On the evolution of massive close binary stars in stellar populations

Pols, O.R.

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Monte-Carlo simulations of binary stellar evolution in young open clusters

O.R. Pols and M. Marinus

Abstract

We present a model for calculating the evolution of close binaries, including mass exchange episodes, in an approximate way. Our model covers all parts of parameter space (initial masses and orbital separations) for binaries with primaries more massive than $2M_\odot$, and calculates the evolution until both stars have become compact objects. The model is described in detail and its approximations and uncertainties are discussed.

The model is applied in Monte-Carlo simulations of a large population of stars, representing the initial population of young open star clusters. The colour-magnitude diagrams of these synthetic clusters reproduce several observed features of real young open clusters that can be attributed to the presence of binaries.

In particular, blue stragglers are produced as remnants of mass exchange in close binaries. We identify five types of blue straggler. Four of these types are binaries with helium-star, white-dwarf, neutron-star, and stripped main-sequence-star companions; the fifth type are single, merged main-sequence stars. We derive the expected numbers and properties of each of these blue-straggler types as a function of cluster age. The numbers and properties of our synthetic blue stragglers (rapid rotation, small or absent radial-velocity variations) are consistent with the observed incidence and characteristics of blue stragglers in clusters younger than about 300 Myr. In contrast, our model appears to be unable to account for all the observed blue stragglers in intermediate-age clusters (between 300 and 1500 Myr).

Other observed features that are reproduced by the inclusion of binaries are: the occurrence of a second main sequence, the appearance of 'yellow straddler' giants between the giant branch and the turn-off point, and the occurrence of 'blue interlopers' below the main sequence.

5.1 Introduction

The observed colour-magnitude diagrams (CMDs) of star clusters show a number of features that cannot be explained by the simple model that a cluster consists only of single stars of the same chemical composition and age. A large number of stars do not fall on the isochrone appropriate for the age of the cluster. Most striking are the blue stragglers, which are bluer
and often brighter than the main-sequence turn-off, indicating that either their lifetime is longer than expected for their mass, or that they were formed later than the other stars of the cluster. Other remarkable features in the CMD are: a broadened and sometimes double main sequence; stars hovering between the turn-off point and the giant branch; red or yellow stars which fall below the giant branch; and sometimes stars to the blue of the main sequence and below the turn-off point. These 'non-conformist' stars cannot all be explained as observational errors or non-members (with the possible exception of the last-mentioned two features, which are often close to the maximum observable magnitude).

Several effects may be responsible for the anomalous positions of these stars (most of them have been proposed as explanations for the blue stragglers in particular, as we will discuss in § 5.2):

1. Binary stars, i.e., unresolved unevolved binaries (and maybe higher-order multiples), as well as the products of mass exchange in close binaries.

2. Variations in age of the member stars (i.e., extended star formation history).

3. Variations in chemical composition within one cluster.

4. Divergence in the evolution of single stars of the same mass and composition, e.g., due to rotation or magnetic fields.

5. Dynamical effects due to the high star densities in clusters.

The assumption that all stars in a cluster have equal initial compositions is well-founded because a cluster is formed from a single molecular cloud. The assumption that all cluster stars are single is clearly wrong, since a large number of spectroscopic binaries are observed among cluster stars. In fact, the fraction and distribution of binaries in young open clusters seems to be similar to that in field stars, so the effects of binaries (unevolved binaries, as well as the effects of mass transfer) should not be neglected.

In this paper we investigate the effect of primordial binaries on the evolution of the CMD of young open clusters by means of a model for close-binary evolution that includes mass exchange. There are two reasons why we limit ourselves, at least in this paper, to young clusters (i.e., not older than the Hyades, about 1 Gyr): (1) star densities are sufficiently low that dynamical effects, like collisions and disruption and formation of binaries by encounters, are not important and do not complicate things, and (2) in young open clusters there is good evidence for the presence of a significant fraction of primordial binaries, making their inclusion not only interesting but, as we shall see, essential for a good understanding of the evolution of the cluster as a whole.

Earlier work along these lines was done by E.P.J. van den Heuvel (1969, unpublished), Klumper (1981) and Collier & Jenkins (1984) who treated, however, only a limited part of the parameter space for close binary evolution. Their work showed that indeed blue stragglers can be produced as a result of binary mass exchange.

In our model for close binary evolution our main objective is to cover all parts of parameter space, i.e., to be complete rather than very accurate. This means that we make the best approximation we can for the evolutionary scenario of a binary for every possible set of orbital parameters. Many detailed calculations of binary evolution have been done during the past 25 years (see, for instance, reviews by Paczynski 1971a, Thomas 1977, and Van den Heuvel 1993). Unfortunately the parameter space is not covered homogeneously by these calculations, and the work has concentrated on systems that are 'easy' to calculate: binaries with more or less stable mass transfer, for which the assumption of conservation of total mass and angular momentum is fairly safe. For these systems there is now a rather clear and quantitative picture of how
5.2 Young open clusters and blue stragglers

Following Abt (1985), we make a broad division of open clusters into old (older than the Hyades, about 1400 Myr by our calibration; see § 5.5), intermediate (older than NGC 2516, up to and including the Hyades) and young (younger than, and including, NGC 2516). An extensive analysis of the colour-magnitude diagrams of 75 young and intermediate-age open clusters was carried out by Mermilliod (1981a). He divided these clusters into 14 age groups, each containing 4 to 8 clusters, and presented composite CMDs for each of these age groups. Although some of these clusters have since been studied in more detail, to our knowledge Mermilliod’s is still the most extensive study of a large sample of young clusters, and therefore it will serve as the basis for the observational test of our simulations (§ 5.5.3).

In Figs. 5.1 and 5.2 we present, as examples of intermediate-age and young open clusters, respectively, composite $(B-V, M_V)$ diagrams of two cluster groups from Mermilliod (1981a). Fig. 5.1 shows the CMD of the Pleiades age group, containing data of 8 clusters, and Fig. 5.2 shows the CMD of the Hyades age group (4 clusters). These Figures can be compared with our simulated CMDs, which will be presented in § 5.5.1. One clearly sees in these observed CMDs the broadened MS band (and, especially in Fig. 5.1, a second main sequence about 0.75 above the single-star sequence), a few ‘yellow straggler’ giants between the giant branch and the top of the main sequence, and several blue stragglers.

Among old open clusters, blue stragglers are quite common and numerous (e.g., M67 has at least 9 and NGC 7789 has 21 blue stragglers, Wheeler 1979b), and relatively well studied. In comparison, they are a rather rare phenomenon in the younger clusters. The first, and so far the only, comprehensive observational study of blue stragglers in young and intermediate open clusters was made by Mermilliod (1982). He presents a list of 39 blue stragglers in 75 clusters and discusses their properties. Mermilliod points out that blue stragglers are common to clusters of all ages, although the frequency-per-cluster is very low for the youngest clusters and increases with age. Blaauw (1992) presents evidence that, in young OB associations, OB runaway stars are blue stragglers in their parent associations.

Mermilliod (1982) finds that many blue stragglers have ‘anomalous’ spectra. In the young clusters, the blue stragglers (with spectral types O6–B4) show emission lines (Be or Of) in about half of the cases, and often have large rotational velocities, $v \sin \iota \gtrsim 100$ km/s. In contrast, the blue stragglers in intermediate-age clusters (with spectral types B5–A1) have no emission lines but often show Ap and Bp characteristics and have values of $v \sin \iota \lesssim 50$ km/s. Additional observations by Abt (1985) confirmed and strengthened the conclusion that intermediate-age blue stragglers have peculiar spectra and small rotational velocities.
Figure 5.1: Composite ($B - V, M_V$) diagram of the Pleiades age group (reproduced from Fig. 10 in Mermilliod 1981a). The solid line is the zero-age main sequence. Blue stragglers are indicated by inverted solid triangles; for an explanation of the meaning of other symbols see Mermilliod (1981a).
Figure 5.2: Composite \((B-V, M_V)\) diagram of the Hyades age group (reproduced from Fig. 2 in Mermilliod 1981a); see Fig. 5.1.
Another remarkable feature is the apparent paucity of binaries among blue stragglers. Among the intermediate-age stragglers, Conti et al. (1974) concluded that the three blue stragglers they had observed (in Praesepe, the Hyades and NGC 6475) have constant radial velocities. Hintzen et al. (1974) were unable to detect variations greater than 3 km/s on a time scale of 75 days for two blue stragglers in NGC 6633. For the young clusters no systematic searches for radial velocity variations of blue stragglers have been undertaken. However, there is one interesting exception: θ Car in IC 2602 (B0Vp) is a spectroscopic binary with a short orbital period and a moderate eccentricity ($P = 2.14$ d and $e = 0.24 \pm 0.08$, Garcia et al. 1988). Furthermore, it has anomalous CNO abundances and a higher He abundance than other cluster members. Another young straggler, HR 3147 in NGC 2516 (B2Vne), is also suspected of radial-velocity variations (Dachs 1972; Abt & Levy 1972).

Several hypotheses have been advanced over the past three decades to explain the existence of blue stragglers (see the reviews by Wheeler 1979a and Livio 1993). One of the earliest and most attractive hypotheses, originally advanced by McCrea (1964), is that blue stragglers are formed by mass transfer in close binaries. This idea poses two stringent observational constraints: the mass of a blue straggler cannot exceed twice the cluster turn-off mass, and they should have a binary companion. If one allows stars to merge in a mass exchange process, however, the last constraint does not apply.

Among the viable alternative hypotheses are non-coeval star formation (Hintzen et al. 1974; Eggen & Iben 1988) and extended main-sequence lifetimes due to extra internal mixing by some unspecified process (Wheeler 1979b). Although there is some evidence for ongoing star formation, in the form of bursts, in several young clusters (Eggen & Iben 1988), the fact that blue stragglers are observed in clusters of all ages implies that, if this were the cause, star formation should be a continuous process, spread over more than $10^9$ years. This is not supported by observations. Extended MS lifetimes due to additional mixing were suggested by Wheeler as the most satisfactory explanation, but this is partly due to the 'vagueness' of the invoked mixing process. Maeder (1987) considers turbulent diffusion due to fast rotation in massive stars as the cause of massive ON-type blue stragglers.

Star collisions are a likely cause of formation of blue stragglers in globular clusters, where star densities are extremely high and dynamical effects play a dominant role (Hut 1993). However, the densities in open clusters are insufficient for collisions to play any part. Mermilliod (1982) found that neither the peculiar compositions of Ap and Bp-type blue stragglers, nor different rotation rates are by themselves sufficient to explain the large colour differences.

The lack of observed radial-velocity variations has often been used as an argument against the mass transfer hypothesis. However, in the simulations of Collier & Jenkins (1984) the majority of blue stragglers produced by mass transfer have variations less than 10 km/s, which is below the detection limit for most searches. Similar results for young and intermediate-age clusters are obtained in the present study.

