The evolution of low-mass close binary systems with a compact component

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II.1. SPIN EVOLUTION OF MAGNETIZED NEUTRON STARS IN LOW-MASS CLOSE BINARIES.
Spin evolution of neutron stars in low-mass close binaries.

E.H.P. Pylyser

Summary:

We present the results of calculations simulating the evolution of the spin period of neutron stars in low-mass X-ray binaries. This evolution depends on the time histories of the magnetic field of the neutron star, and of the mass transfer rate in the binary. We assumed that the magnetic field of the neutron star decays on a timescale of about \(10^7\) yrs until a transition- (or 'bottom-') field is reached. Subsequently, the magnetic field is assumed to decay on a much longer timescale. The evolution of the mass transfer rates in low-mass interacting close binaries was obtained from numerical calculations.

Based on the observed characteristics of some galactic low-mass binary radio pulsars and low-mass X-ray binary systems, we discuss the rotational status of the neutron stars in both types of systems and we make an attempt to reconstruct their rotational histories. Model calculations indicate that the neutron star in PSR 1855+09 has accreted at least 0.04 \(M_\odot\), and that the initial mass of the white dwarf progenitor in this system was about 1.0 to 1.4 \(M_\odot\). We suggest that the neutron star in PSR 1831-00 has not been accreting mass since its formation, and present an alternative formation-scenario for this system, involving an "evaporation"-process for the low-mass companion. Finally, we briefly discuss the present rotational state of the neutron star in some low-mass X-ray binaries.
1. **Introduction**

According to their observed characteristics, galactic X-ray sources can be divided into several subgroups, one of which is that of the low-mass X-ray binaries (LMXBs). These systems consist of a low-mass star, which fills its Roche-lobe, accompanied by a magnetized neutron star (NS). The mass lost by the low-mass component is (partly) accreted by the neutron star.

One aspect of accretion of mass onto a NS is the production of energetic X-radiation. The X-ray characteristics observed in this type of systems depend on the magnetic field of the neutron star. If X-ray pulsations are observed (the system is then a binary X-ray pulsar), the magnetic field of the NS is believed to be strong (i.e. \( > 10^{12} \) Gauss) whereas, when X-ray bursts are observed (the system is then an X-ray burster), the field is believed to be much weaker (i.e. \( < 10^{11} \) Gauss). X-ray pulsations and X-ray bursts are mutually exclusive phenomena (for a review, see Joss and Rappaport, 1984). In many of these X-ray sources, quasi-periodic oscillations have been observed, which in most of the theoretical models (see Lewin, 1986; Lewin et al., 1988) invoke neutron stars with magnetic field strengths of \( 10^9 \) to \( 10^{10} \) Gauss.

Furthermore, in all these systems, with the accretion of matter, also angular momentum is accreted; depending on the strength of the magnetic field of the NS and on the mass-accretion rate, this can result in a significant variation of the spin period of the NS. Depending on both the magnetic field strength of the neutron star and the mass-accretion rate, spin-up or spin-down may occur during an accretion phase. Spin periods of the order of milliseconds can thereby be attained. During non-accreting phases, the NS always spins down due to magnetic dipole radiation (Pacini, 1967).

Evolutionary scenarios leading to the formation of LMXBs and describing their observational properties have received a lot of attention in the past decade. Pylyser and Savonije (1988a,b; see also references therein) have presented the results of extensive numerical calculations describing the possible binary parameters of the progenitor- and remnant-systems of LMXBs. Based on that study, the LMXBs can be divided into two groups, depending on whether the progenitor
systems started mass transfer when their orbital period was approximately $> 12$ hours or $< 12$ hours. The orbital evolution of the former group of LMXBs is determined by the nuclear evolution of the mass-losing component, which leads to the formation of LMXBs with increasing orbital periods (i.e. diverging systems; Webbink et al., 1983; Pylyser and Savonije, 1988a (PS88a) and further references therein). The orbital evolution of the second group of LMXBs is determined by angular-momentum losses, due to gravitational radiation (Landau and Lifschitz, 1959; Paczynski, 1967; Faulkner, 1971) and magnetic braking (Huang, 1966; Verbunt and Zwaan, 1981), which leads to the formation of LMXBs with decreasing orbital periods (i.e. converging systems; Rappaport et al., 1983; Pylyser and Savonije, 1988b (PS88b) and references therein).

Four LMXBs show regular X-ray pulsations, i.e. 4U1627-67 ($P=7.7$ sec), 1E2259+59 ($P=6.98$ sec), Her X-1 ($P=1.24$ sec) and GX1+4 ($P=107$ sec) (see table 1a). It is therefore likely that the neutron stars in these systems all have a (still) strong magnetic field (i.e. $B > 10^{12}$ Gauss), and if one assumes that the magnetic field of a neutron star decays on a timescale $10^6$ to $10^7$ years (e.g. Stollman, 1987), these neutron stars must be relatively young ($< a$ few times $10^7$ yrs). The first two systems have short orbital periods ($P < 1h$), while the latter two systems (will ultimately) evolve with increasing orbital periods. Such diverging LMXBs are the progenitor-systems of low-mass binary radio pulsars (LMBRPs), which mostly consist of a low-mass ($< 0.45 M_\odot$) Helium white dwarf (WD) and a (relatively) rapidly rotating magnetized NS (Rappaport and Joss, 1983; Paczynski, 1983; Savonije, 1983 and the review by van den Heuvel, 1987). If the low mass companion is a WD, this star is thought to be the remnant of the (sub)giant progenitor, which started mass-transfer (case B or possibly AB (see PS88a)) towards a compact object (a WD or a NS).

If the compact accreting component was initially a massive WD ($> 1.2 M_\odot$; Hernanz et al., 1988), it is possible that, as it was accreting matter, it has ultimately collapsed into a neutron star when its mass reached the Chandrasekhar mass ($\sim 1.4 M_\odot$; Nomoto, 1987; Hernanz et al., 1988 and references therein). In such a case, the sudden mass
### Table 1a

<table>
<thead>
<tr>
<th>System</th>
<th>$P_{\text{spin}}$ Sec</th>
<th>$\tau_{\text{obs}}$ yr</th>
<th>log B</th>
<th>$P_{\text{orb}}$</th>
<th>$f(M_1/M_2)$</th>
<th>likely $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Her X-1</td>
<td>1.2 $-$ 3. 10$^5$</td>
<td>3. - 5. 10$^{11}$</td>
<td>1.71 d</td>
<td>?</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>GX 1+4</td>
<td>107 $-$ 50</td>
<td>?</td>
<td>&gt; 10 d</td>
<td>?</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>4U1627-67</td>
<td>7.7 $-$ 5.5 10$^3$</td>
<td>?</td>
<td>41.3 min</td>
<td>1.3 10$^{-6}$</td>
<td>0.02 - 0.1</td>
<td></td>
</tr>
<tr>
<td>1E2259+59</td>
<td>7.0 + 3-5 10$^5$</td>
<td>few 10$^{11}$</td>
<td>38.0 min</td>
<td>0.008</td>
<td>0.02 - 0.4</td>
<td></td>
</tr>
</tbody>
</table>

### Table 1b

<table>
<thead>
<tr>
<th>PSR</th>
<th>$P_{\text{spin}}$ ms</th>
<th>log $P_{\text{obs}}$</th>
<th>log B</th>
<th>$P_{\text{orb}}$</th>
<th>$f(M_1/M_2)$</th>
<th>likely $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0820+02</td>
<td>865</td>
<td>-16.0</td>
<td>11.5</td>
<td>1232.4</td>
<td>0.003</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>1831-01</td>
<td>521</td>
<td>-17.0</td>
<td>10.9</td>
<td>1.81</td>
<td>0.00012</td>
<td>0.06-0.13</td>
</tr>
<tr>
<td>1855+09</td>
<td>5.36</td>
<td>-19.7</td>
<td>8.52</td>
<td>12.33</td>
<td>0.0052</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>1953+29</td>
<td>6.1</td>
<td>-19.5</td>
<td>8.65</td>
<td>117.35</td>
<td>0.0027</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>1957+20</td>
<td>1.6</td>
<td>?</td>
<td>7</td>
<td>0.38</td>
<td>?</td>
<td>few 0.01</td>
</tr>
</tbody>
</table>

- $P_{\text{spin}}$: spin period of the NS; a "-", "+" following the spin period indicates "spin-up", "spin down", respectively
- $P_{\text{orb}}$: orbital period of the system
- $P_{\text{obs}}$: the observed variation of the spin period; $\tau_{\text{obs}} = P_{\text{spin}}/P_{\text{obs}}$
- B: the derived magnetic field strength
- $M_1, M_2$: mass of the compact object and companion, respectively
- $f(M_1/M_2)$: the observed mass-function of the system, i.e.

$$f(M_1/M_2) = \frac{(M_2 \sin i)^3}{a_1^2} = \frac{(a_1 \sin i)^3}{(M_1 + M_2)^2 G P_{\text{orb}}^2},$$

where $i$ is the inclination angle between the plane of the orbit and the plane of the sky, $a_1$ is the semi-major axis and $G$ is the gravitational constant.

Loss from the system, in the form of loss of binding energy of the collapsing compact object and possibly some expelled material, results in an enlargement of the binary system and the system becomes detached (see e.g. van den Heuvel and Habets, 1984). Mass transfer resumes when the expanding radius of the hydrogen shell-burning secondary reaches the Roche-lobe again, and the system can be observed as a wide bright LMXB (Webbink et al., 1983). Mass transfer finally ceases when the hydrogen-rich envelope of the (sub)giant is exhausted. Then, a LMBRF is born.
Up to now, five galactic LMBRPs have been discovered: PSR 1831-00, PSR 0820-02, PSR 1953+29, PSR 1855+09 and PSR 1957+20, the last three being millisecond radio pulsars. Three more such systems have recently been discovered in globular clusters (Lyne et al., 1988). The radio pulses originating from these objects yield the neutron star's rotation period. Table 1b presents the relevant system parameters of the galactic LMBRPs. The evolutionary history of PSR 1957+20 is probably very different from the other LMBRPs. We refer to e.g. Fruchter et al. (1988), Phinney et al. (1988) and Kluzniak et al. (1988) for a detailed description of this system and to van den Heuvel and van Paradijs (1988) and Ruderman et al. (1988) for the processes which are likely to be involved in its formation and future evolution (see also section 2.5 for a short description).

The aim of this work is to investigate, by means of semi-analytical calculations, the evolution of the spin period of a magnetized NS in a LMXB with a realistic mass-transfer profile. A comparison can be made between the calculated spin-period of the neutron star when the mass-transfer phase in a LMXB terminates, and the observed pulse periods of LMBRPs, which we assumed to be their remnant-systems. Similar calculations, in which the mass-losing component is a giant, were performed by de Kool and van Paradijs (1987; KP).

Section 2 describes the assumptions underlying our calculations. The theoretical basis for a description of the evolution of the spin period of both an accreting and non-accreting NS is briefly summarized in sections 2.1 and 2.2. Current ideas about the evolution of neutron star magnetic fields and their importance in the context of the spin-evolution of accreting neutron stars are briefly reviewed (section 2.3). We also discuss (and reconsider) the evolution of the mass-transfer rates in LMXBs (section 2.4). In section 2.5, we discuss the effects of the process of "evaporation" of the low mass secondary by the impinging energetic radiation of a non-accreting millisecond neutron star, and its effects on the evolution of the binary system. Section 3 presents an examination of the effects of the physical processes (and the parameters present in the modelation of these processes) on the evolution of the spin period of an accreting NS. Thereby, we extend and generalize the work performed by KP. In a first approach, the discussions in section 3
are qualitative. Some quantitative results are presented and compared with those obtained in previous work in section 3.5. In section 4, we try to reconstruct the rotational history of some LMBRsPs (section 4.1) and discuss the observed status of some LMXB-pulsars (section 4.2). Section 5 summarizes the obtained results.

