Essays on the measurement sensitivity of risk aversion and causal effects in education

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4 A Parametric Analysis of Prospect Theory's Functionals[*]

4.1 Introduction

After numerous studies systematically falsified the classical expected utility model as descriptive theory of decision making under risk (Allais 1953; Kahneman and Tversky 1979), various new descriptive theories of individual decision making under risk have been developed (Starmer 2000). The most prominent of these non-expected utility models is prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Prospect theory entails two fundamental breakaways from the classical model. Instead of defining preferences over wealth, preferences are defined over changes with respect to a *flexible* reference point, often taken as the status quo. Decision makers are assumed to be less sensitive to changes in outcomes further away from this reference point, which is called *diminishing sensitivity*, and it is assumed that negative changes (losses) hurt more than positive changes (gains), a phenomenon called *loss aversion*. This generalization helps explain phenomena such as the equity premium puzzle (Benartzy and Thaler 1995), downward-sloping labor supply (Goette et al. 2004), the End-of-the-day-Effect in horse race betting (McGlothlin 1956), and the co-existence of appreciable small stake- and moderate large stake- risk aversion (Rabin 2000a). Furthermore, linearity in probability is replaced by a subjective probability weighting function that is assumed to have an inverse-S shape, reflecting increased sensitivity toward changes in probabilities near 0 and 1. This accommodates anomalies of the classical model such as the Allais paradox (1953), the co-

[*] This chapter is based on Booij et al. (2007).

existence of gambling and insurance, betting on long-shots at horse races (Jullien and Salanié 2000), and the avoidance of probabilistic insurance (Wakker et al. 1997).[29]

The generalization that prospect theory entails breaks the one-to-one relationship between utility curvature and risk attitudes that holds under expected utility. Hence, in the prospect theory framework, risk attitudes are jointly determined by utility curvature, *and* subjective probability weighting, where outcomes are defined as changes with respect to the status quo. This adds complexity to the interpretation of the degree of *risk aversion* (preferring the expected value of a prospect to the prospect itself), as it can no longer be summarized into a single index of curvature (Wakker 1994), and it complicates the empirical determination of risk aversion, because of the simultaneous confounding effects of utility curvature and subjective probability weighting (Tversky and Kahneman 1992).

In order to test prospect theory's hypotheses about the specific functional forms, and to quantify the sources of risk aversion, various authors have attempted to empirically determine the prevailing shape for the utility- and probability-weighting functions. These studies deal with the simultaneity problem by either assuming a parametric form for these functions (Tversky and Kahneman 1992; Camerer and Ho 1994; Tversky and Fox 1995; Donkers et al. 2001; Harrison and Rutström 2007; Abdellaoui et al. 2008) or by exploiting a particular design that permits them to be disentangled non-parametrically (Wakker and Deneffe 1996; Abdellaoui 2000; Bleichrodt and Pinto 2000; Abdellaoui et al. 2007b).

Both approaches have their advantages and drawbacks. The parametric approaches are easy to estimate and interpret, but they suffer from a contamination effect: a misspecification of the utility function will also bias the estimated probability weighting function and vice versa (Abdellaoui 2000). For instance, in the parametric estimation of prospect theory, Harrison and Rutström (2007) assume the one parameter probability weighting function introduced by Tversky and Kahneman (1992). This function may be a misspecification if the true weighting function exhibits underweighting for intermediate and large probabilities, and minimal overweighting of small probabilities. Moreover, the authors assume the probability weighting function for gains and losses to be equal. This assumption will directly affect the loss aversion measure if the degree of pessimism differs between both domains.

[29] For a survey of examples of field phenomena that prospect theory can and expected utility cannot explain, see

Donkers et al. (2001) impose the same restriction and use a one parameter weighting function due to Prelec (1998). Both studies find relatively much utility curvature and a low degree of loss aversion compared to the non-parametric approaches, which suggests that the probability weighting function may have been mis-specified. Another disadvantage of the parametric approach is that allowing for unobserved heterogeneity in the model is necessarily parametric which means the results may depend on the choice of the stochastic error process (Wilcox 2008, p. 265).

The non-parametric methods do not have these problems as no functional forms are assumed beforehand and estimation is conducted at the individual level allowing for full heterogeneity. This approach, however, requires data that have a chained nature which may introduce error propagation leading to less precise inference (Wakker and Deneffe 1996; Blavatskyy 2006) and, in theory, an incentive compatibility problem (Harrison and Rutström 2008).

This chapter aims at combining the best of both approaches by parametrically estimating the complete prospect theory model, thereby allowing for decision errors, using a rich dataset that permits the identification of prospect theory's functionals without making stringent parametric assumptions. The results have relevance for the empirical issue of whether the utility for losses is convex (Currim and Sarin 1989; Tversky and Kahneman 1992; Abdellaoui 2000; Etchart-Vincent 2004) or concave (Davidson et al. 1957; Laury and Holt 2000 (for real incentives only); Fehr-Duhda et al. 2006; Abdellaoui et al. 2008) and also whether the prevailing shape of the probability weighting function in the population is inverse S-shaped (Kahneman and Tversky 1992; Wu and Gonzales 1996; Fehr-Duhda et al. 2006), linear (Hey and Orme 1994) or convex (Jullien and Salanié 2000; Goeree et al. 2002; van de Kuilen et al. 2006).

The data that are used in this study are obtained from the same representative internet survey that is used in chapter 3, which consists of 27 matching questions per individual. To reduce the dependence on functional form assumptions we use a three stage estimation procedure that exploits the (gamble-) trade-off method for the elicitation of utilities. This method is robust against subjective probability distortion (Wakker and Deneffe 1996) such

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Camerer (2000).

that the measurement of utility does not depend on the estimates of the probability weights. Our stochastic specification allows for decision errors, and it naturally accommodates the propagation of errors that is introduced by the chaining of the questions that is at the heart the trade-off method (Blavatskyy 2006). Furthermore, the data contains background variables that can be linked to the obtained preference parameters to shed light on how the various components of risk attitudes vary in the population. Finally, a randomly assigned scaling-up of the outcomes by a factor 10 allows us to test whether utility curvature and probability weighting are sensitive to the magnitude of the stakes (Etchart-Vincent 2004).

The analysis confirms and complements the study in chapter 3 (Booij and van de Kuilen 2007) which presents non-parametric estimates of utility curvature and loss aversion obtained from a subset of the same data. The results reiterate the main finding that utility curvature is close to linear and much less pronounced than suggested by classical utility measurements that neglect probability weighting. Diminishing sensitivity is also found, as predicted by prospect theory but contrary to the classical prediction of universal concavity. Utility for gains and losses is found to be closer to linear compared to other parametric studies, suggesting these may be mis-specified, while the parametric results are a little more curved compared to the non-parametric estimates. This suggests that assuming homogeneity leads to a small downward bias, while providing evidence that error propagation is unlikely to greatly affect the results in the non-parametric analysis.

In addition to these results we find evidence of an inverted-S shaped probability weighting function that is more elevated for losses than for gains, suggesting pessimism in both domains. We do not find evidence that the shape or the degree of elevation of the probability weighting functions depend on the magnitude of the stakes, but the weighting function for gains varies with gender and age. The weighting function for losses seems unrelated to any background variables. These results confirm the common finding that females are more risk averse than males, but contrary to classical studies that ascribed this gender difference solely to differences in the degree of utility curvature, our results show that this finding is primarily driven by subjective probability weighting and loss aversion.

The remainder of this chapter is organized as follows. Section 4.2 discusses prospect theory and summarizes the parametric estimates found in the literature. Section 4.3 presents the experimental method and summary statistics of the data, followed by the presentation of the econometric specification in section 4.4. The results are presented in section 4.5. Section 4.6 concludes, followed by the appendix to this chapter that provides tables of additional results and experimental instructions.

4.2 Prospect Theory

4.2.1 Parametric specifications

With prospect theory we refer to the modern (cumulative) version described in section 3.3. To make the model empirically tractable, several parametric shapes have been proposed for the utility- and probability weighting functions. The utility function determines individuals' attitudes towards additional monetary gains and losses. The curvature of this function for gains is often modeled by a power function because of its simplicity and its good fit to (experimental) data (Wakker 2008).[30] Tversky and Kahneman (1992) introduced this function for prospect theory, written as $\mathrm{U}(x) = x^{a}\mathbb{1}(x \geq 0) - \lambda(-x)^{\beta}\mathbb{1}(x < 0)$. Here the parameters α and β determine the curvature of the utility for money gains and losses respectively. The psychological concept of diminishing sensitivity implies that both $\alpha < 1$ and $\beta < 1$, i.e. individuals are decreasingly sensitive to changes further away from the reference point. Less frequently used parametric specifications of the utility function are the exponential and the expo-power utility functions. These functions often have a slightly inferior fit. Their properties are described extensively in Abdellaoui et al. (2007a).

The parameter λ determines the utility ratio between a gain of one Euro and a loss of one Euro:

$$\lambda = -\frac{\mathrm{U}(-1)}{\mathrm{U}(1)}. \tag{4.2.1}$$

[30] In its traditional use in the expected utility model, power utility implies that the fraction of wealth that an agent is prepared to pay to forego a fair gamble over percentages of wealth, is constant. Therefore, the power function is commonly referred to as constant relative risk aversion (CRRA). Under non-expected utility models such as prospect theory this designation is no longer appropriate.

This definition of loss aversion is slightly different from that in (3.4.4). Both can be seen as an approximation of the definition proposed by Köbberling and Wakker (2005) ($\lambda^{KW} \equiv \mathrm{U}'_{\uparrow}(0)/\mathrm{U}'_{\downarrow}(0)$).[31] An individual is defined loss averse when $\lambda > 1$.

