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Accurate measurement of shape at low resolution

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ACCURATE MEASUREMENT OF SHAPE AT LOW RESOLUTION

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Boundaries, edges and contours in image processing with the aim to measure parameters such as length and curvature, have almost exclusively been treated with common 1D-signal processing techniques. The spacing of contour points along the path s is not equidistant however, but is depending on the tangent $\theta(s)$. In effect the apparent spacing is $h'=h/\cos\theta$ (h grid constant). So, any contour operation (implicitly) using $h'=h$ will introduce directionally dependent inaccuracies in the analysis. For example, the frequency response of a linear filter may shift up to 41% to lower frequencies, depending on $\theta(s)$. As a remedy one may fit a curve, resample the digital curve, or match a curve as has been reported, but none of these methods solved the anisotropy of the grid fundamentally.

We propose a solution where the set of original (i.e. predigitized) curves for each configuration of M contour points is reconstructed. The set is different for each different configuration. As we now have the set of originals, we have achieved two goals in one blow: The anisotropy of the grid has become irrelevant, and any parameter value of the curve can be estimated directly from the set of originals. For the noise-free situation the method produces results with the highest possible accuracy and the least possible sensitivity to the orientation of the object relative to the measurement grid.

For any (local) parameter or specific shape clue, e.g. curvature, length, or rectangular corner detection, the method can be implemented efficiently by a table search operation of 2.3^{M-2} entries, requiring no contour tracing.

It is concluded that the method is pre-eminently apt for low resolution, noise free measurement of shape as the method has the smallest possible measurement error.

1. INTRODUCTION

1.1. Analysis of shape

Analysis of shape has importance in almost every application field of image processing. To name a few, in the field of industrial inspection, the assessment of product quality requires the confirmation of a shape with the standard [6]. In the field of robot control, vision is frequently based on the detection of landmarks in the shape of objects and circles with a distinct radius [8]. In the field of microscopy, shape analysis has been used to detect clusters of overlapping cells by searching for kinks in the contour [7], [12] and to describe the general state of the cell [15]. Finally, in medical nuclear imaging (MNI), functioning of the heart is derived from an analysis of the shape of the heart wall, e.g. [1].

In almost all of the selected examples, the resolution in the image is expected to be

low, either intrinsically as in the MNI-case or by choice to keep the computational speed in accordance with the number of images flowing in. In the sequel, we will study the accuracy and precision of measurement of arbitrary local shape features, i.e. those which have a value for each instance s along the contour of the object. An important source of measurement error will appear to be the non equal distances when sampling along the contour, as they are tied to the 2D-grid. The aim of the analysis is to design a method for shape measurement which produces results with the highest possible accuracy and the least sensitivity to the orientation of the object relative to the measurement grid, in analogy with the method described in [2] [3]. Aiming at a general method to combat the influence of the discretisation, noise is not considered here as it is problem specific. Noise reduction procedures on the 2D image or on the contour are to be applied prior to the present method. The procedure as developed will be particularly fruitful for the analysis of images at low resolution, still yielding a high measurement accuracy.

1.2. Contours are non-equidistantly sampled

A most commonly used procedure in shape analysis is to trace the boundary of the object, Object Boundary Quantisation (OBQ). The result is a Freeman *code string*. As an alternative, one could also trace the background rather than the object, but that distinction is not relevant here. The code string resulting from the OBQ is *freed from* jumps when passing through 2π and conceived as a one dimensional signal. Commonly, standard signal processing procedures are then applied to derive shape features from the encoded contour.

Implicitly, in those procedures, it is usually assumed that the code string is a signal with equidistant samples, which is in reality not the case. For, if the orientation at location s anywhere on the contour is given by $\theta(s)$, then the sample points of the contour are spaced a distance $h/\cos \theta(s)$ apart. So, when the local orientation of the contour is in diagonal directions, the spacing of sample points there is 41% larger than in those parts of the contour where θ is in the main directions of the grid.

