Overrated credit risk: three essays on credit risk in turbulent times
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Citation for published version (APA):
Bongaerts, D. G. J. (2010). Overrated credit risk: three essays on credit risk in turbulent times

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Chapter 2

Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market

This chapter is based on Bongaerts, De Jong and Driessen (2010).

2.1 Introduction

The relation between liquidity and asset prices has received considerable attention recently. Much less is known about liquidity effects in derivative markets. This chapter provides a theoretical asset pricing model of liquidity effects in both primary asset markets and derivative markets, incorporating short selling and non-traded risk exposure. We derive new and testable implications for the pricing of liquidity and test these for one of the key derivative markets, the credit default swap market.

This chapter has two main contributions. The first contribution is a new equilibrium pricing model for assets and derivatives that incorporates liquidity risk and short-selling due to hedging of non-traded risk. This model builds on the ‘Liquidity-CAPM’ of Acharya and Pedersen (2005), who only consider investors with long positions in assets that are in positive net supply, in which case illiquidity always leads to lower asset prices and higher expected returns. We show that, if some investors optimally short assets, the effects of liquidity are more complicated and can be zero, positive or negative. Obviously, allowing for short positions is essential for derivative assets, but our framework is also relevant for positive-net-supply assets with considerable short-selling activity. Our theory thus also adds
to the literature on asset pricing and liquidity.\footnote{For the equity market, the pricing of liquidity risk has been studied by Amihud (2002), Acharya and Pedersen (2005), Pastor and Stambaugh (2003), Bekaert, Harvey and Lundblad (2007), and Korajczyk and Sadka (2008), amongst others. De Jong and Driessen (2007), Downing, Underwood and Xing (2005) and Nashikkar and Subrahmanyam (2006) study on the pricing of liquidity in corporate bond markets, amongst others.}

In the equilibrium framework heterogenous investors use some assets to hedge exposure to a non-traded risk factor. We derive that the expected return on these 'hedge assets' can be decomposed into priced exposure to non-hedge asset returns, hedging demand effects, an expected liquidity component, and several liquidity risk premia. The first main theoretical result is that the sign of these liquidity effects depends on heterogeneity in investors' non-traded risk exposure, risk aversion, horizon and wealth. For example, if the investors who are long in the hedge assets are more aggressive (due to either higher aggregate wealth or lower risk-aversion) or have a shorter horizon than the 'short' investors, the effect of expected liquidity on expected returns is positive. The intuition for this result is that the aggressive investors are most sensitive to transaction costs and thus need to be compensated for these costs in equilibrium. This result holds for both positive-net-supply assets and derivatives. Our second main theoretical result is that, if the hedge assets are derivatives in zero net supply, some of the liquidity risk covariances have a zero premium. Specifically, the covariance between the return on a derivative (net of transaction costs) and the return on the total derivative market return (net of market-wide transaction costs) has a zero premium in case of zero-net-supply assets. Another liquidity risk covariance, the covariance between transaction costs on a given derivative and the return on the non-traded risk factor, does carry a risk premium however, whose sign depends on the investors' non-traded risk exposures.

Our model also generates interaction effects between hedging demand and liquidity premia. We show that increased hedging demand on the one hand implies that some investors will short hedge assets to hedge, but that on the other hand this hedging pressure pushes up expected returns which increases the speculative demand of investors. The net effect can either be an increase or decrease in the net aggressiveness of the 'long' versus 'short' investors and the premium on expected liquidity. If hedging demand varies across assets, the model thus generates cross-sectional effects in the pricing of liquidity and we test for such effects empirically. Of course, we also get a standard hedge pressure effect in our model because the covariance of an asset’s return with the non-traded risk factor is priced. We thus add to existing work on hedging pressures in derivative markets (see, for example, De Roon, Nijman and Veld (2000) and Gärleanu, Pedersen and Poteshman (2009)).
by incorporating liquidity effects.

Our second main contribution is an empirical test of this theoretical framework for an important class of derivative assets, credit default swaps (CDS). By now, the CDS market is one of the largest derivative markets with a total notional amount of approximately 31 trillion USD around August 2009 according to the Depository Trust & Clearing Corporation. This has induced researchers and practitioners to use CDS spreads as pure measures of default risk (for example, Longstaff et al. (2005) and Blanco et al. (2005)). However, as discussed below, we find that part of the CDS spread reflects liquidity effects.

Empirical work on liquidity and derivative assets is scarce. Two recent papers empirically assess the impact of liquidity on CDS spreads, Tang and Yan (2007) and Chen, Cheng and Wu (2005). Tang and Yan (2007) regress CDS spreads on variables that capture expected liquidity and liquidity risk, and find that illiquidity leads to higher spreads. Chen et al. (2005) estimate the impact of liquidity and other factors on CDS spreads using a term structure approach. They find that premia for liquidity risk and the expected liquidity premium are earned by the CDS buyer. The identification of the liquidity risk premium comes from the term structure of CDS spreads, whereas our method follows the standard procedure of identifying risk premia from expected excess returns. Another recent paper by Das and Hanouna (2009) develops a framework in which lower equity market liquidity leads to higher CDS prices and confirms this mechanism empirically. Bühler and Trapp (2009) estimate a reduced-form CDS pricing model that allows for an interaction of CDS and bond liquidity effects. Turning to other derivative markets, Çetin, Jarrow, Protter and Warachka (2006) add illiquidity to the standard Black-Scholes framework and Brenner, Eldor and Hauser (2001) investigate the effect of non-tradability on currency derivatives. Deuskar, Gupta and Subrahmanyam (2009) find empirically that illiquid interest rate options trade at higher prices than liquid options, and also find evidence for commonality in liquidity of different options. this chapter contributes to this literature by developing an equilibrium pricing model and testing this structural asset pricing model for the CDS market. Our use of an asset pricing model allows for an immediate interpretation of our results in terms of hedging pressure effects, liquidity and liquidity risk premia.

In addition to our two main contributions, the empirical analysis makes several methodological contributions. We derive expressions for realized and expected excess returns on CDS positions. In particular, we show how to construct expected excess returns from the CDS spread level, corrected for the expected loss where we use Moody’s-KMV Expected Default Frequencies (EDFs) to assess default
probabilities. As argued by Campello, Chen and Zhang (2008) and De Jong and Driessen (2007), this procedure gives more precise estimates of expected returns than averaged realized returns. On the econometric side, we use a repeat sales method to construct unbiased and efficient estimates of portfolio CDS returns from the unbalanced panel of individual CDS quotes.

We use a representative dataset of CDS bid and ask quotes for U.S. firms and banks over a relatively long period (2004-2008). We only rely on the most standard 5-year contracts. Applying the repeat sales method to these data, we construct CDS returns and bid-ask spreads for portfolios sorted on a range of variables capturing default risk and liquidity. The level and innovations of the bid-ask spreads are used to construct measures of expected liquidity and liquidity risk. We estimate our asset pricing model using the Generalized Method of Moments and find significant exposure to expected liquidity and liquidity risk. Our key empirical finding is that sellers of credit protection receive an illiquidity compensation on top of the compensation for default risk. Most of the liquidity effect comes from the expected liquidity component, and the effect of expected liquidity implies that credit protection sellers are more aggressive than protection buyers, either due to more wealth, lower risk-aversion, or shorter horizons. The theoretical model can fully explain the empirical effect of expected liquidity provided that holding periods are not too long. We also confirm that several parameter restrictions imposed by the theory are not violated by our empirical estimates. For example, the theory predicts that the covariance between derivative transaction costs and derivative-market-wide transaction costs carries a zero premium, and we cannot reject this empirically.

We perform an extensive series of robustness checks that support our results. Our benchmark results imply that the expected liquidity effect on expected returns is 0.175% per quarter on average, while across all robustness checks the lowest expected liquidity effect is 0.060% per quarter and still strongly statistically significant. We also control for corporate bond market liquidity and find evidence of hedging demand effects that are in line with our theory, with some evidence for interaction between hedging demand and liquidity effects as discussed above.

Our results are robust to inclusion of the two last quarters of 2008. In this economic crisis period, CDS spreads and bid-ask spreads increase dramatically, amongst others due to a strong increase in counterparty risk and deleveraging of many financial institutions. The liquidity effects remain statistically significant and even become larger in some cases.
The remainder of this chapter is structured as follows. Section 2.2 introduces our theoretical model. Section 2.3 discusses in detail how the model can be applied and estimated for the CDS market. Section 2.4 discusses the data and construction of CDS returns and liquidity measures. Section 2.5 presents the results of the empirical analysis and Section 2.6 concludes.

2.2 A model for pricing liquidity in asset and derivative markets

2.2.1 Motivation

In this section we derive an asset pricing model in a setting with transaction costs, heterogeneous agents and multiple assets. The model has two key ingredients that differentiate it from the liquidity CAPM of Acharya and Pedersen (2005) (henceforth AP). First of all, some of the assets can be derivative assets in zero net supply. Second, agents have exposure to a non-traded risk factor. We introduce this non-traded risk factor so that, in equilibrium, a fraction of the agents optimally hold short positions in some assets in order to hedge the non-traded risk exposure. This contrasts with AP whose assumptions imply that all agents hold long positions in equilibrium.

After deriving the implications of this benchmark setup, we study two extensions. First, we incorporate long-horizon investors who are not exposed to transaction costs. Second, we allow part of the non-traded risk to be idiosyncratic.

In the empirical analysis we apply our model to a setting where agents have a non-traded exposure to credit risk. For example, commercial banks have exposure to non-traded bank loans and illiquid corporate bonds, which they can partially hedge using credit default swaps. Other agents, such as hedge funds or insurance companies, have little or zero exposure to non-traded credit risk and may sell credit default swaps to commercial banks, thus earning a credit risk premium.

Even though we test our model empirically on a derivative market, the theory applies more generally to asset markets with positive supply, but where some agents short assets to hedge their non-traded risk exposure. A pricing model similar to the liquidity CAPM of AP is a special case of our model, since in AP all assets are in positive net supply and agents have no non-traded risk exposure.
2.2.2 Assumptions and notation

1. AGENTS. The economy has $N$ agents with CRRA preferences. Agents live for one period, invest at time $t - 1$ and consume at $t$ (this assumption will be relaxed in one of the model extensions). We allow for heterogeneity in relative risk aversion $\gamma_i$ and initial wealth $w_i$.

2. TRADED ASSETS. There are $K$ traded assets, divided into two subsets without loss of generality. For the first subset of $K_b$ assets, all agents optimally hold long positions in equilibrium. We refer to these assets as basic assets or non-hedge assets. The second subset has $K_h$ assets, and some agents optimally hold short positions in these assets. We refer to these assets as hedge assets. For simplicity, each investor can be either optimally long in all hedge assets or short in all hedge assets. Again, this assumption will be relaxed in one of the model extensions.

Using $\delta_i$ to denote a diagonal matrix with the sign of the hedge asset positions of investor $i$ on the diagonal, we thus have either $\delta_i = I$ or $\delta_i = -I$. Aggregate supply is denoted by $S_b$ for non-hedge assets and $S_h$ for hedge assets. Obviously, for non-hedge assets supply has to be positive. For hedge assets, aggregate supply could be zero, in which case these are derivative assets, but it can also be nonzero. To maintain tractability, we assume that all asset returns are jointly lognormally distributed.

3. NON-TRADED RISK EXPOSURE. Agent $i$ has an exposure of $q_i$ to non-traded background risk with log-return equal to $r_{n,t}$, where $r_{n,t}$ is the return on a single non-traded risk factor. This assumption is necessary to generate nonzero positions in hedge assets. If the exposure $q_i$ varies across agents or if agents have different risk aversion, their hedging demands will differ and they will hold different hedge asset positions if the hedge asset returns correlate with $r_{n,t}$. For example, agents with large positive $q_i$ may hold short positions in hedge assets, while agents with zero or negative non-traded exposure take long positions in the hedge assets. We think of $r_{n,t}$ as the return on a diversified portfolio of very illiquid assets that

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2This assumption is made only to generate tractable expressions that allow for a simple empirical test of liquidity effects. It is straightforward to include more than two groups in the theory.

3Of course, the sign of the optimal portfolio weights is determined in equilibrium. In Appendix [2.9.1](#) we show the necessary conditions (equation (2.29)). Essentially, the 'long' group of investor should have zero or small hedging demand while the 'short' group needs to have sufficient hedging demand to ensure that they always short derivatives.

4In this one-period setup, similar asset pricing implications are obtained without distributional assumptions if we assume mean-variance investors. The assumption of CRRA investors and lognormal returns allows us to extend the model to a setting with long-horizon investors in a simple way.
are not traded in equilibrium. For our credit risk application, these can be bank
loans or illiquid corporate bonds.

4. TRANSACTION COSTS. Following Acharya and Pedersen (2005), the
one-period investors investing at \( t - 1 \) pay transaction costs proportional to the
current price when closing the position at time \( t \). Transaction costs are stochastic.
The percentage costs are denoted by the \( K_b \)-dimensional vector \( C_{b,t} \) for non-hedge
assets and \( K_h \)-dimensional vector \( C_{h,t} \) for hedge assets. Transaction costs represent
both the bid-ask spread and search costs, which are relevant in over-the-counter
markets (see Duffie, Gârleanu and Pedersen (2005)). Both long and short holders
pay transaction costs. This can be motivated by the existence of (implicit) market
makers who only play a role as intermediary, i.e. they earn the bid-ask spread
and hold zero net positions in the hedge assets. Alternatively, our assumption
that both long and short investors pay transaction costs is appropriate in a setting
without market makers if transaction costs fully represent search costs. The net
log-returns of the agents (in excess of the risk-free rate) are then given by the
vectors \( r_{b,t} - c_{b,t} \) for the basic assets and \( r_{h,t} - c_{h,t} \) for the investors who are long
in the hedge assets, and \( -r_{h,t} - c_{h,t} \) for the investors who are short in the hedge
assets, with log-transaction costs defined as \( c_{h,t} = -\ln(1 - C_{h,t}) \).