The probability weighting function captures the degree of sensitivity towards probabilities. Two distinct properties of this function have been put forward, that can be given a psychological interpretation. The first property refers to the degree of curvature of the probability weighting function, which reflects the degree of discriminability with respect to changes in probabilities. This property is closely linked to the notion of diminishing sensitivity, where the probability of 0 (impossibility) and 1 (certainty) serve as reference points. According to this psychological hypothesis, people's behavior becomes less responsive to changes on the probability scale as they move further away from these reference points. This implies an inverse-S shaped weighting function, with relatively much curvature near the probability end points and a linear shape in between. The second property of the probability weighting function refers to its elevation, which determines the degree of attractiveness of gambling (Gonzalez and Wu 1999). For gains (losses), a highly elevated probability weighting function implies that individuals are optimistic (pessimistic), and overweight probabilities relative to the objective probabilities of winning (losing).

Several parametric functions have been proposed to describe the probably weighting function (see Stott 2006 for an overview). The most commonly used specification is the linear-in-log-odds specification, introduced by Goldstein and Einhorn (1987) (GE-87), and given by:

$$w(p) = \delta p^{\gamma} \Big/ \left(\delta p^{\gamma} + (1-p)^{\gamma}\right).$$

The popularity of this function stems from its empirical tractability and the fact that it has two parameters γ and δ, that separately control curvature and elevation respectively. Hence, both parameters can readily be given a psychological interpretation as indexes of discriminability and attractiveness. Another popular specification in which γ and δ have a

[31] Both measures will be almost identical if utility for gains is not significantly more curved over the inteval [1, x_0] than utility of losses over the interval [y_0, −1]. Given the approximate lineairity of utility over these short intervals (Wakker and Deneffe 1996), this is likely to hold approximately.

similar interpretation is the two parameter specification due to Prelec (1998) (Prelec-2), given by:

$$w(p) = \exp\left(-\delta(-\ln p)^{\gamma}\right).$$

The GE-87 specification has an inverted-S shape when $0 < \gamma < 1$. An additional (sufficient) condition for the Prelec-2 function is $0 < \delta < 1$. One-parameter specifications have also been used to describe the probability weighting function, but these cannot set curvature and elevation independently. Estimates of these probability weighting functions will lead to biased inferences if curvature and elevation do not co-vary accordingly.

4.2.2 Empirical evidence

Table 4.1 gives the definition and estimates of the power utility function and some commonly used one- and two-parameter probability weighting functions. All the mentioned studies estimate prospect theory, albeit with varying (parametric) assumptions, incentives, tasks and samples. Although the table is not intended to be exhaustive, it covers most studies that somehow report a parametric measure of utility curvature, loss aversion or probability weighting under prospect theory. Studies that do not report such estimates are not included in the table, which means that not all studies mentioned in the introduction are listed. If multiple measures of loss aversion are reported we take the definition that most closely resembles that of Köbberling and Wakker (2005).

With respect to the shape of the utility function the table reveals four notable features. First of all, the utility for gains is much closer to linearity (a power equal to 1) than what is found in classical utility measurements that do not take probability weighting into account. In that literature estimates just below .5 (Cubitt et al. 2001; Holt and Laury 2002; Harrison et al. 2005b; Andersen et al. 2008) or lower (Barsky et al. 1997; Dohmen et al. 2006) are common. Second, in all studies that report utility curvature for gains and losses, losses are evaluated more linearly than gains, but utility for losses does display diminishing sensitivity ($\beta < 1$) in most studies. This suggests that people become less sensitive towards additional gains more rapidly as compared to additional losses. Third, there is some variability in the estimates, but the power parameters for both domains are always quite close. This suggests that the differences in the estimates between studies most likely stem from differences in the elicitation method and the method of analysis. Fourth, there is significant variation in the coefficient of loss aversion, but it is always estimated to be higher than one.

Table 4.1: Empirical estimates of prospect theory using different parametric functionals

Functional Form, name	**Estimates**				**Properties****				**Authors**
Utility	α	β	λ		E.	T	I	N	
$U(x)=x^{\alpha}\mathbb{1}(x\geq 0)-\lambda(-x)^{\beta}\mathbb{1}(x<0)$	.88	.88	2.25		md	c	n	25	Tversky and Kahneman (1992)
, power.*	.22				ml	c	n	1497	Camerer and Ho (1994)
	.50				ml	c	n	420	Wu and Gonzalez (1996)
	.39	.84			md	c	n	64	Fennema and van Assen (1999)
	.49				md	c	y	10	Gonzalez and Wu (1999)
	.89	.92			md	c	y	40	Abdellaoui (2000)
	.61	.61			ml	b	n	2593	Donkers et al. (2001)
			1.43		md	c	n	45	Schmidt and Traub (2002)
		.97			md	c	n	35	Etchart-Vincent (2004)
	.91	.96			md	c	n	41	Abdellaoui et al. (2005)
	.68	.74	3.2		ml	b	n	1743	Tu (2005)
	1.01	1.05			md	c	y	181	Fehr-Duda et al. (2006)
	.72	.73	2.54		md	c	n	48	Abdellaoui et al. (2007b)
	.81	.80	1.07		ml	c	y	90	Andersen et al. (2006)
	.71	.72	1.38		ml	c	y	158	Harrison and Rutström (2007)
	.86	1.06	2.61		md	c	y	48	Abdellaoui et al. (2008)
Probability weights.	δ^{+}	γ^{+}	δ^{-}	γ^{-}					
$w(p)=\frac{p^{\gamma}}{\left[p^{\gamma}+(1-p)^{\gamma}\right]^{\frac{1}{\gamma}}}$, TK-92.		.61		.69		c	n	25	Tversky and Kahneman (1992)
		.56				c	n	1497	Camerer and Ho (1994)
		.71				c	n	420	Wu and Gonzales (1996)
		.60		.70		c	y	40	Abdellaoui (2000)
		.67				m	n	51	Bleichrodt and Pinto (2000)
		.76		.76		c	y	90	Andersen et al. (2006)
		.91		.91		c	y	158	Harrison and Rutström (2007)
$w(p)=\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$, GE-87.	.84	.68				c	n	420	Wu and Gonzalez (1996)
	.77	.69			md	c	n	40	Tversky and Fox (1995)
	.77	.44				c	y	10	Gonzalez and Wu (1999)
	.65	.60	.84	.65		c	y	40	Abdellaoui (2000)
	.82	.55				m	n	51	Bleichrodt and Pinto (2000)
			1.10	.84		c	n	35	Etchart-Vincent (2004)
	.98	.83	1.35	.84		c	n	41	Abdellaoui et al. (2005)
	.41	1.24				c	y	78	van de Kuilen et al. (2006)
	.87	.51	1.07	.53		c	y	181	Fehr-Duda et al. (2006)
$w(p)=\exp\left(-(-\ln p)^{\gamma}\right)$, Prelec-1.		.74				c	n	420	Wu and Gonzalez (1996)
		.53				m	n	51	Bleichrodt and Pinto (2000)
		.413		.413		b	n	2593	Donkers et al. (2001)
		1.00		.77		b	n	1743	Tu (2005)
$w(p)=\exp\left(-\delta(-\ln p)^{\gamma}\right)$, Prelec-2.	1.08	.53				m	n	51	Bleichrodt and Pinto (2000)
	2.12	.96			ml	m	y	80	Goeree et al. (2002)

Notes: Adopted names and notations do not form a convention, and are used for convenience. As reported in the text, γ mainly controls curvature (sensitivity) and δ mainly controls elevation (attractiveness) of the probability weighting function. +/− denote gains/losses.

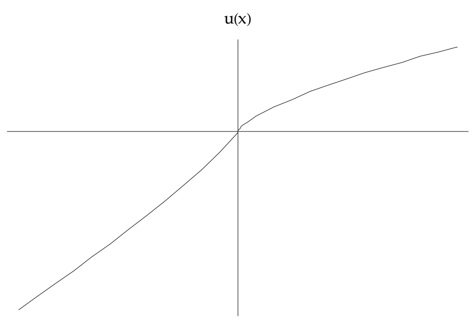
* The utility functional is specified on the complete real axis, where λ represents the loss aversion coefficient which creates a kink in utility at the status quo. The displayed utility function is based on the assumption $\alpha>0$ and $\beta>0$, which is mostly found empirically. The function has a different specification for other parameter values (Wakker 2008).

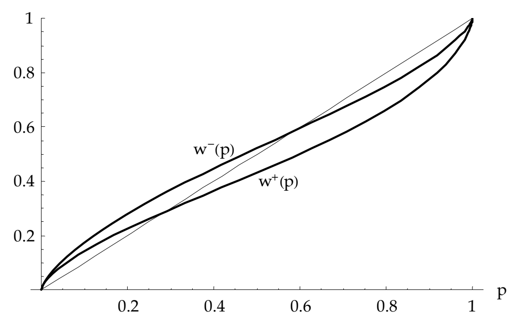
** Properties: E(estimator): m: mean; md: median; ml: maximum likelihood; T(task): c: choice; m: matching; b: both; I(incentives): yes (random lottery incentive scheme/Becker de Groot-Marschak procedure; no (fixed or no payment).

The table conveys three other notable features with respect to the estimated shape of the probability weighting function. The predominant shape is inverse-S, with few studies reporting $\gamma > 1$. Also, for studies that report estimates of both domains, elevation is higher in the loss domain. This is intuitively plausible because it suggests that in both domains individuals display pessimism, i.e. they dislike gambling. Finally, the estimates of elevation show a little less variability than those of curvature, suggesting that curvature is harder to identify empirically.