Wheeler (1979a) has argued that the bluest stragglers in several old open clusters are more massive than twice the turn-off mass ($m_{\odot}$), and that the observed blue-straggler mass function is too steep to be explained by the mass transfer hypothesis. However, it should be emphasized (cf. Collier & Jenkins 1984) that Wheeler's mass estimates are based on the position in the H-R diagram and therefore rather uncertain, and that the great majority of the stragglers have masses less than $2m_{\odot}$. For young and intermediate-age clusters, there is no evidence for blue-straggler masses larger than $2m_{\odot}$ in the list of Mermilliod (1982).

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1Walborn (1979) has found $P = 1.78$ d and $e = 0.44$, but his determination was based on much fewer radial-velocity measurements.
5.3 A model for binary evolution

5.3.1 General overview and description

The evolution of a single star depends, in the simplest approximation, only on its mass $m$ and its initial chemical composition. In contrast, the evolution of a binary star with a certain chemical composition is determined by four parameters instead of one: the initial values of the masses of the two stars $m_1$ and $m_2$, the orbital separation $a$ and the orbital eccentricity $e$. There are some other parameters that may influence the evolution of both single and binary stars, like the rates of rotation, magnetic fields, etc., but their influence is not well understood. They are probably of secondary, if any, importance and in our model they will be ignored. Furthermore, we assume that $e = 0$, at least initially and at times when the stars interact, and we keep the initial composition fixed so the evolution of a binary is assumed to be a function of three input parameters.

The evolution of stars in binary systems differs from that of single stars because the two components can interact. This interaction is most conveniently described in terms of the Roche model, which defines a critical equipotential surface, the Roche lobe, for each star. It is important to realize, however, that the Roche model strictly applies only when the orbit is circular and both components are in synchronous rotation with the orbit. The important quantity in this model is the equivalent Roche-lobe radius $R_{L}$ (or, in short, Roche radius) which is the maximum radius a star can have without having to transfer mass to its companion through the inner Lagrangean point. We shall use Eggleton's (1983) approximation to the Roche radius:

$$r_{L}(q) = \frac{R_{L}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})},$$  \hspace{1cm} (5.1)$$

which gives the Roche radius of the component with mass $m_1$ relative to the orbital separation, as a function of the mass ratio $q = m_1/m_2$.

The phases in the evolution of a binary can be divided according to whether the binary is detached (i.e., both components are inside their Roche lobes) or whether there is mass exchange between the components (i.e., the system is either semi-detached or in contact, when either one or both components overfill their Roche lobes, respectively).

In a detached binary the two stars evolve in much the same way as two single stars of the same mass and composition. Although several types of interaction are possible, they are usually of little or no importance to the structure and evolution of the components. In our model we therefore treat the stars as single stars during detached phases. The main importance of interactions in a detached binary is for the orbital evolution of the system. Tidal interaction tends to synchronize the rotations of the components with the orbit and to circularize the orbit. Hence, it is the efficiency of the tidal interaction which determines the correctness of the Roche model approximation. Since the time scale of tidal interaction becomes very short when one of the stars is close to filling its Roche lobe, the Roche model is usually a good approximation when it really matters, i.e., at the onset of mass exchange phases. We implicitly assume infinitely efficient tidal interaction throughout the model, i.e., synchronous rotation and circular orbits at all times. However, we shall encounter several situations where this is not a good approximation and the Roche model breaks down, so that we have to resort to a different description.

Tidal interaction can also influence the structure of the stars because the equipotential surfaces are not exactly spherical. E.g., baroclinic instabilities can induce additional mixing in the interior, but the extent to which this occurs is poorly known and we therefore neglect this effect in our model. Other types of interaction which are at present not included in the model are magnetic interaction (which is probably unimportant for the internal evolution of the stars, but may influence the orbit through the process of magnetic braking, see below) and
gravitational radiation which also causes orbital angular momentum loss, but is unimportant for massive binaries.

Finally, the influence of mass loss from one or both components (both in the form of stellar winds and of sudden, explosive mass loss in supernovae) on the orbital evolution cannot be neglected and is therefore accounted for in the model. Stellar-wind mass loss slowly increases the orbital separation if it is spherically symmetric. Supernova explosions induce an eccentricity to the orbit and give the whole binary a recoil velocity, or even disrupt the binary into two single stars. Other effects of mass loss in binaries are at present not included in the model, namely: (1) the possible enhancement of stellar-wind mass-loss rates by tidal interaction (Tout & Eggleton 1988), and (2) magnetic braking, i.e., orbital angular momentum losses due to magnetic stellar winds coupled with tidal interaction (Verbunt & Zwaan 1981).

Mass exchange is an extreme form of tidal interaction, which does, however, strongly influence the structure and evolution of the components. We distinguish four modes of mass exchange, according to the degree of stability of Roche-lobe overflow (cf. Webbink 1979, 1985). Our definition is an extension of the modes defined by Webbink. Thus, we do not use the classical division into case A, B and C mass transfer (Kippenhahn & Weigert 1967, Lauterborn 1970), which we find to be less useful as a criterion for the time scale and efficiency of mass transfer, but we shall refer to it where it is appropriate.

The four modes are distinguished by the time scale on which Roche-lobe overflow occurs (the nuclear, thermal, or the dynamical time scale) and by whether Roche-lobe overflow occurs at all. We define the donor, with mass \( m_d \), as the star that is the first to fill its Roche lobe and lose mass to its companion with mass \( m_a \), named the accretor (irrespective of whether it actually accretes any matter). The time scale of mass transfer is determined by the responses to mass loss of the donor radius \( R_d \) and the donor's Roche radius \( R_{d,\text{in}} \). These responses can be described in terms of so-called mass-radius exponents \( \zeta \equiv d \ln R / d \ln m \) (cf. Webbink 1985, Hjellming & Webbink 1987), for either the adiabatic response of the stellar radius (\( \zeta_{\text{ad}} \)), the thermal-equilibrium response (\( \zeta_{\text{th}} \)) and the Roche radius response (\( \zeta_{\text{L}} \)).

Apart from the time scale of mass transfer, which is largely determined by the structure of the donor at the onset of Roche-lobe overflow, a complete description of the mass exchange process requires answers to the following questions: (1) what is the reaction of the accretor to receiving the mass that is transferred to it, and (2) when and how is the mass transfer terminated?

We shall discuss below the four modes of mass exchange and try to answer the questions posed above.

**Mode I: Stable Roche-lobe overflow**

If both \( \zeta_{\text{ad}} > \zeta_{\text{L}} \) and \( \zeta_{\text{th}} > \zeta_{\text{L}} \) (i.e., the donor expands less rapidly, or contracts more rapidly, in response to mass loss than its Roche lobe) Roche-lobe overflow will be stable. Mass transfer is driven by the donor's gradual expansion due to nuclear burning (or, in low-mass binaries, by the contraction of the Roche lobe because of angular momentum loss) and proceeds on the nuclear-expansion time scale of the donor, \( \tau_{\text{exp}} = R / \dot{R} \). The system parameters and the structure of the components therefore change very slowly and the donor will remain in almost complete equilibrium during the mass exchange phase. Mass transfer is terminated when the equilibrium radius of the donor becomes smaller than its Roche radius, i.e., when the donor starts to contract of its own accord. This occurs, e.g., in case the donor is a giant (case B or C) when either the entire envelope has been transferred (low-mass giants) or when a new type of fuel is ignited in the core (e.g., helium ignition in high-mass giants), or in case the donor is a main-sequence star (case A) when hydrogen is exhausted in the centre. The conditions for stable Roche-lobe overflow usually imply that the accretor is more massive than the donor, so that it too will remain in thermal equilibrium. With the low mass-transfer rates it will not have any trouble
accreting the transferred mass. Therefore, the total system mass and angular momentum are usually conserved during mass exchange.

An interesting hybrid case of mode I occurs when the radius of the accretor becomes large enough to fill its own Roche lobe, and a contact binary is formed. This can occur because the mass of the donor decreases, hence its nuclear time scale becomes longer, while the opposite happens to the accretor, so that the accretor actually 'overtakes' the donor (cf. Chapter 6). The evolution of contact binaries is driven not only by mass exchange but also by luminosity transfer within the common envelope surrounding the two Roche lobes (see, e.g., Shu & Lubow 1981). This is still an ill-understood problem, but the large number of observed systems suggests that it can be a very stable process.

**Mode II: Thermally unstable Roche-lobe overflow**

If $\zeta_{ad} > \zeta_{L} > \zeta_{th}$, then in order to achieve thermal equilibrium the donor keeps overfilling its Roche lobe. Mass transfer is driven by the thermal relaxation of the donor and takes place on the thermal or Kelvin-Helmholtz time scale:

$$\tau_{KH} = \frac{Gm^2}{RL}, \quad (5.2)$$

and rapid changes in the system parameters and the structure of the stars occur. Although the donor is not in thermal equilibrium its radius only slightly exceeds the Roche radius, by at most a few percent. Mode II mass exchange usually occurs when the donor is more massive than the accretor and possesses a radiative envelope, i.e., in the first phase of mass transfer in case A and early case B systems. Thermal time scale mass transfer continues until the donor has become less massive than the accretor, after which the donor recovers thermal equilibrium and mass transfer relaxes to the stable mode. Termination of mass transfer is thus governed by the processes discussed above under mode I. In practice, this means that mass transfer ceases when almost the entire envelope has been transferred and only the core of the donor remains.

Because the accretor is usually less massive than the donor, and furthermore the mass-transfer rate is much higher than in mode I, the accretor will not be able to maintain thermal equilibrium. Therefore, its reaction to accretion is an important factor which has to be included in the model. Several calculations of accreting main-sequence stars have been performed (cf. Ulrich & Burger 1976, Flannery & Ulrich 1977, Kippenhahn & Meyer-Hofmeister 1977, Neo et al. 1977, Packet & De Greve 1980). These calculations show that if the accretion time scale $\tau_{acc} = m_a/\dot{m}$ is shorter than the accretor's Kelvin-Helmholtz time scale $\tau_{KH,a}$, the accretor gets out of thermal equilibrium, expands and becomes overluminous. If $\tau_{KH,a} > 10\tau_{acc}$ the expansion becomes much larger than a factor 2. The expansion may eventually cause the accretor to fill its own Roche lobe and form a contact binary. In contrast to mode I, however, this type of contact binary is very unstable because both components are out of thermal equilibrium. Since both stars have to expand to recover thermal equilibrium, very soon the outer Lagrangean surface will be reached and mass transfer becomes non-conservative. Matter will be ejected from the common envelope through the outer Lagrangean point, after which it is no longer forced to corotate with the binary. Because it does not possess enough kinetic energy to escape from the system it will probably form a ring or disk around the binary. Such a disk can extract angular momentum from the binary so that the orbit will shrink.

