Beat-to-beat blood-pressure fluctuations and heart-rate variability in man: physiological relationships, analysis techniques and a simple model

de Boer, R.W.

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Chapter 6

Relationships between short-term blood-pressure fluctuations and heart-rate variability:
1. A spectral analysis approach.

This chapter describes a method to study blood-pressure and heart-rate variability by spectral analysis methods. We show power spectra of RR-intervals, systolic, mean, diastolic and pulse pressures, and also cross-spectra (coherence spectra and phase spectra) of RR-intervals against the various pressure variables. The spectra are discussed in physiological terms.

6.1 Abstract

A method to attribute the short-term variability of blood pressure and heart rate of resting subjects to their various causes, using spectral techniques is presented. Power spectra and cross-spectra are calculated for beat-to-beat values of RR-interval and blood pressure from resting subjects. Interval values as well as systolic, mean, and pulse pressures show variations linked to respiration and to the so-called 10-second-rhythm. The diastolic pressure values are scarcely influenced by respiration in the normal respiratory range (0.20–0.35 Hz), but do show 10-second variability. Relationships between pressure and interval variability become manifest in cross-spectra, which indicate that the 10-second variability in systolic pressure leads the interval variation by two to three beats; however, no such lag is found between the respiration-linked variations in systolic pressure and intervals. It is argued that the technique presented provides a critical test for models of the fast regulation of the cardiovascular system.

6.2 Introduction

Both heart rate and blood pressure in resting subjects are not constant, but fluctuate around mean values. The short-term variations in RR-interval (RRI) and blood pressure (BP) are mainly due to respiration and to the so-called 10-second-rhythm. Respiratory sinus arrhythmia, noticeable as respiration-linked variations in heart
rate, is a well-known phenomenon (e.g., Hirsch and Bishop, 1981), as is the respiratory variation in blood pressure (Dornhorst et al., 1952). The frequency of these variations is in resting persons usually between 0.20 Hz and 0.35 Hz, corresponding to 12-20 respirations per minute.

The origin of the 10-second-waves in the blood pressure is not well understood, although several tentative explanations have been put forward (Penaz, 1978; Koepchen, 1984). Related to these pressure waves are rhythmic variations in heart rate having the same period (Sayers, 1973). Periodical fluctuations in heart rate and blood pressure with still lower frequencies (less than 0.05 Hz, i.e., periods of more than 20 seconds) have been described and are possibly caused by properties of the thermoregulatory system (Sayers, 1973).

The relationship between spontaneous short-term variations in blood pressure and RR-intervals has received little attention in the physiological literature. The aim of our study is to extract information on properties of the cardiovascular system from these pressure and interval fluctuations. In a previous chapter (Chapter 5, or DeBoer et al., 1983) we presented a simple model of the cardiovascular system that connected BP- and RRI-variability. The various sources of variability were not distinguished in that chapter.

In the present chapter we use spectral analysis techniques to differentiate between variability related to the 10-second-rhythm and to respiration. These techniques are useful to shed light on the still unanswered question: are the respiratory pressure waves the cause or the effect of the respiratory arrhythmia, or are the respiratory BP- and RRI-variations both caused directly by some central drive (Melcher, 1976)? A similar question can be asked regarding the 10-second waves (Koepchen, 1984).

We applied spectral analysis methods to blood-pressure and RR-interval data from normotensive, resting subjects. Blood pressure was measured in some subjects by a noninvasive method, in others intra-arterially. In the discussion we compare our method with previously published techniques for applying spectral analysis to blood pressure, and we indicate for which purposes the method is useful. A tentative explanation of part of the results is given in a following chapter (Chapter 7, or DeBoer et al., 1984d).
6.3 Methods

