Economic development and growth in transition countries

Rusinova, D.T.

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APPENDICES

A. A short review of the cross-sectional spatial econometric models

This appendix discusses some technical details used in chapter 2 and is based on Anselin (1988). There are two broad groups of spatial effects: spatial dependence and spatial heterogeneity. Spatial dependence can take the form of a spatial lag or a spatial error, while spatial heterogeneity can occur in the form of heteroskedastic error or coefficient variation.

A.1. Basic spatial dependence models

The *spatial error model* includes a spatially correlated error term and is characterized by the following equations:

$$Y = X\beta + \varepsilon$$

where

$$\begin{aligned} \varepsilon &= \lambda W_2 \varepsilon + \mu \\ \mu &\sim N(0, \sigma^2) \end{aligned}$$

Here W_2 is a spatial weighting matrix. In this case, the spatial effect affects the residuals, which might be correlated due to unobserved factors or random shocks which affect not only one country, but also other countries. If the spatial error autocorrelation is ignored, the OLS estimates, although not biased, will be inefficient (Anselin, 1992).

The *spatial lag model* corresponds to the presence of a spatially lagged dependent variable:

$$\begin{aligned} y &= \rho W_1 y + X\beta + \varepsilon \\ \varepsilon &\sim N(0, \sigma^2) \end{aligned}$$

W_1 is again a spatial weighting matrix.

In fact, these two main models are related. The spatial error model is a nested model in the so-called spatial Durbin model, including a spatial lag and (possibly) spatially lagged explanatory variables (Mur and Angulo, 2005). The unconstrained form of this model is:

$$y = \rho W y + X\beta + WX\eta + \varepsilon$$

This reduces to a spatial error model if the following non-linear restriction on the coefficients holds:

$$\rho\beta = -\eta$$

This restriction is tested by the test of the *Common factor hypothesis* in the spatial error setting. The null hypothesis is that the above restriction is fulfilled; then a simple spatial error model can be estimated instead of the spatial Durbin model. The alternative hypothesis is a model including a spatial lag Wy and spatially lagged variables WX; if the null is rejected they cannot be omitted from the specification.

If the spatial autocorrelation in the dependent variable is ignored, the OLS estimates will be biased and correspondingly the inference from them incorrect (Anselin, 1992).

A.2. Estimation methods

In the spatial lag and spatial error models, the spatial autoregressive parameters (ρ and λ) have to be estimated together with the parameters of the regression β and σ^2. SpaceStat estimates both models through maximum likelihood and a nonlinear optimization procedure, which relies on the assumption of normal errors. Initially the regression parameters β and σ^2 can be expressed as functions of the autoregressive parameter λ; after their substitution in the likelihood function the so-called concentrated likelihood is obtained, which depends only on λ. The concentrated likelihood is then maximized through a bisection search between the two threshold values, 1/wmin and 1/wmax, which are correspondingly the largest and the smallest eigenvector of the weighting matrix. The spatial lag model is estimated in a similar way – the regression parameters are expressed as functions of the autocorrelation parameter ρ; then they are substituted to obtain the concentrated likelihood and it is maximized between the same two boundary values (Anselin, 1992).

Apart from the estimation with maximum likelihood, the spatial error model can also be estimated in Space Stat with moment conditions. The estimation method is based on Kelejian and Prucha (1999) (Anselin, 1999): If u is an i.i.d. error vector, we have the following system of equations:

$$\begin{aligned} E\left[\frac{u'u}{N}\right] &= \sigma^2 \\ E\left[\frac{u'W'Wu}{N}\right] &= \sigma^2\left(\frac{1}{N}\right) tr\left(W'W\right) \\ E\left[\frac{u'Wu}{N}\right] &= 0 \end{aligned}$$

Replacing above u with $e - \lambda We$, where e is a vector of OLS residuals, we obtain a system of three equations in λ, λ^2 and σ^2 (Anselin, 1999).

A.3. Spatial dependence tests

The spatial tests used in this work are the following[1] :

[1] For a detailed discussion and the formulas of these test statistics, see Anselin (1988).

1. Moran's I – statistic. In matrix form the test statistic is given by

$$I = \frac{N}{S_0} \frac{e'We}{e'e}$$

   where e is a vector of OLS residuals, and $S_0 = \sum_i \sum_j w_{ij}$ is a standardization factor, equal to the sum of the weights of the non-zero cross products, and wij are the elements of a spatial weighting matrix W (Anselin, 1988).

   A positive and significant z-value of that test indicates positive spatial autocorrelation; a negative and significant z-value means negative spatial autocorrelation. The test provides little information about the precise nature of the spatial effect because the alternative hypothesis is very general – it can be spatial residual autocorrelation or spatially lagged variable, heteroskedasticity or non-normality of the errors.

2. *The Lagrange Multiplier error test* – an asymptotic test for the presence of spatial residual autocorrelation, also called the LM-ERR test (Anselin and Florax, 1995). It is $\chi^2(1)$-distributed and depends crucially on the normality of the errors.

3. *Kelejian and Robinson test for spatial lag or spatial error*, which is specification-robust and does not depend on the normality of the errors; however, it is a large sample test with low power in small samples (Anselin, 1992).

4. *Lagrange Multiplier test for spatial lag*, asymptotic and dependent on the normality of the errors. Similarly to the other Lagrange Multiplier error test, it is also $\chi^2(1)$-distributed.

5. *Test for a spatial autoregressive (SARMA) specification*, or presence of both spatial lag and spatial error. It is an F-test for the joint significance of the spatial error coefficient λ and the spatial lag coefficient ρ.

A.4. Spatial heterogeneity models

1. The second type of spatial effect is spatial heterogeneity (Anselin, 1988). It can take two forms: varying coefficients or heteroskedasticity in the error terms. These effects can be formalized correspondingly by switching regimes, random coefficient variation or suggesting a functional relationship for the heteroskedasticity in the error terms.

