CHAPTER I

DOES A CORRELATION BETWEEN THE MAGNETIC FIELD AND THE TRANSVERSE VELOCITY EXIST FOR RADIO PULSARS?

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Summary

We examine, by means of a Monte Carlo simulation, whether the correlation between magnetic field strengths and transverse velocities of radio pulsars found by Anderson and Lyne (1983) and Cordes (1985) might be due to observational selection effects. Two selection effects are present in the measurements of field strengths and transverse velocities of pulsars. One is due to the lower cut-off of the radiation flux that can be measured, in combination with the increase in radio luminosity with magnetic field strength for pulsars. The other selection effect is due to the fact that the transverse velocity that can be measured for a pulsar, depends on its distance. The Monte Carlo simulation shows that these selection effects will produce an apparent correlation between the transverse velocity and the magnetic field strength for pulsars. But by comparing these results with the correlation observed by Anderson and Lyne we conclude that the probability that the observed correlation is due to these selection effects (and therefore spurious) is only 3%. The probability that the correlation found by Cordes - who derived the transverse velocities from scintillation data - is due to selection effects is larger (20 to 30 percent); our simulation does not, however, generate pulsars with small fields (i.e. $B < 3 \times 10^{11}$ G) of which a fair number is present in both observed samples. Both simulations, therefore, add support to the alternative possibility that the samples of Anderson & Lyne and of Cordes consist of two populations of radio pulsars, one "normal" consisting of pulsars with relatively large velocities and strong magnetic fields and one "abnormal" population, characterized by low space velocities and weak magnetic fields.

Key words: pulsars - magnetic fields - proper motion - evolution.
1. Introduction

Anderson and Lyne (1983) and, more recently, Cordes (1985) found that there seems to exist a correlation between the magnetic moments (and therefore the magnetic field strengths) of radio pulsars and their transverse velocities (Fig.1). The main conclusion from these investigations is that this correlation is not due to a selection effect, but is real. As a possible explanation Anderson and Lyne argue that an asymmetry in the supernova explosion could give rise to the observed velocities. If the asymmetry is due to a particular magnetic field configuration, this could lead to a correlation between the velocity and the magnetic field strength of the pulsars. Also Cordes (1985) argues that the correlation is most plausibly due to a relationship between the magnetic moment and the impulse given to a neutron star at or near the time of its formation.

It was pointed out by Radhakrishnan (1985) that, although the asymmetry-hypothesis is attractive due to the fact that the kinetic energy of the pulsar is about five orders of magnitude smaller than that of the ejecta of the supernova shell, the energy stored in the magnetic field is about as many orders down from the kinetic energy of the pulsar. Radhakrishnan, therefore, suggested that the correlation, shown in Fig.1a, is the result of the existence of two populations of radio pulsars. Firstly a population of "normal" pulsars with high to medium field strenghts and high velocities and secondly a population of recycled pulsars. These latter ones are old and are believed to have low field strengths (cf. Radhakrishnan and Srinivasan, 1981; Van den Heuvel, 1985). If, furthermore, a pulsar has been recycled in an initially wide binary consisting of a neutron star and a B-emission star, then the space velocity of that pulsar may be quite small (cf. Van den Heuvel and Bonsema, 1984).

Although the explanation given by Radhakrishnan is attractive and appears to fit in the evolutionary scenarios for radio pulsars with weak magnetic fields (Van den Heuvel, 1985), it seems useful to first investigate the possible influence of selection effects on the relation between the magnetic field and the transverse velocity.

In paragraph 2 we will discuss two selection effects that are present in the measurement of pulsars. In paragraph 3 we will give a qualitative account of the influence of these selection effects on the \( (B - v_t) \)-diagram, where \( B \) denotes the surface dipole field strength and \( v_t \) the transverse velocity. In paragraph 4 a description will be given of a Monte Carlo simulation of the expected correlation between the magnetic field and transverse velocity taking into account the selection effects described in paragraph 2. In paragraph 5 the results will be discussed.
Fig. 1a. The transverse velocity plotted versus the magnetic field strength for the 26 pulsars in the sample of Anderson and Lyne (1983). The pulsars 1 to 5 might form a special population (see text). The correlation coefficient for all 26 pulsars is 0.63. For the sample without the pulsars 1 to 5 this coefficient is 0.33.

