Investment Risk, CDS Insurance and Firm Financing *

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Abstract

We develop a model in which investment risk drives the demand for CDS insurance. We show that CDS overinsurance (insurance in excess of renegotiation proceeds) is procyclical and allows for greater financing of firms with positive NPV projects. In bad times, CDS overinsurance triggers the early liquidation of firms with low continuation values. Our analysis explains the benefits of CDS contracting over economic cycles and reconciles evidence showing that CDSs are most beneficial for firms that are safer and have higher continuation values. The model generates a number of empirical predictions and provides insights on the regulation of CDS markets.

Keywords: CDS, Bankruptcy, Moral Hazard, Financing Efficiency, Regulation.
JEL: G33, D86, D61.

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Abstract
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“Some derivatives ought not to be allowed to be traded at all. I have in mind credit default swaps. The more I’ve heard about them, the more I’ve realized they’re truly toxic.”

— George Soros, June 2009.

1 Introduction

The 2000s witnessed a formidable growth in the market for credit default swaps. According to the International Swaps and Derivatives Association (ISDA), the outstanding amount of CDS contracts increased from $3 trillion in 2003 to a peak of $62 trillion in 2007. The 2008–9 Financial Crisis brought attention to these contracts and there is ongoing debate about whether CDSs contributed to the crisis and how to regulate CDS markets.\(^1\) Surprisingly, we know little about the role of CDSs in financial markets and what contracting inefficiencies they address.

It has been argued that CDSs alleviate capital markets frictions by improving banks’ ability to lay off credit risk and allowing for greater credit supply and better credit terms for firms (see, e.g., Duffee and Zhou (2001)). In this case, the introduction of CDSs should benefit riskier firms since they would likely be rationed out of credit markets in the absence of credit risk insurance. In contrast, Figure 1 shows that firms with CDSs outstanding are relatively safer. This puzzling fact invites research into better understanding how CDS markets work and their role in the economy.

A CDS is a bilateral agreement between a debt protection seller and a debt protection buyer. The buyer makes periodic payments to the seller in exchange for compensation in the event a borrower defaults on its debt. Under this contracting scheme, CDSs can give rise to the empty creditor problem (see Hu and Black (2008a,b)). Simply put, lenders protected by CDS may have low incentives to participate in out-of-court restructurings of distressed firms since formal default triggers immediate compensation for their exposure.\(^2\) The incentives to engage in restructuring could be even lower if lenders “overinsure;” that is, their protection payoff sur-

\(^1\)Title VII of Dodd-Frank Act (HR #4173) gives the SEC regulatory authority over swaps, including CDSs. The Act requires the reporting of trades, sets position limits, imposes margin requirements, and moves swaps away from over-the-counter markets into organized exchanges.

\(^2\)See Subrahmanyam et al. (2014) for empirical evidence on the empty creditor problem.
passes the amount of debt that can be recovered in default. In these cases, CDS-insured lenders might collect large profits by forcing distressed firms into bankruptcy even when continuation would be optimal.\(^3\) In all, CDS contracts may alter the dynamics of corporate financing since optimal lending decisions are influenced by expected distress outcomes.

While there is interest in the impact of CDSs on creditor–borrower relations, the literature lacks theoretical treatment concerning important aspects of these contracts. This paper develops a model of the interplay between investment risk and CDS contracting. The key insight of the model is that imperfect asset verification establishes a link between creditors’ choices of CDS protection and underlying economic conditions. Our analysis shows how these choices are made, allowing us to further examine the corporate finance implications of CDS

\(^3\)Numerous accounts blame overinsured CDS lenders for blocking out-of-court restructurings of high profile firms during the Financial Crisis. In 2009 alone, companies in that category included Six Flags, Harrah’s, GM, Chrysler, Unisys, R. H. Donnelley, Abitibi Bowater, Marconi, and Lyondell Basell.
contracting when investment is subject to moral hazard. The rich setup in which we analyze CDS contracting leads to a number of novel empirical predictions and helps explain reported empirical regularities.

In a nutshell, our model shows that the demand for CDS insurance is associated with the implementation of policies that maximize the likelihood that projects succeed and alleviate the empty creditor problem. We show that CDS overinsurance is associated with safer firms and is procyclical (more pronounced in booms). Indeed, our analysis suggests that CDS contracts may have emerged and become popular in the early 2000s by virtue of its overinsurance capabilities. It additionally shows that CDS contracts may boost the availability of credit in the economy. Importantly, we show that while CDSs facilitate borrowing by credit-constrained firms, CDSs will also be observationally associated with their demise in bad times, leading to the “appearance” that CDSs aggravate the impact of economic downturns. As we discuss below, proposed regulatory changes that limit the use of CDS insurance may have the adverse consequence of reducing the availability of credit when firms most need it.

Let us provide context to our framework and discuss the implications of our analysis. In the model, borrowers face a limited commitment problem in that they cannot commit to pay out cash flows from their projects. In effect, borrowers can strategically default even when projects succeed. As is standard in contracting problems of this type, lenders can refuse to renegotiate contracts in default and force firms into costly liquidation; else they can engage in out-of-court restructuring and bargain over the portion of firm continuation values that can be verified. In the presence of CDS contracts, however, lenders have an alternative course of action: they can insure against strategic default. In essence, CDSs trigger an insurance payment by a third party if a “credit event” occurs. As we demonstrate, the innovation brought about by CDSs is that they can be uniquely used to strengthen lenders’ bargaining position by: (1) increasing debt repayments when investments succeed and (2) increasing lenders’ share of proceeds in

\[\text{As defined by the ISDA, credit events include default, debt acceleration, failure to pay, repudiation/moratorium, and bankruptcy. The standard CDS contract does not recognize out-of-court restructuring as a credit event.}\]
default states. Differently put, CDSs can be used to modulate whether lenders will have a stronger bargaining position when projects succeed or when they fail.

If lenders buy CDS protection beyond the maximum amount they can receive in restructurings (i.e., lenders “overinsure” or have so-called “negative net economic ownership”), they pre-commit to forcing defaulting borrowers into bankruptcy. Intuitively, the mechanism works somewhat similarly to standard insurance. CDSs resemble actuarially fairly-priced policies and overinsurance increases both the likelihood that the insured party will require payoffs (immediate compensation for credit events) and the associated insurance premia (CDS fees). Once a credit event happens, the one-time payoff from seeking immediate borrower liquidation is large enough to commit CDS-protected lenders with that course of action. As a result, borrowers are prevented from capturing rents from default–continuation strategies, discouraging them from defaulting strategically. By altering the dynamics of renegotiations and heightening borrowers’ incentives, overinsured lenders maximize regular debt repayments in good investment states (e.g., extraction of higher “debt coupons”).

If lenders buy an amount of CDS protection that equals the maximum payoff under restructurings (“zero net economic ownership”), they do not commit to unconditional liquidation in default states. Instead, they position themselves so as to bargain over surpluses stemming from out-of-court renegotiations. Although zero net economic ownership maximizes the amount of debt repayment consistent with no liquidation, it leaves some surplus for borrowers when verification in default states is imperfect. Because “just-insured” lenders are relatively less inclined to call for bankruptcy if borrowers default, they pay lower fees for their CDS insurance. At the same time, because borrowers retain a fraction of restructuring values and know forced liquidation is less likely to happen, they are more prone to strategically default. This dynamic determines the tradeoffs faced by just-insured lenders. These lenders forego debt repayment surpluses that are extracted when investments succeed in exchange for higher

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5As in any competitive market, the insurance premium schedule is such that, in expectation, the insured party does not make an economic profit (e.g., higher insurance payoffs are associated with higher premia).

6For completeness, in the law and economics literature “positive net economic ownership” refers to the case in which lenders do not completely hedge their economic exposure to borrowers (see Hu and Black (2007)).
renegotiation proceeds when investments fail.

In this setting, the optimal degree of CDS insurance will be a function of tradeoffs between continuation and liquidation values, as well as the probability of investment success. To wit, when values under out-of-court restructuring and liquidation are similar, lenders expect to get the same payoff should firms become distressed. Given a similar bad state payoff, it is not worth it for lenders to position themselves so as to bargain over firm continuation. Instead, lenders are inclined to overinsure in order to maximize gains from good investment states. When continuation values are higher than liquidation values, on the other hand, lenders face a more difficult problem. In this case, as we discuss next, they need to weigh in the likelihood that projects succeed.

When investments are likely to succeed (“boom periods”), the probability that borrowers are in distress is small. Lenders’ payoffs will come mostly from regular debt repayments. To maximize those payoffs, lenders will prefer to take negative net economic ownerships (overinsure with CDS). CDS overinsurance will then maximize debt repayments consistent with borrowers not strategically defaulting. Conditional on distress, however, firms with CDS-overinsured lenders will be promptly liquidated — the empty creditor problem is pronounced in booms.

If the probability of investment failure is higher (“busts”), lenders’ expected payoffs lean more towards outcomes associated with default (out-of-court restructuring and liquidation values). If continuation values are higher than liquidation values, lenders will be inclined to have zero net economic ownership — the empty creditor problem is reduced. In this scenario, zero net economic ownership reduces borrowers’ payoffs when their firms are in distress (as lenders stand to capture restructuring surpluses). On the flip side, if continuation values are low and approximate liquidation values, the gains from renegotiation decline. Lenders will then be more inclined to overinsure. Notably, because investments are more likely to fail in bad states, this dynamic might lead one to “too often” observe CDS-insured lenders forcing firms with low continuation values into bankruptcy during busts.

Having studied the link between CDS demand and the probability that investments suc-
ceed, we examine the optimal insurance policy when the effort that borrowers dedicate to their projects is unobservable. Since borrowers’ effort choices affect cash flows, it is an important determinant of the likelihood of investment success. Our model shows that, because CDS overinsurance leads to the liquidation of defaulting borrowers, it prevents them from appropriating unverifiable funds in default-continuation strategies. This incentivizes them to exert high effort in order to reduce the chances of investment failure, which increases firm value. Notably, CDS overinsurance helps the implementation of high effort in firms with lower continuation values, where borrowers are demotivated due to lower surpluses in non-default states. Moreover, since these firms’ continuation and liquidation values are closer to each other, the inefficiency associated with the empty creditor problem is smaller.

The link between the demand for CDS and the state of the economy described by our model is new to the literature. The implications of a contracting framework that allows for complexities such as commitment and moral hazard problems stand in contrast to the extant notion that CDSs are harmful for allowing lenders to have negative economic ownership in the firms they finance. Additional model analysis shows that, in booms, CDS overinsurance increases financing to levels that exceed financing in economies where lender ownership is constrained to be non-negative. In busts, CDSs increase funding to levels that, at a minimum, equal those in economies where lender ownership is constrained to be non-negative. Naturally, there are more investment failures in downturns and there are more bankruptcies being pushed forward by lenders that are CDS insured during those times. In the absence of a benchmark, however, that casual observation is uninformative about the role played by CDSs in busts.

The theory we propose has several empirical implications and sheds light on recent attempts to find evidence on the empty creditor hypothesis. We show, for example, that CDSs are more beneficial for firms that are safer and have higher continuation values. This result is surprising as one might expect riskier firms to benefit the most from the existence of CDS insurance. Consistent with our model, studies by Ashcraft and Santos (2009) and Hirtle (2009) find that safer, larger firms have benefited the most from CDS contracts (for example, by paying lower
interest on their bank loans once CDSs are written on their bonds). Our theory also predicts
that the beneficial effects of CDSs on firm financing are present even when aggregate credit
is tight. Consistent with this prediction, Saretto and Tookes (2013) find that CDSs increased
corporate leverage and debt maturity even during the 2008–9 crisis. Our model further implies
that the empty credit problem is procyclical; that is, the conditional probability of CDS-led
liquidation given that a firm is in distress is higher (lower) in booms (busts). As such, the
model reconciles results from empirical studies looking at the role of CDSs in influencing the
choice between restructuring and bankruptcy during recent contractions, including the Financial Crisis (e.g., Bedendo et al. (2016)). Finally, we show that CDSs reduce agency problems
by increasing the penalties associated with misbehavior. Relatedly, Shan et al. (2014) find
evidence that covenants are less strict for firms with CDSs outstanding.