2.2.3 Asset pricing implications: Benchmark model

We now derive the main asset pricing implications for this benchmark economy. We
focus on the implications for the hedge asset returns, since these are the relevant
assets for our empirical analysis. Using the standard approximation of log-portfolio
returns (Campbell and Viceira (2002), page 29), investor \( i \) with CRRA preferences
and relative risk aversion \( \gamma_i \) trades off the mean and variance of the log portfolio
return. As shown in Appendix 2.9.1, investor \( i \) thus maximizes

\[
x'_i E(r_b - c_b + \frac{1}{2}\sigma^2_b) + y'_i E(r_h - \delta_i c_h + \frac{1}{2}\sigma^2_{i,h}) - \frac{1}{2}\gamma_i \text{Var}(x'_i(r_b - c_b) + y'_i(r_h - \delta_i c_h) + q_nr_n)
\]

(2.1)

over positions \( x_i \) in non-hedge assets and positions \( y_i \) in hedge assets, where \( \sigma^2_b \) is
the diagonal of \( \text{Var}(r_b - c_b) \) and \( \sigma^2_{i,h} \) the diagonal of \( \text{Var}(r_h - \delta_i c_h) \). To simplify
notation, we drop all time subscripts. In particular, note that the expectation and
variance in (2.1) are conditional on the information set at time \( t - 1 \).

Imposing the market clearing condition and the optimality conditions for each
investor, we can derive the asset pricing equation for the hedge assets.

Theorem I. Given assumptions 1 to 4, investors maximizing the function in
(2.1) and using a Taylor-expansion of $\text{Var}(\hat{r}_h - \hat{c}_h) Var(\hat{r}_h - \delta_h \hat{c}_h)^{-1}$ around $\hat{c}_h = 0$, the vector with expected excess log-returns on the hedge assets, $E(r_h)$, satisfies in equilibrium

$$E(\hat{r}_h) + \frac{1}{2}\sigma^2_{\hat{r}_h} + F_h - AE(\hat{c}_h) = \lambda [\text{Cov}(\hat{r}_h, r_n) - A\text{Cov}(\hat{c}_h, r_n)] + \zeta_0 [E(\hat{c}_h) - AE(\hat{r}_h + \frac{1}{2}\sigma^2_{\hat{r}_h})] + \kappa_0 [\text{Cov}(\hat{c}_h, r_n) - A\text{Cov}(\hat{r}_h, r_n)] + \phi(I - H_1)^{-1}\text{Cov}(\hat{r}_h - \hat{c}_h, \hat{r}_m - \hat{c}_m)$$

(2.2)

where $F_h$ is a vector of convexity corrections (see Appendix 2.9.1) and

$$\hat{r}_h = r_h - \beta_{r_h,r_b}(r_b - c_b), \quad \hat{c}_h = c_h - \beta_{c_h,r_b}(r_b - c_b)$$

(2.3)

where $\beta_{r_h,r_b} = \text{Var}(r_h - c_b)^{-1}\text{Cov}(r_h, r_b - c_b)$ and $\beta_{c_h,r_b} = \text{Var}(r_b - c_b)^{-1}\text{Cov}(c_h, r_b - c_b)$. The market-wide hedge asset return and transaction costs are defined as $\hat{r}_m = S_h\hat{r}_h/t'S_h$ and $\hat{c}_m = S_h\hat{c}_h/t'S_h$ if $S_h \neq 0$ and equal to zero otherwise. The following asset pricing parameters are scalars, and defined as

$$\lambda = \frac{1}{\eta} \sum_{i=1}^{N} w_i q_i$$

(2.4)

$$\zeta_0 = \frac{1}{\eta} \left( \sum_{i: \delta_i = 1} w_i \gamma_i^{-1} - \sum_{i: \delta_i = -1} w_i \gamma_i^{-1} \right)$$

$$\kappa_0 = \frac{1}{\eta} \left( \sum_{i: \delta_i = 1} w_i q_i - \sum_{i: \delta_i = -1} w_i q_i \right)$$

$$\phi = \frac{1}{\eta} t'S_h \geq 0$$

$$\eta = \sum_{i=1}^{N} w_i \gamma_i^{-1} > 0$$

and with $A = (I - H_1)^{-1}H_1$, $H_1 = (C + C')\text{Var}(\hat{r}_h)^{-1}$, and $C = \text{Cov}(\hat{c}_h, \hat{r}_h)$. 

**Proof:** Appendix 2.9.1

Note that the expected return on the hedge assets also enters on the right hand side of (2.2), hence the expected returns are implicitly defined in (2.2).5

This theorem shows that expected returns on the hedge assets are determined

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5It is straightforward to show that equation (2.2) reduces to a model similar to AP’s Liquidity CAPM if $S_h > 0$, $K_b = 0$, $q_i = 0$ and $\delta_i = I$ for all agents. In this case $\lambda$ and $\kappa_0$ are equal to zero and all the liquidity risk premia are in the term $\phi(I - H_1)^{-1}\text{Cov}(\hat{r}_h - \hat{c}_h, \hat{r}_m - \hat{c}_m)$. 

by five terms. First of all, from the definition of $\widehat{r}_h$ in (2.3) we obtain the standard result that covariance with the non-hedge asset returns is priced if these non-hedge assets carry a risk premium ($E(r_h - c_h) > 0$).

The second term concerns the covariance of hedge asset returns (orthogonalized for non-hedge asset returns) with the non-traded risk factor returns $r_n$. Given that $\eta > 0$, the sign of the coefficient $\lambda$ on this covariance depends on the sign of $\sum_i w_i q_i$. For example, if aggregate exposure to non-traded risk is positive we obtain the intuitive result that assets that have positive covariance with the non-traded risk factor have high expected returns, in a similar way as in De Roon et al. (2000).

The third term captures expected liquidity. The sign of this effect depends on $\zeta_0$. The model implies that if the more wealthy or less risk averse investors have long positions in the assets, the coefficient on expected transaction costs is positive and the 'long' holders earn an expected liquidity premium. In the next subsection we illustrate this in a simple example.

The fourth term contains a coefficient $\kappa_0$ on the covariance of hedge asset transaction costs (orthogonalized for non-hedge asset returns) with $r_n$, which depends on the non-traded risk exposure of the short versus long agents. If, for example, the 'short' agents have positive non-traded risk exposure while the 'long' agents have zero non-traded risk exposure, $\kappa_0$ is positive. In this case, only the 'short' agents care about covariance with $r_n$. If the covariance between costs $c_h$ and $r_n$ is negative, the non-traded risk is amplified by the costs because the return on their short position equals $-r_h - c_h$. This weakens the hedge quality of the hedge asset, thus decreasing the demand for hedge assets by the 'short' agents which in turn lowers the expected return. This is the effect that we find empirically.

The final term in (2.2) only enters if aggregate supply $S_h$ is positive. In this case, the covariance with the net market-wide return of hedge assets is priced since the agents have to hold the positive supply in equilibrium. This term is equal to zero if the hedge assets are in zero net supply ($S_h = 0$).

All expectations and covariances in (2.2) are 'corrected' for the covariance matrix between the hedge asset returns and costs through the matrix $A$. In the example below we provide more intuition for the presence of this correction term $A$. If the matrix $\text{Cov}(\widehat{c}_h, \widehat{r}_h)$ is equal to zero, we have $A = 0$ and (2.2) simplifies to a linear asset pricing equation

$$E(\widehat{r}_h) + \frac{1}{2}\sigma^2_{\widehat{r}_h} + F_h = \zeta_0 E(\widehat{c}_h) + \lambda \text{Cov}(\widehat{r}_h, r_n) + \kappa_0 \text{Cov}(\widehat{c}_h, r_n) + \phi \text{Cov}(\widehat{r}_h - \widehat{c}_h, \widehat{r}_m - \widehat{c}_m)$$

(2.5)
In Section 2.3.2.3.2 we discuss in detail how the general model in (2.2) can be estimated using the Generalized Method of Moments (GMM).

2.2.4 Example

We provide more intuition for the expected liquidity effect by a simple example with constant transaction costs $c$. Let there be one hedge asset with return $r$ with $\sigma^2 = \text{Var}(r)$ and two investors. One investor (agent 1) has a positive exposure to the non-traded risk factor $r_n$ and the other investor (agent 2) none, hence $q_1 > 0$ and $q_2 = 0$. There are no other assets. If agent 1 is short an agent 2 long, the asset demands (as a fraction of wealth) of the investors are

$$y_1 = \frac{E(r) + \frac{1}{2} \sigma^2 + c}{\gamma_1 \sigma^2} - \frac{\text{Cov}(r, r_n) q_1}{\sigma^2} \gamma_1$$

$$y_2 = \frac{E(r) + \frac{1}{2} \sigma^2 - c}{\gamma_2 \sigma^2}$$

In a zero net supply market, the wealth-weighted demands have to add up to zero, $w_1 y_1 + w_2 y_2 = 0$, which gives the equilibrium expected returns

$$E(r) + \frac{1}{2} \sigma^2 = \frac{\text{Cov}(r, r_n) w_1 q_1}{w_1 \gamma_1} + \left( \frac{w_2 \gamma_2^{-1} - w_1 \gamma_1^{-1}}{w_1 \gamma_1^{-1} + w_2 \gamma_2^{-1}} \right) c = \rho + \zeta_0 c$$

with the compensation for transaction costs determined by the coefficient $\zeta_0$. For this equilibrium to hold, we need to make sure that $y_1 < 0$ and $y_2 > 0$ in equilibrium. These inequalities are satisfied if $\text{Cov}(r, r_n) q_1 \gamma_1 > 2c$, requiring that the hedge demand has to be sufficiently large relative to the costs $c$ and the speculative demand (which depends on $\gamma_1$). In Appendix 2.9.1 we show the restrictions needed in the general setting.

Figure 2.1 illustrates the equilibrium for two cases, $c = 0$ and $c > 0$. It shows minus the asset demand of agent 1 ($-w_1 y_1$) and the asset demand of agent 2 ($w_2 y_2$), and is drawn such that agent 2 is more aggressive (i.e. less risk averse or more wealthy, $w_2 \gamma_2^{-1} > w_1 \gamma_1^{-1}$), hence her speculative asset demand is steeper than that of agent 1. The equilibrium expected return is obtained at the point where the two lines cross, i.e. $-w_1 y_1 = w_2 y_2$ or $w_1 y_1 + w_2 y_2 = 0$. For $c > 0$, both lines shift downward as transaction costs make the investors want to invest less (in absolute value). The lines now cross at a different point, where the expected return is higher by $\zeta_0 c$. Due to higher wealth and/or lower risk aversion, the demand of the 'aggressive' agent 2 is more sensitive to transaction costs than agent 1. Therefore,
when transaction costs increase, the asset demand of agent 2 reduces more strongly than the asset demand of agent 1. Then, to persuade agent 2 to invest a sufficient amount in the risky asset, the expected return needs to increase so that $\zeta_0 c$ must be positive.

Allowing for stochastic transaction costs, this two-investor example can be used to understand the role of the ”correction term” $A$ for expected returns and costs in (2.2). It is straightforward to show that in this simple example we have

$$\frac{\partial E(r)}{\partial E(c)} = \frac{\zeta_0 + 1}{1 + 2(\zeta_0 - 1)\text{Cov}(c,r)/V(r)} - 1$$

(2.8)

and that this term has a positive dependence on $\text{Cov}(c,r)$ since $|\zeta_0| \leq 1$. For example, empirically we typically have $\text{Cov}(c,r) < 0$. Such negative covariance lowers the speculative demand of agent 2, since it increases the variance of her net return $r - c$. The speculative demand of agent 1 increases, since the return on her short position is $-r - c$. In the context of Figure [2.1] negative covariance thus flattens the demand of agent 2 and steepens the demand of agent 1, leading to a lower coefficient on $E(c)$ and a lower expected return for agent 2. This effect occurs even when both agents have the same wealth and risk aversion (i.e. if $\zeta_0 = 0$).

2.2.5 Model extension I: Long-horizon investors

We extend the model presented above in two directions. Our first extension is that we allow some investors to have a long horizon. Specifically, we add $M$ investors with CRRA preferences and investment horizon till the expiration of the contract, so that they are not exposed to the transaction costs. For simplicity, we assume that any intertemporal rebalancing is costless for these investors. In addition, each period $N$ one-period investors enter the market and close their positions at the end of the period, incurring transaction costs at that moment. In order to generate a stationary equilibrium, we assume that all returns and costs are independent and identically distributed (IID) over time and that the characteristics of the one-period investors are the same each period.

This setup captures in a simple way that some investors essentially hold buy-and-hold portfolios and thus incur no transaction costs, while other investors trade frequently in financial markets, either due to external constraints, liquidity shocks, investor-specific preferences or beliefs.

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6As shown below, these investors have constant portfolio weights in equilibrium. We thus neglect the transaction costs needed to maintain constant portfolio weights.
Given these assumptions, the optimal portfolio choice of the long-horizon investors is constant over time and equal to the myopic portfolio rule. Then, the equilibrium generates the following asset pricing implications:

**Theorem II.** Given assumptions 1 to 4, \( N \) investors maximizing the function in (2.1), \( M \) long-horizon investors, and using a Taylor-expansion of \( \text{Var}(\hat{r}_h - \hat{c}_h)\text{Var}(\hat{r}_h - \delta_i \hat{c}_h) \) around \( c_h = 0 \), the \( K_h \)-dimensional vector with expected excess log-returns on the hedge assets, \( E(r_h) \), satisfies in equilibrium

\[
E(\hat{r}_h) + \frac{1}{2} \sigma^2_{\hat{r}_h} + F_h = \frac{1}{1 + \mu_0} \left( E_{BM}(\hat{r}_h + \frac{1}{2} \sigma^2_{\hat{r}_h}) + F_h \right) + \frac{\mu_1}{1 + \mu_0} \text{Cov}(\hat{r}_h, r_n) \tag{2.9}
\]

where \( E_{BM}(\hat{r}_h) \) denotes the expected returns that hold in the benchmark model (Theorem I), and

\[
\mu_0 = \frac{1}{\eta} \sum_{i=N+1}^{N+M} w_i \gamma_i^{-1} \tag{2.10}
\]

\[
\mu_1 = \frac{1}{\eta} \sum_{i=N+1}^{N+M} w_i q_i
\]

and, as before, \( \eta = \sum_{i=1}^{N} w_i \gamma_i^{-1} \).

**Proof:** Appendix 2.9.1

The effect of the presence of long-term investors is twofold. First of all, given that the long-term investors do not care about transaction costs, all terms involving expected transaction costs and covariation with costs (liquidity risk) are depreciated by the term \( \frac{1}{1 + \mu_0} \), with \( \mu_0 \) capturing the aggressiveness of the long-term investors relative to all short-term investors. Second, the risk premium on the non-traded risk factor \( r_n \) is adjusted to reflect the exposure of long-term investors.

