Beat-to-beat blood-pressure fluctuations and heart-rate variability in man: physiological relationships, analysis techniques and a simple model

de Boer, R.W.

Citation for published version (APA):
Chapter 3
Comparing spectra of a series of point events, particularly for heart-rate variability data

In this chapter we describe and compare different methods that are used to calculate a heart-rate variability spectrum.

3.1 Abstract
Different methods for the spectral analysis of the heart-rate signal -- considered as series of point events -- are used in studies on heart-rate variability. This chapter compares these methods, focussing on the two principal ones: the Interval Spectrum, i.e. the spectrum of the interval series, and the Spectrum of Counts, which is related to the representation of the event series as a series of spikes (delta-functions). Both autospectra are estimated for experimental heart-rate data and are shown to produce similar results. This similarity is proven analytically and it is shown that for small variations in interval length the ratio of these spectra $P_I(f)/P_C(f)$ equals $\sin(\pi f \bar{T})/(\pi f \bar{T})^2$, with $P_I$ and $P_C$ the Interval Spectrum and the Spectrum of Counts respectively, $f$ the frequency and $\bar{T}$ the mean interval length. It is concluded that both autospectra are equivalent for the considered heart-rate data but that, when relating the heart-rate signal to other signals (e.g., respiration, blood pressure) by means of cross spectra, the technique to be used depends on the characteristics of the second signal.

3.2 Introduction
In the study of heart-rate variability many authors have used spectral analysis methods (for a survey see Chapter 1.5). For data from man (Sayers, 1973) as well as from dog (Akselrod et al., 1981) and cat (Chesl et al., 1975) three peaks in the spectrum are usually distinguished. One peak is due to respiration, for man around 0.3 Hz. Often a peak at approximately 0.1 Hz is found, which seems related to the 10 seconds waves as seen in the blood pressure (Schweitzer, 1952). A peak at still lower frequencies is attributed to properties of the thermoregulatory system.
The technique for spectral analysis of heart-rate data is not straightforward, however; the successive heart beats must be considered as a series of events and different methods can be used for the estimation of spectra from such signals. In this chapter we present a survey and a comparison of the different spectra that can be defined. The emphasis is on data from heart-rate variability studies. Characteristics of these data are: (i) the variation of interval lengths is much smaller than the mean length and (ii) the variation of lengths is more or less regular (e.g., due to respiratory influences).

In section 3.3 we discuss the three types of spectra that are used in heart-rate variability studies. In section 3.4 we compare the interval spectrum and the spectrum of counts (Cox and Lewis, 1966), which seem to us the most interesting ones. In section 3.4a both spectra are estimated and compared for the same sets of heart-rate data and in section 3.4b we prove analytically the similarity of the two kinds of spectra for this type of data. In the conclusion we discuss under what conditions each of the two spectra is most useful.

3.3 The power spectrum of a series of point events.
Standard Fourier analysis, i.e. the decomposition of a signal in sinusoids, is not possible for a series of point events. Hence, a power spectrum for such a series must be defined in a different way. In heart-rate variability studies the following three approaches are used to arrive at a useful concept for a spectrum of a point process.

a) A signal that is defined at all times is derived from the point event series (fig. 1a). Several possibilities exist (Chapter 2, or DeBoer et al., 1985b), e.g. the transformation of the event series into a heart-rate signal, fig. 1b. The spectrum of the signal is estimated by equidistant sampling followed by a Discrete Fourier Transform. A number of authors have used this approach (e.g., Kobayashi and Musha, 1982; Luczak and Laurig, 1973; Mulder and Mulder, 1973; Pomeranz et al., 1985; Womack, 1971). A disadvantage of this method is that the signals deduced from the point event series are often not differentiable and sometimes not even continuous (cf. fig. 1b). This causes spurious contributions in the spectrum, in particular in the higher frequencies. It is also a moot point that different procedures are used to derive a signal from the series without a clear preference. This calls for a more canonical definition of a spectrum.
In the literature on stochastic point processes two different spectra are defined: the interval spectrum and the spectrum of counts (Cox and Lewis, 1966; Cox and Isham, 1980; DeKwaadsteniet, 1982). Both spectra are used in heart-rate variability studies and will be discussed in the following.

b) The **interval spectrum** is the spectrum of the series of intervals spaced equidistantly (fig.1c). Standard procedures for spectral estimation (e.g. computation of the periodogram) can be used. Several authors presented heart-rate variability spectra in this way (Chess et al., 1975; Mohn, 1976; Sayers, 1973 and 1980). In a similar way a **heart-rate spectrum** is defined as the spectrum of equidistantly spaced inverse intervals (fig.1d; Mohn, 1976).