Our analysis has direct implications for the debate about the regulation of CDS markets.
Hu and Black (2008a,b) argue that voting in restructuring decisions should be limited to lenders
with positive net economic ownerships. Bolton and Oehmke (2011) suggest that eliminating
negative net economic ownerships (CDS overinsurance) might be beneficial in certain cases
as it would reduce the risk of breakdowns in renegotiations. Oehmke and Zawadowski (2015)
point out that banning naked CDS positions can increase funding costs by increasing short
selling on bond markets. We show that the benefits of banning CDS overinsurance may also
depend on investments prospects. CDS overinsurance in our model is negatively associated
with its costs, such that imposing limits on CDS positions can sometimes end up reducing
firms’ credit capacity when they most need it.\footnote{We note that our work complements Bolton and Oehmke (2011) by considering the impact of the likelihood of investment success on the demand for CDSs (CDS contracting in their paper is independent of the probability that investment succeeds and borrower effort). Our analysis stresses the role CDSs as commitment devices first identified in their study.}

Our paper is related to an infant literature on links between CDSs and creditor–borrower
relations. Most papers in this literature focus on the effect of CDSs on adverse selection and
moral hazard problems. Duffee and Zhou (2001) show that CDSs can alleviate “lemons problems” in credit risk-transfer markets. Parlour and Winton (2013) show how loan sales and
CDSs might jointly emerge in equilibrium, characterizing risk-transfer efficiency. Parlour and Plantin (2008) further investigate the effect of CDS markets on banks’ incentive to monitor (see also Morrison (2005)). Arping (2014) shows that CDSs increase the commitment of lenders to terminate projects in the presence of moral hazard. Campello and Matta (2012) examine the effect of CDSs on risk-shifting incentives. Bolton and Oehmke (2011) show that CDSs help reduce strategic defaults, but often result in empty creditors when firms’ continuation value conditional on default is high. Our paper adds to the existing literature by associating payoffs in non-default states and optimal CDS insurance, which allows us to examine the interplay between the empty creditor problem and the probability of investment success.

The remainder of the paper is organized as follows. Section 2 describes the model setup. In Section 3, we analyze the consequences of CDS contracting on renegotiation and liquidation outcomes. Section 4 characterizes the interplay between CDS contracts, debt repayments, and economic conditions. Section 5 abstracts away from exogenous economic conditions and examines the effect of CDS contracts on borrowers’ effort choices. We present a set of empirical implications in Section 7. Section 8 concludes the paper. Proofs are collected in the Appendix.

2 Model Setup

There are three risk neutral players: a borrower, a lender, and a competitive CDS provider. The game is played in three periods $t = 0, 1, 2$ and there is no discounting. The borrower is penniless, but endowed with a project. He turns to a lender to fund the project.

The time line and structure of the game is depicted in Figure 1. The project needs $I > 0$ units of investment in $t = 0$ and generates outcome $o_1 \in \{0, y_1\}$ in $t = 1$ if funded. In the event the project is not liquidated in $t = 1$, it also generates outcome $o_2 = y_2$ in $t = 2$. Following Hart and Moore (1994, 1998) and Bolton and Scharfstein (1990, 1996), we assume that $o_1$ and $o_2$ are non-verifiable in $t = 0$. Following Bolton and Oehmke (2011), we assume that the lender can make $y_2$ verifiable in $t = 1$. However, we adopt the assumption in Calomiris and Kahn
(1991) and Krasa and Villamil (2000) that the verification technology is imperfect, that is, only a fraction $\delta \in (0, 1)$ of the continuation outcome can be verified by an outside court. As will be clear when we examine the lender’s demand for CDS, this assumption creates a “discontinuity” whereby the lender commits to always liquidate the borrower. This condition allows us to link the demand for CDS to the state of the economy. We also assume that verifying $y_2$ is costly, such that the verifiable continuation value net of verification costs equals $\lambda \delta y_2$, where $\lambda \in (0, 1)$.

The distribution of the short-term outcome $o_1$ depends on the state of the economy $s \in \{l, h\}$, where $h$ and $l$ represent “booms” and “busts”, respectively. The economy is in a boom with probability $q$ and in a bust with probability $1 - q$. The probability that $o_1 = y_1$ is $p_h = p + \tau$ in a boom and $p_l = p$ in a bust. Therefore, the distribution of the short-term outcome is more favorable (in terms of first-order stochastic dominance) in periods of economic expansion. We assume that the state of the economy can be verified. As a consequence, while a contract written in $t = 0$ with repayments due in $t = 1$ cannot depend on the realized short-term outcome, it can be contingent on the state of the economy. The lender makes a take-it-or-leave-it offer to the borrower in the beginning of $t = 0$ with repayments given by $R_s$.

After a contract is signed, the state of the economy is observed by all players. With this information, the lender decides whether to buy a CDS. If the lender buys a CDS, she chooses the repayment that accrues if a “credit event” occurs in $t = 1$, and pays the correspondent endogenous fee $f$ to the CDS provider. We model the payment received by the lender in the event of liquidation according to practice in the CDS market. The lender retains the liquidation value of the investment (interpreted as proceeds from Chapter 11) $\beta I$, where $\beta \in (0, 1)$. In addition, she also receives the compensation amount $\pi$. Since the CDS market is competitive, the premium $f$ is fairly priced. A credit event is said to occur if the borrower is formally in

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8There could be uncertainty regarding the continuation outcome $o_2$ (as in Bolton and Oehmke (2011)). Our results, however, are qualitatively similar in the presence of uncertainty.

9Our results remain the same if the state of the economy is non-verifiable and the lender cannot commit not to offer a new contract upon observing the state. In this case, $R_h$ would denote the face value of debt agreed upon in $t = 0$ and $R_l$ would correspond to the renegotiated repayment after the realization of the state. As we show in the equilibrium analysis, $R_l < R_h$, implying debt-forgiveness in busts. This result is supported by empirical evidence, which shows that debt renegotiations are frequent and driven by macroeconomic fluctuations (see Roberts and Sufi (2009)).
default; that is, if the borrower does not repay the lender and the latter refuses to engage in a voluntary debt renegotiation.

At the beginning of $t = 1$, outcome $o_1$ is realized. If the borrower repays the lender, then no default occurs. In this case, the project continues and generates outcome $o_2 = y_2$ in $t = 2$. The borrower’s payoff is $o_1 - R_s + y_2$ and the lender receives $R_s$. If the borrower does not repay, the lender can either engage in renegotiation or force liquidation. If the lender refuses to renegotiate, the borrower defaults on his debt. The project is liquidated and the lender receives $L(\pi) \equiv \beta I + \pi$, while the borrower receives $o_1$. If the lender adheres to a renegotiation schedule, both the lender and the borrower bargain over the value $\tilde{y}_2 \equiv \lambda \delta y_2$ in $t = 2$. In this case, the lender receives $x$ and the borrower receives $o_1 + (1 - \delta) y_2 + \tilde{y}_2 - x$, where $x$ is determined by the Nash bargaining solution.

### 3 CDS Contracts, Renegotiation, and Default

We start our equilibrium analysis by investigating the outcome that would prevail when the borrower triggers renegotiation and the lender accepts to renegotiate. The parties split the renegotiation surplus according to the Nash bargaining solution, where the borrower and the
lender disagreement payoffs are 0 and $L(\pi)$, respectively. The bargaining outcome is given by

$$x(L(\pi)) = \frac{1}{2} \tilde{y}_2 + \frac{1}{2} L(\pi).$$  \hspace{1cm} (1)$$

From equation (1) one can see that the lender’s (gross) economic ownership — her share of the continuation value — is increasing in both the amount of CDS protection $\pi$ and liquidation value $\beta I$. For the purpose of our analysis, we assume that $x(L(0)) > L(0)$. This implies that, in the absence of CDSs, the lender prefers renegotiation to liquidation. It also implies that liquidation is inefficient (as $\tilde{y}_2 > L(0)$). Given the outcome of renegotiation, the lender refuses to renegotiate if $L(\pi) > x(L(\pi))$ and engages in renegotiation if $L(\pi) \leq x(L(\pi))$.

We note that an increase in $L(\pi)$ not only directly affects the threat point of liquidation (one-to-one), but also the renegotiation outcome (also in a linear fashion, but with slope smaller than one). Therefore, there exists a unique threshold of $L(\pi^*)$ such that the lender is indifferent between liquidation and renegotiation. This lead us to the following proposition:

**Proposition 1.** Suppose the borrower triggers renegotiation. The lender refuses to renegotiate if $L(\pi) > L(\pi^*)$ and engages in renegotiation if $L(\pi) \leq L(\pi^*)$, where $L(\pi^*) = \tilde{y}_2$.

Proposition 1 says that the lender’s maximum payoff consistent with renegotiation is attained when she buys credit protection in the amount of $\pi = \pi^*$. Although CDS protection above $\pi^*$ increases the lender’s economic ownership, it reduces her interest to renegotiate and results in default. The reason is that the lender’s incentive to renegotiate is dictated by her net economic ownership.

The lender’s net economic ownership is a combination of her share of the continuation value and her payoff under bankruptcy.\(^\text{10}\) Accordingly, credit protection above $\pi^*$ builds up negative net economic ownership, while credit protection below that amount results in positive net economic ownership. If the lender’s CDS protection is equal to $\pi^*$, she has zero net economic ownership. For credit protection values at most as high as $\pi^*$, the lender builds up non-negative

\(^{10}\)Hu and Black (2007) define net economic ownership as “...a person’s combined economic ownership of host company shares and coupled assets, and can be positive, zero, or negative.”
net economic ownership and engages in renegotiation. In this case, the maximum renegotiation payoff is given by \( \tilde{y}_2 \). Credit protection in excess of \( \pi^* \) results in negative net economic ownership, in which case the lender refuses to renegotiate and forces the borrower into bankruptcy.

We now derive the borrower’s decision to strategically default given that the project succeeds \( (o_1 = y_1) \). If the borrower repays the lender, his payoff is \( y_1 - R_s + y_2 \). This payoff needs to be compared to that when he does not repay the lender. In this case, if the lender renegotiates, the borrower’s payoff is \( y_1 + (1 - \delta) y_2 + \tilde{y}_2 - x(L(\pi)) \). If the lender does not renegotiate, the borrower’s payoff is \( y_1 \).\(^{11}\) Intuitively, the borrower strategically defaults if the face value of debt \( R_s \) exceeds a threshold. If the lender is expected to renegotiate, the borrower triggers renegotiation if \( R_s > \delta y_2 - \tilde{y}_2 + x(L(\pi)) \). If the lender is expected to liquidate, the borrower triggers renegotiation if \( R_s > y_2 \). These results are summarized in the next proposition.

**Proposition 2.** Suppose the project succeeds. Then:

(1) If the lender has non-negative net economic ownership, the borrower triggers renegotiation if and only if \( R_s > \delta (1 - \lambda) y_2 + x(L(\pi)) \).

(2) If the lender has negative net economic ownership, the borrower triggers renegotiation if and only if \( R_s > y_2 \).