Fig. 1b. The interstellar scintillation speed ($V_{\text{ISS}}$) plotted versus the magnetic field for 70 pulsars in the sample of Cordes (1985). Pulsars indicated by open circles are the same as in the Anderson and Lyne sample. The pulsars 1 to 10 might form a special population (see text). The correlation coefficient for the 69 pulsars (we excluded PSR 0820+02, which is a binary) is 0.20. For the sample without 1 to 10 this coefficient is 0.11.
2. Selection effects

2.1. The Anderson and Lyne sample

That some selection effect is present in the observations by Anderson and Lyne is suggested by Fig. 2, where we have plotted the luminosity, \( L_{400} \), versus the distance, \( d \) for the pulsars in their sample. The luminosity is defined by \( L_{400} = S_{400} d^2 \), with \( S_{400} \) being the mean spectral energy density of incoming radiation at 400 MHz. The values for \( S_{400} \) and \( d \) are taken from Manchester and Taylor (1981). As can be seen, the pulsars with a high luminosity have, on average, a larger distance than those with a low luminosity. This can be understood in terms of flux-limitation. There is a minimum flux \( (S_{400})_{\text{min}} \) that can be observed. The larger the distance to the pulsar, the higher the luminosity must be for it to be observable. In Fig. 2 this selection effect is shown by a tentative cut-off line, given by

\[
\log L_{400} = 2 \log d + \log (S_{400})_{\text{min}} \tag{1}
\]

The presence of a further selection effect is suggested by Fig. 3a where we have plotted the transverse velocity, \( v_t \), versus the distance. It is clear from this Figure that the pulsars that are far away tend to have, on average, higher velocities than those that are close by. This is also due to a selection effect.
because the proper motion of each pulsar was determined from the successive measurements of the position of the pulsar relative to one or more radio sources lying close to the pulsar in the plane of the sky (see Lyne, Anderson and Salter, 1982).

\[ 30 \quad 10 \quad 3.0 \]

\[ 0.1 \quad 0.3 \quad 1.0 \quad 3.0 \]

**Fig. 3a.** The transverse velocity versus the distance for the pulsars in the Anderson and Lyne sample. The straight line indicates the cut-off line given by eq. (3) for \( \log \phi_{\text{min}} = 1.7 \), as used in our computer simulation of Population 1.

It is clear that the angle between different positions cannot be measured if, for a given distance, the transverse velocity of the pulsar is smaller than a certain value. This value increases for increasing distances. We may write

\[ v_t = \frac{\alpha d}{(t_f - t_0)} \quad (2) \]

where \( \alpha \) is the difference in angle between the position of the pulsar at the beginning of the measurement, \( t_0 \), and that at the end, \( t_f \). Therefore, we can define a cut-off line in Fig. 3a by assuming that there is a minimum value of \( \phi = \alpha/(t_f - t_0) = \phi_{\text{min}} \), that can be measured. The cut-off line is then given by

\[ \log v_t = \log \phi_{\text{min}} + \log d \quad (3) \]

We like to emphasize the fact that we are not suggesting that the total sample of Anderson and Lyne was originally larger than 26 pulsars and that they left out
those for which they could not measure the transverse velocity. We are merely pointing out that the low transverse velocities of the pulsars with a large distance can be uncertain as is suggested by the error bars in Fig. 3a.

The selection effects, mentioned above, are in fact exactly the same ones that affect the studies of stellar populations in the galaxy, and the use of high proper motions is indeed a standard method for detecting nearby low-luminosity stars.

