Other people’s money: essays on capital market frictions

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Chapter 4

Sand in the Wheels of Capitalism

On the Political Economy of Capital Market Frictions

Abstract. This chapter develops a positive theory of capital market frictions, arising from a political conflict across different vintages of human capital. Older workers seek a political alliance to restrict the reallocation of capital between sectors, as this reduces their productivity and thus wages. Such an alliance is not feasible in a static framework, but may arise if capital market frictions are persistent over time. We show that a majority of voters chooses to restrict capital mobility if wealth is concentrated, and if technological obsolescence is high.

\footnote{This chapter is based on joint work with Enrico Perotti and Ernst-Ludwig von Thadden. For helpful comments we thank Philippe Aghion, Per Krusell, Enrique Schroth and seminar audiences at the ESWC 2010 Congress in Shanghai, and the EEA 2010 Congress in Glasgow.}
4.1 Introduction

Political economists say that capital sets towards the most profitable trades, and that it rapidly leaves the less profitable non-paying trades. But in ordinary countries this is a slow process.

(Bagehot, 1873)

In a free market, capital moves naturally towards its most profitable use, leaving less productive activities. In reality, capital is reallocated fast in some countries, slow in others. Wurgler (2000) provides evidence that industries with better growth prospects invest more in countries that are more financially developed; these are also the countries in which declining sectors shrink faster. In a neoclassical economic framework, the financial sector should be functional to the needs of industry and trade, and these differences are attributed to institutional frictions in capital markets.

There are clear examples of institutional frictions in capital markets which are hard to reverse. Bankruptcy law, for instance, defines specific conditions to the assignment of assets from declining sectors. While in some countries bankruptcy law is designed to protect financial interests, in others—such as France and Italy—it explicitly instructs the liquidator to reassign capital in a manner which protects employment. As another example, state banking or specific financial regulators may be chosen on a mandate to protect traditional lending. For many years, banking in the U.S. was restricted to be local, assigning control over credit to established interests.

In this chapter, we adopt a political economy approach to explain the emergence and persistence of capital market frictions. In the tradition of classical political economy, we view the rules on capital reallocation as resulting from the political process that is shaped by economic interests. In practice, the allocation of capital across industries is heavily politicized, especially in more democratic countries. Political intervention may be direct, as when the government provides emergency loans or acquires companies outright; or indirect, as when the govern-
ment adopts takeover regulations or bankruptcy laws that affect how much capital is reallocated.

Our starting insight is that labor is less mobile than capital. Here one should think of redeployable capital, such as land. While land can easily be redeployed, it is hard to retrain workers once they have acquired specific human capital. As human capital risk cannot be fully insured—for moral hazard reasons—workers are exposed to the risks that are specific to the sector they work in. The result of human capital specificity is a political conflict between citizens with different vintages of sunk human capital: agents with sector-specific human capital resist the reallocation of capital to newer sectors, as this leads to a reduction in their wages.

We show that in democracies a majority of the population wants to restrict capital mobility when the redistributive risk is large. This will be the case if wealth is concentrated, and if technological obsolescence is high. Young workers are the decisive, or pivotal, group in elections; they do not gain from capital market frictions immediately, but they would like to limit future capital reallocation, anticipating their old age, when they are less productive. A consumption smoothing motive then leads young workers’ preferences to be partially aligned with the preferences of the old workers. Rapid technological change implies that the productivity gap between young and old workers is bigger, and therefore that the motive to impede capital reallocation is stronger.

An alliance against capital reallocation is never a political equilibrium when the capital market friction can be repealed at any time in the future. An alliance against capital reallocation can arise only if capital market frictions are persistent over time. So we posit that capital market frictions may occur if they can be introduced as institutional frictions—as opposed to reversible legislative choices.

4.1.1 Related Literature

Wurgler (2000) provides evidence that capital is reallocated more efficiently in countries with (i) more developed financial markets, (ii) a higher degree of minority investor protection, and (iii) a lesser extent of state ownership in the economy.
Countries with deeper financial markets increase investment more in growing industries, and decrease investment more in declining industries. Our political explanation seeks to explain this pattern by endogenizing the resistance to capital reallocation.

This chapter is a contribution to the literature on the political determinants of financial market regulations and corporate governance, Pagano and Volpin (cf. 2005) or Perotti and von Thadden (2006). This literature emphasizes that the economic interests of capital investors can be subordinated to political considerations. In Pagano and Volpin (2005), labor forms an alliance with inside shareholders, in the contrast of a corporatist alliance with labor against financial investor’s return. In Perotti and von Thadden (2006), a majority limits the ability of shareholders to allocate capital in order to limit risk for other stakeholders. This result arises only in more unequal societies, as in this chapter. Other related papers include Krusell and Ríos-Rull (1996) and Saint-Paul (2002), who study the political support for technological innovation, and labor market flexibility. The capital market plays no role in these papers, while it is central in this chapter. We offer an alternative channel to advance stakeholder interests: capital market frictions.

Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) study the political support for a distortionary welfare state. The welfare state distorts private incentives to invest in education, which in turn gives rise to a constituency that supports the welfare state. Hassler et al. (2003) provide an example of how repeated majority voting in an OLG model can generate persistence in support of an inefficient welfare policy, we provide another. A key difference in our model is that all constituencies vote, whereas the young are disenfranchised in Hassler et al. (2003) Another difference is that our current framework does not allow for dynamic feedback of political choices through incentives as in Hassler et al. (2003); the composition of constituencies is fixed in this chapter.

Our approach is close to Azariadis and Galasso (2002), who study the political support for intergenerational transfers from young to old generations (such as pay-as-you go pension systems). They show that the young generation, who form
a majority, may choose to set positive transfers if they can expect to receive a transfer when old. Our approach differs in two important aspects. First, we study political support for distortionary policies, whereas intergenerational transfers are efficient in Azariadis and Galasso (2002). Second, the young are a majority in Azariadis and Galasso (2002), while the outcome of our voting game is more complex: based on age and wealth differences, we identify four distinct voter classes, none of which forms a majority. Young workers are decisive because their preferences are less extreme than the preferences of other voter classes, not because of their number.

4.2 Model

We use a repeated two-period overlapping-generations model with an infinite horizon. Production requires capital and labor, and takes place in two sectors: a sector of young firms that employ the young generation; and a sector of old firms that employ the old generation. Labor is sector-specific while a fixed supply of capital can be used by all firms. We ignore capital growth in order to focus on the question how capital is allocated among different sectors. Time is denoted by subscripts $t = 0, 1, 2, \ldots$; the different sectors are denoted by superscripts $j = Y, O$.

4.2.1 Production

At each time, there is a unit mass of identical firms in the young sector and a unit mass of identical firms in the old sector. All firms exist for two period; they use a vintage technology to produce a common consumption good that cannot be stored or saved. As young firms use the latest technology, they are more productive than old firms.

Production in each sector is given by a sector-specific productivity factor $\theta^j$ and a general production function $F$; production in the $j$-sector is

$$\theta^j F(K^j, L^j), \ j = Y, O$$
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where $K^j$ and $L^j$ denote the amounts of capital and labor used in sector $j$, $\theta^O < \theta^Y$, and the price of output is normalized to 1. The production function $F$ satisfies the common conditions, i.e., (i) production is increasing in both factors, at a decreasing rate; (ii) capital and labor are complementary factors of production; and (iii) the Inada conditions are satisfied.\footnote{Formally, $F$ satisfies (i) $F_K, F_L > 0$ and $F_{KK}, F_{LL} < 0$, (ii) $F_{LK} = F_{KL} > 0$, and (iii) $\lim_{K \to 0} F_K = \lim_{L \to \infty} F_L = \infty$, and $\lim_{K \to \infty} F_K = \lim_{L \to \infty} F_L = 0$.} Firms maximize profits facing competitive factor and output markets. Firms hire workers in competitive segmented labor markets and sell their output in a competitive output market. Reallocated capital is subject to a politically determined capital market friction.

