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EXAMINING GALACTIC AND EXTRAGALACTIC GAMMA-RAY BURST MODELS USING THE PEAK FLUX DISTRIBUTION

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ABSTRACT

We compare the observed distribution of peak fluxes, F_{peak}, of gamma-ray bursts with simulated distributions. On the basis of the cumulative F_{peak} distribution, we show that a standard candle galactic halo and a standard candle no-evolution cosmological model both reproduce well the distributions presently observed by BATSE and *PVO*. One can distinguish between these two particular models once the BATSE sample size is larger than 1600 bursts. However, because one can always invoke more complex galactic or extragalactic scenarios, it will never be possible to distinguish with certainty between distance scales on the basis of the log N–log F_{peak} distribution alone. If the distance scale could be obtained independently, the observed log N–log F_{peak} distribution can be used to constrain the parameters of a given model even with the presently available sample sizes.

Subject headings: gamma rays: bursts — stars: statistics

1. INTRODUCTION

The results from the Burst and Transient Source Experiment (BATSE) aboard the *Compton Gamma Ray Observatory* have revealed that gamma-ray bursts are distributed uniformly over the sky, with no significant concentration to the Galactic plane or Galactic center and with the number of weak bursts relative to bright bursts much smaller than expected from a uniform distribution in distance (Fishman et al. 1992; Meegan et al. 1992).

The parameter (V/V_{max}) is a useful statistical quantity to determine whether the observed distribution of bursts is drawn from a spatially homogeneous sample ($\langle V/V_{max}\rangle = 0.5$). The results from BATSE give $\langle V/V_{max}\rangle = 0.348 \pm 0.024$ (Meegan et al. 1992). Earlier experiments with much lower sensitivities than BATSE, e.g., *Pioneer Venus Orbiter* (*PVO*), have detected the strong, rarer events. These bursts have $\langle V/V_{max}\rangle \simeq 0.5$ and a slope of -1.5 in the log $N(>F_{peak})$–log F_{peak} diagram, where F_{peak} is the peak flux of the bursts (Epstein & Hurley 1988; Fenimore et al. 1992; Chuang et al. 1992).

The observed distribution of gamma-ray bursts is inconsistent with any Galactic disk models (Mao & Paczyński 1992b; Hakkila & Meegan 1992), while a Galactic halo distribution requires a large core radius $R_c > 14$ kpc in order to have no measurable dipole anisotropy (Mao & Paczyński 1992b). Other constraints, such as the anisotropy from M31 (Hakkila & Meegan 1992), have also been used to place upper limits on the core radius. Cosmological models have been suggested as well (see, e.g., Paczyński 1991b and references therein). For bursts which are standard candles with identical power-law spectra and a burst rate which is constant (per unit comoving volume per unit comoving cosmological time in a Friedmann universe with $\Omega = 1$ and $\Lambda = 0$), Mao & Paczyński (1992a) found that the calculated distribution of burst intensities is consistent with that found by BATSE, which observes mostly weak events, and *PVO*, which observes only strong events. Under these assumptions, the redshift of the observations would be $z_{BATSE} \simeq 1.5$ and $z_{PVO} \simeq 0.2$ (Fenimore et al. 1993 find $z_{BATSE} \simeq 0.8$ and $z_{PVO} \simeq 0.2$.)

In this *Letter*, we examine what information can be extracted from the gamma-ray burst log N–log F_{peak} distribution of simple Galactic halo and cosmological models. First, we examine the samples that BATSE and *PVO* would observe given these two models and confirm that they are consistent with observations. Secondly, we investigate how accurately the parameters of a simple Galactic halo model can be determined from a sample with the approximate size of the presently available BATSE data set (~400 bursts). Finally, we compare our simple Galactic halo and cosmological models and show that these models are indistinguishable given the present BATSE database.

2. THE GAMMA-RAY BURST MODELS

We examine log N–log F_{peak} distributions of two gamma-ray burst (GRB) models. These models are inspired by simple forms of a Galactic halo and cosmological distribution.

