Other people's money: essays on capital market frictions
Bersem, M.R.C.

Citation for published version (APA):
Bersem, M. R. C. (2012). Other people's money: essays on capital market frictions

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Appendix A

Appendix: Proofs

This appendix contains the proofs of Chapter 2 (Incentive-Compatible Sovereign Debt), Chapter 3 (Collective Pension Funds), and Chapter 4 (Sand in the Wheels of Capitalism).

A.1 Proofs of Chapter 2

Proof of Proposition 2.3.3:

Proof. Let \((O_1, I_d)\) be an optimal contract and let \(D\) be the constant value of \(O_1\) when \(I_d(y) = 0\). Consider a new contract \((\tilde{O}_1, \tilde{I}_d)\) given by

\[
\tilde{I}_d(y) = \begin{cases} 
0 & \text{if } \tilde{D} \leq \min\{\gamma y + B, y\} \\
1 & \text{if } \tilde{D} > \min\{\gamma y + B, y\}
\end{cases}
\]

and

\[
\tilde{O}_1(y) = \begin{cases} 
\tilde{D} & \text{if } \tilde{I}_d(y) = 0 \\
\gamma y & \text{if } \tilde{I}_d(y) = 1
\end{cases}
\]

and suppose first that \(\tilde{D} = D\). If \(\tilde{I}_d(y) = I_d(y)\), then the construction of \(\tilde{O}_1\) implies that \(\tilde{O}_1(y) \geq O_1(y)\). If \(\tilde{I}_d(y) < I_d(y)\), i.e. if \(\tilde{I}_d(y) = 0\) and \(I_d(y) = 1\), then it follows
Appendix: Proofs

from proposition 2.3.2 that

\[ O_1(y) \leq D \leq \tilde{O}_1(y) \]

Furthermore, we can rule out \( \tilde{I}_d(y) > I_d(y) \). To see this, suppose that \( y \) is such that \( \tilde{I}_d(y) = 1 \) and \( I_d(y) = 0 \). Then we know that \( O_1(y) = D \), but this cannot be, as we also know that \( \gamma y + B < D \) from \( \tilde{I}_d(y) = 1 \), and an optimal contract must be repudiation proof. This proves that \( \tilde{O}_1(y) \geq O_1(y) \) if \( \tilde{D} = D \).

Now, one can choose \( \tilde{D} \leq D \) such that the investor participation constraint is still satisfied. By construction, the resulting contract \( (\tilde{O}_1, \tilde{I}_d) \) satisfies truthful revelation and is repudiation-proof–like any sovereign debt contract. As \( \tilde{I}_d(y) \leq I_d(y) \), it must be optimal.

Since both \( (O_1, I_d) \) and \( (\tilde{O}_1, \tilde{I}_d) \) are optimal contracts, we have

\[ E(I_d - \tilde{I}_d)B = 0 \]

Consider the state observation function \( I_d(y) \). For all states \( y \in \left[ 0, \frac{D - B}{\gamma} \right) \) we must have \( I_d(y) = 1 \), since \( I_d(y) = 0 \) would mean that \( O_1(y) = D \) which contradicts repudiation-proofness. There may be more states for which \( I_d(y) = 1 \), as we only know that \( I_d(y) \geq \tilde{I}_d(y) \). Let \( T_2 \) denote the set of those states, so

\[ T_2 = \left\{ y \geq \frac{D - B}{\gamma} \mid I_d(y) = 1 \right\} \]

We see that

\[ EI_d = \int_{\frac{D - B}{\gamma}}^{\frac{D - B}{\gamma}} 1 f(y)dy + \int_{\tilde{I}_2}^{\frac{D - B}{\gamma}} 1 f(y)dy \]

furthermore we have

\[ E\tilde{I}_d = \int_{\frac{D - B}{\gamma}}^{\frac{D - B}{\gamma}} 1 f(y)dy \]

Now since \( \tilde{D} \leq D \) and \( B > 0 \), it follows that (i) \( D = \tilde{D} \) and (ii) \( T_2 \) has probability
mass zero; hence we see that $I_d = \bar{I}_d$ almost surely. It follows that, as $EO_1 = E\bar{O}_1$, we must also have that $O_1 = \bar{O}_1$ almost surely, and I conclude that the optimal contract is a sovereign debt contract.

\[\Box\]

