Other people's money: essays on capital market frictions

Bersem, M.R.C.

Publication date
2012

Appendix A

Appendix: Proofs

This appendix contains the proofs of Chapter 2 (*Incentive-Compatible Sovereign Debt*), Chapter 3 (*Collective Pension Funds*), and Chapter 4 (*Sand in the Wheels of Capitalism*).

A.1 Proofs of Chapter 2

Proof of Proposition 2.3.3:

Proof. Let (O_1, I_d) be an optimal contract and let D be the constant value of O_1 when $I_d(y) = 0$. Consider a new contract $(\tilde{O}_1, \tilde{I}_d)$ given by

$$\tilde{I}_d(y) = \begin{cases} 0 & \text{if} \quad \tilde{D} \leq \min\{\gamma y + B, y\} \\ 1 & \text{if} \quad \tilde{D} > \min\{\gamma y + B, y\} \end{cases}$$

and

$$\tilde{O}_1(y) = \begin{cases} \tilde{D} & \text{if} \quad \tilde{I}_d(y) = 0 \\ \gamma y & \text{if} \quad \tilde{I}_d(y) = 1 \end{cases}$$

and suppose first that $\tilde{D} = D$. If $\tilde{I}_d(y) = I_d(y)$, then the construction of $\tilde{O}_1$ implies that $\tilde{O}_1(y) \geq O_1(y)$. If $\tilde{I}_d(y) < I_d(y)$, i.e. if $\tilde{I}_d(y) = 0$ and $I_d(y) = 1$, then it follows from proposition 2.3.2 that

$$O_1(y) \leq D \leq \tilde{O}_1(y)$$

Furthermore, we can rule out $\tilde{I}_d(y) > I_d(y)$. To see this, suppose that y is such that $\tilde{I}_d(y) = 1$ and $I_d(y) = 0$. Then we know that $O_1(y) = D$, but this cannot be, as we also know that $\gamma y + B < D$ from $\tilde{I}_d(y) = 1$, and an optimal contract must be repudiation proof. This proves that $\tilde{O}_1(y) \geq O_1(y)$ if $\tilde{D} = D$.

Now, one can choose $\tilde{D} \leq D$ such that the investor participation constraint is still satisfied. By construction, the resulting contract $(\tilde{O}_1, \tilde{I}_d)$ satisfies truthful revelation and is repudiation-proof–like any sovereign debt contract. As $\tilde{I}_d(y) \leq I_d(y)$, it must be optimal.

Since both (O_1, I_d) and $(\tilde{O}_1, \tilde{I}_d)$ are optimal contracts, we have

$$E(I_d - \tilde{I}_d)B = 0$$

Consider the state observation function $I_d(y)$. For all states $y \in \left[0, \frac{D-B}{\gamma}\right)$ we must have $I_d(y) = 1$, since $I_d(y) = 0$ would mean that $O_1(y) = D$ which contradicts repudiation-proofness. There may be more states for which $I_d(y) = 1$, as we only know that $I_d(y) \geq \tilde{I}_d(y)$. Let T_2 denote the set of those states, so $T_2 = \left\{ y \geq \frac{D-B}{\gamma} \middle| I_d(y) = 1 \right\}$. We see that

$$EI_d = \int_0^{\frac{D-B}{\gamma}} 1 f(y) dy + \int_{T_2} 1 f(y) dy$$

furthermore we have

$$E\tilde{I}_d = \int_0^{\frac{\tilde{D}-B}{\gamma}} 1 f(y) dy$$

Now since $\tilde{D} \leq D$ and $B > 0$, it follows that (i) $D = \tilde{D}$ and (ii) T_2 has probability mass zero; hence we see that $I_d = \tilde{I}_d$ almost surely. It follows that, as $EO_1 = E\tilde{O}_1$, we must also have that $O_1 = \tilde{O}_1$ almost surely, and I conclude that the optimal contract is a sovereign debt contract. $\square$