2.1. *Model A*

Mao & Paczyński (1992b) studied a Galactic halo model in combination with a large range of gamma-ray burst luminosity functions and found that the observable parameters ($\langle V/V_{max}\rangle$, $\langle\cos\theta\rangle$, $\langle\sin^2 b\rangle$) are almost the same as those calculated for a "standard candle" scenario. Therefore, for our analysis, we have assumed that all bursters are "standard candles"; that is, all bursts have the same intrinsic peak luminosity. For the distribution which characterizes the rate of gamma-ray bursts in the Galactic halo, we adopt the form

$$n(R) = \frac{n_0}{1 + (R/R_c)^\beta}, \tag{1}$$

where R is the distance from the Galactic center, R_c is the halo core radius, and β is a dimensionless parameter with $\beta = 2$ corresponding to the dark halo and $\beta = 3$ corresponding to the luminous halo. In order to reconcile these halo distributions with the observable parameters given by BATSE, Mao & Paczyński (1992b) found that the halo core radius had to be $R_c > 14$ kpc for $\beta = 2$ and $R_c > 20$ kpc for $\beta = 3$ (see also

Paczyński 1991a). Because such a large core radius is required, we have assumed that we, as observers, are at the center of the Galaxy. In reality, there may still be a small effect due to our offset from the Galactic center; however, given the uncertainty due to our small sample sizes and the ample freedom (e.g., luminosity functions) in a Galactic model (see § 6), we feel justified in making this simplifying assumption. Therefore, the number of sources with observed peak fluxes in excess of $F_{\rm peak}$ is given by

$$N(>F_{\rm peak}) = 4\pi \int_0^{D(L)} R^2 n(R) dR , \qquad (2)$$

where the distance $D(L) = (L/4\pi F_{\rm peak})^{1/2}$, and the luminosity L is a constant for a standard candle model. Equation (2) can be reduced to an expression characterized by two dimensionless parameters, β and $R_{\rm max}/R_c$; $R_{\rm max}$ is the *maximum* distance out to which a burst of luminosity L can be seen and thereby corresponds to the *minimum* flux in the sample. For β values of 2 and 3, equation (2) can be evaluated analytically.

In this *Letter*, we examine two cases of the above model (hereafter referred to as model A). In each case, the $\langle V/V_{\rm max}\rangle$ of each distribution is within 1 σ of the value observed by BATSE see (§ 1). The cases are (1) $\beta = 2$ and $R_{\rm max}/R_c = 5.0 (\langle V/V_{\rm max}\rangle = 0.321)$, and (2) $\beta = 3$ and $R_{\rm max}/R_c = 2.0 (\langle V/V_{\rm max}\rangle = 0.330)$.

2.2. *Model B*

We have used the same cosmological scenario as Mao & Paczyński (1992a). They have adopted a simple model where (1) the universe is flat with $\Omega = 1$, (2) the bursts are standard candles with identical power-law spectra, and (3) there is no evolution of the GRB population, which implies that the rate of bursts is constant per comoving volume per unit comoving time. The total number of bursts observable per unit time out to the redshift of $z_{\rm max}$ can be evaluated analytically if the slope of the GRB spectrum, γ, equals 0.5, 1.0, or 1.5. The GRB power-law spectra are defined by the equation $\nu L_\nu d\nu = C\nu^\gamma d\nu$ (eqs. [1]–[11] of Mao & Paczyński 1992a). We examine the model with all three values of γ. Their respective log $N(>F_{\rm peak})$–log $F_{\rm peak}$ distributions are normalized to the BATSE result of $\langle V/V_{\rm max}\rangle = 0.348$ (Meegan et al. 1992). Hereafter, this cosmological model will be referred to as model B.

3. COMPARING GAMMA-RAY BURST SAMPLES OF BATSE AND *PVO*

The BATSE and *PVO* experiments have each sampled a different part of the distribution of gamma-ray bursts. *PVO* have detected strong events, revealing that they are distributed isotropically in angle and uniformly in distance (Chuang et al. 1992), while BATSE appears to have reached the "edge" of the distribution on the faint end (Meegan et al. 1992). We examine the distributions that BATSE and *PVO* would have seen if the distribution of GRBs were actually given by our models A and B as described in § 2.