### A.2 Proofs of Chapter 3

#### A.2.1 Proof of Proposition 3.2.1

The dynamic investment problem to solve is\textsuperscript{1}

\[
\max_{\alpha_t, \alpha_{t+1}} Eu(\tilde{b})
\]

such that

\[
\begin{align*}
  w_t &= 0 \\
  w_{t+1} &= \alpha_t (1 + \tilde{r}_t) + (y - \alpha_t) \\
  w_{t+2} &= \alpha_{t+1} (1 + \tilde{r}_{t+1}) + (w_{t+1} + y - \alpha_{t+1}) \\
  \tilde{b} &= w_{t+2}
\end{align*}
\]

First, we rewrite this problem in recursive form

\[
v_t(w_t) = \max_{\alpha_t} Ev_{t+1} ((w_t + y) + \alpha \tilde{r})
\]

where $v_t$, the remaining-value function, is a function of the financial reserve, $w_t$. We know that $v_{t+2}(w) = u(w) = \frac{w^{1-\phi}}{1-\phi}$, as individuals consume their financial reserve in retirement. Note that $v_t(0)$ is the expected utility in retirement of a young individual at time $t$ who invests optimally throughout his life. Optimal investment, $\alpha_t$, is a function of the single state variable, $w_t$.

We consider the trial solution function $v_{t+1}(w_{t+1}) = \gamma_{t+1} \frac{(w_{t+1} + h_{t+1})^{1-\phi}}{1-\phi}$, where $\gamma_{t+1} > 0$ is a scalar and $h_{t+1}$ is the human capital reserve of an individual. Our trial

\textsuperscript{1}We drop the superscripts to save on notation.
Appendix: Proofs

solution implies that

\[ v_{t+1} \left( (w_t + y) + \alpha_t \tilde{r} \right) = \gamma_{t+1} \frac{((w_t + y) + \alpha_t \tilde{r} + h_{t+1})^{1-\phi}}{1 - \phi} \]

\[ = \gamma_{t+1} \frac{(w_t + h_t + \alpha_t \tilde{r})^{1-\phi}}{1 - \phi} \]

so that the first-order condition reads as

\[ \gamma_{t+1} E \tilde{r} (w_t + h_t + \alpha_t \tilde{r})^{-\phi} = 0 \]

Solving for optimal investment in the risky asset yields

\[ \alpha_t (w_t) = a^* (w_t + h_t) \]

where

\[ a^* := \frac{\left( -r^l (1 - p) \right)^{\frac{1}{\phi}} - 1}{r^l - r^h \left( -r^l (1 - p) \right)^{\frac{1}{\phi}}} \quad \text{(A.2.2)} \]

Finally, it follows from (A.2.1) that

\[ v_t (w_t) = \gamma_{t+1} E \frac{(w_t + h_t + a^* (w_t + h_t) \tilde{r})^{1-\phi}}{1 - \phi} \]

\[ = \delta \gamma_{t+1} \frac{(w_t + h_t)^{1-\phi}}{1 - \phi} \]

where

\[ \delta := E \left( 1 + a^* \tilde{r} \right)^{1-\phi} \]

so that our trial solution is correct with \( \gamma_t = \delta \gamma_{t+1} \). Note that

\[ \delta \approx 1 + (1 - \phi) a^* \mu > 1 \]

where we’ve used a first-order approximation. We conclude that, conditional on
Appendix: Proofs

an optimal investment strategy, expected lifetime utility of the young at time $t$ is

$$
n_t(0) = \delta^2 u(h_t)
= \delta^2 u(2y)
$$

\[\square\]

A.2.2 Proof of Proposition 3.2.2

Incomplete markets give rise to additional constraints, $\alpha_t = 0$ and $\alpha_{t+1} \leq w_{t+1}$. To obtain a lower bound for the welfare loss due to credit constraints, we assume that the middle-aged do not face constraints, only the young do. Then the new investment problem is

$$
\max_{\alpha_{t+1}} Eu(\tilde{b})
$$

such that

$$
w_t = 0
w_{t+1} = y
w_{t+2} = \alpha_{t+1} (1 + \tilde{r}_{t+1}) + (w_{t+1} + y - \alpha_{t+1})
\tilde{b} = w_{t+2}
$$

It is easy to see that the middle aged will invest $a^*(2y)$ so that

$$
w_{t+2} = 2y(1 + a^* \tilde{r}_{t+1})
$$
Eu (\tilde{b}) = Eu (2y (1 + a^* \tilde{r}_{t+1})) \\
= E (1 + a^* \tilde{r}_{t+1})^{1-\phi} u(2y) \\
= \delta \ u(2y)