A.2 Proofs of Chapter 3

A.2.1 Proof of Proposition 3.2.1

The dynamic investment problem to solve is[1]

$$\max_{\alpha_t, \alpha_{t+1}} Eu(\tilde{b})$$

such that

$$\begin{aligned} w_t &= 0 \\ w_{t+1} &= \alpha_t(1+\tilde{r}_t) + (y - \alpha_t) \\ w_{t+2} &= \alpha_{t+1}(1+\tilde{r}_{t+1}) + (w_{t+1} + y - \alpha_{t+1}) \\ \tilde{b} &= w_{t+2} \end{aligned}$$

First, we rewrite this problem in recursive form

$$v_t(w_t) = \max_{\alpha_t} Ev_{t+1}\left((w_t + y) + \alpha\tilde{r}\right) \tag{A.2.1}$$

where v_t, the remaining-value function, is a function of the financial reserve, w_t. We know that $v_{t+2}(w) = u(w) = \frac{w^{1-\phi}}{1-\phi}$, as individuals consume their financial reserve in retirement. Note that $v_t(0)$ is the expected utility in retirement of a young individual at time t who invests optimally throughout his life. Optimal investment, α_t, is a function of the single state variable, w_t.

We consider the trial solution function $v_{t+1}(w_{t+1}) = \gamma_{t+1}\frac{(w_{t+1}+h_{t+1})^{1-\phi}}{1-\phi}$, where $\gamma_{t+1} > 0$ is a scalar and $h_{t+1}$is the human capital reserve of an individual. Our trial

[1] We drop the superscripts to save on notation.

solution implies that

$$\begin{aligned} v_{t+1}\left((w_t+y)+\alpha_t\tilde{r}\right) &= \gamma_{t+1}\frac{\left((w_t+y)+\alpha_t\tilde{r}+h_{t+1}\right)^{1-\phi}}{1-\phi} \\ &= \gamma_{t+1}\frac{\left(w_t+h_t+\alpha_t\tilde{r}\right)^{1-\phi}}{1-\phi} \end{aligned}$$

so that the first-order condition reads as

$$\gamma_{t+1}E\tilde{r}\left(w_t+h_t+\alpha_t\tilde{r}\right)^{-\phi}=0$$

Solving for optimal investment in the risky asset yields

$$\alpha_t(w_t)=a^*\left(w_t+h_t\right)$$

where

$$a^* := \frac{\left(\frac{-r^l(1-p)}{r^h p}\right)^{\frac{1}{\phi}}-1}{r^l-r^h\left(\frac{-r^l(1-p)}{r^h p}\right)^{\frac{1}{\phi}}} \tag{A.2.2}$$

Finally, it follows from $(A.2.1)$ that

$$\begin{aligned} v_t(w_t) &= \gamma_{t+1}E\frac{\left(w_t+h_t+a^*\left(w_t+h_t\right)\tilde{r}\right)^{1-\phi}}{1-\phi} \\ &= \delta\gamma_{t+1}\frac{\left(w_t+h_t\right)^{1-\phi}}{1-\phi} \end{aligned}$$

where

$$\delta := E\left(1+a^*\tilde{r}\right)^{1-\phi}$$

so that our trial solution is correct with $\gamma_t=\delta\gamma_{t+1}$. Note that

$$\delta \approx 1+(1-\phi)a^*\mu > 1$$

where we've used a first-order approximation. We conclude that, conditional on an optimal investment strategy, expected lifetime utility of the young at time t is

$$\begin{aligned} v_t(0) &= \delta^2 u(h_t) \\ &= \delta^2 u(2y) \end{aligned}$$

□

A.2.2 Proof of Proposition 3.2.2

Incomplete markets give rise to additional constraints, $\alpha_t = 0$ and $\alpha_{t+1} \leq w_{t+1}$. To obtain a lower bound for the welfare loss due to credit constraints, we assume that the middle-aged do not face constraints, only the young do. Then the new investment problem is

$$\max_{\alpha_{t+1}} Eu\left(\tilde{b}\right)$$

such that

$$\begin{aligned} w_t &= 0 \\ w_{t+1} &= y \\ w_{t+2} &= \alpha_{t+1}\left(1+\tilde{r}_{t+1}\right) + \left(w_{t+1} + y - \alpha_{t+1}\right) \\ \tilde{b} &= w_{t+2} \end{aligned}$$