2.2. The Cordes sample

The selection effect due to flux-limitation is also present in the pulsar sample measured by Cordes (1985). The presence of the second selection effect is less clear. Cordes does not measure the proper motion of the pulsars but the interstellar scintillation speed. These two velocities are correlated but the correlation is not very strong for individual objects, as is clear from a comparison between the proper motions measured by Anderson and Lyne and the scintillation speeds for the same pulsars (see Cordes (1985) and compare Fig. 1a to 1b).

In Fig. 3b we have plotted the scintillation speed (V\textsubscript{ISS}) versus the distance for the 70 pulsars in the Cordes sample. (We have left out PSR 2224+65 for which the measured velocity was 1746 ± 312 km/s).

![Fig. 3b. The interstellar scintillation speed versus the distance for the pulsars in the Cordes sample. The straight line indicates the cut-off line given by eq. 3 for log\textsubscript{10} \text{min} = 1.2, as used in our computer simulation of population 2a.](image)

If we compare Fig. 3b to 3a it is clear that the scintillation speed can be measured to larger distances than the real proper motion, as indicated by Cordes
It is however also clear that the smallest scintillation velocity that can be measured accurately must still depend on the distance to the pulsar. Whether this creates a cut-off line, given by eq. (3), in Fig. 3b is difficult to say because it strongly depends on the accuracy of the low scintillation velocities measured for some of the pulsars with large distances.

3. The \((8 - v_p)\) - diagram

Radio pulsars are born close to the galactic plane, with a 3-dimensional velocity distribution and, presumably, a distribution in initial magnetic field strength, \(10^{12}\) G < \(B_0\) < \(10^{13.5}\) G (cf. Flowers and Ruderman, 1977). As time increases the pulsars move away from the galactic plane and their magnetic field strengths decrease. As usual we adopt for this decay an exponential function: \(B(t) = B_0 \exp(-t/\tau_D)\), where \(\tau_D\) is the decay time of the field.

It was shown by Gunn and Ostriker (1970) and more recently by Lyne, Manchester and Taylor (1985) that there exists a positive correlation between the magnetic field strength of a radio pulsar and its luminosity. The most commonly used relation is given by \(L \propto B^2\), which we will also adopt here. This correlation implies that as time increases and therefore \(B(t)\) decreases, the luminosity of the pulsars becomes smaller. Therefore, radio pulsars finally become undetectable for two reasons. Firstly they move away from the galactic plane and therefore on average their distance to us, \(d\), increases and their flux decreases until the latter drops below a certain threshold value, \(S_{\text{min}}\). Secondly, as time increases the magnetic field strength and therefore the luminosity decreases, which also implies a reduction in flux, for a fixed or increasing distance.

It was pointed out to us by dr. Taylor (1985) that the assumption that \(L\) varies proportional with \(B^2\) may have some problems. A better empirical luminosity dependence is \(L \propto F^{-1} P^{-0.35}\) as found by Proszynski and Przybycien (1985). However, in this first study of the correlation between the magnetic field and the transverse velocity we like to keep the dependence of the luminosity in terms of the magnetic field as simple as possible, first of all because it makes the \((B - v_p)\)-diagram much easier to understand and secondly because the magnetic field is a variable in which we are primarily interested. Furthermore, the relation \(L \propto B^2\) fits what we call the normal pulsar population well (see eg. Lyne et al., 1985) but not the so-called "recycled" pulsars. In this paper we start with the assumption that there is just one population of pulsars and see whether this agrees with what is observed. Since we assume this population to consist of normal pulsars, the relation \(L \propto B^2\) seems a good first approximation.
If all pulsars belong to one class then another important consequence of field decay is that, since pulsars are born with strong fields, a weak field pulsar is on average older than a high field one. This can also be seen from Fig. 1 in Van den Heuvel (1985).