With proposition 4.3.2 we see that the lower bound for the welfare loss due to credit constraints is

\[(\delta^2 - \delta) u(2y)\]

\[\square\]

A.3 Proofs of Chapter 4

A.3.1 Cobb-Douglas Production

We derive the steady state equilibria for a Cobb-Douglas production economy, where \( F \) is given by

\[ F(K,L) = K^\alpha L^\beta \]  \hspace{1cm} (A.3.1)

with \( 0 < \alpha + \beta < 1 \). Steady-state capital allocations for \( c \in [0,\bar{c}] \) are

\[ K^Y* = \left( \frac{\alpha \theta^Y}{r^*(c) + c} \right)^{\frac{1}{1-\alpha}} ; \quad K^O* = \left( \frac{\alpha \theta^O}{r^*(c)} \right)^{\frac{1}{1-\alpha}} \]  \hspace{1cm} (A.3.2)

with \( r^*(c) \) given by capital market clearing condition

\[ \left( \frac{\alpha \theta^Y}{r^*(c) + c} \right)^{\frac{1}{1-\alpha}} + \left( \frac{\alpha \theta^O}{r^*(c)} \right)^{\frac{1}{1-\alpha}} = \bar{K} \]
Appendix: Proofs

The boundary value $\bar{c}$ follows from (4.3.21) and is given by

$$\bar{c} = \alpha \frac{\theta_Y - \theta^O}{(\frac{1}{2} \bar{K})^{1-\alpha}}$$

For $c > \bar{c}$ we have steady state capital allocations

$$K^Y* = \left( \frac{\alpha \theta_Y}{r^*(c) + c} \right)^{\frac{1}{1-\alpha}} \quad ; \quad K^O* = \frac{1}{2} \bar{K}$$

where capital market clearing condition

$$\left( \frac{\alpha \theta_Y}{r^*(c) + c} \right)^{\frac{1}{1-\alpha}} = \frac{1}{2} \bar{K}$$

allows us to obtain the equilibrium interest rate explicitly,

$$r^*(c) = \frac{\alpha \theta_Y}{(\frac{1}{2} \bar{K})^{1-\alpha}} - c$$

Next we determine what steady state equilibria can be politically supported.

A.3.2 Proof of lemma 4.4.2

Consider the equilibrium interest rate in period $t$. For $c_t < \bar{c}_t$, we have $\bar{K}_t^O \geq \bar{K}_t^O$ so that old dirms scale down and the interest rate is given by

$$\left( \frac{\alpha \theta_Y}{r_t + c_t} \right)^{\frac{1}{1-\alpha}} + \left( \frac{\alpha \theta^O}{r_t} \right)^{\frac{1}{1-\alpha}} = \bar{K}$$

Implicit differentiation yields

$$- \frac{1}{1 - \alpha} \frac{K_t^Y}{r_t + c_t} \left( \frac{dr_t}{dc_t} + 1 \right) = \frac{1}{1 - \alpha} \frac{K_t^O}{r_t} \frac{dr_t}{dc_t} = 0 \quad (A.3.3)$$
Appendix: Proofs

which rewrites as

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O (r_t + c_t)}$$

so that $\frac{dr_t}{dc_t} \in (-1, 0)$. For $c_t \geq \bar{c}_t$, we have $K_t^O = \hat{K}_t^O$ and the interest rate is given by

$$\left(\frac{\alpha \theta^Y}{r_t + c_t}\right)^{\frac{1}{1-\alpha}} = \bar{K} - \hat{K}_t^O$$

$$r_t = \frac{\alpha \theta^Y}{(\bar{K} - \hat{K}_t^O)^{1-\alpha} - c_t}$$

and we see that $\frac{dr_t}{dc_t} = -1$. Now for (i), capital in the Y-sector is given by

$$K_t^Y = \left(\frac{\alpha \theta^Y}{r_t + c_t}\right)^{\frac{1}{1-\alpha}}$$

Taking the derivative with respect to $c_t$ gives

$$\frac{dK_t^Y}{dc_t} = -\frac{1}{1-\alpha} \frac{K_t^Y}{r_t + c_t} \left(\frac{dr_t}{dc_t} + 1\right)$$