Due to human capital specificity, the labor market is segmented: old firms hire old workers and pay wages $w_t^O$; young firms hire young workers and pay wages $w_t^Y$. The capital market has two important features: the cost of last period’s retained capital is $r_t$, while firms pay an additional cost, $c_t$, if they wish to employ additional capital this period. The cost $c_t$ represents a pure deadweight loss; it drives a wedge between the interest rate that capitalists receive and the cost of capital that firms pay. We refer to $c_t$ as the capital market friction or, simply the reallocation cost.

As there are two costs of capital in the economy—one for retained capital and one for newly obtained capital—a firm’s capital cost depends on the capital stock at the start of each period. We denote the initial capital stock of $j$-firms in period $t$ by $\hat{K}_t^j$. Profits by firms of age $j$ in period $t$ are then given by

$$\theta^j F(K_t^j, L_t^j) - r_t K_t^j - c_t \max(0, K_t^j - \hat{K}_t^j) - w_t^j L_t^j$$

This is a standard expression for firm profits, except for the third term. Firms pay a marginal cost of capital $r_t + c_t$, if they want to attract capital beyond the initial stock of $\hat{K}_t^j$.

Young firms don’t have retained capital ($\hat{K}_t^Y = 0$) and must attract all capital at a unit cost of $r_t + c_t$. When young firms turn into old firms—and the young
generation turns into the old generation–they retain last period’s capital ($\hat{K}^O_t = K^{Y}_{t-1}$). As old firms are less productive than young firms, there is an economic rationale for capital reallocation from old to young firms. When old firms go extinct, the capital they previously employ comes available to use elsewhere.

### 4.2.2 Agents

At each time, there are two generations of agents, the young and the old, each of unit mass. Young agents work in young firms, old agents work in old firms. All agents inelastically supply labor normalized at $\bar{L} = 1$ per period.

The fixed capital stock, $\bar{K} > 0$, is owned the capitalists, a fraction $\eta$ of the old generation. Capitalists receive all firm profits and interest payments; they are identical and diversified. It follows that capitalists receive

$$w^O_t + \frac{r_t + \Pi_t}{\eta}$$

where $\Pi_t$ denotes aggregate firm profits. When capitalists die, a subset of their children inherits the capital stock; a fraction $\eta$ of the young workers turns into old capitalists.

The population falls into four groups with identical lifetime income: young workers ($YW$), old workers ($OW$), young workers that will be capitalists ($YC$), and old capitalists ($OC$). The fraction of capitalists $\eta \in (0,1)$ is a measure of inequality among the old: higher $\eta$ means more capitalists and less wealth per capitalist.

As income cannot be saved or stored, agents consume all income in each period. The lifetime utility of the young generation at time $t$ is given by

$$U^{YW}_t := u(w^Y_t) + \delta u(w^{O}_{t+1})$$

---

3 All the debt and equity in the economy is owned by the capitalists, who all hold the same portfolio of assets.
for young workers, and by

$$U^Y_t := u(w^Y_t) + \delta u(w^{O}_{t+1} + s_{t+1}) \quad (4.2.2)$$

for young capitalists, where $\delta \in (0, 1]$ is the time discount factor; $u$ is a standard felicicy function with $u' > 0$ and $u'' < 0$; and $s_t := \frac{1}{\eta} (r_t \bar{K} + \Pi_t)$. Remaining lifetime utility of the old generation at time $t$ is then

$$U^{OW}_t = u(w^{O}_t) \quad (4.2.3)$$

for the old workers, and

$$U^{OC}_t = u(w^{O}_t + s_t) \quad (4.2.4)$$

for the old capitalists. Note that agents do not optimize over economic choices, as they don’t save and supply labor inelastically. Instead, agents optimize over political choices, by choosing a capital market friction in each period.

### 4.2.3 Interaction

Firms and agents interact in competitive factor and product markets. The product market is competitive and the price of the unique consumption good is normalized to 1. Each segment of the labor market is competitive and wages, $w^Y_t$, adjust until the markets for young and old workers clear. There is one market for capital; it is competitive in the sense that the interest rate, $r_t$ adjusts until the market clears, but transactions on this market are subject to the reallocation cost, $c_t$.

The reallocation cost is set by a vote: preceding market interaction, agents vote over $c_t$ in each period. We return to the voting process in section 4, when we discuss political equilibrium. Timing in each period is as follows,

1. an initial allocation of capital is inherited from the previous period;
2. agents vote over the capital market friction $c_t$;
3. economic activity results in a new allocation of capital; and
4. agents get their payoff, i.e., their wage and capital income.

The political conflict has two dimensions: there is a class conflict between capitalists and workers; and there is a generational conflict between young and old. Old workers workers stand to lose most from free capital mobility: their wage drops, as capital is reallocated from old firms to young firms. Old capitalists too see their labor income drop, but their capital income increases. Preferences of the young generation depend on the nature of the capital market frictions, in particular, whether they are linked over time. Young workers may vote in favor of a positive reallocation cost, if they expect it to prevail until they are old. We analyze policy preferences and the resulting political equilibria in section 4.4. First we characterize the set of economic equilibria for a given sequence of capital market frictions.

### 4.3 Economic Equilibrium

#### 4.3.1 Existence and Characterization

For a given sequence of capital market frictions,

**Definition.** An economic equilibrium is given by a sequence of factor prices and capital allocations \( E = \{r_t, w_Y^t, w_O^t, K_Y^t, K_O^t \}_{t=0}^\infty \) such that in every period (i) firms maximize profits, and (ii) markets clear.

We prove the existence and uniqueness of an economic equilibrium for any sequence of capital market frictions \( \{c_t\}_{t=0}^\infty \) with \( 0 \leq c_t < \infty \). In each period, firms take all prices as given and maximize the period profits.\(^4\) Young firms start without capital, and pay the reallocation cost on each unit of capital they employ. They solve

\[
\max_{K_Y^t, L_Y^t} \theta^Y F(K_Y^t, L_Y^t) - (r_t + c_t)K_Y^t - w_Y^t L_Y^t
\]

\(^4\)Firms maximize profits that accrue to their owners, i.e. the capitalists. As all capitalists are old, firms maximize current profits only.
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which leads to standard first-order conditions

\[ \theta^y F_K(K^y_t, L^y_t) = r_t + c_t \]  \hspace{1cm} (4.3.1)  
\[ \theta^y F_L(K^y_t, L^y_t) = w^y_t \]  \hspace{1cm} (4.3.2)  

and corresponding capital and labor demand, \( K^y_t(r_t, w^y_t) \) and \( L^y_t(r_t, w^y_t) \). Old firms retain the capital they employed last period; they solve

\[ \max_{K^O_t, L^O_t} \theta^O F(K^O_t, L^O_t) - r_t K^O_t - c_t \max(0, K^O_t - \hat{K}^O_t) - w^O_t L^O_t \]

which leads to standard first-order condition

\[ \theta^O F_L(K^O_t, L^O_t) = w^O_t \]  \hspace{1cm} (4.3.3)  

and corresponding labor demand \( L^O_t(w^O_t, r_t) \). Note that the profit function of old firms is not differentiable at \( \hat{K}^O_t \) and capital demand depends on whether old firms adjust their capital. If old firms acquire extra capital in equilibrium, then \( K^O_t \) must satisfy

\[ \theta^O F_K(K^O_t, L^O_t) = r_t + c_t \]  \hspace{1cm} (4.3.4)  

which is consistent if old firms indeed scale up, i.e., if

\[ \theta^O F_K(K^O_t, L^O_t) < \theta^O F_K(\hat{K}^O_t, L^O_t) \]

or

\[ r_t < \theta^O F_K(\hat{K}^O_t, L^O_t) - c_t \]

Old firms increase their capital if the interest rate is sufficiently small; likewise, old firms decrease their capital if the interest rate is sufficiently big, or

\[ r_t > \theta^O F_K(\hat{K}^O_t, L^O_t). \]

For intermediate values of the interest rate, old firms keep using the capital they
from last period, \( \hat{K}_t^O = K_{t-1}^Y \).