The above discussion holds if the Roche model is valid, at least until the outer Lagrangean surface is reached. It is doubtful, however, whether both stars can remain in synchronous rotation with the orbit during this rapid mass transfer. Especially the accretor, which also receives angular momentum with the matter stream from the donor, will start to rotate much faster than the orbital frequency. Packet (1981) has shown that only a small amount of accreted
mass is necessary to spin up the accretor to near break-up frequency. The rest of the transferred mass may then be stored in a disk around the accretor and will be only slowly accreted. In such a situation, which resembles what is observed in β Lyrae, a common-envelope binary is never formed. Some of the matter in the disk is probably still removed from the binary (maybe in the form of a stellar wind from the accretor), but the reduction of the orbital separation might be less severe.

**Mode III: Dynamically unstable Roche-lobe overflow**

If $\zeta_l > \zeta_{ad}$, the adiabatic response of the donor to mass loss is such that its radius keeps exceeding the Roche radius, and Roche-lobe overflow proceeds on a hydrodynamical time scale:

$$\tau_{dyn} = \frac{R}{c_s}, \quad \text{(5.3)}$$

where $c_s$ is the speed of sound in the envelope. A very large mass-transfer rate results, limited only by the sonic flow through the nozzle at the inner Lagrangean point, and the donor exceeds its Roche lobe by a large amount. Mode III usually occurs when the donor has a deep convective envelope and is more massive than about 0.7 times the accretor,\(^2\) i.e., during the first phase of mass exchange in late case B and in case C binaries. The Roche model probably loses its significance shortly after the onset of dynamically unstable mass transfer, since the donor’s envelope will not be able to maintain synchronous rotation. Furthermore, the accretor will quickly fill its own Roche lobe because of the processes described above under mode II. It is unlikely that the accretor will be able to accrete a large amount of the transferred matter. Most of the transferred matter will therefore be rejected and form a common envelope surrounding both stars, and rotating differentially with respect to the binary (Paczynski 1976). This situation resembles what we will encounter in mode IV below, where the evolution is dominated by frictional forces. However, because of the different circumstances under which mass transfer started, it is not certain that the course and outcome of these two modes will be the same.

**Mode IV: Orbital tidal instability**

The reason why the Roche model is a good approximation is that tidal interaction stabilizes the orbit. However, if the rotational angular momentum of one component exceeds one third of the orbital angular momentum, tidal interaction becomes unstable (Darwin 1908). This can happen during the expansion of a star (the future donor), before it fills its Roche lobe, if it is more than roughly 6 times as massive as its companion (Sparks & Stecher 1974). The orbit will then decay on the tidal time scale (which becomes very short when the donor is close to filling its Roche lobe). The donor cannot maintain synchronous rotation with the orbit, and the ‘accretor’ is dragged into the envelope with a differential velocity. Clearly, the Roche model loses any significance in such a situation and the orbital evolution is determined by frictional forces in the common envelope.

Several hydrodynamical calculations which apply to this spiral-in situation have been reported, see the review by De Kool (1987) and papers by Livio & Soker (1988) and Taam & Bodenheimer (1989, 1991). However, no calculation has yet been done which computes the spiral-in process in a consistent (3-dimensional) way from start to finish. Neither the initial phases, i.e., the formation of a common envelope, nor the final ejection of the envelope have been calculated consistently. The calculations suggest that the energy that is gained from the decay of the orbit can be used fairly efficiently for ejecting matter from the common envelope.

\(^2\)This value applies to a star with a fractional core mass $m_{core}/m = 0.1$. A larger core mass increases $\zeta_{ad}$ and causes mass transfer to stabilise for larger mass ratios $m_d/m_a$ (Hjellming & Webbink 1987).
5.3 A model for binary evolution

5.3.2 Approximations and implementation in the code

5.3.2.1 Detached phases

Both components are treated as single stars as long as the binary is detached. Their models are calculated by means of interpolation between grids of evolutionary models of single stars that have been published in the literature. Since we are also interested in the evolution after one or more mass exchange phases, we not only need models of 'normal' hydrogen-rich stars but also of evolved remnants: helium stars, carbon-oxygen stars and compact objects (white dwarfs and neutron stars). The grids we have used for these different types of stars will be described below.

For both components a single star grid is interpolated, depending on its initial mass $m_0$ and its evolutionary status (i.e., 'normal', helium star or C-O star). The interpolation is performed between models corresponding to equal evolutionary states. Apart from the luminosity $L$ and effective temperature $T_{\text{eff}}$, the following stellar parameters are relevant to our model: the radius $R$ for comparison with the Roche radius $R_L$, the actual stellar mass $m$ and the stellar-wind mass-loss rate $\dot{m}$, the central hydrogen mass fraction $X_c$, the mass of the convective core $m_{cc}$, and the mass of the hydrogen-exhausted core $m_{\text{core}}$.

The two grids are combined into a grid for the binary with orbital separation $a_0$, until both stars have reached the endpoints of their evolution. The evolution of the orbit is calculated with the appropriate formulae presented in the Appendix (eqs. 5.21 to 5.32). At every time step (determined by the shortest time step of the single-star grids) a new separation is calculated, taking into account the stellar-wind mass-loss rates of both stars using eq. (5.27). The Roche-lobe radii are calculated and compared with the stellar radii. When for one of the stars $R \geq R_L$, mass exchange is simulated (see § 5.3.2.2).

After a phase of mass transfer, a subsequent detached episode may follow and interpolation between the stellar grids is resumed. When a star is the remnant of mass exchange, the initial model usually is not a homogeneous, zero-age star (except in some cases where the remnant is the stripped core of the donor). In such cases we assume that the initial model of the remnant is at the same evolutionary state as its precursor (e.g., it has the same value of $X_c$ or $Y_c$).

A different prescription is followed for the remnant of a main-sequence star after a phase of accretion. Such a star is rejuvenated because of its convective core, which grows according to the extra mass it has accreted and thereby mixes fresh hydrogen into the core. Detailed calculations show that the hydrogen profile of a main-sequence star after accretion has approximately the same shape as that of a star which had a constant mass since zero age (Hellings 1983; see also Chapter 6 of this thesis). We therefore use the following prescription for rejuvenation of main-sequence accretors. The total mass of hydrogen burned can be derived from the hydrogen profile left by the retreating convective core, thus:

$$m_H = \int_{X_{c,0}}^{X_{c,f}} m_{cc}(X_c) \, dX_c,$$

where $X_{c,0}$ and $X_{c,f}$ are the initial and final values of $X_c$ (i.e., in our case $X_{c,0} = 0.7$). This amount can be calculated for the accretor over the time preceding the mass-transfer phase, and should be the same after mass transfer. Therefore, because the relation $m_{cc}(X_c)$ for the new accretor mass follows from the interpolated grid, eq. (5.4) can be used for the new accretor mass to determine $X_{c,f}$ with the condition that $m_H$ does not change. This is then the new value of $X_c$ immediately after mass transfer, where the interpolation should be resumed.

When the final model in the grid has been reached before Roche-lobe overflow occurs, the final status of the star is determined on the basis of its initial mass, $m_0$, and the values of two limiting masses, $M_{\text{up}}$ and $M_{cc}$. $M_{\text{up}}$ is the minimum initial stellar mass for non-degenerate carbon ignition, and $M_{cc}$ is the minimum initial mass for non-degenerate ignition of all subsequent
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burning stages. If \( m_0 < M_{up} \) the star forms a degenerate C-O core, which becomes a C-O white dwarf if the envelope is lost before \( m_{core} > 1.4M_\odot \). If the core grows larger than the Chandrasekhar mass, carbon is ignited explosively and the whole star is assumed to be disrupted, leaving no remnant. For \( M_{up} < m_0 < M_{ec} \), the core evolves through carbon burning and becomes a degenerate O-Ne-Mg core. If sufficient mass is left in the envelope for this core to grow to \( m_{core} > 1.4M_\odot \), electron captures on Mg nuclei induce a supernova explosion leaving a neutron star remnant, otherwise the star turns into a O-Ne-Mg white dwarf. For initial masses \( m_0 > M_{ec} \), the core evolves through all burning stages and forms an Fe core which produces a supernova explosion through photodisintegration, and the remnant is assumed to be a neutron star. The values of \( M_{up} \) and \( M_{ec} \) for the different types of star (H-rich, He or C-O) will be given below, along with the description of the evolutionary grids.

If a white dwarf is formed, the envelope is blown away in a stellar wind and the final orbital separation is calculated by eq. (5.27), see the Appendix. When a supernova explosion occurs, the eccentricity and orbital separation of the system are calculated under the assumption of a symmetric explosion (eqs. 5.28 and 5.30), and the system receives a recoil velocity given by eq. (5.29) or eq. (5.31). The impact of the supernova shell is assumed to have no influence on the companion. In the case of a completely disruptive explosion, the companion receives a system velocity equal to its original orbital velocity.

**The evolutionary grids used as input for interpolation**

For hydrogen-rich single stars we use the grids of Maeder & Meynet (1988, 1989) for the initial mass range 0.85\( M_\odot \) to 12\( M_\odot \), and of Maeder (1990) for 15 to 120\( M_\odot \), for an initial chemical composition \( X = 0.70 \) and \( Z = 0.02 \). Their calculations take into account stellar-wind mass loss in massive stars and giant-branch stars, and a moderate amount of overshooting from the convective core. The grids include all burning stages up to and including carbon burning in massive stars (\( m_0 \geq 7M_\odot \)), but only part of the asymptotic giant branch (AGB) for less massive stars. For \( m_0 < 2M_\odot \), only the first giant branch is included up to the helium flash. Since we limit ourselves here only to cluster ages less than the time for a 2\( M_\odot \) star to reach the giant branch, this is sufficient for our purposes. However, for intermediate-mass stars we have extended the AGB phase in a simplified analytic approximation until the envelope has been exhausted.