As the interval series is a function of interval number and not of time, the spectrum is a function of frequency in "cycles per beat" instead of the usual "cycles per second" or Hz. In section 3.4b it is shown that the spectrum may be interpreted in terms of frequency in Hz if the "cycles/beat" are transformed in "cycles/second" by considering the length of the beat to be equal to the mean interval length $\bar{I}$.

---

**Fig.3-1a** Event series representing R-waves of the ECG.
**Fig.3-1b** Heart-rate signal derived from the event series in Fig.1a.
**Fig.3-1c** Interval series derived from Fig.1a. The Discrete Fourier Transform of this signal leads to an estimator for the **interval spectrum**.
**Fig.3-1d** Series of inverse intervals, leading to the **heart-rate spectrum**.
**Fig.3-1e** In this figure the events of fig.1a have been replaced by spikes (delta-functions); the spectrum of this signal gives the **spectrum of counts**.
Some authors presented their results as a function of "cycles/beat" without a transformation in "Hz": Lisenby et al. (1969) invented the term "beatquency" analysis for such a frequency analysis, while Haddad et al. (1984) defined "one cycle/beat" as "one Hz", which is somewhat confusing.

Note that the interval spectrum of a fully regular process (all intervals equal) consists solely of a DC-component.

c) The spectrum of counts is also used in the statistical analysis of series of events. This spectrum can be estimated by a straightforward calculation of the spectrum of the signal in fig.1e, where the events (fig.1a) have been replaced by delta-functions (Cox and Lewis, 1966). Thus the signal is described as

\[ p(t) = \sum_k \delta(t-t_k) \]

where \( t_k \) is the time of the \( k \)-th occurrence of an event. For equal intervals \( \bar{T} = t_k-t_{k-1} \) the spectrum of counts consists of an infinite series of delta-functions, spaced at distance \( 1/\bar{T} \) along the frequency axis. Usually one is only interested in frequencies much lower than the mean repetition frequency of the events, so only the low-frequency part of the spectrum needs to be considered. Two different approaches for the estimation of this part of the spectrum have been proposed:

- The signal is passed through an ideal low-pass filter with cut-off frequency \( f_{max} \); this is equivalent to convolution of the signal \( p(t) \) with the function \( \sin(2\pi f_{max} t)/(\pi t) \) and amounts to replacing each deltafunction at time \( t_k \) by the function \( \sin(2\pi f_{max}(t-t_k))/(\pi(t-t_k)) \). The result is a continuous signal which was named the Low Pass Filtered Event Series or LPFES (Hyndman and Mohn, 1975b). This signal is sampled and the spectrum is calculated by a Digital Fourier Transform. An efficient algorithm was published by French and Holden (1971); see also Peterka et al. (1978). (The French-Holden algorithm is discussed in Appendix A2).

Coenen et al. (1977) described a hardware device to perform the convolution.

- The interesting (i.e. low-frequency) part of the spectrum \( P_C(f) \) of the signal \( p(t) \) can also be computed directly, using as estimator:

\[
P_C(f) = \frac{1}{N\bar{T}} \left\{ \left[ \sum_{k=0}^{N-1} \cos(2\pi f t_k) \right]^2 + \left[ \sum_{k=0}^{N-1} \sin(2\pi f t_k) \right]^2 \right\}
\]

(1)

with \( N \) the number of intervals in the period of observation and \( \bar{T} \) the mean interval length (Rompeleman et al., 1982).
Munemori et al. (1982, 1984) estimated the spectrum of counts with the formula:

\[ P'(f) = \frac{1}{N} \left\{ N + 2 \left[ \sum_{k=1}^{N} \cos(2\pi f(t_k-t_{k-1})) \right] + \sum_{k=2}^{N} \cos(2\pi f(t_k-t_{k-2})) + \ldots \right. 
\left. + \sum_{k=N}^{N} \cos(2\pi f(t_k-t_{k-N})) \right\}. \]

This formula requires of order \( N^2 \) evaluations of the cosine-function per spectral estimate, whereas eqn.1 above requires only of order \( N \) evaluations. The formulae are completely equivalent, as can be seen using trigonometrical identities, and hence eqn.1 is to be preferred for practical computations.