In each period, young and old agents inelastically supply labor, normalized to 1, in their respective labor markets. As both segments of the labor market must clear in equilibrium, we obtain the equilibrium wage rate as a function of capital from (4.3.2) and (4.3.3):

\[
\begin{align*}
  w_t^Y &= \theta^Y F_L(K_t^Y, 1) \\
  w_t^O &= \theta^O F_L(K_t^O, 1).
\end{align*}
\]

Sector wages increase, or decrease, along with an increase, or decrease, of sector capital.

Capital demand of young firms follows from (4.3.1) and, letting \( g(K) := F_K(K, 1) \), we can write

\[
K_t^Y(r_t) = g^{-1}\left(\frac{r_t + c_t}{\theta^Y}\right) \tag{4.3.7}
\]

Our earlier discussion shows that capital demand of old firms is

\[
K_t^O(r_t) = \begin{cases} 
  g^{-1}\left(\frac{r_t + c_t}{\theta^O}\right) & \text{if } r_t < r_t^L \\
  g^{-1}\left(\frac{r_t}{\theta^O}\right) & \text{if } r_t > r_t^L \\
  \hat{K}_t^O & \text{if } r_t^L \leq r_t \leq r_t^L
\end{cases} \tag{4.3.8}
\]

with \( r_t^L \equiv \theta^O g(\hat{K}_t^O) - c_t \), and \( r_t^L \equiv \theta^O g(\hat{K}_t^O) \). Total capital demand then is

\[
\varphi_{c_t}(r_t) := K_t^Y(r_t) + K_t^O(r_t) \tag{4.3.9}
\]

and the capital market clears if \( \varphi_{c_t}(r_t) = \hat{K} \). The following lemma shows that a market clearing interest rate exists and is unique in each period.

**Lemma 4.3.1.** For \( 0 \leq c_t < \infty \), there exists a unique market clearing interest rate \( r_t^* = r_t^*(c_t) \).

**Proof.** For any \( 0 \leq c_t < \infty \), \( \varphi_{c_t} \) is continuous, strictly decreasing, and piecewise differentiable in \( r_t \) (with kinks at \( r_t^L \) and \( r_t^L \)). Furthermore, by the Inada conditions
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we have

\[ \lim_{r_t \to -\infty} \varphi_{c_t}(r_t) = 0 \quad \text{and} \quad \lim_{r_t \to -c_t} \varphi_{c_t}(r_t) = \infty \]

Hence by the continuity of \( \varphi_{c_t} \), there is \( r_t^* > -c_t \) such that

\[ \varphi_{c_t}(r_t^*) = \bar{K} \quad (4.3.10) \]

By the strict monotonicity of \( \varphi_{c_t} \), \( r_t^* \) is unique.

From the market clearing interest rate, \( r_t^* \), equilibrium capital demands and sector wages readily follow.\(^5\) Hence, we have proven the following proposition:

**Proposition 4.3.1.** For any sequence of capital market frictions \( \{c_t\}_{t=0}^{\infty} \) with \( 0 \leq c_t < \infty \), there exists a unique economic equilibrium \( E \).

The previous argument, and in particular capital demand of old firms, given by (4.3.8), shows that the economic equilibrium is characterized by the initial capital of the old firms, \( \hat{K}_t^O \), and the prevailing capital market friction in the capital market, \( c_t \). We characterize firm behavior in equilibrium in the following.

Depending on the equilibrium interest rate, old firms adjust their capital, down- or upward, or they keep the capital of last period. We investigate these cases in turn. Old firms scale up capital if and only if the interest rate is sufficiently small, or \( r_t^* < r_t \), cf. (4.3.8). Market clearing then reads

\[ g^{-1} \left( \frac{r_t^* + c_t}{\theta^O} \right) + g^{-1} \left( \frac{r_t^* + c_t}{\theta^O} \right) = \bar{K} \quad (4.3.11) \]

which implicitly gives the equilibrium interest rate, \( r_t^* \); old firms’ capital is given by

\[ \hat{K}_t^O := g^{-1} \left( \frac{r_t^* + c_t}{\theta^O} \right) \quad (4.3.12) \]

\(^5\)If the market clearing interest rate is negative, we set it to 0, which implies that some capital remains unused. We show in section 4.4.1 that no voter class wishes to set \( c_t \) so high as to induce a negative interest
It follows that old firms scale up if and only if \( \hat{K}_t^O < K_t^O \). Note that, through (4.3.11), old firms’ equilibrium capital, \( K_t^O \), does not depend on \( c_t \).

Similarly, old firms scale down capital if and only if the interest rate is sufficiently big, or \( r_t^* > \hat{r}_t \). Market clearing then reads

\[
g^{-1} \left( \frac{r_t^* + c_t}{\theta^Y} \right) + g^{-1} \left( \frac{r_t^*}{\theta^O} \right) = \bar{K}
\]

which gives the interest rate. Old firms’ capital is given by

\[
\hat{K}_t^O := g^{-1} \left( \frac{r_t^*}{\theta^O} \right)
\]

so that old firms scale down if and only if \( \hat{K}_t^O < \bar{K}_t^O \). Note that equilibrium capital of old firms, \( \bar{K}_t^O = \hat{K}_t^O(c_t) \), is a function of the capital market friction, \( c_t \).

Finally, old firms keep their initial capital if and only if

\[
K_t^O \leq \hat{K}_t^O \leq \bar{K}_t^O(c_t)
\]

in which case the equilibrium interest rate is given by

\[
r_t^* = \theta^Y g(\bar{K} - \hat{K}_t^O) - c_t
\]

This concludes the description of old firms in equilibrium.

Equilibrium behavior of young firms is easily described: as they have no initial capital, they must adjust capital upward. It follows that equilibrium capital of young firms is given by

\[
g^{-1} \left( \frac{r_t^* + c_t}{\theta^Y} \right)
\]

where \( r_t^* \) depends on the equilibrium behavior of old firms, i.e., on \( \hat{K}_t^O \) and \( c_t \).

As an illustration, consider the economic equilibrium if there is no capital market friction, i.e., if \( c_t = 0 \). Then there is only one cost of capital in the economy
and capital market clearing reads

\[ g^{-1}\left(\frac{r_t^*}{\theta^Y}\right) + g^{-1}\left(\frac{r_t^*}{\theta^O}\right) = \bar{K} \]

As old firms are less productive than young firms, old firms employ less capital than young firms in equilibrium if there is no capital market friction. If the capital market friction is positive \((c_t > 0)\), then the decision of old firms to adjust capital can go either way—as we have shown—and there is no guarantee that old firms employ less capital than young firms. But in a dynamic economic equilibrium, firms in the old sector do not scale up, except possibly in the first or second period..