For the heteroskedastic error model we have

$$\varepsilon \sim N(0, \Omega)$$

where the diagonal elements of the error covariance matrix have the form

$$\Omega_{ii} = h_i(z\alpha)$$

In the case of varying slope parameters the variation can be systematically determined through a limited number of "spatial regimes". The benchmark for distinguishing the regimes can be determined on the basis of economic theory or analysis of the data itself. In the case with two spatial regimes the model is described by the equation:

$$\begin{bmatrix} y_i \\ y_j \end{bmatrix} = \begin{bmatrix} X_i & 0 \\ 0 & X_j \end{bmatrix} \begin{bmatrix} \beta_i \\ \beta_j \end{bmatrix} + \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix}$$

Provided there is no spatial dependence, this model can be estimated as ordinary OLS regression. However, simultaneous appearance of spatial dependence and spatial heterogeneity is possible; in this case, the model has to be estimated with maximum likelihood and the corresponding spatial effect – taken into account. Anselin (1988) demonstrates that ignoring the spatial dependence and estimating through a usual OLS procedure can invalidate OLS inference. Depending on the omitted spatial effect we would need to estimate correspondingly, a combined spatial lag – spatial regime model or spatial error – spatial regime model. (Anselin, 1992).

Since in cross-sectional samples spatial heterogeneity might be observationally equivalent to spatial dependence (Abreu et al. 2005), it is necessary to test for the latter as an alternative of the spatial dependence models. In this chapter this is done through two "switching" regimes, i.e. dividing the countries into two groups and assuming variation in all regression coefficients between them. The first division criterion chosen is whether the country belonged to FSU since the sharp difference between Central and Eastern Europe and the CIS countries in terms of economic performance is well documented in the literature (e.g. Campos and Coricelli, 2002)[2]. The second criterion is whether the country is an accession country to the EU or not. Although the estimated coefficients differ between the two regimes, the tests for the stability of coefficients show that this difference is not statistically significant. We can conclude that in our case spatial dependence is more relevant than spatial heterogeneity.

A.5. Spatial weighting matrices

The general form of a spatial weighting matrix is the following:

The use of the spatial weighting matrix is necessary since the variance-covariance matrix has too many parameters to be estimated using only cross-sectional data (Abreu et al., 2005). Its elements are calculated in the following way:

$$W = \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ a_{21} & 0 & \dots & a_{2n} \\ \dots & \dots & 0 & \dots \\ a_{n1} & \dots & \dots & 0 \end{bmatrix}$$

1. In the case of a binary contiguity matrix with row-standardized weights, the element

$$a_{ij} = \frac{d_{ij}}{\sum d_{ij}}$$

   (a) where $d_{ij} = 1$ if countries i and j share a common border $d_{ij} = 0$ otherwise. where d_{ij} is the distance between the capitals of countries i and j;

[2]FSU dummies explain substantial part of the variance for instance in Aslund, Boone, and Johnson (1996); Havrylyshyn (2001); Cornia and Popov (1998).

(b) For the inverse distance matrix

$$a_{ij} = \frac{1000}{d_{ij}}$$

where d_{ij} is the distance between the capitals of the countries i and j.

(c) For the inverse distance matrix with cutoff point

$$a_{ij} = \begin{matrix} \frac{1000}{d_{ij}} if \ d_{ij} < 1000 \\ 0 \ \ if \ \ d_{ij} \geq 1000 \end{matrix}$$

Here it is assumed that the dependence between the spatial units is present only with a distance of up to 1000 km and disappears with higher distances.

B. Extreme bounds analysis

Table 5.1 lists the results from the two steps of the extreme bounds analysis for the variables from the recession sub-period. At the first step, only the war dummy passes the robustness test of Sala-i-Martin (1997) (to show significance in 95% of the cases) Among the remaining variables, those most frequently significant are the schooling, inflation and investment variables and the FSU dummy. On the second step the war dummy is always included in the regressions (see the second part of Table 5.1), and the variables which remain significant most often are war, schooling, the FSU dummy, and CMEA. Here the reform variables are very fragile: although the 1st generation reforms coefficient has a predominantly positive coefficient, both variables perform quite poorly in terms of significance.

For the second period the average growth during the early period (lagged growth) is robustly correlated to current growth. It has a negative sign and is significant in all regressions (see table 5.2). It appears that countries, which have lost most in income during the recession, tend to grow the fastest during recovery, in particular some CIS countries. This finding is similar to the significance of lagged growth found in EBRD (2004) and Falcetti et al. (2005).

If the robust lagged growth is included as compulsory variable in the regressions (see the last two columns of Table 5.2), the only variables which remain significant at least in some regressions are the schooling and investment variables together with the FSU dummy. They render the inflation, reforms and institutional variables insignificant, which points at the superior explanatory power of factor inputs for that period.

Note: Due to the high correlations of the FSU dummy with the initial conditions and the logarithm of inflation, to avoid multicollinearity, the analysis is repeated without the dummy. The relative significance of the variables changes negligibly.

C. Data appendix

FDI flows and stocks

Data about FDI flows and stocks is taken from the 2007 edition of the WiiW database "WIIW database on Foreign Direct Investment in Central, Eastern and Southeast

Table 5.1: Extreme bounds analysis for the recession period

Variable	number of models	mean value	fraction of positive values	fraction of significant positive values	fraction of significant negative values	fraction of significant positive values	fraction of significant negative values
	Without obligatory variables					With war dummy as obligatory variable	
CONST	2431	-11.5	0.17	0	0.42	0	0.27
Initial income	1012	-0.00	0.25	0	0.05	0	0.04
WAR	1012	-7.68	0	0	0.99	0	0.98
Dummy FSU	1012	-4.04	0.12	0	0.30	0	0.40
Log inflation	1012	-2.69	0.17	0.02	0.33	0.02	0.03
investment	1012	0.20	0.98	0.34	0	0	0
1.gen. reforms	1012	0.23	0.63	0.00	0.04	0	0.04
2.gen. reforms	1012	-0.22	0.02	0.01	0.01	0.01	0
Population growth	1012	0.27	0.69	0	0	0	0
Secondary enrolment	1012	0.26	1	0.76	0	0	0.04
CMEA trade	1012	-0.12	0.09	0	0.212	0	0.41
Over-industrialization	1012	-0.07	0.08	0	0	0	0
Years under communism	1012	-0.06	0.29	0	0.07	0	0.4

Table 5.2: Extreme bounds analysis for the recovery period

Variable	number of models	mean value	fraction of positive values	fraction of significant positive values	fraction of significant negative values	fraction of significant positive values	fraction of significant negative values
Constant	1012	-1.87	0.27	0.24	0.04	0.004	0.13
Lagged growth	1012	-0.29	0	0	1	0	1
Log inflation	1012	-0.03	0.54	0	0.01	0	0
Investment	1012	0.09	1	0.06	0	0.11	0
Enrolment	1012	0.05	1	0.20	0	0.52	0
Population change	1012	-0.19	0.25	0	0	0	0
War dummy	1012	-1.25	0	0	0	0	0
1. gen. reforms	1012	0.14	0.77	0	0	0	0
2. gen. reforms	1012	-0.19	0.12	0	0	0	0
Dummy FSU	1012	0.96	1	0.46	0	0.10	0
Income in 1994	1012	0.0001	0.81	0	0	0	0
Institutions	1012	-0.06	0.41	0	0	0	0

Europe". The main source of the data are the national banks of the corresponding countries. The statistics in the balance of payments are collected in the standardized way described in the Balance of Payments Manual by IMF. There, capital investment abroad is regarded as foreign direct investment if the purpose is to establish and maintain permanent relations with a foreign company (the share of the foreign investor must make up at least 10% of the target firm's equity capital).