As the spectrum of counts and the interval spectrum are the most common spectra in the study of point event series, we concentrate on these spectra in the following.

3.4 Comparison of the interval spectrum, the heart-rate spectrum and the spectrum of counts of a point process.

In this section we present experimental and theoretical comparisons of the interval spectrum, the heart-rate spectrum and the spectrum of counts of series of point events. Heart-rate data are used in the examples and we are concerned only with series in which the spread of interval lengths is small.

3.4.1 Experimental comparison of the spectra.

For fig.2 we used 940 successive heart intervals, derived from the R-waves in the electrocardiogram of a healthy young person, breathing freely. The first 400 intervals are shown in fig.2a. The mean interval length was 0.93 s. Similar data when breathing at a fixed rate of 0.16 Hz are shown in fig.3a (340 intervals with a mean of 0.94 s). From these data we calculated the interval spectra (figs.2b,3b), the heart-rate spectra (fig.2c,3c) and the spectra of counts (figs.2d,3d,3e) in the following way:

- The interval spectrum \( P_i(f) \) was estimated by the periodogram using a Fast Fourier Transform. The intervals \( I_k \) were first normalized as \( \bar{I}_k = (I_k - \bar{I})/\bar{I} \), \( \bar{I} \) being the mean interval length. We added zeros to achieve 1024 datapoints (zero-padding) and divided the spectral values by 0.875 to compensate for the 10% cosine taper that was used (Bendat and Piersol, 1971; see also appendix 3.A1 for more details).
on the spectral estimation procedure we used). The frequency axis was scaled by considering the intervals to be spaced at distances equal to the mean interval length $\bar{T}$ (Sayers, 1973). So frequency-values in Hz were obtained and, as the effective sampling frequency is $1/\bar{T}$, the maximum frequency in the spectrum is $1/2\bar{T}$ ($0.54$ Hz and $0.53$ Hz for figs.2b and 3b, resp.). This procedure will be justified in section 4.2.
Thus the periodogram was calculated as:

\[ P_I(f) = \frac{1}{0.875} \frac{2\pi}{N} \cdot S_I(f) \cdot S_I^*(f) \]

(2)

with \[ S_I(f) = \sum_{k=0}^{N-I} \exp(-2\pi if\Delta f) \] for \( f = 0,1/(1024\Delta f),2/(1024\Delta f),\ldots,1/(2\Delta f) \).

No frequency smoothing was performed in fig.3b, while a 27 point rectangular window was used in fig.2b (equivalent to a 25 point window for the unpadded data).\( P_I(f) \) is the power spectrum; in all figures we show the amplitude spectrum \((P(f))^{1/2}\) to stress the higher harmonics.

- The heart-rate spectrum was calculated in the same way, but now the inverse interval values were used.

- The spectrum of counts \( P_C(f) \) was calculated using a slightly modified version of eqn.1. The mean value was subtracted from the signal \( p(t) = \sum_k \delta(t-t_k) \) in addition we gave each spike an impulse content equal to \( I \), so the signal to be transformed becomes dimensionless: \( p'(t) = \sum_k I \delta(t-t_k) - I \). The subtraction of the mean value removes the large DC-component and its side lobes from the spectrum. A factor two was added as we only consider positive frequencies. \( P_C(f) \) was thus computed as the transform of \( p'(t) \):

\[ P_C(f) = \frac{2\pi}{N} \left\{ \sum_{k=0}^{N-1} \left[ \sum_{k=0}^{N-1} \cos(2\pi ft_k) - \sin(\pi ft_{N-1}) \sin(\pi ft_0) \right] \right\}^2 + \]

\[ \left\{ \sum_{k=0}^{N-1} \sin(2\pi ft_k) - \sin(\pi ft_{N-1}) \sin(\pi ft_0) \right\}^2 \]

(11)

In figs.2d,3d \((P_C(f))^{1/2}\) is presented up to 0.5 Hz for frequencies at distance 0.001 Hz apart. In fig.2d a 27 point rectangular window was used to smooth the spectrum. The width of this window is thus equal to the one used in fig.2b. Whereas the interval spectrum -- being a Digital Fourier Transform -- is periodical and limited in frequency-range, the spectrum of counts is not. This is shown in fig.3e, where the spectrum of counts up to 2.5 Hz is presented at distances 0.005 Hz (no smoothing). The mean repetition frequency of the heart-rate signal is apparent from the large contributions to the spectrum around 1.06 Hz and 2.13 Hz (the mean interval length being 0.94 s).