Pulsars that are located in the upper lefthand corner of the \((B - v_t)\) diagram (Fig. 1) have a weak field and are therefore on average older than pulsars in the upper and lower righthand corners. Their low field strength implies a low luminosity and the fact that they have a high transverse velocity and are old suggests that their distances are large, on average. As a result these pulsars have a very low flux and, therefore, a small probability of detection, with respect to those in the other parts of the diagram. For example, the pulsars in the lower lefthand corner have a low luminosity but because of their small velocity are closer to us, on average, and therefore they have, on average, a higher observed flux. This effect could explain the absence of observed pulsars in the upper lefthand corner of Fig. 1. This possibility was already mentioned by Anderson and Lyne.

In the reasoning given so far we have thought of the transverse velocity as the real absolute velocity in 3-dimensional space, which it is not. Furthermore, we have related velocity and distance in a simple way, without considering the fact that pulsars can move in our direction and thereby decrease their distance. Therefore, the explanation given so far cannot by itself fully explain the observed correlation between \(B\) and \(v_t\). Furthermore, as has already been pointed out by Anderson and Lyne, it cannot account for the absence of pulsars in the lower righthand corner: i.e. the absence of pulsars with a high field strength and low transverse velocity.

We think that the latter can be explained in the following way. As is clear from Fig. 2, pulsars with a high luminosity and therefore strong field tend, on average, to be farther away than pulsars with a weak field. A larger distance implies that the transverse velocity of the pulsar must be large for it to be measured. Pulsars in the lower righthand corner have a strong field and are, therefore, on average far away, while at the same time their transverse velocity is small, and therefore difficult to measure. Therefore, they are expected to be absent in the \((B - v_t)\) diagram.

4. A computer simulation

In order to derive a quantitative value for the expected correlation between the transverse velocities and magnetic field strength that might be expected due to the above-mentioned selection effects we have made a Monte Carlo simulation of the expected pulsar population in the neighbourhood of the Sun.
First of all we have set up a coordinate system for the solar neighbourhood. The x and y axes are chosen parallel to the plane of our galaxy and the z axis is perpendicular to it. The Sun is in the center of the system and has the coordinates (0,0,0). We assume that pulsars are born in a box, centered on the Sun, with the following dimensions: \(-3 < x < 3, -3 < y < 3\) and \(-0.05 < z < 0.05\). (Here x, y and z are measured in kiloparsecs (kpc)). The probability of a pulsar being born was chosen to be the same for each position within the box. Outside the box this probability was set equal to zero.

Each pulsar was assumed to be born with an initial magnetic field strength, \(B_0\). The value of \(\log B_0\) was chosen from a gaussian probability distribution.

\[
P(\log B_0)\,d\log B_0 = \frac{1}{\sigma_B (2\pi)^{1/2}} \exp\left[-\frac{1}{2} \frac{(\log B_0 - \langle \log B_0 \rangle)^2}{\sigma_B^2}\right] \, d\log B_0 \tag{4}
\]

We have assumed \(\langle \log B_0 \rangle\) equal to 12.5 and \(\sigma_B = 0.3\).

At birth each pulsar was assigned a velocity, which was selected from a gaussian distribution, given by

\[
f(v_x, v_y, v_z) \, dv_x \, dv_y \, dv_z = \frac{1}{\sigma_v^3 (2\pi)^{3/2}} \exp\left[-\frac{1}{2} \frac{v_x^2 + v_y^2 + v_z^2}{\sigma_v^2}\right] \, dv_x \, dv_y \, dv_z \tag{5}
\]

The value for \(\sigma_v\) was chosen to be consistent with the distribution of transverse velocities as measured by Lyne et al. (1982): i.e. \(\sigma_v = 100\) km/s.