The grids of Maeder & Meynet for 2\( M_\odot \) < \( m_0 \) < 7\( M_\odot \) include only the early-AGB phase. In order to extend the grids to include the double-shell burning phase, we approximate the luminosity by the Paczyński (1970) core mass-luminosity relation:

\[
L = L_0(m_{core} - m_{core,0}),
\]

with \( L_0 = 5.92 \times 10^4 \) \( L_\odot \) and \( m_{core,0} = 0.522 \) \( M_\odot \). The luminosity of the burning shells is given by:

\[
L = \varepsilon_{eff} \frac{d m_{core}}{dt},
\]

where \( \varepsilon_{eff} \) is the effective energy release due to the double shell of hydrogen and helium burning (\( \varepsilon_{eff} \approx 5.0 \times 10^{18} \) erg/s). At the same time, the stellar mass decreases due to stellar-wind mass loss, for which we use the Reimers (1975) formula:

\[
\frac{d m}{dt} = -\eta \kappa_R \frac{LR}{m},
\]

where \( \kappa_R = 4 \times 10^{-13} \left( M_\odot/L_\odot R_\odot \right) M_\odot/\text{yr} \), and \( \eta \) is a parameter whose value is of order unity. We have used \( \eta = 2 \) in the calculations. The radius \( R \) is found from a linear approximation of

**The value of** \( M_{up} \) **and** \( M_{ec} \) **for the different type of star (H-rich, He or C-O) will be given below, along with the description of the evolutionary grids.**

**If a white dwarf is formed, the envelope is blown away in a stellar wind and the final orbital separation is calculated by eq. (5.27), see the Appendix. When a supernova explosion occurs, the eccentricity and orbital separation of the system are calculated under the assumption of a symmetric explosion (eqs. 5.28 and 5.30), and the system receives a recoil velocity given by eq. (5.29) or eq. (5.31). The impact of the supernova shell is assumed to have no influence on the companion. In the case of a completely disruptive explosion, the companion receives a system velocity equal to its original orbital velocity.**
the evolutionary track along the Hayashi line:

$$\log T_{\text{eff}} = C + \lambda \log L + \mu \log m,$$

(5.8)

where $\lambda$ and $\mu$ are found by fitting the slope of the early-AGB phases in the Maeder & Meynet grids. We find $\lambda = -0.1$ and $\mu = 0.12$. The constant $C$ is found by fitting eq. (5.8) to the last model in the grids.

The evolutionary track is found by integrating eqs. (5.5) to (5.8) until $m = m_{\text{core}}$, which is the final white dwarf mass. If $m_{\text{core}} > 1.4 M_\odot$, there would be a disruptive explosion of the core in the case of a C-O white dwarf (see above). However, our choice of $\eta = 2$ ensures that this situation never occurs. The values of the limiting initial masses $M_{\text{up}}$ and $M_{\text{ec}}$ following from the grids of Maeder & Meynet are $M_{\text{up}} = 6.6 M_\odot$ and $M_{\text{ec}} = 8.0 M_\odot$.

For bare helium stars, we use the grids of Paczyński (1971b) for masses $0.5 - 16 M_\odot$, supplemented by those of Habets (1986) for $2 - 4 M_\odot$. The $2 M_\odot$ and $4 M_\odot$ grids which these two sets have in common agree remarkably well, even though quite different input physics has been used. The initial composition for both these grids is $X = 0, Z = 0.03$, so that they do not correspond exactly to the remnants of the Maeder grids. However, the differences due to composition are small compared to other uncertainties in the model. Habets (1986) finds equivalent values for the limiting initial masses in his calculations of $M_{\text{up}} = 1.9 M_\odot$ and $M_{\text{ec}} = 2.2 M_\odot$. Neither of the helium star grids include stellar-wind mass loss, which is important for massive helium stars, i.e., Wolf-Rayet stars. In order to be complete, we have applied the mass-loss rate parameterization for WR stars from Langer (1989) to these grids, i.e.:

$$\dot{m} = \kappa_L \frac{L^2}{m(1 - L/L_{\text{Edd}})}$$

(5.9)

with $\kappa_L = 9 \times 10^{-10} (M_\odot/L_\odot^2) M_\odot/\text{yr}$, and $L_{\text{Edd}}$ is the Eddington luminosity of the star. Note that this is not a self-consistent method because the mass loss was applied after the grids had been calculated.

Carbon-oxygen stars (the stripped cores of helium stars) evolve extremely fast and no detailed calculations have been done for these objects. When such a star is produced in our simulations, we simply approximate it by assuming a radius $R = 0.045 m_0^{0.6}$ and a luminosity $L = 900 m^3$ (cf. Paczyński & Kozłowski 1972), and a lifetime of $10^4$ years. The minimum mass for a carbon star to evolve through all subsequent burning phases is $M_{\text{ec}} = 1.4 M_\odot$.

For white dwarfs we assume a radius $R = 0.0079 m^{-1/3}$ and, for the moment, $L = 0$, but we plan to include a cooling track for white dwarfs into our simulations. Neutron stars are assumed to have $m = 1.4 M_\odot$ and $R = 10 \text{ km}$, and also $L = 0$.

5.3.2.2 Mass exchange

When either star starts overfilling its Roche lobe, the mode of mass exchange is determined according to the prescription of § 5.3.1. This requires knowledge of the radius-mass exponents $\zeta_{\text{ad}}, \zeta_{\text{th}}$ and $\zeta_L$, which depend on the structure of the donor. Hjellming & Webbink (1987) have determined $\zeta_{\text{ad}}$ for different polytropic configurations, and Hjellming (1989) has calculated $\zeta_{\text{ad}}$ and $\zeta_{\text{th}}$ for a large grid of actual stellar models. In the code, we use only an extremely simple parameterization of $\zeta_{\text{ad}}$ and $\zeta_{\text{th}}$, which could certainly be refined using Hjellming’s results. For the adiabatic response the only distinction we make is between radiative and convective envelopes. Radiative envelopes contract very rapidly, and we arbitrarily set $\zeta_{\text{ad}} = 15$ for star models bluer than the base of the giant branch. For models representing stars on the giant branch we set $\zeta_{\text{ad}} = -\frac{1}{3}$, which is appropriate for fully convective stars. Hjellming & Webbink show that $\zeta_{\text{ad}}$ increases when a compact core is present, and even becomes greater than 0 if $m_{\text{core}}$
is large enough. The thermal response of main-sequence stars is derived from the mass-radius relationship for massive stars, i.e., $\zeta_{\text{th}} = 0.55$. For more evolved models in thermal equilibrium (e.g., giants) we set $\zeta_{\text{th}} = 0$, since the radius generally depends to only a very small extent on the total stellar mass (eqs. 5.5 and 5.8, which apply to highly evolved giants, imply $\zeta_{\text{th}} = -0.24$). Models for evolved stars which are already expanding in order to achieve thermal equilibrium, i.e., stars in the Hertzsprung gap, should have $\zeta_{\text{th}}$ large and negative since the removal of mass will not prevent further expansion. Therefore we set $\zeta_{\text{th}} = -10$ in such cases. The response of the Roche lobe is calculated under the assumption that the total mass and angular momentum is conserved (cf. eq. 5.23 in the Appendix), which is appropriate at least in the initial stages of mass transfer. In that case $\zeta_{\text{L}}$ can be derived from eq. (5.1) and and depends only on the mass ratio $q = m_{d}/m_{a}$. Finally, the criterion for the occurrence of mode IV is that $q > q_{\text{cr}}$, where $q_{\text{cr}}$ depends on the structure of the donor through the gyration radius $k_{g}$. We use an average value of $k_{g}$, appropriate for giants, of $k_{g}^{2} = 0.15$, leading to $q_{\text{cr}} = 6.7$.

According to the mode of mass exchange, determined as outlined above, mass transfer is simulated as follows.

**Mode I** mass transfer is at the moment treated in the same way as mode II, i.e., as if proceeding on the thermal time scale. This is clearly wrong, but does not have serious consequences for massive binaries in most cases. The first mass-transfer phase is almost always unstable, since the donor is always the most massive star (cf. §5.3.1). Unstable (mode II) mass transfer is often followed by a phase of stable mass transfer, but in massive binaries this is usually a short-lived phase. The major exception is when the donor is a main-sequence star, i.e., case A mass transfer, when the stable phase can last for the remaining main-sequence lifetime of the donor. In such cases, we simply assume that the donor mass remains constant and approximate its structure by a normal main-sequence star of the same mass, using the grids of Maeder & Meynet (1989) to interpolate its luminosity and effective temperature, etc. A realistic modeling of stable mass transfer requires a fine grid of equilibrium models of different masses and at different evolutionary stages (e.g., similar to the models calculated by Giannone et al. 1968). We plan to produce such a grid, using calculations such as described in Chapter 6 of this thesis, and to include this in future calculations.

**Mode II** mass transfer is simulated by assuming a constant mass-transfer rate:

$$\dot{m} = \frac{m_{d}}{\tau_{\text{KH,d}}}$$

which approximates to within a factor 2 the peak mass-transfer rate encountered in detailed calculations (cf. Paczyński 1971a, Van den Heuvel 1993). The reaction of the accretor to this transfer rate is taken into account as follows.

When the accretor is a main-sequence star, the expansion due to accretion (cf. §5.3.1) is calculated by an analytic approximation of the results of Kippenhahn & Meyer-Hofmeister (1977), Neo et al. (1977) and Packet & De Greve (1980). Their calculations yield, as a function of the accretion rate $\dot{m}$, the ratio of the maximum radius of the expanded accretor $R_{\text{max,a}}$ to the equilibrium main-sequence radius $R_{\text{ma,a}}$, for different initial accretor masses. We find that the results can be reasonably fitted to a relation of the following form:

$$\log \log \frac{R_{\text{max,a}}}{R_{\text{ma,a}}} = A \log \frac{\dot{m}}{\dot{m}_{\text{KH,a}}} + B,$$

(5.11)

where $\dot{m}_{\text{KH,a}}$ is the Kelvin-Helmholtz time scale accretion rate of the accretor, defined as:

$$\dot{m}_{\text{KH,a}} \equiv \frac{\dot{m}_{a}}{\tau_{\text{KH,a}}}$$

(5.12)
5.3 A model for binary evolution

Table 5.1: Parameters for radius expansion

<table>
<thead>
<tr>
<th>$m_0/M_\odot$</th>
<th>$A$</th>
<th>$B$</th>
<th>references$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.599</td>
<td>-1.374</td>
<td>KMH</td>
</tr>
<tr>
<td>3.0</td>
<td>0.719</td>
<td>-1.647</td>
<td>PG</td>
</tr>
<tr>
<td>5.0</td>
<td>1.006</td>
<td>-2.030</td>
<td>KMH, Neo</td>
</tr>
<tr>
<td>10.0</td>
<td>1.619</td>
<td>-2.560</td>
<td>KMH, PG</td>
</tr>
<tr>
<td>17.0</td>
<td>2.212</td>
<td>-3.056</td>
<td>KMH</td>
</tr>
</tbody>
</table>

$^1$KMH: Kippenhahn & Meyer-Hofmeister (1977); Neo: Neo et al. (1977); PG: Packet & De Greve (1980)

In Table 5.1 we give the values of $A$ and $B$ of the fits to several initial accretor masses. For an arbitrary accretor mass we interpolate $A$ and $B$ linearly in mass, or extrapolate if the mass falls beyond the mass limits for which the calculations have been made. Unfortunately the fit for the $17M_\odot$ accretor is not very good, so the extrapolation to higher masses does not give very reliable results.