Data about FDI flows stems from the Financial account in the balance of payment, and comprises three items: equity capital, other capital, and reinvested earnings. The yearly figures represent net values, a net difference between the increases and decreases in foreign direct investment. An increase in the foreign direct investment in the host country can result from the acquisition of equities and shares in capital, from receipt of principal of a loan or from reinvested profit. Correspondingly, a decrease can be the result of disposal of equities and shares, repayment of loans, and the non-residents share in the loss of the company.

Data about FDI stocks comes from the international investment positions, also compiled by the National banks who rely on company surveys for obtaining this type of information. These data is available usually with one year delay, but national banks may calculate the stocks by aggregating flows. It is recorded each time in the end of the year and is measured in thousands of euro. Since data on both indicators is not available for all countries, in those cases where data on FDI flows is missing, we calculate it by subtracting the corresponding stocks, and vice versa - for those years where FDI stocks are not available we obtain them using the stocks for the available years and the FDI flows from the balance of payments.

In the cases of Slovenia and Poland, since the data series from the WiiW database are too short, we complement the dataset with national bank data for recent years (2005 and 2006). For Bulgaria, since the database does not contain data on Bulgarian FDI stocks, we complemented those with data from the Bulgarian National Bank, retrieved in Bulgarian leva and correspondingly converted into euros with the fixed BNB exchange rate.

When using sectorally disaggregated FDI data, we differentiate between FDI to manufacturing and FDI to services. For Romania, since there is no separate data about FDI to manufacturing, we use FDI in total industry, including manufacturing, mining and quarrying and energy.

Bilateral FDI data used to calculate the weighted foreign R&D are also from the WiiW database.

Imports of capital goods

The imports of capital goods are from the UN comtrade database, which contains data about bilateral trade between all countries, measured in dollars. In order to convert the quantities in euros, we use the nominal effective exchange rate reported by ECB.

Labour productivity

For measuring labour productivity, we use total real GDP in euro, and divide it by the total employment. For most countries, both output and employment are contained in

Eurostat data. An exception is Croatia, for which output data is taken from the AMECO database, and since it is expressed in national currency, we use the nominal effective exchange rate of ECB. Total employment for Croatia stems from the IMF IFS database.

Investment

Investment data are given by the gross capital formation series taken from the ESA database for all countries except Croatia, where we retrieve data from AMECO. Since Croatian investment data is expressed in domestic currency, we first convert it into euro using the ECB nominal effective exchange rate.

R&D stocks

Data about the R&D stocks is extracted from the Eurostat database. The database contains total (private and public) R&D expenditure as a percent of the GDP. From this series, we first obtain the absolute amounts of R&D expenditure by multiplying it with GDP, then we construct the R&D stocks by using the perpetual inventory method explained in detail in Coe and Helpman (1995): it assumes a constant rate of depreciation equal to 5% and obtains the initial value of the R&D stock by extrapolating the yearly R&D expenditures.

D. GMM estimation

The small sample size leads to a number of problems and modifications in our system GMM estimation. First, the number of GMM-type instruments tends to grow quadratically with the number of periods T, and therefore if we were to use the full number of instruments, it would require a panel with large number of groups N. When N is small, the system easily becomes overidentified, with the number of instruments larger than the number of groups. Therefore, we face a trade-off: on one hand, we need a number of instruments equal to the number of regressors in order to be able to estimate the equation. On the other hand however, too large number of instruments leads to overidentification and two negative effects: the Hansen test for overidentifying restrictions becomes unreliable and the instruments fail to isolate the endogenous part of the variables[3]. To avoid these effects, a known rule of thumb is that the number of instruments has to be kept lower than the number of groups (Roodman 2006). In order to meet this requirement, we have to keep the number of instruments low, but then the number of regressors also has to be kept small. For this reason, and in order not to lose degrees of freedom, we do not use time dummies in the short-run regressions; moreover, tests indicate their joint insignificance.

For reducing the instrument count, we use two options in system GMM estimation incorporated by Roodman (2006) into the STATA routine xtabond2: "collapsing" of the instrument matrix and imposing a lag limit for the instruments. The "collapsed" form of the instrument matrix reduces the number of instruments by including only one instrument per time period, while still using the complete information available from all

[3] For a comprehensive and detailed discussion on the pitfalls of system-GMM in panel data like overidentification or violation of the additional moment conditions, see Roodman (2006) and Roodman (2007)

time lags. The second strategy is to limit the lags used to the most recent ones. We dispose of the higher-order lags as instruments, since they are likely to be weak instruments: with increasing the lag order the quality of the instrument decreases (Acharya and Keller, 2007). The maximal lag length used in our GMM regressions was usually 2 or 3.

The decision whether to use the system or difference GMM methods took into account one caveat of the system GMM, namely that it exploits an additional moment condition introduced by Blundell and Bond (1998), which is a non-trivial one:

$$\Delta \left[E y_{i,t-1} \varepsilon_{it} \right] = 0$$

Roodman (2006) demonstrates that this moment condition is equivalent to the following restriction:

$$E \left[\left(y_{i1} - \frac{\mu_i}{1-\alpha} \right) \mu_i \right] = 0$$

The interpretation of the last condition is that the initial deviations of the countries from the steady state are not correlated with the country fixed effects. In other words, the additional condition requires that the units have achieved steady state before the study period (Roodman, 2006). However, it is easy to see that this might be violated in our case. For instance, the country fixed effects might easily capture unobserved transition-specific factors like institutions, policy of reform and advance in restructuring. In the beginning of our sample, some countries (e.g. Bulgaria and Romania) were starting to recover from severe economic crises and were therefore supposedly far from steady state. Moreover, the crises are to a large extent attributable exactly to ill-designed policy and lack of real reform, which would be also reflected in the country's fixed effects.