Evidently, the fact that the spacing of contour sample points is not equidistant will have a strong effect on the filter response when the contour is smoothed or analyzed. In the case of a linear filter of the contour string with the filter response expressed in terms of the grid constant h , an apparent shift in the frequency response of the filter will occur to the amount of $h/\cos(\theta(s))$, where $\theta(s)$ is the orientation of the contour at s . The error goes up to 41% for the diagonal directions^{*1}. This direction dependency of the outcome does not only occur for linear filtering but in general for any other operation based on the sequence of contour pixels perceived as if they were sampled equidistantly.

1.3. Approaches to anisotropy in analyzing shape

In [15] the bending energy of the contour is computed by *linear filtering* the contour and then computing the differential. Then, according to Hooke's law, the local bending energy is proportional to the inverse of the curvature squared. The method may also be integrated to one linear filter [12] yielding an efficient one pass procedure through the contour data, but the main objection to this approach is the fact that the filter response is depending on the local orientation of the contour, as was discussed above.

In [6] a set S of N digital pixels representing a circular line is described by *locating* the center $\underline{M}_k = (x_M, y_M)$ of the arc in an *iterative* way. To that end, the overall distance discrepancy between the position of S and the edge of the disc with radius R_k around $\underline{M}_k$ is minimized by shifting in each iteration k the position of $\underline{M}_k$ and changing the value of radius R_k . From an arbitrarily chosen starting point, the procedure requires the iterative computation of:

$$R_k = - \frac{1}{N} \sum_{i=1}^N \| \underline{x}_i - \underline{M}_k \|$$

$$\underline{M}_{k+1} = \frac{1}{N} \sum_{i=1}^N \underline{r}_i - R_k \frac{\underline{r}_i - \underline{M}_k}{\|\underline{r}_i - \underline{M}_k\|}$$

until

$$\|\underline{M}_k - \underline{M}_{k+1}\| < \epsilon.$$

For the example in the reference with $\pm 25\%$ uniform noise in the radius, the number of iterations may go up to $K=150$ to arrive at 1% of $\underline{M}$ for a 400 pixel arc of 0.8π rad. This number decreases rapidly when the arc is larger: $K=16$ for 1.2π rad and $K=5$ for 1.6π rad. For an arc of 0.4π rad, which is just short of a right angle, no convergence seems to be obtained. With these figures it should be noted that the number of iterations will depend on the initial guess $\underline{M}_0$. The computational effort required in the total procedure is considerable as in each iteration a norm is computed twice. No figures on convergence and on the number of iterations are given for other than the rather large values of the noise the reference considers, including the noise-free situation when only discretisation effects remain.

In this paper we argue that apart from image noise, the discretisation leads to an unavoidable measurement error of a different characteristic than the noise in the image and hence this error should be treated differently.

In [1] in noisy scintigram images, the orientation and size of a *matching* ellipse circumscribing the heart is searched via a Hough transform. In principle, the image is transposed into the 5-dimensional parameter space, one dimension to each of the 5 parameters of an ellipse (x_M, y_M, θ, a, b). They argue, that generally the search for the maximum likelihood values in this space can, however, be restricted to neighborhoods around the maxima in x_M, y_M and θ separately.

It is a powerful method, especially for noisy images, in principle suited for complete ellipses only. In spite of the restricted search, the computational effort is considerable, however, and the method yields one globally fitted ellipse insensitive to small local variations in the heart's shape.

In [9] corners are detected by *fitting* a B-spline to the data pixels. The authors achieve, in conjunction with their preceding paper [8], subpixel accuracy in detecting corners in the image. In their search for high resolution, they consider the discrete position of the pixels on the grid as a "quantisation noise", for which "we need to introduce some smoothing".

In this paper, we will argue, however, that rather than smoothing the digital data point positions, their position, known to be confined to the grid, should be exploited to achieve the maximum (subpixel) accuracy.

To get around the dependence on the orientation, in [11] *resampling* of the 8-connected digital contour was proposed by resampling the line connecting the pixels of the contour. The resampling can be implemented by inserting new pixels in between grid points: 4 new points for the main directions of the grid and 6 for the diagonal directions. The effect on the Freeman chain code would be to replace every even code by 5 copies and every odd code by 7 copies. The rationale is that $7/5$ approximates $\sqrt{2}$, hence reducing the non-isotropy of the grid. It was found that the non-equidistance in the spacing decreases from -9.97% to $4.95\%^2$. The virtue of the method in the reference is that it recognizes the non-isotropy of the contour points, but the method has two disadvantages as well. One: the problem is not solved fundamentally, as a new (non-uniform) quantisation is introduced. Two, and this is the major objection: there is no convincing argument to resample the line drawn between the contour pixels as this line is neither the original line nor the digitized line, the latter being a set of pixels rather than a line. Why not resample a line connecting every other contour pixel, or fit a curve through the contour pixels? The residual error of 4.95% (or at minimum 2.6%) ensures that deriving properties from a spline fitted through the contour pixels will generally give more accurate results.