We now calculate the maximum accretion rate $\dot{m}_{\text{max},a}$ by assuming that the accretor must remain within its own Roche lobe, i.e., by the condition $R_a \leq R_{L,a}$, where $R_a$ is the instantaneous radius of the expanded accretor, and $R_{L,a}$ is the Roche-lobe radius of the accretor. It is further assumed that $R_a = R_{\text{max},a}(\dot{m})$ during the whole accretion process. This is clearly an overestimate, since the maximum radius is attained at only one point in time and the accretor radius can be smaller before and after this point. $\dot{m}_{\text{max},a}$ is then given by inverting eq. (5.11):

$$\dot{m}_{\text{max},a} = \dot{m}_{KH,a} \left(10^{-B} \cdot \log \frac{R_{L,a}}{R_{\text{ms},a}}\right)^{1/A}.$$  \hspace{1cm} (5.13)

If $R_a = R_{L,a}$ at some stage, both stars fill their Roche lobes and a fraction $\alpha$ of the transferred mass is assumed to be ejected from the system, given by:

$$\alpha = 1 - \frac{\dot{m}_{\text{max},a}}{\dot{m}}.$$ \hspace{1cm} (5.14)

If $R_a < R_{L,a}$, mass transfer is conservative: $\alpha = 0$. The evolution through the mass-transfer phase can now be calculated by assuming that the ejected mass fraction $\alpha$ has a certain specific angular momentum $j = \beta J/M$, where $J$ is the orbital angular momentum of the system and $M$ is the total mass of the system (see the Appendix). Note that $\alpha$ changes with time through the dependence on $R_{L,a}$ and hence on the separation $a$, which is determined by the value of $\beta$. We divide the mass-transfer phase into a number of equal steps, and for each step calculate $\alpha$ with eq. (5.14) and the orbital separation with eq. (5.22). It turns out that 20 steps suffice to give accurate results with errors less than a few per cent.

The specific angular momentum of the ejected matter is an uncertain parameter. In dynamical calculations of particle trajectories, assuming synchronous rotation until the outer Lagrangean point is reached, very high values of $\beta \gtrsim 6$ are attained (see, e.g., Flannery & Ulrich 1977). This would imply an enormous rate of reduction of the separation; however, the assumption of corotation seems unrealistic. In the present simulation we have chosen a constant value of $\beta = 3$, mainly because this value gives the best fit of the final orbital separation at the boundary with mode IV (i.e., for $q = q_{\alpha}$) for the assumed value of the spiral-in efficiency parameter, $\alpha_{CE} = 1$ (see below). Other values need to be investigated as well.
The above radius expansion calculations do not apply to evolved stars. For the moment we assume that the accretion rate onto evolved stars is limited by the Eddington rate:

\[ L_{\text{Edd}} = \frac{Gm_\alpha \dot{m}_{\text{max},\alpha}}{R_\alpha} \]  

(5.15)

which is a good approximation when the accretor is a compact star (neutron star or white dwarf). We assume that the ejected fraction \( \alpha \) leaves the system in the form of a jet or stellar wind from the compact object, which implies \( \beta = m_\alpha / m_\alpha \) and we calculate the orbital evolution with eq. (5.25). For other cases of accreting stars we have no prescription at hand, and we use the same recipe as for accreting compact stars. For instance, accretion of hydrogen-rich material onto a helium star will probably lead to ignition of the H-burning shell and expansion to giant dimensions. However, such cases are relatively rare compared to accreting main-sequence stars and compact stars.

As outlined in § 5.3.1, the course and outcome of mode III mass transfer are subject to major uncertainties. In the present calculations, we model this mode with the same prescriptions as for mode II, but with a different time scale for mass transfer. The mass-transfer rate is assumed to be constant and of magnitude:

\[ \dot{m} = \frac{m_d}{(\tau_{\text{KH,d}} \tau_{\text{dyn,d}})^{1/2}} \]  

(5.16)

where \( \tau_{\text{dyn,d}} \) is the time scale of sound travel through the donor. This is a good approximation to the peak transfer rate when mass transfer proceeds according to Roche-lobe overflow (Paczyński & Sienkiewicz 1972). This leads to a very high mass-transfer rate, and it is no surprise that the value of \( \alpha \) calculated with eq. (5.14) is almost always very close to unity. However, when the initial mass ratio is close to unity \( \alpha \) can be as small as 0.9, and it is not inconceivable that in such systems some accretion does indeed take place despite the very short time scale involved.

Mode IV mass transfer is treated according to the spiral-in prescription by Webbink (1984). The final orbital parameters are calculated under the assumption that the orbital energy is converted with some efficiency \( \alpha_{\text{CE}} \) into potential energy of the envelope of the donor, which has to be dispersed to infinity, i.e.:

\[ \frac{Gm_{d,0}(m_{d,0} - m_d)}{\lambda \tau_{L,d,a_{0}}} = \alpha_{\text{CE}} \left( \frac{Gm_\alpha m_\alpha}{a} - \frac{Gm_{d,0}m_\alpha}{a_0} \right). \]

In this equation and in the next, a subscript ‘0’ indicates values before spiral-in and no subscript indicates values after spiral-in; \( \lambda \) is a factor describing the relative binding energy of the donor’s envelope, which is a function of its structure; and \( \tau_{L,d,a_{0}} \) is the donor’s initial Roche radius. This leads to the following ‘energy equation’ for the evolution of the orbit (see also Bhattacharya & Van den Heuvel 1991):

\[ \frac{a}{a_0} = \frac{m_d}{m_{d,0}} \cdot \frac{m_\alpha}{m_\alpha + 2(m_{d,0} - m_d)/\lambda \tau_{L,d,a_{0}} \alpha_{\text{CE}}}. \]  

(5.17)

It is assumed that the accretor remains inert, irrespective of its evolutionary state, i.e., its mass \( m_\alpha \) is constant. For the value of \( \lambda \), which depends on the initial structure of the donor, we use an average value of 0.5 throughout. The efficiency parameter \( \alpha_{\text{CE}} \) is a free parameter whose value is probably between 0.1 and 1, as follows from the observed incidence and properties of close white-dwarf binaries (De Kool 1990, Bragaglia et al. 1990). Similar values are suggested by hydrodynamical calculations (Livio & Soker 1988, Taam & Bodenheimer 1989, 1991). The
models which are described in detail in the following section all have an assumed value of \( \alpha_{\text{CC}} = 1 \).

We have also tried models in which mode III is treated in the same way as mode IV, i.e., by using the spiral-in prescription (eq. 5.17). This applies if dynamically unstable mass transfer leads to formation of a common envelope dominated by frictional forces due to differential rotation of the 'inner' binary with respect to the envelope. This prescription leads to a smaller final orbital separation than the alternative recipe, described above. However, our results concerning blue stragglers are not affected by these different prescriptions for the dynamical mode.

The final mass of the donor after mass exchange is assumed to be the core mass \( m_{\text{core}} \), irrespective of the mode of mass transfer. In the case of a main-sequence donor, which has not yet formed a condensed core (i.e., case A mass transfer), we use the value of the convective core mass \( m_{\text{cc}} \) at the time of Roche-lobe overflow as the final mass after mass exchange. The new evolutionary state of the donor is that of the core, while the state of the accretor is assumed not to have changed, only its mass. After mass transfer, both stars are again treated as single stars along the lines of § 5.3.2.1 until a next phase of mass exchange occurs.

However, if the new equilibrium radii do not fit the Roche lobes the binary is assumed to have merged into a single star. Except in the case of two merged main-sequence stars, we have as yet no description for merged products which can be 'weird' objects, and their evolution is assumed to have terminated. In the case of two merged main-sequence stars we make the following assumptions: the mass of a merger is equal to the total mass of the binary at the time when it came into contact, and its structure is similar to that of a MS star rejuvenated by accretion. I.e., it has a convective core with a depleted amount of hydrogen equal to the sum of the amounts burnt in the two original cores, and we use the grids Maeder & Meynet to interpolate their luminosity and effective temperature. Hence, we do not take into account either additional mixing of the interiors or systematic mass loss during the merging process.

5.4 Simulating open clusters

The evolution of an open cluster is simulated in three steps, as follows:

1. Generating a realistic zero-age population. This is done by picking a number of systems at random from an input distributions of primary masses by means of a Monte-Carlo method. For a realistic binary fraction of these systems, a secondary mass and an orbital separation are generated in a similar way. The input distributions are described below, in § 5.4.1.

2. Evolving each system independently up to a sufficiently advanced age, by means of the evolutionary model described in § 5.3. One thus constructs for each system a grid of masses and separations, and of the luminosity \( L \) and effective temperature \( T_{\text{eff}} \) of both components, as a function of time.

3. Taking a ‘snapshot’ of all these grids at a certain age \( t \). This should be representative of the stellar population of a ‘real’ open cluster since all stars were formed at approximately the same time. An observable colour-magnitude diagram can be constructed of this population by converting \( L \) and \( T_{\text{eff}} \) of each star to magnitudes and colours. This conversion is described in § 5.4.2.

5.4.1 The distributions of input parameters

We need the zero-age distribution \( N(m_1, a, q) \) of the three input parameters for the evolutionary model: \( m_1, m_2 \) (or, equivalently, the initial mass ratio \( q \)), and \( a \). We assume, as usual, that
this distribution can be decomposed into independent factors: 

\[ N(m_1, a, q) = \Psi(m_1) \Gamma(a) \Phi(q) \]

All these distributions have been derived from observations, mainly of field stars, since theoretical considerations do not give reliable answers. One further assumption is therefore that the distributions in open clusters do not differ much from those of field stars.

**The initial mass function \( \Psi(m) \)**

The initial distribution of stellar masses, or initial mass function (IMF), has been discussed extensively by Miller & Scalo (1979) and Scalo (1986). We will use a power-law distribution, \( \Psi(m) \propto m^{-\alpha_m} \), with an exponent \( \alpha_m = 2.7 \). This is suggested by Scalo as a good approximation for \( m > 1M_\odot \). We generate a total number of \( N \) systems with primary masses between \( m_{\text{min}} \) and \( m_{\text{max}} \). Since we will simulate clusters with ages up to the age of the Hyades, we are mainly interested in primary masses \( > 2M_\odot \) because stars of lower mass do not evolve off the main sequence within this age. We have chosen a value of \( m_{\text{min}} \) somewhat lower than \( 2M_\odot \) in order to see the effect of main-sequence stars on the H-R diagrams, but as high as possible in order to minimize computing time because otherwise low-mass stars would dominate the population because of the steepness of the IMF.