To accurately combine the BATSE and *PVO* data is technically difficult and beyond the scope of this *Letter*. (For a careful discussion, see Fenimore et al. 1993). In order to make a rough (yet sensible) comparison, we use the yearly GRB rates of BATSE and *PVO* as a means to compare the sensitivity of the two instruments. The yearly rate of BATSE (corrected for duty cycle) is $N \approx 800$ yr^{-1} (Meegan et al. 1992); for *PVO* it is $\sim$23 yr^{-1} (Epstein & Hurley 1988; Fenimore, Epstein, & Ho 1992). Using the ratio $N_{\rm BATSE}/N_{PVO} \approx 35$, we can calculate the expected peak flux above which *PVO* would detect bursts given the distribution of model A or B. We assume that BATSE can detect bursts with peak fluxes from the sample minimum all the way to infinity. *PVO* detected $\sim$200 bursts while BATSE has detected $\sim$400. Therefore, to simulate the BATSE and *PVO* data, we generate samples from models A and B with sample sizes and limiting fluxes indicated above.

Standard Monte Carlo techniques are used to generate random samples of GRB peak fluxes according to models A and B. The samples are analyzed using the Kolmogorov-Smirnov (K-S) test; that is, the $F_{\rm peak}$ values are converted to an *observed* cumulative distribution $S_N(F_{\rm peak})$ which is compared to the *model* cumulative distribution $P(F_{\rm peak})$. The difference between the two distributions is determined by the K-S d, defined as

$$d = \max_{-\infty < F_{\rm peak} < \infty} | S_N(F_{\rm peak}) - P(F_{\rm peak}) | . \qquad (3)$$

The quantity d is a measure of the goodness of the fit of P to S_N; the smaller it is, the better the fit.

One thousand samples which represent the BATSE and *PVO* data are generated according to each case of models A and B (§ 2). Using the K-S test, each sample is "best-fitted" to a single power-law function to determine the slope of the distribution. The best-fit power law of each sample is the function which gives the *minimum* K-S d. The normalized form of the cumulative power law ($F_{\rm min}$ is the minimum peak flux in each sample) is

$$N(>F_{\rm peak}) = \left(\frac{F_{\rm peak}}{F_{\rm min}}\right)^\alpha . \qquad (4)$$

3.1. *Model A*

For each of the two cases of model A, the distributions of the best-fit power-law exponent, α, for samples which represent BATSE and *PVO*, are plotted in Figure 1. For case 1, the

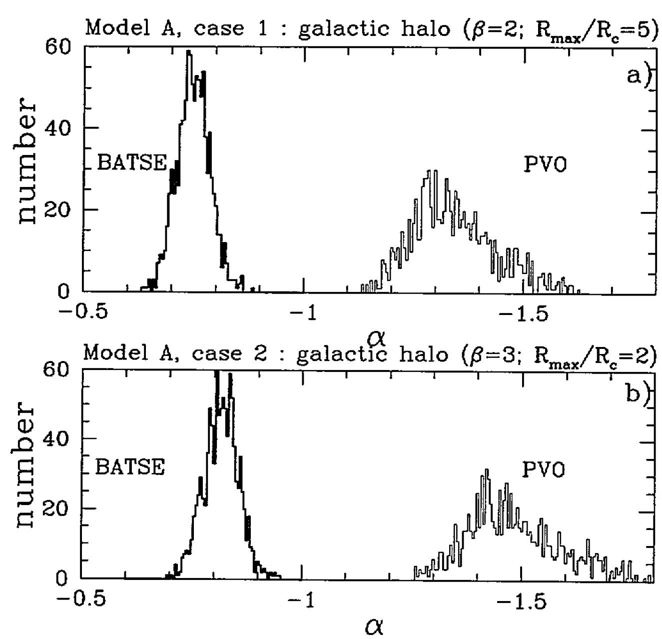

FIG. 1.—For each case of model A (a simple Galactic halo model), the distribution of the best fit power-law exponents, α, to samples which represent the data sets observed by BATSE and *PVO* (see § 3).