2. Assumptions.

2.1. Spin evolution of an accreting magnetized neutron star.

Mass transferred through the inner Langrangian point of an interacting binary is expected to form an accretion disk around the neutron star (see e.g. Petterson, 1983; Henrichs, 1983). After leaving the donor star matter falls in at the outer edge of the disk, where a shock-front is created (the "hot spot"), and subsequently flows inwards as a result of the local viscosity. Matter at the inner boundary of the disk interacts and gets coupled to the magnetic field of the neutron star and then falls down along the field lines towards the neutron star surface. If a significant field is present, the interaction region between the field and the disk is instrumental for the transfer of angular momentum from the inner boundary of the disk towards the magnetic field. The torque which results from this interaction is transferred by the field to the neutron star causing the spin period of the neutron star to change.

The calculations presented in section 3 are based on a refined accretion-disk model as developed by Ghosh and Lamb (1978, 1979), which includes magnetic and viscous effects in the disk-field interaction. Ghosh and Lamb find that the magnetic coupling between the star and the material outside the inner edge of the accretion disk can be appreciable, i.e. the interaction region is not infinitely thin. The spin-up torque exerted on rapidly rotating neutron stars is therefore substantially lower than that on more slowly rotating neutron stars and spin-down of the neutron star, even while accretion is still continuing, is possible for high rotation rates and low accretion rates.

Largely based on the presentation given by de Kool and van Paradijs
(1987), we briefly summarize the above refined disk model. The calculated torque is expressed as a function of the 'fastness parameter' $\omega_s$, defined as the ratio of the angular velocity of the neutron star $\omega_*$ and the Keplerian angular velocity at the magnetospheric radius $r_m$:

$$\omega_s = \frac{\Omega_s}{\Omega_K(r_m)} = 1.44 \times 10^{-3} \times B_9^{-6/7} \times M_8^{-3/7} \times M_*^{-5/7} \times 1/P$$  \hspace{1cm} (1)$$

Here, $r_m$ is the radius of the magnetospheric boundary, $B_9$ is the magnetic field strength of the neutron star in units of $10^9$ Gauss, $M_8$ is the mass-accretion rate in units of $10^{-8} M_\odot/yr$, $M_*$ is the mass of the neutron star in solar masses, and $P$ is the rotation period of the neutron star in seconds.

The evolution of the NS can be qualitatively understood by considering the evolution of the relative positions of the magnetospheric radius $r_m$ and the co-rotation radius $r_{co}$, at which the Keplerian and stellar angular velocities are equal.

In the more simple theoretical approximations of the description of the interaction between the accretion disk and the magnetic field (i.e. Pringle and Rees, 1972; Davidson and Ostriker, 1973; Savonije, 1978), equilibrium is reached when $r_m = r_{co}$. If, for example, due to a decay of the magnetic field, or to an increase of the mass-accretion rate, $r_m$ decreases and becomes $< r_{co}$, the neutron star will be spun up by the particles at radii in the range $r_m$ to $r_{co}$, which have angular velocity $\omega(r) > \omega_* = \omega_{co}$, until (due to spin-up) the decreasing radius $r_{co}$ becomes equal to $r_m$, again. If this occurs on a timescale shorter than the timescales governing the variation of $B$ and/or the accretion rate, the NS will continue to rotate near equilibrium. If, on the other hand, the variation of the magnetic field strength occurs too rapid to allow the NS to adjust its spin-period to the continuously varying situation, the spin-period of the NS will depart from equilibrium.

Although the theory of Ghosh and Lamb (1978, 1979) is more refined, the relative evolution of $r_m$ and $r_{co}$ determines the spin evolution of the neutron star in a similar way.
The net accretion torque in the theory of Ghosh and Lamb is given by:

\[ N = N_0 \cdot n(\omega_s), \]

where \( N_0 \) is the angular-momentum flux carried by matter flowing through the inner boundary of the disk at radius \( r_m \). If \( j_X \) is the specific angular momentum of matter in a Keplerian orbit at radius \( r_m \) then

\[ N_0 = j_X(r_m) \cdot \dot{M} \]

\( n(\omega_s) \) is determined by Ghosh and Lamb as:

\[ n(\omega_s) = 1.39 \cdot (1 - \omega_s) \cdot (4.03 \cdot (1 - \omega_s) \cdot 173 - .878) / (1 - \omega_s) \]

(2)

We notice that the dimensionless torque \( n(\omega_s) \) becomes zero when \( \omega_s \) approaches 0.35. The star is then said to rotate at its equilibrium period \( P_{\text{eq}} \) for the present values of the physical parameters. \( P_{\text{eq}} \) can be expressed as:

\[ P_{\text{eq}} = (2.7 \text{ ms}) \cdot B_9^{6/7} \cdot \left( \frac{M}{M_{\text{edd}}} \right)^{-3/7} \cdot M_*^{-5/7} \]

(3)

\( (\text{Ghosh and Lamb, 1979; Henrichs, 1983; Stollman, 1987}). \)

\( M_{\text{edd}} \) is the Eddington limiting accretion rate for neutron stars, i.e.

\( \dot{M} = (1.5 \ 10^{-8} \ M_\odot /\text{yr}) \cdot M_* \). The equilibrium period is (becomes) thus shorter for higher (increasing) mass-accretion rates and neutron star masses, and for weaker (decreasing) magnetic fields.

It is important to notice that although the neutron star is spinning at \( P_{\text{eq}} \), mass-accretion may still be continuing. Since most of the parameters which determine \( P_{\text{eq}} \) (i.e. \( B_9 \), \( \dot{M} \) and \( M_* \)) evolve with time, \( P_{\text{eq}} \) is also continuously changing. So, although the neutron star is spinning at its equilibrium period, \( P \) may be different from zero.

Assuming a constant moment of inertia of the accreting neutron star (i.e. \( 10^{45} \ \text{gcm}^2 \)), the variation in the spin period is determined by:

\[ P = (-1.054 \ 10^{-10} \ \text{s/yr}) \cdot B_9^2 \cdot n(\omega_s) / (\omega_s^2 \cdot M_*) \]

(4)
The timescale for spin-up, defined as the time necessary to spin-up a neutron star from rest \( (\omega_s = 0) \) to its equilibrium period (at which \( n(\omega_s) = 0 \), i.e. no torque is acting on the neutron star) is:

\[
\tau_{su} = \frac{\Omega_{eq}}{Q(\omega_s=0)} = (3.43 \times 10^6 \text{ yrs}) \times B_9^{-8/7} \times M_8^{-3/7} \times M_*^{-5/7}
\]

For decreasing values of all the determining physical quantities, the spin-up timescale increases. The thus defined spin-up timescale \( \tau_{su} \) only indicates how rapidly the spin period evolves towards its equilibrium period \( P_{eq} \). The observed time scale of spin-variation \( \tau_{obs} \) will therefore only be of the same order of \( \tau_{su} \), if the rotation period of the NS is significantly different from its equilibrium period. If the NS rotates at (or near) equilibrium, \( \tau_{obs} \approx \frac{\tau_p}{P} \neq \tau_{su} \). For example, a NS rotating at \( P_{eq} \), with a field which does not decay and accreting mass at a rate which varies on a timescale \( \tau_{ev} \), defined as \( M/M \) yields \( \tau_{obs} \approx \frac{\tau_p}{P} \tau_{ev} \) (Henrichs, 1983).

In the case of very low magnetic field strengths, the magnetic boundary can become smaller than the radius of the neutron star (we assume \( R_* = 1 \times 10^6 \text{ cm.} \)). In such a case the model of Ghosh and Lamb is no longer valid. We then use a description (also used by de Kool and van Paradijs (1987)), as follows:

\[
N = j_K(R_*) \times \dot{M}
\]

and

\[
\dot{P} = (-4.35 \times 10^{-5} \text{ s/yr}) \times P^2 \times M_8
\]

Equation 3 shows that the rotation rate of an accreting magnetized neutron star can easily evolve into the range of milliseconds, as is confirmed by the observation of several millisecond binary radio pulsars (Table 1b), which form a later evolutionary phase of the low-mass X-ray binaries.
Finally, we note that according to, for example, Papaloizou and Pringle (1978), Harding (1983) and Shapiro et al. (1983), the neutron star becomes unstable to non-radial oscillations whenever its rotation period is shorter than about 1.5 milliseconds (this actually depends on the equation of state which is used). In such a case, spin angular momentum is effectively radiated away in the form of gravitational waves. The spin period does then not decrease any further.

2.2. Spin evolution of non-accreting magnetized neutron stars.

We assume that, when the neutron star does not accrete, it loses angular momentum due to magnetic dipole radiation (Pacini, 1967). When the system ends its life as a LMXB, i.e. after the contraction of the remnant of a mass-losing (sub)giant, due to the exhaustion of its hydrogen-envelope (PS88a, and references therein), the system detaches and a LMBRP is formed.

Another non-accretion phase may occur after an AIC of a white dwarf to a NS (see section 1). Mass is then suddenly lost from the system which results in an increase of the binary separation; the system becomes detached (see e.g. van den Heuvel and Habets, 1984). During some time the neutron star spins down and the magnetic field of the newly formed neutron star will decay. Mass transfer resumes when the expanding radius of the hydrogen shell-burning secondary reaches the Roche-lobe again, after which the system is observed as a LMXB.

Finally, we indicate that the top of the period gap (at 3 hours) in the distribution of orbital periods of cataclysmic variables and LMXBs may be produced by termination of the mass transfer phase in these systems. We refer to e.g. Pylyser and Savonije (1988b; and references therein) for a discussion on the formation of this period gap. During the subsequent detached phase, the system could be visible as a low-mass binary radio pulsar, until mass transfer resumes, due to orbital angular momentum loss resulting from gravitational radiation (but, see also van den Heuvel and van Paradijs, 1988).

For the calculations in section 3, we use the description of spin-
down of non-accreting neutron stars as presented by Stollman (1987). Starting the calculations with a spin period $P_0$ and a magnetic field strength $B_0$, the change in spin period is given by:

$$\dot{P} = \beta/\tau_D \cdot \exp(-2 \cdot t/\tau_D)/(1-\exp(-2 \cdot t/\tau_D)+P_0^2/\beta^2)^{1/2}$$  

(6)

where $\beta^2 = B_0^2 \cdot \tau_D \cdot \alpha$

and $\alpha$ is defined as: $B(t)^2 = \alpha \cdot P(t) \cdot \dot{P}(t)$

$\tau_D$ and $\dot{P}(t)$ are the decay timescale of the magnetic field and the spin period derivative at time $t$, respectively. A canonical value for $\alpha = 1 \cdot 10^{39}$ Gauss$^2$/sec (Manchester and Taylor, 1977).

Summarizing sections 2.1 and 2.2, the spin evolution of a neutron star in a low-mass close binary in which the NS was formed by an accretion induced collapse (AIC) can be divided into three subsequent phases:

1) a spin-down phase due to magnetic dipole radiation during the post-AIC phase until mass-accretion resumes.
2) a spin-up and/or spin-down phase during the subsequent mass-transfer phase between both components
3) a spin-down phase, by magnetic dipole radiation, after the mass-transfer phase, as a low-mass binary radio pulsar.

2.3. Evolution of the magnetic field of the neutron star.

The evolution of the magnetic field of neutron stars in low-mass binary systems has recently been the subject of much discussion. Based on studies of newborn radio pulsars (Lyne et al., 1985; Stollman, 1987; and references therein), neutron star magnetic field strengths have been found to decay on time scales of $10^6$-7 yrs. Whether this decay timescale remains constant when the field decays or varies with the age of the neutron star cannot be inferred from these single radio pulsars, since their radio luminosity decreases with time, finally rendering them
unobservable (Taylor and Stinebring, 1986). However, a comparison of the formation rates of low-mass X-ray binaries and their probable remnant systems, i.e. the wide binary radio pulsars provides strong evidence for the existence of a "bottom" value for the magnetic fields of neutron stars in low-mass binary systems (Bhattacharya and Srinivasan, 1986; van den Heuvel et al., 1986). Direct observational support for the existence of a "bottom-field" for the magnetic fields of neutron stars is provided by the high age of the white dwarf companions of e.g. PSR 0655+64 (Kulkarni, 1986, 1987) and PSR 1855+09 (Wright and Loh, 1986).