In this paper, we propose to conceive (each part of) the contour as the set of pixels delimiting the set of original curves which may have generated exactly this digital pattern. Then, each contour pixel serves to delimit the set best, and, as each specific arrangement of contour pixels is associated with smallest possible set of originals, the anisotropy of the grid is resolved as well as is theoretically possible.

2. THE METHOD FOR HIGH ACCURACY

2.1. Characterization of circular arcs

The procedure we will pursue is as follows.

First, we decompose the *original* in circular arcs in such way that the tangent and the local radius remain unaltered, see figure 1, by requiring smoothness in first and second

Figure 1: Decomposition of the original in local tangent circular arcs, with c_i and r_i denoting the center and radius of the i 'th circular arc.

derivatives. To this demand there is a unique solution for continuously differentiable curves. We have preferred to use circular arcs as they are the only directionally invariant solution to the decomposition. An arc is fitted to each location s in the original curve. After the discretisation we have lost precise knowledge which exact curve was the original generating the digitized curve. We can only know the set of originals anyone of which may have generated that part of the digitized contour, and we can only do so for the N locations of the contour where pixels are.

Hence, we search for the set of all originals which may have generated this particular digitized contour. Consider for each of the N pixels of the digitized contour, the M neighborhood of pixels of the grid. Some of the M pixels are on the inside of the original (the in-pixels), some of them fall outside (the ex-pixels). The only relevant pixels on the in-side are the set (S^I) of contour pixels in the M neighborhood, and their opposing ex-pixels, the set (S^E) , see figure 2.

Figure 2: A possible configuration j of an M -neighborhood, with two of the members of the set of original discs. The configuration will be used in the figures 3, 4, and 5.

Now, consider a circular disc $\mathfrak{d}(x_d, y_d, r_d)$ without edge, given by,

$$\mathfrak{d} = \{ (x, y) \mid (x - x_d)^2 + (y - y_d)^2 < r_d^2 \},$$

where $\setminus$ indicates the open set and its complement by $\underline{\mathfrak{d}}(x_d, y_d, r_d)$,

$$\underline{\mathfrak{d}} = \{ (x, y) \mid (x - x_d)^2 + (y - y_d)^2 \geq r_d^2 \}.$$

where underlining indicates the complement, and bold indicates the closed set. Also define the edge, which is the circle d as:

$$d = \{ (x, y) \mid (x - x_d)^2 + (y - y_d)^2 = r_d^2 \}.$$

The closed disc $\mathfrak{d}$, is equal to:

$$\mathfrak{d} = \mathfrak{d} \cup d, \text{ hence also,} \\ \mathfrak{d} = \mathfrak{d} \cap \neg d.$$

The complement of $\mathfrak{d}$ is $\underline{\mathfrak{d}}$ with:

$$\mathfrak{d} \cup \underline{\mathfrak{d}} = \mathbb{R}_2 \quad \text{and} \quad \mathfrak{d} \cap \underline{\mathfrak{d}} = \emptyset, \text{ and,} \\ \mathfrak{d} \cup \underline{\mathfrak{d}} = \mathbb{R}_2 \quad \text{and} \quad \mathfrak{d} \cap \underline{\mathfrak{d}} = \emptyset.$$

We search for the set (D^j) of all original circles $\mathfrak{d}_i^j$ corresponding to a specific configuration $(S^I, S^E)^j$ of digitized pixels in the M neighborhood. The index j numbers all different configurations of the contour pixels in an arbitrary but unique way. The total number of different configurations which can be found in the M neighborhood depends on the restrictions one poses on the contour. For the time being, we will study one arbitrary configuration j' and further omit index j .