so that $\frac{dK_t^Y}{dc_t} < 0$ for $c_t \in [0, \bar{c}_t]$ and $\frac{dK_t^Y}{dc_t} = 0$ for $c \geq \bar{c}_t$. Wages in the Y-sector are

$$w_t^Y = \theta^Y \beta (K_t^Y)^\alpha$$

Taking the derivative with respect to $c_t$ yields

$$\frac{dw_t^Y}{dc_t} = -\frac{\beta}{1-\alpha} K_t^Y \left(\frac{dr_t}{dc_t} + 1\right)$$

so that $\frac{dw_t^Y}{dc_t} < 0$ for $c_t \in [0, \bar{c}_t]$; $\frac{dw_t^Y}{dc_t} = 0$ for $c_t > \bar{c}_t$. For (ii), capital in the O-sector is given by

$$K_t^O = \left(\frac{\alpha \theta^O}{r_t}\right)^{\frac{1}{1-\alpha}}$$
for \( c_t < \bar{c}_t \); and
\[
K_t^O = \hat{K}^O
\]
for \( c_t \geq \bar{c}_t \). Hence \( \frac{dK_t^O}{dc_t} > 0 \) for \( c_t \in [0, \bar{c}_t) \) and \( \frac{dK_t^O}{dc_t} = 0 \) for \( c_t \geq \bar{c}_t \). Wages in the O-sector are
\[
w_t^O = \theta^O \beta \left( K_t^O \right)^\alpha
\]
Taking the derivative with respect to \( c_t \) gives
\[
\frac{dw_t^O}{dc_t} = -\frac{\beta}{1 - \alpha} K_t^O \frac{dr_t}{dc_t}
\] (A.3.5)
so that \( \frac{dw_t^O}{dc_t} > 0 \) for \( c_t \in [0, \bar{c}_t) \) and \( \frac{dw_t^O}{dc_t} = 0 \) for \( c_t \geq \bar{c}_t \). For (iii), first consider profits. Let \( \pi_t^Y \) and \( \pi_t^O \) denote profits in the Y- and O-sector respectively. Then
\[
\pi_t^Y = \theta^Y \left( K_t^Y \right)^\alpha - (r_t + c_t)K_t^Y - w_t^Y
\]
and
\[
\pi_t^O = (\theta^O)(K_t^O)^\alpha - r_tK_t^O - w_t^O
\]
We take the derivative of \( \pi_t^Y \) with respect to \( c_t \) and obtain
\[
\frac{d\pi_t^Y}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \left( \frac{dr_t}{dc_t} + 1 \right) K_t^Y
\]
so that \( \frac{d\pi_t^Y}{dc_t} < 0 \) for \( c_t \in [0, \bar{c}_t) \) and \( \frac{d\pi_t^Y}{dc_t} = 0 \) for \( c_t \geq \bar{c}_t \). Similarly, for O-sector profits, we get
\[
\frac{d\pi_t^O}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \frac{dr_t}{dc_t} K_t^O
\]
so that \( \frac{d\pi_t^O}{dc_t} > 0 \) for \( c_t \in [0, \bar{c}_t) \) and \( \frac{d\pi_t^O}{dc_t} > 0 \) for \( c_t \geq \bar{c}_t \). Turning to total profits, \( \Pi_t = \pi_t^Y + \pi_t^O \), we have
\[
\frac{d\Pi_t}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \left[ \left( \frac{dr_t}{dc_t} + 1 \right) K_t^Y + \frac{dr_t}{dc_t} K_t^O \right]
\] (A.3.6)
Appendix: Proofs

and we see that $\Pi_t$ is decreasing in $c_t$ iff

$$\frac{dr_t}{dc_t} \tilde{K} + K_t^Y \geq 0$$

Recall that for $c_t \in [0, \bar{c}_t)$ we have

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O \left( \frac{r_t + c_t}{r_t} \right)}$$

so that $-\frac{dr_t}{dc_t} \leq K_t^Y$ and total profits are nonincreasing in $c_t$. For $c_t \geq \bar{c}_t$, we have $\frac{dr_t}{dc_t} = -1$ so that total profits are increasing in $c_t$. This result is due to the fact that $r_t$ declines in $c_t$ while the allocation of capital does not change in this range of capital market frictions. Hence the cost of capital goes down for O-firms, stays the same for Y-firms, as production in both sectors remains the same. While profits may increase in $c_t$, capital income cannot. Recall that capital income is given by