An interesting feature of this extension is that it can generate liquidity effects even if investors are identical in terms of wealth and risk aversion. The coefficient on expected liquidity is

\[
\zeta_0 = \frac{\sum_{i:T_i=1, \delta_i=1} w_i \gamma_i^{-1} - \sum_{i:T_i=1, \delta_i=-L} w_i \gamma_i^{-1}}{\sum_{i:T_i=1} w_i \gamma_i^{-1} + \sum_{i:T_i=L} w_i \gamma_i^{-1}} \tag{2.11}
\]

where \( T_i = 1 \) denotes one-period investors and \( T_i = L \) long-horizon investors. In the model with only short-term investors the expected liquidity coefficient \( \zeta_0 \) is zero if buyers and sellers have the same aggregate wealth-weighted risk aversion.
However, if the sellers are long-term investors and buyers short-term, the expected liquidity effect is positive. In this case, only the short-term buyers care about transaction costs and need to be compensated for this by the long-term sellers.

2.2.6 Model extension II: Idiosyncratic background risk and hedging demand

The second extension incorporates that (part of) the non-traded background risk is idiosyncratic, by assuming that each investor \(i\) has exposure of \(q_i\) to background risk with log-return equal to \(r_{n,t} + \varepsilon_{i,t}\), where \(\varepsilon_{i,t}\) is independent from \(r_{n,t}\) and independent across investors but potentially correlated with the asset returns. In addition, we introduce a third group of investors to generate heterogeneity in hedging demand across hedge assets. This group of investors is long for a fixed subset of the hedge assets and short in the other hedge assets. For this ‘long/short’ group we then have \(\delta_i = D\), where \(D\) is a diagonal matrix with diagonal elements equal to 1 or \(-1\). This will lead to an interaction effect between hedging demand and liquidity effects as discussed below.

**Theorem III.** Given assumptions 1 to 4, the presence of idiosyncratic background risk, and the presence of ‘long/short’ investors, all investors maximizing the function in (2.1) and using a Taylor-expansion of \(\text{Var}(\hat{r}_h - \hat{c}_h)\text{Var}(\hat{r}_h - \delta_i\hat{c}_h)^{-1}\) around \(\hat{c}_h = 0\), the \(K_h\)-dimensional vector with expected excess log-returns on the hedge assets, \(E(\hat{r}_h)\), satisfies in equilibrium

\[
E(\hat{r}_h) = E_{BM}(\hat{r}_h) + \zeta_1 [B_1E(\hat{c}_h) - B_2E(\hat{r}_h + \frac{1}{2}\sigma_{\hat{r}_h}^2)] + \kappa_1 [B_1\text{Cov}(\hat{c}_h, r_n) - B_2\text{Cov}(\hat{r}_h, r_n)] + \frac{1}{\eta} (I - H_1)^{-1} \left[ \sum_i w_i q_i (I - 2H_11_{\delta_i = -1} + (H_2 - H_1)1_{\delta_i = D}) \text{Cov}(\hat{r}_h - \hat{c}_h, \varepsilon_i) \right]
\]

where \(E_{BM}(\hat{r}_h)\) denotes the expected returns that hold in the benchmark model (Theorem I), \(1\) is the indicator function,

\[
\zeta_1 = \frac{1}{\eta} \sum_{i: \delta_i = D} w_i q_i^{-1} \geq 0
\]

\[
\kappa_1 = -\frac{1}{\eta} \sum_{i: \delta_i = D} w_i q_i
\]

and with the following matrix definitions: \(H_2 = (DC + C'D)\text{Var}(\hat{r}_h)^{-1}\), \(B_1 = (I - H_1)^{-1}(D + H_2D - H_1 - H_1D)\), and \(B_2 = (I - H_1)^{-1}H_2\). If \(C = \text{Cov}(\hat{c}_h, \hat{r}_h) = 0\,
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(2.12) simplifies to

\[
E(\hat{r}_h) = E_{BM}(\hat{r}_h) + \zeta_1 DE(\hat{c}_h) + \kappa_1 DCov(\hat{c}_h, r_n) + \frac{1}{\eta} \left[ \sum_i w_i q_i Cov(\hat{r}_h - \hat{c}_h, \varepsilon_i) \right]
\]

(2.14)

**Proof:** Appendix 2.9.1.

Theorem III adds three new terms to the model. First of all, the final term in (2.12) and (2.14) shows how the presence of idiosyncratic background risk generates asset-specific and investor-specific hedging pressures. For example, if a wealthy investor has large and positive \( q_i Cov(\hat{r}_{jh} - \hat{c}_{jh}, \varepsilon_i) \) for asset \( j \), this investor will likely short this asset, paying a premium to those willing to buy the asset and thus increasing the expected return.

The second key implication of Theorem III is that the introduction of 'long/short' investors generates additional effects for expected liquidity and liquidity risk. There are several reasons why some investors are optimally long in some hedge assets and short in the other. For example, if they only have moderate non-traded risk exposure they may only short those hedge assets that covary most strongly with the non-traded risk factor. In addition, the investor-specific hedging effects in Theorem III can also generate strong hedging demand for only a subset of hedge assets.

The effect of this on liquidity premia can be seen most easily when \( C = 0 \). In this case, assets for which the long/short investors are long have a coefficient on expected costs equal to \( \zeta_0 + \zeta_1 = (\sum_{i: \delta_i = 1 \lor \delta_i = D} w_i \gamma_i^{-1} - \sum_{i: \delta_i = -I \lor \delta_i = D} w_i \gamma_i^{-1}) / \eta \). For the remaining hedge assets, the long/short investors have a short position, and the expected liquidity coefficient is \( \zeta_0 - \zeta_1 = (\sum_{i: \delta_i = 1} w_i \gamma_i^{-1} - \sum_{i: \delta_i = -I \lor \delta_i = D} w_i \gamma_i^{-1}) / \eta \). In both cases the coefficient captures the net aggressiveness of long versus short investors. A similar effect occurs for liquidity risk.

In which assets the long/short investors are long or short depends on a complex interaction between hedging demands and expected returns. A high hedging demand for an asset on the one hand implies that some investors will short hedge assets to hedge, but on the other hand this hedging pressure pushes up expected returns which increases the speculative demand of other investors. The net effect in equilibrium can be either a long or a short position. For example, if high hedging demand for an asset is caused by only a few agents who have large idiosyncratic background risk covarying with this hedge asset, the net effect will be that most other investors will be long in this hedge asset to exploit the higher expected return (pushed up by hedging demand). In this case we expect that the long/short
2.3 Applying the theory to the CDS market

2.3.1 Motivation and theoretical predictions

We apply the theory developed above to the market for credit risk. In this market, the key background risk is the systematic credit risk of illiquid corporate bonds and bank loans. We focus on credit default swap (CDS) contracts as assets to hedge this risk. In principle, one could also short liquid corporate bonds to hedge credit risk exposure. Our theory does not restrict that some of the hedge assets are corporate bonds. However, in practice shorting corporate bonds may be difficult and costly (see Nashikkar and Pedersen (2007)). In addition, Gârleanu and Pedersen (2009) document that margins on (unfunded) corporate bond positions are considerably higher than those for CDS trades. Hence, we abstain from including corporate bonds as hedge assets and focus on the pricing of CDS contracts.

Table 2.1 is a summary of figures reported by Mengle (2007), providing an overview of buyers and sellers in the CDS market. It shows that insurance companies and funds (pension funds, hedge funds and mutual funds) are net protection sellers in the CDS market, while banks are net protection buyers for their loan portfolio. This suggests that banks typically use the CDS market to hedge credit exposure, while long-term investors buy credit risk to earn a credit risk premium. This supports our assumption that investors are either hedgers (always buying insurance) or speculators who have little or no exposure to non-traded risk and take on credit risk to earn a risk premium.

Linking this market to our theoretical model, the background credit risk is proxied by a general credit index. Credit default swap (CDS) contracts are the hedge assets in zero net supply. We include as non-hedge asset (with positive net supply) the U.S. equity market index. We also construct a proxy to capture hedging demand. The precise definitions of these variables are discussed below.
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For this application to the CDS market we can deduce the expected sign for most parameters of the structural model. We define a long position in a CDS to be the case where the investor takes on credit risk, in other words when he sells protection and receives the CDS premium. The non-traded risk factor is given by the return on a portfolio exposed to credit risk, thus giving a negative return in case of defaults or other credit events. Given these definitions, we expect that investors have either a zero exposure \( q_i \) to non-traded credit risk (for example institutional investors) or a positive exposure (commercial banks). We thus expect \( \lambda \geq 0 \) and \( \kappa_1 \leq 0 \). In addition, investors with zero \( q_i \) are likely the 'long' investors (taking on credit risk and earning a risk premium), while those with positive \( q_i \) will be the 'short' investors (hedging credit risk). This implies \( \kappa_0 > 0 \). Finally, as shown in (2.4), the theory always implies \( 0 \leq \zeta_1 \leq 1 \), and \( |\zeta_0| \leq 1 \), and, in case of zero-net-supply markets, \( \phi = 0 \). In sum, we expect the following signs and restrictions for the model parameters

\[
|\zeta_0| \leq 1, \ 0 \leq \zeta_1 \leq 1, \ \lambda \geq 0, \ \kappa_0 \geq 0, \ \kappa_1 \leq 0, \ \phi = 0 \quad (2.15)
\]

2.3.2 Estimation method

For estimation purposes it is useful to write the asset pricing model in Theorem III as a beta-pricing model and invert the model to back out the expected hedge asset return. This gives the estimation equation

\[
E(r_h) + \frac{1}{2}\sigma_{\hat{r}_h}^2 = \beta_{\hat{r}_h,r_n}\psi + X\alpha + W(\zeta) \left\{ (A + \zeta_0 I)E(\hat{c}_h) + B_1 E(\hat{c}_h)\zeta_1 + \left[ \beta_{\hat{r}_h,r_n} - A\beta_{\hat{c}_h,r_n} \right] \tilde{\lambda} + \left[ \beta_{\hat{c}_h,r_n} - A\beta_{\hat{r}_h,r_n} \right] \tilde{\kappa}_0 + \left[ B_1 \beta_{\hat{c}_h,r_n} - B_2 \beta_{\hat{r}_h,r_n} \right] \tilde{\kappa}_1 \right\} \quad (2.16)
\]

with

\[
W(\zeta) = (I + \zeta_0 A + \zeta_1 B_2)^{-1}
\]

\[
\beta_{\hat{r}_h,r_n} = \frac{Cov(\hat{r}_h, r_n)}{Var(r_n)} \quad (2.17)
\]

\[
\beta_{\hat{c}_h,r_n} = \frac{Cov(\hat{c}_h, r_n)}{Var(r_n)}
\]

and \( \psi = E(r_h - c_h), \ \tilde{\lambda} = Var(r_n)\lambda, \ \tilde{\kappa}_0 = Var(r_n)\kappa_0 \) and \( \tilde{\kappa}_1 = Var(r_n)\kappa_1 \). The equation imposes \( F_h = 0 \) since this term is numerically small (see Appendix 2.9.1). All expectations and covariances are conditional upon the information set at time \( t - 1 \), implying that we have to focus on the innovations in transaction costs when calculating betas. The matrix \( X \) denotes any additional explanatory variables (for example, hedging demand or bond market characteristics), including an intercept.
This theoretical model can then be estimated using the Generalized Method of Moments (GMM), since (2.16) defines a system of \(K_h\) moment conditions (with \(K_h\) denoting the number of hedge assets) and parameter vector \((\psi, \alpha, \zeta_0, \zeta_1, \hat{\lambda}, \tilde{\kappa}_0, \tilde{\kappa}_1)\). In the estimation, we replace all expectations and betas with their sample counterparts in (2.16) and then perform GMM estimation by minimizing the (weighted) sum of squared errors over the parameters. The system in (2.16) is nonlinear because \(W(\zeta)\) depends on the model parameters. But given an initial estimate of \(\zeta\), the equations are linear in the parameter vector and can be estimated by weighted least squares. Iterating this procedure yields the GMM estimates. We weight the errors according to the diagonal of the error covariance matrix of (2.16). Following Shanken (1992) we correct the standard errors for the fact that betas, expected costs and expected returns are estimated (see Appendix 2.9.3 for an outline of this procedure).

In asset pricing tests, expected returns are typically estimated by the sample average of realized excess returns. However, given the short sample period this will give noisy estimates of expected returns in our case. Instead, following Campello et al. (2008) and De Jong and Driessen (2007), we use an ex-ante measure of expected returns by correcting CDS spread levels for the expected default losses (the exact procedure is discussed in detail below). This procedure gives the expected simple return \(E(R_h)\) on a CDS contract. Using the lognormality assumption, we back out the expected log return from this expected simple return via
\[
E(r_h) + \frac{1}{2}\sigma^2 = \ln(1 + E(R_h)).
\]

2.4 Data and construction of main variables

In this section we describe in detail which data we use and how we construct (expected) CDS returns, liquidity measures, and the non-traded risk factor, all of which are needed to estimate the asset pricing model.

2.4.1 Data description

We use CDS data from Datastream, which in turn sources its data from CMA. The data start in January 2004 and contain daily consensus bid, mid and offer CDS.

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7 We do not perform the full GMM where the weighting matrix is the inverse of the covariance matrix of pricing errors because these exhibit considerable cross-correlations (see Cochrane (2005), page 194, for a discussion).

8 The leading Bloomberg data platform obtains its CDS data also from CMA. Nashikkar, Subrahmanyam and Mahanti (2009) compare these CMA data with CDS data provided by GFI group and finds that they are "substantially in agreement".
spreads from a panel of thirty leading financial institutions. In addition, they also report a so called veracity score, which indicates whether the quotes provided are based on market quotes/trades or are derived. We collect these data for the U.S. market (corporates and financials) from January 2004 up to December 2008.

Since we only want to base our analysis on actual market data, we discard all derived quotes. Moreover, we remove all quotes for which both bid and ask are stale. We only select CDS contracts of U.S. companies in U.S. dollars on senior unsecured debt with a maturity of 5 years and a Modified Restructuring (MR) clause. After doing this, we end up with 422,779 daily observations on 595 entities.

Our CDS sample captures a significant part of the market for credit risk. This follows from the fact that the 595 firms for which we have CDS contracts cover 46% of the corporate bond market in terms of amount issued.

To create characteristics for our regressions and to base our sorts on, we match our CDS data to several other data sources. First we match the CDS data to issuer rating data in Moody’s Default Risk Services Corporate database. To obtain leverage and total debt outstanding, we match our data to Compustat. We also use corporate bond data from TRACE in order to construct several proxies for corporate bond liquidity. For each firm with CDS quote data, we calculate the total amount of corporate bonds issued by this firm, the total corporate bond trading volume for this firm, and total bond turnover (volume divided by amount issued).