It is striking that figs.2b,c,d are rather similar. In all cases the spectrum consists of a low frequency component, a peak around 3.1 Hz and a peak in the region of
Fig. 3-3a
340 heart intervals from a healthy young person, breathing at a fixed rate of 0.16 Hz.

Fig. 3-3b
Interval spectrum calculated from the data of fig. 3a. This spectrum is not smoothed.

Fig. 3-3c
Heart rate spectrum

Fig. 3-3d
Spectrum of counts in the range 0-0.5 Hz.

Fig. 3-3e
Spectrum of counts in the range 0-2.5 Hz. Note the large contributions around the mean repetition frequency (1/0.94 s = 1.06 Hz) and around twice this value (2.12 Hz).
the mean breathing frequency (0.25 Hz). For frequencies above 0.2 Hz the spec-
trum of counts is somewhat larger than the other ones. The spectra in figs.3b,c,d
contain only contributions at multiples of the fixed respiratory frequency (0.16 Hz).
The higher harmonics are more pronounced in fig.3d than in fig.3b,c, but the
difference is small. The peaks in the interval and heart-rate spectrum appear to
be slightly wider than the ones in the spectrum of counts. In the next section we
explain why the spectra are so similar for these data.

3.4.2 Analytical comparison of the spectra.

How is it that the spectrum of counts, the heart-rate spectrum and the interval
spectrum are so similar in shape for the data presented in figs.2,3 (cf. Mohn, 1976,
fig.4)? To answer this question, we compare in the following the crude estimators
for the interval spectrum and the spectrum of counts.

The interval spectrum -- or rather the spectrum of normalized intervals \( I_k/I \) -- is
estimated by the periodogram

\[
P_1(f') = \frac{2}{N} \sum I_k \exp(-2\pi i f'k) \quad f' = 0, 1/N, \ldots, 1/2.
\]

All summations are to be taken from \( k = 0 \) till \( k = N-1 \). We put \( f = f'/I \) (\( f = 0,
1/N, \ldots, 1/2 \)), so

\[
C_1(f) = \sum I_k \exp(-2\pi i f k).
\]

The spectrum of counts is estimated as

\[
P_C(f) = \frac{2}{N} \sum C_C(f) C_C^*(f)
\]

with

\[
C_C(f) = \sum I \exp(-2\pi i f t_k).
\]

From these expressions the similarity of the spectra is not evident at first sight.
The next analysis shows under which conditions the spectra are alike. We put
\( t_k = kI + \delta_k \), so \( \delta_k \) is the deviation from a regular train and
\( I_k = t_k - t_{k-1} = I + \delta_k - \delta_{k-1} \). We assume that the deviations \( \delta_k \) are sinusoidally modu-
lated:

\[
\delta_k = \delta \sin(2\pi f_m \tau + \phi), \quad f_m < 1/2I.
\]

This implies a sinusoidal modulation of the intervals as well:

\[
t_k = I + 2\delta \sin(2\pi f_m \tau + \phi) \cos(2\pi (k-1/2)f_m \tau + \phi).
\]

Using the complex representation for \( \delta_k \) we find for the interval spectrum (with
\( \delta_k = \delta \exp(2\pi i f_m \tau + i \phi) \) :
The amplitude of the first part of this expression (DC-component) is:

\[ I = \frac{\sin(\pi f \Delta t)}{\pi f \Delta t} \]

This is equal to \( N \) for \( f = n/\Delta t \) \((n = 0, 1, 2, \ldots)\) and remains finite for \( N \to \infty \) for all other frequencies. The contribution of the DC-term will be neglected in the following as it can easily be removed by subtracting the mean from the interval values.