Furthermore, the observed values for the spin period and its derivative in the system PSR 1831-00 indicate that its "age" (i.e. $P/P$) is of the order of a few times $10^9$ yrs, while its derived magnetic field strength is still $8 \times 10^{10}$ Gauss (see also section 4.1.2). These observations indicate that the bottom-field may be different in each system.

Van den Heuvel (1986) reviewed two possible interpretations of this phenomenon. In one of these (Kulkarni, 1986, 1987), the magnetic field of the neutron star possibly consists of two distinct fields: a rapidly decaying ($\tau_B \sim 5 \times 10^6$ yrs), strong surface field, which is anchored in the crust of the neutron star (the "crustal" field), and a much weaker component seated in the superconducting interior which decays very slowly or perhaps not within the age of the Galaxy (the "core" field). Depending on the age of the NS, the observed magnetic field is then either the "crustal" or the "core" field.

Another interpretation is based on a model of non-decaying neutron star magnetic fields (i.e. Kundt, 1986; Blair and Candy, 1988) and on the specific binary nature of the systems: the fields of (single) neutron stars do not decay, but are presumably either weakened by the occurrence of mass-accretion during the previous mass-transfer phase (Blonding and Freese, 1986) or, independent of the circumstances, remain constant (see also Sang and Chanmugam, 1988; Kundt, 1986). The observed torque-decay of single pulsars could then possibly be explained by the occurrence of the process of alignment of the rotation-axis and the field-axis (see Lyne et al., 1988). Although no definitive observational distinction between both interpretations could be made up to now, de Kool and van Paradijs (KP) adopted the first interpretation in their calculations of the evolution of the spin period of accreting neutron stars. The calculations presented in section 3 are based on the
2.4. The evolution of the mass-accretion rate onto the neutron stars.

The spin period calculations presented in this work differ from those presented by KP in the adopted evolution of the accretion rates. KP determined the accretion rates semi-analytically based on a model for bright low-mass X-ray sources (see Webbink et al., 1983). This model satisfactorily describes the evolution of wide low-mass galactic X-ray binaries and their observational properties, and explains naturally the formation of binary radio pulsars as their remnants (Rappaport and Joss, 1983; Paczynski, 1983 and Savonije, 1983, 1987). However, this semi-analytical approximation is strictly valid only for mass losing giants which developed He-cores of mass between about 0.20 and 0.45 $M_\odot$ (i.e. where hydrogen is burned in a thin shell) at the onset of mass-transfer. It is not valid for (sub)giants which start mass transfer shortly after the core hydrogen-burning phase (i.e. while a thick hydrogen burning shell develops; see PS88a; Côté and Pilyser, 1988). In PS88a, a series of numerical calculations of (sub)giants starting mass transfer between the end of the core hydrogen-burning phase (or even earlier) and the base of the giant branch are presented. For all these numerical calculations, the mass transfer rates were stored as a function of time and of a few additional system-parameters (see also Côté and Pilyser, 1988), one of which is the mass of the donor star (see for example fig. 8a in section 3.3.4). These numerically obtained mass transfer rates will be used as accretion rates onto the neutron star in section 3, in order to follow the spin evolution of an accreting neutron star. Unless explicitly mentioned, we will assume that all mass lost by the (sub)giant will be accreted by the neutron star (i.e. we make the so-called conservative assumptions).

It is important to note that any numerically obtained history of mass transfer rate is dependent on the mass of the accompanying compact component. In many of the calculations presented in section 3, we have included the initial mass of the NS as a free parameter. The chosen mass of the NS may be different from that used by PS88a. In section 3.4, we discuss the effects of this difference on the evolution of the spin.
period of the NS and use a simple description to correct the numerically obtained mass transfer rates whenever such a correction is thought to be necessary.

Finally, we emphasize that, although the mass-accretion rates may be varying relatively rapidly (see later, figure 8a), the mass-accretion rates considered here are mean rates on timescales of at least 1. to $10^6$ yrs. Fluctuations over timescales of the order $10^2$ to $10^3$ yrs are only considered qualitatively (section 4). It can not be excluded that instabilities in either the mass transfer rate (dictated by the evolution of the low-mass companion) or the accretion rate (dictated by the evolution of the disk) even quench the accretion of mass onto the NS, interrupting the X-ray phase. We shall demonstrate that such variations on short timescales can have important consequences.

2.5. The process of evaporation of a low-mass companion by a non-accreting, rapidly rotating neutron star.

In the previous section, we mentioned that the spin-period of an accreting NS can relatively easily decrease to a few milliseconds. It can be shown that, if for one of the reasons presented above (sections 2.2 and 2.4), mass-accretion stops, the outflowing radiation of a millisecond neutron star is so strong, that even a fraction of the impinging energy can be sufficient to result in the evaporation of (part of) the hydrogen-envelope of the orbiting low-mass companion (Ruderman et al., 1988).

The recently observed LMBRP, PSR 1957+20 (Fruchter et al., 1988) is a perfect example, which formed the inspiration for the above process of evaporation (van den Heuvel and van Paradijs, 1988; Phinney et al., 1988; Kluzniak et al., 1988). Whenever the process of evaporation is started, it can not be interrupted. If the evaporation timescale $\tau_{\text{evap}}$ is much shorter than the spin-down timescale $\tau_{\text{sd}}$, the low-mass secondary will be completely evaporated before the spin period increased significantly. If $\tau_{\text{evap}} > \tau_{\text{sd}}$, the evaporation process will terminate due to the decrease in energy outflow of the neutron star. If, after the termination of the evaporation process, the low-mass companion still has a significantly massive remnant hydrogen envelope ($M_{\text{env}}$ at least about a
few 0.01 $M_\odot$), mass transfer towards the neutron star can resume, and mass accretion can again spin up the neutron star.

It can be shown that the latter process of evaporation is of significant importance in the estimate of the lifetime and birthrate of LMXBs (Côté and Pylyser, 1988).

3. Computational results.

3.1. Introduction and working scheme.

In the following sub-sections, we present the results of calculations of the evolution of the spin period of accreting neutron stars in wide low-mass X-ray binaries. For this purpose, we create various neutron star accretion situations in which the different parameters present in the magnetic field decay model and the mass-accretion sequences are varied. We discuss the essential effects of these variations on the spin period evolution of the accreting neutron star. Table 2 lists the parameters and the ranges in which they will be varied.

Table 2

<table>
<thead>
<tr>
<th>Par.</th>
<th>range</th>
<th>units</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>$1. \times 10^{12}$ - $1. \times 10^{13}$</td>
<td>Gauss</td>
<td>1</td>
</tr>
<tr>
<td>$\tau_{B1}$</td>
<td>$1. \times 10^6$ - $1. \times 10^7$</td>
<td>yrs</td>
<td>2</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$1. \times 10^8$ - $1. \times 10^9$</td>
<td>Gauss</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_{B2}$</td>
<td>$1. \times 10^{10}$ - $\infty$</td>
<td>yrs</td>
<td>4</td>
</tr>
<tr>
<td>$t_{det}$</td>
<td>$5. \times 10^6$ - $5. \times 10^7$</td>
<td>yrs</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta M$</td>
<td>$0.001$ - $0.5$</td>
<td>$M_\odot$</td>
<td>6</td>
</tr>
<tr>
<td>$M_{NS}$</td>
<td>$0.6$ - $1.6$</td>
<td>$M_\odot$</td>
<td>7</td>
</tr>
<tr>
<td>$M$</td>
<td>$1. \times 10^{-7}$ - $1. \times 10^{-11}$</td>
<td>$M_\odot$/yr</td>
<td>8</td>
</tr>
</tbody>
</table>

$B_1$ and $B_2$ are the initial "crustal" and "core" field, respectively. $\tau_{B1}$ and $\tau_{B2}$ are the decay timescales of $B_1$ and $B_2$, respectively. $t_{det}$ is the duration of the detached phase after the assumed AIC. $M_{NS}$ is the initial neutron star mass, i.e. before accretion and $\Delta M$ is the total amount of mass to be accreted by the neutron star. $M$ is the mass-transfer rate between the components, used as the accretion rate onto the neutron star.

We will adopt the idea that the NS is formed by an AIC, followed by

References
1) Flowers and Ruderman, 1977
2) Lyne et al., 1985
3) Bhattacharya and Srinivasan, 1986
4) Sutanto and van den Heuvel, 1985
5) Horne et al., 1986
6) Pylyser and Savonije, 1988
a detached phase. The process of evaporation of the secondary, as discussed in section 2.5 is not included in the calculations and discussions presented in this section.

All the calculations are performed according to the following scheme. First, we choose values for $B_1$, $B_2$, $\tau_{B1}$, $\tau_{B2}$, $M_{NS}$, $\Delta M$, and the duration $t_{det}$ of the detached phase after the assumed AIC. We also select a mass-transfer sequence describing the evolution of an interacting binary system evolving towards a LMBRP (as presented by PS88a).

The initial spin period of the neutron star after the AIC is always chosen to be 50 ms (we refer to KP for arguments).

The accretion sequence is started at the point in the mass-transfer phase where the donor star mass $M_d$ is equal to the mass of its final WD-remnant after the end of the mass-transfer phase augmented with $\Delta M$. As an example: in sequence G25 (see PS88a and figure 8a), the mass of the WD after the mass-transfer phase is 0.24 $M_\odot$. If the chosen total amount of mass to be accreted by the NS is 0.5 $M_\odot$, the accretion sequence is started at the point of the mass-transfer phase where the donor star has a mass of 0.74 $M_\odot$, using the corresponding stored mass-transfer rate as an accretion rate.

3.2. Qualitative description of the evolution of the spin period and the relation to the underlying physical processes.

Figure 1 presents, for some selected sequences, the spin behaviour of an accreting NS as a function of time in terms of $\omega_s$, the fastness parameter (fig. 1a), and $P$, the spin period (fig. 1b).

The evolution of the field and/or mass-accretion rate in these calculations are chosen somewhat arbitrarily, but with the aim to understand and demonstrate the effects of the physical processes determinating the spin-evolution of the neutron star. Table 3 lists the assumptions about the evolution of the magnetic field and the accretion rate in the calculations presented in this sub-section. In all of them, the initial mass of the NS is assumed to be 1.3 $M_\odot$, its initial spin period is 50 ms, and the total amount of mass accreted by the NS is assumed to be 0.25 $M_\odot$. In order to examine the effects of the early
Figures 1a,b: The evolution of the fastness parameter $\omega_s$ (upper part) and the corresponding evolution of the spin period of the accreting neutron star (lower part) as a function of the time elapsed since the onset of mass-accretion, for some selected sequences. The assumptions about the evolution of the magnetic field strength and the mass-accretion rate for each sequence are listed in table 3. The open dots along the curves indicate when the (eventually) decaying magnetic field has reached its bottom-value. Whenever $\omega_s > 0.35$, the neutron star spins down, while spin up of the neutron star is characterized by $\omega_s < 0.35$. 

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**Figure 1c**: The evolution of some of the characteristic timescales during the accretion phase in sequence BSML. $\tau_p$ is defined as $P/P$, $\tau_{ev}$ as $M/M$ and $\tau_{su}$ in section 2.1 (eqn. (5)). $\tau_{p}$ is displayed in dashed form, when the NS spins down. $\tau_{B1}$ is the decay timescale of the "crustal" field.