Proposition 2 implies that, when the project succeeds, an increase in \( \pi \) (and hence \( L(\pi) \)) reduces the borrower’s incentive to strategically trigger renegotiation as long as the lender has positive net economic ownership (i.e., \( \pi < \pi^* \)). In other words, an increase in the amount of CDS protection continuously increases the threshold value for repayment \( R_s \). At \( \pi = \pi^* \) (zero net economic ownership), the threshold value for \( R_s \) hits a discontinuity as the lender’s economic ownership becomes negative. As a result, one needs to consider only the following two cases in the analysis of optimal CDS demand: (1) the lender has zero net economic ownership, i.e., \( \pi = \pi^* \); or (2) the lender has negative net economic ownership, i.e., \( \pi > \pi^* \). By choosing the amount of CDS protection, \( \pi \), the lender modulates her net economic ownership in the firm.\(^{11}\)

\(^{11}\)We assume that the borrower does not strategically default when he is indifferent between diverting the realized cash flow and reporting the true outcome.
4 CDS Contracts and the Economy

In the last section we identified the key tradeoff faced by the lender in our CDS model. Although overinsurance allows the lender to receive higher debt repayments when investment is successful, it comes at the cost of triggering bankruptcy when investment fails. Since CDSs are fairly priced (i.e., the CDS fee equals the expected CDS payment), the lenders’ ex ante payoff (before CDS is purchased) under bankruptcy, $\beta I$, is smaller than her share of the continuation value under renegotiation, $\delta \lambda y_2$. An important factor influencing this tradeoff is the state of the economy. In this section, we study how the lender chooses the optimal level of credit protection over the economic cycle. We then characterize the optimal contractual repayments.

4.1 Debt Repayments

It might be possible for the lender to sustain a higher debt repayment without strategic default. This would likely require the lender to overinsure. The drawback of this strategy is that overinsurance leads to liquidation, which provides a lower payoff ex ante (before the insurance decision is made) compared to renegotiation. Therefore, the lender is more willing to overinsure provided that the probability of project success is sufficiently high. Since the lender’s payoff in booms relies more heavily on the repayment $R_h$ rather than on distress outcomes (distress is less often in economic upturns), CDS-overinsurance becomes more attractive in economic expansions. Proposition 3 characterizes the lender’s optimal choice of CDS protection.

Proposition 3. The lender’s net economic ownership is determined as follows:

1. For $R_s > y_2$, the lender chooses to have zero net economic ownership.

2. For $R_s \in (\delta y_2, y_2]$, the lender chooses to have negative net economic ownership if and only if

$$R_s > \frac{\delta \lambda y_2 - \beta I (1 - p_s)}{p_s}.$$

3. For $R_s \leq \delta y_2$, the lender chooses to have zero net economic ownership.
According to Proposition 3, the lender does not overinsure if she chooses \( R_s > y_2 \). The reason is that a repayment \( R_s > y_2 \) causes the borrower to strategically default, implying that renegotiation takes place independent of the outcome \( o_1 \). If the lender overinsures, she refuses to renegotiate and the borrower defaults. The payoff of the lender is \( \beta I \). If the lender does not build up negative economic ownership, her payoff is \( \tilde{y}_2 > \beta I \). Therefore, the lender is better off without overinsurance.

If the lender chooses \( R_s \in (\delta y_2, y_2] \), then Proposition 3 shows that overinsurance is attractive for the lender provided that she is able to receive a high debt repayment. In this case, if the lender chooses \( \pi = \pi^* \), renegotiation is always triggered and the lender’s payoff is given by \( \delta \lambda y_2 \). If the lender overinsures, she receives \( R_s \) when the project succeeds and receives \( \beta I \) when the project fails. Therefore, the repayment \( R_s \) must be high enough so as to compensate for foregone renegotiation proceeds. This translates into the requirement that \( R_s > \frac{\delta \lambda y_2 - \beta I (1 - p_s)}{p_s} \).

Since the right-hand-side of the inequality is decreasing in \( p_s \), the lower the repayment must be in other to compensate her for the loan in booms. One key observation is that the higher the recovery value \( \beta I \), the smaller the repayment necessary to induce overinsurance.

A repayment \( R_s \leq \delta y_2 \) is insufficient to induce overinsurance. In this situation, zero net economic ownership is enough to avoid strategic default. Accordingly, overinsurance only decreases the lender’s payoff since foregone renegotiation proceeds are higher than bankruptcy proceeds.

### 4.2 The State of the Economy

The analysis in the preceding subsection shows the possibility for both zero and negative net economic ownership for \( R_s \in (\delta y_2, y_2] \). However, the lender optimally chooses a repayment in this range only if it results in overinsurance. This result follows from the fact that, if she chooses zero net economic ownership, then a repayment in this range is strictly dominated by a repayment of \( R_s = \delta y_2 \). To see this point, note that if the lender chooses \( R_s \in (\delta y_2, y_2] \), then renegotiation is always triggered and her payoff is \( \tilde{y}_2 \). If the lender chooses \( R_s = \delta y_2 \), then the borrower
does not strategically default and the lender’s payoff in state $s$ is strictly greater than $\bar{y}_2$:

$$\Pi_s(\delta y_2) \equiv [p_s \delta + (1 - p_s) \delta \lambda] y_2. \quad (2)$$

Two results follow from the preceding analysis. First, the lender chooses $R_s \in (\delta y_2, y_2]$ only if she overinsures. Second, the lender never chooses $R_s > y_2$. Thus, we are able to characterize the lender’s debt repayment choice by comparing repayments $R_s \in (\delta y_2, y_2]$ with those $R_s \leq \delta y_2$. Since the lender’s payoff is increasing in the repayment, this amounts to comparing (2) with

$$\Pi_s(y_2) \equiv (1 - p_s) \beta I + p_s y_2, \quad (3)$$

which leads us to the first main result of our paper:

**Proposition 4.** Let $R^*_s$ be the optimal repayment in state $s$ and $p \equiv \frac{\delta y_2 - \beta I}{(1 - \delta)(1 - \lambda)} y_2 - \beta I$. Then $R^*_s = y_2$ if $p_s > p$ and $R^*_s = \delta y_2$ if otherwise. That is, debt repayments are procyclical and the lender’s net economic ownership is countercyclical. In addition, the lower the continuation value $y_2$, the higher the liquidation value $\beta I$, the higher the renegotiation costs (lower $\delta \lambda$), and the safer the borrower (higher $p$), the more likely the lender is to overinsure.

Proposition 4 results from the lender’s fundamental tradeoff. On one hand, if the lender chooses the high debt repayment $y_2$, then negative net economic ownership is required in order to preclude strategic default. However, since CDSs are fairly priced — thus are zero-profit investments ex ante — this overinsurance implies liquidation and a low payoff to the lender in the event of default. On the other hand, if the lender chooses the low debt repayment $\delta y_2$, zero net economic ownership is sufficient to discourage strategic default. In this case, CDSs yield the highest surplus in renegotiation consistent with no liquidation.

In booms, the lender’s payoff leans more heavily on that repayments rather than on distress outcomes. In this case, a high debt repayment associated with overinsurance becomes more attractive. This implies that the empty creditor problem is procyclical. To wit, higher probabilities of successful investments strengthen the beneficial effects of CDSs on limited
commitment problems. This boosts the income that can be pledged to lenders, increasing firm
debt capacity. As a result, a larger number of projects with positive NPV receive financing.
Because the probabilities of investment failure are smaller in booms, the appearance of the
empty creditor problem is reduced during those times. The countercyclicality of net economic
ownership is a positive feature of CDS markets; however, these markets still present inefficien-
cies. The reason is that lenders might overinsure even if the project can be financed with zero
net economic ownership. The exact cases when this occurs along with the effect of policies
limiting CDS positions are examined in the next subsection.

4.3 Efficiency and Regulatory Constraints on CDS Markets

In our model, efficiency requires no strategic default and no liquidation given default. To see
this, suppose the realized outcome is \( o_1 = y_1 \). If the borrower does not strategically default,
then total welfare is \( y_1 + y_2 \). If strategic default takes place, then total welfare under renego-
tiation is \( y_1 + (1 - \delta) y_2 + \tilde{y}_2 < y_1 + y_2 \). Accordingly, strategic default is inefficient. Failure to
renegotiate when \( o_1 = 0 \) is also inefficient. Total welfare under renegotiation is \( (1 - \delta) y_2 + \tilde{y}_2 \).
However, if the lender chooses to liquidate the project, total welfare is \( \beta I < (1 - \delta) y_2 + \tilde{y}_2 \).

Since strategic default does not occur in equilibrium, inefficiency issues are related to liqui-
dation following default. Zero net economic ownership does not cause liquidation and increases
the lender’s payoff, allowing for funding of projects with larger investment requirements. There-
fore, zero net economic ownership is always efficient. This result questions the reform proposals
made by Hu and Black (2008a,b), who argue that lenders’ CDS positions should be limited to
positive net economic ownerships. Under that proposed reform, our model says that restruc-
turing proceeds would be inefficiently reduced when zero net economic ownership is optimal.

The results in the last subsection state that overinsurance does arise, resulting in liquidation.
It follows that overinsurance is inefficient ex post. This does not imply that overinsurance
is ex ante inefficient. If overinsurance provides the lender with a higher payoff than that ob-
tained with zero net economic ownership, it might be that a project can be financed only
if the lender overinsures. It follows that proposals to restrict net economic ownership to be non-negative improve welfare if and only if the project can be financed with zero net economic ownership. Let \( \Pi (R_h, R_l) \equiv q \Pi_h (R_h) + (1 - q) \Pi_l (R_l) \). A project can be financed with zero net economic ownership if and only if \( I \leq \Pi (\delta y_2, \delta y_2) \). With this benchmark, we characterize efficiency in the CDS market as well as welfare-improving features of limits on CDS positions.

**Proposition 5.** The following holds regarding efficiency and intervention in CDS markets:

1. For \( p \geq 1 \), there is no overinsurance (inefficiency) and restricting net economic ownership to be nonnegative is innocuous.

2. For \( p < p \leq 1 \), overinsurance (restricting net economic ownership to be nonnegative) is efficient (inefficient) if and only if \( \Pi (\delta y_2, \delta y_2) < I \leq \Pi (y_2, y_2) \).

3. For \( p \leq p < 1 \):
   
   (i) if \( \tau \leq p - p \), there is no overinsurance (inefficiency) and restricting net economic ownership to be nonnegative is innocuous;

   (ii) if \( \tau > p - p \), overinsurance (restricting net economic ownership to be nonnegative) is efficient (inefficient) if and only if \( \Pi (\delta y_2, \delta y_2) < I \leq \Pi (y_2, \delta y_2) \).

4. Zero net economic ownership (restricting net economic ownership to be positive) is efficient (inefficient).

The first result of Proposition 5 states that overinsurance does not occur for firms with low renegotiation costs, high continuation values, and low liquidation proceeds. The second and third results say that, if the previous features do not hold, then overinsurance occurs only if firms are sufficiently safe or if the economy is in a boom period. In essence, Proposition 5 says that overinsurance is more likely the smaller is the expected ex post inefficiency associated with it. That is, overinsurance is used when it allows for profitable projects to be financed and the costs of liquidation upon default are small.
Putting these results together, our analysis reveals that the empty creditor problem is procyclical. Although CDS overinsurance leads to bankruptcy when the borrower is distressed, the incidence of overinsurance is higher in booms, when the probability of distress is small. Notably, the dynamics of the demand for CDS over the business cycle works so as to minimize the empty creditor problem. This makes the task of designing policies — especially those that limit CDS positions — to reduce the empty creditor problem particularly challenging.

5 CDSs, Debt Repayments, and Effort

In the last section we identified the key tradeoff faced by the lender in our CDS model. Although overinsurance allows the lender to receive higher debt repayments when investment is successful, it comes at the cost of triggering bankruptcy when investment fails. Bankruptcy gives the lender the liquidation value $\beta I$, which is smaller than her share of the continuation value under renegotiation, $\delta \lambda y_2$. To our knowledge, we are the first to link CDS demand to the probability that investment succeeds.

We are now in a position to investigate the effect that CDSs have on the incentives of borrowers to make their investments profitable. Since the borrower’s choice of effort affects the probability that investment succeeds, it is an important factor influencing the tradeoff highlighted in the previous paragraph. Instead of examining the effect of exogenous economic conditions on CDS demand, in this section we study how CDSs influences the borrower’s effort choice, hence investment success.