6.3.1 Data acquisition

Two sets of blood pressure and interval data were used. One set comprised data obtained by the technique of indirect registration of finger blood pressure after Penaz (Penaz, 1973, see appendix A). This technique enables continuous, noninvasive recording of blood pressure, measured as the imposed cuff-pressure needed to keep the vascular volume in the finger constant at its unloaded value. The apparatus we used was developed by Wesseling and coworkers ('Fin.A.Press'; Wesseling et al., 1982, 1985; Settels and Wesseling, 1985) and has been validated by comparison with intra-arterial blood pressure data (Molhoek et al., 1983). An apparatus based on the same principle has been described and evaluated by Yamakoshi et al. (1980, 1982). During data collection the ECG and a respiratory signal from a nose-thermistor were recorded as well. The subjects were seated in a comfortable chair, while 20 minutes registrations were made. The pressure signal was digitized and processed by computer; the RR-intervals were derived from the ECG. In addition, intra-arterial blood pressure data were available from a project for quantifying the 24-hour heart-rate and blood-pressure variability in normal subjects and in hypertensive patients (VanMontfrans et al., 1982, 1984). Here the pressure was measured in the brachial artery; the RR-interval was estimated as the time between two successive systolic upstrokes of the pressure signal. We analyzed signals that had been recorded during periods of rest (e.g., reading, watching tv), as evidenced by the subjects detailed diary.

6.3.2 Data reduction

We extracted the following beat-to-beat values from the pulsatile pressure signal (each occurring during RR-interval $I_n$; see Fig. 1): systolic pressure $S_n$, diastolic pressure $D_n$, pulse pressure $P_n$, and mean pressure $M_n$. In this way a set of five numeric values was obtained for each heart beat: $I_n$, $S_n$, $M_n$, $D_n$, $P_n$.

![Fig.6-1  ECG and blood pressure registration, explaining the notation as used in this paper. Systolic ($S_n$), diastolic ($D_n$), pulse ($P_n$) and mean ($M_n$) pressure occur during RR-interval $I_n$.](image-url)
6.3.3 Power spectra

Power spectra were estimated for all five variables. For the spectral analysis we considered successive values of S, M, D, and P each to be equidistantly spaced. This leads to spectra that are functions of 'cycles per beat'. We took this spacing equal to the mean interval length \( \bar{t} \); the spectra can then be interpreted as functions of cycles per second, or Hz (Sayers, 1973; Chapters 3, 4, or DeBoer et al., 1984 and 1985a).

After subtraction of a linear trend from the data, a Digital Fourier Transform (DFT) technique was used to estimate the spectra. Owing to the apparent sampling distance of \( \bar{t} \), the spectrum is defined for the range 0-1/2\( \bar{t} \) Hz; in our figures we always present the range 0-0.5 Hz. As our interest is in the short-term fluctuations, i.e. periods shorter than approximately 20 s, we did not consider the part of the spectrum where the frequency is less than 0.05 Hz. The spectrum was smoothed by a 31-point triangular window, leading to about 48 degrees of freedom in the estimation (Chapter 3.A1). 10 - 15 minutes of registration were used for the estimation of each spectrum, corresponding to 500 - 1000 heart beats. For further details on the estimation of the spectrum see Chapter 3 or DeBoer et al., 1984. In those registrations where the respiratory signal was available we also estimated the respiratory spectrum by Fourier transformation of the thermistor signal.

6.3.4 Cross-spectra

Cross-spectra were calculated between the blood pressure variables S, M, D, and P, and the RR-interval I (Jenkins and Watts, 1968, Chapter 9). The pressure values \( B_n \) (B = S, M, D or P) were transformed into \( X_B(f) \), using a DFT; similarly \( X_I(f) \) was calculated from the interval values. The power spectra of \( B \) and I are \( C_{BB}(f) = X_B(f)^2 \) and \( C_{II}(f) = X_I(f)^2 \), respectively. The cross-spectrum of \( B \) against I is defined as \( C_{BI}(f) = X_B(f)X_I(f)^* \), where the asterisk denotes the complex conjugate. We wrote \( C_{BI}(f) = L(f) - iQ(f) \), with \( L(f) \) the co-spectrum and \( Q(f) \) the quadrature spectrum; the smoothed squared-coherence estimator is:

\[
k^2(f) = \frac{(L(f))^2 + (Q(f))^2}{C_{BB}(f)C_{II}(f)}
\]

where the bar denotes smoothing (triangular in our case). The smoothed estimator of the phase spectrum is:

\[
\Phi(f) = \arctan\left(\frac{\overline{Q(f)}}{\overline{L(f)}}\right)
\]
The Fortran-function ATAN2 was used to obtain values for $\Phi(f)$ in the range $-180^\circ$ to $180^\circ$.

6.3.5 Note on the interpretation of cross-spectra

The interpretation of a power spectrum is straightforward: it shows the amount of variation in the data for each frequency. A cross-spectrum between two signals can be considered to consist of two parts, the coherence spectrum and the phase spectrum. The (squared) coherence spectrum $k^2(f)$, having values between 0 and 1, is a measure for the correlation between the variations of two signals around the frequency $f$. It is to be compared to the squared correlation coefficient $r^2$ as used in linear regression analysis. The phase spectrum $\Phi(f)$ indicates at each frequency $f$ the phase-difference (lead or lag) between the signals. All phase spectra we present have been scaled in the region $-180^\circ$ till $180^\circ$.

In our figures a negative value of $\Phi(f)$ implies the pressure variation (S, M, D, P) to lead the interval variation I at this frequency; for a positive phase the reverse holds.

If the coherence is low for a certain frequency, the phase at this frequency cannot be estimated reliably. This is to be compared to the unreliability of the estimation of a regression coefficient if the correlation coefficient is low. As our estimate has about 48 degrees of freedom, the 95% confidence interval for the phase-angles is $\pm 7^\circ$, $\pm 13^\circ$ and $\pm 26^\circ$ for $k^2(f)$ equal to 0.8, 0.5 and 0.2, respectively (Jenkins and Watts, 1968, figure 9.3).

The phase between two signals is only defined up to a multiple of $360^\circ$, which implies that in phase spectra the horizontal line at $\Phi=180^\circ$ may be equated with the line at $\Phi=-180^\circ$. In the phase spectra lines between successive values were suppressed if the difference was larger than $180^\circ$, because we supposed that in these cases the phase had passed over the $-180^\circ$ border to reappear at $+180^\circ$, or vice versa. If no such method is used, confusing vertical lines may appear in the phase spectra.

For illustrative purposes, in Appendix 1 cross-spectra are shown for simulated data, consisting of the sum of two sinusoids of different frequencies.

6.4 Results

To give an impression of the amount of variability found in data from different subjects, we show results from three subjects. The results in Fig.2 and Fig.3 are from men, 32 years and 26 years of age, respectively. Blood pressure was measured using the noninvasive method (section 3.1). Fig.4 is from a healthy woman.
Fig. 6-2  For legend see opposite page.
Fig. 6-2a Power spectrum of intervals (solid line) and respiration (dashed). The scale of the respiratory spectrum is in arbitrary units. The interval spectrum shows peaks at the respiratory frequency and around 0.1 Hz (10-second-rhythm).

Fig. 6-2b Power spectrum of systolic pressures (solid) and mean pressures (dashed). Again, two peaks are visible.

Fig. 6-2c Power spectrum of diastolic pressures (solid) and pulse pressures (dashed). In the spectrum of diastolic pressures the respiratory peak is absent.

Fig. 6-2d Squared coherence spectrum $k^2(f)$ (dashed line, between 0 and 1), and phase spectrum $\Phi(f)$ (solid line, between $-180^\circ$ and $180^\circ$) of systolic pressure vs. interval. A high coherence implies a strong link between pressure and interval variations, as is the case around 0.1 Hz and in the respiratory region. When the coherence is high (0.9), the phase is reliably estimated (heavy line). The phase is negative when pressure variations lead interval variations (e.g., at 0.1 Hz). In the region of respiratory frequencies, the phase difference between pressure and interval variations is small.