Therefore, in all cases we ran a full set of diagnostic tests for the validity of instruments. In cases where the test statistics of the system GMM were significant (e.g. significant Sargan test of overidentifying restrictions or difference-Sargan tests), we had to consider the possibility that the additional moment condition might be violated. In these cases, we estimated the regressions with the difference-GMM method, which does not use the additional moment condition, and it usually gave satisfactory values of the diagnostic tests.

E. Proofs of propositions.

Proof. (Proof of Proposition 1)

If we denote with k_{1i} the total capital available to a rent-seeker at the end of period 1, and with $k_0^{(r)}$ and $k_{1i}^{(r)}$ only the share of capital used in rent-seeking in periods 1 and 2, then the shares of capital used in producing are correspondingly $k_0 - k_0^{(r)}$ and $k_{1i} - k_{1i}^{(r)}$.Then the budget constraints are: ■

$$c_{1i} + k_{1i} = k_0 (1-\delta) + \theta_1 s_i k_0^{(r)} + r_1 \left(k_0 - k_0^{(r)} \right) + w_1 \mathbb{I}_{k_0^{(r)}=0}, \tag{E.1}$$

$$c_{2i} = k_{1i} (1-\delta) + \theta_2 s_i k_{1i}^{(r)} + r_2 \left(k_{1i} - k_{1i}^{(r)} \right) + w_2 \mathbb{I}_{k_{1i}^{(r)}=0}. \tag{E.2}$$

In the above constraints, we have substituted Π_i the definition of the rent-seeking function from 4.4.

The decision in this case is more complicated than in the case of workers: the rent-seekers can choose k_0^r and k_1^r in addition to k_1, that results in the following FOCs:

$$\begin{aligned} k_0^r &: \frac{\theta_1 s - r_1}{c_1} \gtreqless 0 \\ k_1^r &: \beta\frac{\theta_2 s - r_2}{c_2} \gtreqless 0 \\ k_1 &: \frac{1}{c_1} = \beta\frac{r_2 + 1 - \delta}{c_2} \end{aligned}$$

The last condition is the Euler equation. We see that there is only one agent indifferent between working and rent-seeking, as the solution of the individual problem is at the corner, and each rent-seeker invests all her capital into one use. In a given period, any of the potential rent-seekers can either work or extract rent, depending on which of the two activities would generate more income for them. Therefore, if in a given period an agent chooses to rent-seek, we have $k_{1i}^{(r)} = k_{1i}$ and the budget constraints reduce to the ones given by..., and in case the agent works, then $k_{1i}^{(r)} = 0$ and the maximization problem is identical to that of a worker.

Proof. (Proof of Proposition 2) ∎

We can compute the consistency condition (4.26) for the first period to get

$$\begin{aligned} \theta_1 k_0 \frac{S}{\lambda}\int_{\gamma_1}^{1} (i + \lambda - 1)\, di &= \gamma_1 T_1 \qquad \text{(E.3)} \\ \theta_1 k_0 \frac{S}{\lambda}\left(\frac{1-\gamma_1^2}{2} + (\lambda - 1)(1-\gamma_1)\right) &= \gamma_1 T_1 \\ \theta_1 k_0 S\left(\frac{1+\gamma_1}{2\lambda} + 1 - \frac{1}{\lambda}\right) &= \frac{\gamma_1}{1-\gamma_1} T_1 \end{aligned}$$

Now we can go back to the arbitrage condition (4.20) and see what it implies for γ_1:

$$\begin{aligned} \theta_1 k_0 (\gamma_1 + \lambda - 1)\frac{S}{\lambda} &= r_1 k_0 + w_1 - T_1 \\ \gamma_1 &= \frac{r_1 k_0 + w_1 - T_1}{\theta_1 k_0 S}\lambda - \lambda + 1 \\ \frac{\gamma_1}{\lambda} &= \frac{r_1 k_0 + w_1 - T_1}{\theta_1 k_0 S} - 1 + \frac{1}{\lambda} \\ \frac{\gamma_1}{\lambda} &= \frac{\alpha A k_0^{\alpha-1} k_0 + (1-\alpha) A k_0^{\alpha} - T_1}{\theta_1 k_0 S} - 1 + \frac{1}{\lambda} \\ \frac{\gamma_1}{\lambda} &= \frac{A k_0^{\alpha} - T_1}{\theta_1 k_0 S} - 1 + \frac{1}{\lambda} \end{aligned}$$

Substituting (E.3) brings us to

$$\begin{aligned}
\frac{\gamma_1}{\lambda} &= \frac{Ak_0^\alpha - \theta_1 k_0 S\left(\frac{1+\gamma_1}{2\lambda} + 1 - \frac{1}{\lambda}\right)\frac{1-\gamma_1}{\gamma_1}}{\theta_1 k_0 S} - \left(1 - \frac{1}{\lambda}\right) \\
\frac{\gamma_1}{\lambda} &= \frac{Ak_0^{\alpha-1}}{\theta_1 S} - \left(1 - \frac{1}{\lambda}\right) - \left(\frac{1+\gamma_1}{2\lambda} + 1 - \frac{1}{\lambda}\right)\frac{1-\gamma_1}{\gamma_1} \\
x &:= \frac{Ak_0^{\alpha-1}}{\theta_1 S} \\
\frac{\gamma_1}{\lambda} &= x - \left(1 - \frac{1}{\lambda}\right)\left(1 + \frac{1-\gamma_1}{\gamma_1}\right) - \frac{1+\gamma_1}{2\lambda}\frac{1-\gamma_1}{\gamma_1} \\
\gamma_1 &= x\lambda - (\lambda - 1)\frac{1}{\gamma_1} - \frac{1+\gamma_1}{2}\frac{1-\gamma_1}{\gamma_1} \\
\gamma_1^2 &= x\lambda\gamma_1 - (\lambda - 1) - \frac{1-\gamma_1^2}{2} \\
\frac{1}{2}\gamma_1^2 &= x\lambda\gamma_1 - \lambda + \frac{1}{2}
\end{aligned}$$

The two solutions of this quadratic equation are: $x\lambda - \sqrt{x^2\lambda^2 - 2\lambda + 1}$ and $x\lambda + \sqrt{x^2\lambda^2 - 2\lambda + 1}$