To formalize ideas, the distribution of the short-term outcome $o_1$ now depends on the borrower’s effort level, which is chosen after the contract is signed and before the lender buys CDSs. Since the effort level is non-observable, the contract is simply specified by debt repayment $R_1$. The borrower chooses either to exert high effort, $e_H$, or low effort, $e_L$. The probability that $o_1 = y_1$ is $p_H$ if the borrower chooses $e = e_H$, and $p_L$ if the borrower chooses $e = e_L$, where $p_H > p_L$. If the borrower chooses $e_L$, he derives a private benefit $B > 0$. Effort
choices are not observed by the lender, who has belief $\mu$ that the borrower chooses $e = e^H$. All else in the model remains unchanged. We first study how the lender chooses an optimal level of credit protection given that she does not observe the borrower’s effort choice. We then analyze how the lender’s choice of debt repayment affects the equilibrium of this CDS–Effort subgame.

5.1 The CDS–Effort Subgame

In this subsection we characterize the equilibria of the CDS-Effort subgame. First, we derive the lender’s optimal CDS demand given her expectation regarding the borrower’s effort choice. Next, we determine the borrower’s best effort choice given the lender’s choice of CDS protection. Finally, we combine both the lender’s and the borrower’s best responses to find the equilibria.

If the borrower chooses high effort, he increases the probability that the project succeeds. This weights the lender’s expected payoff more towards the repayment $R_1$, making CDS-overinsurance more attractive given the greater bargaining power this position entails. The problem is that the lender does not observe the borrower’s effort level and must make her decision on the amount of CDS under uncertainty. Proposition 6, which is a variant of Proposition 3, characterizes the lender’s optimal decision regarding the level of CDS protection.

Proposition 6. The lender’s net economic ownership is determined as follows:

1. For $R_1 > y_2$, the lender chooses to have zero net economic ownership.

2. For $R_1 \in (\delta y_2, y_2]$,

   (i) the lender chooses to have negative net economic ownership if

   $$R_1 > R(\mu) = \frac{\delta \lambda y_2 - \beta I [\mu (1 - p_H) + (1 - \mu) (1 - p_L)]}{\mu p_H + (1 - \mu) p_L},$$

   (ii) the lender chooses to have zero net economic ownership if $R_1 \leq R(\mu)$.

3. For $R_1 \leq \delta y_2$, the lender chooses to have zero net economic ownership.

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According to Proposition 6, the lender does not overinsure if she chooses $R_1 > y_2$. The reason is that a repayment $R_1 > y_2$ causes the borrower to trigger strategically default, implying that renegotiation takes place independent of the outcome $o_1$. If the lender overinsures, she refuses to renegotiate and the borrower defaults. The payoff of the lender is $\beta I$. If the lender does not build up negative economic ownership, her payoff is $\tilde{y}_2 > \beta I$. Therefore, the lender is better off without overinsurance.

If the lender chooses $R_1 \in (\delta y_2, y_2]$, then Proposition 6 shows that overinsurance is attractive for the lender provided that she is able to receive a high debt repayment. In this case, if the lender chooses $\pi = \pi^*$, renegotiation is always triggered and the lender’s payoff is given by $\delta \lambda y_2$. If the lender overinsures, she receives $R_1$ when the project succeeds and receives $\beta I$ when the project fails. Given the lender’s expected probability of success $\mu p_H + (1 - \mu) p_L$, the repayment $R_1$ must be high enough so as to compensate for foregone renegotiation proceeds. This translates into the requirement that $R_1 > R(\mu)$. Since $R(\mu)$ is decreasing in $\mu$, the more the lender believes the borrower is exerting low effort, the higher the repayment must be in order to compensate her for the loan. Another important observation is that the higher the recovery value $\beta I$, the smaller the repayment necessary to induce overinsurance.

A repayment $R_1 \leq \delta y_2$ is insufficient to induce overinsurance. In this situation, zero net economic ownership is enough to avoid strategic default. Accordingly, overinsurance only decreases the lender’s payoff since foregone renegotiation proceeds are higher than bankruptcy proceeds.

Proposition 6 described the lender’s best choices of CDS insurance given her beliefs about the borrower’s effort choice. To find the equilibria of this subgame, we need to derive the borrower’s effort choices given the lender’s amount of insurance. This is given by Proposition 7.

**Proposition 7.** Let $\Delta \equiv y_1 - \frac{B}{(p_H - p_L)}$. The borrower’s choice of effort is determined as follows:

1. For $R_1 > y_2$, the borrower chooses high effort if and only if $y_1 (p_H - p_L) \geq B$.

2. For $R_1 \in (\delta y, y_2]$,
(i) if the lender has negative net economic ownership, the borrower chooses high effort if and only if \( R_1 \equiv y_2 + \Delta \geq R_1 \).

(ii) if the lender has non-negative net economic ownership, the borrower chooses high effort if and only if \( y_1 (p_H - p_L) \geq B \).

(3) For \( R_1 \leq \delta y_2 \), the borrower chooses high effort if and only if

\[
R_1 \equiv \delta y_2 + \Delta \geq R_1.
\]

We assume that it is optimal to implement high effort in the absence of strategic default and liquidation given default. In this efficient world, investment \( I \) should be made if and only if \( \Pi \geq I \), where \( \Pi \) is defined as

\[
\max \{p_H (y_1 + y_2) + (1 - p_H) [(1 - \lambda) y_2 + \bar{y}_2], p_L (y_1 + y_2) + (1 - p_L) [(1 - \lambda) y_2 + \bar{y}_2] + B \}.
\]

Conditional on the project having positive NPV (i.e., \( \Pi > 0 \)), high effort should be induced if and only if \( y_1 (p_H - p_L) + \delta (1 - \lambda) y_2 \geq B \). Along with this condition, we also assume that \( y_1 (p_H - p_L) < B \).\(^{12}\) These assumptions imply that verification costs are sufficiently high such that it is optimal do induce high effort in order to avoid those costs.

It follows from Proposition 7 that, if the lender has zero net economic ownership, the borrower’s compensation in the event the project succeeds must be sufficiently high to induce him to exert high effort. Alternatively, if the lender chooses a debt repayment that is sufficiently high, then she must build up negative net economic ownership to induce high effort. If the lender overinsures, she can credibly threat to reject renegotiation and force the borrower into bankruptcy. This reduces the borrower’s payoff when investment fails and creates a compensation scheme that induces high effort.

Proposition 8 characterizes the equilibria of the CDS–Effort subgame. While the propo-

\(^{12}\)If we assume otherwise, then according to Proposition 7 it would follow that high effort is always implemented.
tion seems fairly involved, it reveals a number of economically interesting results.

**Proposition 8.** Let \( R(0) \in (\delta y_2, y_2) \). The equilibria of the CDS–Effort subgame are determined as follows:

1. For \( R_1 > y_2 \), the lender chooses zero net economic ownership and the borrower chooses low effort.

2. Let \( R_1 \in (\delta y_2, y_2] \).
   
   (i) For \( R_1 \in (R(0), y_2] \): (a) if \( R_1 > \overline{R}_1 \), the lender chooses negative net economic ownership and the borrower chooses low effort; (b) if \( R_1 \leq \overline{R}_1 \), the lender chooses negative net economic ownership and the borrower chooses high effort.
   
   (ii) For \( R_1 \in (\delta y_2, R(0)] \): (a) if \( R_1 > \overline{R}_1 \), the lender chooses zero net economic ownership and the borrower chooses low effort; (b) if \( R_1 \leq \overline{R}_1 \), there is one equilibrium in which the lender chooses negative net economic ownership and the borrower chooses high effort, and another in which the lender chooses zero net economic ownership and the borrower chooses low effort.

3. For \( R_1 \leq \delta y_2 \):
   
   (i) if \( R_1 > \overline{R}_1 \), the lender chooses zero net economic ownership and the borrower chooses low effort;

   (ii) if \( R_1 \leq \overline{R}_1 \), the lender chooses zero net economic ownership and the borrower chooses high effort.

If the lender charges a repayment that is too high (i.e., \( R_1 > y_2 \)), then she chooses to have zero net economic ownership. According to Proposition 8, this debt repayment is insufficient to induce the borrower to exert high effort. If the lender chooses a debt repayment below \( R_1 \leq \delta y_2 \), then she also prefers a zero net economic ownership position. The reason is that the lender receives \( \delta R_1 \) when the project succeeds under both negative and zero net economic ownership.
ownerships. On the other hand, if the lender overinsures and the project fails, she receives the liquidation value $\beta I$, as opposed to the renegotiation surplus $\tilde{y}_2 > \beta I$. The borrower’s effort choice depends on his payoff when the project succeeds. If the repayment chosen by the lender is above $\overline{R}_1$, then the borrower exerts low effort and derives benefit $B$. If the lender’s repayment is sufficiently low ($R_1 \leq \overline{R}_1$), then the borrower has enough incentives to choose high effort in order to increase the project’s probability of success.

The analysis is slightly more involved when the lender’s choice of debt repayment lies in the interval $(\delta y_2, y_2]$. If the debt repayment is sufficiently high ($R_1 > R(0)$), the lender prefers to overinsure. This result follows from the fact that the debt repayment received when the project succeeds is large enough so as to compensate for the foregone renegotiation proceeds when the project fails. If the debt repayment is such that $\overline{R}_1 < R_1 \leq R(0)$, the lender chooses zero net economic ownership. The intuition for nonexistence of an equilibrium with overinsurance is as follows. Proposition 7 says the lender overinsures if and only if her belief that the borrower chooses high effort is sufficiently high, such that $R_1 > R(\mu)$. Proposition 8 says that for $R_1 > \overline{R}_1$, the borrower’s optimal choice of effort is $e_L$, which implies that the lender’s updated belief is $\mu = 0$. However, because $R_1 \leq R(0)$ the lender is better off without overinsurance.

If $R_1 \leq \overline{R}_1 \leq R(0)$, then there are two equilibria. On the one hand, if the lender anticipates that the borrower will choose high effort (i.e., $R_1 > R(\mu)$), then according the Proposition 6 she overinsures. From Proposition 7 we know that overinsurance induces the borrower to exert high effort, which results in $\mu = 1$. This reinforces the lender’s willingness to assume a negative net economic ownership as $R_1 > R(\mu) \geq R(1)$. On the other hand, if the lender anticipates the borrower will choose low effort, then $R_1 \leq R(\mu)$. Proposition 6 says that the lender chooses to have zero net economic ownership and Proposition 7 implies that he borrower chooses low effort. Accordingly, the lender’s updated belief is $\mu = 0$, which confirms her decision to not overinsure since $R_1 \leq R(\mu) \leq R(0)$.

Proposition 8 shows that the lender can choose a debt repayment schedule from two different sets. If she chooses a repayment from the set of low values, then just-insurance is enough
to avoid strategic default. On the other hand, to achieve the same outcome when choosing from the set with high values, she must overinsure. Regardless of the set from which the lender chooses the repayment, she needs to select a value that is sufficiently small (within the relevant range) in order to induce high effort.

### 5.2 Debt Repayment and Effort

The analysis in the preceding subsection shows the possibility for equilibria with both zero and negative net economic ownership in the CDS–Effort subgame for $R_1 \in (\delta y_2, y_2]$. In particular, equilibria with zero net economic ownership occur when $R_1 \in (\delta y_2, R(0)]$.