**Lemma 4.3.2.** In economic equilibrium, old firms do not scale up if \(t \geq 2\).

**Proof.** Consider an arbitrary economic equilibrium and fix \(t \geq 2\). We prove that \(\hat{K}_t^O \geq K_t^O\), so that old firms do not scale up in period \(t\). For our argument we consider the economic equilibrium at time \(t-2\) and assume that \(c_{t-2} = 0\). Then old firm equilibrium capital is given by

\[ K_{t-2}^{O*} = K^O \]

and is minimized across all state-parameter pairs \((\hat{K}_{t-2}^O, c_{t-2})\). By market clearing, it follows that young firm capital \((K_{t-2}^{Y*})\) is maximized across state-parameter pairs, and we denote it by \(\bar{K}^Y\). Note that \(\bar{K}^Y\) exceeds \(K^O\), as young firms are more productive than old firms. Moving forward one period, it follows that \(\hat{K}_{t-1}^O > K^O\), which means that old firms do not scale up in period \(t-1\). Hence equilibrium old firm capital satisfies \(K_{t-1}^{O*} \leq \bar{K}^Y\), and market clearing implies that equilibrium young firm capital satisfies \(K_{t-1}^{Y*} \geq K^O\). We have shown that the minimum value \(K_{t-1}^{Y*}\) can take in equilibrium exceeds \(K^O\). \(\square\)

The intuition for Lemma 4.3.2 is that, independent of the sequence of capital market frictions, young firms choose to employ more capital than what they will need when they are old firms.
4.3.2 Steady States

For arbitrary sequences of frictions \( \{c_t\}_{t=0}^{\infty} \), capital market activity has little structure. As we are mostly interested in stable political outcomes that yield constant sequences of \( c_t \), we look for steady state equilibria.

**Definition.** A steady state is an economic equilibrium such that 
\[
(K^Y_t, K^O_t) = (K^Y_{t+1}, K^O_{t+1})
\]
for all \( t \).

In a steady state, the amount of capital that is reallocated from old firms to young firms is constant over time; it is positive by lemma 4.3.2. The next proposition gives all steady states that exist for a constant sequence of capital market frictions.

**Proposition 4.3.2.** Given a constant sequence of capital market frictions \( \{c\}_{t=0}^{\infty} \), there exists a bound \( \bar{c} > 0 \) such that

1. for \( c \in [0, \bar{c}] \), the unique steady state equilibrium is given by

\[
K^{Y*} = g^{-1} \left( \frac{r^* + c}{\theta^Y} \right), \quad K^{O*} = g^{-1} \left( \frac{r^*}{\theta^O} \right)
\]  

(4.3.16)

with \( r^* \) given by the market clearing condition,

\[
g^{-1} \left( \frac{r^* + c}{\theta^Y} \right) + g^{-1} \left( \frac{r^*}{\theta^O} \right) = \bar{K}
\]  

(4.3.17)

(a) for \( c \geq \bar{c} \), the unique steady state equilibrium is given by

\[
K^{Y*} = K^{O*} = \frac{1}{2} \bar{K}
\]  

(4.3.18)

with \( r^* = \theta^Y g(\frac{1}{2} \bar{K}) - c \).

**Proof.** For 1: Let time \( t \) capital allocations be given by (4.3.16). We check that the same allocation of capital results in period \( t + 1 \), for which it suffices to check
that old firms scale down in period $t + 1$. Old firms scale down if and only if

$$\hat{K}^O_{t+1} = K^*_t > \hat{K}^O_{t+1}$$  \hspace{1cm} (4.3.19)

where $\hat{K}^O_{t+1} = \hat{K}^O(c)$ is the equilibrium cut-off level defined by (4.3.14). Note that condition (4.3.19) holds for $c = 0$, since without frictions the more productive young firms attracts more capital than the old firms. Taking the total derivative of (4.3.17) with respect to $c$ shows that $r^*_t + c$ is strictly increasing in $c$. It follows that $K^*_t$ is strictly decreasing in $c$. Furthermore, $\hat{K}^O_{t+1}$ is strictly increasing in $c$ as noted after (4.3.14). Hence, the proposed equilibrium is a steady state for $c \in [0, \bar{c}]$, with $\bar{c}$ defined by

$$K^*_t(\bar{c}) = \hat{K}^O(\bar{c})$$  \hspace{1cm} (4.3.20)

Now consider the steady state for $c = \bar{c}$. By the definition of $\bar{c}$, we have $\hat{K}^O = K^O$ which implies that old firms do not adjust capital, and so $K^O = \hat{K}^O$. Since we also have $\hat{K}^O = K^Y$, it follows that $K^O = K^Y = \frac{1}{2} \hat{K}$ in this boundary case steady state. With (4.3.16), we then obtain $r^*$ and $\bar{c}$:

$$r^* = \theta^O g \left( \frac{1}{2} \hat{K} \right)$$

and

$$\bar{c} = (\theta^Y - \theta^O) g \left( \frac{1}{2} \hat{K} \right)$$  \hspace{1cm} (4.3.21)

For 2: let $c > \bar{c}$ and the time $t$ allocations be given by (4.3.18). This allocation induces the same capital allocation in period $t + 1$, if

$$K^O \leq \frac{1}{2} \hat{K} < \hat{K}^O_{t+1}$$

The first inequality holds trivially by (4.3.11) and (4.3.12), the second holds by the definition of $\bar{c}$ since $\hat{K}^O_{t+1} = \hat{K}^O(c)$. Finally, note that every economic equilibrium is unique by lemma 4.3.1. \hfill \Box

Proposition 4.3.2 gives all steady state equilibria that exist for constant sequences
of capital market frictions \( \{c_t\}_{t=0}^{\infty} \). We denote these steady states by \( E_c \).\(^6\) As a benchmark, consider the steady state equilibrium that results if the capital market is frictionless, \( E_0 \). If the sequence of capital market frictions is given by \( c_t = 0 \) for all \( t \), then all capital has the same rental price. It follows from proposition 4.3.2 that young firms’ capital is maximized in this steady state—and old firms’ capital minimized. As total capital is constant and young firms are more productive than old firms, \( E_0 \) is the steady state in which maximum output is achieved. Hence \( c = 0 \) would be chosen by a social planner if lump sum transfers are available.

In any period, a positive capital market friction reduces output, an economic inefficiency. The economic inefficiency, however, does not constitute a Pareto inefficiency, as old workers stand to benefit from it. We rule out vote buying as a means to restore the economically efficient outcome, i.e., we rule out that the capitalists compensate the workers to vote for a free (or frictionless) capital market. As Acemoglu (2003) has argued, there is an essential hold-up problem that prevents such trades from taking place: if the workers vote for \( c_t = 0 \), the capitalists have no incentive to compensate them ex post; likewise, the workers have no incentive to vote for \( c_t = 0 \), if they receive the compensation upfront.

### 4.4 Political Equilibrium

We endogenize the sequence of capital market frictions by treating the frictions as politically determined. To include politics, we extend the economic model by a simple majority vote in each period. In order to obtain closed form solutions, we assume that production is Cobb-Douglas, with decreasing returns to scale; production is given by

\[
F(K, L) = K^\alpha L^\beta
\]

with \( 0 < \alpha + \beta < 1 \). We derive steady state capital allocations and factor prices in appendix A.3.1.

\(^6\)From the Proof of Prop 4.3.2, we see that a steady state with \( K^Y = K^O = \frac{1}{2} \bar{K} \) also exists for non constant sequences \( \{c_t\}_{t=0}^{\infty} \) as long as \( \bar{c} < c_t < \infty \). We do not consider these in the following.
4.4.1 Policy Preferences

Voter preferences follow from the lifetime utility functions (4.2.1) - (4.2.4) and equilibrium factor prices. Remember that for a given value of the reallocation cost, $c_t$, the economic equilibrium at time $t$ is fully characterized by the initial capital of old firms, $\hat{K}_t^O$. The next lemma establishes two useful properties of the equilibrium interest rate

**Lemma 4.4.1.** In each period, the equilibrium interest rate $r_t^*$ is strictly decreasing in the reallocation cost $c_t$, and $r_t^* + c_t$ is nondecreasing in $c_t$.