The coefficients of loss aversion reported in Table 4.1 range from 1.07 to 3.2. Hence, all studies find evidence of loss aversion, albeit to varying degrees. This may be caused by differing definitions of loss aversion and different elicitation contexts. Figure 4.1 plots a power utility function and a GE-87 probability weighting function for gains and losses corresponding to the average of the estimates found in Table 4.1. The next section describes the data that will enable us to identify utility curvature and probability weighting for a representative sample.

Figure 4.1: Utility and probability weighting functions for average estimates





Note: Figure based on the average of the estimates from Table 4.1. $(\alpha,\beta,\gamma)=(.69,.86,2.07)$ and $(\delta^{+},\gamma^{+},\delta^{-},\gamma^{-})=(.76,.69,1.09,.72)$.

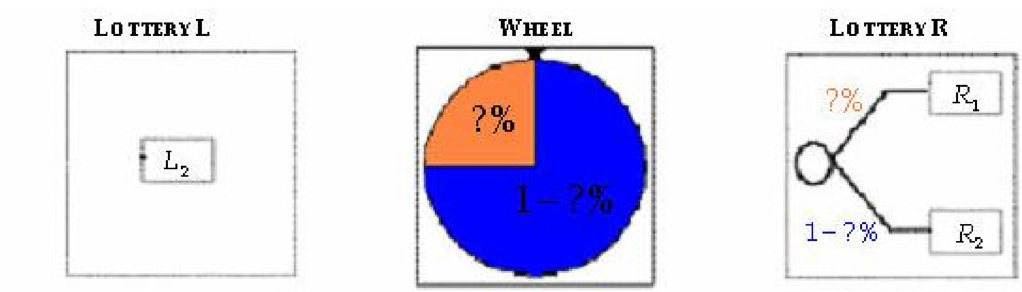
4.3 The Data

4.3.1 *Survey design*

Participants. For the elicitation of both utility curvature and subjective probability weighting we used the same internet questionnaire that was used in chapter 3. There, only the first part of the questionnaire (Q1 – Q16) is used to non-parametrically identify utility for a representative sample of the Dutch population (see section 3.5 for details). The second part of the questionnaire (Q17 – Q27) consists of questions that, in conjunction with the first part, allow for the determination of the probability weighting function for gains and losses, which is the aim of this chapter.

Procedure. After completion of the first part, respondents first read experimental instructions for the second part (section 4.7.2), followed by a practice question. In this part indifference was obtained through probability matching, i.e. in Figure 4.2 subjects were asked to report the (missing) probability that would make them indifferent between two particular lotteries, where the parameters (L_2, R_1, R_2) differed between questions. After filling in a specific number the areas in the wheel were filled accordingly and the respondent was asked to confirm his choice or reconsider.

Figure 4.2: The Framing of the Prospect Pairs in Part II



Note: The specific parameter values varied between the questions, see Table 4.2.

Table 4.2 gives a full description of the parameter values of the questions of the second part. The ten main questions Q18 – Q27 re-used the answers $x_0,\ldots, x_6, y_0,\ldots, y_6$ given in the first part, and asked the respondents to give the probability that would make them indifferent between the two prospects (Figure 4.2). For example, if an individual responded with $\underline{x_1}$=€180 and $\underline{x_6}$=€800 in the first part, then Q18 would elicit the probability $\underline{p_1}$ that made him indifferent between prospects (€180) and ($\underline{p_1}$: €800, €100).

The questions of the second part allow for the non-parametric determination of the subjective probability weighting functions at the individual level if one assumes that no stochastic errors have been made in the elicitation of the indifference outcomes ($x_0,\ldots, x_6, y_0,\ldots, y_6$) in the first part. To see this, consider the domain of gains and assume that there is no stochastic error component in the subjects' responses. Then, under prospect theory, the reported probabilities p_i satisfy $w(p_i) = ((\mathrm{U}(x_i) - \mathrm{U}(x_0))/((\mathrm{U}(x_6) - \mathrm{U}(x_0))$. Given that the outcomes $x_0,\ldots, x_6$ comprise a standard sequence of outcomes, there holds $\mathrm{U}(x_i) - \mathrm{U}(x_i) = \mathrm{U}(x_{i+1}) - \mathrm{U}(x_i)$ for $i = 1,\ldots,5$. This implies that $((\mathrm{U}(x_i) - \mathrm{U}(x_0))/((\mathrm{U}(x_6) - \mathrm{U}(x_0)) = i/6$, and hence $w(p_i) = i/6$ (Abdellaoui 2000). However, in the presence of error this correspondence need no longer hold because the outcomes $x_0,\ldots, x_6$, are then, in general, not equally spaced in utility units. The econometric specification we use explicitly accounts for this in the analysis of the responses to these questions.

Table 4.2: The Obtained Indifferences in Part II

Matching Question	Prospect L		Prospect R
	(L_2)		(p: R_1, R_2)
17 (practice)	(250)	~	($\underline{r}$: 750,−100)
18*	(x_1)	~	($\underline{p_1}$: x_6, x_0)
19*	(y_1)	~	($\underline{q_1}$: y_6, y_0)
20*	(x_2)	~	($\underline{p_2}$: x_6, x_0)
21*	(y_2)	~	($\underline{q_2}$: y_6, y_0)
22*	(x_3)	~	($\underline{p_3}$: x_6, x_0)
23*	(y_3)	~	($\underline{q_3}$: y_6, y_0)
24*	(x_4)	~	($\underline{p_4}$: x_6, x_0)
25*	(y_4)	~	($\underline{q_4}$: y_6, y_0)
26*	(x_5)	~	($\underline{p_5}$: x_6, x_0)
27*	(y_5)	~	($\underline{q_5}$: y_6, y_0)

Note: Underlined outcomes are the matching probabilities and questions marked with an asterisk were presented in randomized order. The specific values x_i and y_i are obtained in the first part.

4.3.2 *Summary statistics*

For the estimation of utility curvature and loss aversion we use the questions from the first part of the experiment (see section 3.5) for the same sample as in chapter 3, i.e. we use only individuals that gave monotonous responses in the first part. The order of the probability matching questions that were posed in the second part was completely random, meaning that subjects could not have an easy comparison with questions that had outcomes close in magnitude. This increases the likelihood of an inconsistent answer. Moreover, these questions are likely to be more cognitively demanding for respondents. Hence, in the second part we allowed for one mistake (meaning a violation of dominance) in the subjects' answers before classifying them as inconsistent. Furthermore, we only considered individuals in the second part if they had been classified consistent in the first, because the questions in the second part were determined by the first. Of the remaining data we removed some outlying answers that clearly indicated either a mistake or lack of understanding (denoted by *Outlier*). As in chapter 3, we estimated a sample-selection equation and used the inverse of the predicted probabilities as weights in the econometric analysis to control for a potential bias due to sample selectivity (see section 4.7.1). This procedure yields unbiased estimates if sample selection is random conditional on the selection variables. Most coefficients were not greatly affected by this procedure, except for the measure of loss aversion, which is adjusted upwards as in chapter 3 because men and higher educated people are over-represented in the sample. The sample selection process was already discussed in more detail in section 3.6.1. Table 4.3 gives the summary statistics of the selected sample.

Table 4.3: Summary Statistics (unweighted)

	x_i		y_i		p_i		q_i		b	
i	**High**	**Low**	**High**	**Low**	**High**	**Low**	**High**	**Low**	**High**	**Low**
1	€ 1993	€ 205	€ −851	€ −86	27.3 %	31.6 %	21.5 %	21.5 %	€ 4016	€ 386
	(602)	(94)	(231)	(36)	(15.3)	(17.1)	(12.9)	(12.2)	(1604)	(150)
2	€ 3000	€ 319	€ −1243	€ −126	41.4 %	41.3 %	33.1 %	33.9 %	c	
	(1131)	(184)	(431)	(59)	(17.5)	(17.4)	(14.8)	(16.3)		
3	€ 4060	€ 441	€ −1664	€ −168	51.6 %	53.0 %	42.8 %	43.7 %	€ −1569	€ −157
	(1692)	(313)	(634)	(83)	(17.4)	(17.4)	(16.5)	(16.4)	(612)	(59.0)
4	€ 5161	€ 576	€ −2075	€ −211	62.5 %	63.0 %	53.4 %	53.4 %	d	
	(2311)	(561)	(856)	(106)	(18.7)	(19.3)	(20.1)	(19.1)		
5	€ 6283	€ 727	€ −2494	€ −254	75.9 %	74.5 %	68.6 %	66.9 %	€ 1842	€ 180
	(2980)	(865)	(1069)	(130)	(19.2)	(19.9)	(21.3)	(21.7)	(833)	(87.6)
6	€ 7447	€ 893	€ −2920	€ −298						
	(3713)	(1244)	(1297)	(156)						
N	383	431	330	360	184	182	147	125	210	228
Non-R.	187	188	210	212	126	158	112	145	1	1
Non-M.	388	340	422	389	73	91	71	90	54	69
Outlier	13	5	9	3					5	8
Total	971	964	971	964	383	431	330	360	270	306

Note: Standard deviations in parentheses.

The table readily shows some apparent features of the data. The differences between subsequent outcomes of the standard sequence are gradually increasing, suggesting mild concavity in the utility for gains and mild convexity for losses. Also, the probabilities reported in the gain domain are all uniformly higher than the ones in the loss domain suggesting more elevation in the probability weighting function for losses. This is consistent with pessimism with respect to gambling in both domains. Finally, the outcomes between the high and the low treatments are mostly close to a scaling up by a factor 10, suggesting no difference between treatments.