However, the lender chooses a repayment in this range only if $R_1 \leq \overline{R}_1$ and the equilibrium played is one that results in overinsurance. This result follows from the fact that if the equilibrium for $R_1 \in (\delta y_2, R(0)]$ involves zero net economic ownership and low effort, then a repayment in this range is strictly dominated by a repayment of $R_1 = \delta y_2$. To see this point, note that if the lender chooses $R_1 \in (\delta y_2, R(0)]$, then renegotiation is always triggered and her payoff is $\tilde{y}_2$. If the lender chooses $R_1 = \delta y_2$, then the borrower does not strategically default and the lender’s payoff is strictly greater than $\tilde{y}_2$:

$$\Pi(\delta y_2) = (p_L \delta + (1 - p_L) \delta \lambda) y_2.$$  

(4)

As a consequence of the preceding analysis, the lender chooses $R_1 \in (\delta y_2, y_2]$ only if she overinsures. The lender’s dilemma within this range is whether to choose a low repayment $R_1 \leq \overline{R}_1$ consistent with high effort, or require a high repayment $R_1 > \overline{R}_1$ at the expense of inducing low effort. To streamline our subsequent analysis, we assume that the equilibrium played in (ii)(b) of Proposition 8 is the one that results in negative net economic ownership.\(^\text{13}\)

We are then able to characterize the lender’s debt repayment choice.

If the lender chooses $R_1 \leq \delta y_2$, then she faces a tradeoff between (1) requiring a repayment

\(^{13}\)If the equilibrium played is the one that results in zero net economic ownership, then if $\overline{R}_1 \leq R(0)$, the lender chooses $R_1 \in (\delta y_2, y_2]$ only if $R_1 > \overline{R}_1$ (i.e., only if she induces low effort).
that is consistent with the borrower exerting high effort, and (2) demanding a higher repayment that induces the borrower to choose low effort and increases the probability of failure and renegotiation. If the lender chooses the former, her payoff is given by:

$$\Pi (R_1) \equiv p_H \Delta + [p_H \delta + (1 - p_H) \delta \lambda] y_2. \quad (5)$$

If the lender chooses \( R_1 \in (\delta y_2, y_2] \), she faces a similar tradeoff. In particular, when the lender chooses \( R_1 = y_2 \), her payoff is

$$\Pi (y_2) \equiv (1 - p_L) \beta I + p_L y_2, \quad (6)$$

while if she chooses \( R_1 = \overline{R_1} \), her payoff is

$$\Pi (\overline{R_1}) \equiv p_H \Delta + (1 - p_H) \beta I + p_H y_2. \quad (7)$$

Proposition 9 derives the conditions under which the lender chooses to induce high effort.

**Proposition 9.** The level of effort induced by the lender is determined as follows:

1. For \( R_1 \in (\delta y_2, y_2] \), the lender chooses \( R_1 = \overline{R_1} \) over \( R_1 = y_2 \) if and only if

   $$y_2 \geq \overline{y}_2 \equiv -\frac{p_H}{p_H - p_L} \Delta + \beta I.$$

2. For \( R_1 \leq \delta y_2 \), the lender chooses \( R_1 = R_1 \) over \( R_1 = \delta y_2 \) if and only if

   $$y_2 \geq \overline{y}_2 \equiv -\frac{p_H}{\delta (p_H - p_L) (1 - \lambda)} \Delta.$$

Proposition 9 shows that, within a given range, it is optimal for the lender to induce high effort if and only if the project’s continuation value is sufficiently large. Since there is no
strategic default in equilibrium, the lender’s payoff is partially dependent upon the debt re-
payment received when the project succeeds. Recall, Proposition 2 showed that the higher
the project’s continuation value, the higher the debt repayment consistent with no strategic
default. The probability that the project succeeds is thus partially dependent upon the effort
that the borrower chooses. In order for the lender to increase the probability of success, she
must give up some debt repayment to induce the borrower to exert high effort.

At the same time, increases in the project’s continuation value improve the tradeoff terms
in favor of inducing high effort. A higher probability of success makes the lender’s payoff
more sensitive to the continuation value. As Proposition 7 showed, higher continuation values
increase the debt repayment consistent with the borrower exerting high effort. Proposition 9
shows that for large enough continuation values, the lender prefers to induce the borrower to
exert high effort.

5.3 CDSs and Effort

Although Proposition 9 describes the lender’s tradeoff between payoffs associated with debt
repayment and investment failure, it does not shed light on the lender’s choice to have zero or
negative net economic ownership — the demand for CDS. We describe this choice in turn.

The lender overinsures if she chooses a debt repayment in the range $(\delta y_2, \ y_2]$, and assumes
a zero net economic ownership if she chooses a debt repayment such that $R_1 \leq \delta y_2$. Propo-
sition 10 characterizes the lender’s optimal repayment choice when the liquidation value is
sufficiently small. It describes an important tradeoff faced by the lender in our model that we
discuss in turn. A more complete characterization is given in the Appendix.

**Proposition 10.** Let $c(p) = \delta \lambda + \delta (1 - \lambda) p$. If $\beta I$ is sufficiently small, the lender’s repayment
choice is characterized by cutoffs $y_2^* < y_2 < y_2^{**} < \overline{y}_2 \leq y_2^{***}$ such that:
If $p_H > p_L \geq c(p_H) > c(p_L)$, the lender chooses

\[ R_1 = \begin{cases} 
  y_2, & \text{for } y_2 < y_2^* \\
  \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}] \\
  R_1, & \text{for } y_2 \geq y_2^{**} 
\end{cases} \]

(2) If $p_H \geq c(p_H) > c(p_L) > p_L$, the lender chooses

\[ R_1 = \begin{cases} 
  y_2, & \text{for } y_2 < y_2^* \\
  \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}] \\
  \delta y_2, & \text{for } y_2 \in [y_2^{**}, y_2^{***}] \\
  R_1, & \text{for } y_2 \geq y_2^{***} 
\end{cases} \]

(3) If $c(p_H) > p_H > c(p_L) > p_L$, the lender chooses

\[ R_1 = \begin{cases} 
  y_2, & \text{for } y_2 < y_2^* \\
  \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}] \\
  R_1, & \text{for } y_2 \geq y_2^{**} 
\end{cases} \]

(4) If $c(p_H) > c(p_L) \geq p_H > p_L$, the lender chooses

\[ R_1 = \begin{cases} 
  y_2, & \text{for } y_2 < y_2^* \\
  \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}] \\
  R_1, & \text{for } y_2 \geq y_2^{**} 
\end{cases} \]

If the lender chooses to have negative net economic ownership, she refuses to renegotiate in default and forces the borrower into bankruptcy. This maximizes debt repayments when investment is successful, but reduces the payoff to the liquidation value of the project when the borrower enters distress. If the lender chooses to have zero net economic ownership, she gives up some debt repayment in the event investment succeeds in exchange for maximum
renegotiation proceeds in the event it fails. These dynamics are determined by the probability of investment success $p_H$ and $p_L$ and project’s continuation value.

The first result of Proposition 10 states that, if $p_H > p_L \geq c(p_H) > c(p_L)$, then the lender’s payoff leans more heavily towards outcomes associated with investment success (a portion of the project’s cash flows). As a result, the extra debt repayment extracted when the project succeeds compensates for the forgone renegotiation proceeds when the project fails.

The second result from Proposition 10 says that, when economic conditions are such that $p_H \geq c(p_H) > c(p_L) > p_L$, there is a range of continuation values for which the lender prefers not to overinsure. If the continuation value is low, the tradeoff faced by the lender disappears. The lender’s payoffs when the project fails are approximately the same regardless of her net economic ownership. Therefore, the lender overinsures to maximize her payoff (debt repayment) in the event the project succeeds. In addition, the continuation value is insufficient for high effort to be optimal and the resulting probability of success is $p_L$. As the continuation value increases, the opportunity cost of overinsurance also increases. Since the probability of success is still relatively low, expected forgone proceeds from renegotiation are sizeable. Accordingly, it becomes optimal for the lender to have zero net economic ownership. On the other hand, if the continuation value is sufficiently high, then inducing the borrower to exert high effort is attractive for the lender. In this case, overinsurance becomes optimal as it increases the debt repayment consistent with high effort.

Another implication of Proposition 10 is that overinsurance is more likely to be associated with firms that are safer (higher probability of investment success) and larger (higher continuation values). Along with the fact that the main difference between CDS and standard insurance is that CDS allows for protection beyond economic interest, Proposition 10 helps characterize the types of CDS positions we often observe: CDS are written on safer, larger firms and many times leave lenders “overinsured” in their exposures to firms they lend to.

Proposition 10 also shows that the lender prefers to overinsure when continuation values are low. On the other hand, if $c(p_H) > c(p_L) \geq p_H > p_L$, the lender chooses to have zero net
economic ownership for continuation values that are large. The reason is that, in this case, the lender’s payoff is weighted more towards outcomes associated with project failure. This increases the expected forgone renegotiation proceeds and the opportunity cost under negative net economic ownership.

The fourth result of Proposition 10 affirms that, when the continuation value is sufficiently low, the lender’s payoff when the project fails is the same regardless of her net economic ownership. Thus, the lender prefers overinsurance in order to maximize her payoff in the event the project succeeds. Since the continuation value is low, inducing high effort is not optimal for the lender. Increasing the continuation value raises the attractiveness of inducing the borrower to exert high effort. As a result, the lender chooses to overinsure and to induce high effort. At high continuation values, in contrast, the tradeoff between debt repayment and renegotiation proceeds faced by the lender is sizeable. Because the probability of success is low, the opportunity cost of overinsurance is large and it becomes optimal for the lender to have zero net economic ownership.

These results suggest that, for riskier firms, CDS overinsurance emerges where credit constraints are most likely to bind. Indeed, CDS overinsurance eases the financing of profitable projects with relatively low continuation values, projects that would likely be underfunded (“financially constrained”) in tight credit markets without CDS contracts. On the flip side, the lender does not overinsure when project continuation values are high. This optimal CDS policy reduces the empty creditor problem exactly when its drawback is potentially sizeable; that is, when the probability of distress is high. Importantly, zero net economic ownership does not come at the cost of leaving profitable firms underfunded since, in expectation, firms with high continuation values would likely receive funding even in the absence CDS overinsurance.

5.4 Efficiency and Regulatory Constraints on CDS Markets

From a welfare standpoint, it is important to characterize the efficiency gains associated with the existence of CDS contracts. It is also important to understand how constraints on CDSs
— in especial, constraints on CDS overinsurance — may affect credit markets and firms. The analysis of this section considers these issues and sheds light on the economic effects of proposed regulatory changes in CDS markets. The comparison benchmark we use is the equilibrium that obtains in the absence of CDSs.

5.4.1 Equilibrium without CDS Markets

Proposition 2 shows that in the absence of CDSs the maximum repayment consistent with no strategic default is given by \( R_1 = \delta (1 - \lambda) y_2 + x (L (0)) \). In order to implement high effort, the repayment chosen by the lender must be such that

\[
p_H (y_1 - R_1 + y_2) + (1 - p_H) [(1 - \delta) y_2 + \bar{y}_2 - x (L (0))] \geq p_L (y_1 - R_1 + y_2) + (1 - p_L) [(1 - \delta) y_2 + \bar{y}_2 - x (L (0))] + B.
\]

At the same time, the lender chooses to induce high effort if and only if

\[
p_H [\delta (1 - \lambda) y_2 + x (L (0)) + \Delta] + (1 - p_H) [x (L (0))] \geq p_L [\delta (1 - \lambda) y_2 + x (L (0))] + (1 - p_L) [x (L (0))],
\]

which holds if and only if \( y_2 \geq \bar{y}_2 \).

It follows that the borrower’s effort in the absence of CDSs is similar to the effort level that obtains when CDS overinsurance is not allowed. This result is important in order to examine the efficiency properties of CDSs as well as proposals to cap CDS insurance.

5.4.2 Efficiency

In our model, efficiency requires: (1) no strategic default, (2) no liquidation given default, and (3) implementation of high effort. To see this, suppose the realized outcome is \( o_1 = y_1 \). If the borrower does not call for renegotiation (i.e., \( \bar{o}_1 = y_1 \)), then total welfare is \( y_1 + y_2 \). If strategic
Table 1. CDS–Effort Equilibrium Outcomes

The table entries represent the equilibrium levels of CDS insurance (overinsurance vs. no-overinsurance) and effort level (low vs. high) for different combinations of investment success probability (across rows) and continuation values (across columns). O, NO, LE, and HE denote, respectively, overinsurance, no-overinsurance, low effort, and high effort.