**Proof.** (i) Let $0 < c_t < \infty$. By lemma 4.3.1, there is a unique $r_t^*(c_t)$ such that

$$\varphi_{c_t}(r_t^*(c_t)) = \bar{K}$$

Now consider a $c_t' > c_t$. An inspection of (4.3.9) shows that $\varphi_{c_t'} < \varphi_{c_t}$ uniformly, so $\varphi_{c_t'}(r_t^*(c_t)) < \bar{K}$. Again by lemma 4.3.1, there is a unique $r_t^*(c_t')$ such that

$$\varphi_{c_t'}(r_t^*(c_t')) = \bar{K}$$

Because $\varphi_{c_t'}$ is strictly monotonically decreasing, it follows that $r_t^*(c_t') < r_t^*(c_t)$.

(ii) Suppose that $r_t^*(c_t) + c_t$ is decreasing in $c_t$. Then result (i) implies that $K_t^{Y^*}$ and $K_t^{O^*}$ must increase in $c_t$ which contradicts market clearing. □

The following lemma summarizes the comparative statics of factor prices, sector capital and voter class income with respect to $c_t$. In order to have the necessary generality to analyze out-of-steady-state deviations, the lemma is formulated for arbitrary economic equilibria.

**Lemma 4.4.2.** Let $\bar{c}_t$ be defined by

$$\dot{K}_t^O = \dot{K}_t^O(\bar{c}_t). \tag{4.4.2}$$

\footnote{From lemma 4.3.2 we know that $\dot{K}_t^O \geq K_t^O$, because old firms do not scale up in equilibrium.}
Then (i) $K^Y_t$ and $w^Y_t$ are strictly decreasing in $c_t \in [0, \bar{c}_t]$ and constant for $c_t > \bar{c}_t$; (ii) $K^O_t$ and $w^O_t$ are strictly increasing in $c_t \in [0, \bar{c}_t]$ and constant for $c_t > \bar{c}_t$; (iii) $w^O_t + s_t$ is decreasing in $c_t$.

Proof. See appendix A.3.2

The function $\bar{K}^O_t$ that yields the equilibrium cut-off value $\bar{K}^O_t(\bar{c}_t)$ has been defined after (4.3.14). Note that $\bar{c}_t$ is the boundary value of the capital market friction at which old firms keep their initial capital $\bar{K}^O_t$: for lower values of the friction, old firms scale down.\(^8\) The proof of the lemma is straightforward but long and we provide it in the appendix.

Lemma 4.4.2 shows that total income of old capitalists is decreasing in $c_t$, even as wages may be increasing; the positive wage effect is dominated by the negative capital income effect.\(^9\) Note that once $c_t$ is fixed for a given period, the equilibrium interest rate $r_t^*$ and all other time $t$ equilibrium values readily follow. Hence we may write lifetime utility of agents as a function of the capital market frictions that prevail today and tomorrow:

$$U^i_t = U^i_t(c_t, c_{t+1}) \text{ for } i \in \{OW, OC, YW, YC\}$$

We note that income of the old generation does not, in fact, depend on $c_{t+1}$. The following definition of single peakedness goes back to Black (1948).

**Definition.** Policy preferences of voter class $i \in \{OW, OC, YW, YC\}$ are single-peaked if the following statement is true:

for any

$$c^i \epsilon \underset{c_t}{\arg \max} U^i_t(c_t, c_{t+1})$$

if $c'' \leq c' \leq c^i$, or if $c^i \leq c' \leq c''$, then $U^i_t(c'', c_{t+1}) \leq U^i_t(c', c_{t+1})$.

\(^8\)Since we consider deviations out-of-steady-state here, we may restrict attention to values $\bar{K}^O_t \geq \frac{1}{2} \bar{K}$ in the following (cf. proposition 4.3.16). It follows that $\bar{c}_t \geq \bar{c}$, where $\bar{c}$, given by (4.3.20), is the boundary value of the steady state friction for which no reallocation of capital takes place.

\(^9\)This holds for any value of $\eta$. 

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As is well-known, with single-peaked preferences we can apply a median voter theorem: for existence of an equilibrium of the simple majority vote, it suffices that all voters have single-peaked preferences. Preferences of the old generation can easily be characterized.

**Lemma 4.4.3.** In each period, preferences of the old generation are single-peaked, and their preferred policies are

\[
\text{arg max}_{c_t} U_t^{OW}(c_t, c_{t+1}) = [\bar{c}_t, \infty)
\]

for the old workers, and

\[
\text{arg max}_{c_t} U_t^{OC}(c_t, c_{t+1}) = 0
\]

for the old capitalists.

**Proof.** The preferences over \(c_t\) follow from lemma 4.4.2. Because the utility of the \(OC\) and \(OW\) class is monotonic in \(c_t\), it follows that they are also single peakedness.

Turning to the preferences of the young, we note that lifetime utility of the young generation depends on current policy \(c_t\) as well as future policy \(c_{t+1}\). Keeping tomorrow’s friction fixed, we obtain the young’s preferences over \(c_t\).

**Lemma 4.4.4.** Preferences of the young generation over \(c_t\) are single-peaked, and preferred policies are

\[
\text{arg max}_{c_t} U_t^{YW}(c_t, c_{t+1}) = 0
\]

and

\[
\text{arg max}_{c_t} U_t^{YC}(c_t, c_{t+1}) = 0
\]

**Proof.** Let \(c_{t+1}\) be fixed. The \(YW\) class and the \(YC\) class have the same income \(w_t^Y\) in period \(t\). By lemma 4.4.2, \(w_t^Y\) is strictly decreasing in \(c_t\). Hence, the utility of the \(YW\) and \(YC\) are monotonically decreasing in \(c_t\) which is sufficient for single peakedness.
A majority consisting of old capitalists and all the young favor a capital market without frictions. The intuition is that wages in the Y-sector and capitalist income are maximized in a frictionless capital market. Importantly, we have derived preferences keeping future policy fixed. If the future friction $c_{t+1}$ depends on the currently prevailing friction $c_t$, then preferences of the young change while preferences of the old are still given by lemma 4.4.3.

Lemmas 4.4.3 and 4.4.4 show that young capitalists achieve maximum lifetime utility if $c_t = 0$ and $c_{t+1} = 0$. Young workers on the other hand achieve maximum utility if $c_t = 0$ and $c_{t+1} \geq \bar{c}_{t+1}$. Young workers prefer a frictionless capital market when young and wish to see the maximum friction when old.

We turn to the question in which steady state the utility $U_j^i$ of the different voter classes $i \in \{OW, OC, YW, YC\}$ is maximized. Note that in steady state the reallocation cost is time independent, as are the utilities, so that we may write $U^i = U^i(c)$. It follows from the above that $U^{YC}$ and $U^{OC}$ are maximized in the steady state without friction, $E_0$. Old worker utility $U^{OW}$ is maximized in steady states with no capital reallocation, i.e. $E_c$ with $c \geq \bar{c}$. Young workers are the only group who may have more interesting preferences as we show in the following.

Consider young worker utility $U^{YW} = u(w^Y) + \delta u(w^O)$. Steady state wages follow from (4.3.5) and (4.3.6) and the steady state capital allocations we derived in section A.3.1. We have

$$w^Y = \theta^Y \beta \left( \frac{\alpha \theta^Y}{r(c)+c} \right)^{\frac{a}{1-a}}; \quad w^O = \theta^Y \beta \left( \frac{\alpha \theta^O}{r(c)} \right)^{\frac{a}{1-a}}$$

for $c \in [0, \bar{c}]$, and

$$w^Y = \theta^Y \beta \left( \frac{1}{2} \bar{K} \right)^{\frac{a}{1-a}}; \quad w^O = \theta^Y \beta \left( \frac{1}{2} \bar{K} \right)^{\frac{a}{1-a}}$$

for $c > \bar{c}$. The following lemma gives the preferred steady state policy of the young worker.