Fig. 6-2e Squared coherence spectrum (dashed line) and phase spectrum (solid line) of mean pressure vs. interval.

Fig. 6-2f Squared coherence spectrum (dashed line) and phase spectrum (solid line) of diastolic pressure vs. interval. The phase spectrum shows no trend.

Fig. 6-2g Squared coherence spectrum (dashed line) and phase spectrum (solid line) of pulse pressure vs. interval.

(23 years of age), whose blood pressure was measured intra-arterially. For the estimation of the spectra 950, 1020 and 650 beats were used in figs. 2, 3 and 4, respectively.

6.4.1 Power spectra (Figs. 2a-c, 3a-c, 4a-c)

Figs. 2a, 3a, 4a show spectra of respiration (dashed) and of intervals (solid line). The respiration spectrum shows two peaks around 0.25 Hz and 0.30 Hz in fig.2a, while the narrower peak in fig.3a indicates a more regular respiration. In the case of fig.4 no respiratory signal was available (see section 3.1). In the interval spectrum peaks at the respiratory frequency and around 0.1 Hz (10-second-rhythm) are clearly visible in all three figures. A low-frequency component is present in all spectra.
Fig. 6-3a-g As Fig. 2a-g, for blood pressure and interval data from another subject.
In figs.2b, 3b and 4b the spectra of the systolic pressure values (solid line) and of the mean pressure (dashed) are shown. Peaks due to respiratory influence and to the 10-second-rhythm are visible, but the amplitudes of the peaks are rather different for the three subjects.

Figs.2c, 3c and 4c show the spectra of diastolic pressures (solid) and of pulse pressures (dashed). In the pulse-pressure spectrum the 0.1 Hz-peak is smaller than the respiratory peak, but in the spectrum of diastolic pressures the respiratory influence is small (fig.3c) or absent (figs.2c, 4c).

6.4.2 Cross-spectra (Figs.2d-g, 3d-g, 4d-g)
In these figures the dashed line represents the value of the squared coherence $k^2(f)$ between 0 and 1. The solid line is the estimated phase ($-180^\circ$ till $+180^\circ$; positive if the interval leads). A heavy line indicates that the coherence is high ($k^2(f) \geq 0.5$) and that hence the phase is reliably estimated (section 3.5). In all cross-spectra a high coherence value is seen around 0.1 Hz and in the region of the respiratory frequency. The coherence $k^2(f)$ reaches values up to 0.95, indicating a close link between variation of the RR-interval and of the different pressure parameters. For low frequencies ($f < 0.05$ Hz), the coherence is small, indicating dissociation of blood-pressure and interval variability in this range.

The phase spectra show a positive trend for the spectra of $S$ against $I$ (Figs.2d, 3d, 4d), $M$ against $I$ (Figs.2e, 3e, 4e), and somewhat less for the spectrum of $P$ against $I$ (Figs.2f, 3f, 4f). Variations in systolic pressure lead interval variations by approximately $60^\circ$-$90^\circ$ for 0.1 Hz (Figs.2c, 3d, 4d). For higher frequencies, i.e. in the range of normal respiration (0.30 Hz), the phase difference between $S$ and $I$ becomes small. The phase seems to increase for still higher frequencies, but here the coherence is low and therefore the estimated phase is not reliable. However, in other subjects who breathed at higher frequencies, definite positive phase-differences were seen in this frequency-region. The phase spectrum of $D$ against $I$ (figs.2f, 3f, 4f) does not show a trend, but fluctuates around $-90^\circ$.