Define the notion $\subset$ pronounce IN by:

$$\text{A set } S \text{ of pixels } P \text{ is said to be } \subset \text{ a disc } d \text{ iff:} \\ S \subset d \iff \forall P \in S \Rightarrow P \in d.$$

In concordance with the definition of the OBQ-discretisation algorithm, the arc-element d_i is a member of the requested set D , iff:

$$d_i \in D \iff (S^I \subset d_i) \wedge \neg (S^E \subset d_i).$$

Thus,

$$D = \{ d_i \mid (S^I \subset d_i) \wedge (S^E \subset d_i) \}.$$

Call the set D^I , the set of all arc-elements including S^I , and D^E the set of all arc-elements excluding S^E . Then,

$$D^I = \{ d_i \mid (S^I \subset d_i) \},$$

$$D^E = \{ d_i \mid (S^E \subset d_i) \}, \text{ and,}$$

$$D = D^I \cap D^E.$$

The set D thus indicates the set of all original circular arcs through the M -neighborhood, see figure 3.

Figure 3: Extreme members of the set $D(r)$ of all original circular arcs which could have generated the digitized configuration j (left), and the set $M(r)$ of middlepoints of all originals. Note that $D(r)$ and $M(r)$ here are indicated for only one value of r .

Similarly, denote the sets of all centers of arcs given the radius r of the arc by $M^I(r)$, $M^E(r)$ and $M(r)$ respectively.

For this paper, consider a digital pattern j which is convex and not straight in the sense of the linearity conditions [14], so the sets D^I , D^E and D are finite.

2.2. Convex-r

At this point, we use the concept of the convex- r hull. The convex- r hull $H(r)$ for radius r , defined by Verbeek and Groen in [13], is a generalization of the convex hull around the object T such that the sides of the hull are curved with radius r . The definition is given for continuous space and connected objects. Extending the definition of the convex- r for pixel sets, and adding a sign to it for later use, we use the definition:

A set T around a pixel set S is convex- r^+ if
 $S \subset T$ and
 T is connected and
 $\forall P, Q \in T \Rightarrow T \subset (\cap \{ \forall d \mid P, Q \subset d(x,y,r) \})$.

As concerns the hull $H^+S(r)$ we extend the definition for pixel sets and for later use into convex- r^+ hull:

The set $H^+S(r)$ is the convex- r^+ hull of pixel set S iff
 H^+S is convex- r^+ around S and
 $(\forall C \text{ convex-}r^+ \text{ around } S \mid S \subset C) \Rightarrow H^+S \subset C$.

Introducing in the proofs of the reference, the pixel set S in the continuous set T , it can be shown that H^+S equals:

$$H^+S(r) = P^+P^+S(r),$$

The pseudo-object transformation P^+C is given by

$$P^+C(r) = \{ \cup \forall P \mid (\forall Q \in C) \subset d(x_P, y_P, r) \},$$

(where C is either a pixel set or a connected set). The transformation renders the object covering the space when swinging a wheel with radius r around S . P^+S has the property that it is convex- r^+ around S , see the reference. Expressed in terms of mathematical morphology, P^+ and H^+ can be written as,

$$P^+C = C \oplus \underline{d} = d \ominus C',$$

(where $'$ denotes mirroring through an arbitrary origin) [5], and:

$$H^+C = d \ominus (d \ominus C')' = d \ominus (d \ominus C).$$

2.3. Set of originals

From the definitions, it is clear that:

$$M^I = P^+S^I, \text{ and,} \\ D^I = H^+S^I,$$

and hence D^I is convex- r^+ . And, for the determination of D^I (and hence D^I) the only pixels of S^I that matter are on H^+S^I (the border of H^+S^I), as $P^+H^+S^I = P^+S^I$. Which pixels are on H^+S^I depends on r , which cannot go under (but not necessarily reach) a minimum:

$$r_{mn} = \text{MAX } e/2, \quad e = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2}, \quad \forall P, Q \in S^I$$

For D^E , we need to define $H^-S(r)$ by:

$$H^-S(r) = P^-P^-S(r),$$

where

$$P^-C(r) = \{ \cup \forall P \mid (\exists Q \in C) \subset d(x_P, y_P, r) \}.$$