We have a natural restriction:

$$1 - \lambda < \gamma_1 < 1, \lambda < \frac{1}{2} \tag{E.4}$$

The first root is negative, so only the second root remains relevant:

$$\gamma_1 = x\lambda + \sqrt{x^2\lambda^2 - 2\lambda + 1}$$

It can also be checked whether it satisfies (E.4):

$$\begin{aligned}
1 - \lambda &< x\lambda + \sqrt{x^2\lambda^2 - 2\lambda + 1} < 1, \lambda < \frac{1}{2} \\
1 - \lambda - x\lambda &< \sqrt{x^2\lambda^2 - 2\lambda + 1} < 1 - x\lambda \\
1 - x\lambda &> 0 \to x < \frac{1}{\lambda} \\
x^2\lambda^2 - 2\lambda + 1 &< 1 - 2x\lambda + x^2\lambda^2 \\
1 - 2\lambda &< 1 - 2x\lambda \\
x &< 1
\end{aligned}$$

This is an intuitive result: for an interior solution we need rent-seeking to be more attractive than working for the most able rent-seeker, otherwise everyone works. The other condition is

$$
\begin{aligned}
1-\lambda-x\lambda &< \sqrt{x^2\lambda^2-2\lambda+1} \\
1-2\lambda-2x\lambda+2x\lambda^2+\lambda^2 &< -2\lambda+1 \\
-2x\lambda+2x\lambda^2+\lambda^2 &< 0 \\
\lambda &< 2x\left(1-\lambda\right) \\
x &> \frac{1}{2}\frac{\lambda}{1-\lambda}
\end{aligned}
$$

This condition states that the least able rent seeker always prefers work.

Thus, for the first period all the variables are computable analytically and can be summarized in the following:

$$r_1 = \alpha A k_0^{\alpha-1} \tag{E.5}$$

$$w_1 = \left(1-\alpha\right) A k_0^{\alpha}$$

$$\gamma_1 = x\lambda + \sqrt{x^2\lambda^2-2\lambda+1}, \text{ where } x := \frac{Ak_0^{\alpha-1}}{\theta_1 S} \tag{E.6}$$

$$T_1 = \theta_1 k_0 S\left(\frac{1+\gamma_1}{2\lambda}+1-\frac{1}{\lambda}\right)\frac{1-\gamma_1}{\gamma_1} \tag{E.7}$$

$$\frac{1}{2}\frac{\lambda}{1-\lambda} < \frac{Ak_0^{\alpha-1}}{\theta_1 S} < 1 \tag{E.8}$$

Proof. (Proof of proposition 3) ■

For the second period the system is different from the first one.

$$\theta_2 \frac{S}{\lambda} \int_{\gamma_2}^{1} (i+\lambda-1)\, k_{1i} di = \gamma_2 T_2;$$

$$\theta_2 \tfrac{S}{\lambda} \left(\begin{array}{c} \int_{\gamma_1}^{\gamma_2} (i+\lambda-1) \left(\frac{\beta}{1+\beta} (1+\theta_1 s_i) k_0 - \frac{1}{1+\beta} \frac{W_2}{R_2} \right) di + \\ + \int_{\gamma_2}^{1} (i+\lambda-1) \frac{\beta}{1+\beta} k_0 (1+\theta_1 s_i) di \end{array} \right) \qquad \gamma_1 < \gamma_2;$$
$$= \gamma_2 T_2,$$

$$\theta_2 \tfrac{S}{\lambda} \int_{\gamma_2}^{1} (i+\lambda-1) \tfrac{\beta}{1+\beta} k_0 (1+\theta_1 s_i) di = \gamma_2 T_2 \qquad \gamma_1 \geq \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{1}{1+\beta} \left(-\tfrac{W_2}{R_2} \int_{\gamma_1}^{\gamma_2} (i+\lambda-1) di + \beta k_0 \int_{\gamma_1}^{1} (i+\lambda-1)(1+\theta_1 s_i) di \right) = \gamma_2 T_2, \quad \gamma_1 < \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{\beta}{1+\beta} k_0 \int_{\gamma_2}^{1} (i+\lambda-1)(1+\theta_1 s_i) di = \gamma_2 T_2 \qquad \gamma_1 \geq \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{1}{1+\beta} \left(\begin{array}{c} -\frac{W_2}{R_2} \left(\frac{\gamma_2^2 - \gamma_1^2}{2} + (\lambda-1)(\gamma_2-\gamma_1) \right) + \\ + \beta k_0 \int_{\gamma_1}^{1} (i+\lambda-1) \left(\theta_1 \frac{S}{\lambda} (i+\lambda-1) + 1 \right) di \end{array} \right) \qquad \gamma_1 < \gamma_2;$$
$$= \gamma_2 T_2,$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{\beta}{1+\beta} k_0 \int_{\gamma_2}^{1} \left(\theta_1 \tfrac{S}{\lambda} (i+\lambda-1)^2 + i + \lambda - 1 \right) di = \gamma_2 T_2 \qquad \gamma_1 \geq \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{1}{1+\beta} \left(\begin{array}{c} -\frac{W_2}{R_2} \left(\frac{\gamma_2^2 - \gamma_1^2}{2} + (\lambda-1)(\gamma_2-\gamma_1) \right) + \\ \beta k_0 \left(\begin{array}{c} \theta_1 \frac{S}{3\lambda} \left((1+\lambda-1)^3 - (\gamma_1+\lambda-1)^3 \right) \\ + \frac{1-\gamma_1^2}{2} + (\lambda-1)(1-\gamma_1) \end{array} \right) \end{array} \right) = \gamma_2 T_2, \quad \gamma_1 < \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{\beta}{1+\beta} k_0 \left(\begin{array}{c} \theta_1 \frac{S}{3\lambda} \left((1+\lambda-1)^3 - (\gamma_1+\lambda-1)^3 \right) + \\ + \frac{1-\gamma_1^2}{2} + (\lambda-1)(1-\gamma_1) \end{array} \right) = \gamma_2 T_2 \qquad \gamma_1 \geq \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{1}{1+\beta} \left(\begin{array}{c} -\frac{W_2}{R_2} \left(\left(\frac{\gamma_2+\gamma_1}{2} + \lambda - 1 \right) (\gamma_2-\gamma_1) \right) + \\ + \beta k_0 \left(\begin{array}{c} \theta_1 \frac{S}{3\lambda} \left(\lambda^3 - (\gamma_1+\lambda-1)^3 \right) \\ + \left(\frac{1+\gamma_1}{2} + \lambda - 1 \right)(1-\gamma_1) \end{array} \right) \end{array} \right) = \gamma_2 T_2, \quad \gamma_1 < \gamma_2;$$

$$\theta_2 \tfrac{S}{\lambda} \tfrac{\beta}{1+\beta} k_0 \left(\begin{array}{c} \theta_1 \frac{S}{3\lambda} \left(\lambda^3 - (\gamma_1+\lambda-1)^3 \right) + \\ + \left(\frac{1+\gamma_1}{2} + \lambda - 1 \right)(1-\gamma_1) \end{array} \right) = \gamma_2 T_2 \qquad \gamma_1 \geq \gamma_2;$$

In this way we have implicitly expressed T_2 from other variables, and obtained a constraint on the rent seeking.