**The distribution of orbital separations \( \Gamma(a) \)**

Although observational selection effects are severe, the observations appear to be consistent with a roughly flat distribution of \( \log a \) between a few \( R_\odot \) and several hundred AU, cf. Popova et al. (1982) and Abt (1983). However, Griffin (1985) and Duquennoy & Mayor (1991) find a distribution that increases with \( \log a \) up to \( a \sim 40 \) AU, and decreases for larger orbital separations. We use a flat distribution in \( \log a \): \( \Gamma(a) \propto a^{-1} \), for \( a_{\text{min}} < a < a_{\text{max}} \). We assume that \( a_{\text{min}} \) is determined by the minimum size of the Roche lobe that just fits a zero-age star: \( a_{\text{min}} = R_0(m)/r_L(q) \), and that \( a_{\text{max}}/a_{\text{min}} \) is a constant, for which we adopt a value of 5000 in most of our simulations. This value, corresponding to \( a_{\text{max}} \approx 100 \) AU for a \( 2M_\odot \) primary, makes sure that about two thirds of all binaries are close, i.e., will interact at some stage. Although much wider binaries do exist, in any case among field stars, many of these might well have an unresolved close companion as well and we are yet unable to simulate triple systems. Furthermore, a wider system has a good chance of being resolved as a visual binary in the most nearby clusters (with distances less than 200pc, assuming a resolving power of 0.5).

**The initial mass-ratio distribution \( \Phi(q) \) and the binary fraction \( f_b \)**

The observed distribution of \( q \) is also very much affected by selection effects, especially for \( q < 0.4 \). After a careful study of the selection effects among spectroscopic binaries, Hogeveen (1991, 1992) concludes that the \( q \)-distribution decreases from \( q \approx 0.3 \) towards \( q = 1 \), and that the peak at \( q = 1 \) found in earlier studies is due entirely to selection effects. For \( q \lesssim 0.3 \), however, the selection effects become too severe to determine the distribution with any certainty, but Hogeveen suggests that the distribution flattens towards smaller mass ratios. Hogeveen finds that the best fit to observations is achieved with a distribution \( \Phi(q) \propto q^{-\alpha_q} \) with \( \alpha_q = 2.0 \) for \( q > 0.3 \) and \( \Phi(q) = \text{constant} \) for \( q < 0.3 \). Other recent studies (Trimble 1990, Tout 1991) also find distributions that decrease towards \( q = 1 \), but with varying exponents: anywhere between IMF-like (\( \alpha_q = 2.7 \)) to flat (\( \alpha_q = 0 \)) distributions.

In our simulations we use a somewhat different definition of \( \Phi(q) \). Suppose that a close binary is formed from a single gas cloud of mass \( M \) that splits into two parts, with masses \( m_1 = \frac{1}{2}(1+z)M \) and \( m_2 = \frac{1}{2}(1-z)M \). The way in which the cloud is divided (i.e., the distribution \( n(z) \) of splitting parameters \( z \) over the interval \([-1, 1]\)) then determines the mass ratio distribution. For reasons of symmetry one expects to have \( n(-z) = n(z) \), and a continuous
5.4 Simulating open clusters

Figure 5.3: Mass-ratio distributions \( \Phi(q_0) \) produced with the procedure described in § 5.4.1, by assuming \( \sigma = 0 \) (dashed line) and \( \sigma = 10 \) (solid line). Also shown for comparison is Hogeveen's distribution (dotted line), normalised to the same fraction of binaries on the interval \( q \in [0.3, 1] \) as the \( \sigma = 10 \) distribution.

derivative of \( n(x) \) at \( x = 0 \). In order to simulate different \( q \)-distributions, we generate \( x \) in \([0, 1]\)
from the distribution \( n(x) \propto 1 + \sigma x^2 \). A larger value of \( \sigma \) will produce more unequal masses, i.e., a steeper \( q \) distribution. The advantage of the resulting \( q \)-distribution over using a power-law distribution (apart from the very weak physical 'basis') is that it yields a finite \( \Phi(q) \) at \( q = 0 \) without having to define a different shape for small \( q \).

One of the first attempts to derive the mass ratio distribution was made by Kuiper (1935), who found that a flat distribution of \( x \) (i.e., arbitrary splitting or \( n(x) = 1 \)) provides a good fit to the observations. This assumption, i.e., \( \sigma = 0 \), leads to \( \Phi(q) = 2(1 + q)^{-2} \). This is still in good agreement with observations, which is why we will use it in one of our models. We find that Hogeveen's distribution for \( q > 0.3 \) is closely reproduced by taking \( \sigma = 10 \). This produces, however, about twice as many binaries with \( q < 0.3 \) than with Hogeveen's distribution. In Fig 5.3 the \( q \)-distributions for different values of \( \sigma \) are plotted, together with Hogeveen's distribution.

Another important parameter is the fraction of close binaries among stellar systems, to which the \( q \)-distribution should be normalised. In view of the selection effects for small \( q \) this fraction is difficult to determine observationally. Hogeveen (1991) finds from a sample of bright stars that 19% of all stellar systems are spectroscopic binaries, and estimates that this is a complete sample of close binaries with \( q > 0.3 \). From the papers of Abt & Levy (1978) and Wolff (1978) it can be estimated that 25 to 30% of the B-type stars are primaries of a close companion \( (P < 10 \) years) with \( q > 0.25 \) (cf. Pols et al. 1991). We will see in the next section that other estimates of the fraction of binaries can be obtained from the CMD's of open clusters.

We now have to define a binary fraction \( f_b \), in accordance with the above normalization considerations. Because of our assumed \( a \)-distribution, the fraction of close binaries is then 0.67\( f_b \). We find \( f_{0.3} \Phi(q) dq = 0.538 \) for \( \sigma = 0 \) and 0.244 for \( \sigma = 10 \). These numbers should be multiplied by 0.67\( f_b \) to give the fractions \( C \) of all systems that are close binaries with \( q > 0.3 \).

For \( \sigma = 0 \), a value of \( f_b = 0.75 \) yields \( C = 0.27 \), in agreement with the upper limits imposed by observations. For \( \sigma = 10 \), we have taken \( f_b = 1 \) which gives \( C = 0.16 \), which seems to be a lower limit. In this model there are no truly single stars but 39% of all systems have \( q < 0.1 \), which will be indistinguishable from a single star by any means of observation.
5 Simulations of binary evolution in young open clusters

Table 5.2: Parameters for simulated models

<table>
<thead>
<tr>
<th>model</th>
<th>$m_{\text{min}}$</th>
<th>$m_{\text{max}}$</th>
<th>$N$</th>
<th>$N_{&gt;2M_{\odot}}$</th>
<th>$f_b$</th>
<th>$a_{\text{max}}/a_{\text{min}}$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>1.331</td>
<td>60</td>
<td>2000</td>
<td>1000</td>
<td>0.75</td>
<td>5000</td>
<td>0.0</td>
</tr>
<tr>
<td>A17</td>
<td>1.466</td>
<td>60</td>
<td>17000</td>
<td>10000</td>
<td>0.75</td>
<td>5000</td>
<td>0.0</td>
</tr>
<tr>
<td>B17</td>
<td>1.466</td>
<td>60</td>
<td>17000</td>
<td>10000</td>
<td>1.0</td>
<td>5000</td>
<td>10.0</td>
</tr>
</tbody>
</table>

5.4.2 Conversion to colour-magnitude diagrams

A comparison between simulated and observed clusters is possible only after conversion of luminosity and effective temperature to observable colours and magnitudes. The available data for young open clusters consist mostly of photometry in the Johnson $U$, $B$ and $V$ bands. We therefore wish to construct diagrams of $M_V$ against both $B-V$ and $U-B$. The advantage of the $U-B$ colour over $B-V$ is that it gives a much better discrimination of temperature for young open clusters, in which the brightest stars have $B-V < 0$ (spectral types O, B, or early-A). In the $(M_V, B-V)$ diagram these early-type stars clump up along an almost vertical line. On the other hand, the calibration of the $U$-band is known to be uncertain which makes the photometry in $U-B$ less reliable.

The conversion of effective temperature to bolometric correction ($BC$) and colours depends on two additional parameters: the chemical composition and the effective surface gravity $g$. As part of the Yale isochrone tables (cf. Green et al. 1987), Green (1988) has constructed an extensive conversion grid of $T_{\text{eff}}$ (between 2800K and 20000K) into $BC$, $U-B$, $B-V$ and other colours, for a range in log $g$ (between 0.0 and 6.0, in cm sec$^{-2}$) and metallicity [Fe/H]. This conversion is based on an empirical recalibration of synthetic colours and $BC$ from Kurucz (1979) and Vandenberg & Bell (1985). We have decided to use this table (for solar metallicity, [Fe/H] = 0) because it provides a fine grid in $T_{\text{eff}}$ and log $g$ and because it compares well with other, observationally determined conversions published in the literature (cf. Flower 1977, Hayes 1978, Böhm-Vitense 1981). The differences between Green’s (1988) conversion and others are less than $0.01$ in $BC$, and less than $0.02$ in $B-V$. The deviations in $U-B$ are somewhat larger, up to $0.01$, and for $15000K < T_{\text{eff}} < 20000K$ the Yale values seem to be systematically bluer than those of other conversions by $0.05$ to $0.10$.

Unfortunately, no values are available for $T_{\text{eff}} > 20000K$. We have supplemented the grid with synthetic colours and $BC$ from Kurucz (1979) for $20000K < T_{\text{eff}} \leq 50000K$ and $3.0 \leq \log g \leq 5.0$. These values match well at $20000K$, but some smoothing had to be applied because the derivatives were discontinuous.

In the next section we will present the results of a number of simulations with different sets of parameters. Table 5.2 summarizes the parameters used in these different models.

5.5 Results

5.5.1 Simulated colour-magnitude diagrams

In Figure 5.4 we present the resulting CMDs of the simulation of model A2, with 2000 systems, at a number of subsequent ages. These synthetic CMDs can be compared with the observed diagrams from Mermilliod (1981a), some of which were presented in § 5.2. In particular, notice the similarities between Fig. 5.4f and Fig. 5.1, and between Fig. 5.4k and Fig. 5.2. Several effects of the inclusion of binaries and their evolution are clearly visible and can be divided into two sorts:
5.5 Results

Effects of unresolved, non-interacting (wide) binaries

Firstly, the main sequence is smeared out above the ordinary, single-star sequence up to $-0^m75$, and at this limiting magnitude a secondary 'ridge' is visible. Such a second main sequence has been observed in a number of nearby open clusters, in particular the Pleiades, the Hyades and Praesepe; see also Figs. 5.1 and 5.2. At first glance these stars are expected to be unresolved binaries with components of equal luminosity and mass, but this second main sequence appears to be present even with a mass-ratio distribution which decreases towards $q = 1$, as we have assumed here. An explanation for this phenomenon was given already by Haffner & Heckmann (1937, see their Fig. 4)\(^3\) for the observed main-sequence stars in Praesepe. The second main sequence results from the way in which the fluxes of the two stars combine: for $0.3 < q < 0.6$ the presence of the companion causes an excursion almost horizontally to the right in the CMD, while for $q > 0.6$ it makes the system move upwards and to the left again. As a result, binaries with $q > 0.6$ are located in a band between $0^m6$ and $0^m75$ above the single-star main sequence. Systems with $q < 0.3$ merge in with the single-star main sequence because the contribution from the companion is negligible.