It is easy to see that the middle aged will invest $a^*(2y)$ so that

$$w_{t+2} = 2y\left(1 + a^* \tilde{r}_{t+1}\right)$$

and

$$
\begin{aligned}
Eu\left(\tilde{b}\right) &= Eu\left(2y\left(1+a^{*}\tilde{r}_{t+1}\right)\right) \\
&= E\left(1+a^{*}\tilde{r}_{t+1}\right)^{1-\phi}u(2y) \\
&= \delta \quad u(2y)
\end{aligned}
$$

With proposition 4.3.2 we see that the lower bound for the welfare loss due to credit constraints is

$$\left(\delta^{2}-\delta\right)u(2y)$$

□

A.3 Proofs of Chapter 4

A.3.1 Cobb-Douglas Production

We derive the steady state equilibria for a Cobb-Douglas production economy, where F is given by

$$F(K,L) = K^{\alpha}L^{\beta} \tag{A.3.1}$$

with $0 < \alpha+\beta < 1$. Steady-state capital allocations for $c \in [0,\bar{c}]$ are

$$K^{Y*} = \left(\frac{\alpha\theta^{Y}}{r^{*}(c)+c}\right)^{\frac{1}{1-\alpha}}; \quad K^{O*} = \left(\frac{\alpha\theta^{O}}{r^{*}(c)}\right)^{\frac{1}{1-\alpha}} \tag{A.3.2}$$

with $r^{*}(c)$ given by capital market clearing condition

$$\left(\frac{\alpha\theta^{Y}}{r^{*}(c)+c}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^{O}}{r^{*}(c)}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

The boundary value $\bar{c}$ follows from (4.3.21) and is given by

$$\bar{c} = \alpha \frac{\theta^Y - \theta^O}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}}$$

For $c > \bar{c}$ we have steady state capital allocations

$$K^{Y*} = \left(\frac{\alpha\theta^Y}{r^*(c)+c}\right)^{\frac{1}{1-\alpha}};,\quad K^{O*} = \frac{1}{2}\bar{K}$$

where capital market clearing condition

$$\left(\frac{\alpha\theta^Y}{r^*(c)+c}\right)^{\frac{1}{1-\alpha}} = \frac{1}{2}\bar{K}$$

allows us to obtain the equilibrium interest rate explicitly,

$$r^*(c) = \frac{\alpha\theta^Y}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}} - c$$

Next we determine what steady state equilibria can be politically supported.

A.3.2 Proof of lemma 4.4.2

Consider the equilibrium interest rate in period t. For $c_t < \bar{c}_t$, we have $\hat{K}_t^O \geq \bar{K}_t^O$ so that old dirms scale down and the interest rate is given by

$$\left(\frac{\alpha\theta^Y}{r_t+c_t}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^O}{r_t}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

Implicit differentation yields

$$-\frac{1}{1-\alpha}\frac{K_t^Y}{r_t+c_t}\left(\frac{dr_t}{dc_t}+1\right) - \frac{1}{1-\alpha}\frac{K_t^O}{r_t}\frac{dr_t}{dc_t} = 0 \tag{A.3.3}$$

which rewrites as

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O(\frac{r_t+c_t}{r_t})}$$

so that $\frac{dr_t}{dc_t} \varepsilon(-1,0)$. For $c_t \geq \bar{c}_t$, we have $K_t^O = \hat{K}_t^O$ and the interest rate is given by

$$\left(\frac{\alpha\theta^Y}{r_t + c_t}\right)^{\frac{1}{1-\alpha}} = \bar{K} - \hat{K}_t^O$$

$$r_t = \frac{\alpha\theta^Y}{\left(\bar{K} - \hat{K}_t^O\right)^{1-\alpha}} - c_t$$

and we see that $\frac{dr_t}{dc_t} = -1$. Now for (i), capital in the Y-sector is given by

$$K_t^Y = \left(\frac{\alpha\theta^Y}{r_t + c_t}\right)^{\frac{1}{1-\alpha}}$$

Taking the derivative with respect to c_t gives

$$\frac{dK_t^Y}{dc_t} = \frac{-1}{1-\alpha}\frac{K_t^Y}{r_t + c_t}\left(\frac{dr_t}{dc_t} + 1\right)$$

