

\[
v_e(r) = v_e(a) \exp\left(-\left(\frac{r-a}{R_a}\right)^2\right)
\]

(11)
to mimick the effects of a diffusion process. In this way the angular momentum of the entire envelope can be derived from \( v_e(a) \), and the value of this quantity is always chosen in such a way that the total angular momentum of star and envelope remains constant. Using eq. (7) the evolution of the orbit can be followed simultaneously with that of the star. (Energy source terms such as accretion onto the neutron star or turbulent viscous dissipation due to tides or differential rotation are also considered by TBO but are generally found to be less important than the frictional luminosity \( E_{\text{dis}} \).) TBO consider two cases: case i), in which the common envelope forms while the 16 \( M_\odot \) star is a yellow giant, at the onset of He-burning. In this evolutionary phase the envelope is radiative, and not very extended. In case ii) the spiral in starts when the massive star has evolved to a red giant with an extended convective envelope (\( R=535 R_\odot \)).

In the first case the neutron star spends most of the time in the outer envelope because the density is very low there, which makes the
frictional energy losses very small. After about $3 \times 10^3$ yr the star starts to enter denser layers, and the orbital decay time is drastically reduced. Because of the very large frictional luminosity ($10^{42}$ ergs/sec) the temperature gradient in the envelope becomes super-adiabatic and convection sets in. It is found that this convection is able to transport all heat deposited in the envelope to the surface in a sufficiently short time, and so avoid a build-up of thermal energy (and pressure) around the position of the neutron star that could drive off the envelope. The spiral-in continues until the neutron star is very close to the He-core of the giant, and the layers around the neutron star have been spun up to the velocity of the neutron star. When the relative velocity disappears the frictional luminosity drops to zero, and further evolution proceeds on the timescale of the tidal and viscous dissipation. In fact the situation becomes very similar to that in the MMH model. This is where TBO stopped the calculations.

In case ii) the neutron star passes through the outer layers of the giant much more quickly because the density is higher than in case i), and the entire spiral in process takes only about 20 yrs. Similarly to case i) the frictional luminosity increases as the star enters denser layers, but now the low densities and temperatures in the extended red giant envelope cause $\tau_c$, the timescale for convective energy transport to be much longer than in case i) ($10^{-1}$ versus $10^{-3}$ yr). When the orbital separation is reduced to only $3.5 \, R_\odot$ the orbital decay time becomes shorter than $\tau_c$. The energy deposited can no longer be transported to the surface quickly enough, and a pressure build-up at the radius of the neutron star then causes the envelope to acquire large outward velocities. When the calculations were stopped the layers just outside the orbit had velocities in excess of the escape velocity.

From these calculations it would appear that a simple recipe exists to predict the outcome of spiral-in evolution, based only on the total available energy and the efficiency of the convective energy transport. We will see below, however, that this result is severely dependent on the assumption of spherical symmetry that TBO had to use.

Delgado (1980) performed calculations very similar to those of TBO, but now considering a binary consisting of a $25 \, M_\odot$ blue supergiant and a $1 \, M_\odot$ neutron star. In this work it is argued that if the total luminosity of the giant exceeds the Eddington luminosity very large wind mass loss rates might occur that modify the further evolution of the common envelope. To test this he calculated the evolution using the
expression for wind mass loss due to Chiosi et al (1978), modified to make the mass loss increase by a factor $10^3$ when the luminosity becomes equal to $L_{\text{edd}}$. Because of numerical difficulties the evolution could only be followed to the time at which the neutron star had crossed 3.5 percent of the mass of the giant, which at that time had expanded to about 12 times its original radius. The frictional luminosity at this point exceeded the luminosity at the surface of the star, which led Delgado to the conclusion that the outer part of the envelope would be ejected, and that further evolution would proceed in similar steps, the neutron star entering the giant by a small mass fraction and subsequently blowing this off. However, this is only conjecture which is not directly supported by the results of the calculations.