The assumptions for the sequence labeled BCMC are very simple, i.e. both the field strength and the mass-accretion rates are constant ($B = 4 \times 10^8$ G; $\dot{M} = 1 \times 10^{-10}$ $M_\odot$/yr, respectively). In this case, the (assumed) initial spin period of 50 ms is long with respect to the equilibrium period ($P_{eq} = 9.8$ ms, according to eqn. (3)). The torque on the NS is positive; after a few times $10^8$ yrs, the equilibrium period
<table>
<thead>
<tr>
<th>Seq.</th>
<th>Assumed evolution of the B-field</th>
<th>Assumed evolution of the accretion-rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCM</td>
<td>constant: i.e. B=4. $10^8$ Gauss</td>
<td>constant: i.e. M=1. $10^{-10}$ $\dot{M}_\odot$/yr</td>
</tr>
<tr>
<td></td>
<td></td>
<td>----------------------------------------</td>
</tr>
<tr>
<td>BSMC</td>
<td>&quot;standard&quot;: as in BCM (section 2.3)</td>
<td>constant: as in BCM</td>
</tr>
<tr>
<td></td>
<td>$B_1$=3. $10^{12}$ G</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_{B1}$=5.3 $10^6$ yr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_2$=4. $10^8$ G</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_{B2}$=1. $10^{10}$ yr</td>
<td></td>
</tr>
<tr>
<td>BSML</td>
<td>&quot;standard&quot;: linearly decreasing from 5.5 $10^{-10}$ to 1. $10^{-10}$ $\dot{M}_\odot$/yr as a function of the mass of the donor star</td>
<td></td>
</tr>
<tr>
<td></td>
<td>as in BSMC</td>
<td></td>
</tr>
<tr>
<td>BCMS</td>
<td>constant: &quot;standard&quot;: Sequence G25, numerically calculated sequence of mass-transfer rates (obtained from PS88a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>as in BCM</td>
<td></td>
</tr>
</tbody>
</table>

is reached (i.e. $\omega_s = 0.35$; the torque $n(\omega_s) = 0.$) and since neither the field nor the accretion rate vary (the mass of the NS increases only marginally during the accretion phase), equilibrium is maintained until mass-accretion stops. The final equilibrium period is 9.8 ms.

Sequence BSMC demonstrates the effects of the evolution of the magnetic field. The mass-accretion rate is again kept constant, but the magnetic field (of initially 3. $10^{12}$ Gauss) is assumed to decay on a timescale of 5.3 $10^6$ yrs until a bottom field of 4. $10^8$ Gauss is attained (see section 2.3).

This sequence is similar to the sequences presented by KP (1987): after the short detached phase, the NS is spun down to its equilibrium period (according to eqn. (3), $P_{eq}$ is 17.4 sec) on a timescale shorter than 1. $10^4$ yrs (i.e. $\tau_{su}$; see figure 1c). Subsequently, the NS spins at its equilibrium period, which decreases on a timescale $\tau_{B1}$ ($\tau_p = \tau_{B1}$; figure 1c), due to the field decay. After a few times $10^7$ yrs, the NS has increasing difficulty to maintain its rotation period at the rapidly decreasing equilibrium period. The fastness parameter $\omega_s$ starts decreasing, thereby increasing the torque on the NS. When finally, the spin-up time scale $\tau_{su}$ becomes comparable to or even exceeds the decay-
timescale $\tau_{B_1}$, the NS can no longer rotate at its equilibrium period, until the decaying field approaches its "bottom"-value (i.e. $B_2$). Once the field has stopped decaying, $\omega_s$ increases again and the NS evolves towards its (new) equilibrium period, now determined by the bottom strength of the field and the mass-accretion rate (i.e. $P_{eq} = 9.8$ ms). From the above it appears that the NS rotates out of equilibrium as long as its magnetic field decays approximately between a few times $10^{10}$ Gauss and its bottom-value. The subsequent spin-evolution is qualitatively similar to the evolution described in sequence BCMC, since from now on, the (core) field is assumed to be constant.

We note that the evolution through the minimum value of $\omega_s$ is less discontinuous than in the calculations of KP. This is due to our more smooth approximation of the transition phase from a rapidly decaying crustal field to the almost constant core field (see also the discussion of fig. 12).

In sequence BSML, the assumptions about the field evolution are identical to those in sequence BSMC; however, the mass-accretion rate decreases linearly as a function of the decreasing mass of the donor star, from about $5.5 \times 10^{-10} M_{\odot}/\text{yr}$ at the initiation of mass transfer to $1.0 \times 10^{-10} M_{\odot}/\text{yr}$ at the end of the mass transfer phase. Since the mass-transfer rate in sequence BSML is at any time higher than in sequence BSMC, the torque necessary to maintain the NS at its equilibrium period before the effects of field decay become important, is sufficiently strong during a longer period of time than in sequence BSMC. Figure 1c shows that until then, the evolution of the spin period is dictated by the decay timescale $\tau_{B_1}$ of the magnetic field. The subsequent evolution away from equilibrium is as in sequence BSMC. Due to the higher mass-accretion rates, the minimum value of $\omega_s$ is slightly higher. The evolution towards equilibrium is comparable to the first two sequences until equilibrium is approached and $B = B_2$ is constant (see fig. 1c). Equation (3) shows that, subsequently, the equilibrium period increases due to the decrease of the mass-accretion rate. Although the NS is in fact rotating at its equilibrium spin-period, a continuous negative torque is acting on the NS ($n(\omega_s) < 0.$ and $\omega_s > 0.35$) and the NS spins down as long as $M$ decreases. In such a situation, the observed spin-down timescale $\tau_{obs}$ (i.e. $\tau_p$) is determined by the decrease of the mass-accretion rate and is of the order of $\tau_{ev}$ (see fig. 1c).
The evolution of a sequence BCMS, in which the field is again kept constant (at $4 \times 10^8$ Gauss) and in which the sequence describing the mass-accretion rate originates from the numerical calculations performed by PS88a (i.e. G25), is almost similar to the evolution of sequence BCMC. A discussion of some specific characteristics of the mass transfer rates is given below, where fig. 8a is presented. We note here the wavy character of the evolution of the fastness parameter $\omega_s$ and the spin period at the end of the mass-accretion phase, which reflects the wavy character of the evolution of the mass-transfer rate itself. A decreasing mass-transfer rate produces an increasing $\omega_s$ (and thus decreasing torque) and inversely. The final strong increase of the fastness parameter corresponds with the termination of the mass-transfer phase, the mass-accretion rate rapidly fading away. As a result, the NS ultimately spins down.

Figures 2a, b and 2c present the evolution of a "standard" model: the assumptions regarding the evolution of the magnetic field are as in sequence BSMC (or BSML) and regarding the mass-transfer rates as in BCMS. All the calculations performed in the following subsections will be qualitatively similar to one another. As such, the evolution of the spin-period of the NS, described in figure 2 is qualitatively a combination of the sequences BSMC and BCMS as described above: the initial and final evolution are identical to the initial evolution of sequence BSMC and the final evolution of system BCMS, respectively.

We notice the existence of two little dips in the curve describing $\omega_s$ (see the arrows). These are caused by variations in the rate of change of the mass-accretion rate. We will come back to this phenomenon in a discussion on the spin evolution of the NS in Her X-1 (section 4.2).

In all cases presented so far, the magnetic field did not decay significantly (anymore) during the final phase of mass-accretion.

In some cases, however, the amount of mass which is accreted by the neutron star is sufficiently low and/or the accretion rate is sufficiently high, that mass-accretion ends before the magnetic field has decayed significantly. Figure 3, curves (a), (b) and (c) present the
Figures 2a, b and c: The evolution of the numerically computed mass-transfer rate in sequence G25 (see PS88a), as a function of the time elapsed since mass-accretion started. In this case, 0.5 $M_\odot$ is accreted by the NS. Some specific characteristics of the evolution of the mass transfer rate are discussed later (fig. 8a). Assuming a "standard" evolution for the magnetic field B (see section 2.3), the corresponding evolution of the fastness-parameter and the spin period, during the accretion phase are shown in figures 2b and 2c. Due to the wavy character of the evolution of the mass-accretion rate, the final evolution of the spin period of the NS is characterized by alternating spin-up and spin-down phases.

Figures 3a, b and c: The evolution with time of the mass-accretion rate, the fastness parameter and the spin period, respectively, for a case in which the magnetic field of the NS has not yet decayed significantly. Only 0.005 $M_\odot$ was accreted at a relatively high rate (sequence I25), in order to terminate mass transfer before significant decay of the field could occur. The rapidly decreasing mass-accretion rate, which causes a rapidly increasing equilibrium period, results in a rapid spin down phase at the end of the accretion phase. During this final phase, the NS is rotating at equilibrium.
evolution of such a case. They depict the evolution of the mass-accretion rate, the fastness parameter and the spin-period, respectively, as a function of time elapsed since the AIC. The total amount of mass to be accreted onto the 1.3 $M_\odot$ neutron star is in this case 0.005 $M_\odot$, and the mass-transfer sequence used describes the evolution of sequence I25 in PS88a (see also figure 8a). The initial evolution is similar to the spin evolution of sequence BSMC (or BSML) as described above. However, the accretion phase lasts only about $4 \times 10^6$ yrs. Before the field has decayed significantly, the mass-accretion rate rapidly decreases at the end of the transfer phase. The field can thereby be regarded as constant, and the spin-evolution is qualitatively similar to the final part of the evolution of, for example, sequence BSML, in which the neutron star spins down due to the (relatively) rapidly decreasing mass-accretion rate. During the whole final accretion phase, $\tau_{ev} \gg \tau_{su}$, showing that, although $M$ varies relatively rapidly, the NS is rotating in equilibrium.

Evolutionary sequences intermediate between, for example, BSML and the last presented sequence (i.e. in which mass-accretion terminates when the NS is rotating out of equilibrium, due to the rapid decay of the magnetic field) are of course also possible, depending on the choice of the parameters. In Section 3.3.5, we present such intermediate cases.

3.3. Effects of the variation of the free parameters.

In order to examine the influence of the values of the free parameters used in these calculations, we present sets of accretion-sequences, in which the parameter-values are varied separately in a range dictated by results in previous work (see section 3.1).

To provide a comparison, we define the following list of "standard"-values: $B_1 = 3 \times 10^{12}$ Gauss

$B_2 = 4 \times 10^8$ Gauss

$\tau_{B1} = 5.3 \times 10^6$ yrs.

$\tau_{B2} > 10^{10}$ yrs.

$t_{det} = 1.07$ yrs.

$M_* = 1.3 M_\odot$

$P_{spin,i} = 50$ ms

$10^2$
3.3.1. Variation of $B_1$, $T_{\text{det}}$ and $t_{\text{det}}$

A study of the influence of the different values of the parameters $B_1$ and $\tau_{B_1}$ on the evolution of the spin-period of an accreting NS, is only relevant as long as $B = B_1 \gg B_2$. Therefore, we ascertained that in all cases presented in this sub-section, the strength of the magnetic field $B_1$ was always $\gg B_2$ at the end of the mass transfer phase (as it was the case in the final sequence discussed in section 3.2.).

\[ \text{Figure 4:} \text{ The evolution of an accreting NS in the spin period-magnetic field plane for a different initial "crustal" field strength. The field was assumed to decay on a timescale of } 5.3 \times 10^6 \text{ yrs. In all cases, the mass of the NS was assumed to be } 1.3 \, M_\odot \text{ and } 0.005 \, M_\odot \text{ was accreted at a rate determined by sequence G25 (see also fig. 8a). The detached phase, due to the AIC lasted } 1.10^7 \text{ yrs. } P_i \text{ and } P_f \text{ are the initial and final spin period of the NS, respectively. } P_{\text{det}} \text{ is the spin period at the end of the detached phase. The magnetic field strengths are denoted in a similar way.} \]

First, we examine the effects of different values of the initial field strengths $B_1 (= B_1^*)$. In all sequences, after a detached phase of $\sim 1.10^7$ yrs, $0.005 \, M_\odot$ was transferred towards a $1.3 \, M_\odot$ neutron star, using mass-transfer sequence G25 (PS88a). Figure 4 presents the various relations that exist between the spin period of the NS and the magnetic field strength $B$ during the evolution. The final spin-period $P_f$ is given as a function of $B_1$ and $B_f$, which are the initial and final strength of the magnetic field, respectively. $P_{\text{det}}(B_{\text{det}})$ relates the period and the magnetic field at the end of the detached phase. The initial period $P_i$, equal to 50 ms in each calculated sequence, as a function of $B_1$ is labeled $P_i(B_1)$. Dashed curves present (a part of) the spin-evolution as a function of the magnetic field strength in some of the cases. The spin period of the NS starts at the relation $P_i(B_1)$.
evolves towards the relation \( P_{\text{det}}(B_{\text{det}}) \) during the detached phase, after which accretion begins. On a short timescale (< \( 10^5 \) yrs) the neutron star evolves towards its equilibrium period (by spinning down). Once equilibrium is achieved, it is maintained while the NS spins up. After a final spin-down phase due to the rapidly decreasing mass-accretion rate at the end of the accretion phase, the final configuration of the NS is described by the relation \( P_{\text{f}}(B_{\text{f}}) \).