Finally, we construct a proxy for hedging pressure effects. For each firm with CDS quote data, we calculate the total dollar amount of syndicated loans outstanding by this firm for each quarter in our sample. Data on syndicated loans are obtained from Dealscan. Since credit lines are likely to be largely drawn in distress scenarios, we assumed a 100% usage at default.

### 2.4.2 Portfolio construction

As is usual in the asset pricing literature, we fit the model to different test portfolios rather than to individual assets. We perform five sequential sorts on the cross-section of CDS contracts, always sorting first on credit rating, and then on either (i) firm leverage, (ii) total syndicated loan amount, (iii) total debt outstanding, (iv) CDS bid-ask spread, and (v) CDS quote frequency. The credit rating and leverage should generate variation in market risk exposure, the bid-ask spread and quote frequency are included to capture CDS liquidity variation, and total loan
amount captures hedging pressure effects. In order to have enough observations in each portfolio, we construct five rating categories: Aaa to Aa, A, Baa, Ba, and B to Caa. Within each rating category, we create quartile portfolios based on the second sorting variable. In total, we thus have five sequential sorts with five times four portfolios per sort. We re-sort portfolios each quarter, and calculate the sorting variables over the previous quarter. Given that our data start in January 2004, our first portfolio observations are thus for the second quarter of 2004.

To illustrate the patterns in the CDS spread and bid-ask spread, we present the (geometric) average CDS spread across all firms in Figure 2.2. Before mid-2007, the market-wide CDS spread showed only moderate variation, but since July 2007 spreads increased dramatically, with a peak around November 2008. Figure 2.2 also shows the bid-ask spreads for the sort on rating and bid-ask spreads, where we average each bid-ask spread quartile across all ratings. We see a small peak at the Ford/GM downgrade\(^9\) and a strong increase in bid-ask spreads during the crisis, with the most extreme movements around the Lehman default in September 2008.

\subsection*{2.4.3 CDS portfolio return and transaction costs}

Calculating CDS portfolio returns is nontrivial. For some CDS contracts we observe quotes every day, while for other contracts some missing observations occur. We therefore adopt a technique called weighted repeat sales that originates from the real estate literature (for example Bailey, Muth and Nourse (1963) and Case and Shiller (1987)). This method calculates a mid-quote CDS spread index for all CDS contracts in a specific portfolio. Let \( k(i) \) be the portfolio that contains constituent \( i \). We assume that the spread mid-quote of a five-year CDS contract \( p_{i,t} \) is given by

\[
p_{i,t} = CDS_{k(i),t} + u_{i,t},
\]

where \( CDS_{k(i),t} \) is the portfolio CDS spread level (which is to be estimated) and \( u_{i,t} \) is a CDS-specific error term. The repeat sales method employs regression analysis to estimate the innovations in the portfolio CDS spread at different points in time as regression coefficients.\(^{10} \) The portfolio bid-ask spread is obtained by equally weighting all bid-ask spreads of all CDS contracts in a portfolio in each week.

\(^{9}\)Acharya, Schaefer and Zhang (2008) provide evidence that liquidity risk for Ford and GM bonds increased correlations within the entire CDS market.

\(^{10}\)To construct portfolio CDS spread levels from the CDS spread innovations obtained by the repeat sales procedure, we match the accumulated spread change to the level of the portfolio spread at the start of each calendar year.
We then transform the portfolio CDS spreads to excess returns. In the CDS market, bid and ask quotes are given in terms of the CDS spread. Denote this bid-ask spread $s_{k,t}$. Then, consider an investor at time $t-\Delta t$ who sells protection using CDS contract $k$, at a CDS spread $CDS_{k,t-\Delta t} - \frac{1}{2}s_{k,t-\Delta t}$, paid in quarterly periods. One week later at time $t$, the investor buys an offsetting contract at $CDS_{k,t} + \frac{1}{2}s_{k,t}$ which generates a net cash flow of $-\frac{1}{4}(CDS_{k,t} - CDS_{k,t-\Delta t}) - \frac{1}{4}(s_{k,t-\Delta t} + \frac{1}{2}s_{k,t})$ each quarter until default or maturity. The value of this stream at time $t$ is the value of a portfolio of defaultable zero-coupon bonds, which gives the excess holding return net of costs:

$$R_{k,t} - c_{k,t} = -\frac{1}{4}(\Delta CDS_{k,t} + \frac{1}{2}s_{k,t-\Delta t} + \frac{1}{2}s_{k,t}) \sum_{j=1}^{T-t} B_t(t+j)Q_{k,t}^{SV}(t+j) + \frac{\Delta t}{4} (CDS_{k,t-\Delta t} - \frac{1}{2}s_{k,t-\Delta t}),$$

(2.19)

where $\Delta CDS_{k,t} = CDS_{k,t} - CDS_{k,t-\Delta t}$, $Q_{k,t}^{SV}(t+j)$ is the risk-neutral survival probability up to time $t+j$ and $B_t(t+j)$ is the price of a risk-free zero-coupon bond maturing at $t+j$. We also include the 'accrued' spread in (2.19), although this term is very small for our weekly return frequency. Time is measured in quarters (the payment frequency) and $T$ is the final payment date. Since we initiated the contract at zero cost, (2.19) directly defines an excess return. Equation (2.19) also shows that proportional transaction costs are equal to

$$c_{k,t} = \frac{1}{4} \left( \frac{1}{2}s_{k,t-\Delta t} + \frac{1}{2}s_{k,t} \right) \sum_{j=1}^{T-t} B_t(t+j)Q_{k,t}^{SV}(t+j) + \frac{\Delta t}{4} \frac{1}{2}s_{k,t-\Delta t},$$

(2.20)

We thus obtain transaction costs by multiplying the bid-ask spreads with a 'duration' factor. This shows that the CDS bid-ask spread (scaled by duration) is already a proportional transaction cost and that it is not necessary to divide this bid-ask spread by the CDS spread to obtain proportional costs: for any notional amount $N$, the dollar payoff equals $N(R_{k,t} - c_{k,t})$. Note that part of these costs

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11 Of course, if default occurs between $t-\Delta t$ and $t$, the excess return on the CDS is equal to (minus) the loss given default. However, if we assume that these individual jumps-to-default are not priced, we can ignore these cases for estimating portfolio betas.

12 This method is very close to the one used by Longstaff, Pan, Singleton and Pedersen (2009), who discount each cash flow with the risk-free rate plus CDS spread, whereas we discount with the risk-free rate and multiply with the risk neutral survival probabilities. Appendix 2.9.5 provides details on the risk-free rate data and the construction of the risk-neutral default probabilities.

13 To see this in a numerical example, consider a CDS with bid CDS spread of 50 basis points and ask CDS spread of 60 basis points. The bid-ask spread is then equal to 10 basis points. A round-trip CDS transaction would then cost 10 basis points (scaled by the duration) times the
(due to \( s_{k,t-\Delta t} \)) are known at \( t - \Delta t \). To assess liquidity risk we therefore only include the part of \( c_{k,t} \) that is not known at \( t - \Delta t \), while the full costs \( c_{k,t} \) are included for the expected liquidity calculation.

### 2.4.4 Non-traded credit risk factor and non-hedge asset

For our empirical analysis, we also need to measure the return on the non-traded credit risk factor \( r_n \). As mentioned above, we think of \( r_n \) as the return on bank loans and illiquid corporate bonds. Since these returns are, by their very nature, not observable, we construct a proxy for this systematic credit risk using returns on all commonly available credit instruments: corporate bond indices and CDS portfolios. Weekly bond index holding returns are obtained for the intermediate-maturity Lehman Brothers U.S. corporate bond indices per credit rating. The average maturity of all indices is approximately five years. Five-year benchmark treasury returns from Datastream are subtracted from the corporate bond returns to construct excess returns.

In order to capture the systematic credit risk factor in these bond indices and CDS portfolios, we perform a principal component analysis (PCA). We take the excess holding returns of all our CDS portfolios and the excess holding returns on Lehman corporate bond indices for the ratings AAA up to CCC and calculate the first principal component in these returns based on their correlation matrix. Since all these instruments have exposure to systematic credit risk, we would expect the first principal component to be a good measure for systematic credit risk. The loadings on this first component are all close to each other. This first component explains 52% of the variation. As a robustness check we use the return on the investment-grade CDX index as a proxy for the non-traded credit risk factor.

For the non-hedge asset we use returns on the S&P 500 equity index. We do not include transaction costs for this index since S&P futures can be traded with costs that are negligible for our purpose.\(^{14}\)

### 2.4.5 Time-series model of liquidity

The betas in the asset pricing model are defined as the ratio of conditional covariances and variances, i.e. the (co)variances of the unpredictable shocks (innovations) in returns and costs. As in Acharya and Pedersen (2005), we assume that returns

have no serial correlation and correct for persistence in the liquidity level by taking the residuals of an autoregressive model as the liquidity innovations. We incorporate the lagged portfolio CDS spread as explanatory variable in the time-series model for liquidity, because, as shown in Figure 2.2, it seems that bid-ask spreads are higher when CDS spreads increase. We thus estimate for each portfolio the regression

\[ c_{k,t} = a + b_1 c_{k,t-1} + b_2 CDS_{k,t-1} + \varepsilon_{k,t} \]  

(2.21)

where \( c_{k,t} \) denotes the portfolio-level transaction costs (bid-ask spread times 'duration' factor). Empirically, we indeed find a positive coefficient on the lagged CDS spread equal to 0.047 with t-statistic 4.5 (both averaged across all portfolios), showing that it is important to correct for this level-dependency. The coefficient on the lagged transaction cost \( b_1 \) equals 0.74 with t-statistic of 15.5 (again averaged across portfolios).

### 2.4.6 Expected CDS returns

To calculate the expected excess return \( R_{k,t,T} \) on a CDS portfolio at time \( t \), we calculate the expectation under the real world measure of all cash flows resulting from the CDS contract when held till maturity \( T \), discounted at the risk-free rate,

\[ E_t(R_{k,t,T}) = \frac{1}{4} CDS_{k,t} \sum_{j=1}^{T-t} B_t(t+j)P_{SV}^{SV}(t+j) - \]

(2.22)

\[ L \sum_{j=1}^{T-t} B_t(t+j)P_{SV}^{SV}(t+j-1)P_{k,t}^{def|SV}(t+j), \]

where \( L \) is the expected loss rate in case of default, set at the market-standard rate of 60\%, \( P_{SV}^{SV}(t+j) \) is the real-world survival probability up to time \( t+j \) and \( P_{k,t}^{def|SV}(t+j) \) is the probability of a default in period \( t+j \) conditional on survival up to time \( t+j-1 \). Constructing expected excess returns in this way rather than averaging realized excess returns allows us to achieve more accurate estimates and lower standard errors for risk premia. Since we use the expected return to maturity for this calculation, the underlying assumption we make here is that the term structure of expected CDS returns is flat.

Real-world default probability estimates, needed to construct these expected excess returns, are obtained from Moody’s-KMV EDF database. We have data on 1-year and 5-year EDFs, which we use to construct 1-year and 5-year default probabilities, and we interpolate linearly to obtain the intermediate default probabilities.
We prefer using Moody’s-KMV EDFs over rating-implied default probabilities because especially in the last two years of the sample (2007 and 2008) we observe a strong increase in the EDFs. It is not obvious how to adjust for these new market circumstances when using rating-based historical default frequencies.

We construct these default probabilities at the portfolio level, by taking weighted averages of all individual default probabilities, where the weight is the number of quotes per issuer relative to the total number of quotes in the portfolio.

We construct the expected return in (2.22) over a five-year holding period for each week in the sample. An estimate of the unconditional expected excess return is then obtained by averaging these weekly expected returns over all weeks in the sample. These unconditional expected returns are then used in the estimation of the asset pricing model.

2.4.7 Alternative CDS liquidity measures

In the benchmark analysis we use the bid-ask spread as liquidity measure. As a robustness check, we use two alternative liquidity measures. Our data set consists of bid and ask quotes. Due to the absence of volume data, we can not construct Amihud’s (2002) ILLIQ measure of price impact directly, but we can create proxies for price impact. Our first alternative measure (ILLIQ-1) divides the absolute value of the weekly portfolio CDS return by the number of quotes observed for this portfolio in the same week. If the number of quotes is correlated with the actual trading volume, this measure captures price impact of trade in a similar way as Amihud’s ILLIQ. Second, we construct a similar measure at the individual CDS level. We take the sum of the absolute values of all observed changes in the CDS spread in a given week, and divide this by the number of quotes observed for this CDS contract in that week. This measure is then aggregated to the portfolio level (ILLIQ-2). The advantage of this latter measure is that it takes the average of individual CDS data, which reduces the noise in the portfolio-level liquidity estimate. On the other hand, individual CDS spread changes are more volatile than portfolio-level CDS returns, which increases the noise in the ILLIQ-2 measure. Since these liquidity measures should capture transaction costs on CDS contracts, we normalize these such that they have the same overall mean and variance as the bid-ask spreads. These two alternative liquidity measures are similar to Tang and Yan’s (2007) ‘volatility-to-volume’ ratio. They divide the volatility of CDS spreads by the total number of quotes and trades observed in each month.
CHAPTER 2. DERIVATIVE PRICING WITH LIQUIDITY RISK

2.5 Empirical Results

This section presents the results of estimating the model on CDS returns. We first discuss the estimated betas, expected returns and transaction costs that go into the asset pricing model. Then we present the benchmark estimates of the structural asset pricing model. We provide a range of robustness checks and present two extensions of the empirical model by including bond liquidity and hedging pressure variables.

2.5.1 Betas, costs and expected returns

For the sake of brevity, we show expected CDS returns, costs, and betas only for the portfolios sequentially sorted on credit ratings and bid-ask spreads in Tables 2.2 and 2.3. Expected returns are reported for a quarterly holding period and costs are equal to the bid-ask spread times the duration factor (see (2.20)). Table 2.2 shows that bid-ask spreads are typically higher for lower-rated firms. We also see that expected returns are higher for lower ratings, as expected. In addition, for each rating category, expected returns are always monotonically increasing with bid-ask spread. Averaged across ratings, the high-liquidity portfolios have an expected return of 0.14% per quarter versus 0.29% per quarter for the low-liquidity portfolios. This provides some first informal evidence that illiquidity may affect expected returns in a positive way. This is further illustrated in Figure 2.3, where we plot expected return pricing errors for a two-factor model with equity and credit risk only (and no liquidity variables) against the expected liquidity per portfolio for all 100 portfolios used in our analysis. The scatter plot shows that a model without liquidity variables generates substantial pricing errors between -0.3% and 0.3% per quarter. Most importantly, these pricing errors exhibit a positive relation with expected liquidity, in line with the strongly positive estimate for \( \zeta_0 \) reported below.