V.4.3 The model of Livio and Soker

A different, semi-analytical approach (but also based on the Bondi-Hoyle approximation to describe the interaction between star and envelope) was taken by Livio and Soker (1984a,b). They modelled the common envelope evolution of a binary initially consisting of a very evolved low-mass (0.88 $M\odot$) giant and a companion of planetary mass (0.001-0.025 $M\odot$). An advantage of starting with such a low-mass companion is that the use of the Bondi-Hoyle approximation is very well justified in this case because the accretion radius is much smaller than any of the length scales associated with the giant structure. Neglecting all structural changes in the giant envelope (which again can be justified by the small mass of the companion) they calculate the decay of the orbit of the planet due to friction and tidal dissipation, and at the same time follow the change in mass of the planet due to accretion and thermal evaporation. Cases with and without angular momentum transfer from planet to envelope are considered. It is found that planets below a certain critical mass (the exact value of which is determined by the assumptions used) evaporate completely. Above this critical mass accretion dominates, and the mass of the planet increases while it spirals in. If the planet is sufficiently massive to start with, it is able to accrete the entire envelope (0.16 $M\odot$) before colliding with the core, and a binary consisting of a white dwarf and a main sequence dwarf remains. Such a binary is an excellent candidate for a cataclysmic variable progenitor. The lower mass planets will collide with the giant core, presumably being disrupted in the process. Although
some aspects of this model may be far from reality, such as the assumption that the planet can accrete all matter falling on to it and continues to obey a simple mass radius relation in spite of the high temperature of the accreting gas, this scenario is a very interesting one in view of the probable ubiquity of the progenitor binaries.

Livio and Soker also calculated a few cases in which the planet was not circling inside the giant envelope but in the giant wind, to investigate whether spiral-in evolution can be induced by strong wind mass losses. It was found that the planet always spiralled out, because the widening of the orbit caused by the reduction in mass of the giant was stronger than the decay due to friction. By comparing the timescale for spiral-out (which is equal to the wind mass loss timescale $m_g/\dot{m}_w$, $m_g$ being the mass of the giant and $\dot{m}_w$ the wind mass loss rate) to the orbital decay time scale we find that spiral out results from wind mass loss as long as

$$\frac{\alpha \beta}{\sqrt{2}} \frac{m_g}{m_p} \left( \frac{\alpha}{R} \right)^{1/2} > 1$$

(12)

Here $m_p$ is the mass of the planet, $\alpha$ the ratio of the velocity of the star relative to the wind to the orbital velocity ($\alpha \sim 1$), and $\beta$ the ratio of the wind velocity to the escape velocity at the surface of the giant. We can see that this expression is independent of the actual wind mass loss rate. Since in most cases $m_g$ will be much larger than $m_p$ the condition is generally well satisfied unless $\beta < 1$, i.e. the wind is extremely slow.

V.4.4 The model of Tutukov and Yungelson

For completeness we should also mention the paper by Tutukov and Yungelson (1979), who also attempt a semianalytical approach to the spiral-in problem. Unfortunately, the physical basis used in this work is incorrect, since the authors assume that the frictional force between star and envelope scales with the relative velocity squared, which is correct for a solid non-gravitating body moving through a gas, but not for gravitational accretion flow, in which this force scales with approximately the inverse of the relative velocity squared (see eq. 5 and 6). For this reason we shall not consider this model any further.
V.4.5 A two-dimensional model