**Lemma 4.4.5.** Steady state preferences of the young worker are single-peaked.
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Their preferred policies are

• \( c^{YW} = 0 \) if \( \frac{u'(w^Y)}{\delta u'(w^O)} > \frac{r(c)}{r(c)+c} \) on \((0, \bar{c})\),

• \( c^{YW} = \bar{c} \) if \( \frac{u'(w^Y)}{\delta u'(w^O)} < \frac{r(c)}{r(c)+c} \) on \((0, \bar{c})\),

• otherwise \( c^{YW} \) is given by

\[
\frac{u'(w^Y)}{\delta u'(w^O)} = \frac{r(c^{YW})}{r(c^{YW})+c^{YW}} \tag{4.4.3}
\]

Proof. Consider \( U^{YW}(c) \) on \((0, \bar{c})\). Taking the derivative of \( U^{YW} \) with respect to \( c \), we get

\[
\frac{dU^{YW}}{dc} = u'(w^Y)\frac{dw^Y}{dc} + \delta u'(w^O)\frac{dw^O}{dc} \tag{4.4.4}
\]

Substituting \( \frac{dw^Y}{dc} \) and \( \frac{dw^O}{dc} \) in (4.4.4) yields

\[
\frac{dU^{YW}}{dc} = -\frac{\beta}{1-\alpha} \left[ u'(w^Y)\theta^Y\left(\frac{K^Y}{r+c}\right)^\alpha \left(\frac{dr}{dc} + 1\right) + \delta u'(w^O)\theta^O\left(\frac{K^O}{r}\right)^\alpha \frac{dr}{dc} \right]
\]

where \( K^Y \) and \( K^O \) are the steady state capital allocations given by (A.3.2) and \( r \) follows from \( K^Y + K^O = \bar{K} \). Implicit differentiation of the capital market clearing condition with respect to \( c \) shows that

\[
\frac{dr}{dc} + 1 = -\left(\frac{\theta^O}{\theta^Y}\right)^{\frac{1}{1-\alpha}} \left(\frac{r+c}{r}\right)^{\frac{1}{1-\alpha}} \frac{dr}{dc}
\]

which we use to obtain

\[
\frac{dU^{YW}}{dc} = -\frac{\beta}{1-\alpha} \frac{1}{dc} \frac{dr}{dc} K^O[\delta u'(w^O) r - u'(w^Y) (r + c)]
\]

Since \( r \) and \( K^O \) are nonnegative and \( \frac{dr}{dc} < 0 \), the first part of this expression is
positive for all c. Hence $\frac{dU^{YW}}{dc} \geq 0$ if and only if

$$\delta u'(w^O) r - u'(w^Y) (r + c) \geq 0 \quad (4.4.5)$$

Note that $u'(w^Y) (r + c)$ strictly increases in $c$ while $\delta u'(w^O) r$ strictly decreases in $c$. It follows that if for some $c$ condition (4.4.5) is satisfied with equality then it is the unique utility maximizing steady state policy. Otherwise, either $\delta u'(w^O) r - u'(w^Y) (r + c) > 0$ for all $c \in (0, \bar{c})$ so that $\text{Arg max} U^{YW}(c) = [\bar{c}, \infty)$; or $\delta u'(w^O) r - u'(w^Y) (r + c) < 0$ for all $c \in (0, \bar{c})$ so that

$$\text{Arg max} U^{YW}(c) = \{0\}$$

Finally note that preferences are single peaked. ☐

Lemma 4.4.5 shows that young workers may prefer a steady state with a positive capital market friction. A positive $c$ allows the young worker to smooth consumption over his lifetime; the capital market friction works as a savings technology. Equation (4.4.3) can be interpreted as an optimal savings condition. If utility is linear, for example, there is no consumption smoothing motive and (4.4.3) cannot be satisfied. Condition (4.4.3) shows that $c^{YW}$ depends on the functional form $u$, the discount rate $\delta$ and technological factors $\theta^Y$ and $\theta^O$. Implicit derivation of $c^{YW}$ gives the following comparative statics.

**Lemma 4.4.6.** $c^{YW}$ is increasing in $\delta$; decreasing in $\theta^O$; and increasing in $\theta^Y$.

Lemma 4.4.6 is intuitive: the consumption smoothing motive is increased if future consumption is valued more (increase in $\delta$) and if the wage gap between young and old age is bigger (decrease in $\theta^O$; increase in $\theta^Y$). This concludes our description of voter preferences.
4.4.2 Majority Voting

In this section we analyze equilibria under pure majority rule. A pure majority rule is defined by three characteristics: (i) democracy is direct so that voters directly choose their preferred capital market friction; (ii) voters vote sincerely; and (iii) there is an open agenda so that all alternatives are considered in the vote. Every period $t$, all agents cast a vote over the capital market friction $c_t$.

Formally, an action at time $t$ for a member of voter class $i$ is a capital market friction $a_t^i \in [0, \infty)$, we also refer to actions as votes. At time $t$, the publicly known history of the game is $h_t = (c_0, \ldots, c_{t-1}) \epsilon H_t$ with $H_t = [0, \infty)^I$. A strategy for voter class $i$ at time $t$ is given by a mapping $v_t^i : H_t \rightarrow [0, \infty)$.

**Definition.** A political economic equilibrium is a sequence of factor prices and allocations $E = \{r_t, w_t^Y, w_t^O, K_t^Y, K_t^O\}_{t=0}^\infty$, supplemented with a voting strategy profile $v = \{v_t^{YW}, v_t^{YC}, v_t^{OW}, v_t^{OC}\}_{t=0}^\infty$, such that (i) $v$ is an equilibrium of the voting game; and (ii) $E$ is an economic equilibrium given the sequence of reallocation costs $\{c_t\}_{t=0}^\infty$ in the outcome of $v$.

We focus on steady state equilibria that can be politically supported, i.e. steady state equilibria $E_c$ for which there is a voting strategy $v$ such that $\{E_c, v\}$ is a political economic equilibrium.

**Open-Loop Equilibrium**

As a benchmark we consider the political equilibrium that results if voters play open-loop strategies, i.e. strategies that do not depend on history.

**Proposition 4.4.1.** If voters play open-loop strategies, then the unique political economic equilibrium is given by $E_0$ and the voting strategy profile $v = \{v_t^{YW} = 0, v_t^{YC} = 0, v_t^{OW} = \bar{c}_t, v_t^{OC} = 0\}_{t=0}^\infty$.

**Proof.** Strategy profile $v$ follows from sincere voting and voter preferences derived in lemma 4.4.3 and lemma 4.4.4. We see that a majority of agents votes for $c_t = 0$ in every period. The corresponding steady state is $E_0$. \qed
Capital market frictions cannot arise in the political equilibrium under open-loop strategies. Capitalists achieve maximum lifetime utility in this equilibrium. The young worker class, while decisive, does not achieve her maximum lifetime income (which is achieved for \( c = c^{YW} \), cf. lemma 4.4.5). Open-loop strategies leave no scope for cooperation between young workers in different periods. This result is reminiscent of results in Sjoblom (1985) and Azariadis and Galasso (2002), who show that social security cannot be supported if voters play open-loop strategies.