<table>
<thead>
<tr>
<th>Investment Success Probability</th>
<th>Continuation Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_H &gt; p_L \geq c(p_H) &gt; c(p_L) )</td>
<td>(&lt; y_2^* )</td>
</tr>
<tr>
<td>( p_H \geq c(p_H) &gt; c(p_L) &gt; p_L )</td>
<td>O–LE</td>
</tr>
<tr>
<td>( c(p_H) &gt; p_H &gt; c(p_L) &gt; p_L )</td>
<td>O–LE</td>
</tr>
<tr>
<td>( c(p_H) &gt; c(p_L) \geq p_H &gt; p_L )</td>
<td>O–LE</td>
</tr>
</tbody>
</table>

default takes place \( (\delta_1 = 0) \), then total welfare is \( y_1 + (1 - \delta) y_2 + \tilde{y}_2 < y_1 + y_2 \). Accordingly, strategic default is inefficient. Failure to renegotiate when \( a_1 = 0 \) is also inefficient. Total welfare under renegotiation is \( (1 - \delta) y_2 + \tilde{y}_2 \). However, if the lender refuses to engage in renegotiation, the lender defaults and total welfare is \( \beta I < (1 - \delta) y_2 + \tilde{y}_2 \). Finally, implementation of high effort is efficient under the assumption that \( y_1 (p_H - p_L) + \delta (1 - \lambda) y_2 \geq B \).

In order to assess the efficiency properties of CDS contracts, we need to examine the equilibrium levels of effort and insurance as functions of the project’s continuation value and probability of success. We can use Proposition 10 to compile a table that helps illustrate the problem.

Table 1 shows the equilibrium levels of CDS insurance (overinsurance vs. no-overinsurance) and effort (low vs. high) for various combinations of investment success probability and continuation values. Each entry has the CDS–Effort equilibrium outcome that obtains for a continuation value that is lower than the level specified in the column heading. One can readily see from the table that overinsurance is increasing in the probability of investment success. The table also suggests that effort is increasing in the project’s continuation value, probability of success, and the level of CDS insurance.

To give context to the results in Table 1, recall that in the absence of CDSs, \( \pi = 0 \). In this case, the lender’s share of the continuation value resulting from renegotiation is \( x(L(0)) \). This value is smaller than \( \tilde{y}_2 \), which is her share when she just-insures, i.e., \( \pi = \pi^* \). The borrower receives a greater share of the continuation value when he calls for renegotiation, which
increases his incentive to strategically default. The maximum debt repayment consistent with no strategic default is therefore smaller in the absence of CDSs. Because the effort levels implemented without CDSs and with just-insurance are the same, it follows that it is always efficient for the lender to have zero net economic ownership. CDS (just-)insurance improves debt capacity and does not cause the empty creditor problem.

If the level of effort implemented is the same with and without CDSs, then overinsurance is inefficient if the project can be financed in the absence of CDS contracts. The cause of this inefficiency is the empty creditor problem. Although CDSs increase lenders’ payoffs and debt capacity, they also bring the threat of inefficient liquidation. If a project cannot be financed with an amount of \( \pi^* \) of CDS protection, overinsurance is efficient if it allows the project to be financed.

Within this context, Table 1 depicts the efficiency role of CDS insurance; in particular, CDS overinsurance. Despite the fact that overinsurance may lead to the empty creditor problem, in equilibrium, overinsurance is more likely to emerge when the probability of investment success is high (see upper part of Table 1). In addition, CDS overinsurance helps implement the efficient level of effort (more often than not, overinsurance is associated with high effort in Table 1). Indeed, without CDSs (or when only just-insurance is allowed), high effort is only implemented for continuation values above \( \bar{y}_2 \). Recall, a concern with CDS overinsurance is that losses brought about by the empty creditor problem are increasing in continuation values. However, Table 1 shows that this effect is partially offset by the fact that effort is also increasing in continuation values, which reduce the probability of inefficient liquidation.

Finally, note that the inefficiency of empty creditors is higher when the verification cost is lower (\( \lambda \) is higher), which results in higher forgone renegotiation proceeds. However, from Proposition 10 one can see that the cutoffs \( c(p^*) \), \( c(p_H) \), and \( c(p_L) \) are increasing in \( \lambda \). This makes the equilibria depicted in the lower part Table 1 more likely to obtain, implying less overinsurance.\(^\text{14}\)

\(^\text{14}\)Although not depicted in Table 1, note that the expected inefficiency of CDS overinsurance is reduced by a better verification technology (higher \( \delta \)) and a higher recovery rate (\( \beta \)). The former implies higher verification
5.4.3 Constraints on CDS

According to the analysis of the last subsection, it is efficient for the lender to have zero net economic ownership. This result questions the reform proposals supporting that lenders’ CDS positions should be limited to positive net economic ownerships (e.g., Hu and Black (2008a,b)). Under that proposed reform, our model says that restructuring proceeds would be inefficiently reduced when zero net economic ownership is optimal.

If the level of effort implemented under both zero and negative net economic ownerships are the same, overinsurance is inefficient if the project can be financed with just-insurance. In this case, proposals to restrict net economic ownership to be non-negative would increase welfare. However, as pointed by our model, overinsurance minimizes agency problems by allowing the implementation of high effort. When this happens, the gains brought about by overinsurance in terms of higher probability of success can offset the losses caused by the empty creditor problem. Our analysis suggests that banning CDS overinsurance may thus be unwarranted.

To characterize this latter point, we need to start by considering equilibria that result in overinsurance and high effort in the absence of constraints on CDSs. These equilibria must then lead to low effort if we ban CDS overinsurance. These scenarios are described in Table 1 by the outcomes with overinsurance and continuation values above $y_2$ and below $\bar{y}_2$. Total welfare with negative net economic ownership is given by

$$W_- \equiv p_H (y_1 + y_2) + (1 - p_H) \beta I,$$  \hfill (8)

while welfare with zero net economic ownership is equal to

$$W_0 \equiv p_L (y_1 + y_2) + (1 - p_L) [1 - \delta (1 - \lambda)] y_2.$$

Since $W_- > W_0$ for $p_H$ sufficiently high, and $W_- < W_0$ for $p_H$ close to $p_L$, there exists a cutoff $p_H^* > p_L$ such that for $p_H > p_H^*$ it holds that $W_- > W_0$, and for $p_H < p_H^*$ we have costs and reduces the proceeds from renegotiation, while the latter increases the proceeds from liquidation.
$W_0 < W$. This result says that a policy to cap net economic ownership to be nonnegative can reduce welfare.

The results derived in this section are summarized in Proposition 8.

**Proposition 11.** The following holds regarding intervention and efficiency in CDS markets:

1. For continuation values below $\frac{y_2}{y_2}$ and above $\frac{y_2}{y_2}$, overinsurance (restricting net economic ownership to be nonnegative) is inefficient (efficient) if and only if the project can be financed without overinsurance. The inefficiency (efficiency) of overinsurance is increasing (decreasing) in the projects’ continuation value and decreasing in its probability of success.

2. For continuation values between $\frac{y_2}{y_2}$ and $\frac{y_2}{y_2}$, there exists a cutoff $p_H > p_L$ such that overinsurance (restricting net economic ownership to be nonnegative) is inefficient (efficient) if and only if $p_H < p_H$.

3. Just-insurance (restricting net economic ownership to be positive) is efficient (inefficient).

Proposition 11 shows that for continuation values that are either sufficiently high or small enough, CDS markets can be inefficient if they lead to overinsurance and if projects can be financed without CDSs. However, the inefficiency caused by the empty creditor problem is likely to be small in these cases. High continuation values are associated with high effort and high probability of success, which reduces the probability of default and liquidation.

For low continuation values, the inefficiency of empty creditors is reduced since forgone renegotiation proceeds under liquidation are small. Our results suggest that constraining the lender’s net economic ownership to be nonnegative need not reduce the inefficiencies caused by the empty creditor problem. In fact, when the probability of success is low, overinsurance only occurs for borrowers with low continuation values (inefficiency due to empty creditors is small). For continuation values in the intermediary range, not allowing for CDS overinsurance can be inefficient whether or not projects can be financed by overinsured creditors. Since
overinsurance minimizes the moral hazard problem and helps the implementation of high effort, it increases projects’ payoffs. This is particularly true when agency problems are severe and the state of the economy is such that projects are likely to succeed (“booms”).

To sum up, although CDS overinsurance may cause the empty creditor problem, our model shows that overinsurance is more likely to be observed when expected inefficiencies associated with empty creditors are lowest. In addition, we show that the efficient effort levels are generally induced along with overinsurance, further reducing the probability of default and inefficient liquidation. The model implies that these effects have an impact on credit availability, suggesting that they need to be more fully appreciated by researchers in the field and policymakers.

6 Empirical Implications

We dedicate this section to the discussion of model implications. We do so presenting a non-exhaustive list of testable empirical predictions, some of which are summarized in Table 1. We propose that examining these predictions would deepen our understanding of the CDS markets and their impact on corporate financing and economic efficiency.

Implication #1: The incidence of negative net economic ownership is decreasing in firm risk.

This implication follows from the fact that safe firms have high probability of success, which implies that lenders’ payoff lean more heavily on outcomes associated with investment success. As a result, the extra debt repayment extracted when the project succeeds compensates for the forgone renegotiation proceeds when the project fails.

Implication #2: Net economic ownership is countercyclical.

This implication is driven by firms for which the probability of investment success is closely tied to economic conditions (“cyclical firms”). Booms result in higher probability of success, strengthening the dependence of lenders’ payoff on success outcomes. During booms, cyclical firms perform well, resembling safer firms. Overinsurance thus becomes optimal. During busts, however, those firms display significantly higher probabilities of default, which makes
the opportunity cost of overinsurance high.

**Implication #3:** The incidence of negative net economic ownership is higher for firms with low continuation values, high liquidation values, and high renegotiation costs.

Lenders to these firms anticipate that the payoffs at stake in out-of-court renegotiations are small relative to those in court. This results arises from (either) low continuations values, costly renegotiation, or high tangibility.

**Implication #4:** Among firms with CDSs written on their debt, the probability of bankruptcy given default is decreasing in firm risk.

This implication follows from Implication #1 along with the following results: (1) negative net economic ownership leads to bankruptcy given default; and (2) zero negative net economic ownership leads to successful out-of-court renegotiation.

**Implication #5:** Firms’ probability of bankruptcy given default is higher in booms and lower in busts; i.e., the probability of bankruptcy given default is procyclical.

This implication is a corollary of Implication #4 and suggests that CDS-led bankruptcy (out-of-court restructuring) probabilities given distress are higher (lower) in booms. The opposite holds for busts. Simply put, the empty creditor problem is procyclical.

**Implication #6:** Among firms with CDSs written on their debt, the incidence of agency costs is smaller for firms with higher probability of success.

This implication follows from the fact that negative net economic ownership minimizes managers’ payoffs under default. The latter implies that managers have higher incentives to avoid default by implementing high effort, hence increasing the probability of success.

While our model’s predictions are new and have not been directly taken to the data, some reported empirical regularities are consistent with our theory. We argue, for example, that CDSs are more beneficial for firms that are safer and have higher continuation values. This result is interesting and stands in contrast to common intuition that riskier firms would benefit
the most from the existence of CDS markets. Consistent with our theory, however, studies by Ashcraft and Santos (2009) and Hirtle (2009) find that safer and larger firms have benefited the most from CDS contracts (for example, by paying lower spreads on their bank loans).