The present pulsar population was simulated by creating a pulsar once every 1000 years for \(15 \times 10^6\) years. For each ith pulsar the position \(\mathbf{r}_i = (x_i, y_i, z_i)\) was evaluated by

\[
\mathbf{r}_i = \mathbf{r}_{0i} + \mathbf{v}_it_i \tag{6}
\]

where \(\mathbf{r}_{0i}\) is the place of birth and \(t_i\) is the age of the pulsar. Once the position and the velocity are known it is possible to calculate the transverse velocity, \(v_T\), which is the velocity perpendicular to the line of sight. Since the age of the pulsar was known we could evaluate its magnetic field strength by

\[
B_i = B_{0i} \exp(-t_i/\tau_D) \tag{7}
\]

The value of \(\tau_D\) was set equal to \(2 \times 10^6\) yr (see Radhakrishanan, 1985).

In this way we created a population of \(1.5 \times 10^4\) pulsars. For each pulsar the position with respect to the Sun (and therefore its distance, \(d\)), the transverse velocity, \(v_T\), and its magnetic field strength were known. In order to evaluate the luminosity for each pulsar we assumed that \(L_{400} \propto B^2\) and fitted the radio pulsars from the catalog of Manchester and Taylor (1981) with a distance less
than 3 kpc to this relation. We found

\[ L_{400} = S_{400}d^2 = 23 \left( B / 10^{12} \right)^2 \]  

(8)

To simulate the real pulsar population somewhat better we distributed the luminosities around the logarithm of the average value as given by eq. (8). For this we used the following distribution

\[ p(\log L_{400})d\log L_{400} = \frac{1}{\sigma_L(2\pi)^{1/2}} \exp\left[ -\frac{1}{2} \left( \frac{\log L_{400} - \langle \log L_{400} \rangle}{\sigma_L} \right)^2 \right] d\log L_{400} \]  

(9)

Here \( \langle \log L_{400} \rangle \) is calculated from equation (8) and the value for \( \sigma_L \) we chose equal to 0.25.

It is generally believed that pulsars do not pulse forever, but die after a certain time, depending on their magnetic field strength and rotation period, \( P \) (see e.g., Ruderman and Sutherland, 1975). This is represented by a death-line in the \( B \) versus \( P \) diagram. This death-line can be parameterized in terms of the initial field strength, \( B_0 \), once the decay time, \( \tau_d \), of the field is known. Assuming a decay time of \( 2 \times 10^6 \) years, we found for each \( B_0 \) a minimum magnetic field strength, \( B_{\text{min}} \), given by

\[ \log B_{\text{min}} = 2.0 \log B_0 - 13.9 \]  

(10)

Once the field strength of a pulsar, that was born with a field \( B_0 \), had become smaller than \( B_{\text{min}} \), the pulsar was excluded from our population.

Another important reduction in the number of pulsars, that can be seen, is caused by the fact that the radiation of a radio pulsar is strongly beamed. We have assumed that, due to this beaming, 1 pulsar in 5 could be seen.

The pulsar population thus created would be the one that could be observed if there were no other selection effects present. As we mentioned above, however, we have to take account of two important selection effects. First of all there is a minimum flux, \( S_{\text{min}} \), that can be observed. For the Anderson and Lyne sample we have set \( S_{\text{min}} \) equal to 15 mJy. This is the flux observed from PSR 0943+10 as given by Manchester and Taylor (1981) and is the lowest observed in the Anderson & Lyne sample. For the sample of Cordes we have set \( S_{\text{min}} \) equal to 10 mJy. For our own populations we calculated the flux of each pulsar by evaluating its luminosity through eqs. (8) and (9) and its distance through eq. 6 (i.e., \( d_1 = | \tau_i | \)). If \( (S_{400})_i \) was less than \( S_{\text{min}} \) we excluded the pulsar from our sample.

The other selection effect that has to be taken into account is the cut-off
line in the $v_L$ versus distance diagram (see eq. (2)). From the Anderson and Lyne sample it is difficult to find an exact value for $\log \Phi_{\min}$, due to the large observational errors in the transverse velocities near the cut-off line. The value we have taken is $\log \Phi_{\min} = 1.7$ for which the line is drawn in Fig. 3a. This value is somewhat large but is still consistent with the errors of all points in the diagram except for one (i.e., PSR 2016+28) for which the error in the transverse velocity is larger than the measured value and is therefore very unreliable.