so that $\frac{dK_t^Y}{dc_t} < 0$ for $c_t \in [0, \bar{c}_t)$ and $\frac{dK_t^Y}{dc_t} = 0$ for $c \geq \bar{c}_t$. Wages in the Y-sector are

$$w_t^Y = \theta^Y \beta \left(K_t^Y\right)^{\alpha}$$

Taking the derivative with respect to c_t yields

$$\frac{dw_t^Y}{dc_t} = -\frac{\beta}{1-\alpha}K_t^Y\left(\frac{dr_t}{dc_t} + 1\right) \tag{A.3.4}$$

so that $\frac{dw_t^Y}{dc_t} < 0$ for $c_t \in [0, \bar{c}_t]$; $\frac{dw_t^Y}{dc_t} = 0$ for $c_t > \bar{c}_t$. For (ii), capital in the O-sector is given by

$$K_t^O = \left(\frac{\alpha\theta^O}{r_t}\right)^{\frac{1}{1-\alpha}}$$

for $c_t < \bar{c}_t$; and

$$K_t^O = \hat{K}^O$$

for $c_t \geq \bar{c}_t$. Hence $\frac{dK_t^O}{dc_t} > 0$ for $c_t \in [0, \bar{c}_t)$ and $\frac{dK_t^O}{dc_t} = 0$ for $c_t \geq \bar{c}_t$. Wages in the O-sector are

$$w_t^O = \theta^O \beta \left(K_t^O\right)^\alpha$$

Taking the derivative with respect to c_t gives

$$\frac{dw_t^O}{dc_t} = -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t} \tag{A.3.5}$$

so that $\frac{dw^O}{dc_t} > 0$ for $c_t \varepsilon [0, \bar{c}_t)$ and $\frac{dw_t^O}{dc_t} = 0$ for $c_t \geq \bar{c}_t$. For (iii), first consider profits. Let π_t^Y and π_t^O denote profits in the Y- and O-sector respectively. Then

$$\pi_t^Y = \theta^Y \left(K_t^Y\right)^\alpha - (r_t + c_t) K_t^Y - w_t^Y$$

and

$$\pi_t^O = (\theta^O)(K_t^O)^\alpha - r_t K_t^O - w_t^O$$

We take the derivative of π_t^Y with respect to c_t and obtain

$$\frac{d\pi_t^Y}{dc_t} = \frac{\alpha + \beta - 1}{1-\alpha} \left(\frac{dr_t}{dc_t} + 1\right) K_t^Y$$

so that $\frac{d\pi_t^Y}{dc_t} < 0$ for $c_t \in [0, \bar{c}_t)$ and $\frac{d\pi_t^Y}{dc_t} = 0$ for $c \geq \bar{c}_t$. Similarly, for O-sector profits, we get

$$\frac{d\pi_t^O}{dc_t} = \frac{\alpha + \beta - 1}{1-\alpha} \frac{dr_t}{dc_t} K_t^O$$

so that $\frac{d\pi_t^O}{dc_t} > 0$ for $c_t \in [0, \bar{c}_t)$ and $\frac{d\pi_t^O}{dc_t} > 0$ for $c_t \geq \bar{c}_t$. Turning to total profits, $\Pi_t = \pi_t^Y + \pi_t^O$, we have

$$\frac{d\Pi_t}{dc_t} = \frac{\alpha + \beta - 1}{1-\alpha} \left[\left(\frac{dr_t}{dc_t} + 1\right) K_t^Y + \frac{dr_t}{dc_t} K_t^O\right] \tag{A.3.6}$$

and we see that Π_t is decreasing in c_t iff

$$\frac{dr_t}{dc_t}\bar{K}+K_t^Y \geq 0$$

Recall that for $c_t \in [0, \bar{c}_t)$ we have

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O(\frac{r_t+c_t}{r_t})}$$

so that $-\frac{dr_t}{dc_t}\bar{K} \leq K_t^Y$ and total profits are nonincreasing in c_t. For $c_t \geq \bar{c}_t$, we have $\frac{dr_t}{dc_t} = -1$ so that total profits are increasing in c_t. This result is due to the fact that r_t declines in c_t while the allocation of capital does not change in this range of capital market frictions. Hence the cost of capital goes down for O-firms, stays the same for Y-firms, as production in both sectors remains the same. While profits may increase in c_t, capital income cannot. Recall that capital income is given by