The second part of equation (3) contains a similar sum, with \((fm - f)\) instead of \(f\). So now:

\[ C_I(f) = N \delta \exp(\phi) (1 - \exp(-2\pi ifm)) \text{ at } f = fm, \]

while for \( f \neq fm \) and \( N \to \infty \), \( C_I(f) \) remains of order one. Thus the spectrum contains a spike at frequency \( fm \) as is to be expected.

Let us now make the additional assumption that \( \delta \ll \Delta t \), i.e. the event train is rather regular. We obtain for the spectrum of counts:

\[ C_C(f) = \sum \delta \exp(-2\pi ik(fm - f)) \approx \frac{1}{\Delta t} \sum \delta \exp(-2\pi ik\Delta t) - 2\pi ifm \sum \delta \sin(\pi fm) \exp(\pi ifm) \]

The first part of this expression is equal to the similar one in (3) and will be neglected as well. So \( C_C(f) \approx -2\pi ifm \delta \sin(\pi fm) \exp(\phi) \) at \( f = fm \), while for all other frequencies in our range of interest and \( N \to \infty \), \( C_C(f) \) remains of order 1. Taking into account higher order terms in \( \delta \) in (3), we find harmonics at frequencies \( 2fm, 3fm, \ldots \)

The ratio of the powerspectra at \( f = fm \) is \( P_I(fm)/P_C(fm) = (\sin(\pi fm)/\pi fm)^2 \), which for slow modulation, i.e. \( fm \ll f \Delta t \), approximates the value 1. Hence for small sinusoidally modulated deviations \( \delta \), both types of spectra are similar; they have a peak at the modulation-frequency \( fm \) with relative amplitude \( \delta fm \Delta t \) (spectrum of counts) and \( \delta \sin(\pi fm) \) (interval spectrum). In case the deviations cannot be described as sinusoids, a similar result holds due to the superposition principle which may be applied to (3) and (4).
The above analysis proves that for slowly modulated event series not too different from a regular pulse train, the interval spectrum and the spectrum of counts are equal to first order. In the calculation of the interval spectrum the intervals must be considered to be spaced at distances equal to the mean interval length \( \bar{T} \); in this way the Interval Spectrum, which is fundamentally a function of "cycles per beat", may be considered as a function of "cycles per second" or Hz. A similar calculation shows the heart-rate spectrum to be equal to both other spectra under the stated conditions. These conditions (slow and small modulation of the interval values) are often valid in heart-rate data.

3.5 Conclusions.

In this chapter the interval spectrum and the spectrum of counts of heart-rate variability data were compared. Both spectra lead to equivalent results (figs. 2 and 3). We explained this likeness analytically. The spectra were shown to be similar for event series that are slowly and slightly modulated. The absolute amplitudes of the peaks in the spectra can be compared only if the spectra are correctly scaled, i.e.

- if the spectrum of counts is calculated as the Fourier transform of the dimensionless signal \( \sum \delta(T - t_j) \)
- if the interval spectrum is calculated as the Digital Fourier Transform of the numbers \( \frac{I_k}{\bar{T}}, k = 0, 1, ..., N-1 \), which are considered to be spaced at distance \( \bar{T} \).

The smaller value of the interval spectrum (fig. 2b) as compared to the spectrum of counts (fig. 2d) for frequencies above 0.2 Hz can be explained by the theoretical ratio \( \sin(\pi \bar{T}) / (\pi \bar{T}) \) between these spectra. This ratio has value 0.68 for \( f = 0.5 \) Hz, which corresponds roughly with the figures. A difference in the amplitude of the higher harmonics in the spectra from data with a fixed respiration rate (figs. 3) is apparent. However, this difference does not lead to a preference for one of the spectra, as harmonics may appear in both spectra in various ways and so their presence or absence cannot be a decisive criterion.

For a sinusoidally modulated interval series, i.e., of the form \( I_k = \bar{T} + \delta \sin(2\pi f m n) \), the interval spectrum consists of one harmonic and the spectrum of counts of many (section 4.2). On the other hand, physiological event series are often considered as originating from an Integral Pulse Frequency Modulation (IPFM) model (Bayly, 1968; Hyndman and Mohn, 1975b; Rompelman et al., 1982; Chapter 2, or DeBoer et al., 1985b). For a sinusoidally modulated input-signal of an IPFM model, the interval
The spectrum of the generated event series consists of many harmonics, whereas the spectrum of counts has a single peak at the frequency of the sinus, plus peaks and sidebands around multiples of the mean repetition frequency (Bayly, 1968; Mohn, 1976; Chapter 4, or DeBoer et al., 1985a). Besides, a slightly modified IPFM model, e.g. with a refractory period (Hyndman and Mohn, 1975b), produces harmonics anyway. Finally, there is little reason to assume the respiratory influence on heart rate to be perfectly sinusoidal, so harmonics of unknown amplitude can be expected, as well in the spectrum of counts as in the interval spectrum.