It proves useful in the following to express the capital of the agent who is indifferent between rent-seeking and working in the second period as:

$$k_{\gamma_2} = \begin{cases} \frac{\beta}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda} (\gamma_2 - 1 + \lambda)\right), & \gamma_1 < \gamma_2; \\ \frac{1}{1+\beta} \left(\beta (R_1 k_0 + W_1) - \frac{W_2}{R_2}\right), & \gamma_1 \geq \gamma_2. \end{cases}$$

Then for the second period we have the following system of 4 equations with 4 unknowns γ_2, r_2, w_2, T_2, (the last one is expressed above).

$$\begin{array}{rl} K_1^{(s)} = \left(\frac{r_2}{\alpha A}\right)^{\frac{1}{\alpha-1}} \gamma_2 & \gamma_1 < \gamma_2; \\ \frac{\gamma_2}{1+\beta} \left(\beta (R_1 k_0 + W_1) - \frac{W_2}{R_2}\right) = \left(\frac{r_2}{\alpha A}\right)^{\frac{1}{\alpha-1}} \gamma_2, & \gamma_1 \geq \gamma_2. \end{array} \quad \text{(E.9)}$$

$$\gamma_2 = \left(\frac{(1-\alpha) A}{w_2}\right)^{\frac{1}{\alpha}} \left(\frac{r_2}{\alpha A}\right)^{\frac{1}{\alpha-1}} \gamma_2$$

$$\left(\theta_2 \frac{S}{\lambda} (\gamma_2 - 1 + \lambda) - r_2\right) k_{1\gamma_2} = W_2 \quad \text{(E.10)}$$

Here the first expression is the capital market equilibrium. The second is the labor market equilibrium, and the third is the arbitrage condition.

For the case when $\gamma_1 < \gamma_2$ (we can derive the conditions under which this is true after solving the system) we get the following expression

$$\frac{1}{1+\beta} \left(\begin{array}{c} \left(\gamma_1 \beta W_1 - \gamma_2 \frac{W_2}{R_2}\right) + \\ +\beta k_0 \left(\left(\theta_1 \frac{S}{\lambda} \left(\lambda - 1 + \frac{\gamma_2+\gamma_1}{2}\right) + 1\right)(\gamma_2 - \gamma_1) + \gamma_1 R_1\right) \end{array} \right) = \left(\frac{r_2}{\alpha A}\right)^{\frac{1}{\alpha-1}} \gamma_2 \quad \text{(E.11)}$$

$$\left(\frac{\alpha A}{r_2}\right)^{\frac{1}{\alpha-1}} = \left(\frac{(1-\alpha) A}{w_2}\right)^{\frac{1}{\alpha}}$$

$$\left(\theta_2 \frac{S}{\lambda} (\gamma_2 - 1 + \lambda) - r_2\right) \frac{\beta}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda} (\gamma_2 - 1 + \lambda)\right) = w_2 - T_2 \quad \text{(E.12)}$$

$$\theta_2 \frac{S}{\lambda} \frac{1}{1+\beta} \left(\begin{array}{c} -\frac{w_2 - T_2}{R_2} \left(\left(\frac{\gamma_2+\gamma_1}{2} + \lambda - 1\right)(\gamma_2 - \gamma_1)\right) \\ +\beta k_0 \left(\theta_1 \frac{S}{3\lambda} \left(\lambda^3 - (\gamma_1 + \lambda - 1)^3\right) + \left(\frac{1+\gamma_1}{2} + \lambda - 1\right)(1 - \gamma_1)\right) \end{array} \right) = \gamma_2 T_2 \quad \text{(E.13)}$$

The resulting system has to be solved numerically. $\gamma_2 = \frac{1+\gamma_1}{2}$

For the opposite case, $\gamma_1 \geq \gamma_2$ the system is

$$\frac{1}{1+\beta} \left(\beta (R_1 k_0 + W_1) - \frac{W_2}{R_2}\right) = \left(\frac{r_2}{\alpha A}\right)^{\frac{1}{\alpha-1}}$$

$$\left(\frac{\alpha A}{r_2}\right)^{\frac{1}{\alpha-1}} = \left(\frac{(1-\alpha) A}{w_2}\right)^{\frac{1}{\alpha}}$$

$$\left(\theta_2 \frac{S}{\lambda} (\gamma_2 - 1 + \lambda) - r_2\right) \frac{1}{1+\beta} \left(\beta (R_1 k_0 + W_1) - \frac{W_2}{R_2}\right) = W_2$$

$$\theta_2 \frac{S}{\lambda} \frac{\beta}{1+\beta} k_0 \left(\theta_1 \frac{S}{3\lambda} \left(\lambda^3 - (\gamma_1 + \lambda - 1)^3\right) + \left(\frac{1+\gamma_1}{2} + \lambda - 1\right)(1 - \gamma_1)\right) = \gamma_2 T_2$$

Here the last expression explicitly determines the size of rents.