For the neutron stars with the strongest fields, the spin-period variations at the onset of accretion are the strongest, as is to be expected from the expression for the equilibrium period (eqn. (3)).

![Graph showing spin period vs. magnetic field strength](image)

**Figure 5:** As in figure 4, but assuming a different decay timescale for the "crustal" field. The initial strength of the "crustal" field was assumed to be \( 3 \times 10^{12} \) Gauss.

The influence of the different values assumed for the decay time scale \( \tau_{B_1} \) of the magnetic field \( B_1 \) on the spin evolution of the NS is illustrated in figure 5. It depicts the evolution of the spin-period as a function of the magnetic field strength for different decay time scales.
The spin period and magnetic field strength at the end of the detached phase (i.e. $P_{\text{det}}$ and $B_{\text{det}}$) and at the end of the accretion phase ($P_f$ and $B_f$) are given. Since, for the longer decay time scales, the fields are systematically stronger than in case of shorter time scales, variations of the spin period at the onset of accretion are the strongest in case of the longest decay-timescales. For decay timescales $\tau_{B_1} < 4 \times 10^6$ yrs, the spin period at the end of the detached phase is longer than the corresponding equilibrium period when accretion begins, and the NS starts spinning up.

The qualitative effect of varying the duration of the detached phase is very similar to that of the variation of, e.g. the decay timescale, since the duration of the detached phase as well as the decay timescale, both only determine the field strength at the onset of accretion. Therefore, we did not perform separate calculations in which the duration of the detached phase is varied.

3.3.2 Variation of $B_2$ and $\tau_{B_2}$.

Arguments have been presented against a significant decay of the core field $B_2$ (see section 2.1). We therefore assume that $\tau_{B_2}$ is at least $10^{10}$ yrs, and have not made calculations in which this parameter is varied.

The variation of the final spin period as a function of the value of the core field strength is given in figure 6a. The assumed values of the other parameters are indicated in the figure. Figure 6b depicts the evolution of the spin period of the NS as a function of the remnant mass of the donor star. At the left hand axis of each curve, the value of $B_2$ is given. The neutron star is rotating at equilibrium at the end of mass accretion phase and the final spin period is again a strongly increasing function of the magnetic field strength (see eqn (3)). In addition, one observes that variations in the mass accretion rate yield stronger variations in the spin period when the field is stronger.
Figures 6a,b: Figure 6a (upper left) presents the final spin period of the NS as a function of the assumed initial value of the core field strength. The values for the model parameters are listed in the figure. In figure 6b (lower left), the evolution of the spin period is displayed as a function of the mass of the donor star. Along each curve, the value of the assumed strength of the core field is given in units of $10^8$ Gauss.

Figures 7a,b (figures at the right side): As in figures 6a,b, respectively, but with the initial mass of the NS as a parameter.
3.3.3. Variation of the mass of the NS.

Figures 7a and 7b are very similar to figures 6a and 6b, respectively. The varied parameter, however, is now the mass of the neutron star, which is varied from 0.6 \( M_\odot \) to 1.6 \( M_\odot \). More massive neutron stars rotating at equilibrium yield shorter spin periods, which is expected from the expression of the equilibrium period (eqn(3)).

3.3.4. Variation of the mass-transfer rates.

The final phase of mass-transfer between an initially 1.5 \( M_\odot \) donor star and a 1.0 \( M_\odot \) compact object, as obtained from the numerical calculations is shown as a function of the remnant mass of the donor star in Fig. 8a. The initial evolutionary state of the 1.5 \( M_\odot \) is different in each sequence. The highest mass-transfer rates originate from the systems with the most evolved donor stars. In addition, for systems with more evolved donor stars, an extensive dip in the mass-transfer rate at the end of the mass-transfer phase becomes visible. This dip occurs, for the sequences with the more evolved donor stars, at a systematically higher mass of the donor star. This peculiarity in the evolution of the mass-transfer rate in binary systems with donor stars of similar initial mass was already observed in the early binary calculations presented by Kippenhahn et al. (1967). These authors ascribed this decrease in the mass-transfer rate to the stellar contraction due to the passage of the rising thin hydrogen burning shell through a hydrogen abundance discontinuity. Such a discontinuity in the hydrogen abundance can be built up either during the preceding phase of mixing in the outer layers, when an extensive convective envelope is formed, or during the phase of core-hydrogen burning in the convective core of a star with initial mass \( M_* > 1.2 \ M_\odot \) (Iben, 1967).

The evolution of the mass-transfer rates originating from the least evolved donor stars (i.e. sequence B25) does not present such a dip. This is due to the fact that the formation of a rising hydrogen burning shell does not occur before the end of the mass-transfer phase.

Figure 8b shows the corresponding spin-period evolution of the neutron star, assuming the "standard" parameter-values. In total, 0.5 \( M_\odot \)
was accreted by the NS in these calculations. The shortest spin periods are obtained for systems with the largest accretion rates during most of the accretion phase.

Figures 8a,b: The final evolution of the mass transfer (c.q. accretion) rate (upper part) for the different sequences in set 25 (see PS88a) and the corresponding evolution of the spin period (lower part) as a function of the mass of the donor star. The specific characteristics of the evolution of the mass transfer rate are explained in the text.
3.3.5. Variation of the total amount of mass accreted by the NS.

Finally, for the above presented set of "standard" parameter-values, we vary the amount of mass accreted by the NS. Figures 9a and 9b present the evolution of the fastness parameter and the spin period as a function of time for two different amounts of mass accreted by the NS. The mass-accretion rate is determined by sequence 125 (see fig. 8a). Figure 9a shows a sequence in which the NS had not yet reached its equilibrium spin when the accretion stopped (i.e. with $\Delta M = 0.05 \, M_\odot$).

**Figures 9a,b:** As in figs. 2b and 2c. In each sequence presented in figure 9, the total amount of mass accreted by the NS is different. One sequence presents a case for which the neutron star does not rotate at equilibrium when mass-accretion stops (i.e. when $\Delta M = 0.05 \, M_\odot$). At that time, the magnetic field has not yet reached its bottom-strength.

3.4. Corrections to the mass-transfer rates.

The working-scheme presented in section 3.1 contains implicitly an arbitrary choice of the moment at which the assumed AIC occurs during the mass-transfer phase. That moment depends on the choice of the total amount of mass to be accreted by the NS. We may, for example, as extremes, choose to transfer either 0.001 $M_\odot$ or 0.6 $M_\odot$ from the donor star. Nevertheless, in each case, a certain initial mass for the NS is chosen (e.g. 1.3 $M_\odot$). The mass transfer rates obtained from the calculations of PS88a, however, depend on the mass of the neutron star at that moment in the numerical calculations, and that mass is usually not the same as the initial mass that we have chosen to start the accretion calculations. A correction for this has to be made. It is
necessary to examine whether and to what extent the accretion calculations are sensitive to this correction. To this end, we use a prescription derived from Webbink et al. (1983) to correct our numerically obtained mass-transfer rates:

\[ \dot{M}(M_d/M_{NS}) \sim (5/6 - M_d/M_{NS})^{-1} \]

or

\[ \dot{M}(M_d,M_{NS,2}) = \dot{M}(M_d,M_{NS,1}) \times \frac{(5/6 - M_d/M_{NS,1})}{(5/6 - M_d/M_{NS,2})} \]

\[ \frac{\dot{M}, M_d, M_{NS,1} \text{ and } M_{NS,2}}{are \text{ the } mass \text{ transfer rate (depending on the masses of both components), the mass of the donor star, the mass of the neutron star in the numerical calculations and the mass of the neutron star in the calculations for the evolution of the spin period, respectively. This description is valid only for mass ratios } q = M_d/M_{NS} < 0.8, \text{ a restriction which is fulfilled in all the calculations presented in the foregoing sections. Figures 10a, b and 10c show the evolution of the mass transfer rate, the fastness parameter and the corresponding evolution of the spin period of the NS as function of time for a case, in which the magnetic field has not stopped decaying at the end of the mass transfer phase (i.e. } B = B_1). \text{ Figures 11a, b and 11c are similar, but now } B = B_2, \text{ which has stopped decaying at the end of the mass transfer phase. Both figures show the effects of the correction made} \]
Figures 11a, b and c: As in figures 2a, b and c and figures 10a, b and c respectively.

to the mass transfer rate, i.e. the dashed curves present the original numerically obtained mass transfer rates (in figures 10a and 11a) and the corresponding spin-period evolution (in figures 10c and 11c) of an initially 1.0 $M_\odot$ NS, accreting mass at an uncorrected rate. The full lines present the calculations with mass transfer rates corrected for the mass-difference of the NS.
The labels of the curves correspond with the evolutionary sequences in PS88a from which the mass-transfer rates were used (see figure 8a).

From eqn. (7), it is clear that the corrected and uncorrected mass-transfer rates become almost identical for low mass ratios $q<1$, i.e. at the very end of the mass transfer phase. This can be observed in figs. 10a and 11a. Figs. 10c and 11c show that the evolution of the spin period of the neutron star, determined by the corrected and uncorrected mass-transfer rates, closely follows the evolutionary behaviour of the mass transfer rates, i.e. the differences in the final spin period are rather insignificant. However, the evolution during the accretion phase is quantitatively different. Figure 10c, for example, shows that, as long as the neutron star spins up, the spin period of the neutron star is shortest in the case of properly corrected mass-accretion rates. Indeed, since the corrected mass-accretion rates are higher, slightly shorter spin periods are obtained (see section 3.3.4).
3.5. Some quantitative Results.

In all the following calculations, the mass-transfer rates have been corrected as described in section 3.4.

Figure 12: The relation between the final pulse period of the NS, the total amount of mass accreted by the NS and the final orbital period of the system for all sequences in set 25 (PS88a). Each point is the result of one sequence calculation. All points, for which the total amount of mass accreted by the NS was the same were connected by fully drawn curves. Each curve presents (or should present, eventually also outside of the range of orbital periods presented in this work, see KP) a minimum, which migrates to shorter orbital periods and longer spin periods for decreasing amounts of accreted mass. At the right hand side of that minimum, the magnetic field of the NS has not yet reached its bottom-strength, and possibly, the NS does not rotate in equilibrium, when mass-accretion terminates. At the left hand side of the minimum, however, the NS rotates at equilibrium, and the magnetic field of the NS has reached its bottom-strength. The positions of the galactic low-mass binary radio pulsars in this plane are indicated by a "*".
Fig. 12 presents the final spin period of the neutron star, calculated with the standard set of parameter values presented in section 3.3 and with the mass-transfer rates of figure 8a, as a function of final orbital period of the system. Each point in Figure 12 represents the outcome of one sequence calculation in which a certain amount of mass is accreted. Full curves are drawn through all points, for which a same amount of mass was accreted by the neutron star.