We state that it cannot be decided which spectrum is to be preferred, though the spectrum of counts has the advantage of being related to the physiologically attractive IPFM model. In our opinion two questions are of importance in deciding which spectrum to use in heart-rate variability studies: how easily can the spectrum be computed, and why is the spectrum needed?

As to ease of calculation, the interval spectrum has the advantage that it can be computed directly with standard Fast Fourier Techniques; the spectrum of counts requires special computer programs (see French and Holden, 1971; Hyndman and Mohn, 1975b; Rompelman et al., 1982). The computation times of the two spectra need not be too different and will not be prohibitive. We computed for this chapter the spectrum of counts in a straightforward but lengthy way, as we first of all wanted to stress the fundamental differences and similarities of the spectra. For the same reason we computed the spectrum of counts at frequencies 0.001 Hz apart in figs.2d and 3d, though probably only values at $1/N\bar{t}$ apart do contain independent information (DeKwaadsteniet, 1982, p.35). Besides, if one computes $P_{C}(t)$ only for frequencies $f_k = k/N\bar{t}$ ($k = 1,2,3,..$), the contribution of the DC-component is indescernible, even when calculating the Fourier transform of the signal $p(t)$ (eqn.1) instead of $P(t)$ (eqn.1').

Both the interval spectrum and the spectrum of counts are useful if one is interested only in the presence and relative amplitudes of periodicities in the event series. However, a preference for one of the two spectra exists if one wants to study relationships between the event-series and another signal. If the heart-beat event series is to be related with a continuous signal like respiration or temperature and one needs cross-spectra or phase-spectra, the spectrum of counts should be used as it is the transform of a function of time. The interval spectrum, being
the transform of a signal which is essentially a function of interval number, is less applicable in this case. However, if one is interested in the relationship between heart interval and another variable that is defined on a beat-to-beat basis, e.g. systolic pressure, then the interval spectrum is the most logical choice.

In conclusion: the two described spectra are equally useful if periodicities in the heart-rate series are sought. The resulting spectra are similar. A choice between the two spectra can be made on grounds of ease of computation. Only if one needs to relate these periodicities to another signal, a definite choice for one of the spectra must and can be made.
3.1 Appendix 1: Some practical notes on the computation of the power spectrum

We estimated the Interval Spectrum by the periodogram, which was calculated by a Fast Fourier Transform (Bendat and Piersol, 1971; Kay and Marple, 1981). Another possible approach for the calculation of heart-rate variability spectra is the Auto-Regressive (AR-) method for spectral estimation (Box and Jenkins, 1976), as used in HRV studies by various authors (Bartoli et al., 1985; Baselli et al., 1985c; Cerutti et al., 1985; Kitney et al., 1984a,b). The latter method sometimes has a better spectral resolution (Kay and Marple, 1981). We stuck to the well-known periodogram technique because we needed to calculate cross spectra as well (Chapters 6, 7), and no standard algorithm was known to us to calculate complex cross spectra using the AR spectral estimation technique. In addition, the resolution of the periodogram-approach was sufficient for our purposes.

A few points of our spectral estimation procedure will be discussed in more detail:
1. Trend removal, 2. tapering, 3. zero-padding, 4. smoothing and reliability (number of degrees of freedom), 5. aliasing, and 6. data-editing.

1. **Trend removal**, including subtraction of the mean value, is used to avoid disproportionately large contributions in the low-frequency band. This low-frequency power can spread into higher-frequency regions, due to the limited length of the data record. For example: if a pure DC-component of value $A$ is present in a record of length $T$, its contribution to the power-spectrum is $(A \cdot \sin(\pi f T)/(\pi f T))^2$. The side-lobes of this contribution will mix with the other components in the spectrum, especially if $A$ is large or $T$ is short or both. We removed from the data the mean value and a linear trend.