Proof. (Proof of proposition 4)

The Euler equation is unchanged as compared to the exogenous case:

$$k_{1i} = \frac{\beta}{1+\beta} k_0 \left(1 + \theta_1 s_i\right).$$

■

From the first-order condition with respect to the contribution we get

$$\begin{aligned}
\frac{1}{c_1} &= \beta \frac{\frac{1}{2}\sqrt{\frac{\theta_1}{BO}} s^i k_1}{c_2} \\
k_1 + \left(\hat{\theta} + \sqrt{\frac{\theta_1 O}{B}}\right) s^{\gamma_i} k_1 &= \frac{1}{2}\beta\sqrt{\frac{\theta_1}{BO}} s^{\gamma_i} k_1 \left(k_0 + \theta_1 s^{\gamma_i} k_0 - k_1 - o^i\right) \\
1 + \left(\hat{\theta} + \sqrt{\frac{\theta_1 O}{B}}\right) s^{\gamma_i} &= \frac{1}{2}\beta\sqrt{\frac{\theta_1}{BO}} s^{\gamma_i} \left(k_0 + \theta_1 s^{\gamma_i} k_0 - k_1 - o^i\right) \\
1 + \left(\hat{\theta} + \sqrt{\frac{\theta_1 O}{B}}\right) s^{\gamma_i} &= \frac{1}{2}\beta\sqrt{\frac{\theta_1}{BO}} s^{\gamma_i} \left(k_0 + \theta_1 s^{\gamma_i} k_0 - \frac{\beta}{1+\beta} k_0 \left(1 + \theta_1 s^{\gamma_i}\right) - o^i\right) \\
\frac{1}{s^i} + \hat{\theta} + \sqrt{\frac{\theta_1 O}{B}} &= \frac{1}{2}\beta\sqrt{\frac{\theta_1}{BO}} \left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) - o^i\right), s^i = \frac{S}{\lambda}\left(i - 1 + \lambda\right) \\
\left(\frac{1}{s^i} + \hat{\theta}\right)\sqrt{\frac{BO}{\theta_1}} + O &= \frac{1}{2}\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) - o^i\right)
\end{aligned}$$

Rewritten without radicals:

$$\left(\frac{1}{s^i} + \hat{\theta}\right)^2 \frac{BO}{\theta_1} = \left(\frac{1}{2}\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) - o^i\right)\right)^2 - O\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) - o^i\right) + O^2$$

For the indifferent rent-seeker the equation takes the following form:

$$\begin{aligned}
\left(\frac{1}{\frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)} + \hat{\theta}\right)^2 \frac{BO}{\theta_1} &= \left(\frac{1}{2}\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)\right)\right)\right)^2 \\
&\quad - O\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)\right)\right) + O^2
\end{aligned}$$

which is a quadratic equation:

$$\begin{aligned}
AO^2 + DO + C &= 0, \text{ where:} \\
A &= 1 \\
D &= -\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)\right)\right) - \left(\frac{1}{\frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)} + \hat{\theta}\right)^2 \frac{B}{\theta_1} \\
C &= \left(\frac{1}{2}\beta\left(\frac{1}{1+\beta} k_0 \left(1 + \theta_1 \frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right)\right)\right)\right)^2
\end{aligned}$$

and the solution is

$$O = \frac{1}{2}\left(-D \pm \sqrt{D^2 - 4C}\right)$$

In order to understand which root is relevant, we use the budget constraint:

$$\begin{aligned} o^i &< k_0 + \theta_1 s^i k_0 - c_1 - k_1 \\ o^i &< k_0 + \theta_1 s^i k_0 - \frac{\beta}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) \\ o^i &< \frac{1}{1+\beta} k_0 \left(1 + \theta_1 s^i\right) \end{aligned}$$

Due to monotonicity we have that the contribution of the most able rent-seeker is the highest, so we can write

$$O < \frac{1-\gamma_2}{1+\beta} k_0 \left(1 + \theta_1 S\right)$$

If we assume that $O + D < 0$, then the relevant root is $-D - \sqrt{D^2 - 4C}$. The corresponding conditions for this inequality are as follows:

$$O = \frac{1}{2}\left(\begin{array}{c} \beta\left(\frac{1}{1+\beta}k_0\left(1+\theta_1\frac{S}{\lambda}\left(\gamma_2-1+\lambda\right)\right)\right)+\left(\frac{1}{\frac{S}{\lambda}\left(\gamma_2-1+\lambda\right)}+\hat{\theta}\right)^2\frac{B}{\theta_1} \\ -\sqrt{\begin{array}{c}\left(\beta\left(\frac{1}{1+\beta}k_0\left(1+\theta_1\frac{S}{\lambda}\left(\gamma_2-1+\lambda\right)\right)\right)+\left(\frac{1}{\frac{S}{\lambda}\left(\gamma_2-1+\lambda\right)}+\hat{\theta}\right)^2\frac{B}{\theta_1}\right)^2 \\ -\left(\beta\left(\frac{1}{1+\beta}k_0\left(1+\theta_1\frac{S}{\lambda}\left(\gamma_2-1+\lambda\right)\right)\right)\right)^2\end{array}} \end{array}\right)$$

We introduce the following notation to simplify the above expression:

$$\begin{aligned} x &: = \frac{S}{\lambda}\left(\gamma_2 - 1 + \lambda\right), y := \frac{1}{1+\beta}k_0, z := y\left(1 + \theta_1 x\right), X := \left(\frac{1}{x} + \hat{\theta}\right)\sqrt{\frac{B}{\theta_1}} \\ O &= \frac{1}{2}\left(\beta z + \left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1} - \sqrt{\left(\beta z + \left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1}\right)^2 - (\beta z)^2}\right) \\ O &= \frac{1}{2}\left(\beta z + \left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1} - \sqrt{\left(\left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1}\right)^2 + 2\beta z\left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1}}\right) \\ O &= \frac{1}{2}\left(\beta z + \left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1} - \left(\frac{1}{x}+\hat{\theta}\right)\sqrt{\frac{B}{\theta_1}}\sqrt{\left(\frac{1}{x}+\hat{\theta}\right)^2\frac{B}{\theta_1} + 2\beta z}\right) \\ O &= \frac{1}{2}\left(\beta z + X^2 - X\sqrt{X^2 + 2\beta z}\right) \end{aligned}$$

Therefore, we obtain the solution numerically.

Once we know the total contribution, the individual can be found as a solution to another quadratic equation:

$$\left(\frac{1}{s^i}+\hat{\theta}\right)^2\frac{BO}{\theta_1} = \left(\frac{1}{2}\frac{\beta}{1+\beta}k_0\left(1+\theta_1 s^i\right)\right)^2-\frac{\beta}{1+\beta}k_0\left(1+\theta_1 s^i\right)o+ +o^2-O\beta\left(\frac{1}{1+\beta}k_0\left(1+\theta_1 s^i\right)-o\right)+O^2$$

In fact, the individual contribution only matters for the redistribution.