Figure 12 shows that with the exception of curves A, E and F, all curves have a minimum. This minimum migrates to shorter spin periods and longer orbital periods when the total amount of accreted mass increases. In fact, the same behaviour is expected for curves E and F: their minimum probably occurs at still longer orbital periods (see later). Similarly, according to the above qualitative description, curve A is expected to contain a minimum at very short final orbital periods. The mere existence of systems with such a short final orbital period is, however, highly improbable, since the sequence with the shortest orbital period (i.e. sequence B25) already presents the evolution of a binary system evolving very closely to the bifurcation between diverging and converging systems (see introduction), and starting mass transfer with still shorter initial periods would almost certainly result in the generation of a converging system (PS88a) and would not produce the wide LMBRP's studied in this section.

Figure 12 displays the same quantities as Fig. 2 in KP (1987). In fact, their figure presents the far right hand-side part, inaccessible to us, of our fig. 12 (P_{\text{orb}} > 20 \text{ d}). These authors started their calculations with a 0.8 M_\odot donor star and a 1.3 M_\odot mass-accreting neutron star. Unfortunately, no final donor star masses or total amounts of accreted mass are mentioned by KP. However, WRS (1983) and Savonije (1983, 1987) indicate that BRP's with a binary period of about 100 days (at which KP find a minimum in their curve) contain WD's (i.e. donor-remnants) of mass about 0.30 M_\odot. Thus, approximately 0.5 M_\odot was transferred between the components in most of the calculations performed by KP. The left hand part of the curve in their Fig. 2 should thus approximately match the right hand part of the F-curve in the above Fig. 12, which indeed, it does quite well.

The increase of the final spin period at higher orbital periods (in both our figure 12 and figure 2 of KP) is due to the fact that at the
end of the mass transfer phase, the "crustal" field has not yet decayed sufficiently to become insignificant (because of the much higher mass transfer rates). The total field is thus still stronger than the "core" field, which results in relatively longer spin periods, as already mentioned by KP.

The sharp transition from a slow decrease of the spin period towards a strong increase as a function of the orbital period at the right hand part of figure 2 of KP in fact corresponds to a smooth transition through a minimum, as observed in our Fig. 12. The discontinuous character of the curve in Fig. 2 of KP is due to the abruptness of the introduction of another decay timescale when the magnetic field has decayed below a value $B_{tr}$ (transition field; $B_2$ in our notation). The magnetic fields of the neutron stars in the systems at the right hand side of the discontinuity have not yet reached the transition field strength, and decay rapidly, while the fields of neutron stars at the very left of the discontinuity just reach the transition field before the end of mass transfer, and subsequently decay very slowly. In this work, however, we considered two distinct and separate fields, i.e. the "crustal" field which is decaying rapidly ($B_1$) and the "core" field which decays very slowly ($B_2$), which were simply added to form the total field. Technically, this construction yields a more smooth transition from a rapidly decaying to a very slowly decaying total field, and therefore we observe a smooth minimum (instead of a discontinuity) in Fig. 12.

Figure 12 indicates that more evolved systems (with longer orbital periods) form LMBRPs (shortly after the end of the mass transfer phase) with a great diversity in possible pulse periods, depending on the total amount of mass accreted. However, for an accreted amount of $>0.05 M_\odot$, the variation in the final spin period is relatively quite small. BRPs with very small orbital periods ($P_{\text{orb}} < 2$ days) have a relatively limited range in spin periods from about 15 ms to 100 ms.

Similar results are obtained when analogous calculations are performed with the other mass transfer sequences obtained from PS88a (i.e. diverging systems in set 17, 20 and 33).
4. Applications to observed systems.

4.1. The galactic low-mass binary radio pulsars.

Five of these systems have been observed so far. Their system characteristics are presented in Table 1b. Three of them (PSR 0820+02, PSR 1953+29 and PSR 1957+20) have orbital periods outside of the range of final orbital periods obtained in the calculations of PS88a. Therefore, no numerically obtained mass-transfer rates are available, and no quantitative discussion on the spin-evolution of these systems will be made.

On the other hand, we discuss in more detail the spin evolution of the NS in PSR 1831-00 ($P_{\text{orb}} = 1.81$ day) and PSR 1855+09 ($P_{\text{orb}} = 12.3$ days) and their formation scenario.

4.1.1. PSR 1855+09

4.1.1.1. Introduction

The circular orbit and the relatively short spin period of the NS in PSR 1855+09 (Segelstein et al., 1986; see table 1 for its observational characteristics) are definite signatures of, respectively, tidal interaction and mass transfer between both components, after the formation of the NS.

Wright and Loh (1986) found the companion of PSR 1855+09 to be a 0.3 $M_\odot$ white dwarf (WD) with a surface temperature of 5900 K, implying that this component is older than about $2 \times 10^9$ years.

A system like PSR 1855+09 fits easily in the formation scenario for LMBRP's described in section 1 (see also e.g. Rappaport and Joss, 1983; Savonije, 1983; and Paczynski, 1983), according to which these systems are the remnant-systems of bright wide low-mass X-ray binaries.

Based on a semi-analytical description of that scenario (WRS, 1983), Savonije (1987) obtained a mass of 0.22 $M_\odot$ for the WD in PSR 1855+09, and a rather low upper limit of 1.20 $M_\odot$ on the mass of the neutron star (see also Joss et al., 1987). He estimates the
mass transfer rate at the end of the mass transfer phase at about
$7 \times 10^{-10} \, M_\odot/\text{yr}$.

The numerical simulations of low-mass close binary evolution
performed by PS88a indicate that low-mass close binaries containing
radio pulsars, with orbital periods of about 12 days, are formed with
companions which have a mass of about $0.25 \, M_\odot$ (see table 1 of PS88a).
Although the latter value for the WD-mass is only marginally different
from the value derived by Savonije (1987), it yields an upper limit on
the mass of the neutron star of about $1.50 \, M_\odot$, which is in better
agreement with standard values for the masses of a NS than the upper
limit of $1.2 \, M_\odot$ derived by Savonije (1987). In the following discussions
we will use therefore these numerically derived values for the masses of
both components. From the calculations performed by PS88a, it can be
concluded that the initial period of the system was about 1.2 days.

In the following sub-sections, the possible evolution of the spin
period and the magnetic field of the NS is reconstructed from its birth
until its presently observed spin period (5.36 ms) and magnetic field
strength ($3.3 \times 10^8$ Gauss), adopting the same assumptions and working
scheme as presented in sections 2 and 3, respectively.

Constraints on the total amount of mass accreted by the neutron
star and the minimum accretion rate at the end of the mass transfer
phase are determined in section 4.1.1.2. Section 4.1.1.3 summarizes the
conclusions of section 4.1.1.

4.1.1.2. Spin evolution of the neutron star

As mentioned in section 2, neutron stars are likely to be formed
with a strong magnetic field (e.g. about $3. \times 10^{12}$ Gauss) and spinning
with a period of a few tens of milliseconds. The subsequent spin-history
of the neutron star in a low-mass binary is then as follows (section
2b): 1) a first spin-down phase due to magnetic dipole radiation during
the detached phase following the AIC; 2) the accretion phase, and 3) the
final spin-down phase, due to magnetic dipole radiation.

In the following discussions, we start from the system
configuration as observed today and make an attempt to determine the
evolution of the different system parameters backwards in time.

i) Spin-down of the NS after the end of the mass-transfer phase.

We first estimate the duration of the spin-down phase (due to magnetic dipole radiation) after the end of the mass transfer phase. By making use of the observed spin period, the estimated duration of the spin-down phase and the theoretical description of the spin-down evolution of the neutron star (section 2.2), it is then possible to determine the spin period of the neutron star at the end of the accretion phase. To do so, we estimate the age of the He-WD companion, formed at the end of the mass transfer phase.

In order to determine the age (or the cooling time) of the low-mass He-WD, we calculate its luminosity. We assume a radius for the observed He-WD (of mass $0.25 \, M_\odot$) of $0.0185 \, R_\odot$ (Hamada and Salpeter, 1961) and since its effective temperature is about $5900 \, K (\pm 1400, -1000)$ (Wright and Loh, 1986), the stellar luminosity ($\log L/L_\odot$) ranges then from at maximum $-3.05$ to at minimum $-3.75$.

From the cooling curve of a $0.3 \, M_\odot$ He-WD (Iben and Tutukov, 1986; their figure 2), the above luminosity range of the WD yields a cooling time of $1.4 \, 10^9$ to $3.2 \, 10^9$ yrs. Since the final cooling evolution of a $0.6 \, M_\odot$ WD and a $0.3 \, M_\odot$ WD (as presented by Iben and Tutukov (1986)) are already approximately the same, we do not expect the cooling phase of a $0.25 \, M_\odot$ He-WD to differ significantly from that of a $0.3 \, M_\odot$ He-WD.

During the cooling phase of the WD the accompanying neutron star is spinning down. It is not possible to determine whether or not the magnetic field of the neutron star has already reached its bottom-value, which we assume to be its presently observed field-strength, at the end of the mass-accretion phase. If it did not, it is in principle possible that mass-accretion terminated without having the neutron star rotating in equilibrium (section 3.3.5). In the following section, however, we find that, although the NS was possibly not strictly rotating with the equilibrium period at the end of the accretion phase, it was very close to, and the magnetic field of the NS must have attained its bottom-strength of $3.3 \, 10^8$ Gauss, shortly after the end of the accretion phase.

If one assumes that indeed the field had reached its bottom-value at the end of mass-accretion, a rotation period for the neutron star at
that time of about 1.5 to 4.3 ms can be expected, depending on the
duration of the spin-down phase after the end of the transfer phase and
on the assumed value of $\tau_{B2}$.

ii) Spin evolution of the NS before and during mass transfer.

In all realistic cases, i.e. those with a duration of the detached
phase ($t_{\text{det}}$) between $5 \times 10^6$ and $5 \times 10^7$ yrs (see Sutantyo and van den
Heuvel, 1985), the pulse period of the neutron star at the end of the
detached phase was between 0.57 and 0.69 sec, assuming standard values
for the initial field strength and the decay timescale.

We generate optimal conditions for spin up of the NS to
sufficiently short rotation rates by adopting weak magnetic fields
during the accretion phase (i.e. by adopting long detached phases (e.g.
$5 \times 10^7$ yrs) and long accretion phases (i.e. high $\Delta M$) and/or by adopting
a relatively weak "core" (bottom-) field (i.e. $3 \times 10^8$ G, the observed
strength of the field). We further assume that the bottom-field has a
decay time $\tau_{B2} > 10^{10}$ yrs.

We adopted the calculated mass-transfer rates (PS88a) between
components of wide low-mass binary systems for which the orbital period
at the end of the transfer phase was about 12 days, to obtain a
realistic view of both the mass-accretion rate and the spin-period
evolution of the neutron star in this system. We use the sequences of
mass transfer rates, obtained in the systems G25 and H25 of PS88a, which
have corresponding final periods of 9.68 and 15.7 days, respectively.
Interpolation between the results obtained for these systems allows a
comparison with the observed spin period of PSR 1855+09. In addition to
the mass-transfer rates obtained from the above sequences, which
originates from sub-giants that have an initial mass of 1.5 $M_\odot$, we
constructed two sequences of mass-transfer rates, i.e. X17 and X20,
which are approximate extrapolations of the mass-transfer rates obtained
from the sequences C17-D17 and C20-D20 in PS88a, respectively. The aim
of the construction of these two X-sequences is to have an idea of the
spin-evolution of the neutron star under mass-accretion from an
initially 1.0 $M_\odot$ donor star.