$$s_t = \frac{r_t \tilde{K} + \Pi_t}{\eta}$$

For $c_t \in [0, \bar{c}_t)$, we have $\frac{d\Pi_t}{dc_t} \leq 0$ so that $\frac{ds_t}{dc_t} \leq 0$. For $c_t \geq \bar{c}_t$ we have

$$\frac{ds_t}{dc_t} = \frac{1}{\eta} \left( \frac{1}{K_t^O} \right)$$

so that $\frac{ds_t}{dc_t} < 0$. Now, old capitalists income is given by

$$w_t^O + s_t$$
Appendix: Proofs

Let $c_t \in [0, \bar{c}_t)$, then we have

$$\frac{d(w_t^O + s_t)}{dc_t} = -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t} + \frac{1}{\eta} \left( \frac{\beta}{1-\alpha} \frac{dr_t}{dc_t} \bar{K} + \frac{\alpha + \beta - 1}{1-\alpha} K_t^Y \right)$$

$$< -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t} + \frac{\beta}{1-\alpha} \frac{dr_t}{dc_t} \bar{K}$$

$$< 0$$

Next, for $c_t \geq \bar{c}_t$, $\frac{dw_t^O}{dc_t} = 0$ and $\frac{ds_t}{dc_t} < 0$ which shows part (iii).

### A.3.3 Proof of Lemma 4.4.7

To prove lemma 4.4.7, we first prove two auxiliary lemmas that give the economic equilibrium in periods $t$ and $t + 1$. Since we consider out-of-steady-state dynamics we assume that the equilibrium at time $t - 1$ is given by steady state allocations for some arbitrary $c \in [0, \bar{c}]$. The first lemma describes the economic equilibrium after a downward change in policy:

Let the economic equilibrium at time $t - 1$ be given by steady state values for some $c \leq \bar{c}$. Consider a downward change in policy $c_t = c_{t+1} \leq c$, then

$$K_t^Y = \left( \frac{\alpha \theta^Y}{r_t^* + c_t} \right)^{\frac{1}{1-\alpha}}$$

and

$$K_{t+1}^O = \left( \frac{\alpha \theta^O}{r_t^*} \right)^{\frac{1}{1-\alpha}}$$

with $r_t^*$ given by $K_t^Y + K_t^O = \bar{K}$, furthermore $K_{t+1}^Y = K_{t+1}^O$ and $K_{t+1}^O = K_t^O$.

Since $c_t \leq c$, we have $\bar{K}^O(c) \geq \bar{K}^O(c_t)$, where $\bar{K}^O$ is the equilibrium cut-off value function defined after (4.3.14). It follows that

$$\bar{K}^O_t = K_{t-1}^Y \geq \bar{K}^O(c_t)$$

so that $K_t^Y$ and $K_t^O$ are as posed. Note that because $c_t \leq c$, we have $K_t^O < K_{t-1}^O$ by
Appendix: Proofs

Lemma 4.4.1 and hence $K_t^Y > K_{t-1}^Y$ by market clearing. We must verify that the same allocation obtains in period $t + 1$. Moving forward one period we have

$$\hat{K}_{t+1}^O = K_t^Y > \hat{K}^O(c_{t+1})$$

so that again old firms scale down, the same interest rate obtains (i.e. $r_t^* = r_{t+1}^*$), and allocations are as posed.

Lemma A.3.3 shows that a downward change in policy results in steady state values that correspond to a lower reallocation cost. Informally, we can say that changing policy downward moves the economy to a new steady state corresponding to the new value of the friction $c_t$. The same need not be true for an upward policy change as the next lemma shows.

Let the economic equilibrium at time $t - 1$ be given by steady state values for some $c \leq \bar{c}$. Consider an upward change in policy $c_t = c_{t+1} > c$, then

(i) if $c_t \leq \bar{c}$ we have

$$K_t^Y = \left( \frac{\alpha \theta^Y}{r_t^* + c_t} \right)^{\frac{1}{1-a}} ; \quad K_t^O = \left( \frac{\alpha \theta^O}{r_t^*} \right)^{\frac{1}{1-a}}$$

with $r_t^*$ given by $K_t^Y + K_t^O = \hat{K}$, furthermore $K_{t+1}^Y = K_t^Y$ and $K_{t+1}^O = K_t^O$;