For the sample of Cordes we have used two values for $\log \Phi_{\min}$: namely 1.2 (for which the cut-off line, according to eq. 3, is drawn in Fig. 3b) and minus infinity, which implies that the second selection effect is not present. We have chosen the value of 1.2 for $\log \Phi_{\min}$ because, if the second selection effect is present in Fig. 3b then this seems a reasonable value. It excludes 4 pulsars with very small velocities ($v_{iss} < 25 \text{ km/s}$) at large distances ($d > 2 \text{ kpc}$) (see Fig. 3b). We expect that these measurements can be very inaccurate. For example, one of these 4 pulsars, PSR 0611+22, was also measured by Anderson and Lyne. They found for this pulsar a transverse velocity of $117 \pm 63 \text{ km/s}$, which is larger than the value found by Cordes ($v_{iss} = 10 \pm 2 \text{ km/s}$).

For each pulsar in our population we calculated $\log \Phi_1 = \log v_L - \log d$. If $\log \Phi_1 < \log \Phi_{\min}$ we excluded the pulsar from our population.

5. Results and discussion

With the Monte Carlo method described above we have created two pulsar populations. The first population was created by applying the selection effects valid for the Anderson and Lyne sample (i.e., $S_{\min} = 15 \text{ mJy}$ and $\log \Phi_{\min} = 1.7$) while the second population was created by applying the selection effects valid for the Cordes sample (i.e., $S_{\min} = 10 \text{ mJy}$, $\log \Phi_{\min} = 1.2$ and minus infinity). All the other variables, as given in paragraph 4, were kept the same for both populations.

5.1. Computer population 1

The results of the Monte Carlo simulation of population 1 are presented in the Figs. 4 to 8. In Fig. 4 we have plotted the magnetic field strength, B, versus the transverse velocity for all the pulsars in population 1. From this population we have randomly chosen 30 pulsars to simulate the sample of Anderson and Lyne. We have done this a hundred times and for each time we have calculated the best straight line fit through these 30 points in the $(B - v_L)$ - diagram.
Fig. 4. The transverse velocity versus the magnetic field strength for the population 1, generated by our Monte Carlo simulation. From these a hundred different samples of 30 pulsars were randomly chosen. For each sample the best straight line fit was calculated. Plotted are the lines with the lowest slope (a), an average slope (b) and the highest slope (c). Also plotted (i.e. dashed line) is the best line fit to the 21 pulsars from the Anderson & Lyne sample without the pulsars 1 to 5.

Fig. 5. The distribution of correlation coefficients for the 100 randomly chosen samples of 30 pulsars, from population 1. The expected value lies between 0.3 and 0.4 and there is a 3% probability of finding a correlation coefficient larger than 0.6.
In Fig. 4 we have plotted the line with the smallest slope (a), an average slope (b) and the largest slope (c). From this figure it is clear that some correlation can be expected by randomly choosing 30 pulsars from a complete population.

Furthermore, we have calculated for each sample of 30 pulsars the correlation coefficient and plotted the distribution of these coefficients in Fig. 5, for the 100 different samples. To show the importance of the second selection effect (i.e., the fact that there is a cut-off line in the \((v_t - d)\) diagram) we calculated the distribution of correlation coefficients without taking this effect into account. The result is presented in Fig. 6.

**Fig. 6.** The same as Figure 5 but now the population of pulsars is calculated without taking account of the cut-off line in Figure 3. In this way the correlation becomes much less strong.

The average correlation coefficient has shifted to lower values and the shape of the the distribution has changed substantially. Therefore, the probability of finding a strong correlation between the transverse velocity and the magnetic field strength has decreased with respect to the case in which the second selection effect has been taken into account.