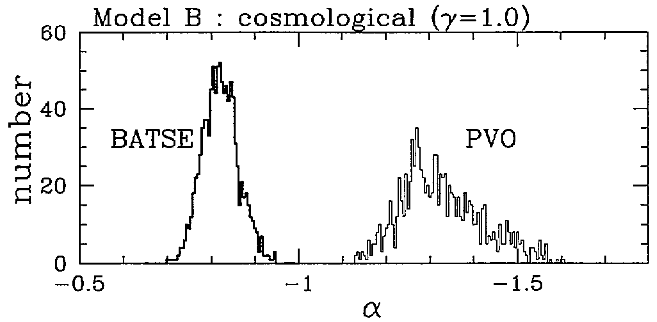

FIG. 2.—For model B (a simple cosmological model) with $\gamma = 1.0$, the distribution of the best-fit power-law exponents, α, to samples which represent data sets observed by BATSE and *PVO* (see § 3).

average values (and standard deviations) of α are -1.34 ± 0.09 (*PVO*) and -0.75 ± 0.04 (BATSE), and for case (2), -1.48 ± 0.10 (*PVO*) and -0.82 ± 0.04 (BATSE). Case 1 is reasonably consistent (less than 2 σ) with the slopes observed by BATSE (approximately -0.8; see Fig. 1a of Meegan et al. 1992) and *PVO* (approximately -1.48; Chuang et al. 1992), while case 2 is in perfect agreement.

3.2. *Model B*

The distributions of the best-fit power-law exponent, α, for samples which represent BATSE and *PVO* are almost identical for model B with any of the three values of γ (see § 2). Therefore, we plot in Figure 2 the distributions of α for the $\gamma = 1.0$ model. The average values (and standard deviations) of α for this model are -1.33 ± 0.09 (*PVO*) and -0.82 ± 0.04 (BATSE). Model B reproduces perfectly the slope observed by BATSE and reasonably reproduces (less than 2 σ) the slope observed by *PVO*.

4. DETERMINING THE PARAMETERS OF OUR GALACTIC HALO MODEL

We examine cumulative distributions of GRB peak fluxes based on a simple Galactic halo model (model A). Using the K-S test, we estimate the accuracy with which parameters of this model can be determined from a randomly generated sample of 400 bursts.

In order to determine the accuracy of the parameters, 1000 random samples (of 400 bursts each) are generated according to model A for the two cases given in § 2. Each random sample is fitted to the functional form of model A (eq. [2]) using the K-S test. The best-fit parameters of the sample are those values which give the *minimum* K-S d (eq. [3]).

From the 1000 samples, a probability distribution can be determined for each model parameter. For samples with 400 bursts, this distribution is not necessarily Gaussian but can be highly skewed with a tail toward high values. Therefore, we parameterize the probability distribution by determining the mode or the "most probable value," as this is the value which is most likely to be observed, rather than determining the mean of the distribution which tends to be biased towards higher values. We define a "1 σ" confidence interval around this parameter by determining the points in the distribution such that the area in each tail is 16%. This confidence interval represents the accuracy with which we can determine the model parameters from *one* log N–log F_{peak} distribution.

One randomly generated sample of 400 bursts (and the best-fit model) for each case of model A is shown in Figure 3. From