6.5 Discussion
We presented power spectra of RR-interval data and blood pressure data from three resting subjects during unrestrained respiration. The spectra were estimated from successive beat-to-beat values of RR-interval $I$ and of systolic, mean, diastolic and pulse pressure ($S$, $M$, $D$ and $P$, respectively). We also presented coherence and
Fig. 6-4a-g  As Fig. 2a-g, for blood pressure and interval data from another subject. In this case the blood pressure was measured intra-arterially, and no respiratory signal was available.
phase spectra of the various pressure variables against RR-interval. Although a variability exists between spectra from different subjects, the following results were consistently found:

1. A peak at the respiratory frequency exists in the interval spectrum and in the spectrum of the pressure variables S, M and P. This peak is often absent in the spectrum of diastolic pressures D (cf. Fiser et al., 1978; Penaz et al., 1978b).
2. A peak at the frequency of the 10-second-rhythm is seen in all spectra, but is often small in the spectrum of pulse pressures P.
3. A high coherence exists between pressure variations and interval variations around 0.1 Hz as well as in the region of respiratory frequency.
4. The phase spectra of S and M against I show a trend from negative values around 0.1 Hz (i.e. pressure leads) to positive values above 0.35 Hz (i.e. interval leads). This trend is less outspoken in the phase spectrum of P against I. For 0.1 Hz, systolic pressure variations lead interval variations by 60°-90°, or approximately 1/5th period. This corresponds to 10/5=2 s, or two to three beats (the mean interval being 0.93 s, 0.82 s and 0.71 s in figs.2,3 and 4, respectively).
5. The phase spectrum of D against I fluctuates around -90°.

All spectra show the presence of low-frequency variation in both interval and pressure data. A distinct peak, attributed to thermoregulation (Sayers, 1973), was not always seen (but see, e.g., fig.3b). This may be partly due to the spectral-averaging procedure we used. The coherence between interval and pressure was low for frequencies under 0.05 Hz.

An explanation of some of the above results is given in a following chapter (Chapter 7, or DeBoer et al., 1985d). Here we only relate the presented spectral method to other published techniques, and we discuss the relevance of the method. As to the reliability of our results, the following can be said. For the purpose of spectral analysis, the interval values and the pressure values were considered to be equidistantly spaced at distance \( \bar{I} \) from each other. This technique is justified if the interval variations are small and if no heavy trend in the interval values occurs (Chapter 3, or DeBoer et al., 1984). The Penaz-method we used for obtaining indirect blood pressure measurements has been validated on several occasions (Molhoek et al., 1983; Yamakoshi et al., 1980 and 1982). The reliability of this method is best in resting subjects with relatively constant blood pressure values, as is the
case in our study. The spectra as calculated from intra-arterial pressure values (fig.4) are essentially equal to the spectra derived from indirect measurements (figs.2,3).

The blood pressure spectra presented in this chapter are not related to the spectrum of the pulsatile pressure signal (Taylor, 1966; Daniels et al., 1983). Whereas our approach is concerned with the fluctuation of the pressure variables around their mean values, the spectrum of the pressure signal is dominated by its pulsatile shape. Hence both kind of spectra are useful for different purposes.

Only few papers are known to us which present cross-spectra of blood pressure against interval (or heart rate). Zwiener and coworkers (1978, 1982) and the group of Wesseling and Settels (Settels, 1980; Wesseling et al., 1983) computed cross-spectra of noninvasively recorded mean blood pressure against interval, and against heart rate, respectively. Both groups obtained a mean blood pressure signal by low-pass filtering of the pulsatile signal. Penaz and coworkers (Penaz et al., 1978b; Fiser et al., 1978) showed cross-spectra of indirectly measured systolic, diastolic and pulse pressure against interval. They interpolated between successive systolic values, and similarly for diastolic and pulse pressures. In all papers high coherences were found around 0.1 Hz and in the region of respiratory frequencies. This corresponds with our results.