The most recent development in the study of common envelope or spiral-in evolution has been the application of two-dimensional numerical hydrodynamics (Bodenheimer and Taam, 1984, hereafter BT). The great difficulty in this type of calculations is the variety of length and time scales that is involved. To describe the structure of the giant properly, very small zones have to be taken near the core of the giant, where density and temperature change on a small length scale. The Courant-Friedrichs condition for stability of any explicit method for the solution of the hydrodynamical equations requires that the time step be smaller than the sound crossing time of a grid zone. Since in the zones closest to the core the temperature (and hence also the sound speed) is largest, the stability of the calculations in these inner regions require such small timesteps that following the entire common envelope phase becomes impossible in terms of calculation time. To investigate the possible effects of dropping the assumption of spherical symmetry anyhow, BT therefore considered the following limited problem. They started with a one-dimensional giant model from the calculations of TBO, taken at a time when the neutron star had already spiralled in very close to the core, and had an orbital period sufficiently short that the computation could cover at least one orbital period. This giant model was then transferred to a two-dimensional grid in the \((r,z)\) coordinates of a cylindrical \((r,\phi,z)\) coordinate system. (In this way rotational symmetry about the orbital angular momentum vector is assumed.) To avoid extremely small timesteps the core of the giant was then replaced by a point mass and a solid inner boundary. Further reduction of the computational effort was reached by not including the entire giant envelope in most calculations, but only the inner part. This does not influence the results, since even with these simplifications the time it took for the induced motions to reach the edge of the grid (1-2 orbital periods) was about all that could be calculated. One model was constructed on a larger grid in which the entire envelope was included, to check if any unexpected events occurred in the time that the motions needed to reach the surface of the star. The energy and angular momentum transferred from orbit to envelope by friction, which are calculated in the same way as by TBO, is distributed over an annular region consisting of the four zones closest to the position of the neutron star. The gravitational potential is calculated by solving the Poisson equation.
simultaneously with the hydrodynamical equations. An example of the results is illustrated in figs 8a,b where the velocity and angular momentum distribution in the red supergiant envelope after slightly more than one orbital period is shown. It is found that an equatorial outflow develops, in which the velocities after some time exceed the local escape velocity. Physically, this result can be understood as follows. Material close to the position of the neutron star first receives a kick in the direction of the orbital motion, which will make it move outwards. The effect of this initial kick soon becomes unimportant relative to the velocity gained by the buoyancy force acting on the material, because it has also been heated by dissipation, and has gained an entropy excess relative to its surroundings. Since the Mach number of the relative motion between star and envelope is not very high (see fig. 6) the thermal energy gained by the material is of the same order as the original thermal energy, and hence the buoyancy force is of the same order as the local gravitational force, but working in the opposite direction. Hence the material is accelerated radially outward with an acceleration comparable to the local gravitational acceleration. Since the outer layers are already convectively unstable, this acceleration can continue over a long path. This description in terms of buoyancy forces (which is not used by BT) gives an explanation of the gradual acceleration of material from the vicinity of the neutron star to velocities exceeding the local escape velocity further out in the envelope. It is interesting to note that the buoyancy mechanism does not operate when the relative motion is subsonic, because in this case the entropy of the matter passing close to the star does not increase very much.

In the work of TBO it was found that the spin-up of the layers in the envelope around the neutron star reduced the relative velocity, and hence the rate of orbital decay. In the two-dimensional models the material that receives the angular momentum immediately moves outwards and is replaced by low angular momentum material. This means the spiral-in will proceed even more rapidly than in the spherically symmetric case. The simplifications made by BT that will probably affect the outcome of their calculations most severely are:

1) **The symmetric, hydrostatic starting model.** Since the structure of the envelope changes completely in one orbital period, this can not be consistent with the earlier evolution. Especially he effectiveness of the buoyancy forces may depend on the initial hydrostatic structure.
Fig. 8a. The velocity field in the giant envelope as calculated by Bodenheimer and Taam (1984), with a two-dimensional hydrodynamics code. The largest velocities exceed the local escape velocity.

Fig. 8b. A contour plot of the angular momentum distribution in the same model as fig. 8a. Since the initial angular momentum distribution was independent of z, the height above the equatorial plane, the circulation pattern from 8a can also be traced in the deformation of the contours.
2) The short computed time interval. Since this interval is even shorter than one dynamical time of the envelope, one can not really conclude from the present results what the flow pattern developing on a longer timescale will be like.

3) The rotational symmetry. Even if the initial evolution could be properly described by a cylindrically symmetric model which is followed by a very sudden increase in frictional luminosity as the star hits denser layers near the giant core, the change in envelope structure in one orbital period is not consistent with rotational symmetry.

4) The size of the region in which the energy and angular momentum is deposited. For unclear reasons BT chose this region to be much smaller than the accretion radius of the neutron star. If the energy is distributed over a much larger amount of matter smaller pressure gradients and hence smaller outflow velocities might result.

In view of these difficulties the results of the two-dimensional calculations should not be used quantitatively. The conclusion that one-dimensional models can not describe the dominant physical processes is however very firm.

V.5 CONCLUSIONS

We conclude that the detailed studies of common envelope evolution made so far have yielded no more than an inventory of the physical processes that may be involved. The main difficulty is the complexity of the three-dimensional hydrodynamics with very different length- and time scales that is involved. This can probably only be resolved by using numerical techniques, on a scale which is presently unattainable.

Returning to the question posed in the introduction regarding the outcome of common envelope evolution, we can now conclude that there is no justification whatsoever for the assumption that the gravitational energy liberated by the spiral-in is efficiently used to eject the envelope, which is the basis of most predictions of the outcome of the CE phase (Chapter III.2, Iben and Tutukov 1984). In the 1-D models it is found that most of this energy is radiated away, in the 2-D models (at least as far as can be judged from the present results) it is not efficiently used, since a small fraction of the mass carries away a large excess of energy.