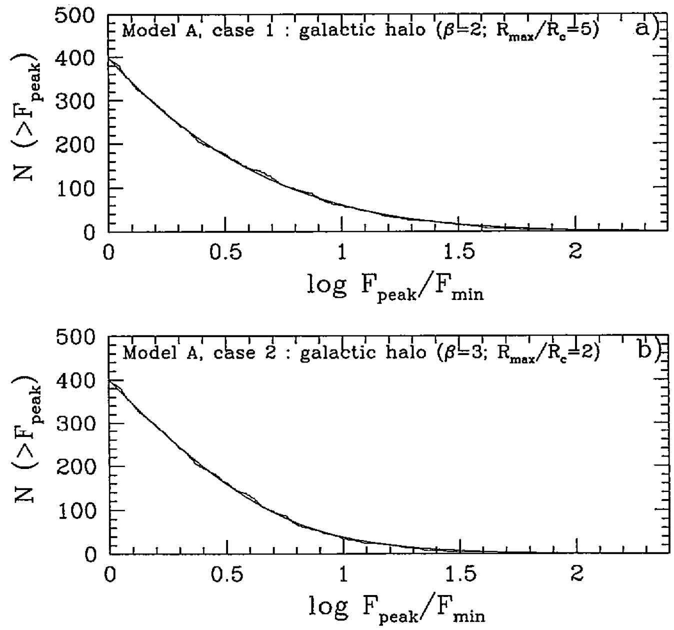

FIG. 3.—A randomly generated sample (400 bursts) of the two cases of model A (a simple Galactic halo distribution). The solid lines indicate the best-fit model A function to each sample (see § 4). The best-fit parameters of model A (β and R_{max}/R_c) for these samples are 2.04 and 4.51 (case 1) and 3.08 and 1.92 (case 2). Notice that we use a linear scale (rather than the traditional log scale) on the y-axis because the K-S d measures a linear distance; therefore, with this representation, it is easier to see the deviations of each sample from the best-fit theoretical curve.

the analysis of the 1000 samples, the most probable values of the parameters and their "1 σ" confidence limits (as described above) are for case 1 $\beta = 2.1^{+0.3}_{-0.3}$ and $R_{max}/R_c = 4.9^{+4.2}_{-1.4}$ and for case 2 $\beta = 3.0^{+0.5}_{-0.5}$ and $R_{max}/R_c = 1.9^{+0.5}_{-0.1}$. In both cases, the parameter R_{max}/R_c is more difficult to determine (that is, has a much larger error) due to its highly skewed probability distribution. For a sample size of $\sim$3000 bursts, the error in β is $\sim$7%, and it is therefore possible to distinguish at a 3 σ level between the dark halo (case 1) and a luminous halo (case 2).

5. DISTINGUISHING BETWEEN OUR GALACTIC HALO AND COSMOLOGICAL MODELS

Given the present BATSE sample of over 400 bursts, can we differentiate between our simple galactic halo and cosmological models from the log N–log F_{peak} distribution? To investigate this, we compare models A and B using the K-S test.

The procedure to compare the two distributions is as follows. We generate samples from model B by randomly drawing 400 peak fluxes. These samples are then fitted (using the K-S test) to the functional form of model A (eq. [2]). The best-fit parameters are determined for each sample by minimizing the K-S d (eq. [3]). This procedure is repeated on 1000 different samples, and the resulting K-S d values are recorded. From the fits of the 1000 samples of model B, we determine the "most probable values" of the two parameters (β and R_{max}/R_c) of model A in the same manner as described in § 4.

To compare models A and B, we generate another 1000 samples (of 400 bursts each), now from model A with parameters equal to the "most probable" parameters determined above from the samples of model B. The samples of model A are also fitted to the functional form of model A (eq. [2]). The resulting distribution of K-S d values is then compared to the distribution of K-S d values determined from the fits to the samples of model B.

The percentage overlap of the two resulting K-S d distributions is used as a statistical measure of the probability that models A and B are indistinguishable from a log N–log $F_{\rm peak}$ distribution. The probability that a given log N–log $F_{\rm peak}$ distribution could be classified as model A when it was actually model B (or vice versa) is equal to approximately one-half (depending on the exact shapes of the two K-S d distributions) of the percentage overlap. This can be seen in the limit that the two K-S d distributions are identical. The fractional overlap would be 100%, and there would be a 50% probability of misclassifying the log N–log $F_{\rm peak}$ distribution as either model A or model B. We assume that we can distinguish between the two models when the percentage overlap is $\sim 10\%$; that is, there is only an $\sim 5\%$ chance that the BATSE sample could ever be incorrectly classified.