In terms of mathematical morphology:

$$P^-C = C \oplus d, \text{ and,}$$

$$H^-C = \underline{(C \oplus d)} + d = (C \oplus \underline{d}) \ominus \underline{d}.$$

which is the dilation and the closing operation, respectively. Similarly with D^I , now:

$$M^E = P^-S^E, \text{ and,}$$

$$D^E = H^-S^E$$

We will call $\mathcal{D}^E$ convex- r^- , as it has the arc to the inside of the figure. For the determination of $\mathcal{D}^E$ (and hence $\mathcal{D}^E$) the only pixels of S^E that matter are on H^-S^E . Finally:

$$\mathcal{D} = H^+S^I \cap H^-S^E \text{ and,}$$

$$\mathcal{M} = P^+S^I \cap P^-S^E.$$

Note that both $\mathcal{D}$ and $\mathcal{M}$ are functions of r , and that they are different for each different discretisation pattern j as well.

Figure 4: The region of R_2 , where middlepoints of original arcs lie for pattern j . The region is a convex polygon, the boundaries of which are formed by midperpendiculars between pairs of pixels; one from the set of inside pixels, one from the set of outside pixels.

2.4. Properties of $\mathcal{D}^j$

Thus $\mathcal{D}^j(x_M, y_M, r)$ is the set of each pattern j . A few properties of $\mathcal{D}$ are (given here without proof):

- 1) For the assumed convex, non-straight string j , $\mathcal{M}$ lies in R_2 on the convex side of the curve, with $\mathcal{M}^I$ on the outside and $\mathcal{M}^E$ on the inside.
- 2) The union of all $\mathcal{M}(r)$ varied over r is a (not curved) polygon in R_2 , the boundaries of which are formed by midperpendicular lines of pairs of pixels; one from S^I , the other one from S^E , see figure 4.
- 3) The integral $A(r)$ of $\mathcal{M}(r)$ over x and y is a continuous function of r , limited to $r_{mn} < r < r_{mx}$, (where the last limit comes from the restriction to convex, non-straight patterns j). The function expresses the 'number' of middlepoints of the set of originals for each value of r . As we have assumed uniform density in x_M and y_M , it will give the probability of an arc with radius r to have generated the digitized pattern when normalised, see figure 5.

Figure 5: The probability $p_j(r)$ of an original as a function of r for pattern j .

Numerically $p(r)$ is easily expressible in the coordinates of the corner points of M . The function is generally also differentiable, apart from the instances where a $P \in S^I$ with increasing r enters H^+S^I or when a $Q \in S^E$ leaves H^-S^E . That implies, when searching for the set of all originals with fixed r associated with pattern j , it is sufficient to determine the order in which the pixels of j are entering H^+S^I and H^-S^E with increasing r .

4) With increasing r , a pixel P of pixel set S will enter the side between two already existing elements A and B of H^+ , iff P is in the convex- r^+ area of A and B . The convex- r^+ area is the area bounded by the arc connecting A and B , and the straight line between A and B . With increasing r , P will enter when $r=r'$, where r' is the radius of the circle circumscribing the triangle APB .

5) If j is a straight part of the string, in the sense that it obeys the linearity conditions [14], $\mathcal{D}$ includes also $R=\infty$, and H^-S^E is partially "wrapped" through infinity to the opposing side of j . If j is a concave, not straight part of the string then $\mathcal{D}$ is on the opposing, convex, side of j , now with M^I on the inside and M^E on the outside.

So, with these properties, it is possible to compute for every pattern j in neighborhood M the set of all corresponding originals $\mathcal{D}^j$, each one of which could have generated j at the moment of discretisation.

2.5. Accurate anisotropic local shape features

Any feature expressible in circular arc parameters (x_M, y_M, r) such as the location of the center of the circle, the local curvature of the curve, the local bending energy, the local tangent to the curve, or the position (i.e. path length s) can now be computed from $\mathcal{D}^j$. Call this feature in the continuous domain $f(x_M, y_M, r)$.

The feature value g^j for pattern j is found by evaluating, for every j , the expression:

$$g^j = \int \int \int_{D^j(x_M, y_M, r)} f(x_M, y_M, r) p(x_M, y_M, r) dx_M dy_M dr$$

In [2] it is proved for sampling a digitized curve on a grid that this estimation scheme yields a Best Linear Unbiased Estimate (BLUE), and that among all BLUEstimators, it has minimal overall MSE, as D^j is unique to each different j . Therefore this value is called *optimal BLUE*.