$$s_t = \frac{r_t\bar{K}+\Pi_t}{\eta}$$

For $c_t \in [0, \bar{c}_t)$, we have $\frac{d\Pi_t}{dc_t} \leq 0$ so that $\frac{ds_t}{dc_t} \leq 0$. For $c_t \geq \bar{c}_t$ we have

$$\frac{ds_t}{dc_t} = \frac{1}{\eta}\left(-\bar{K}+\frac{1-\alpha-\beta}{1-\alpha}K_t^O\right)$$

so that $\frac{ds_t}{dc_t} < 0$. Now, old capitalists income is given by

$$w_t^O + s_t$$

Let $c_t \in [0, \bar{c}_t)$, then we have

$$\begin{aligned}
\frac{d(w_t^O + s_t)}{dc_t} &= -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t} + \frac{1}{\eta}\left(\frac{\beta}{1-\alpha}\frac{dr_t}{dc_t}\bar{K} + \frac{\alpha+\beta-1}{1-\alpha} K_t^Y\right) \\
&< -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t} + \frac{\beta}{1-\alpha}\frac{dr_t}{dc_t}\bar{K} \\
&< 0
\end{aligned}$$

Next, for $c_t \geq \bar{c}_t$, $\frac{dw_t^O}{dc_t} = 0$ and $\frac{ds_t}{dc_t} < 0$ which shows part (iii).

A.3.3 Proof of Lemma 4.4.7

To prove lemma 4.4.7,we first prove two auxiliary lemmas that give the economic equilibrium in periods t and $t+1$. Since we consider out-of steady-state dynamics we assume that the equilibrium at time $t-1$ is given by steady state allocations for some arbitrary $c \in [0, \bar{c}]$. The first lemma describes the economic equilibrium after a downward change in policy:

Let the economic equilibrium at time $t-1$ be given by steady state values for some $c \leq \bar{c}$. Consider a downward change in policy $c_t = c_{t+1} \leq c$, then

$$K_t^Y = \left(\frac{\alpha\theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}}$$

and

$$K_{t+1}^O = \left(\frac{\alpha\theta^O}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^Y + K_t^O = \bar{K}$, furthermore $K_{t+1}^Y = K_t^Y$ and $K_{t+1}^O = K_t^O$.

Since $c_t \leq c$, we have $\bar{K}^O(c) \geq \bar{K}^O(c_t)$, where $\bar{K}^O$ is the equilibrium cut-off value function defined after (4.3.14). It follows that

$$\hat{K}_t^O = K_{t-1}^Y \geq \bar{K}^O(c_t)$$

so that K_t^Y and K_t^O are as posed. Note that because $c_t \leq c$, we have $K_t^O < K_{t-1}^O$ by lemma 4.4.1 and hence $K_t^Y > K_{t-1}^Y$ by market clearing . We must verify that the same allocation obtains in period $t+1$. Moving forward one period we have

$$\hat{K}_{t+1}^O = K_t^Y > \bar{K}^O(c_{t+1})$$

so that again old firms scale down, the same interest rate obtains (i.e. $r_t^* = r_{t+1}^*$), and allocations are as posed.

Lemma A.3.3 shows that a downward change in policy results in steady state values that correspond to a lower reallocation cost. Informally, we can say that changing policy downward moves the economy to a new steady state corresponding to the new value of the friction c_t. The same need not be true for an upward policy change as the next lemma shows.