The results of the Galactic halo (model A) fittings to the cosmological (model B) samples are the same within 1 σ for all three γ values. The most probable values of the model A parameters and their "1 σ" confidence limits for the samples of model B with $\gamma = 1.0$ are $\beta = 2.5^{+0.6}_{-0.4}$ and $R_{\rm max}/R_c = 1.9^{+1.2}_{-0.9}$. In Figure 4*a*, we show one randomly generated sample of the $\gamma = 1.0$ model B distribution and the best-fit model A. In Figure 4*b*, the shaded histogram is the distribution of K-S d values from the fits of model A to the 1000 samples of model B; the heavy-lined histogram is the distribution of K-S d values from the fits of model A to the 1000 random samples of model A (with parameters determined from the model B samples). The percentage overlap of the two distributions is 83%, indicating that the log N–log $F_{\rm peak}$ distributions of the two models are practically indistinguishable. That is, there is an $\sim 42\%$ probability that a given log N–log $F_{\rm peak}$ distribution could be improperly classified as model A (or model B). To reduce the probability to $\sim 5\%$ would require a sample size of ~ 1600 bursts (to $< 1\%$, 2000 bursts).

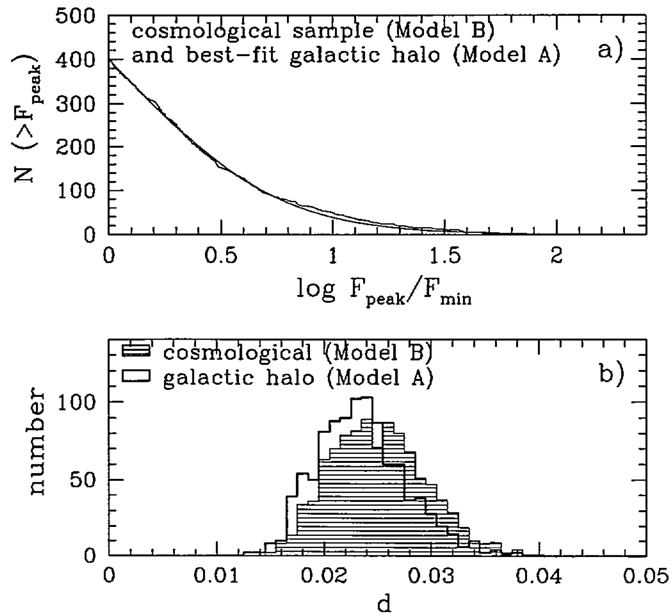

FIG. 4.—(*a*) Randomly generated sample (400 bursts) from model B (a simple cosmological model) with $\gamma = 1.0$. The best-fit model A function (a simple Galactic halo model) to this sample is indicated with a solid line. The best-fit parameters (β and $R_{\rm max}/R_c$) are 3.37 and 1.82. (*b*) Resulting K-S d distributions from fits of model A to the samples of model B (*shaded histogram*) and to the samples of the best-fit model A (*heavy-lined histogram*). The percentage overlap of the two distributions is 83%, indicating that the log N–log $F_{\rm peak}$ distributions of model A and model B are practically indistinguishable (see § 5 for details).

6. DISCUSSION

In this *Letter*, we have examined two distinct gamma-ray burst models. Model A is based on a standard candle Galactic halo distribution, while model B is based on a standard candle no-evolution cosmological distribution. These models both produce log N–log $F_{\rm peak}$ distributions that are consistent with that observed by BATSE and *PVO*. We have shown that considerably larger samples of gamma-ray bursts (> 1600 bursts) than those presently available are needed to differentiate between these two models. However, even though we can distinguish between them with improved statistics, it will never be possible to differentiate with certainty between a Galactic halo and a cosmological *distance scale* from the log N–log $F_{\rm peak}$ distribution alone, because there are far too many degrees of freedom (including luminosity functions, halo densities, or cosmological evolution) involved in either scenario. However, if the distance scale is obtained by other means, the log N–log $F_{\rm peak}$ distribution can be used to study particulars of a given model, such as the variation of number density with distance in the Galactic halo or the evolution of gamma-ray bursts in the cosmological scenario. The statistical analysis presented in this *Letter* provides a means of determining the sample sizes necessary to make quantitative statements on particular model parameters. For example, we have already shown that it will be possible to distinguish between a dark and luminous halo for a BATSE sample size of ~ 3000 bursts. In a forthcoming paper, we explore new (model-independent) techniques for parameterizing the log N–log $F_{\rm peak}$ distribution (Wijers & Lubin 1993).

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