For the a_priori distributions of curves, $p(x_M, y_M, r)$, we assume here independence of $p(x_M, y_M)$ and $p(r)$. Moreover, we assume a uniform distribution for $p(x_M, y_M)$, and a reciprocal one for $p(r)=1/r$ (as then the total bending energy of an object is independent of magnification).

For most features, f does not depend on x_M, y_M (e.g. $f(r)=E.r^{-2}$ for bending energy) and D^j can be reduced to M^j :

$$g^j = \int_{M^j(r)} f(r) P_{x_M y_M}(r) p(r) dr$$

An estimator simpler to compute is the most probable value ML:

$$g_{ML}^j = \max_{M^j(r)} p(r)/|f'(r)|,$$

with f' denoting the first derivative of f .

2.6. Implementation

The values of g^j can be computed once and stored in a table, one entry to each configuration j in the M neighborhood. For a simply connected contour (i.e. the connections between two subsequent pairs of contour points differ $-\pi/4$, 0 or $\pi/4$ and no other value), it is easy to compute that the table has length 2.3^{M-2} (i.e. 54 for $M=5$, 486 for $M=7$ and 4374 for $M=9$).

An entry in the table is provided by tracing the directional differential of the points in the M neighborhood.

As the contour of an arbitrary binary object can be made simply connected by first applying an opening -this is a sufficient but not necessary condition-, local shape measures can be computed with or without tracing the contour:

prior to the analysis, compute tables of optimal best linear unbiased estimates or most probable value estimates for one (or more) features g .

then during runtime analysis, apply either one of the following:

2.7. Difference with fitting

The presented method is not equivalent to locally fitting circular arcs to the discrete points, nor does it produce the same results.

For the case of length measurement of straight contours, this was analyzed in [4]. There, the equivalent BLUE-method gives an error in the estimate the length of straight edges proportional to the theoretical minimum of $n^{-3/2}$, where n is the number of pixels along the edge. In contrast, fitting a straight line through the pixels of the contour or measuring the Euclidean distance between the endpoints, an error was found proportional to n^{-1} , which is inferior to BLUE-estimation.

The fact that fitting a MSE line through the pixels gives a result worse than the BLUE-method is due to the fact that the set of originals corresponding to one specific digital pattern over which the averaging of the feature to measure takes places, in the BLUE-method is tighter. In fact, it is the smallest one possible, a property not shared by the MSE-fitting. As a consequence there are different patterns of discretisation which have the same MSE-fitted line (and hence the same measurement result), whereas the BLUE value will be specific to each different discretisation pattern.

3. CONCLUSION

We have proposed a method to compute accurate, anisotropic local shape measures from digitized lines or contours. For every digital digitization pattern j , the set of all original continuous curves decomposed in circular arcs is derived. This set is used to compute the span of feature values in the original, predigitized domain. Treating every different pattern j in the M neighborhood as different, yielding a different outcome for the property to estimate as well, the method delivers optimal (in MSE-sense) Best Linear Unbiased Estimates for that original value. As an alternative, ML-values may also be used.

Currently the method is being implemented in an efficient table search. It will be compared with existing methods as discussed above. It is argued that in a noise-free situation, the method has the highest possible accuracy and the lowest possible sensitivity to orientation of the figure on the grid. Given the high accuracy of the method, it can be afforded to use low resolution analysis. Therefore, it is particularly suited for real time analysis, as in the analysis of the heart motion wall (as proposed by Duncan and others), and industrial inspection applications (cartography, product quality assessment).

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FOOTNOTES AND REFERENCES

*1 As the a priori average distance between sample points can be computed as $1.1107h$, it is better to design the linear filter response on the basis of $1.11h$ than on the basis of h . Still the average non-equidistance would be 11%, with a maximum of 27% [4].

*2 This figure can be reduced by using coefficients other than 5 and 7 to a theoretical minimum value of 2.6%, as it is equivalent to the relative error in the Kulpa method for measuring the length of a line, see [4].

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