From Fig. 5 we see that the expected value for the correlation coefficient is between 0.3 and 0.4. Anderson and Lyne found a correlation coefficient of 0.63 for their sample. From Fig. 5 we see that the probability of finding a coefficient larger than 0.6 is 3%. As can be seen from Fig. 1a the strong correlation found by Anderson and Lyne is due to the five pulsars in the lower corner designated by the numbers 1 to 5. If we compare Fig. 1a to Fig. 4 we see that our Monte Carlo simulation does not create pulsars with field strengths of less than about \(3.8 \times 10^{11}\) G and velocities smaller than 27 km/s. Our calculation,
therefore, cannot represent a true simulation of the present total pulsar population. As was mentioned above, Radhakrishnan has suggested that these pulsars are members of a different subset because they have been recycled (see also Tutukov et al. 1984, who suggest, as an alternative, that these originate from very wide binaries). Since our calculations do not include the possibility of recycling we have, as an experiment, excluded pulsars 1 to 5 from the Anderson & Lyne sample and compared the modified correlation coefficient with our own calculations. The 21 pulsars which are show a much weaker correlation. We find a coefficient of 0.33, which is roughly the same value that we expect from our simulations (see Fig. 5). The best straight line for these 21 pulsars is given by $\log v_t = 0.29 \log B - 1.44$. We have plotted this line in Fig. 4 (see dashed line). As can be seen this line is consistent with our calculations.

To show that our simulation is indeed affected by the selection effects that are present in Fig. 2 and 3a, we have plotted the luminosity versus the distance in Fig. 7 and the transverse velocity versus the distance in Fig. 8. From Fig. 7 it is clear that pulsars with a high luminosity tend to have a larger distance and from Fig. 8 the same thing is clear for pulsars with a high transverse velocity.

5.2. Computer population 2

To simulate the Cordes sample we have generated two computer populations of pulsars: namely 2a and 2b. In population 2a we have assumed the second selection effect to be present and we have adopted the value of 1.2 for $\log \psi_{\min}$ as mentioned above. For population 2b we not included the second selection effect. The results of the Monte Carlo calculation for population 2 are presented in Figs. 9 to 11. In Fig 9 we have plotted the the magnetic field strength, $B$, versus the transverse velocity, $v_t$, for all the pulsars in population 2a. From these pulsars we have randomly chosen 100 samples of 70 pulsars to simulate the sample of Cordes (1985). For each of the 100 samples we have again calculated the best straight line fit and in Fig. 9 we have plotted the lines with the largest (c), average (b) and smallest (a) slope. If we compare Fig. 9 to Fig. 4 we see that the expected correlation is weaker for the samples of 70 pulsars in population 2a. This is clearly due to the weaker selection effects in comparison to population 1 and because of the larger samples. We have also calculated the distribution of correlation coefficients for the 100 samples. This distribution is plotted in Fig. 10. We have done the same calculations for population 2b, in which the cut-off line in the $v_t$ versus distance diagram is not present.
Fig. 7. The luminosity $L_{400}$ versus the distance for population 1 pulsars.

Fig. 8. The transverse velocity versus the distance for population 1 pulsars.
Fig. 9. The transverse velocity versus magnetic field strength for population 2a generated by our Monte Carlo simulation. From this population a 100 different samples of 70 pulsars were randomly chosen. For each sample the best straight line fit was calculated. Plotted are the lines with the smallest slope (a), the average slope (b) and the largest slope (c).

Fig. 10. The distribution of correlation coefficients for the 100 randomly chosen samples of 70 pulsars, from population 2a.
In Fig. 11 we plotted the distribution of the expected correlation coefficients for 100 randomly chosen samples of 70 pulsars from our computer-generated population 2b.