Another unintuitive prediction of our model is the procyclicality of the empty creditor problem. This implies that the conditional probability of CDS-led bankruptcy given that a firm is in distress is higher in booms. Interestingly, starting with the prior that CDSs aggravate the empty credit problems in busts — the opposite of our model’s prediction — Bedendo et al. (2016) fail to find evidence that the CDS contracting leads to a higher incidence of bankruptcies (relative to out-of-court restructurings) during the Financial Crisis.

A number of other predictions listed above can be directly taken to the data. Empirical research on CDS is still in its infancy and this strikes us as setting in which models describing rich sets of creditor–borrower relations are particularly useful in guiding empirical work.

7 Concluding Remarks

Financial innovation is the hallmark of capital markets in developed economies. The 2008–9 crisis has brought renewed interest in innovation and regulation of financial markets. A great deal of attention, in particular, has been given to CDS contracts as these derivatives seemed to play a role in the demise of numerous banks and industrial firms during the crisis. Examples range from Lehman Brothers to GM and Six Flags. While we might have observed a high degree of association between bankruptcies and CDS contracts during the crisis, in the absence of a theoretical framework it is hard to conclude that CDSs led to excessive, inefficient liquidation in that period.

We develop a model of optimal CDS contracting when investment is subject to moral hazard and wealth verification is imperfect. To our knowledge, our is the first to show how lenders choose between debt payments and restructuring proceeds — accounting for the state of the economy — when selecting the optimal amount of CDS protection. We show that CDS overin-
urance is more likely to occur in booms, when it boosts firm debt capacity and increases the number of projects with positive NPV that receive funding. CDS contracts alleviate credit rationing during recessions, but in those times CDS overinsurance may prompt the liquidation of firms with less promising prospects (firms that would likely be rationed in the absence of CDS). Our model demonstrates that the empty creditor problem is procyclical. Moreover, it shows that the casual observation that CDS contracts are associated with bankruptcies in the crisis does not imply that those contracts harm financial efficiency.

A number of recent proposals aim at limiting the amount and ownership of CDS contracts on a firm’s debt. Our paper cautions about the potential effects of these proposals on the availability of credit and on financing efficiency. Complex contracts such as CDSs are inexorably linked to the forms of financing arrangements we will be seeing in future years, as financial markets become more sophisticated and integrated. For this reason, one needs to better understand how these contracts work and the types of inefficiencies they address. In this way, one might be able to more fully benefit from what such contracts have to offer.
Appendix A

Proof of Proposition 3. Suppose \( R_s > y_2 \). In this case borrower always triggers renegotiation. The lender’s payoff is \( x(L(\pi)) - f \) if he buys a CDS with \( \pi \leq \pi^* \) and \( L(\pi) - f \) if he buys a CDS with \( \pi > \pi^* \). In the former case, since a credit event never occurs and the CDS provider is competitive, \( f = 0 \). The lender’s payoff is maximized when he chooses \( \pi = \pi^* \), which yields him a payoff of \( \delta \lambda y_2 \). In the latter case, the competitive CDS provider charges \( f = \pi \) and the lender’s payoff is \( \beta I \) regardless of his CDS position. Therefore, since \( \delta \lambda y_2 > \beta I \), the lender buys a CDS with \( \pi = \pi^* \).

Suppose \( R_s \in (\delta y_2, y_2] \). If the lender buys a CDS with \( \pi \leq \pi^* \), then

\[
R_s > \delta y_2 = \delta (1 - \lambda) y_2 + \delta \lambda y_2 = \delta (1 - \lambda) y_2 + x(L(\pi^*)) \geq \delta (1 - \lambda) y_2 + x(L(\pi)).
\]

Therefore, the borrower always triggers renegotiation. The lender’s payoff is \( \delta \lambda y_2 - f \). Because a credit event never occurs, the competitive CDS provider charges \( f = 0 \). Therefore, the lender’s payoff is \( \delta \lambda y_2 \). If the lender buys a CDS with \( \pi > \pi^* \), then since \( R_s \leq y_2 \), the borrower does not trigger strategic renegotiation. The lender’s payoff is \( [p_s R_s + (1 - p_s) (\beta I + \pi)] - f \). Since the breakeven condition for the competitive CDS provider is \( f = \pi (1 - p_s) \), the lender’s payoff is \( p_s R_s + (1 - p_s) \beta I \). Therefore, the lender buys a CDS with \( \pi > \pi^* \) if and only if

\[
p_s R_s + (1 - p_s) \beta I > \delta \lambda y_2 \iff R_s > \frac{\delta \lambda y_2 - \beta I (1 - p_s)}{p_s}.
\]

Suppose \( R_s \leq \delta y_2 \). In this case the borrower never strategically defaults. In the lender buys a CDS with \( \pi \leq \pi^* \), his payoff is \( p_s R_s + (1 - p_s) x(L(\pi)) - f \). Since a credit event never occurs, the competitive CDS provider charges \( f = 0 \). Because the lender’s payoff is increasing in \( \pi \), it is optimal to choose \( \pi = \pi^* \). Therefore, the lender’s payoff is \( p_H R_1 + (1 - p_H) \delta \lambda y_2 \). If the lender buys a CDS with \( \pi > \pi^* \), then his payoff is \( p_s R_s + (1 - p_s) (\beta I + \pi) \). Because the competitive CDS provider charges \( f = \pi (1 - p_s) \), the lender’s payoff is \( p_s R_s + (1 - p_s) \beta I \). Clearly, it is optimal for the lender to demand \( \pi = \pi^* \).

Proof of Proposition 7. For \( R_1 > y_2 \), the borrower’s payoff if he chooses high effort is given by

\[
p_H (y_1 + (1 - \delta) y_2) + (1 - p_H) (1 - \delta) y_2,
\]

whereas his payoff if he chooses low effort is given by

\[
p_L (y_1 + (1 - \delta) y_2) + (1 - p_L) (1 - \delta) y_2 + B.
\]

Therefore, the borrower chooses high effort if and only if \( y_1 (p_H - p_L) \geq B \). Suppose \( R_1 \in (\delta y_2, y_2] \). If the lender has a CDS with \( \pi > \pi^* \), the borrower’s payoff if he chooses high effort is \( p_H (y_1 - R_1 + y_2) \) and if he chooses low effort is \( p_L (y_1 - R_1 + y_2) + B \). Therefore, the borrower chooses high effort if and only if \( y_1 + y_2 - \frac{B}{p_H - p_L} \geq \)
If the lender has a CDS with $\pi \leq \pi^*$, the borrower’s payoff if he chooses high effort is

$$p_H [y_1 + (1 - \delta) y_2] + (1 - p_H) (1 - \delta) y_2.$$ 

The borrower’s payoff if he chooses low effort is

$$p_L [y_1 + (1 - \delta) y_2] + (1 - p_L) (1 - \delta) y_2 + B.$$ 

Therefore, the borrower chooses high effort if and only if

$$y_1 (p_H - p_L) \geq B.$$ 

Suppose $R_1 \leq \delta y_2$. The borrower’s payoff is he chooses high effort is

$$p_H [y_1 - R_1 + y_2] + (1 - p_H) (1 - \delta) y_2.$$ 

The borrower’s payoff is he chooses low effort is

$$p_L [y_1 - R_1 + y_2] + (1 - p_L) (1 - \delta) y_2 + B.$$ 

Therefore, the borrower chooses high effort if and only if

$$y_1 + \delta y_2 - \frac{B}{p_H - p_L} \geq R_1,$$ 

which concludes the proof.

**Proof of Proposition 8.** One must note that (1) and (3) follow directly from Propositions 4 and 5. For (2), let $R_1 \in (R(0), y_2]$. One must note that $R_1 > R(0) \geq R(\mu)$, which implies that the lender buys a CDS with $\pi > \pi^*$. If $R_1 > \overline{R_1}$, then the borrower chooses $e_L$, which implies $\mu = 0$. Therefore ($\pi > \pi^*, e = e_L, \mu = 0$) is the unique equilibrium of the CDS–Effort game, which establishes (a). If $R_1 \leq \overline{R_1}$, then the borrower chooses $e_H$ and we have $\mu = 1$. As a result, $R_1 > R(0) \geq R(1)$ and ($\pi > \pi^*, e = e_H, \mu = 1$) is the unique equilibrium of the CDS–Effort game, which establishes (b).

Let $R_1 \in (\delta y_2, R(0)]$. Suppose $R_1 > \overline{R_1}$. If $\mu$ is such that $R_1 > R(\mu)$, the lender chooses $\pi > \pi^*$. The borrower chooses $e_L$, which implies $\mu = 0$. But then we have a contradiction since $R_1 \leq R(0)$ and choosing $\pi = \pi^*$ is optimal for the lender. If $\mu$ is such that $R_1 \leq R(\mu)$, the lender chooses $\pi = \pi^*$. The borrower chooses $e_L$, which implies $\mu = 0$. Therefore, $R_1 \leq R(\mu) \leq R(0)$ and ($\pi = \pi^*, e = e_L, \mu = 0$) is the unique equilibrium of the CDS–Effort game.

Suppose $R_1 \leq \overline{R_1}$. If $\mu$ is such that $R_1 > R(\mu)$, the lender chooses $\pi > \pi^*$. The borrower chooses $e_H$, which implies $\mu = 1$. Therefore, $R_1 > R(\mu) \geq R(1)$ and ($\pi > \pi^*, e = e_H, \mu = 1$)
is an equilibrium. If $\mu$ is such that $R_1 \leq R(\mu)$, the lender chooses $\pi = \pi^*$. As a consequence, the borrower chooses $e_L$, which implies $\mu = 0$. Therefore, $R_1 \leq R(\mu) \leq R(0)$ and $(\pi = \pi^*, e = e_L, \mu = 0)$ is an equilibrium. \qed

Proof of Proposition 9. Suppose $R_1$ is such that $R_1 \leq \delta y_2$. Without loss of generality, let $R_1 \in [0, \delta y_2]$. If the lender chooses $R_1 = R_1^1$, then the borrower chooses $e_H$, the lender buys a CDS with $\pi = \pi^*$, and consistency of beliefs implies $\mu = 1$. We know that the borrower does not call for strategic renegotiation. The lender’s expected payoff is $p_H R_1 + (1 - p_H) \delta y_2$. If the lender chooses $\delta y_2 = R_1$, the borrower chooses $e_L$, the lender buys a CDS with $\pi = \pi^*$, and consistency of beliefs implies $\mu = 0$. We know that the borrower does not strategically default. The lender’s expected payoff is $p_L R_1 + (1 - p_L) \delta y_2$. The lender chooses $R_1 = R_1^1$ if and only if $y_2 \geq \overline{y}_2 \equiv -\frac{\mu}{\delta(p_H - p_L)[1 - \lambda]} \Delta$.