Comparison of the phase spectra in these papers with the ones we present is not well possible. In the papers of Zwiener and coworkers and of Penaz and coworkers phase spectra are only presented for angles between 90° and -90°; in our opinion, this is an unnecessary restriction, because the phase spectrum is unambiguously defined between 180° and -180°. In addition, in these papers the alignment of pressure values and interval values is different from ours (fig.1). Such a difference in alignment leads to disparate phase spectra. For example, whereas we bring together the pressure values occurring during one RR-interval (Fig.1), this might also be done with the values occurring between two systolic upstrokes -- which implies a shift of one beat for the diastolic values, corresponding to one-third period or 120° phase shift at \( f = 1/3T_0 = 0.33 \) Hz and 36° shift at \( f = 1/10T_0 = 0.1 \) Hz. The resulting phase spectrum of D against I would be entirely different from ours. Likewise, the conversion of systolic, diastolic and interval values into continuous signals can easily lead to a shift of one beat (Chapter 2, or DeBoer et al., 1985b) and hence to a different phase spectrum. If heart rate values (inverse intervals) are used instead
of intervals, all phase angles will be augmented by 180°.

The use of beat-to-beat values in cross-spectral analysis is in our opinion advantageous as it keeps separated the interval-variability and the pressure-variability. In addition, the obtained phase spectra are open for physiological interpretation (Chapter 7, or DeBoer et al., 1985d). Another feature of our approach is the large data reduction obtained (only a few values per heart beat).

The cross-spectral technique we presented is useful in the study of data from subjects who have a free, hence irregular respiration. In this case the fluctuations in blood pressure and heart rate must necessarily be studied with statistical techniques, e.g., with spectral analysis methods. If, however, the subject has a prescribed respiratory rhythm, relations between respiration-linked interval and pressure variations can in principle be found without using cross-spectral methods.

Any realistic model that explains the 10-second-rhythm (Hyndman et al., 1971; Kitney, 1980; Wesseling et al., 1983, 1985), or the respiration-induced variations (Kitney et al., 1982a) in blood pressure and RR-intervals should be able to produce power and cross-spectra that are similar to the ones shown in figs. 2, 3 and 4. So the presented cross-spectral technique can be used as a check for such models. More data are needed concerning the changes in power and cross-spectra caused by, e.g., pathophysiological conditions or pharmacological intervention. The 0.1 Hz peak in the interval spectra of conscious dogs disappeared after pharmacological blockade of the parasympathetic nervous system (Akselrod et al., 1981). Will the 0.1 Hz peak in the blood pressure spectra also be absent under these conditions? Akselrod and coworkers (1981) wondered about the small number of efforts to characterize mathematically the physiological mechanisms that generate the — clinically relevant — beat-to-beat fluctuations in haemodynamic parameters. We agree with them.
6. Appendix A1 Cross-spectra of simulated data, and the effect of smoothing

In this appendix we present a few cross-spectra of simulated data to give the reader some feeling for properties of these spectra. This seems useful because cross-spectra are at first sight not easily interpreted. In fig. A1 we show power, coherence and phase spectra of simulated data, consisting of the sum of two sinusoids and noise. "Intervals" I, "systolic-pressures" S and "mean values" M were generated by:

\[
I_n = 1 + 0.01 \sin(2\pi n/10) + 0.015 \sin(2\pi n/4) + \delta_n,
\]

\[
S_n = 100 + \sin(2\pi (n/10-1/10)) + 2 \sin(2\pi (n/4+1/4)) + \epsilon_n,
\]

\[
M_n = 75 + \eta_n,
\]

with \( \delta_n, \epsilon_n \) and \( \eta_n \) independent, Gaussian noise with mean zero and standard deviation 0.01, 2, 2, respectively. Hence, I and S contain, apart from the noise, two sinusoids at frequencies 1/(10.1)=0.1 Hz and 1/(4.1)=0.25 Hz. At 0.1 Hz I leads one-tenth of a period, or 36°, with respect to S; at 0.25 Hz I lags 90°. The signal M consists only of white noise. (N.B. these simulated signals are not meant to resemble actual blood-pressure and interval data.)