Table 2.2 also gives t-stats for the expected CDS returns using Newey-West with 24 lags. These are generally high despite our relatively short sample period, which is due to our 'forward-looking' way of backing out expected returns from CDS spreads. Finally, Table 2.2 reports estimated values for \( \text{Corr}(\hat{r}_h, \hat{c}_h) \). We see that both the diagonal and off-diagonal elements of this matrix vary across portfolios, but in some cases differ substantially from zero and are typically negative. Hence, for our empirical analysis we do not impose that \( \text{Cov}(\hat{r}_h, \hat{c}_h) = 0 \) and rely on the general case of Theorem I.

Turning to the betas in Table 2.3, CDS returns have a positive exposure to
equity market returns ($\beta_{r, r_b} > 0$), which is higher for CDS contracts with lower ratings and higher bid-ask spreads. Since we defined a 'long' return as the return for a credit protection seller a positive sign for $\beta_{r, r_b}$ can be expected: when equity prices increase, CDS spreads go down. As expected, all betas are below one, since especially high-rated bonds have small market risk (see also Elton, Gruber, Agrawal and Mann (2001)). These betas are always significant for the reported sort. Across all sorts, these betas are significant at the 5% level for 97 out of 100 portfolios. We observe a similar pattern for the exposure to the proxy for systematic non-traded credit risk ($\beta_{\hat{r}, r_n} > 0$), significant at the 5% level for all portfolios. As expected, the return on selling credit protection is positively related to our proxy for systematic credit risk.

The exposure of transaction costs to equity market returns and systematic credit risk is typically negative, suggesting that bid-ask spreads increase in bad times. This is mainly the case for portfolios with low ratings and high bid-ask spreads, and significant at the 5% level for 30 out of 100 portfolios for equity index returns ($\beta_{c, r_b}$) and for 44 out of 100 portfolios for the credit risk factor ($\beta_{\hat{c}, r_n}$).

### 2.5.2 Benchmark results

We estimate the asset pricing model for a benchmark setting where we use (i) the bid-ask spread to construct expected liquidity and liquidity risk variables, (ii) a sample period from April 2004 until June 2008 and (iii) assume a holding period of one quarter. The choice of holding period, which is required as there are no data on the typical turnover frequency in CDS markets, implies that we insert expected returns over a quarterly horizon in (2.16). In the benchmark setup we exclude the two final quarters of 2008. Because of the events around the default of Lehman Brothers in September 2008, these two quarters exhibit different behavior compared to the rest of the sample (see Figure 2.2). Below, we perform robustness checks on all choices for this benchmark setup. To assess the fit of the model, we define a cross-sectional $R^2$ as one minus the ratio of the variance of pricing errors divided by the variance of the expected CDS returns, where we weight both pricing errors and returns with the optimal GMM weights. Hence, this $R^2$ measures the improvement over a model with constant expected CDS returns across portfolios.

Table 2.4 presents the parameter estimates and $R^2$ obtained by applying GMM to the moment conditions in (2.16), for the case without heterogenous pricing of liquidity ($\zeta_1 = \kappa_1 = 0$) and with an intercept $\alpha_0$. The key result is that we find a strong positive effect of expected liquidity on expected CDS returns, both
statistically and economically significant. For the benchmark case with quarterly holding periods, the coefficient \( \zeta_0 \) equals 0.69 with a t-stat of 13.5. The high statistical precision illustrates the usefulness of constructing forward-looking expected returns from CDS spreads. Note that \( |\zeta_0| < 1 \), in line with the theory (equation (2.15)). The coefficient on liquidity risk \( \kappa_0 \) is also significantly positive, again in line with theoretical predictions. Turning to the premia on equity market risk \((\psi)\) and credit risk \((\lambda)\), we find positive and significant estimates for both risk factors. The exposures to these two risk factor are however strongly correlated cross-sectionally. This becomes evident when we exclude the credit risk premium (specification (2) in Table 2.4). This leads to a higher estimate for the equity risk premium (increasing from 0.22% to 0.51% per quarter), but the decrease in \( R^2 \) is minimal. Overall, the model explains about 90% of the cross-sectional variation in expected returns. Table IV - specification (3) shows that, if we exclude liquidity and liquidity risk from the model, this \( R^2 \) goes down to 20%, which shows that the liquidity effects are not simply picking up the effects of equity and credit risk premia.

The intercept is significantly negative at about −11 basis points per quarter. It turns out that without an intercept the model has problems fitting the expected returns on high-rated and liquid portfolios. These portfolios had very low CDS spreads during the period before the crisis, which is in line with anecdotal evidence that high-rated assets were overpriced in the pre-crisis period. A negative intercept helps to fit such portfolios better.

To assess the economic significance of these estimates, we decompose the total expected CDS portfolio return into parts explained by market risk premia, expected liquidity, liquidity risk and a residual. We use (2.16) to explicitly solve for the expected CDS return. Figure 2.4 presents this decomposition for the rating / bid-ask spread sort. It shows that both the expected liquidity effect and market risk effect increase steadily for lower ratings and less-liquid portfolios, while the effect of liquidity risk is only relevant for portfolios with lowest liquidity. Given that \( \beta_{ch, rn} \) is negative for these portfolios and \( \kappa_0 > 0 \), liquidity risk decreases the expected CDS return. This is line with our argument in Section 2.3.2.3.1. A negative \( \beta_{ch, rn} \) lowers the hedge demand of short investors (who care most about this covariance) and this decreases expected returns.

Taking the average across the 20 portfolios in Figure 2.4 (weighted by their optimal GMM weights), we find that equity and credit risk premia together account for 0.060% return per quarter, expected liquidity for 0.175%, and liquidity risk for -0.005%. This shows the large economic significance of expected liquidity.
2.5. EMPIRICAL RESULTS

2.5.3 Backing out structural parameters

With these parameter estimates, we can back out the relative weights of the two investor groups. We consider two cases. First, we use the benchmark setting presented in Theorem I. Using the estimates of specification (1) in Table 2.4 and the definition of $\zeta_0$ in (2.11), the estimates imply that the 'long' investors (taking on credit risk) represent 84.7% of the total wealth-weighted risk tolerance $\eta$, while the 'short' investors (hedgers) represent 15.3%. This allows us to interpret the strong positive coefficient on expected liquidity $\zeta_0$. Given that the 'long' investors are much more aggressive (either due to more wealth or lower risk aversion), they require a large compensation for expected liquidity, as explained in the simple example in Section 2.2.2.2.4.

Alternatively, we can relate the parameter estimates to the extension of the theory with long-term investors (Theorem II). For example, we can make the identifying assumption that investors that are short in all hedge assets (hedgers) are always long-term investors. This seems conceivable: commercial banks hedge credit risk using CDS contracts and may not need to trade afterwards. Mathematically, this implies that $\sum_{i: \delta_i = -1} w_i \gamma_i^{-1} = 0$, and from $\zeta_0/(1 + \mu_0) = 0.694$ it follows that the long-term hedgers represent 69.4% of the total wealth-weighted risk-tolerance (equal to $\eta(1 + \mu_0)$) and the short-term risk takers 30.6%.

In sum, the positive expected liquidity coefficient is consistent with two settings, one where protection sellers are more aggressive than hedgers, and an alternative setting where protection sellers have shorter horizons than hedgers.

An important assumption in these two explanations is the holding period of the short-term investor. The benchmark setting uses a quarterly holding period. From Theorem I it follows that if $Cov(\hat{r}_h, \hat{c}_h) = 0$ the empirical model can be translated to a different frequency simply by multiplying the coefficients with the holding period (relative to the quarterly period). Theorem I also shows that this is not exactly true in the general setting, but given that the correlations between $\hat{r}_h$ and $\hat{c}_h$ are only moderately different from zero, we do not expect large deviations from the simple multiplication effect. Table 2.4 reports results for monthly and annual holding periods, thus measuring expected returns over shorter or longer horizons. It confirms our intuition: the parameters estimates are approximately scaled by the holding period. This implies that the relative importance of the different premia remains the same irrespective of the holding period and that the empirical model has similar explanatory power across holding periods. The holding period does however affect the interpretation of the coefficients in terms of the theoretical
model. The theory constrains the coefficient on expected liquidity $\zeta_0$ to be below one. For the annual holding period, the estimate for $\zeta_0$ is larger than one. This implies that the empirical estimates are in line with the theoretical predictions as long as the holding period is not too long (roughly less than six months). If the actual holding period would be really one year, setting $\zeta_0$ at its maximum level of one (as opposed to the unrestricted estimate of 2.05) implies that the theoretical model explains about $1/2.05 \cdot 100\% \approx 49\%$ of the empirically observed effect of expected liquidity. It is also useful to note that the bid-ask spread probably does not capture all transaction costs of a CDS trade. For example, search costs are not explicitly incorporated. Also, our bid-ask spreads are for 'on-the-run' five-year CDS contracts, and unwinding a shorter-maturity CDS contract after some time may be more costly. This could partially explain the high coefficient on the bid-ask spread.

### 2.5.4 Robustness checks

Table 2.5 provides a range of robustness checks on the benchmark results of Table 2.4.

First, we use an alternative proxy for systematic credit risk: the returns on the investment-grade CDX index. This leads to similar effects for liquidity risk and expected liquidity, but a smaller and less significant estimate on the credit risk premium. The overall $R^2$ is essentially unchanged.

Second, we change the sample period. We add the two final quarters of 2008 and report estimates for the full sample period April 2004 to December 2008. This leads to an even larger coefficient on expected liquidity, but a somewhat smaller effect of liquidity risk (still significant). We also consider a pure crisis sample: July 2007 to December 2008 and again find very similar results for the liquidity variables. The cross-sectional $R^2$ is a bit lower when we include the crisis period, which is not surprising since there are some extreme movements in CDS spreads and bid-ask spreads in this period.

Third, instead of the bid-ask spread, we use the two alternative measures for liquidity discussed in Section 2.4.2.4.7, the proxies for price impact ILLIQ-1 and ILLIQ-2. The results show that we find similar liquidity effects for both alternative liquidity measures. Compared to the bid-ask spread, both alternative liquidity measures generate slightly lower values for the $R^2$.

Fourth, we use different time-series models for transaction costs. Our benchmark model has an autoregressive specification with the lagged transaction cost
and the lagged CDS spread as explanatory variables (denoted as ARX model in the table). As an alternative, we use pure autoregressive models with two or four lags but without the lagged CDS spread (denoted as (AR(2) and AR(4) models in the table). We find that all results are very similar to the benchmark case.

Fifth, we take out the intercept from the asset pricing model. It turns out that this is the only robustness check that does substantially affect the estimate for the expected liquidity premium, lowering it from 0.694 to 0.243. It remains strongly statistically significant with a t-stat of 7.2, and the economic effect equals 0.060% per quarter across the 20 rating/bid-ask-spread portfolios, compared to 0.175% in the benchmark case. The estimate for the equity risk premium becomes negative and statistically significant. This is because in the absence of a constant term, the model exploits the strong cross-sectional correlation between the equity and credit risk betas to mimic the constant term. The sum of the market and credit premia is positive for all portfolios however at an average of 0.076% per quarter across portfolios. Figure 2.5 shows the decomposition of expected CDS returns across rating/bid-ask spread portfolios for the specification without an intercept.

Finally, we incorporate $\beta^c_h \hat{c}^m = \frac{\text{Cov}(\hat{c}^h, \hat{c}^m)}{\text{Var}(\hat{c}^m)}$ as explanatory variable. According to the theoretical model, this ‘CDS-market’ liquidity risk should not carry a risk premium ($\phi = 0$ in (2.16)). We include this beta in the empirical model as a specification test, in particular because in positive-net-supply markets this exposure is expected to be priced (Acharya and Pedersen, 2005). To make sure that the exposure to $\beta^c_h \hat{c}^m$ captures systematic risk, we check whether there is commonality in liquidity across CDS portfolios by performing a principal component analysis to the portfolio liquidity innovations. We find that the first factor explains 16.6% of the liquidity variation, which gives some evidence for liquidity commonality. This complements existence evidence of liquidity commonality in other markets (see, for example, Chordia, Roll and Subrahmanyam (2000) for equity markets). Table 2.5 shows a positive estimate for the premium on $\beta^c_h \hat{c}^m$, but insignificant with a t-stat of 0.09. Hence, we do not find evidence that this form of liquidity risk is priced. In addition, the effect of expected liquidity hardly changes when including $\beta^c_h \hat{c}^m$.

In sum, throughout all specifications and robustness checks, we find very strong evidence that expected liquidity positively affects expected CDS returns. The effect of liquidity risk (exposure to $\beta^c_h r_n$) is small, but mostly statistically significant.

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15 We do not include the ‘total’ CDS-market beta $\frac{\text{Cov}(\hat{r}_h - \hat{c}_h, \hat{r}_m - \hat{c}_m)}{\text{Var}(\hat{r}_m - \hat{c}_m)}$ because the market-wide CDS return exhibits substantial correlation with the factor capturing systematic non-traded credit risk.
2.5.5 Bond liquidity and hedging pressure

We extend the empirical specification of expected CDS returns in two further ways. First, we include several proxies for bond market liquidity as a portfolio characteristic in the asset pricing model. Second, we include a variable capturing hedging pressure effects and allow this variable to interact with the liquidity effects.

Bond market liquidity may affect the pricing of CDS contracts due to an effect described in Bühler and Trapp (2009). CDS contracts often allow for physical delivery of the underlying bonds in case of default. If the delivered bonds are more liquid, their value will be higher, which lowers the effective loss in case of default and in turn should lead to a lower CDS spread and lower expected return (given a fixed loss rate). The maintained assumption is that the current bond liquidity level predicts the bond liquidity in case of a default event. We construct three corporate bond liquidity measures at the firm level: the total amount issued, total volume and turnover. Portfolio-level liquidity measures are then created by averaging these bond-liquidity measures across all firms underlying the CDS contracts. Including these portfolio-level bond-liquidity measures as portfolio characteristic, Table 2.6 shows that we find no evidence for a significant negative effect of bond liquidity on the expected CDS return. For bond turnover we actually find a positive and significant coefficient.