4.4 The econometric model

Following Wakker and Deneffe (1996) and Abdellaoui (2000), Booij and van de Kuilen (2007, chapter 3) exploit the sequential nature of the questions to analyze the shape of the utility function non-parametrically. This approach has the advantage of being robust against probability weighting and allowing for full heterogeneity in preferences, i.e. they estimate the shape of the utility curve for each individual without making any prior parametric assumption. The disadvantage of this approach is that individual error is not explicitly accounted for statistically, and potential error propagation is not modeled. Also, for the second part, errors generating monotonicity violations would yield uninterpretable weighting functions (recall that we allow for one monotonicity violation in the second part) if errors are not modeled in the analysis. Moreover, Wilcox (2008, pp. 264-265) shows that individual level estimation can suffer from a finite-sample bias leading to biased predictions. By smoothing out errors a parametric approach can alleviate these problems (Currim and Sarin 1989), albeit at the cost of having to make auxiliary assumptions.

Under prospect theory, as described in section 3.3, the questions in the experiment yield the following equations

$$w^{+}(\tfrac{1}{2})(\mathrm{U}(x_{i,n})-\mathrm{U}(x_{i-1,n}))=(1-w^{+}(\tfrac{1}{2}))(\mathrm{U}(G_n)-\mathrm{U}(g_n))\cdot e_{i,n}^{o+}\cdot\eta_n^{+} \qquad i=1,\ldots,6, \tag{4.4.1}$$

$$w^{-}(\tfrac{1}{2})(\mathrm{U}(y_{i,n})-\mathrm{U}(y_{i-1,n}))=(1-w^{-}(\tfrac{1}{2}))(\mathrm{U}(L_n)-\mathrm{U}(l_n))\cdot e_{i,n}^{o-}\cdot\eta_n^{-} \qquad i=1,\ldots,6, \tag{4.4.2}$$

$$\mathrm{U}(x_{i,n})-\mathrm{U}(x_{0,n})=w^{+}(p_{i,n})\left(\mathrm{U}(x_{6,n})-\mathrm{U}(x_{0,n})\right)\cdot e_{i,n}^{p+} \qquad i=1,\ldots,5, \tag{4.4.3}$$

$$\mathrm{U}(y_{i,n})-\mathrm{U}(y_{0,n})=w^{-}(p_{i,n})\left(\mathrm{U}(y_{6,n})-\mathrm{U}(y_{0,n})\right)\cdot e_{i,n}^{p-} \qquad i=1,\ldots,5, \tag{4.4.4}$$

$$w^{+}\left(\tfrac{1}{2}\right)\left(\mathrm{U}\left(b_n\right)-\mathrm{U}\left(x_{1,n}\right)\right)=\left(1-w^{+}\left(\tfrac{1}{2}\right)\right)\mathrm{U}\left(x_{0,n}\right)\cdot e_n^{b}\quad , \tag{4.4.5}$$

$$w^{-}\left(\tfrac{1}{2}\right)\left(\mathrm{U}\left(c_n\right)-\mathrm{U}\left(y_{1,n}\right)\right)=\left(1-w^{-}\left(\tfrac{1}{2}\right)\right)\mathrm{U}\left(y_{0,n}\right)\cdot e_n^{c}\quad , \tag{4.4.6}$$

$$w^{+}\left(\tfrac{1}{2}\right)\left(\mathrm{U}\left(d_n\right)-\mathrm{U}\left(x_{0,n}\right)\right)=w^{-}\left(\tfrac{1}{2}\right)\left(\mathrm{U}\left(y_{0,n}\right)-\mathrm{U}\left(y_{1,n}\right)\right)\cdot e_n^{d}\quad , \tag{4.4.7}$$

where we allow for a multiplicative stochastic error ($e_{i,n}^{\bullet}$), including individual specific effects $\eta_n^{\bullet}$ that capture differences in probability weighting between individuals *n*. In the superscripts, *o* and *p* denote outcomes and probabilities respectively, and the + and – signs denote the gain and the loss domain. The letters b, c, *d*, refer to the corresponding loss aversion questions (see section 3.5).

The errors are assumed to be independently log-normally distributed with different variances, i.e. $e^{\bullet}_{i,n} \sim LN\left(0, \sigma_i^{\bullet 2}\right)$. This is a Fechner model on the log of the value scale, similar to the model employed by Donkers et al. (2001). We chose a multiplicative specification over an additive one (e.g. Blavatskyy 2006) because it naturally satisfies monotonicity. An additive specification would require a truncated error distribution to satisfy monotonicity (Blavatskyy 2007), which is numerically much more involved. Also, we chose the common Fechner structure over a random preference specification or a "trembling hand" specification, two other popular stochastic models (Wilcox 2008). In the first stochastic framework it would be hard to eliminate individual effects, while it is unclear how to implement the second in a continuous outcome context. The consequences of different error specifications in models of decision making under risk has attracted increased attention since the seminal paper by Hey and Orme (1994). There is, however, currently no consensus in the literature on what error structure to use (Hey 1995; Loomes and Sugden 1995; Carbone and Hey 2000; Blavatskyy 2007).

In order to eliminate the probability weighting terms and potential individual specific effects, subsequent outcome equations can be divided by one another. To make the current study consistent with chapter 3, loss aversion is estimated using all questions around the zero outcome. Taking logarithms then gives

$$\varepsilon^{o+}_{i,n} \equiv \ln\left(\frac{e^{o+}_{i+1,n}}{e^{o+}_{i,n}}\right) = \ln\left(\frac{\mathrm{U}(x_{i+1,n}) - \mathrm{U}(x_{i,n})}{\mathrm{U}(x_{i,n}) - \mathrm{U}(x_{i-1,n})}\right) \qquad i = 1,\ldots,5, \tag{4.4.8}$$

$$\varepsilon^{o-}_{i,n} \equiv \ln\left(\frac{e^{o-}_{i+1,n}}{e^{o-}_{i,n}}\right) = \ln\left(\frac{\mathrm{U}(y_{i+1,n}) - \mathrm{U}(y_{i,n})}{\mathrm{U}(y_{i,n}) - \mathrm{U}(y_{i-1,n})}\right) \qquad i = 1,\ldots,5, \tag{4.4.9}$$

$$\varepsilon^{p+}_{i,n} \equiv \ln\left(e^{p+}_{i,n}\right) = \ln\left(w^{+}\left(p_i\right)\frac{\mathrm{U}(x_{6,n}) - \mathrm{U}(x_{0,n})}{\mathrm{U}(x_{i,n}) - \mathrm{U}(x_{0,n})}\right) \qquad i = 1,\ldots,5, \tag{4.4.10}$$

$$\varepsilon^{p-}_{i,n} \equiv \ln\left(e^{p-}_{i,n}\right) = \ln\left(w^{-}\left(q_i\right)\frac{\mathrm{U}(y_{6,n}) - \mathrm{U}(y_{0,n})}{\mathrm{U}(y_{i,n}) - \mathrm{U}(y_{0,n})}\right) \qquad i = 1,\ldots,5, \tag{4.4.11}$$

$$\varepsilon^{LA}_{n} \equiv \ln\left(e^{d}_{n} \cdot e^{c}_{n} / e^{b}_{n}\right) = \ln\left(\frac{\left(1 - w^{+}\left(\frac{1}{2}\right)\right)}{\left(1 - w^{-}\left(\frac{1}{2}\right)\right)}\frac{\left(\mathrm{U}\left(d_n\right) - \mathrm{U}\left(x_{0,n}\right)\right)}{\left(\mathrm{U}\left(b_n\right) - \mathrm{U}\left(x_{1,n}\right)\right)}\frac{\left(\mathrm{U}\left(c_n\right) - \mathrm{U}\left(y_{1,n}\right)\right)}{\left(\mathrm{U}\left(y_{0,n}\right) - \mathrm{U}\left(y_{1,n}\right)\right)}\frac{\mathrm{U}\left(x_{0,n}\right)}{\mathrm{U}\left(y_{0,n}\right)}\right), \tag{4.4.12}$$

where *LA*, denotes loss aversion. Under the assumptions of (4.4.1)-(4.4.7), the transformed error terms, collected in $\varepsilon_n = \left(\varepsilon'^{o+}_{n}, \varepsilon'^{p+}_{n}, \varepsilon'^{o-}_{n}, \varepsilon'^{p-}_{n}, \varepsilon^{LA}_{n}\right)' = (\varepsilon^{o+}_{1,n}, \ldots, \varepsilon^{o+}_{5,n},\ \varepsilon^{o-}_{1,n}, \ldots, \varepsilon^{o-}_{5,n},\ \varepsilon^{p+}_{1,n}, \ldots, \varepsilon^{p+}_{5,n},$

$\varepsilon_{1,n}^{p-},\dots,\varepsilon_{5,n}^{p-},\ \varepsilon_{n}^{LA})'$, are normally distributed with zero mean and covariance matrix Σ. This matrix has off-diagonal elements equal to zero, except for the outcome equations (4.4.1) and (4.4.2). The first differencing applied to these equations generates a correlation between the subsequent error terms. For example, assuming constant error variance, the covariance matrix for positive outcomes is a tridiagonal matrix equal to

$$\Sigma^{o+} = \operatorname{cov}\left[\varepsilon_{1,n}^{o+},\dots,\varepsilon_{5,n}^{o+}\right] = 2\sigma^2 \begin{pmatrix} 1 & -\frac{1}{2} & 0 & \cdots & 0 \\ -\frac{1}{2} & 1 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -\frac{1}{2} \\ 0 & \cdots & 0 & -\frac{1}{2} & 1 \end{pmatrix}. \tag{4.4.13}$$

In this example the correlation between each subsequent error is $-\frac{1}{2}$.[32] In general the first off-diagonal elements will vary. Hence, we will assume the covariance matrices of the outcome domains (Σ^{o+}, Σ^{o-}) to be fully flexible in the empirical analysis. Because the questions of the second part are not chained we simply assume the matrixes Σ^{p+}, Σ^{p-} to have equal (non-zero) diagonal and off-diagonal elements. By assuming non-zero off diagonal elements, within-subject correlation in the answers is accounted for. The mean of the diagonal and off-diagonal elements are given by $\bar{\sigma}$ and $\bar{\rho}$ respectively.