Proof. of proposition 5 ■

This can be demonstrated trivially using the expression $\theta_2(O)=\hat{\theta}+\sqrt{\frac{\theta_1 O}{B}}$. Given $B\to\infty$, then we have $\theta_2(O)=\hat{\theta}=\theta_1-\mu$.Since now rent-seekers can have no influence on the second period institutional value, it is optimal to set $O=0$. (In the comparative statics section, it is also demonstrated numerically that total contribution O decreases with increasing B). With these modifications, all equations of the model become identical to the exogenous model.

F. Derivations for variables of interest in simulations

We derive the expressions for the following two additional economic variables which are of interest: the difference in incomes and the period 2 total output.

Income difference

The income difference is given by the ratio:

$$ID=\frac{Yr}{Yw}$$

Here Yr and Yw are respectively the income from rent-seeking of the average rent-seeker (with $i=\frac{1+\gamma 2}{2}$) and the income from working.

$$ID=\frac{s^{\frac{1+\gamma 2}{2}}\theta_2 k_1^I}{(r_2 k_{1w}+w_2-T_2)}$$

Using the expression for s_i given by its distribution

$$s_i=\frac{(i-1)\,S}{\lambda}+S$$

we obtain

$$ID=\frac{\left(\frac{(\gamma 2-1)S}{2\lambda}+S\right)\theta_2 k_1^I}{(r_2 k_{1w}+w_2-T_2)}$$

Total output

Another variable we trace is the second - period total output. In the first period, total second - period output is simple:

$$Y_1 = A\left(\gamma_1 k_0\right)^{\alpha} \gamma_1^{1-\alpha} = A\gamma_1 \left(k_0\right)^{\alpha}$$

where k_w is the workers' capital. In the opposite case total second - period output is

$$\begin{aligned} Y_2 &= AK^{\alpha}L^{1-\alpha} \\ Y_2 &= A\left(\gamma_1 k_{1w} + \int_{\gamma 1}^{\gamma 2} k_{1i} di\right)^{\alpha} \gamma_2^{1-\alpha} \end{aligned}$$

where k^{add} is the following integral (under k^i we have substituted the capital of those that are rent-seeking in period 1 and then working)

$$\begin{aligned} k^{add} &= \int_{\gamma 1}^{\gamma 2} \left(\frac{\beta}{1+\beta}\left(1+\theta_1 s_i\right) k_0 - \frac{1}{1+\beta}\frac{W_2}{R_2}\right) di \\ k^{add} &= \int_{\gamma 1}^{\gamma 2} \left(\frac{\beta}{1+\beta}\left(1+\theta_1\left(\frac{\left(i-1\right)S}{\lambda}+S\right)\right) k_0 - \frac{1}{1+\beta}\frac{W_2}{R_2}\right) di \\ &= \frac{\beta k_0}{1+\beta}\int_{\gamma 1}^{\gamma 2}\left(1+\frac{\theta_1 iS}{\lambda}-\frac{\theta_1 S}{\lambda}+\theta_1 S\right) di - \frac{\left(\gamma_2-\gamma_1\right)}{1+\beta}\frac{w_2-T_2}{r_2+1} \\ &= \frac{\beta k_0}{1+\beta}\left(\frac{\theta_1 S}{2\lambda}\left(\gamma_2^2-\gamma_1^2\right)+\left(1-\frac{\theta_1 S}{\lambda}+\theta_1 S\right)\left(\gamma_2-\gamma_1\right)\right) - \frac{\left(\gamma_2-\gamma_1\right)}{1+\beta}\frac{w_2-T2}{r_2+1} \\ & \frac{\beta k_0\left(\gamma_2-\gamma_1\right)}{1+\beta}\left(\frac{\theta_1 S}{2\lambda}\left(\gamma_2+\gamma_1\right)+\left(1-\frac{\theta_1 S}{\lambda}+\theta_1 S\right)-\frac{w_2-T2}{\left(r_2+1\right)\beta k_0}\right) \end{aligned}$$

Now we can substitute:

$$Y_2 = A\left(\gamma_1 k^w + k^{add}\right)^{\alpha} \gamma_2^{1-\alpha}$$

Total productive investment

Total productive investment is given by the sum of the investment of workers and those period-1 rent-seekers that switch to working in period 2.

$$
\begin{aligned}
K_1^p &= k_{1w}\gamma_1 + \int\limits_{\gamma 1}^{\gamma 2} k_{1i} di \\
K_1^p &= \left(\frac{\beta}{1+\beta}\left(\left(r_2+1\right)k_0 + w_1 - T_1\right) - \frac{w_2 - T_2}{r_2+1}\right)\gamma_1 + \int\limits_{\gamma 1}^{\gamma 2}\left(\frac{\beta}{1+\beta}\left(1+\theta_1 s_i\right)k_0 - \frac{1}{1+\beta}\frac{W_2}{R_2}\right)di \\
K_1^p &= \left(\frac{\beta}{1+\beta}\left(\left(r_2+1\right)k_0 + w_1 - T_1\right) - \frac{w_2 - T_2}{r_2+1}\right)\gamma_1 + \\
&\quad \frac{\beta k_0\left(\gamma_2-\gamma_1\right)}{1+\beta}\left(\frac{\theta_1 S}{2\lambda}\left(\gamma_2+\gamma_1\right) + \left(1 - \frac{\theta_1 S}{\lambda} + \theta_1 S\right) - \frac{w_2 - T2}{\left(r_2+1\right)\beta k_0}\right)
\end{aligned}
$$

Total rent-seeking investment

Total rent-seeking investment is given by the sum of investment of the double rent-seekers: those period 1 rent-seekers who continue rent-seeking in period 2. The integral is calculated analogously to the above case of productive investment:

$$
\begin{aligned}
K_1^r &= \int\limits_{\gamma 2}^{1} k_{1i} di \\
K_1^r &= \int\limits_{\gamma 2}^{1}\left(\frac{\beta}{1+\beta}\left(1+\theta_1 s^i\right)k_0\right)di \\
K_1^r &= \frac{\beta k_0\left(\gamma_2-\gamma_1\right)}{1+\beta}\left(\frac{\theta_1 S}{2\lambda}\left(\gamma_2+\gamma_1\right) + \left(1 - \frac{\theta_1 S}{\lambda} + \theta_1 S\right)\right)
\end{aligned}
$$