Therefore, the occurrence of a second main sequence does not imply that the mass-ratio distribution has a peak at $q = 1$. Although, in principle, the distribution of stars above the single-star sequence contains information about the $q$-distribution, it is difficult to derive definitive results about the shape. Several papers have been devoted to this subject (Bettis 1975, Jaschek 1976, Dabrowski & Beardsley 1976, Stauffer 1984). These authors found so-called photometric binary frequencies in several nearby clusters between 20% and 50%, depending on the method used to define the single-star sequence. These frequencies apply to binaries with $q > 0.3$, and include also wide, unresolved binaries. Note that the average value of about 35% binary systems with $q > 0.3$ agrees very well with the close binary frequency that we have assumed (see § 5.4.1).

A second prominent effect of unresolved binaries, in the somewhat older clusters with well-defined giant branches ($t > 200 \text{ Myr}$), is the occurrence of stars between the giant branch and the top of the main sequence. These 'yellow straddlers' are the result of adding the fluxes of a giant and an (almost) equally bright main-sequence star, yielding a combined colour in between, 'straddling' the Hertzsprung gap. This effect is also clearly observed in a number of clusters (e.g., see Fig. 5.2), and several of these stars indeed have composite spectra. Like the stars in the second main sequence, the yellow straddlers are binaries with nearly equal masses, $q > 0.7$. The distribution of stars across the Hertzsprung gap could thus serve as a test of the mass-ratio distribution: the closer to the main sequence, the more equal are the component masses. It should be noted that yellow giants in younger clusters ($t < 200 \text{ Myr}$; see also Fig. 5.1) are probably He-burning giants in a blue loop and not necessarily binaries.

Effects of mass exchange (close) binaries

The most obvious effect of the inclusion of mass exchange on the observed CMD is the appearance of blue stragglers. These synthetic blue stragglers are main-sequence stars whose masses have increased significantly due to accretion and now have a mass larger than the turn-off mass. Furthermore, the accretion of matter has rejuvenated the stars because extra hydrogen is mixed into the convective core, which has increased in size. They now appear to be bluer and often brighter than the turn-off. Blue stragglers produced by mass exchange come in five types, i.e., MS stars with helium-star (He), white-dwarf (WD), and neutron-star (NS) companions, double MS stars, and merged MS stars. These types are indicated by different symbols in Fig. 5.4. In § 5.5.2 we will discuss in more detail these different types of blue straggler, their observable properties and their progenitors.

\(^3\)Recently, Romani & Weinberg (1991) re-derived this result theoretically for globular clusters.
Figure 5.4: Synthetic cluster colour-magnitude diagrams, $M_V$ versus $U - B$ (left panels) and $B - V$ (right panels), resulting from the simulation of cluster model A2 containing 2000 systems (see Table 5.2), at a number of subsequent ages. The various symbols depict different types of stellar systems, as indicated: MS$^*$ = main sequence star; G = giant; He$^*$ = helium star; WD = white dwarf; NS = neutron star. The turn-off masses $m_{to}$ are also given for each age. Arrows on the right edges indicate red (super)giants that fall off the scale. (a) Synthetic CMD at $t = 5$ Myr.
Figure 5.4: (b) Synthetic CMD at $t = 10.0$ Myr. See Fig. 5.4a for an explanation of the symbols.
Simulations of binary evolution in young open clusters

Figure 5.4: Synthetic CMDs at t = 20 Myr. The scale of $B-V$ in this Figure has been compressed to show the positions of the red supergiants at $B-V \approx 2$. See Fig. 5.4a for an explanation of the symbols.
Figure 5.4: (d) Synthetic CMDs at $t = 40$ Myr. The scale of $B-V$ in this Figure has been compressed to show the positions of the red supergiants at $B-V \approx 2$. See Fig. 5.4a for an explanation of the symbols.
Simulations of binary evolution in young open clusters

Figure 5.4: (c) Synthetic CMDs at $t = 80$ Myr. See Fig. 5.4a for an explanation of the symbols.
Figure 5.4: (f) Synthetic CMDs at $t = 120$ Myr. See Fig. 5.4a for an explanation of the symbols.
Simulations of binary evolution in young open clusters

Figure 5.4. Synthetic CMDs at $t = 160$ Myr. See Fig. 5.4a for an explanation of the symbols.
Figure 5.4: (a) Synthetic CMDs at \( t = 320 \, \text{Myr} \). See Fig. 5.4b for an explanation of the symbols.
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Figure 5.4: Synthetic CMDs at $t = 640$ Myr. See Fig. 5.4a for an explanation of the symbols.
Figure 5.4: (k) Synthetic CMDs at $t = 1280.0$ Myr. See Fig. 5.4a for an explanation of the symbols.
The post-MS remnants of blue stragglers sometimes appear in the CMD as well, as anomalously bright giants (e.g., in Fig. 5.4f). The brightest star in the Pleiades, Alcyone, may be such a remnant. Its position in the CMD corresponds to a larger mass than the turn-off, and this star is considered by Mermilliod (1982) as a blue straggler although, strictly speaking, it is not because it is redder than the turn-off.

A second effect is the depletion of giants. Stars which encounter their Roche lobe on their way to the giant branch will not contribute to the giant population: the observed number of giants will be smaller than in the case of clusters that have only wide binaries or single stars. Since most giants in young open clusters are in the core He-burning phase, the depletion is to first order caused by systems with orbital periods in the range for case A and case B binaries (the closest ones, and also the progenitors of blue stragglers). Thus, the observed number of giants provides information on the fraction of systems with small orbital periods, and can in principle serve as a diagnostic of the period distribution (cf. § 5.5.3).

The third main effect is the appearance in the CMD of stars bluer than the main sequence, but far below the turn-off. In some cases, especially at high ages (e.g., in Fig. 5.4k), these 'blue interlopers' consist of a low-mass MS star plus a relatively massive He star companion. The He star contributes significantly to the energy distribution and produces a composite colour bluer than the main sequence, especially in $U - B$. In other cases, apparent in cluster of all ages, these systems contain a He star and a compact object (a WD or a NS), and their colour is even bluer. These systems are the remnants of non-conservative case B mass transfer (i.e., in progenitors with small initial mass ratios). Their occurrence and number depends sensitively on the assumptions made about non-conservative mass transfer and spiral-in (we have assumed $\alpha_{CE} = 1$ here), and could be used as a test of these scenarios. In a few clusters (Hyades, Praesepe) some stars are indeed observed below the main sequence, as we will discuss in § 5.5.3. However, the fact that these stars are much fainter that the turn-off often makes them difficult to observe.

One should realize that the theoretical positions of these blue interlopers in the CMD are not very certain because of two simplifying assumptions. The He star models assume a pure He star, while in reality a H-rich layer is left on top of the He core. This means that the effective temperature should be somewhat lower and that the star becomes less blue. Secondly, in converting $L$ and $T_{\text{eff}}$ to colours and magnitudes, values for solar composition are used and the higher He abundance is not taken into account.

Other products of mass exchange do not occupy peculiar positions in the CMD, they 'disappear' in the main-sequence band or the giant branch. These are products of non-conservative or quasi-conservative evolution, and their masses did not increase to values above the turn-off mass. These stars contain He-star, white-dwarf or neutron-star companions (provided that the latter systems are not ejected form the cluster in the supernova explosion). As these systems are not photometrically apparent as evolved binaries, their detection depends on spectroscopic observations, and possibly X-ray emission. The original secondary may appear as a spun-up Be star (cf. Pols et al. 1991, Chapter 3 of this thesis) or a star with a peculiar composition, or these systems may be spectroscopic binaries, possibly CVs or LMXBs. Their predicted numbers depend on the details of non-conservative mass transfer, especially in the lower main sequence.

### 5.5.2 Blue stragglers

In order to study in some more detail the blue stragglers produced in our simulations, we have performed more extensive calculations of 17000 systems in total, of which 10000 with $m > 2M_\odot$ (models A17 and B17). These yield a much larger number of blue stragglers which makes it possible to study these stars statistically.
5.5 Results

We have defined the following criterion to select a blue straggler from the other stars in the cluster:
\[ M_V < \max(M_{V,TAMS} + 1.5, -3.0), \]  
(5.18)
and
\[ B-V < (B-V)_{bl} - \delta_{col}, \quad \text{or} \quad U-B < (U-B)_{bl} - \delta_{col}. \]  
(5.19)

\( M_{V,TAMS} \) is the visual magnitude of the terminal-age main sequence (TAMS), and \((B-V)_{bl}\) and \((U-B)_{bl}\) are the colour indices of the bluest point of the single-star isochrone; these quantities are determined from theoretical isochrones found by interpolating the evolutionary grids of Maeder & Meynet at a certain age \( t \). This criterion simulates the expected observational selection criterion for considering a star as a blue straggler (in practice, this means that blue stragglers are selected on the basis of \( U-B \), except at large ages). A suitable value for \( \delta_{col} \) depends on the scatter in the upper part of the observed CMDs. We have considered the values \( \delta_{col} = 0^m05 \) and \( 0^m10 \).

A different criterion for selecting blue stragglers is on the basis of the mass: theoretically, a main-sequence star with \( m > m_{to} \), where \( m_{to} \) is the turn-off mass, must have been formed by mass exchange in our model. Note that this is not of much use as an observational criterion, since the mass of a star is usually very difficult to measure with sufficient accuracy, but it can be used as a test criterion to see whether the colour criterion actually yields the right number of blue stragglers.

In Fig. 5.5a and b we have plotted, as a function of age, the number of blue stragglers selected by different criteria that are produced in simulations A17 and B17, respectively. Each bin of \( \Delta \log t = 0.1 \) is an average of 20 ‘snapshots’ so that statistical fluctuations are minimized. Apparently, selection by \( \delta_{col} = 0^m05 \) corresponds best to the number of MS stars with \( m > m_{to} \), although the shape of the time dependence is different. This is caused by the relation between colours and \( T_{eff} \), as can be seen in Fig. 5.4: for \( t < 40 \) Myr and \( t > 600 \) Myr the slope \( d(U-B)/d \log T_{eff} \) is large, and even some stars with \( m < m_{to} \) (and below the turn-off magnitude) are selected. Almost all these stars are the products of mass exchange, however, so they can be considered as ‘true’ blue stragglers in that sense.

The criterion \( \delta_{col} = 0^m10 \) appears to always underestimate the number of stars with \( m > m_{to} \). However, in the following we shall consider only the blue stragglers selected with this more conservative criterion, because this corresponds best to the real observational criterion. Only in very few clusters the errors in photometry, and other effects like differential absorption, are so small that a more selective criterion is usable.