To estimate the model we assume two popular parametric specifications. For utility we take the common power specification, with a loss aversion factor λ, as specified by Kahneman and Tversky (1979). For the subjective weighting of cumulative probabilities we take the frequently used linear-in-log-odds specification as first employed by Goltstein and Einhorn (1987). These parametric families have been shown to have a good fit to experimental data (Gonzalez and Wu 1999; Abdellaoui 2008).[33] The probability weighting functions of both domains are allowed to differ as is assumed in the modern version of prospect theory. We have

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[32] To see this, consider the covariance of two subsequent errors in the gain domain: $\operatorname{cov}\left[\varepsilon_{1,n}^{o+},\varepsilon_{2,n}^{o+}\right] = \operatorname{cov}\left[\ln e_{1,n}^{o+} - \ln e_{0,n}^{o+}, \ln e_{2,n}^{o+} - \ln e_{1,n}^{o+}\right] = \operatorname{cov}\left[\ln e_{1,n}^{o+},\ln e_{2,n}^{o+}\right] - \operatorname{cov}\left[\ln e_{1,n}^{o+},\ln e_{1,n}^{o+}\right] - \operatorname{cov}\left[\ln e_{0,n}^{o+},\ln e_{2,n}^{o+}\right]$ $+\operatorname{cov}\left[\ln e_{0,n}^{o+},\ln e_{1,n}^{o+}\right] = 0-\sigma^2-0+0 = -\sigma^2$. The correlation then becomes: $corr\left[\varepsilon_{1,n}^{o+},\varepsilon_{2,n}^{o+}\right] =$ $\operatorname{cov}\left[\varepsilon_{1,n}^{o+},\varepsilon_{2,n}^{o+}\right] \Big/ \sqrt{\left(\operatorname{var}\left[\varepsilon_{1,n}^{o+}\right]\cdot\operatorname{var}\left[\varepsilon_{2,n}^{o+}\right]\right)} = -\sigma^2 \Big/ \sqrt{\left(2\sigma^2\cdot 2\sigma^2\right)} = -\frac{1}{2}$.

[33] In the context of discrete choice Stott (2006) shows that the more parsimonious one parameter specifications often provide a sufficient fit in terms of the Akaike information criterion.

$$\mathrm{U}(x;\alpha,\beta,\lambda)=\begin{cases} x^{\alpha} & x\geq 0 \\ -\lambda(-x)^{\beta} & x<0 \end{cases} \tag{4.4.14}$$

$$\begin{aligned} w^{+}(p;\delta^{+},\gamma^{+}) &= \frac{\delta^{+}p^{\gamma^{+}}}{\delta^{+}p^{\gamma^{+}}+(1-p)^{\gamma^{+}}} \\ w^{-}(p;\delta^{-},\gamma^{-}) &= \frac{\delta^{-}p^{\gamma^{-}}}{\delta^{-}p^{\gamma^{-}}+(1-p)^{\gamma^{-}}}. \end{aligned} \tag{4.4.15}$$

This gives the log-likelihood function:

$$\ell(\alpha,\beta,\lambda,\delta^{+},\gamma^{+},\delta^{-},\gamma^{-})=\sum_{n=1}^{N}-\tfrac{1}{2}\left\{\ln 2\pi+2\ln|\mathbf{\Sigma}|+\varepsilon_n'\mathbf{\Sigma}^{-1}\varepsilon_n\right\}. \tag{4.4.16}$$

To estimate the model we split up the likelihood and use a three stage procedure (limited-information maximum likelihood, LIML) to estimate utilities, and subsequently the probability weighting function and loss aversion. This has two advantages. First of all, it will ensure that the estimated utility curve will not suffer from a functional-form misspecification bias due to misspecification of the probability weighting function. This is precisely what Wakker and Deneffe's (1996) trade-off method is designed for. Using full-information maximum likelihood would eliminate this advantage by re-introducing an interaction between the estimation of probability weighting and utility curvature. Also, the outcome matching questions (Part I) are generally believed to be easier to respond to and give higher quality data. Hence we base the estimate of utility only on the questions from the first part. In the second stage the probability weighting functions are estimated using the estimates of utility from the first stage. Loss aversion is estimated in the final stage, taking the estimated utility and probability weighting functions as given. Table 4.4 summarizes the estimation strategy.

Table 4.4: Estimation Strategy

	1ST STAGE (OUTCOMES)	**2ND STAGE (PROBABILITIES)**	**3RD STAGE (LOSS AV.)**
Gains	Obtain $\hat{\alpha}$	Obtain $(\hat{\delta}^{+},\hat{\gamma}^{+})$	Obtain $\hat{\lambda}$
Losses	Obtain $\hat{\beta}$	Obtain $(\hat{\delta}^{-},\hat{\gamma}^{-})$	
cov	Σ^{o+}, Σ^{o-}	Σ^{p+}, Σ^{p-}	σ^{LA}

By splitting up the estimation we cannot determine the correlations between the errors of the different question modules, i.e. utilities and probabilities, gains and losses. This is unfortunate since it would be interesting to know whether there is unobserved heterogeneity that affects the answers in both domains in a structural way, but it does bias the results.[34] The standard errors in the second and third stages are corrected for the uncertainty in the first stage estimates by using the adjustment specified by Murphy and Topel (1979).[35]

4.5 Results

The model as such assumes homogeneity in preferences. A certain degree of heterogeneity can be implemented, however, by parameterizing the preference parameters $\boldsymbol{\phi} = \left(\alpha, \beta, \lambda, \delta^{+}, \gamma^{+}, \delta^{-}, \gamma^{-}\right)'$ by a linear combination of regressors, i.e. $\boldsymbol{\phi} = \boldsymbol{B}'\boldsymbol{X}$. Hence, apart from estimating the average shape of utility and probability weighting we can test whether there are significant differences in these preferences with respect to variables such as age, gender, education and income. The first row of estimates in Table 4.5 gives the results of the model with only a constant, while the second gives the model with the set of demographic variables that appear to be associated with prospect theory's parameters.

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[34] Note that for the same reason we would not be able to estimate any correlation between random coefficients if they were specified. This is done in Tu (2005), who is unable to identify most correlations, but the ones he does indicate a negative correlation in risk aversion caused by the outcome and probability domain. However, Tu's model is not non-parametrically identified, so it is unclear whether this correlation is genuine or stems from non-linearity.

[35] The correction specified by Murphy and Topel (1979) amounts to calculating $\hat{V}_2^{MT} = \hat{V}_2 + \hat{V}_2\left[\hat{C}\hat{V}_1\hat{C}' - \hat{R}\hat{V}_1\hat{C}' - \hat{R}\hat{V}_1\hat{R}'\right]\hat{V}_2$ where $\hat{V}_1$ and $\hat{V}_2$ are the respective first- and second-stage covariance estimates, and $\hat{\mathrm{C}} = \sum_{i=1}^{n}\left(\frac{\partial \ln(f_{i2})}{\partial \hat{\beta}_2}\right)\left(\frac{\partial \ln(f_{i2})}{\partial \hat{\beta}_1'}\right)$ and $\hat{\mathrm{R}} = \sum_{i=1}^{n}\left(\frac{\partial \ln(f_{i2})}{\partial \hat{\beta}_2}\right)\left(\frac{\partial \ln(f_{i1})}{\partial \hat{\beta}_1'}\right)$.

Table 4.5: Maximum likelihood estimates

	Preference parameter						
	Gains			Losses			Loss. Av.
	α	δ^+	γ^+	β	δ^-	γ^-	λ
Constant only	0.859***	0.772***	0.618***	0.826***	1.022***	0.592***	1.576***
	(0.018)	(0.051)	(0.038)	(0.018)	(0.083)	(0.061)	(0.098)
Low Amounts	−0.071**						0.009
	(0.032)						(0.147)
Female		−0.103*	−0.074				0.251*
		(0.065)	(0.062)				(0.157)
Age	0.003***	−0.004	−0.006***				0.003
	(0.001)	(0.004)	(0.002)				(0.005)
High Education							−0.318***
							(0.117)
ln(*Income*+1)							−0.059*
							(0.044)
Constant	0.776***	0.999***	0.954***	0.826***	1.022***	0.592***	1.766***
	(0.053)	(0.195)	(0.099)	(0.018)	(0.083)	(0.061)	(0.411)
Weighted Average	0.863	0.779	0.608	0.826	1.022	0.592	1.750
$\bar{\sigma}^2$	0.188***	0.267***		0.219***	0.302**		0.574***
$\bar{\rho}$	−0.354***	0.133***		−0.363***	0.062		
ℓ	−13870.9	−16080.7		−14431.0	−16896.4		−2195.1
N	814	366		690	272		438

Note: Murphy-Topel standard errors in parenthesis. Significance levels (one-sided tests): *: 10%;**: 5%; ***: 1%.

4.5.1 *Utility curvature*

The estimated power for gains ($\hat{\alpha} = 0.859$) and for losses ($\hat{\beta} = 0.826$) are displayed in the first row of Table 4.5. Both parameters are significantly below one (z = 8.04, p-value = 0.000 and z = 9.87, p-value = 0.000), and they are not significantly different from one another (z = 1.39, p-value = 0.166). Our estimates are closer to linearity as compared to the parametric studies of Harrison and Rutström (2007) and Donkers et al. (2001), who find $(\hat{\alpha}, \hat{\beta}) = (.71, .72)$ and $(.61, .61)$ respectively, which suggests that their parametric specifications may be inappropriate for separating utility from probability weighting. The estimates confirm *diminishing sensitivity*, both with respect to losses and to gains (Tversky and Kahneman 1992; Abdellaoui 2000; Abdellaoui et al. 2007b), and we cannot reject equal curvature in both domains in favor of the more recent hypothesis of *partial reflection* (Wakker et al. 2007).