**Subgame Perfect Equilibrium**

Once we allow for richer voting strategies, cooperation between subsequent young generations can be achieved. In particular, any steady state in which the young workers get a higher utility than their open loop utility can be supported as a political equilibrium.

**Proposition 4.4.2.** There exists \( c^* \in [0, \bar{c}] \) such that for every \( c \in [0, c^*] \), the steady state equilibrium \( E_c \) can be politically supported.

**Proof.** Consider the set

\[ D := \{ c \in [0, \infty) \mid U^{YW}(c, c) \geq U^{YW}(0, 0) \} \]

\( D \) is nonempty, as \( 0 \in D \); \( D \) is closed, as \( U^{YW} \) is continuous; and \( D \) is convex, as \( U^{YW} \) is single-peaked. It follows that \( D \) is a closed interval which contains \( c^{YW} \), the preferred lifetime policy of the young workers (cf. lemma 4.4.5). Now, let \( c^* := \min \{ \bar{c}, \sup D \} \) and choose an arbitrary \( c \in [0, c^*] \). Then \( E_c \) can be politically supported. Consider the voting strategy profile \( \nu^* = \{ v_t^{YW}, v_t^{YC}, v_t^{OW}, v_t^{OC} \}_{t=0}^{\infty} \) such
that

\[
\begin{align*}
\nu^W_t &= \begin{cases} 
  c & \text{if } c_{t-s} = c \text{ for } s = 1, \ldots, t \\
  0 & \text{otherwise}
\end{cases} \\
\nu^C_t &= \nu^{OC}_t = 0 \\
\nu^O_t &= \bar{c}_t
\end{align*}
\]

With this strategy profile the voting game equilibrium is \( c \) in every period provided the play started with \( c_0 = c \). The best deviation of the \( YW \) class at time \( t \) is to set \( c_t = 0 \) given that \( \nu^W_{t+1} = 0 \). This deviation is not profitable since \( U^W_t(0,0) \leq U^W_t(c,c) \). It remains to check that \( \nu^W_{t+1} = 0 \) is incentive compatible. It is because \( U^W_{t+1}(0,0) > U^W_{t+1}(c_t+1,0) \) for any \( c_t+1 > 0 \). We have shown that \( \nu^* \) is a subgame perfect strategy profile of the repeated voting game. We conclude that \( \{ E_c, \nu^* \} \) is a political economic equilibrium.

The political equilibria with subgame-perfect voting strategies can be interpreted as arising from social contract which allows for cooperation between current and future young. The familiar logic for cooperation is that voters cooperate as long as they expect future generations to honor the social contract if they do so themselves.

Since there are multiple equilibria of the repeated voting game, there is no guarantee that cooperation will be achieved. Following the literature on dynamic political economy models, we can reduce the multiplicity of equilibria in two ways. First, in the spirit of Azariadis and Galasso (2002), we may impose more structure on the political model. Second, we may restrict the solution concept as in Hassler et al. (2003) and related work that studies policies that can be sustained without commitment (e.g. Klein, Krusell, and Ríos-Rull, 2008). We pursue both routes in the following.
Policy Persistence

We restrict the political model and assume that voters cannot overturn policy in every period. Instead, voters at time $t$ set a persistent policy that lasts throughout their lifetime (i.e. $c_t = c_{t+1}$). Note that this means that the next generation of voters is disenfranchised.\(^\text{10}\)

As agents vote sincerely, actions follow from voter preferences derived in section 4.4.1. Recall that utility of old capitalists is maximized for $c_t = 0$. Utility of young capitalists is maximized for $c_t = 0$ and $c_{t+1} = 0$. Hence all capitalists vote for a zero friction, or

$$a_t^{OC} = a_t^{YC} = 0$$

If capitalists form a majority (i.e. $\eta > \frac{1}{2}$), the policy outcome is $c_t = c_{t+1} = 0$. Consequently $E_0$ is the unique steady state that can be politically supported if capitalists are a majority. If capitalists are a minority (i.e. $\eta < \frac{1}{2}$)–the more realistic case that we focus on in the following–then worker preferences are decisive for the political equilibrium. We have seen before that utility of the old worker class is maximized if all capital $\hat{K}_t^O$ is retained in the old firms where they work. It follows that

$$a_t^{OW} = \tilde{c}_t$$

As for the young worker class, they face a tradeoff: a higher capital market friction leads to a decrease in their wage when young ($w^Y_t$) and an increase in their wage when old ($w^O_{t+1}$). If young voters can set the policy for two periods, then they will vote as if choosing among steady state utility levels.\(^\text{11}\)

**Lemma 4.4.7.** Consider a vote at time $t$ under policy persistence. Then young workers choose $a_t^{YW} = c^{YW}$, where $c^{YW}$ is given by lemma 4.4.5.

**Proof.** See appendix A.3.2 \(\square\)

\(^{10}\)However, we show that disenfranchised median voters are better off. The intuition is that the YW class achieves higher utility if the next generation can be bound to its choice.

\(^{11}\)It is not a priori clear whether the economy reaches a new steady state right after the vote at time $t$. We show in the appendix that it does.
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The young worker is the median voter, with preferences in between those of all the capitalist (who are a minority) and the old workers. We have the following proposition,

**Proposition 4.4.3.** The unique political equilibrium if voters choose persistent policies is given by $E_{cYW}$ and voting strategy profile $v = \{v_t^{YW} = c^{YW}, v_t^{YC} = 0, v_t^{OW} = \tilde{c}_t, v_t^{OC} = 0\}_{t=0}^{\infty}$.

*Proof.* Strategy profile $v$ follows from sincere voting and lemmas 4.4.3, 4.4.4 and 4.4.7. All preferences are single peaked so that the median voter is decisive. It follows that $c_t = c^{YW}$ in every period and the corresponding economic equilibrium is $E_{cYW}$.

The unique political equilibrium that arises if voters choose persistent policies features the capital market friction $c^{YW}$, which may be strictly positive (cf. lemma 4.4.5). The young worker class, which is pivotal in the vote, achieves maximum lifetime utility in this political equilibrium.

**Markovian Policies**

Markovian equilibria are subgame-perfect equilibria in which the the policy variable is a time-invariant function of the state variable. In the present context, the natural state variable is $k_t := \dot{K}^O_t = K^Y_{t-1}$. We restrict the choice of the decisive voter class to a Markovian policy function,

$$c_t = \mu(k_t) \quad (4.4.6)$$

and assume that $\mu(\cdot)$ is time invariant and differentiable.

The transition function $T$ gives next period’s state variable as a function of this period’s state variable and current policy. The transition function follows from firm behavior derived in section 4.3.1, evaluated for a Cobb-Douglas production function, so

$$T : [0, \bar{K}] \times [0, \infty) \to [0, \bar{K}]$$
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is given by

\[ T(k_t, c_t) = \begin{cases} 
\frac{(\theta^r)^{\frac{1}{1-\alpha}}}{(\theta^r)^{\frac{1}{1-\alpha}} + (\theta^o)^{\frac{1}{1-\alpha}}} \bar{K} & \text{if } k_t < K^O \\
\bar{K} - k_t & \text{if } K^O \leq k_t < \bar{K}^O(c_t) \\
\left(\frac{\alpha\theta^r}{r^*(c_t) + c_t}\right)^{\frac{1}{1-\alpha}} \text{ with } r^*(c_t) \text{ given by} & \\
\left(\frac{\alpha\theta^o}{r^*(c_t) + c_t}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^o}{r^*(c_t)}\right)^{\frac{1}{1-\alpha}} = \bar{K} & \text{if } k_t \geq \bar{K}^O(c_t)
\end{cases} \]

Our transition function takes a simple form as it is stationary and does not depend on future policy \(c_{t+1}\). Young workers now solve

\[
\mu(k_t) = \arg\max_{c_t} U_t^{YW}(c_t, c_{t+1}; k_t)
\]

subject to
\[
c_{t+1} = \mu(k_{t+1}), \quad c_t \in [0, \infty]
\]

\[
k_{t+1} = T(k_t, c_t)
\]

In words, the function \(\mu\) must yield a \(c_t\) such that utility of the YW class is maximized, taking into account any effect the choice of \(c_t\) has on future policy through the state variable \(k_{t+1}\).