2. **Tapering** is used to diminish the amplitude of the side lobes in the spectrum. It consists of a multiplication of the data-record with a tapering window. We applied a 10 per cent cosine taper after removal of the trend. The variance in the data is somewhat decreased by tapering; as the area of the squared tapering window is 0.875 times the area of a rectangular window, this factor 0.875 is needed to correct the spectral estimate (see eqn.2).

3. **Zero-padding** consists of supplementing the available data-points with zero-values up to a full power of two (1024 in our case), in order to permit the use of Fast Fourier Transform techniques. Its effect is an apparent increase in spectral resolution: $\Delta f = 1/(N_T r)$, with $N_T r$ the number of transformed points (including padded zeros). To preserve the correct scaling of the spectrum, the perio-
dogram must be multiplied by \( \frac{N_{tr}}{N_d} \) with \( N_d \) the number of original data points.

4. **Smoothing** of the raw spectrum is necessary because each spectral estimate has only two degrees of freedom (512 spectral estimates are derived from 1024 data points). Hence, some averaging procedure is needed to obtain more reliable estimates. Two equivalent possibilities exist: a) Spectra can be calculated over sections of the total data-set, and these spectra are subsequently averaged, or b) the spectrum is calculated over the full data-set, followed by averaging of neighbouring spectral values (which are now closer to each other than in case a). In this chapter we used the second approach with a rectangular averaging window of width \( 2M+1 \):

\[
\bar{P}_t(f_k) = \sum_{m=-M}^{M} P(f_{k+m})/(2M+1)
\]

For random noise input, neighbouring spectral values are independent and the number of degrees of freedom becomes \( 2(2M+1) \). The averaging over \( (2M+1) \) spectral values relates to the unpadded data; after zero-padding the averaging concerns \( (2M+1)(\frac{N_{tr}}{N_d}) \) values.

In Fig. A12 and in all later chapters triangular smoothing is used:

\[
\bar{P}_t^T(f_k) = \sum_{m=-M}^{M} P(f_{k+m})(M+1-|m|)/(M+1)^2
\]

To estimate the equivalent number of degrees of freedom for triangular smoothing, a \( \chi^2 \)-distribution is assumed for the averaged spectral values \( \bar{P}(f_k) \) (Blackman and Tukey, 1958). For random noise input, the expected value of the average is \( \bar{P}(f_k) \) and the variance is

\[
\hat{P}_t^2(f_k) \sum_{m=-M}^{M} \frac{(M+1-|m|)^2}{(M+1)^2} = \hat{P}_t^2(f_k)(2M^2+4M+3)/(3(M+1)^3)
\]

Under the assumption of a chi-square distribution, the equivalent number of degrees of freedom \( DF \) equals \( 2\times\text{average}^2/\text{variance} \):

\[
DF = \frac{6(M+1)^3}{(2M^2+4M+3)} = 3(M+1) + O(1/M^2).
\]

Hence, the number of degrees of freedom for triangular smoothing is around 75\% of the number for rectangular smoothing over the same frequency range. An example may illustrate the calculations involved: if 750 intervals are available (mean
value $0.8 \text{ s}$, and if a 31-point triangular smoothing is performed, the number of degrees of freedom is 48 and the smoothing extends over a range of $31 \times 1024 / 750 = 42.3$ spectral values. The spacing between the spectral values is $\Delta f = 1 / (N_{tr} \bar{I}) = 0.00122 \text{ Hz}$, and so the width of the smoothing band is $42.3 \times 0.00122 \text{ Hz} = 0.052 \text{ Hz}$. Under the stated conditions, two peaks in the spectrum at a distance much smaller than 0.05 Hz will not be resolved. The 95% confidence interval for the spectral values will for 48 degrees of freedom amount to $(0.7 \hat{P}(f), 1.5 \hat{P}(f))$ (Jenkins and Watts, 1968). However, this large confidence interval does not imply that also the area of spectral peaks can only be determined with such limited precision.

In fig.6-A2 (Chapter 6) some examples are given of spectra which are not smoothed.

5. **Aliasing** will occur in the interval spectrum and in the heart rate spectrum if the frequency of the interval modulation $f_{\text{mod}}$ is greater than the Nyquist-frequency $f_{\text{Nyq}}$ which equals half the sampling frequency: $f_{\text{Nyq}} = 1 / (2 \bar{I})$.