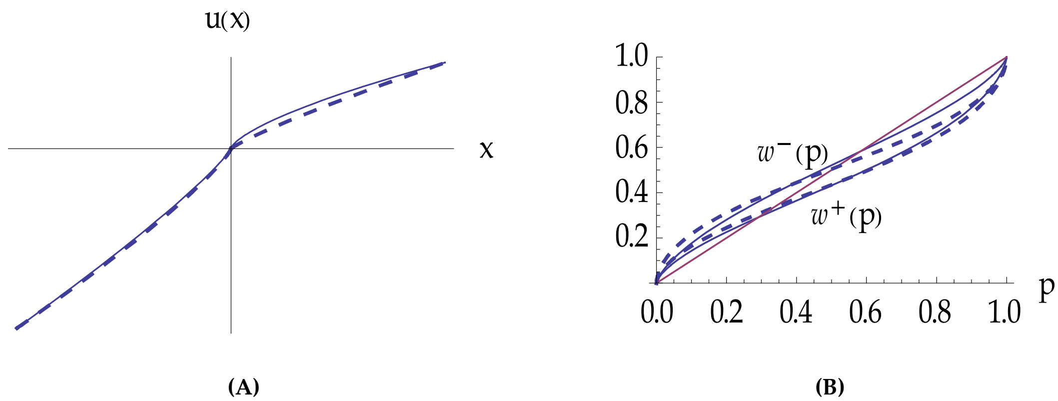
These results are qualitatively similar to those obtained in chapter 3. Those estimates, based on fitting a power function to individual level data, are somewhat closer to linearity ($\hat{\alpha} = 0.94$ and $\hat{\beta} = 0.92$ is found), but still significantly below one, and not significantly different from each other. This suggests that assuming homogeneity in utility curvature may lead to a small downward bias in the estimate of the average,[36] while also providing evidence that any potential bias in the non-parametric analysis due to error propagation is unlikely to be of high magnitude.

If we compare the coefficients to the average estimates of the literature reported in Table 4.1 ($\bar{\hat{\alpha}} = 0.69$ and $\bar{\hat{\beta}} = 0.86$ respectively), we find that the estimated power coefficient for gains is significantly higher (z = 9.61, p-value = 0.000) while that of losses is significantly different at the 10% level only (z = 1.94, p-value = 0.053). It should be noted that most recent estimates of utility curvature are much closer to linearity (Abdellaoui 2000; Etchart-Vincent 2004; Abdellaoui et al. 2005; Fehr-Duda et al. 2006; Abdellaoui et al. 2007b; Andersen et al. 2006; Abdellaoui et al. 2008) than what is suggested by the average estimate calculated from Table 4.1. Hence our estimates fall within the range of contemporaneous estimates that find the power of value function to be between .8 and 1. Figure 4.3 plots the estimated utility function (dashed line) and the average found in the literature (solid line). Indeed the estimated utility curve for losses is very close to the literature average, while that of gains is a little more linear.

Table 4.5 also shows a significant treatment effect for gains. The low amounts treatment for gains (*Low Amounts*) is associated with a power coefficient that is .071 lower than for outcomes that are scaled up by a factor ten, suggesting that utility is more pronounced for low outcomes. This is not often found in the literature, though Cohn et al. (1975) and Blake (1996) report similar results. The effect is driven by the fact that, for gains, the last two mean elements of the standard sequence for low amounts are a bit higher than those in the high-amount treatment divided by 10 (see Table 4.3). However, it should be noted that no significant difference was found in the non-parametric estimates. Because both approaches diverge, we will not draw strong conclusions with respect to this result.

[36] Effectively the non parametric-estimates of chapter 3 allow for full heterogeneity in preferences, while the pooled estimation conducted in this paper, does not. It is *a priori* not evident which method of analysis would yield the highest estimates, but it is clear that, because the model is non-linear, taking the average of estimates will yield a different result from estimating the average directly.

Figure 4.3: Estimated utility and probability weighting functions



Note: The parameters of the solid lines are based on the averages of the estimates in Table 4.1. Dashed lines depict the current estimates. The loss aversion parameter is assumed to equal to the average estimate of $\lambda = 2.09$ form Table 4.1.

4.5.2 Loss Aversion

Table 4.5 shows a loss aversion coefficient of $\hat{\lambda} = 1.58$, which is lower than the parametric estimate of $\hat{\lambda} = 2.25$ obtained by Tversky and Kahneman (1992), and the non-parametric estimate of $\hat{\lambda} = 2.54$ that was found by Abdellaoui et al. (2007b), based on Köbberling and Wakker's (2005) definition (they find values below 2 for the other, global, definitions). Also, the obtained loss aversion parameter is lower than the average (non-parametric) estimate of $\hat{\lambda} = 1.87$ obtained in chapter 3, where estimation is conducted at the individual level. A similar effect is reported by Abdellaoui et al (2008, p. 259) who find a pooled estimate of loss aversion that is lower than the average of the individual estimates. The obtained loss aversion is significantly larger than one (z = 5.88, p-value = 0.000), and it is consistent with the recent estimates of Schmidt and Traub (2002), Johnson et al. (2006), Harrison and Rutström (2007) and Abdellaoui et al. (2008, pooled estimate) who find values of 1.43, 1.85, 1.38 and 1.60 respectively. These and our results provide evidence that people weight a particular loss less than twice as heavy as a commensurable gain when making decisions. This is an interesting finding because Tversky and Kahneman's (1992) original estimate of 2.25 seems to serve as the focal point estimate of loss aversion for many researchers, while many recent estimates find values below two.

Some studies have reported a decrease in the degree of loss aversion with the size of outcomes (Bleichrodt and Pinto 2002 (health); Abdellaoui 2007b). Our point estimate of .004 for the *Low Amount* treatment (Table 4.5) does not provide additional support for this result.

4.5.3 Probability weighting

For both domains we estimated the elevation parameter δ, and the curvature parameter γ of the GE-87 probability weighting function specified in (4.4.15). The estimated elevation parameters point at pessimism with respect to gambling in both domains. For gains we find $\hat{\delta}^+ = 0.772$, which is significantly lower than 1 (*z* = 4.46, *p*-value = 0.000). This implies that a probability of a half is weighted by $\hat{w}^+(½) = 0.436$, which points to sizeable underweighting. The estimated weight is close to Tversky and Kahneman's (1992) original estimate of $\hat{w}^+(½) = 0.421$ and it is not significantly different from the average estimate in the literature (*z* = .23, *p*-value = 0.818). For losses the point estimate is $\hat{\delta}^- = 1.022$ which is higher than one, suggesting pessimism also in the loss domain ($\hat{w}^-(½) = 0.505 > .5$), but we cannot reject the hypothesis that $\delta = 1$ (*z* = 0.27, *p*-value = 0.787). The elevation of the weighting function for losses is significantly higher than that of gains (*z* = 4.54, *p*-value = 0.000) as was also found by Abdellaoui (2000), Abdellaoui et al. (2005) and Fehr-Duda et al. (2006), and we cannot reject the hypothesis that the elevation parameter is different from the literature average (Table 4.1) of $\bar{\hat{\delta}}^- = 1.09$ (*z* = .81, *p*-value = 0.418). Contrary to Etchart-Vincent (2004), who find more elevation for losses with higher stakes, we did not find any effect of the magnitude of the stakes on the degree of pessimism of the respondents.

The shape of the probability weighting function is primarily determined by γ, with $\gamma < 1$ generating an inverse-S shape, and $\gamma > 1$ a convex shape. Most studies that report a parametric estimate of the GE-87 weighting function find evidence of an inverse-S shaped weighting function but, as mentioned in the introduction, some studies have found a convex shaped weighting function. Interestingly, the point estimates for the degree of curvature in both domains are very similar, $\hat{\gamma}^+ = 0.618$ and $\hat{\gamma}^- = 0.592$, and we cannot reject the hypothesis that both are equal (*z* = .12, *p*-value = 0.907). Linearity, which requires $\gamma = 1$, is clearly rejected in favor of the hypotheses that both parameters are below one (*z* = 10.02, *p*-value = 0.000 and *z* = 6.65, *p*-value = 0.000), which means that we have found significant evidence for an inversely-S shaped weighting function in both domains. The degree of curvature we find is slightly higher than the average estimate in the literature. For gains the estimate is about .07 lower than the literature average ($\bar{\hat{\gamma}}^+ = 0.69$), which is significant at the

10% level (z = 1.90, p-value = 0.058). The estimate for losses is about .13 lower than the literature average ($\bar{\gamma}^{-} = 0.72$), which is significant at the 5% level (z = 2.09, p-value = 0.037). These results are illustrated graphically by the plot in Figure 4.3, where the estimated weighting functions are slightly more pronounced than the literature averages for probabilities near 0 and 1, while they are hardly distinguishable from the literature averages for intermediate probabilities.