In the analysis of his sample of 71 pulsars Cordes has excluded 12 pulsars which "are clear outliers with respect to the bulk of the objects". For the reduced sample of 59 pulsars he finds a correlation coefficient of 0.53 and we would therefore not expect this correlation to be purely due to selection effects, as can be seen from Figs. 10 and 11. However, we think that excluding those pulsars that do not seem to fit a presupposed relation is not a correct procedure. We have therefore calculated the correlation coefficient for a sample of 69 pulsars (leaving out PSR 2224+65 because of its extremely large velocity of 1746±312 km/s and PSR 0820+02, which is in a binary). We found a value of .20 and comparing this value with the expected distributions of correlation coefficients we see that the probability of finding a coefficient larger than 0.20 is about 20% for our population 2a and about 30% for population 2b. The observed correlation in the sample of 69 pulsars could therefore be purely due to selection effects and it is even possible that the second selection effect is not needed to explain the observed correlation. If, however, we compare Fig. 1b to 9 we see that our computer simulation does not generate pulsars with magnetic fields less than B ~ 3 10^{11} Gauss and velocities less than v ~ 22 km/s. Such pulsars are, however, present in the Cordes sample and are numbered 1 to 10 in Fig. 1b. The pulsars 1 to 5 are the same as in the Anderson and Lyne sample and are thought to be recycled. Pulsar 6 is PSR 1937+214, which is the millisecond pulsar and is also thought to be recycled. Pulsar 10, PSR 0820+02, can safely be
excluded because it is a member of a binary. We exclude the other three pulsars simply because we do not expect them from any of our computer simulations and they therefore also seem to be part of a special population, like the pulsars 1 to 5 in the Anderson and Lyne sample. When we exclude these pulsar 1 to 10 from the Cordes sample we find a reduced correlation coefficient of 0.11, which is in very good agreement with the value expected from our Monte Carlo simulations (see Figs. 10 and 11).

5.3. Conclusion

Although one of the selection effects (i.e. flux limitation combined with the correlation between luminosity and field strength) was already mentioned by Anderson and Lyne, our Monte Carlo simulation shows the importance of the second effect, i.e., the fact that pulsars with a large distance need to have a large transverse velocity in order for their proper motions to be measurable. This selection effect is stronger than the first one, since it relates directly to one of the measured variables, \( v_t \), and the importance of it is clearly shown by the difference in the distribution of the correlation coefficients between Figs. 5 and 6.

Our conclusion is that the correlation found by Anderson and Lyne (1983), between the transverse velocity and the magnetic field strength for radio pulsars, is more strongly influenced by the presence of pulsars 1 to 5 than by the selection effects discussed here. We have shown that the probability that a correlation coefficient as large as the value 0.6, reported by Anderson and Lyne, is purely a consequence of selection effects, is only 3%. We conclude, therefore, that our calculations provide support for the idea of Radhakrishnan (1985) that the pulsars with numbers 1 to 5 in Fig. 1a form a separate population, with an origin different from that of the other pulsars. If they are excluded from the Anderson and Lyne sample, the reduced correlation coefficient is very close to that obtained from our Monte Carlo calculations. The agreement is still reasonable if one or two of these pulsars, numbers 1 to 5, are included, but certainly not all of them.

The correlation in the sample of 70 pulsars measured by Cordes (1985) is much weaker and could be expected from the influence of only the first selection effect (see Fig. 11). But our Monte Carlo simulation does not produce pulsars with weak fields and low velocities, while these are clearly present in the Cordes sample. If we, however, exclude these weak field and/or low velocity pulsars (i.e., numbers 1 to 10 in Fig. 1b) we are left with a sample of 60 pulsars for which the correlation between field strength and transverse velocity can be fully understood in terms of selection effects. The pulsars that are left
out include the numbers 1 to 5 from the Anderson and Lyne sample.

Thus, while the present study can explain a weak correlation between transverse velocity and magnetic field for normal strong field pulsars as due to selection effects, the observations of Anderson and Lyne and of Cordes can be understood only by the hypothesis that there is a "special" population of pulsars with weak fields and low space velocities, as was proposed by Radhakrishnan (1985). This population appears to make up some 15 to 20 percent of all pulsars.

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