(ii) if $\bar{c} < c_t \leq \bar{c}_t$ we have

$$K_t^Y = \left( \frac{\alpha \theta^Y}{r_t^* + c_t} \right)^{\frac{1}{1-a}} ; \quad K_t^O = \left( \frac{\alpha \theta^O}{r_t^*} \right)^{\frac{1}{1-a}}$$

with $r_t^*$ given by $K_t^Y + K_t^O = \hat{K}$, furthermore $K_{t+1}^Y = K_t^O$ and $K_{t+1}^O = K_t^Y$; and

(iii) if $c_t > \bar{c}_t$ we have

$$K_t^Y = \hat{K} - \hat{K}_t^O ; \quad K_t^O = \hat{K}_t^O$$

furthermore $K_{t+1}^O = K_t^O$ and $K_{t+1}^Y = K_t^Y$.

For (i): suppose $c < c_t \leq \bar{c}$. Then also $c_t \leq \bar{c}_t$, where $\bar{c}_t$ is given by (4.4.2).
Hence we have

\[ K_{t-1}^Y = \hat{K}_t^O \geq \bar{K}^O(c_t) \]

by the monotonicity of \( \bar{K}^O \). It follows that old firms scale down and \( K_t^Y \) and \( K_t^O \) are as posed. Consider period \( t+1 \). Since \( c_t = c_{t+1} \leq \hat{c} \), we have

\[ K_t^Y(c_t) = \hat{K}_{t+1}^O \geq \bar{K}^O(c_{t+1}) \]

by the definition of \( \hat{c} \), given by (4.3.21), and the monotonicity in of \( K^Y \) and \( \bar{K}^O \) in \( c_t \). Hence \( K_t^Y \) and \( K_t^O \) are as posed.

For (ii): suppose \( \hat{c} < c_t \leq \bar{c} \). Then we have

\[ \hat{K}_t^O \geq \bar{K}^O(c_t) \]

so that old firms scale down and allocations are as posed. Moving forward one period it follows from \( c_t = c_{t+1} \leq \bar{c} \) that

\[ K_t^Y(c_t) < \bar{K}^O(c_{t+1}) \]

Hence old firms do not adjust capital and \( K_{t+1}^O = K_t^Y \). By market clearing then

\[ K_{t+1}^Y = K_{t+1}^O. \]

For (iii): suppose \( \bar{c} < c_t \), then \( \hat{K}_t^O < \bar{K}^O(c_t) \) so that old firms do not adjust capital. It follows that \( K_t^O = \hat{K}_t^O \) and, by market clearing, \( K_t^Y = \hat{K} - \hat{K}^O \). In period \( t+1 \), since \( \hat{c} < c_t = c_{t+1} \) we have \( \hat{K}_{t+1}^O < \bar{K}^O(c_{t+1}) \) and so \( K_{t+1}^O = K_t^Y \) and \( K_{t+1}^Y = K_t^O \).

Lemma A.3.3 shows that a small upward change in policy \( (c_t \leq \hat{c}) \) results in steady state allocations that correspond to a higher friction. Now, consider lifetime utility \( U_t^{YW} \) of the young worker at time \( t \). Lemma A.3.3 also implies that young workers will not vote for a higher friction than \( \hat{c} \). To see this note that \( K_t^Y \) and \( K_{t+1}^O \) are strictly decreasing in \( c_t \) for \( \hat{c} < c_t \leq \bar{c} \); they are constant in \( c_t \) for \( c_t > \bar{c} \). Hence young workers strictly prefer \( c_t = \hat{c} \) over any \( c_t > \hat{c} \).

With the auxiliary lemmas, we can now proof lemma 4.4.7. The choice of a
Appendix: Proofs

Persistent friction, $c_t = c_{t+1} \in [0, \infty)$, uniquely determines the equilibrium interest rates, $r_t^*$ and $r_{t+1}^*$, and hence the economic equilibrium at time $t$ and $t + 1$ (cf lemma 4.3.1). Let $c_t^{YW}$ denote the preferred policy of the $YW$ class. We have shown that $c_t^{YW} \in [0, \bar{c}]$ and that, if this policy is set, the economy attains steady state values corresponding to the friction $c_t^{YW}$. It follows that $c_t^{YW} = c_t^{YW}$, where $c_t^{YW}$ is given by lemma 4.4.5. By sincere voting we have $a_t^{YW} = c_t^{YW}$, which concludes the proof.