Our second extension concerns potential hedging pressure effects, motivated by the presence of idiosyncratic background risk (Theorem III). To proxy for hedging pressures, we use the amount of loans issued in the syndicated loan market. If large investors, for example commercial banks, are stuck with a large fraction of syndicated loans of a given firm, this could lead to increased hedging demands which push up the expected return on the CDS written on this firm, as shown formally in Theorem III.

Similar to the bond liquidity variables, we first construct the loan amount at the firm level, and then create a portfolio-level loan amount by averaging across all firms corresponding to the CDS contracts in the portfolio. This variable is then included as characteristic in the asset pricing model. Specification (7) in Table 2.6 shows that high hedging demand indeed leads to significantly higher expected returns. Hence, if a firm has taken on a large amount of syndicated loans over the sample period, we find higher CDS spreads and thus a higher expected CDS return.

It turns out that hedging demand is positively correlated with the bond liquidity measures. Firms with many syndicated loans outstanding typically also have
large and liquid bond issues trading in financial markets. To disentangle bond liquidity and hedging effects, we therefore simultaneously include the loan amount and bond liquidity measures in the asset pricing model in specifications (4)-(6) of Table 2.6. Using amount issued or volume as bond liquidity measures, we still find a positive and significant coefficient on the hedging pressure variable, in line with our intuition and Theorem III. When controlling for bond turnover, the hedging effect is positive but insignificant. Specifications (4)-(6) also show that the bond liquidity effects become more negative once we control for hedging effects, but we still find mixed evidence for the Bühler-Trapp effect of bond liquidity on CDS returns.

Finally, we incorporate interaction effects between hedging demand and liquidity as derived theoretically in Theorem III, by including nonzero $\zeta_1$ and $\kappa_1$. As discussed in subsection 2.2.2.2.6, the net effect of hedging demand on the liquidity premia can be positive or negative. In terms of the theory, the long/short group with position signs given by the matrix $D$, could be either long or short when hedging demand for those assets is high.

Empirically, we proceed as follows. We again use the amount of syndicated loans outstanding as proxy for hedging demand. Specifically, for each CDS portfolio we set the corresponding diagonal element of $D$ equal to 1 if the loans outstanding variable is above the overall median level, and equal to $-1$ otherwise. Specifications (8) and (9) in Table 2.6 show that high hedging demand increases the coefficient on expected liquidity and decreases the coefficient on liquidity risk. Theoretically, this is in line with the case that high hedging demand generates more investors with long positions, since this generates a positive coefficient for $\zeta_1$ and negative for $\kappa_1$. As discussed in Section 2.3.2.3.1, high hedging demand pushes up the expected return, which may induce more investors to hold long positions in the hedge asset (especially if the hedging demand is concentrated within a few investors). Indeed, we found above that loans outstanding had a direct positive effect on expected returns. The liquidity-hedging interaction effect is only significant for liquidity risk ($\kappa_1$) and small and insignificant for expected liquidity ($\zeta_1$), irrespective of whether we control for the direct hedging effect on expected returns. The increase in $R^2$ is also small, showing that the interaction effect of hedging pressure and liquidity is economically modest.

Finally, it is useful to note that for both extensions the effects of expected returns might not be in line with the theoretical predictions.

\footnote{Obviously, specifying $D$ in the opposite way (1 in case of low hedging demand and $-1$ in case of high demand) would generate exactly the opposite results: a negative $\zeta_1$ and positive $\kappa_1$, which would thus not be in line with the theoretical predictions.}
liquidity and liquidity risk are essentially unchanged. Both effects remain strongly significant.

2.6 Conclusion

We develop a theoretical asset pricing model with liquidity risk, incorporating (i) positive-net-supply assets and derivatives, (ii) heterogeneous investors, (iii) optimal short-selling, and (iv) background risk. Our model builds on Acharya and Pedersen (2005) whose model has investors with long positions in positive net-supply assets. Our first main theoretical result is that the sign of liquidity effects depends on investor heterogeneity in wealth, risk aversion, non-traded risk and horizon. Second, we derive that some liquidity risk premia should be equal to zero if the assets are derivatives in zero net supply.

We test this model for one of the key derivative markets: the market for credit default swaps. The nonlinear asset pricing model is estimated by applying GMM to a sample of CDS bid and ask quotes over a sample period from 2004 to 2008. We find significant and robust evidence that liquidity affects CDS prices, in a way that is generally consistent with the predictions of the theoretical model. Our results thus suggest that CDS spreads cannot be used as frictionless measures of default risk, as is often done in the recent literature.

On the theoretical side, several extensions of this chapter are possible. In our model, investors trade in derivatives due to heterogenous exposure to non-traded risk. Obviously, there may be other reasons for why people trade derivatives, such as heterogeneity in beliefs or intertemporal hedging demands that would arise in a multi-period setting. Further, explicitly modeling the preferences and constraints of a market maker (who earns the bid-ask spread) may also generate interesting effects. On the empirical side, our model can be applied to other markets, either derivative markets or positive-net-supply markets with considerable short-selling activity.
### Table 2.1: Buyers and sellers in the CDS market

This table shows the fraction of CDS contracts held by various parties. Source: 2006 Survey of the British Bankers Association, reported in Mengle (2007).

<table>
<thead>
<tr>
<th></th>
<th>Buy protection</th>
<th>Sell protection</th>
<th>Net position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks - Loan portfolio</td>
<td>20%</td>
<td>9%</td>
<td>11%</td>
</tr>
<tr>
<td>Banks - Trading activity</td>
<td>39%</td>
<td>35%</td>
<td>4%</td>
</tr>
<tr>
<td>Insurers</td>
<td>6%</td>
<td>17%</td>
<td>-11%</td>
</tr>
<tr>
<td>Funds</td>
<td>32%</td>
<td>39%</td>
<td>-7%</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>28%</td>
<td>32%</td>
<td>-4%</td>
</tr>
<tr>
<td>Pension funds</td>
<td>2%</td>
<td>4%</td>
<td>-2%</td>
</tr>
<tr>
<td>Mutual Funds</td>
<td>2%</td>
<td>3%</td>
<td>-1%</td>
</tr>
<tr>
<td>Corporates &amp; other</td>
<td>3%</td>
<td>2%</td>
<td>1%</td>
</tr>
</tbody>
</table>
Table 2.2: Descriptive statistics CDS returns and transaction costs

The table presents summary statistics for the CDS returns and transaction costs for the sequential sort on credit rating (five categories) and bid-ask spread (four quartiles). Panel A presents expected CDS returns for a quarterly horizon in percentages, constructed by averaging equation (2.22) over all weeks in our sample. Panel A also contains average proportional transaction costs per portfolio, defined as the bid-ask spread times a duration factor (equation (2.20)). In square brackets t-stats of the weekly averages are given, calculated using Newey-West with 24 lags. Panel B contains the correlation matrix of the time series of CDS returns and transaction cost innovations, \( Corr(\hat{r}_h, \hat{c}_h) \), presenting both the ‘diagonal’ elements and average of each row of this 20 by 20 matrix. Sample period is April 2004 until June 2008.

Panel A: Descriptive statistics expected transaction costs and expected CDS returns

<table>
<thead>
<tr>
<th>Rating/Bid-ask spread</th>
<th>Expected CDS return (%/quarter)</th>
<th>Expected transaction cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Q2</td>
</tr>
<tr>
<td>Aaa to Aa</td>
<td>-0.038</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>[-0.88]</td>
<td>[2.17]</td>
</tr>
<tr>
<td>A</td>
<td>0.047</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>[4.35]</td>
<td>[4.28]</td>
</tr>
<tr>
<td>Baa</td>
<td>0.077</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>[6.52]</td>
<td>[4.91]</td>
</tr>
<tr>
<td>Ba</td>
<td>0.174</td>
<td>0.241</td>
</tr>
<tr>
<td></td>
<td>[5.49]</td>
<td>[5.21]</td>
</tr>
<tr>
<td>B to Caa</td>
<td>0.434</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td>[5.24]</td>
<td>[7.45]</td>
</tr>
</tbody>
</table>

Panel B: Time-series correlations transaction cost innovations and CDS returns

<table>
<thead>
<tr>
<th>Rating/Bid-ask spread</th>
<th>Diagonal</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Q2</td>
</tr>
<tr>
<td>Aaa to Aa</td>
<td>0.121</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>-0.187</td>
<td>-0.083</td>
</tr>
<tr>
<td>A</td>
<td>-0.102</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>-0.014</td>
<td>-0.075</td>
</tr>
<tr>
<td>B to Caa</td>
<td>-0.012</td>
<td>-0.023</td>
</tr>
</tbody>
</table>
Table 2.3: Beta estimates

The table presents beta estimates for the sequential sort on credit rating (five categories) and bid-ask spread (four quartiles). The upper left panel contains the beta of CDS portfolio returns with respect to S&P 500 index returns ($\beta_{r_h,r_b}$). The upper right panel contains the beta of transaction cost innovations (residuals of the AR-X model in equation (2.21)) with respect to the S&P 500 index returns ($\beta_{c_h,r_b}$). The lower left panel contains the beta of CDS returns (orthogonalized for S&P 500 index returns) with respect to the PCA factor capturing systematic credit risk ($\hat{\beta}_{r_h,r_n}$). The lower right panel contains the beta of transaction cost innovations (orthogonalized for S&P 500 index returns) with respect to the PCA systematic credit risk factor ($\hat{\beta}_{c_h,r_n}$). Sample period is April 2004 until June 2008. t-statistics are given in square brackets.

<table>
<thead>
<tr>
<th>Rating/Bid-ask spread</th>
<th>Low</th>
<th>Q2</th>
<th>Q3</th>
<th>High</th>
<th>$\beta_{r_h,r_b}$</th>
<th>$\beta_{c_h,r_b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa to Aa</td>
<td>0.041</td>
<td>0.048</td>
<td>0.054</td>
<td>0.190</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>[8.52]</td>
<td>[7.34]</td>
<td>[6.85]</td>
<td>[5.27]</td>
<td>[-1.38]</td>
<td>[-2.2]</td>
</tr>
<tr>
<td>A</td>
<td>0.046</td>
<td>0.045</td>
<td>0.061</td>
<td>0.134</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>Baa</td>
<td>0.062</td>
<td>0.069</td>
<td>0.077</td>
<td>0.12</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>[8.64]</td>
<td>[9.57]</td>
<td>[8.48]</td>
<td>[8.82]</td>
<td>[-1.69]</td>
<td>[-1.12]</td>
</tr>
<tr>
<td>Ba</td>
<td>0.113</td>
<td>0.159</td>
<td>0.198</td>
<td>0.245</td>
<td>0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>[7.17]</td>
<td>[8.76]</td>
<td>[7.32]</td>
<td>[7.79]</td>
<td>[-0.08]</td>
<td>[-1.08]</td>
</tr>
<tr>
<td>B to Caa</td>
<td>0.285</td>
<td>0.271</td>
<td>0.218</td>
<td>0.396</td>
<td>-0.003</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>[7.72]</td>
<td>[5.63]</td>
<td>[4.62]</td>
<td>[5.97]</td>
<td>[-1.00]</td>
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<table>
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<tr>
<th>Rating/Bid-ask spread</th>
<th>Low</th>
<th>Q2</th>
<th>Q3</th>
<th>High</th>
<th>$\hat{\beta}_{r_h,r_n}$</th>
<th>$\hat{\beta}_{c_h,r_n}$</th>
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<td>Aaa to Aa</td>
<td>0.186</td>
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<td>[9.04]</td>
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<tr>
<td>A</td>
<td>0.267</td>
<td>0.244</td>
<td>0.349</td>
<td>0.821</td>
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<td>0.000</td>
</tr>
<tr>
<td>Baa</td>
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<td>0.473</td>
<td>0.648</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
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<td>[11.75]</td>
<td>[16.54]</td>
<td>[14.63]</td>
<td>[12.36]</td>
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<td>[-0.89]</td>
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<tr>
<td>Ba</td>
<td>0.762</td>
<td>0.969</td>
<td>1.095</td>
<td>1.000</td>
<td>-0.004</td>
<td>-0.008</td>
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<td>B to Caa</td>
<td>1.713</td>
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<td>1.838</td>
<td>2.482</td>
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</table>
Table 2.4: Benchmark GMM estimation results

The theoretical asset pricing model in Theorem I is estimated by applying GMM to the moment condition (2.16) for 100 CDS portfolios (sorted first on credit rating, and then either on leverage, total debt, total syndicated loan amount, bid-ask spread, or quote frequency). Moment conditions are weighted by the diagonal of the optimal GMM weighting matrix. The model also includes an intercept. t-stats (in square brackets) are calculated in GMM fashion using Newey-West with 24 lags, and incorporating that expected returns, costs and betas are estimated, following Shanken (1992), see Appendix 2.9.3. The table also reports a cross-sectional $R^2$, where pricing errors are weighted using the GMM weights. Finally, the table reports results for a benchmark holding period of one quarter, and results for holding periods of one month and one year. Sample period is April 2004 until June 2008.