Let the economic equilibrium at time $t-1$ be given by steady state values for some $c \leq \bar{c}$. Consider an upward change in policy $c_t = c_{t+1} > c$, then

(i) if $c_t \leq \bar{c}$ we have

$$K_t^Y = \left(\frac{\alpha\theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^O = \left(\frac{\alpha\theta^O}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^Y + K_t^O = \bar{K}$, furthermore $K_{t+1}^Y = K_t^Y$ and $K_{t+1}^O = K_t^O$;

(ii) if $\bar{c} < c_t \leq \bar{c}_t$ we have

$$K_t^Y = \left(\frac{\alpha\theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^O = \left(\frac{\alpha\theta^O}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^Y + K_t^O = \bar{K}$, furthermore $K_{t+1}^Y = K_t^O$ and $K_{t+1}^O = K_t^Y$; and

(iii) if $c_t > \bar{c}_t$ we have

$$K_t^Y = \bar{K} - \hat{K}_t^O \quad ; \quad K_t^O = \hat{K}_t^O$$

furthermore $K_{t+1}^O = K_t^Y$ and $K_{t+1}^Y = K_t^O$.

For (i): suppose $c < c_t \leq \bar{c}$. Then also $c_t \leq \bar{c}_t$, where $\bar{c}_t$ is given by (4.4.2).

Hence we have

$$K_{t-1}^Y = \hat{K}_t^O \geq \bar{K}^O(c_t)$$

by the monotonicity of $\bar{K}^O$. It follows that old firms scale down and K_t^Y and K_t^O are as posed. Consider period $t+1$. Since $c_t = c_{t+1} \leq \bar{c}$, we have

$$K_t^Y(c_t) = \hat{K}_{t+1}^O \geq \bar{K}^O(c_{t+1})$$

by the definition of $\bar{c}$, given by (4.3.21), and the monotonicity in of K^Y and $\bar{K}^O$ in c_t. Hence K_{t+1}^Y and K_{t+1}^O are as posed.

For (ii): suppose $\bar{c} < c_t \leq \bar{c}_t$. Then we have

$$\hat{K}_t^O \geq \bar{K}^O(c_t)$$

so that old firms scale down and allocations are as posed. Moving forward one period it follows from $\bar{c} < c_t = c_{t+1}$ that

$$K_t^Y(c_t) < \bar{K}^O(c_{t+1})$$

Hence old firms do not adjust capital and $K_{t+1}^O = K_t^Y$. By market clearing then $K_{t+1}^Y = K_t^O$.

For (iii): suppose $\bar{c}_t < c_t$, then $\hat{K}_t^O < \bar{K}^O(c_t)$ so that old firms do not adjsut capital. It follows that $K_t^O = \hat{K}_t^O$ and, by market clearing, $K_t^Y = \bar{K} - \hat{K}^O$. In period $t+1$, since $\bar{c} < c_t = c_{t+1}$ we have $\hat{K}_{t+1}^O < \bar{K}^O(c_{t+1})$ and so $K_{t+1}^O = K_t^Y$ and $K_{t+1}^Y = K_t^O$.

Lemma A.3.3 shows that a small upward change in policy ($c_t \leq \bar{c}$) results in steady state allocations that correspond to a higher friction. Now, consider lifetime utility U_t^{YW} of the young worker at time t. Lemma A.3.3 also implies that young workers will not vote for a higher friction than $\bar{c}$. To see this note that K_t^Y and K_{t+1}^O are strictly decreasing in c_t for $\bar{c} < c_t \leq \bar{c}_t$; they are constant in c_t for $c_t > \bar{c}_t$. Hence young workers strictly prefer $c_t = \bar{c}$ over any $c_t > \bar{c}$.

With the auxiliary lemmas, we can now proof lemma 4.4.7.The choice of a persistent friction, $c_t = c_{t+1} \in [0, \infty]$, uniquely determines the equilibrium interest rates, r_t^* and r_{t+1}^*, and hence the economic equilibrium at time t and $t+1$ (cf lemma 4.3.1). Let c_t^{YW} denote the preferred policy of the YW class. We have shown that $c_t^{YW} \varepsilon [0, \bar{c}]$ and that, if this policy is set, the economy attains steady state values corresponding to the friction c_t^{YW}. It follows that $c_t^{YW} = c^{YW}$, where c^{YW} is given by lemma 4.4.5. By sincere voting we have $a_t^{YW} = c^{YW}$, which concludes the proof.