In all the calculations, we ascertained that the mass of the NS at
the end of the accretion phase was 1.5 $M_\odot$ and its magnetic field

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### Table 4

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- **$M_{NS}$**: the initial mass of the neutron star
- **$\Delta M$**: the amount of mass accreted by the neutron star
- **$B_{2i}$**: the initial (i) core field, the total field at the end of the accretion phase (acc), and the final (f) total field strength of the neutron star, respectively.
- **$B_{acc}$**: the spin period of the NS at the end of the accretion phase (acc) and the final (f) total field strength of the neutron star, respectively.
- **$B_{f}$**: the final (f) variation of the spin period of the NS at the end of the calculation.
- **$P_{acc}$**: the initial mass of the neutron star
- **$P_{f}$**: the final (f) variation of the spin period of the NS at the end of the accretion phase
- **$P_{f}^*$**: if $Y$: the NS rotates at equilibrium at the end of the accretion phase
- **equil.**: if $N$: the NS does not rotate at equilibrium at the end of the accretion phase

If an additional "~" indicates that either the NS has just reached its equilibrium period (if $Y$), or that the NS is not yet rotating at equilibrium, but the magnetic field has reached its bottom-value.

**G25-H25**: The presented results were obtained, after interpolation between both sequences G25 ($P_{orb}=9.7$ d) and H25 ($P_{orb}=15.7$ d), in order to model the system PSR 1855+09, which has a period of 12.3 days.
strength about $3.3 \times 10^8$ Gauss. The spin-down phase after the end of the mass-transfer phase was assumed to have lasted $1.4 \times 10^9$ yrs. Table 4 lists the obtained results.

Interpolation between the results, obtained with the use of the mass-transfer rates of sequences G25 and H25 (i.e. $M_{d,1} = 1.5 \ M_\odot$), yields pulse periods for the NS in PSR 1855+09, which are slightly too long (i.e. 5.6 ms). Since, for all the calculations presented in table 4, we have adopted the most favourable (and still acceptable) configuration of parameter-values, there is no alternative to decrease the final spin-period than to assume a higher mass-accretion rate during the accretion-phase, especially at the end of the accretion phase. The final mean mass-transfer rate in system H25, which has an orbital period of 15.7 days, is sufficient to obtain spin-periods < 5 ms, and is about $8. \times 10^{-10} \ M_\odot/yr$. The interpolated $\langle M \rangle$ (i.e. between G25 and H25) for a system with a period of 12.3 days is $5. \times 10^{-10} \ M_\odot/yr$ (see also Côté and Pylyser, 1988), and is insufficient. The minimum final mass transfer rate, necessary to spin-up the NS up to 5.3 ms (as observed) must therefore lie between 5. and $8. \times 10^{-10} \ M_\odot/yr$, in agreement with the value $7. \times 10^{-10} \ M_\odot/yr$, obtained from eqn (2) and by Savonije (1987). Such a mass-transfer rate at the end of the mass-transfer phase, occurring in a system with a final orbital period of about 12 days, can only be obtained by assuming that the initial mass of the donor star is slightly lower than the adopted 1.5 $M_\odot$.

The mass-transfer rates originating from an initially 1.0 $M_\odot$ star are higher than those obtained for an initially 1.5 $M_\odot$ star with a same final orbital period. By using the same assumptions and parameter-values as described above, we obtain much shorter final spin-periods for the NS, i.e. shorter than the observed spin period (see table 4). In fact, it is possible to relax some of the assumed parameter-values in the model calculations. From the calculations performed with the most optimal conditions, we find that at minimum, 0.04 $M_\odot$ must have been accreted by the NS in the case of sequence X17.

In all the cases for which the spin evolution of the NS could be modelled and for which more than about 0.2 $M_\odot$ was accreted, the NS was rotating at equilibrium at the end of the mass-accretion phase. For the calculations with $\Delta M < 0.2 \ M_\odot$, the spin period was only slightly out of equilibrium, which explains the slightly higher spin period of the NS.
for the cases in which only 0.1 $M_\odot$ was accreted.

4.1.1.3 Discussion and conclusions

The observed spin period and magnetic field strength of the NS in PSR 1855+09, combined with the use of up to date theoretical descriptions of the evolution of the spin period of a NS and its magnetic field allows an estimate of the range of the initial mass-values of the donor star and the amount of mass accreted by the NS. If we adopt an evolutionary scenario for the magnetic field decay as described in section 2.3, it can be shown that in case that the initial mass of the donor star was 1.0 $M_\odot$, its neutron star companion had to accrete at least some 0.04 $M_\odot$ in order to be spun-up to a rotation period as observed today. In case that the donor star was a 1.5 $M_\odot$ star, the final mean accretion rate corresponding with the final orbital period of the progenitor system of PSR 1855+09 is not sufficient to spin up the NS to its observed period. We argue that the initial mass of the donor star must have been in the range 1.0 - 1.4 $M_\odot$.

Finally, we add the following comment. As discussed in section 2.5, the process of evaporation of the secondary (van den Heuvel and van Paradijs, 1988; Ruderman and Shaham, 1988; Kluźniak et al., 1988; Phinney et al., 1988), which occurs in the vicinity of a pulsar with a high energy flux, can in some circumstances, seriously alter the "standard" evolution of LMXBs, as described in section 1. Although the presently observed characteristics of PSR 1855+09 definitely necessitate a phase of mass-transfer (with accretion of at least 0.04 $M_\odot$, see above), it cannot be excluded that prior to the final mass transfer phase, part of the secondary's hydrogen-rich envelope was evaporated.

Such a scenario is only possible if the timescale for evaporation, $\tau_{\text{evap}}$, is longer than the timescale for neutron star spin-down, $\tau_{\text{sd}}$ (see van den Heuvel and van Paradijs, 1988; section 2.5).

4.1.2. PSR 1831-00.

Dewey et al. (1986) reported the discovery of this 0.521 second
LMBRP. The WD-companion has not directly been observed, which makes an estimate of the duration of the spin-down phase after the end of the accretion phase, as done in the previous section for PSR 1855+09, impossible. However, an estimate of the most likely mass of the WD, from the pulsar's radial velocity orbit is given by Dewey et al. (1986) and yields $0.06 \text{ to } 0.13 \, M_\odot$. The spin period variation $\dot{P}$ is determined to be $1.43 \times 10^{-17} \, \text{s s}^{-1}$ (Taylor and Dewey, 1988), corresponding to a magnetic field strength of $8 \times 10^{10} \, \text{Gauss}$. The resulting age of the neutron star, defined as $P/\dot{P}$ is about $1.5 \times 10^9 \, \text{yrs}$. Thus, the observed field strength must in fact be the strength of the bottom field of the NS in this system.

i) Viability of a scenario with mass transfer and accretion.

If the system PSR 1831-00 was formed according to the "standard" scenario for LMBRPs (see also section 4.1.1), its orbital period of 1.8 days in combination with the calculations performed by PS88a indicate that the mass of the WD must then be about $0.17 \, M_\odot < M_{WD} < 0.20 \, M_\odot$ (PS88a).

The relatively short orbital period and observed low mass of the WD in this system ($< 0.17 \, M_\odot$, Dewey et al., 1986) indicate that, if mass transfer occurred, the progenitor-system has been starting mass transfer towards the NS with an initial period $P_i$ close to (but longer than) the corresponding bifurcation period $P_{\text{bif}}$ (i.e. $P_i = 0.7 \text{ to } 0.8 \, \text{days}$; see PS88a for a discussion).

The low eccentricity of the orbit ($< 0.002$) indicates that during at least some $10^7 \, \text{yrs}$, effective tidal interaction, possibly accompanied with mass transfer between both components must have occurred. An estimate of the final mass-accretion rate of $4 \times 10^{-11} \, M_\odot/\text{yr}$ on the NS in PSR 1831-00 has been made by Savonije (1987).

As for the case of PSR 1855+09, we used the mass-accretion sequences from PS88a to reconstruct the orbital period of PSR 1831-00. We examine sequence B20 ($M_{d,i} = 1.0 \, M_\odot$; $M_{a,i} = 1.0 \, M_\odot$; $P_{\text{orb,f}} = 1.72 \, \text{d}$). The mass-transfer rate at the end of this sequence is $4.5 \times 10^{-11} \, M_\odot/\text{yr}$ (in excellent agreement with Savonije, 1987).

The equilibrium period of a NS with a magnetic field $B = 8 \times 10^{10}$
Gauss and accreting at a rate of \(4 \times 10^{-11} M_\odot/\text{yr}\) is \(> 1\) second, which shows that the observed spin period of 0.521 sec. cannot be reached during a "standard", continuous mass-transfer phase, unless \(M\) was a factor 10 higher. We therefore consider the following formation scenario, in which we assume that no mass-transfer occurred and consequently no accretion on the NS took place, after its formation.

ii) Evolutionary scenario without mass-accretion onto the NS.

Very recently, it has been suggested that a process of "evaporation" of the secondary in a low-mass close binary could take place, provided that the outflowing energy flux of the NS in these systems is sufficiently strong to power this process (Ruderman et al., 1988; section 2.5). The hydrogen-rich envelope of the secondary is then lost from the system without the occurrence of mass-accretion onto the NS, and consequently without spin-up of the NS.

We examine here whether a scenario without mass-transfer in PSR 1831-00 is viable.

Let us assume that the NS is formed during the AIC of a massive WD, when the mass of the secondary is about 0.30-0.40 \(M_\odot\) and its core mass about 0.12-0.15 \(M_\odot\), respectively (which is the case in the calculations of PS88a). The binary period is then about 1.2-1.6 days, respectively and increases suddenly due to mass-loss from the system, produced by the supernova-explosion. The eccentricity of the binary is also increased. If a small amount of mass is lost from the system (i.e. about 0.3 to 0.15 \(M_\odot\), respectively), the orbital period does not increase too much (i.e. \(P_{\text{post-sn}} \sim 1.8\) days) and the secondary still fills a large part of the Roche-lobe. Tidal interaction then remains very effective. Using the description of Savonije and Papaloizou (1985) for the evolution of the eccentricity of binary systems, in which the secondary has an extended convective envelope, the timescale for circularisation is a few times \(10^7\) yrs.

If the newly formed NS rotates with a spin period \(< 10\) to 15 ms, the large energy flux of the NS may be sufficient to evaporate the remaining hydrogen-rich envelope (i.e. 0.15 to 0.25 \(M_\odot\)) on a timescale shorter than the spin-down timescale of neutron stars (see the

Once the evaporation process has terminated, a bare He-core as the remnant of the original low-mass star is left and the neutron star spins down secularly, reaching its presently observed spin-period after some $10^8$ to $10^9$ yrs.

iii) Conclusions

The observations clearly indicate that the NS in system PSR 1831-00 is old and that consequently, in our field-decay picture, the observed field strength must be the value of the bottom field. Combination with the numerical result that a system like PSR 1831-00 should not have accreted mass at a rate $> 4.5 \times 10^{-10} M_\odot/\text{yr}$ shows that the so obtained equilibrium period is longer than the observed spin period. Therefore, either the mass transfer rate was a factor of 10 higher, which would indicate that the numerical determination of $M$ in PS88a is incorrect, or the system has been formed in a different way. We point out that a scenario in which no more mass is transferred towards the NS after its formation is viable, if the envelope of the remnant of the secondary is evaporated by the energetic outflowing radiation of the NS.

4.2. The X-ray pulsars in low-mass X-ray binaries.

Four LMXB's show periodic X-ray variations (Table 1a), indicating the presence of a still strong magnetic field. In the following subsections, we briefly discuss their rotational history and their evolutionary status.

4.2.1. The long orbital period LMXBs ($P_{\text{orb}} > 1 \text{ day}$).

GX1+4 and Her X-1 are binary systems in which a (sub-)giant is (or possibly has been; in the case of GX1+4) transferring mass towards its neutron star companion.
Sutantyo et al. (1985) investigated the possible evolutionary scenarios leading to the formation of a system like Her X-1. Based on the various observational characteristics, they estimated that the NS was born some $2 \times 10^7$ yrs ago. They concluded that relatively recently, probably an AIC occurred in this system, whereby some $0.3 M_\odot$ was expelled from the system.

The about $1.9 M_\odot$ (sub)giant resumed mass transfer very recently ($< 10^5$ yrs) towards its $1.2 M_\odot$ neutron star companion (see Joss and Rappaport, 1984 for the mass-estimates and Sutantyo et al., 1985, for additional remarks).