The calculated power and cross-spectra for these simulated signals are presented in fig. A1. 980 values were used for the calculations; the triangular smoothing window had a width of 31 spectral values or 0.032 Hz, which leads to 48 degrees of freedom (section 3.1). The power spectra of I and S (figs. A1a,b, solid lines) are seen to contain the expected peaks at 0.1 Hz and 0.25 Hz; the spectrum of M (fig. A1b, broken line) shows no peaks. The coherence spectrum of I against S (fig. A1c, broken line) has high values only at 0.1 Hz and 0.25 Hz. The phase spectrum (solid line) has for these frequencies the values +40° and -89°, which corresponds quite well with the theoretical values of +36° and -90°. As was to be expected, the coherence spectrum of I against M (fig. A1d, broken line) has low values over the whole frequency range and accordingly the phase spectrum (solid line) is indefinite.

It may be concluded that the results of the spectral estimation procedure as used in fig. A1 give the correct answers for these simulated signals.
Fig. 6-A1  Power spectra (a,b) and cross-spectra (c,d) of simulated 'interval' values I and 'pressure' values S and M. Both intervals and systolic values consist of the sum of two sinusoids (0.1 Hz and 0.25 Hz) and noise. The simulated mean-pressure values consist only of noise. The power spectra of the interval values (fig.A1a), of the systolic-pressure values (fig.A1b, drawn line) and of the mean-pressure values (fig.A1c, dashed line) are shown. The numbers indicate the mean value and standard deviation (s and mm Hg for intervals and pressure, respectively). Horizontal: frequency (Hz), vertical: power (s²/Hz and mm Hg²/Hz).

The cross-spectrum of systolic values against intervals has an appreciable coherence (fig.A1c, dashed line, left hand scale) only around the frequency of the sinusoids (0.1 Hz and 0.25 Hz). The values of the phase spectrum for these frequencies (drawn line, right-hand scale (degrees)) correspond with the theoretical ones (see text). The coherence spectrum of mean pressure against intervals has a low value throughout (fig.A1d, dashed line), and accordingly the phase spectrum (drawn line) is undetermined and wanders.
An aspect of cross-spectra is illustrated in fig. A2; here the power spectra of I (fig. A2a) and S (fig. A2b) and the cross-spectrum of I against S (fig. A2c) are given for the data of fig. 2, but without smoothing (Chapter 3.A1.4). The power spectra are heavily fluctuating, but their general shape is still recognizable (cf. fig. 2a,b). The unsmoothed estimator of the squared-coherence is (section 6.3.4):

$$k^2(f) = \left| C_{Bf}(f) \right|^2 / (C_{BB}(f) C_{If}(f)) = \left| X_B \cdot X_f \right|^2 / (X_B \cdot X_B^T \cdot X_f \cdot X_f^T) = 1$$

Hence the coherence has value 1 over the whole frequency range if no smoothing is performed (fig. A2c, broken line); this is to be compared to a correlation coefficient equal to 1 if only two points are considered in a linear regression. The shape of the heavily fluctuating phase spectrum (solid line) still has some resemblance to fig. 2d. Comparison of fig. A2 and fig. 2 shows that the smoothing of the spectra is indeed useful, but that even without smoothing some characteristics of the spectrum remain visible.

Fig. 6-A2 Unsmoothed power and cross-spectra for the data of fig. 2. Fig. A2a and fig. A2b are the power spectra of intervals and systolic pressures, respectively (cf. fig. 2a,b). Fig. A2c shows the unsmoothed coherence spectrum (dashed line) and phase spectrum (drawn line; cf. fig. 2d). The coherence spectrum has value 1 throughout, as predicted by theory. Both the power spectra and the phase spectra are recognizable, notwithstanding the lack of smoothing.