To develop some intuition for the solution of this problem, consider trivial Markovian policy functions of the form

\[
\mu(k_t) = C
\]

Then the optimization problem reduces simply to

\[
\max_{c_t} U_t^{YW}(c_t, C)
\]

which, by lemma 4.4.4, is uniquely solved for \(c_t = 0\). It follows that

\[
\mu(k_t) = 0
\]
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is the solution to the young worker’s problem. Consequently, \( c_t = 0 \) is the outcome of the voting game for any \( t \). Thus we have shown that \( E_0 \), the steady state without frictions, is politically supported by the trivial Markovian voting strategy \( \mu(k_t) = 0 \). A priori, there could be other steady states that are politically supported. But we are able to rule this out in the following. Note that in steady state we have

\[
K^Y(c) \geq \hat{K}^O(c)
\]

which implies that the transition function \( T \) reduces to

\[
h(c) = T(k,c) = \left( \frac{\alpha \theta^Y}{r+c} \right)^{\frac{1}{1-a}}
\]

a strictly decreasing function of \( c \) on \([0,\bar{c}]\). Since we must also have

\[
c = \mu(h(c))
\]

we see that \( \mu(k) = h^{-1}(k) \). But this cannot be a solution to the young worker’s optimization problem since

\[
\arg \max_{c_t} U_t^{yw}(c_t, c_t)
\]

is independent of \( k \), cf. lemma 4.4.5. We conclude that there are no steady states, besides \( E_0 \), that can be politically supported using Markovian policy functions. The impossibility to politically support steady states (other than \( E_0 \)) through Markovian voting strategies results from the fact that the pivotal young workers’ optimal choice is independent of the state variable. Hence, the form of \( c_t \) imposed by (4.4.6) can only lead to trivial solutions.
4.4.3 Illustration: CRRA Utility

As an illustration, we solve for the political equilibrium under persistent policy voting with CRRA utility. Let the felicity function \( u \) be given by

\[
u_g(w) := \begin{cases} \frac{w^{1-g}}{1-g} & \text{for } g > 0, g \neq 1 \\ \ln w & \text{for } g = 1 \end{cases}
\]

We derive the policy outcome if voters choose persistent policies (cf. also proposition 4.4.3)

**Lemma 4.4.8.** If voters choose persistent policies, the unique outcome \( c^{YW} \) of the voting game in every period is given by

\[
c^{YW} = \begin{cases} 0 & \text{if } \delta A^{\frac{-g}{1+g(\gamma-1)}} \leq 1 \\ (\delta A^{\frac{-g}{1+g(\gamma-1)}} - 1) r & \text{if } 1 < \delta A^{\frac{-g}{1+g(\gamma-1)}} < \frac{\theta Y}{\theta r} \\ \tilde{c} & \text{if } \delta A^{\frac{-g}{1+g(\gamma-1)}} \geq \frac{\theta Y}{\theta r} \end{cases}
\]

with \( \gamma := \frac{1}{1-\alpha} \) and \( A := (\frac{\theta O}{\theta r})^\gamma \).

**Proof.** Taking the derivative of \( U^{YW} \) with respect to \( c \) we have shown (cf. lemma 4.4.5) that \( \frac{du^{YW}}{dc} \geq 0 \) if and only if

\[
u'(w^Y) (r + c) - \delta u'(w^O) r \leq 0 \quad (4.4.7)
\]

With CRRA utility this rewrites as

\[
\beta^{-g} \alpha^{-g(\gamma-1)} (\theta Y)^{\frac{-g\gamma}{1+g(\gamma-1)}} [(r + c)^{1+g(\gamma-1)} - \delta A^{-g} r^{1+g(\gamma-1)}] \leq 0
\]

so that lifetime utility of the young is increasing in the reallocation cost iff

\[
\left( \frac{r + c}{r} \right) \leq \delta A^{\frac{-g}{1+g(\gamma-1)}} \quad (4.4.8)
\]
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The utility maximizing policy now follows from this condition. First note that \( \left( \frac{r+c}{r} \right) \) is increasing in \( c \); and that we have \( \left( \frac{r+c}{r} \right) = \frac{\theta^Y}{\theta^O} \), so that

\[
1 \leq \frac{r+c}{r} \leq \frac{\theta^Y}{\theta^O}
\]

for \( c \epsilon [0, \bar{c}] \). We see that if

\[
\delta A^{\frac{-g}{1+r(Y-1)}} \leq 1
\]

then \( U^{YW} \) is decreasing in \( c \) so that \( c^{YW} = 0 \). Likewise if

\[
\delta A^{\frac{-g}{1+r(Y-1)}} \geq \frac{\theta^Y}{\theta^O}
\]

then \( U^{YW} \) is increasing in \( c \) so that \( c^{YW} = \bar{c} \). Finally if

\[
1 < \delta A^{\frac{-g}{1+r(Y-1)}} < \frac{\theta^Y}{\theta^O}
\]

then \( \frac{dU^{YW}}{dc} \) switches sign on \([0, \bar{c}]\) and \( c^{YW} \) is given by

\[
c^{YW} = \left( \delta A^{\frac{-g}{1+r(Y-1)}} - 1 \right) r(c^{YW})
\]

\[ \square \]

4.5 Conclusion

In this chapter we model political support for distortionary capital market frictions. We show that workers in democracies may successfully oppose the reallocation of capital to newer sectors, as this affects their labor rents.

Besides class difference, we identify age difference as a source of political conflict. The political conflict exists as long as capital and labor are complementary factors of production, as long as human capital is less mobile than physical capital, and as long as human capital risk cannot be fully insured.

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We identify young workers as the decisive voter class—under the plausible assumption that capitalists are a minority. Young workers are decisive because their preferences are less polarized than preferences of other groups in society. Young workers are hurt by capital market frictions in the short term, but may still favor them to smooth consumption over their lifetime. Young workers prefer a higher friction (i) if technology grows at a faster pace,(ii) if they place more weight on the future, and (iii) if they are more risk averse; the result holds as long as young workers expect the capital market friction to persist.

A special case is when capitalists form a majority; in this case, our model predicts that no capital market frictions arise. Broad capital market participation is found in some democracies, in particular those with funded pension schemes. It would be interesting to see if capital reallocation is less restricted in democracies with fully funded pension systems.

The voting process is critical for the outcome of the voting game: the political equilibrium depends on the ability of a current majority to establish a persistent policy. When policies may be overturned in each period, the model features multiple equilibria. By contrast, the equilibrium prediction is the unique outcome favored by the young worker if the outcome of the vote is irreversible.

Our main contribution is to explain why unrestricted capital mobility may be opposed in democracies as a result of the wealth and age distribution in a country’s population. We identify young workers as the decisive class in society. What makes them pivotal is not their number—they are a minority just as every other voter class—but the fact that their preferences are the least extreme.

A crucial assumption in our model is that agents cannot save, so a capital market friction is necessary for self insurance. If we reinterpret the drop in wages as a decline in employment, then our findings hold even if households can store their income.\textsuperscript{12} The capital market friction then becomes an unemployment insurance that young workers take out in order to have a quiet life.

\textsuperscript{12}A simple way to introduce unemployment in our model is through sticky wages.
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References