4.5.4 Demographics

The dataset also contains background characteristics of the respondents such as their age, gender, education and income. Table 4.5 gives the results of including regressors into the model, where most of the insignificant variables have been removed. The significance levels are reported for one-sided tests. Most of the variation in the behavioral parameters appears idiosyncratic, in particular for the domain of losses, where we do not find a significant effect for any variables. In the gain domain, we find a mild associations of age (+0.003) with utility curvature, and a substantial gender effect on the elevation (−0.103) of the probability weighting function. This last result is interesting because traditionally gender differences in risk taking behavior have been ascribed to differences in utility curvature (e.g. Barsky et al. 1997). The analysis of chapter 3 already showed that loss aversion may explain much of the gender differences in risk attitudes, which is also found here (+0.251) and in other studies (e.g. Schmidt and Traub 2002). The current analysis further refines this by showing that part of this effect is also caused by differences in probability weighting. This is consistent with a recent study of Fehr-Duda et al. (2006), who report a significant gender difference in the elevation parameter of the GE-87 probability weighting function for gains but not for losses. These authors also find curvature to differ between the sexes, which we do not.

Older people seem to value money more linearly, with a 50 year age difference being associated with a power that is .15 higher. This effect works to reduce risk aversion, but it is countered by more non-linear weighting of probabilities (−.30) that, in general, work to increase risk aversion. The total effect of these estimates depends on the prospects under study. For prospects that entail a small probability of a large gain one may find risk aversion to decrease with age, while in those that do not, increasing risk aversion is more likely, which is what is usually found (Pålsson 1996; Donkers and van Soest 1999; Halek and Eisenhauer 2001; Hartog et al. 2002).

Education, defined as having a higher vocational or academic education, does not affect utility curvature, nor is it associated with a more linear weighting of probabilities. This latter effect is surprising if we view expected utility as the rational model of choice under risk. From that perspective one may expect higher educated individuals to weight probabilities more linearly, which is not what we find. Education is associated with a lower degree of loss aversion (−.318), which suggest that the reduction in risk aversion with years of schooling that is often observed (Donkers et al. 2001; Hartog et al. 2002; Dohmen et al. 2006) stems mainly from lower sensitivity to losses (e.g. Gächter et al. 2007).

Finally, the included (log) income variable showed a mild negative association with loss aversion, which is consistent with Gächter et al. 2007. Hence, we conjecture that mainly the loss aversion component of risk attitudes is driving the decrease in (absolute) risk aversion with income that is often found (Donkers et al. 2001; Hartog et al. 2002). The non-parametric analysis of chapter 3 does not reveal such an association, however, so this statement remains speculative.

The (weighted) average predictions $\boldsymbol{B}'\overline{\boldsymbol{X}}$ are very close to the estimated parameters in the model with only a constant, except for loss aversion. The average prediction of $\overline{\lambda}$ =1.75 there is higher than the estimated 1.58 in the model with only a constant, again suggesting that assuming homogeneity lowers the estimated loss aversion.

4.5.5 Stochastics

Table 4.3 shows considerable variability in the answers to the questions, which is picked op by the estimated error variances. The estimated covariance-correlation matrices for the outcome equations are given in Table 4.6, where the diagonal elements correspond to the estimated variances, and the off-diagonal elements correspond to the estimated correlations between the error terms. The average variance $\overline{\sigma}^2$ is 0.188 for gains and 0.219 for losses. This means that the probability that a given utility difference is twice as high (low) as the next one is about 5% (5%). This may not seem very much, but it implies that in a standard sequence of six elements, there is about a 40% probability that there will be at least two subsequent utility increments that differ by a factor two. Although part of this variability is driven by between subject heterogeneity, this result suggests that the assumption of a standard sequence without error is questionable.

Both estimated matrices have a tridiagonal structure, with the one off-diagonal correlation coefficients on average equal to $\bar{\rho} \approx -0.35$ (Table 4.3) and the other correlations equal to zero. The negative correlations are a little weaker than the predicted correlation of $-\frac{1}{2}$ that follows under the assumption of equal variance, which means that not all underlying variances σ_i are equal. There does not appear to be much difference in the average variability of the answers for losses and for gains. The variances of the probability weighting questions are a little higher, 0.267 and 0.302 for gains and losses respectively, which confirms that these questions are more demanding for respondents. For gains there appears to be some positive correlation between the individual answers ($\bar{\rho} = 0.133$), but not for losses.

Table 4.6: Estimated Variance-Correlation Matrices of outcome sequences

	Gains						**Losses**				
	ε_1^{o+}	ε_2^{o+}	ε_3^{o+}	ε_4^{o+}	ε_5^{o+}		ε_1^{o-}	ε_2^{o-}	ε_3^{o-}	ε_4^{o-}	ε_5^{o-}
ε_1^{o+}	0.194***					ε_1^{o-}	0.328***				
	(0.017)						(0.035)				
ε_2^{o+}	−0.258***	0.207***				ε_2^{o-}	−0.281***	0.220***			
	(0.045)	(0.022)					(0.045)	(0.023)			
ε_3^{o+}	−0.006	−0.375***	0.203***			ε_3^{o-}	−0.007	−0.362***	0.177***		
	(0.016)	(0.064)	(0.027)				(0.014)	(0.045)	(0.018)		
ε_4^{o+}	−0.006	−0.006	−0.408***	0.188***		ε_4^{o-}	−0.007	−0.007	−0.380***	0.193***	
	(0.016)	(0.016)	(0.075)	(0.026)			(0.014)	(0.014)	(0.045)	(0.021)	
ε_5^{o+}	−0.006	−0.006	−0.006	−0.373***	0.150***	ε_5^{o-}	−0.007	−0.007	−0.007	−0.430***	0.176***
	(0.016)	(0.016)	(0.016)	(0.057)	(0.015)		(0.014)	(0.014)	(0.014)	(0.052)	(0.019)

Note: The off-diagonal elements are correlation coefficients. Standard errors in parenthesis and significance levels */**/***: 10/5/1%. Test on equality of diagonal elements is rejected for both gains (z = 3.15, p-value = 0.042) and losses (z = 4.67, p-value = 0.000). Equality of the one-off diagonal elements is not rejected for gains (z = 1.98, p-value = 0.272), but is rejected for losses (z = 3.09, p-value = 0.023).

4.6 Summary and Conclusion

This study presents the first, representative, large-scale parametric estimation of prospect theory's functionals, the utility function of money gains and losses, and the subjective probability weighting functions. Unlike previous large scale parametric studies, the richness of the questionnaire allows for estimation of these curves without making too restrictive parametric assumptions, while allowing for response error in the individual answers. The results qualitatively confirm the non-parametric results of chapter 3 and suggest that utility is mildly concave for gains and mildly convex for losses, implying diminishing sensitivity and suggesting that classical utility measurements that neglect probability weighting, are overly concave.

A direct comparison with the non-parametric measures suggests that assuming homogeneity leads to a small downward bias, while providing evidence that a potential bias in a non-parametric analysis due to error propagation is unlikely to be large. Also our estimates are closer to linearity as compared to parametric studies that impose more stringent parametric assumptions (e.g. Donkers et al. 2001; Harrison and Rutström 2007), suggesting the utilities obtained in these studies may suffer from a contamination bias. Further, we find evidence that probabilities are weighted non-linearly, with an inverse-S shape, and that both functions display pessimism (low elevation for gains, high elevation for losses). Hence, these results externally validate probability weighting that was found in a laboratory context (Wu and Gonzalez 1996; Abdellaoui 2000). The obtained degree of loss aversion, as operationalized by Tversky and Kahneman (1992), is 1.6. This is somewhat lower than their estimate of 2.25, but consistent with contemporaneous evidence (Schmidt and Traub 2002; Johnson et al. 2006, Abdellaoui 2008). Furthermore, we found that neither the degree of utility curvature, nor the degree of loss aversion, is altered by scaling up monetary outcomes. The same holds for the probability weighting functions, that do not appear to be affected by the magnitude of the stakes, contrary to what Etchart-Vincent (2004) finds for the loss domain.

By including background characteristics our estimation procedure gives more background as to what causes risk aversion differences between groups in the population. This analysis suggests that the common finding that women are more risk averse than males (Byrnes and Miller 1999) stems from differences in probability weighting and loss aversion, and not from differences in utility curvature. Also, the reduction of risk aversion that is associated with a higher level of education (Donkers et al. 2001; Dohmen et al. 2006) does not derive from utility curvature but from differences in loss aversion. The robustness of these results should be confirmed by further research, but they are indicative of the different channels through which risk taking behavior is associated with background variables.

Two disadvantages of the study are the lack of real incentives and the use of matching tasks in stead of choice tasks. Hypothetical tasks have been found, in some settings, to prime more erratic, and sometimes different behavior, than similar tasks involving real stakes (Camerer and Hogarth 1999; Holt and Laury 2002). Moreover, matching tasks have been found to increase the number of inconsistent answers, suggesting that these tasks are more cognitively demanding (Luce 2000, Hertwig and Ortmann 2001). This is confirmed by our data where for gains 37% of all individuals gave one or more inconsistent answer. These individuals were excluded from the analysis, leading to sample selection. To correct for this the analysis were conducted by using the inverse of the probability of appearing in the sample as weight. Given that our results blend in well with the results from laboratory experiments, providing evidence for diminishing sensitivity both with respect to outcomes and to probabilities, and also producing plausible relationships with demographic variables, we are confident that the obtained measures give a good representation of the average curvature of prospect theory’s functionals.