<table>
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<tr>
<th>Holding period</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.106</td>
<td>-0.113</td>
<td>-0.012</td>
<td>-0.333</td>
<td>-0.040</td>
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<tr>
<td>$(\alpha_0)$</td>
<td>[-11.59]</td>
<td>[-13.18]</td>
<td>[-0.39]</td>
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<tr>
<td>Equity risk</td>
<td>0.224</td>
<td>0.512</td>
<td>-0.032</td>
<td>0.660</td>
<td>0.073</td>
</tr>
<tr>
<td>$(\psi)$</td>
<td>[4.46]</td>
<td>[4.38]</td>
<td>[-0.82]</td>
<td>[3.53]</td>
<td>[3.16]</td>
</tr>
<tr>
<td>Credit risk</td>
<td>0.066</td>
<td>0.226</td>
<td></td>
<td>0.462</td>
<td>0.015</td>
</tr>
<tr>
<td>$(\tilde{\lambda})$</td>
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<td>[8.58]</td>
<td></td>
<td>[4.69]</td>
<td>[1.66]</td>
</tr>
<tr>
<td>Expected liquidity</td>
<td>0.694</td>
<td>0.763</td>
<td></td>
<td>2.050</td>
<td>0.270</td>
</tr>
<tr>
<td>$(\zeta_0)$</td>
<td>[13.51]</td>
<td>[14.04]</td>
<td></td>
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<td>[14.42]</td>
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<tr>
<td>Liquidity risk</td>
<td>1.819</td>
<td>1.72</td>
<td></td>
<td>5.978</td>
<td>0.753</td>
</tr>
<tr>
<td>$(\tilde{\kappa}_0)$</td>
<td>[3.25]</td>
<td>[2.80]</td>
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<td>[3.40]</td>
<td>[4.13]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.899</td>
<td>0.892</td>
<td>0.203</td>
<td>0.880</td>
<td>0.652</td>
</tr>
</tbody>
</table>
Table 2.5: Robustness and specification tests

The table reports estimates of the asset pricing model in Theorem 1, obtained by applying GMM to the moment condition (2.16) for 100 CDS portfolios (sorted first on credit rating, and then either on leverage, total debt, total syndicated loan amount, bid-ask spread, or quote frequency). The setting described in Table 2.4 (included as specification (1) for convenience) is changed in the following ways. First, the PCA factor is replaced by returns on the IG CDX index. Second, results are presented for two alternative sample periods: April 2004 to December 2008 ('full') and July 2007 to December 2008 ('crisis'). Third, two alternative measures for liquidity are used instead of the bid-ask spread (see Section 2.4.2.4.7). Fourth, the AR-X time series model for transaction costs is replaced by either an AR(2) model or an AR(4) model. Fifth, the intercept is excluded from the model. Finally, a premium on the cost-cost beta ($\beta_{ch,cm}$) for the CDS market is included.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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<th>(10)</th>
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<tbody>
<tr>
<td>Non-traded risk</td>
<td>PCA</td>
<td>CDX</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
<td>PCA</td>
</tr>
<tr>
<td>Sample period</td>
<td>base</td>
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<td>full</td>
<td>crisis</td>
<td>base</td>
<td>base</td>
<td>base</td>
<td>base</td>
<td>base</td>
<td>base</td>
</tr>
<tr>
<td>Liquidity measure</td>
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<td>BAS</td>
<td>BAS</td>
<td>BAS</td>
<td>ILLIQ-1</td>
<td>ILLIQ-2</td>
<td>BAS</td>
<td>BAS</td>
<td>BAS</td>
<td>BAS</td>
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<tr>
<td>Cost innovations</td>
<td>ARX</td>
<td>ARX</td>
<td>ARX</td>
<td>ARX</td>
<td>ARX</td>
<td>ARX</td>
<td>AR(2)</td>
<td>AR(4)</td>
<td>ARX</td>
<td>ARX</td>
</tr>
<tr>
<td>Intercept $(\alpha_0)$</td>
<td>-0.106</td>
<td>-0.115</td>
<td>-0.114</td>
<td>-0.135</td>
<td>-0.290</td>
<td>-0.154</td>
<td>-0.108</td>
<td>-0.108</td>
<td>-0.108</td>
<td>-0.106</td>
</tr>
<tr>
<td>Equity risk $(\psi)$</td>
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<td>0.428</td>
<td>0.104</td>
<td>0.202</td>
<td>0.527</td>
<td>0.183</td>
<td>0.279</td>
<td>0.302</td>
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<tr>
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<td>[3.76]</td>
<td>[5.15]</td>
<td>[1.46]</td>
<td>[4.51]</td>
<td>[4.75]</td>
<td>[-4.18]</td>
<td>[3.64]</td>
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<tr>
<td>Credit risk $(\lambda)$</td>
<td>0.066</td>
<td>0.023</td>
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<td>0.093</td>
<td>0.020</td>
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<td>0.054</td>
<td>0.162</td>
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</tr>
<tr>
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<td>[1.02]</td>
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<td>[2.05]</td>
<td>[1.90]</td>
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<td>Expected liquidity $(\zeta_0)$</td>
<td>0.694</td>
<td>0.717</td>
<td>0.900</td>
<td>0.923</td>
<td>0.990</td>
<td>0.925</td>
<td>0.707</td>
<td>0.710</td>
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<td>[7.23]</td>
<td>[11.42]</td>
</tr>
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<td>Liquidity risk $(\kappa_0)$</td>
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<td>1.087</td>
<td>0.279</td>
<td>0.936</td>
<td>0.880</td>
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<td>1.394</td>
<td>1.400</td>
<td>1.427</td>
<td>1.844</td>
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<td>[2.79]</td>
<td>[1.38]</td>
<td>[4.19]</td>
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<td>[2.61]</td>
<td>[2.63]</td>
<td>[5.75]</td>
<td>[4.67]</td>
</tr>
<tr>
<td>Liquidity risk $(\phi_{ch,cm})$</td>
<td>0.899</td>
<td>0.890</td>
<td>0.770</td>
<td>0.775</td>
<td>0.721</td>
<td>0.868</td>
<td>0.878</td>
<td>0.879</td>
<td>0.577</td>
<td>0.899</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.899</td>
<td>0.890</td>
<td>0.770</td>
<td>0.775</td>
<td>0.721</td>
<td>0.868</td>
<td>0.878</td>
<td>0.879</td>
<td>0.577</td>
<td>0.899</td>
</tr>
</tbody>
</table>
Table 2.6: Bond market liquidity and hedging demand effects

The left panel reports estimates of the theoretical asset pricing model of Theorem I, obtained by applying GMM to the moment condition (2.16) for 100 CDS portfolios (sorted first on credit rating, and then either on leverage, total debt, total syndicated loan amount, bid-ask spread, or quote frequency) and adding corporate bond liquidity as a portfolio characteristic. Corporate bond liquidity is measured using either the log of total dollar amount issued for corporate bonds, the log of dollar corporate bond trading volume, or log of corporate bond turnover (volume divided by amount issued). These measures are constructed using TRACE data at the firm-level (for those firms underlying CDS contracts), and then aggregated to the portfolio level. In the middle panel another portfolio characteristic is added to this model, the log of the dollar amount of syndicated loans outstanding (‘loan amount’). This variable is constructed again at the firm level using Dealscan data, and then aggregated to the portfolio level. Finally, the right panel presents estimates for the model extension in Theorem III with interaction between hedging demand and liquidity premia ($\zeta_1$ and $\kappa_1$). Specifically, the diagonal elements in matrix $D$ in (2.16) are set to 1 for portfolios with above-median loan amount, and −1 for portfolio with below-median loan amount.

<table>
<thead>
<tr>
<th>Bond liquidity</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
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<tbody>
<tr>
<td>Interception ($\alpha_0$)</td>
<td>-0.107</td>
<td>-0.112</td>
<td>-0.109</td>
<td>-0.104</td>
<td>-0.109</td>
<td>-0.111</td>
<td>-0.112</td>
<td>-0.107</td>
<td>-0.110</td>
</tr>
<tr>
<td>Equity risk ($\psi$)</td>
<td>0.225</td>
<td>0.231</td>
<td>0.135</td>
<td>0.198</td>
<td>0.211</td>
<td>0.143</td>
<td>0.217</td>
<td>0.223</td>
<td>0.219</td>
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<tr>
<td>($\psi$)</td>
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<td>[2.60]</td>
<td>[4.23]</td>
<td>[4.27]</td>
<td>[4.23]</td>
</tr>
<tr>
<td>Credit risk ($\bar{\lambda}$)</td>
<td>0.065</td>
<td>0.059</td>
<td>0.070</td>
<td>0.063</td>
<td>0.0580</td>
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</tr>
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<td>($\bar{\lambda}$)</td>
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<td>[2.65]</td>
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<td>[2.46]</td>
<td>[2.6]</td>
<td>[2.18]</td>
<td>[2.26]</td>
<td>[2.17]</td>
</tr>
<tr>
<td>Expected liquidity ($\zeta_0$)</td>
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<td>0.732</td>
<td>0.726</td>
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<td>0.743</td>
<td>0.745</td>
<td>0.725</td>
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<td>($\zeta_0$)</td>
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<td>[12.90]</td>
<td>[12.54]</td>
<td>[13.76]</td>
<td>[13.59]</td>
<td>[13.9]</td>
<td>[13.5]</td>
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<td>Interaction ($\zeta_1$)</td>
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<td>0.005</td>
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<tr>
<td>($\zeta_1$)</td>
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<td>[0.005]</td>
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<tr>
<td>Liquidity risk ($\kappa_0$)</td>
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<td>1.797</td>
<td>1.622</td>
<td>1.814</td>
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<td>[3.23]</td>
<td>[2.77]</td>
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<tr>
<td>($\kappa_1$)</td>
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<td>[-3.07]</td>
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<td>Loan amount ($\alpha_1$)</td>
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<td>0.012</td>
<td>0.010</td>
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<tr>
<td>($\alpha_1$)</td>
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<td>[3.97]</td>
<td>[1.45]</td>
<td>[3.24]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond liquidity ($\alpha_2 \times 10$)</td>
<td>0.004</td>
<td>0.039</td>
<td>0.566</td>
<td>-0.068</td>
<td>-0.023</td>
<td>0.050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\alpha_2 \times 10$)</td>
<td>[0.09]</td>
<td>[1.07]</td>
<td>[4.74]</td>
<td>[-1.54]</td>
<td>[-0.61]</td>
<td>[4.26]</td>
<td></td>
<td></td>
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<tr>
<td>$R^2$</td>
<td>0.899</td>
<td>0.903</td>
<td>0.916</td>
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<td>0.909</td>
<td>0.918</td>
<td>0.909</td>
<td>0.910</td>
<td>0.912</td>
</tr>
</tbody>
</table>
2.8. Figures

Figure 2.1: Asset pricing equilibrium with transaction costs

The figure illustrates the asset pricing equilibrium with constant liquidity costs, as in the example of Section 2.2.2.4. The figure graphs (minus) the asset demand of an investor with hedging needs \((-w_1y_1)\) who is short in the asset and a less risk averse investor with no hedging demand \((w_2y_2)\) who is long. The solid lines reflect the situation without transaction costs, the dashed lines reflect the situation with transaction costs \(c\). \(\rho\) is the equilibrium expected return without transaction costs, \(\rho + \zeta_0 c\) is the equilibrium expected return with transaction costs.
Figure 2.2: Time-series of CDS spreads and bid-ask spreads

The graph presents the time-series of weekly CDS spreads (geometric average across all CDS portfolios) from April 2004 to December 2008 on a log scale in annual basis points. Also included are time series of bid-ask spreads for four bid-ask spread quartiles. These are calculated from the sequential sort on credit rating and bid-ask spread, with bid-ask spreads averaged across portfolios with different credit ratings.
On the y-axis, the graph has pricing errors from a two-factor asset pricing model with equity market risk and systematic credit risk (PCA factor) as factors (specification (3) in Table IV), for 100 CDS portfolios (sorted first on credit rating, and then on leverage, total debt, total syndicated loan amount, bid-ask spread, or quote frequency). The x-axis has the expected liquidity of each portfolio (in percentage), calculated as the average of the weekly transaction costs of each portfolio. All returns are in percentages for a quarterly period.
Using the benchmark GMM estimates in Table 2.4 specification (1), for the model for expected CDS returns in equation (2.16), the graph decomposes expected portfolio CDS returns into an intercept, market risk premia (sum of equity risk premium and credit risk premium), expected liquidity, liquidity risk and a pricing error. Results are presented for the sequential sort on credit rating (five categories) and bid-ask spread (four quartiles). All returns are in percentages for a quarterly period.
Using the GMM estimates of a model without intercept in Table 2.5 specification (9) and the model for expected CDS returns in (2.16), the graph decomposes expected portfolio CDS returns into market risk premia (sum of equity risk premium and credit risk premium), expected liquidity, liquidity risk and a pricing error. Results are presented for the sequential sort on credit rating (five categories) and bid-ask spread (four quartiles). All returns are in percentages for a quarterly period.
2.9 Appendix

2.9.1 Proof of Theorems I to III

We work out the market clearing conditions for the most general case (including long-horizon investors and idiosyncratic background risk). Theorems I to III are all special cases of these market clearing conditions.

One-period agent $i$ maximizes at time $t-1$ utility over terminal wealth (at time $t$)

$$\max_{x_i, y_i} E_{t-1} \left( \frac{W_{i,t}^{1-\gamma}}{1-\gamma} \right)$$

(2.23)

where $W_{i,t} = W_{i,t-1}(1 + R_{i,p,t})$ with $R_{i,p}$ the total portfolio return. If the portfolio return is lognormal, the optimization can be written as

$$\max_{x_i, y_i} \left\{ E_{t-1}(r_{i,p,t}) + \frac{1}{2}(1 - \gamma)V_{t-1}(r_{i,p,t}) \right\}$$

(2.24)

where log portfolio returns (in excess of the risk free rate) are denoted by $r_{i,p,t}$.

The simple gross return on the background risk for investor $i$ is assumed to be $q_i e^{r_{n,t} + \epsilon_i,t}$. We then use Campbell and Viceira’s (2002) approximation of the log portfolio return: assuming that individual asset returns are jointly lognormal, the portfolio return is approximately lognormal. Dropping all time subscripts again, we have

$$r_{i,p} \approx x'_i (r_b - c_b) + y'_i (r_h - \delta_i c_h) + q_i (r_n + \epsilon_i) + \frac{1}{2}(x'_i y'_i q_i) \sigma_i^2 - \frac{1}{2} \text{Var}(x'_i (r_b - c_b) + y'_i (r_h - \delta_i c_h) + q_i (r_n + \epsilon_i))$$

(2.25)

where $r_b$, $r_h$ and $r_n$ are defined in excess of the risk-free rate, and $\sigma_i^2$ is a vector with the diagonal of $\text{Var}((r_b - c_b) (r_h - \delta_i c_h) q_i (r_n + \epsilon_i))$.