The neutron star rotates with a spin period of 1.24 seconds (Ogelman and Truemper, 1988) and showed a steady spin-up behaviour ($\tau_{\text{obs}} = 3 \times 10^5$ yrs) until 1982. If the feature in the X-ray spectrum of Her X-1 is interpreted as a cyclotron line, the magnetic field strength of the NS is between 3. and 5. $10^{12}$ Gauss (Truemper et al., 1978; Voges et al., 1982). During 1982, observations with the Japanese satellites Hakuchou and Tenma indicated that the spin-up behaviour of Her X-1 became less evident (Nagase et al., 1984) and even reversed in 1983, i.e. the NS then spun down.

Since the mass ratio of Her X-1 is $> 1$, mass transfer occurs (or will occur, in the relatively near future) on a timescale close to the thermal timescale of the donor star, as shown in the evolutionary calculations presented by PS88a (e.g. their sequences C33 and C33"). During this initial phase of mass-transfer, the mass transfer rate may increase up to $4 \times 10^{-7} M_\odot/yr$. This indicates that the mass-transfer rate in Her X-1, which until 1982 was about $10^{-9} M_\odot/yr$ (from its X-ray luminosity of about $2 \times 10^{37}$ ergs/s, Rappaport and Joss, 1983b) is probably still increasing gradually.

Henrichs (1983) argued that the NS in Her X-1 rotates at equilibrium, and that the observed timescale of spin-up ($= \tau_{\text{obs}}$) in this system displays the spin-up behaviour of the equilibrium period, as a result of the rapidly (timescale about $10^5$ yrs) increasing mass accretion rate. (The calculations in section 3 show a qualitatively similar behaviour, although $P_{\text{eq}}$ is then decreasing due to a relatively
rapidly decreasing mass-accretion rate, during the final phase of accretion).

Since, in the case of Her X-1, $\tau_{\text{obs}} \approx 10^5$ yrs; Henrichs, 1983; Savonije, 1978) > $\tau_{\text{su}}$ (about 1 $\times$ 10$^3$ - 1 $\times$ 10$^4$ yrs; see e.g. figure 1c), it is indeed likely that the NS is rotating at equilibrium and that its spin period is determined by the evolution of the mass transfer rate, which increases on a timescale $\tau_{\text{ev}} = \tau_{\text{obs}}$. If it is rotating at equilibrium with its $M$ of about $10^{-9}$ M$_\odot$/yr, its magnetic field strength would be 3 $\times$ 10$^{11}$ Gauss. In view of the rather strong bottom-field strength of 8 $\times$ 10$^{10}$ Gauss observed in PSR 1831-00, it is not impossible that the observed field strength of the neutron star in Her X-1 is in fact the bottom-field strength for this neutron star.

In order to understand the recent spin-down behaviour of Her X-1, and assuming that the field is still decaying on a timescale $\tau_{\text{su}}$, we recall figure 2b and relate some of the characteristics found during the accretion phase after the strong field decay phase of the crustal field with similar phenomena, possibly occurring before the strong field decay phase. During the accretion phase following the rapid decay of the crustal field, the NS is rotating at equilibrium and nevertheless, slight deviations in the rate of change of the mass transfer rate (on timescales of < 10$^6$ yrs) result in a variation of $\omega_s$ (in fact $\omega_m$; see the dips, arrows in figure 2b) and thus $n(\omega_s)$. Such a variation is strong enough to result in a short-lasting spin-down phase (see for example the second dip in figure 2b) because the NS was rotating close to its equilibrium period, i.e. $n(\omega_s) = 0$. Such a behaviour may be expected whenever a NS is rotating at equilibrium, i.e. also in the phase before the NS is evolving towards the phase of rotation out of equilibrium, when the field decays rapidly. As mentioned in section 2.4, the mass transfer rates used in this work are mean mass transfer rates over 10$^6$ to 10$^7$ yrs, although the short timescale behaviour of the accretion rate onto the NS may well be erratic on a timescale $\tau_{\text{ev}} = 10^2$ to 10$^3$ yrs, < $\tau_{\text{su}}$, possibly even in the initial phase of strong increase of $\dot{M}$. We suggest that the spin-down behaviour, observed in Her X-1 is due to such erratic behavior induced by small variations in the rate of change of the mass-accretion rate.

In case the observed field of the neutron star in Her X-1 should be
its bottom field, the same scenario holds for the spin-down behaviour of Her X-1. However, the field has then already stopped decaying.

4.2.1.2 GX1+4

The binary system GX1+4 consists of a M6III symbiotic giant star orbiting a NS with an orbital period of at least 10 days (Davidson et al., 1977). Cutler et al. (1986) presented an elliptical binary model for GX1+4, based on a possible periodic enhanced spin-up behaviour of the NS in this system. They argued that the orbital period of the system, based on this assumption is about 304 days. Webbink et al. (1983) indicated that the mass transfer rate in this system is possibly about $5 \times 10^{-3} \, M_\odot/\text{yr}$. The rotation period of the NS was about 122 seconds in 1980 (or possibly twice as much; see Doty et al., 1981; Cutler et al., 1986; but Strickman et al., 1980; Koo and Haymes, 1980) and was decreasing on a rather variable timescale with a mean of about 40-50 yrs. Until then, the NS did obviously not rotate at equilibrium. In 1983, the source disappeared as an X-ray source, while also its H$\alpha$-emission vanished, possibly indicating that mass-transfer stopped (Hall and Davelaar, 1983; Whitelock et al., 1983, respectively). Shortly later, however, the H$\alpha$-emission feature was again observed (Whitlock, 1984).

Henrichs (1988; private communication) advanced the possibility that the giant in this system is a pulsating star (i.e. a Mira-variable). Between pulses, i.e. during the quiet phases, the NS spins-down, possibly aided by strong wind-accretion (compare with the spin periods of NS in massive X-ray binary systems, e.g. van Paradijs, 1983), which would explain the still relatively long spin-period of the NS in 1980, despite the subsequent accretion phase by Roche-lobe overflow. When Roche-lobe overflow resumes due to the expansion of the giant, mass-accretion guarantees spin-up towards $P_{\text{eq}}$ on a very short timescale (i.e. $\tau_{\text{su}}$).

A scenario as described above indicates that the mass of the giant is (much) smaller than that of the NS. If $q > 1$, the mass transfer through Roche-lobe overflow would in any case decrease the orbital period, and would guarantee a continuous (although possibly variable) transfer of matter on approximately a thermal timescale between both
components, whether the giant is pulsing or not. We may thus argue that the present mass of the donor star in GX1+4 is $< 1.4 \, M_\odot$, for a standard mass of the NS-companion. Since $1.0 \, M_\odot$-giants have an age close to the age of the Galaxy (see PS88a), its initial mass must have been $> 1.0 \, M_\odot$. Furthermore, as already indicated by Webbink et al (1983), since $\tau_{\text{SU}}$ is possibly at maximum $\sim 10^3 - 10^4$ yrs (figure 1c), and the neutron star in GX1+4 obviously did not yet reach its equilibrium period, we may argue that mass-accretion by Roche-lobe overflow (until 1983) did not start earlier than some $10^3$ to $10^4$ years ago.

4.2.2. The short orbital period LMXB's ($P_{\text{orb}} < 1 \, \text{hr}$).

4.2.2.1. 4U1626-67

The most recent study on 4U1626-67 was performed by Levine et al. (1988). The still unconfirmed orbital period is 41 minutes. The mass of the donor star lies between 0.02 and 0.1 $M_\odot$ and its core hydrogen abundance must be very low (i.e. $X_C < 0.02$; PS88a,b and references therein). The derived mass transfer rate is about 2. to 8. $10^{-10} \, M_\odot/\text{yr}$, if it is assumed that the neutron star is rotating at equilibrium, has a surface field of a few $10^{12}$ Gauss, and that it has a mass of 1.4 $M_\odot$ (Levine et al., 1988; eq. (3)). However, whether the NS is rotating at equilibrium is still an open question, since the observed spin-up timescale $\tau_{\text{obs}}$ of 4U1626-67, i.e. 5.5 $10^3$ yr (Table 1a) is of the same order of $\tau_{\text{SU}}$ (figure 1c), which indicates that the NS may still be evolving towards equilibrium. Another scenario is possible if one assumes that the neutron star is spinning at its equilibrium period and that it is accreting mass at a mean rate of a few $10^{-10} \, M_\odot/\text{yr}$, this rate being variable on a timescale of $\tau_{\text{ev}} \approx \tau_{\text{obs}}$.

4.2.2.2 1E2259+59

The binary system 1E2259+59 has been observed to regularly pulse with a period of 6.98 seconds by Hanson et al. (1988) and Koyama et al. (1987; and references therein). Despite detailed analyses, the orbital period of 2300 seconds, reported by Fahlman and Gregory (1983) could not
be confirmed by subsequent observations. However, if the orbital period is indeed as short as 38 minutes, the donor star in this system could be very much similar to that in 4U1626-67. The mass transfer rate derived for this system is $3.6 \times 10^{-11} \, M_\odot/\text{yr}$ (Hansson et al., 1988; Koyama et al., 1987), which in combination with the 6.98 second pulse period yields a surface field strength of about $7 \times 10^{11}$ Gauss, if the neutron star is assumed to be rotating at equilibrium.

Two characteristics in this system are rather peculiar and complicate the search for an evolutionary scenario. Firstly, the system is located at the center of a supernova-like shell G109-1.0, which is believed to be about $10^4$ yrs old, while the field strength of the NS indicates that the NS would be much older (if one assumes a standard field decay scenario). Secondly, the NS is spinning down on a timescale of $3.5 \times 10^5$ yrs (Table 1a). The long timescale indicates that the NS could be spinning near equilibrium and that the spin-down character could imply that the accretion rate is decreasing on a timescale $\tau_{\text{ev}} \approx \tau_{\text{obs}}$.

As yet, no viable evolutionary scenario has been advanced that explains all above characteristics of this peculiar binary system.

5. Conclusions

We have studied in detail the spin evolution of a mass accreting magnetized neutron star, and the relative importance of the evolution of the magnetic field and the mass accretion rate during the accretion phase.

The initial evolution of the spin period of a standard NS is completely dominated by the (decay of the) magnetic field. After a first phase of rotation at equilibrium when the field of the NS is still strong, the spin period of the NS departs from equilibrium during the phase of strong decay of the field. Once strong field decay has terminated, the spin period of the neutron star return to equilibrium. The subsequent evolution of the spin period is then entirely determined by the evolution of the mass-accretion rate. If the accretion rate is rapidly decreasing, the NS will spin down, since the equilibrium period of the NS increases. If the mass-accretion rate varies, a neutron star
rotating at equilibrium will evolve through alternating spin-up and
spin-down phases (see e.g. Her X-1).

Depending on the total amount of mass accreted by the NS, mass-
accretion may terminate in various phases of the spin-evolution phase
described above. If the field has not yet decayed significantly, or when
it has reached its bottom-field, the final spin-period equals the
equilibrium period. If, however, mass-accretion stops during the rapid
field decay phase, the NS is not rotating at its equilibrium period.

Model calculations show that the NS in PSR 1855+09 must have
accreted at least 0.04 $M_\odot$ during the previous mass-accretion phase. In
addition, it is likely that the initial mass of the donor star in
PSR 1855+09 was in the range 1.0 - 1.4 $M_\odot$.

The observed characteristics of PSR 1831-00, combined with data
from numerical calculations may be consistent with the idea that the
neutron star in this system has not been accreting mass since its
formation. We present an evolutionary scenario in which no mass transfer
between the components has occurred since the formation of the neutron
star, and in which the envelope of the accompanying subgiant remnant has
been evaporated by the energetic radiation from the young neutron star
that was formed in an accretion induced collapse.

The characteristics of the four pulsating X-ray sources in low-mass
binaries are briefly interpreted in terms of the spin-up model presented
here.

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