Suppose he chooses $R_1 \in (\delta y_2, y_2]$. If the lender chooses $R_1 = R_1^1$, then the borrower chooses $e_H$, the lender buys a CDS with $\pi > \pi^*$, and consistency of beliefs implies $\mu = 1$. The borrower does not strategically default. The lender’s expected payoff is $p_H R_1 + (1 - p_H) \beta I$. If the lender chooses $R_1 = y_2$, then the borrower chooses $e_L$, the lender buys a CDS with $\pi > \pi^*$, and consistency of beliefs implies $\mu = 0$. The borrower does not call for strategic renegotiation. The lender expected payoff is $p_L y_2 + (1 - p_L) \beta I$. The lender chooses $R_1 = R_1^1$ if and only if $p_H R_1 + (1 - p_H) \beta I \geq p_L y_2 + (1 - p_L) \beta I \iff y_2 \geq \overline{y}_2 \equiv -\frac{\mu}{p_H - p_L} \Delta + \beta I$. If $R_1 > y_2$ then the borrower chooses $e_L$, the lender buys a CDS with $\pi = \pi^*$, and consistency of beliefs implies $\mu = 0$. The borrower triggers strategic renegotiation. The lender’s payoff is $\delta \lambda y_2$. \qed

Proof of Proposition 10. We have max $\{ \Pi(y_2) : \Pi(\overline{R}_1) \} = \Pi(y_2)$ and max $\{ \Pi(\delta y_2) : \Pi(\overline{R}_1) \} = \Pi(\delta y_2)$ for $y_2 < \overline{y}_2$. Since $\Pi(y_2) > \Pi(\delta y_2)$ for $y_2$ small, there exists $y_2^*$ such that $\Pi(y_2^*) = \Pi(\delta y_2)$ if and only if the slope of $\delta y_2$ is higher than that of $\Pi(y_2)$. This is true if and only if $c(p_L) > p_L$. In this case, direct calculations show that $y_2^* < y_2$ if and only if the liquidation value is sufficiently small, i.e., $\beta I < \frac{\mu}{(p_H - p_L)[1 - c(p_H)]} \Delta$. We also have that max $\{ \Pi(y_2) : \Pi(\overline{R}_1) \} = \Pi(\overline{R}_1)$ and max $\{ \Pi(\delta y_2) : \Pi(\overline{R}_1) \} = \Pi(\delta y_2)$ for $y_2 \leq \delta y_2$. Since $\Pi(\delta y_2) > \Pi(\overline{R}_1)$ for $y_2$ small, there exists $y_2^*$ such that $\Pi(\delta y_2^*) = \Pi(\overline{R}_1)$ if and only if the slope of $\Pi(\overline{R}_1)$ is higher than that of $\Pi(\delta y_2)$. This is true if and only if $p_H > c(p_L)$. Finally, we have that max $\{ \Pi(y_2) : \Pi(\overline{R}_1) \} = \Pi(\overline{R}_1)$ and max $\{ \Pi(\delta y_2) : \Pi(\overline{R}_1) \} = \Pi(\overline{R}_1)$ for $y_2 \geq \overline{y}_2$. Since $\Pi(\overline{R}_1) > \Pi(R_1)$ for $y_2$ small, there exists $y_2^*$ such that $\Pi(\overline{R}_1) > \Pi(R_1)$ if and only if the slope of $\Pi(R_1)$ is higher than that of $\Pi(\overline{R}_1)$. This is true if and only if $c(p_H) > p_H$.

Case 1 ($p_H > p_L \geq c(p_H) > c(p_L)$): In this case, the lender chooses $R_1 = y_2$ for $y_2 < y_2$ and $R_1 = \overline{R}_1$ for $y_2 \geq \overline{y}_2$. It must follow that $R_1 = \overline{R}_1$ for $\overline{y}_2 \leq y_2 < \overline{y}_2$. To see this, suppose otherwise, i.e., $y_2^*$ is such that either $y_2^* \geq \overline{y}_2$ or $y_2 < y_2^* < \overline{y}_2$. If the latter holds then, because $\Pi(y_2) > \Pi(\delta y_2)$ for all $y_2$, we have $\Pi(y_2) > \Pi(\delta y_2) > \Pi(\overline{R}_1)$ for $y_2 < y_2 < y_2^*$. But this contradicts the definition of $y_2$. If the former holds then, because $\Pi(\overline{R}_1) > \Pi(R_1)$ for all $y_2$, we have $\Pi(\delta y_2) > \Pi(\overline{R}_1) > \Pi(R_1)$ for $\overline{y}_2 \leq y_2 \leq y_2^*$. This contradicts the definition of $\overline{y}_2$.

Case 2 ($p_H \geq c(p_H) > p_L \geq c(p_L)$): The analysis is the same as in case Case 1.
Case 3 \( (p_H \geq c(p_H) > c(p_L) > p_L) \): In this case, the lender chooses \( R_1 = \overline{R}_1 \) for \( y_2 \geq \overline{y}_2 \). There are two cases to consider: (1) \( y_2^* \geq \overline{y}_2 \) and (2) \( y_2^* < \overline{y}_2 \). If (1) holds, then we are back to Cases 1 and 2 as \( R_1 = y_2 \) for \( y_2 < \overline{y}_2 \) and \( R_1 = \overline{R}_1 \) for \( y_2 \leq y_2 < \overline{y}_2 \). The former follows by assumption since \( \Pi (y_2) > \Pi (\delta y_2) \) for all \( y_2 \) such that \( y_2 < \overline{y}_2 \), which establishes. For the latter, suppose otherwise, i.e., \( y_2^* > \overline{y}_2 \). First, let \( y_2 < y_2^* < \overline{y}_2 \). If \( y_2^* \) is such that \( y_2^* \leq y_2^* < y_2^* \), then we have \( \Pi (y_2) \geq \Pi (\delta y_2) > \Pi (\overline{R}_1) \) for \( y_2 < y_2 < y_2^* \), which contradicts the definition of \( y_2^* \). If \( y_2 \geq y_2^* \), then we have \( \Pi (y_2) > \Pi (\delta y_2) > \Pi (\overline{R}_1) \) for \( y_2 < y_2 < y_2^* \), which also contradicts the definition of \( y_2^* \). Second, let \( y_2^* \geq \overline{y}_2 \). In this case, because \( \Pi (y_2) > \Pi (\delta y_2) \) for all \( y_2 \), we have \( \Pi (\delta y_2) \geq \Pi (\overline{R}_1) > \Pi (\overline{R}_1) \) for \( \overline{y}_2 \leq y_2 \leq y_2^* \), which contradicts the definition of \( y_2^* \).

If (2) holds, then \( R_1 = y_2 \) for \( y_2 < y_2^* \) and \( R_1 = \delta y_2 \) for \( y_2^* \leq y_2 < y_2^* \). This implies \( y_2 < y_2^* < \overline{y}_2 \) such that \( R_1 = \delta y_2 \) for \( y_2 \leq y_2 < y_2^* \) and \( R_1 = \overline{R}_1 \) for \( y_2^* \leq y_2 < \overline{y}_2 \). To see this, suppose otherwise. i.e., either \( y_2^* \geq \overline{y}_2 \) or \( y_2^* < \overline{y}_2 \). If \( y_2^* \geq \overline{y}_2 \) then, since \( \Pi (\overline{R}_1) > \Pi (\overline{R}_1) \) for all \( y_2 \), we have that \( \Pi (\delta y_2) \geq \Pi (\overline{R}_1) > \Pi (\overline{R}_1) \) for \( \overline{y}_2 \leq y_2 \leq y_2^* \), which contradicts the definition of \( y_2^* \). If \( y_2^* < \overline{y}_2 \), then we have that \( \Pi (\overline{R}_1) > \Pi (\delta y_2) \Pi (y_2) \) for \( y_2^* < y_2 < \overline{y}_2 \), which contradicts the definition of \( y_2^* \).

Case 4 \( (c(p_H) > c(p_L) \geq p_H > p_L) \): In this case, the lender chooses \( R_1 = \delta y_2 \) for \( y_2 \leq y_2 < \overline{y}_2 \). This implies that \( y_2^* \leq \overline{y}_2 \). To see this, suppose otherwise, i.e., \( y_2^* > \overline{y}_2 \). Since \( \Pi (\delta y_2) > \Pi (\overline{R}_1) \) for all \( y_2 \), we have that \( \Pi (\delta y_2) > \Pi (\overline{R}_1) \Pi (\overline{R}_1) \) for \( \overline{y}_2 \leq y_2 < y_2^* \). But this contradicts the definition of \( y_2^* \). Therefore, it follows that \( y_2^* \leq \overline{y}_2 \), which implies that \( R_1 = \overline{R}_1 \) for \( y_2 \geq \overline{y}_2 \). Finally, it must be that \( y_2^* < \overline{y}_2 \). To see this, suppose otherwise, i.e., \( y_2^* \geq \overline{y}_2 \). If \( y_2^* > \overline{y}_2 \), then we have that \( \Pi (y_2) \geq \Pi (\overline{R}_1) > \Pi (\overline{R}_1) \) for \( y_2 \leq y_2 \leq y_2^* \), which contradicts the definition of \( y_2^* \). If \( y_2^* \geq \overline{y}_2 \), then \( \Pi (y_2) \geq \Pi (\delta y_2) \Pi (\overline{R}_1) \) for \( \overline{y}_2 \leq y_2 < y_2 \), which also contradicts the definition of \( y_2^* \).

Case 5 \( (c(p_H) > p_H > p_L \geq c(p_L)) \): This case is impossible. To see this note that, by the definition of \( c(p) \), \( p \geq c(p) \) if and only if \( p \geq \delta \lambda + \delta (1 - \lambda) p \), which is true if and only if \( p \geq \frac{\delta \lambda}{1 - \delta (1 - \lambda)} > p_H \). But this implies that \( p_L \geq \frac{\delta \lambda}{1 - \delta (1 - \lambda)} > p_H \), which contradicts the assumption that \( p_H > p_L \).

Case 6 \( (c(p_H) > p_H > c(p_L) > p_L) \): There are two cases to consider: (1) \( y_2^* \geq \overline{y}_2 \) and (2) \( y_2^* < \overline{y}_2 \). If (1) holds, then from Case 3 we know that \( R_1 = \overline{R}_1 \) for \( y_2^* \leq \overline{y}_2 \) and \( R_1 = \overline{R}_1 \) for \( y_2 \leq \overline{y}_2 \). It also follows that \( y_2^* \geq \overline{y}_2 \). To see this, suppose otherwise, i.e., \( y_2^* < \overline{y}_2 \). If \( y_2^* \geq \overline{y}_2 \), then we have that \( \Pi (\overline{R}_1) \geq \Pi (\overline{R}_1) > \Pi (\delta y_2) \) for \( y_2 \leq y_2^* < \overline{y}_2 \), which contradicts the definition of \( y_2^* \). If \( y_2^* < \overline{y}_2 \), then \( \Pi (\overline{R}_1) > \Pi (\overline{R}_1) > \Pi (\delta y_2) \) for \( y_2 \leq y_2 < \overline{y}_2 \), which also contradicts the definition of \( y_2^* \).

If Case 2 holds, then from Case 3 we know that \( R_1 = \overline{R}_1 \) for \( y_2 < \overline{y}_2 \) and \( R_1 = \delta y_2 \) for \( y_2 < \overline{y}_2 \). If \( y_2^* \geq \overline{y}_2 \), then the lender chooses \( R_1 = \delta y_2 \) for \( y_2 \leq \overline{y}_2 \). This implies that we are back to Case 4, from which it follows that \( y_2^* \leq \overline{y}_2 \) and the lender chooses \( R_1 = \overline{R}_1 \) for \( y_2 \geq \overline{y}_2 \). If \( y_2^* < \overline{y}_2 \), then it must be that \( \overline{y}_2 < y_2^* \). To see this, suppose otherwise, i.e., \( y_2^* \leq \overline{y}_2 \). For \( y_2^* > \overline{y}_2 \), it follows that \( \Pi (\overline{R}_1) > \Pi (\delta y_2) > \Pi (y_2) \) for \( y_2^* \leq y_2 < \overline{y}_2 \), which
contradicts the definition of $y_2$. For $y_2^* \leq y_2^*$, it follows that $\Pi(R_1) > \Pi(\delta y_2) > \Pi(y_2)$ for $y_2^* < y_2 < y_2$, which also contradicts the definition of $y_2$. Therefore, the lender chooses $R_1 = \delta y_2$ for $y_2 \leq y_2 < y_2^*$ and $R_1 = R_2$ for $y_2^* \leq y_2 < y_2$. Finally, it follows that $y_2^* \geq y_2$. To see this suppose otherwise, i.e., $y_2^* < y_2$. If $y_2^* \geq y_2^*$, then we have that $\Pi(R_1) > \Pi(R_2) > \Pi(\delta y_2)$ for $y_2^* \leq y_2 < y_2^*$, which contradicts the definition of $y_2$. If $y_2^* < y_2^*$, then we have that $\Pi(R_1) > \Pi(R_2) \geq \Pi(\delta y_2)$ for $y_2^* \leq y_2 < y_2$, which also contradicts the definition of $y_2$. \qed
References