We then orthogonalize the hedge asset returns for the net return on the non-hedge assets, and optimize with respect to $\hat{x}_i$ and $y_i$ with $\hat{x}_i = x_i - (\beta'_i r_{h,r_b} - \delta_i \beta'_i c_h, r_b) y_i$. Dropping terms that do not depend on $x_i$ or $y_i$ and using (2.25), the objective function in (2.24) can be rewritten as

$$\max_{\hat{x}_i, y_i} \left[ \frac{\hat{x}_i'}{\sigma_{i,h}^2} \text{Var}(\hat{x}_i' (r_b - c_b) + y_i' (r_h - \delta_i c_h) + q_i (r_n + \epsilon_i)) \right]$$

(2.26)

with $\sigma_{i,h}^2 = \text{diag}(\text{Var}(\hat{r}_h - \delta_i \hat{c}_h))$. Taking derivatives with respect to $y_i$ gives the
first order condition for investor $i$

$$E(\hat{r}_h) + \frac{1}{2} \sigma^2_{r,h} - \delta_i E(\hat{c}_h) - \gamma_i \text{Var}(\hat{r}_h - \delta_i \hat{c}_h)y_i - \gamma_i \text{Cov}(\hat{r}_h - \delta_i \hat{c}_h, r_n + \epsilon_i)q_i = 0 \quad (2.27)$$

with solution for the optimal portfolio weights

$$y_i = \gamma_i^{-1} \text{Var}(\hat{r}_h - \delta_i \hat{c}_h)^{-1}[E(\hat{r}_h) + \frac{1}{2} \sigma^2_{r,h} - \delta_i E(\hat{c}_h) - \gamma_i \text{Cov}(\hat{r}_h - \delta_i \hat{c}_h, r_n + \epsilon_i)q_i] \quad (2.28)$$

This result can be used to show the restrictions implied by the assumptions in the theoretical model on the three groups of investors (long, short, long/short). These restrictions are given by

$$y_i \geq 0 \text{ if } \delta_i = I, \quad y_i \leq 0 \text{ if } \delta_i = -I, \quad Dy_i \geq 0 \text{ if } \delta_i = D \quad (2.29)$$

where $E(\hat{r}_h)$ are the equilibrium expected returns derived below. Although these are complicated expressions, it is clear that to satisfy these restrictions one needs heterogeneity in risk aversion and/or hedging demand, and hedge assets that correlate sufficiently with the non-traded risk. This is also illustrated by the simple example in Section 2.2.2.2.4, where the constraint simplifies to $q_1 \gamma_1 \text{Cov}(r, r_n) > 2c$.

The portfolio problem for the long-horizon investors is particularly simple. These investors have an arbitrarily long horizon and maximize utility of terminal wealth, similar to (2.23). Given the assumptions that rebalancing is costless for these investors and that returns are IID, Campbell and Viceira (2002) show that the optimal portfolio weights are constant and equal to the myopic portfolio rule

$$y_i = \gamma_i^{-1} \text{Var}(\hat{r}_h)^{-1}[E(\hat{r}_h) + \frac{1}{2} \sigma^2_{r,h} - \gamma_i \text{Cov}(\hat{r}_h, r_n + \epsilon_i)q_i] \quad (2.30)$$

with $\sigma^2_{r,h} = \text{diag}(\text{Var}(\hat{r}_h))$. Given the assumption of IID returns and that the one-period investors have the same characteristics each period, pre-multiplying by
wealth gives the equilibrium pricing condition for each period

$$\left[\sum_{i:T_i=1} w_i \gamma_i^{-1} V(\delta_i)^{-1} + \sum_{i:T_i=L} w_i \gamma_i^{-1} V_r^{-1}\right] (E(\hat{\gamma}_h) + \frac{1}{2} \sigma_{\hat{\gamma}_h}^2) +$$ (2.31)

$$\frac{1}{2} \left[\sum_{i:T_i=1} w_i \gamma_i^{-1} V(\delta_i)^{-1} (\sigma_{i,h}^2 - \sigma_{\hat{\gamma}_h}^2) \right] -$$

$$\left[\sum_{i:T_i=1} w_i \gamma_i^{-1} V(\delta_i)^{-1} \right] E(\hat{\gamma}_h) -$$

$$\left[\sum_{i:T_i=1} w_i q_i V(\delta_i)^{-1} + \sum_{i:T_i=L} w_i q_i V_r^{-1}\right] \text{Cov}(\hat{\gamma}_h, r_n) +$$

$$\left[\sum_{i:T_i=1} w_i q_i V(\delta_i)^{-1} \delta_i \right] \text{Cov}(\hat{\gamma}_h, r_n) -$$

$$\left[\sum_{i:T_i=1} w_i q_i V(\delta_i)^{-1} \text{Cov}(\hat{\gamma}_h - \delta_i \hat{c}_h, \varepsilon_i) + \sum_{i:T_i=L} w_i q_i V_r^{-1} \text{Cov}(\hat{\gamma}_h, \varepsilon_i)\right] = S_h$$

where $V(\delta_i) = V(\hat{\gamma}_h - \delta_i \hat{c}_h)$ and $V_r = V(\hat{\gamma}_h)$, and where $T_i = 1$ denotes one-period investors and $T_i = L$ long-horizon investors. We multiply this equation on both sides by $V(I) = V(\hat{\gamma}_h - \hat{c}_h)$ and divide by $\eta = \sum_{i=1}^N w_i \gamma_i^{-1}$, and then apply a Taylor approximation to $V(I) V(\delta_i)^{-1}$ around $\hat{c}_h = 0$ (see Appendix 2.9.2). Using that

$$V(I) S_h = \text{Cov}(\hat{\gamma}_h - \hat{c}_h, (\hat{\gamma}_h - \hat{c}_h)' S_h)$$ (2.32)

the resulting equation directly leads to Theorems I to III, which are all special cases of the general setup of this appendix. In all theorems, an extra term due to the convexity correction for log returns is defined as follows:

$$F_h = \frac{1}{2\eta} (I - H_1)^{-1} V(I) \left[\sum_{i:T_i=1} w_i \gamma_i^{-1} V(\delta_i)^{-1} (\sigma_{i,h}^2 - \sigma_{\hat{\gamma}_h}^2) \right] .$$ (2.33)

This term would be equal to zero if transaction costs are constant. Empirically, this term is small, so that we can safely neglect the term $F_h$ in the empirical analysis. Specifically, in case $\text{Cov}(\hat{\gamma}_h, \hat{c}_h) = 0$, (2.33) simplifies to $F_h = \frac{1}{2} \text{Var}(\hat{c}_h)$ which is smaller than 0.2 basis points per quarter (except for one low-rating/low-liquidity portfolio for which it equals 1.3 basis points). For the crisis period, $F_h$ is larger but still below 1 basis point for almost all portfolios.
2.9.2 Taylor approximations

This appendix provides a Taylor approximation to the matrix $V(I)V(\delta_i)^{-1}$ around $\hat{c}_h = 0$. Empirically, transaction costs are small, in the sense that the standard deviation of $c$ is smaller than the standard deviation of $r$. Write $\hat{c}_h = \xi \tilde{c}$ so that

$$V(\delta_i) = V_r - \xi (\delta_i \tilde{C} + \delta_i \tilde{C}') + \xi^2 V_{\tilde{c}}$$

(2.34)

where $V_{\tilde{c}} = Var(\tilde{c})$ and $\tilde{C} = Cov(\tilde{c}, \tilde{r}_h)$ and use a Taylor approximation around $\xi = 0$

$$V(I)V(\delta_i)^{-1} \approx V(I)V(\delta_i)^{-1}_{|\xi=0} + \xi \frac{\partial (V(I)V(\delta_i)^{-1})}{\partial \xi}$$

(2.35)

We have

$$\frac{\partial V(\delta_i)^{-1}}{\partial \xi} = -V(\delta_i)^{-1} \frac{\partial V(\delta_i)}{\partial \xi} V(\delta_i)^{-1} = V(\delta_i)^{-1}(\delta_i \tilde{C} + \delta_i \tilde{C}' - 2\xi V_{\tilde{c}})V(\delta_i)^{-1}$$

(2.36)

Evaluating this in $\xi = 0$ we obtain

$$V(I)V(\delta_i)^{-1} \approx I + \xi (V_r V_r^{-1}(\delta_i \tilde{C} + \delta_i \tilde{C}')V_r^{-1}) + \xi (-\tilde{C} + \tilde{C}')V_r^{-1}$$

(2.37)

$$= I + (\delta_i C + \delta_i C')V_r^{-1} - (C + C')V_r^{-1}$$

If $\delta_i = -I$, this simplifies to $I - 2H_1$, and if $\delta_i = D$ this equals $I + H_2 - H_1$. Finally, using a similar derivation we obtain that $V(I)V_r^{-1} \approx I - H_1$.

2.9.3 GMM standard errors

In this appendix we outline the calculation of standard errors along the lines of Shanken (1992) for the GMM estimation of (2.16). To avoid complicated notation, we discuss a simplified example; the full derivation is available on request. The simplified example is given by

$$E(r) = X\alpha + E(c)\zeta + \beta \theta$$

(2.38)

with $\beta = (\beta_{x,r}, \beta_{x,r}, \beta_{c,r}, \beta_{c,r})$ and $\theta = (\psi, \lambda, \kappa)'$. In practice, the regressors are replaced by estimates and the second step regression is

$$\overline{e} = X\alpha + \tilde{c}\zeta + \tilde{\beta}\theta + \eta$$

(2.39)

with

$$\eta = (\overline{e} - E(r)) - (\overline{e} - E(c))\zeta - (\tilde{\beta} - \beta)\theta$$

(2.40)
and $\overline{er} = T^{-1} \sum_{t=1}^{T} er_t$, with $er_t$ our weekly estimate for the expected CDS returns in (2.22), and likewise for $\overline{\pi}$.

The second step estimates for $(\alpha, \zeta, \theta)$ are given by $(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}\overline{er}$ with $Z = (X \overline{\pi} \widehat{\beta})$ and $\Sigma^{-1}$ the weighting matrix. The standard errors of these estimates are given by $(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}Var(\eta)\Sigma^{-1}Z(Z'\Sigma^{-1}Z)^{-1}$ with

$$Var(\eta) = Var(\overline{er}) + \zeta^2Var(\overline{\pi}) + Var(\widehat{\beta}\theta)$$

(2.41)

The elements $Var(\overline{er})$ and $Var(\overline{\pi})$ can be estimated using a Newey-West procedure,

$$Var(\overline{er}) \approx T^{-2} \sum_{t} \sum_{k} w_k (er_t - \overline{er})(er_{t-k} - \overline{er})$$

(2.42)

and likewise for $\overline{\pi}$, and

$$Var(\widehat{\beta}\theta) \approx T^{-2} \sum_{t=1}^{T} (u_t\theta)(u_t\theta)'$$

(2.43)

where $u_t = (Y'Y)^{-1}Y'\nu_t$, with $\nu_t$ the regression errors of the first step estimation of $\widehat{\beta}$, and $Y$ the regressor variable (either $r_b$ or $r_n$).

### 2.9.4 Repeat sales method

This appendix contains details on the repeat sales method used to form returns on CDS portfolios. Let $k(i)$ be the portfolio that contains constituent $i$ and let $T$ the number of periods in our sample. For constituent $i$, we assume that the spread quote of a CDS contract $p_{i,t}$ is given by

$$p_{i,t} = \text{CDS}_{k(i),t} + u_{i,t}$$

(2.44)

where $\text{CDS}_{k(i),t}$ is the portfolio spread level (which is to be estimated) and $u_{i,t}$ is a quote specific error term. $u_{i,t}$ has mean zero and constant variance $\sigma_u$ and is uncorrelated with the other variables and its own lags. To illustrate the approach, suppose we have three transactions in constituent $i$, say at times $s$, $s'$ and $s''$ with
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$s < s' < s''$. We can then specify spread innovations

\[ \Delta p_{i,s,s'} = p_{i,s'} - p_{i,s} = \sum_{j=2}^{T} x_{i,j,s,s'} \Delta \text{CDS}_{k(i),j} + (u_{i,s'} - u_{i,s}) \]  

(2.45)

\[ \Delta p_{i,s',s''} = p_{i,s''} - p_{i,s'} = \sum_{j=2}^{T} x_{i,j,s',s''} \Delta \text{CDS}_{k(i),j} + (u_{i,s''} - u_{i,s'}) \]

where \( x_{i,j,s,s'} \) is a dummy that defines whether \( j \in [s, s'] \). The error covariance matrix is given by

\[ \text{Var}(\Delta p_{i,s,s'}) = 2\sigma_u^2, \quad \text{Var}(\Delta p_{i,s',s''}) = 2\sigma_u^2, \quad \text{Cov}(\Delta p_{i,s,s'}, \Delta p_{i,s',s''}) = -\sigma_u^2. \]  

(2.46)

We can write our spread innovations for all constituents of \( k(i) \) up to time \( T \) in matrix form as

\[ \Delta p = x \Delta \text{CDS}_{k(i)} + v \]  

(2.47)

where \( v = \Delta u \). The best linear unbiased estimator of \( \Delta \text{CDS}_{k(i)} \) is given by

\[ \hat{\Delta \text{CDS}}_{k(i)} = (x'M^{-1}x)^{-1}x'M^{-1}\Delta p, \]  

(2.48)

where \( M \) is the (sparse, block diagonal) covariance matrix of \( v \). Empirically, \( \sigma_u \) is unknown. However, because \( M \) is known up to a scalar which drops out, it turns out to be possible to consistently estimate \( \Delta \text{CDS}_{k(i)} \) without knowledge of \( \sigma_u \) using regression.

2.9.5 Risk-free rates and default probabilities

To construct excess returns from CDS spread changes, we need risk-free discount rates. Lando and Feldhütter (2008) argue that despite the AA default risk premium present in LIBOR rates, the best estimates of risk-free rates are obtained from swap rates. Therefore, we use daily data on the 3-month LIBOR based swap curve with a maturity of 1 up to 6 years. Swap rates are obtained from Datastream. To construct zero-coupon rates, we assume that these are piece-wise constant and subsequently bootstrap these rates from the observed term structure of swap rates.

To obtain the risk-neutral default probabilities, also needed to construct excess returns, we assume for simplicity that CDS spreads only reflect default risk, that the risk-neutral default intensity is constant over the maturity period and that there is a deterministic loss rate \( L = 60\% \). We then solve the CDS pricing equation under these assumptions to obtain the default intensity and compute the risk-
neutral probabilities (Duffie and Singleton (2003)):

$$ CDS_{k,t} = 4 \sum_{j=1}^{(T-t)} Q_{k,t}^{SV}(t+j-1)Q_{k,t}^{def|SV}(t+j)B_t(t+j) \left/ \sum_{j=1}^{(T-t)} Q_{k,t}^{SV}(t+j)B(t,t+j) \right. $$

where $Q_{k,t}^{def|SV}(t+j)$ is the risk neutral probability of a default in period $t+j$ conditional on survival up to time $t+j-1$. We calculate these probabilities for each CDS portfolio and for each week in the empirical analysis.

Naturally, there is an inconsistency in assuming that CDS prices are only driven by default risk where the goal is to identify a non-default component. However, if we iterate our estimation procedure, by correcting the CDS spread and $\lambda$ for the estimated liquidity effect and re-estimating the model, we find results that are extremely close to the results reported here. Note that we only need $Q_{k,t}^{SV}$ to calculate excess returns.