![Fig. 3-Ala Interval spectrum for heart-rate data from a subject breathing at a frequency (0.48 Hz) which is higher than half the heart rate. Due to aliasing the apparent respiratory peak is at 0.40 Hz (see text). Mean interval length is 1.14 s, corresponding to a heart rate of 0.88 Hz = 53 bpm. The standard deviation is 0.06 s. Horizontal: frequency (Hz), vertical: power ($s^2$/Hz)](image)

![Fig. 3-Ab Spectrum of counts for this record. The broken line represents the respiratory spectrum. No aliasing. (N.B.: Figs.1-2 show power spectra of the intervals $I_k$, not the amplitude spectra of normalized intervals $I_k/I$ as in the rest of this chapter.)](image)
An example of aliasing is shown in Fig. A1a: the mean interval length is 1.14 s, so $f_{\text{Nyq}} = 0.437$ Hz, while the subject had a fixed respiratory rate of 0.475 Hz. This explains the peak in the spectrum at $0.437 - (0.475 - 0.437) = 0.40$ Hz. The spectrum of counts for the same data set is shown in fig. A1b. This spectrum shows a peak at the respiratory frequency (0.475 Hz) and possibly a peak at the sideband frequency $0.40$ Hz = $1/(1.14 - 0.475)$. For more examples of aliasing, and for a discussion of sideband frequencies in the spectrum of counts, see Chapter 4.

Finally: normal beat-to-beat differences of R-R intervals in rest are less than around 100 ms. Hence important changes of the HRV spectrum are seen if even a few halved or doubled intervals are present. These may be due to trigger artefacts, missed beats and extrasystoles. A certain amount of data-checking and data-editing is therefore necessary.


The French-Holden algorithm is an efficient way to transform an event-series signal $p(t) = \sum_k \delta(t-t_k)$ into equidistantly sampled values of its low-pass filtered version $s(t)$ (French and Holden, 1971). These values are needed if one wants to use a Fast Fourier Transform to compute the low-frequency part of the spectrum of counts.

The signal $p(t)$ when passed through an ideal low-pass filter with cut-off frequency $f_{\text{max}}$, becomes the so-called Low-Pass Filtered Event Series or LPFES (Hyndman and Mohn, 1975b):

$$s(t) = \sum_k \sin(2\pi f_{\text{max}}(t-t_k))/(t-t_k)$$

If values of $s(t)$ are to be calculated at $N$ equidistant values $t_n = n \Delta t$ ($n = 1, 2, ..., N$), a straightforward application of this formula requires $N \cdot K$ sine-evaluations and divisions, with $K$ the number of events. French and Holden noted that if $\Delta t$ is chosen equal to $1/2f_{\text{max}}$, the amount of sine-evaluations is reduced to $K$, because then:

$$\sin(2\pi f_{\text{max}}(n \Delta t-t_k)) = \sin(2\pi f_{\text{max}}(n/2f_{\text{max}}-t_k)) = (-1)^{n+1}\sin(2\pi f_{\text{max}}t_k),$$

so only one sine-evaluation per spike is needed. The sampled values of the LPFES-signal $s(t)$ can thus be described as:

$$s(n \Delta t) = (2f_{\text{max}}/\pi) \sum_k (-1)^{n+1}\sin(2\pi f_{\text{max}}t_k)/(n-2f_{\text{max}}t_k)$$

(If $t_k$ is such that $n = 2f_{\text{max}}t_k$, the expression after the summation-sign reduces to $\pi$. )
Using the French-Holden algorithm and a Fast Fourier Transform, we estimated the (power) spectrum of counts up to $f_{\text{max}} = 0.5$ Hz. It is shown in Fig. A2 (dashed line), together with the (power) spectrum of intervals (solid). These spectra are smoother than the amplitude spectra in Figs. 2b, d because of the triangular smoothing which we used in Fig. A2.

A comparison of spectra of counts calculated by means of the straightforward equation 1 and by means of the French-Holden algorithm is given by Figs. 3c and Alb in Chapter 4.

**Fig. 3-A2** Interval Spectrum (solid line) and Spectrum of Counts (dashed) as calculated by the French-Holden algorithm. Data as in Fig. 2. The difference at higher frequencies can be explained by the theoretical factor $(\sin(\pi f)/\pi f)^2$ between the spectra.