4.7 Appendix to chapter 4

4.7.1 *Sample selection probit-equation*

Table 4.7: Sample Selection Equations

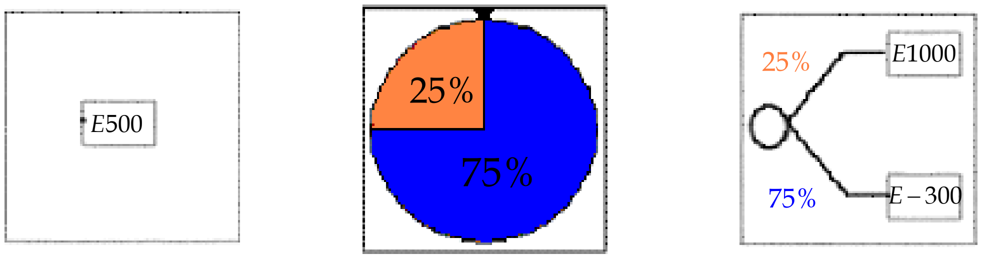
Variable	Frac.	Outcomes		Probabilities		Loss Aversion
		Gains	Losses	Gains	Losses	
Low Amounts Treatment	50%	0.144**	0.101*	0.0123	−0.092	0.088
		(0.058)	(0.059)	(0.18)	(1.23)	(0.065)
Female	46%	−0.052	−0.079	−0.188**	−0.219**	−0.116*
		(0.060)	(0.061)	(2.65)	(2.82)	(0.067)
Lower Secondary Education	26%	0.127	0.029	−0.264	−0.370*	0.218
		(0.142)	(0.143)	(1.53)	(1.99)	(0.177)
Higher Secondary Education	14%	0.335**	0.148	0.169	0.106	0.507***
		(0.151)	(0.152)	(0.96)	(0.56)	(0.182)
Intermediate Vocational Training	19%	0.046	−0.148	−0.127	−0.291	0.147
		(0.146)	(0.148)	(0.73)	(1.55)	(0.180)
Higher Vocational Training	25%	0.258*	0.039	0.205	0.0662	0.394**
		(0.143)	(0.144)	(1.23)	(0.37)	(0.175)
Academic Education	11%	0.488***	0.305*	0.568**	0.447*	0.681***
		(0.158)	(0.158)	(3.16)	(2.36)	(0.187)
Age 35-44	18%	−0.157*	−0.202**	−0.0621	0.0567	−0.284***
		(0.092)	(0.093)	(0.61)	(0.53)	(0.099)
Age 45-54	22%	−0.234***	−0.281***	−0.258*	−0.195	−0.325***
		(0.088)	(0.089)	(2.56)	(1.79)	(0.095)
Age 55-64	18%	−0.313***	−0.340***	−0.273*	−0.418***	−0.524***
		(0.094)	(0.096)	(2.52)	(3.36)	(0.106)
Age 65+	19%	−0.462***	−0.428***	−0.638***	−0.592***	−0.632***
		(0.095)	(0.096)	(5.44)	(4.63)	(0.108)
€ 1.150≤Income<€ 1.800	25%	0.196*	0.269**	0.298	0.480*	0.319**
		(0.118)	(0.122)	(1.94)	(2.52)	(0.146)
€ 1.800≤Income<€ 2.600	31%	0.200*	0.253**	0.325*	0.509**	0.381***
		(0.115)	(0.119)	(2.16)	(2.71)	(0.143)
Income≥€ 2.600	35%	0.344***	0.411***	0.418**	0.699***	0.526***
		(0.115)	(0.119)	(2.80)	(3.79)	(0.142)
Catholic	30%	0.014	0.008	0.0128	0.000	−0.007
		(0.068)	(0.069)	(0.16)	(0.01)	(0.077)
Protestant	20%	0.160**	0.126	0.136	0.083	0.215**
		(0.077)	(0.078)	(1.52)	(0.85)	(0.084)
Constant		−0.503***	−0.510***	−1.024***	−1.280***	−1.199***
		(0.176)	(0.179)	(4.75)	(5.21)	(0.217)
N		1935	1935	1935	1935	1935

Notes: Standard errors allow for clustering within households. */**/***: significant at the 10/5/1% level.

4.7.2 *Experimental instructions (Part II)*

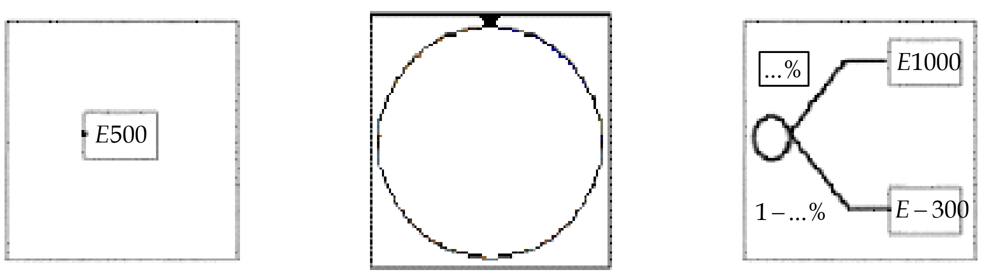
The first part of this experiment has now finished. In the second part of this experiment each question will again be presented on a separate page, with two lotteries Lottery L (Left) and Lottery R (Right) presented at the top. In between the two lotteries you will again be presented with a wheel to illustrate the probabilities. In this part of the experiment, however, Lottery L will always yield a fixed amount with certainty. Below the illustrated lotteries, there will again be text explaining the question. The next screen will show you an example of a question that you could get in the second part of this experiment.

Lottery L Wheel Lottery R



As you can see, in this example Lottery L always yields 500 euros. Lottery R on the other hand, gives with probability 25% a profit of 1000 Euro, and with a probability of 75% a loss of 300 Euro. You should again imagine that, if we were to turn the wheel and the black pointer would be in the orange area, Lottery R would yield 1000 euros. In case the black pointer would be in the blue area, Lottery R would yield a loss of 300 euros.

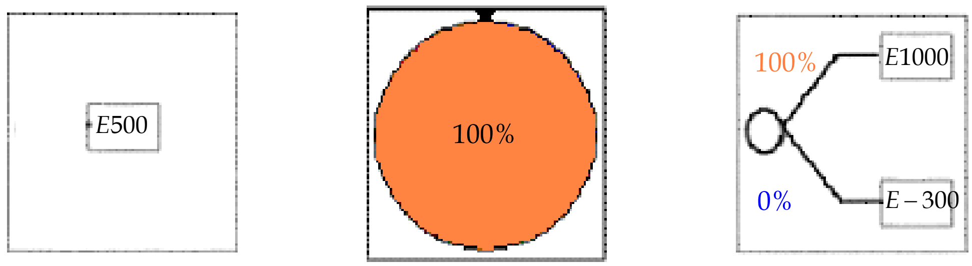
Lottery L Wheel Lottery R



In the previous example you may have preferred Lottery L to Lottery R or the other way around. In the second part of this experiment, however, the probabilities of the prizes in lottery L will be missing, such as in the example given above.

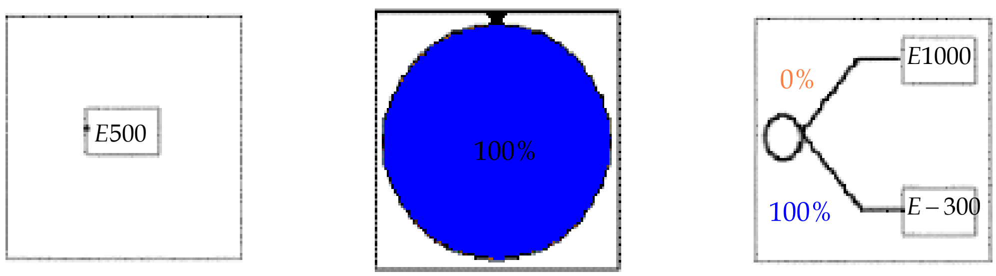
In the second part of this experiment we will ask you in each question to state the value of the missing probability (in whole percentages, from 0% to 100%) for the upper prize of Lottery R that would make you value both equally.

Lottery L Wheel Lottery R



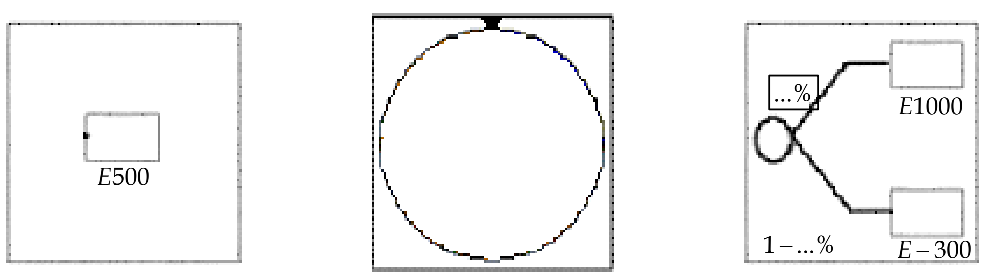
Imagine that the probability of the upper prize of lottery R is equal to 100%. This would give the lotteries presented above. Lottery L will thus always give a profit of 500 Euro, while Lottery R will always give a 1000 euros. Given that Lottery L will always yield less than Lottery R, most people will prefer Lottery R to Lottery R.

Lottery L Wheel Lottery R



Imagine now, however, that the probability of the upper prize of lottery R is equal to 0%. This would give the lotteries presented above. Lottery L will thus always give a profit of 500 Euro, while Lottery R will always give a loss of 300 euros. Given that Lottery L will always yield more that Lottery R, most people will now prefer Lottery L to Lottery R.

Lottery L Wheel Lottery R



Hence, there is a value of the missing probability somewhere between 0% and 100% for which you would value both lotteries equally. In the questions that follow we will ask you for which value of the missing probability you value Lottery L and Lottery R equally, This missing probability can be different for everybody and is your own preference. How this

works precisely will become clear in the practice question that will start if you click on the CONTINUE button below. If something it not clear to you, you can read the explanation of this experiment again by pressing the BACK button below.
[Practice question]

The practice question is now over. The questions you will encounter during this experiment are very familiar to the practice question. If you click on the BEGIN button below, the experiment will start. If you want to go through the explanation of the second part of this experiment again, click on the EXPLANATION button. Good luck.