Fig. 5.6 shows (for \( \delta_{col} = 0^m10 \)) the cumulative number distribution, as a function of age, of the five different types of blue straggler produced: MS+He-star binaries, MS+WD binaries, MS+NS binaries, double MS binaries, and merged main-sequence stars (mergers). The first three types are remnants of case B mass transfer. The MS+He-star blue stragglers are present at all ages (cf. also Fig. 5.4); they are the progenitors of the MS+NS and MS+WD systems. The MS+NS blue stragglers are present in clusters with ages up to about 40 Myr (cf. Fig. 5.4b–d), because in older clusters the blue-straggler progenitor masses (always just above the turn-off mass) become too small for forming a neutron star. Instead, MS+WD blue stragglers appear in the cluster population (Fig. 5.4d–g). For \( t \geq 400 \) Myr there are no more MS+WD blue stragglers, because the lifetime of the preceding MS+He-star phase becomes too long. It should be added that the MS+NS blue stragglers, although they are recognized by our selection criterion, will in general not be recognized observationally as cluster members because they are runaway objects. They are in most cases ejected from the cluster because they have received a recoil velocity in the supernova explosion. However, they may be recognized as runaway stars (see below).
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Figure 5.5: Number of blue stragglers as a function of cluster age, produced in the simulations of model A17 (a) and model B17 (b). Each cluster model contains 10000 systems with primary masses larger than $2\,M_\odot$ (see Table 5.2). The different curves represent the number of 'blue stragglers' selected with different criteria, i.e., on the basis of their colour: $\delta_{\text{col}} = 0.10$ (solid line) and $\delta_{\text{col}} = 0.05$ (dashed line), and on the basis of their (primary) mass: $m > m_{10}$ (dotted line); see § 5.5.2 for further explanation.

The mergers and MS+MS binaries are remnants of case A mass transfer. It is obvious that these systems constitute a large proportion of potential blue stragglers at ages $t \gtrsim 40$ Myr, especially the mergers. Their relative number is clearly dependent on the assumed $q$-distribution: in model B17 more than half of the blue stragglers are case A remnants. However, as described in § 5.3, our assumptions for case A are rather coarse and provisional. Although we are fairly confident of the formation rates calculated with our model, the positions of these objects in the H-R diagram are rather uncertain. In particular, the double MS binaries actually consist of one rejuvenated star and one stripped MS star which is still transferring mass on a long time scale, like known massive Algol-type binaries (cf. Chapter 6). In the simulations we do not take this slow mass transfer into account: the masses are assumed to remain constant after rapid mass transfer. Furthermore, we assume the stripped star to be normal for its mass in luminosity, effective temperature, etc. (cf. § 5.3.2.2), which means that the luminosity of the stripped star is underestimated.

For the mergers we have made very simple assumptions, as outlined at the end of § 5.3.2.2. In particular, we have neglected both mass loss from the system and internal mixing during the merging process. Whereas the first effect would result in a less massive and less luminous remnant star, large-scale mixing will tend to homogenize the star. This would result in a longer MS lifetime and in a higher surface He abundance, giving the star a bluer position in the H-R diagram.

In Figs. 5.7 through 5.12 we present cumulative number distributions of the different types of simulated blue stragglers (as in Fig. 5.6), as a function of some (potentially) observable quantities. The four panels in each of these Figures contain averaged distributions over the indicated time intervals. Each system is weighted with the time, relative to the time bin width $\Delta t$, during which it is recognized as a blue straggler. The numbers should be interpreted as
5.5 Results

The expected number that will be observed in the cluster at an arbitrary point in time in the respective time bins.

In Fig. 5.7 we present the distributions over the colour difference $\Delta(U-B) = (U-B) - (U-B)_\text{iso}$ between a blue straggler and the bluest point of the isochrone. There are clear differences between the various types of blue straggler: e.g., the MS+He binaries have large excess colours, and will therefore be most easily recognized, while the mergers are concentrated closer to the main sequence (see also Fig. 5.4). Furthermore, the input $q_0$-distribution also causes a difference: blue stragglers produced in model B17 are much more concentrated towards the main sequence, because their average mass is smaller than that of the stragglers in model A17.

Fig. 5.8 represents the distributions over the ratio of primary (MS star) mass $m_p$ relative to the cluster turn-off mass $m_{10}$. As expected, the blue straggler primary masses are between $m_{10}$ and $2m_{10}$, but the mergers always have $m_p < 1.5m_{10}$. In Fig. 5.9 we have plotted the distributions over the present mass ratio $q \equiv m_q/m_p$, where $m_q$ is the secondary mass. These mass ratios are small, usually $q < 0.2$ (of course, $q = 0$ for a merger). Fig. 5.10 shows the distributions over the orbital period $P$ of the binary blue straggler systems, and in Fig. 5.11 the distributions over the orbital velocity $v_{\text{orb}}$ of the primary (blue straggler) components in these systems are plotted. The quantity $v_{\text{orb}} \sin i$, where $i$ is the (unknown) orbital inclination, is a measure for the radial-velocity variations that can be expected.

Most interestingly, the expected radial-velocity variations are quite small, $v_{\text{orb}} < 10$ km/s for the majority of the systems at all ages. The systems for which radial-velocity variations should be most easy to detect are the double MS binaries (they also have the shortest periods, and should have a lobe-filling companion). In addition, for $t > 100$ Myr also the MS+WD binaries have short orbital periods and large $v_{\text{orb}}$.

Figure 5.6: Cumulative distribution of the numbers of different types of blue straggler, as a function of cluster age, for models A17 (a) and B17 (b). These types of blue straggler are: MS+He star binaries, MS+WD binaries, MS+NS binaries, double MS star binaries, and MS mergers; they are indicated by different shades of gray. The blue stragglers have been selected with the criterion $\delta_{\text{col}} = 0^\circ 10$, i.e., the upper envelope is the same as the solid line in Fig. 5.5.
Figure 5.7: Cumulative number distributions of the different types of blue straggler, over the difference in colour $\Delta(U-B) = (U-B) - (U-B)_{bl}$ between the blue stragglers and the bluest MS stars on the single-star isochrone. Shown are the averaged numbers over four different time intervals, as indicated, for models A17 (a) and B17 (b). The gray-scale code for the blue-straggler types is the same as in Fig. 5.6.
Figure 5.8: Cumulative number distributions, similar to Fig. 5.7, over the ratio $m_p/m_t$ of blue straggler primary mass to cluster turn-off mass, for models A17 (a) and B17 (b).
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Figure 5.9: Cumulative number distributions, similar to Fig. 5.7, over the mass ratio $q$ of the companion (secondary) to the blue straggler (primary), for models A17 (a) and B17 (b).
Figure 5.10: Cumulative number distributions, similar to Fig. 5.7, over the log of orbital period $P$ of the blue straggler systems, for models A17 (a) and B17 (b).
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Figure 5.11: Cumulative number distributions, similar to Fig. 5.7, over the log of the orbital velocity $v_{\text{orb}}$ of the primary (blue straggler) component of the systems, for models A17 (a) and B17 (b).
Figure 5.12: Cumulative number distributions, similar to Fig. 5.7, over the initial mass ratio $q_0$ of the progenitors of the blue straggler systems, for models A17(a) and B17 (b).
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Figure 5.13: Distribution of the runaway system velocities $v_{sys}$ of the MS+NS binary blue stragglers, averaged over three different time intervals, for models A17 (a) and B17 (b). The selection criterion is $\delta_{col} = 0.05$.

In Fig. 5.12 the distributions over the initial mass ratio $q_0 \equiv m_2/m_1$ of the progenitor system are plotted. As expected, blue stragglers are produced mainly from binaries with initial mass ratios $> 0.5$; the stars observed in the second main sequence are therefore also potential blue straggler progenitors. However, the progenitors of the mergers, and also of the MS+WD systems, peak between $q_0 = 0.4$ and 0.6.

Blue stragglers as runaway stars

In a recent paper, Blaauw (1992) showed that seven well-studied OB runaway stars that could be traced back to their parent association, have (1) abnormally high helium abundances and (2) high rotational velocities. Furthermore, of four of these stars the luminosity could be determined very accurately and they appear to be blue stragglers in their parent association. The runaway velocities of these stars are mostly between 30 and 50 km/s, but exceed 100 km/s in a few cases.

As was mentioned above, the MS+NS systems receive a recoil velocity due to the supernova explosion of the He star in the progenitor system. In Fig. 5.13 we have plotted for the MS+NS blue stragglers, selected for this purpose with $\delta_{col} = 0.05$, the distribution of their system velocities $v_{sys}$ at a number of subsequent age intervals. Clearly, the distribution evolves as a function of age: the system velocities are largest for the youngest clusters, because the He
star that explodes is relatively more massive and a larger fraction of the total binary mass is ejected (cf. Appendix, eq. 5.31). It should be noted that we have assumed symmetric supernova explosions, whereas there is growing evidence that asymmetries in the explosions themselves play an important role in the runaway velocities of radio pulsars (Dewey & Cordes 1987, Bailes 1989, Van den Heuvel 1993). The inclusion of this effect might increase the runaway velocities in our simulations.

5.5.3 Attempt at a comparison with observations

A detailed, statistical comparison of our simulated CMDs with observed ones is difficult for a number of reasons. Firstly, the data are sparse: at most one or two blue stragglers and only a few giants are observed per cluster, and the photometry of main-sequence stars is usually incomplete. Secondly, the zero-age distributions of mass, mass ratio and separation may differ from one cluster to another, as do the binary fraction and star density. What we have assumed about these distributions in our simulations should be taken as a grand average over many clusters. Thirdly, clusters evolve dynamically (although actual physical interactions between stars and binaries are very unlikely): a cluster will disperse in the course of time since it is only loosely bound. The Hyades, for example, are recognisable as a cluster mostly by merit of velocity measurements, and not because they form a distinct group in the sky. Stars are continuously ‘evaporating’ from a cluster, and if there were a preference for low-mass stars to be ejected, the shape of the mass function would change as a function of age.

The first two problems are somewhat resolved by collecting several clusters into age groups, as was done by Mermilliod (1981a): the statistics get better and differences between individual clusters are smoothed out to some extent. In order to compare our results statistically with the observations we have performed star counts in the composite CMDs presented by Mermilliod (1981a). Although Mermilliod explicitly warns against using his diagrams for counting stars because of incompleteness, we expect that the upper parts of the diagrams are sufficiently complete to give meaningful numbers. We have counted the number of ‘bright’ MS stars $n_{\text{MSb}}$, i.e., stars in the main-sequence band above a cut-off magnitude $M_{V,\text{cut}}$ given by

$$M_{V,\text{cut}} = \max(M_{V,\text{TO}} + 1.5, -3.0)$$

(i.e., the same magnitude as the selection criterion for blue stragglers, eq. 5.18). The absolute visual magnitude of the TAMS or turn-off, $M_{V,\text{TO}}$, was estimated for each age group from the empirical isochrones in Figs. 2 and 3 of Mermilliod (1981b). We estimate the uncertainty in $M_{V,\text{TO}}$ to be about $\pm 0.2$.

The results of our star counts are presented in Table 5.3. We give an error estimate for $n_{\text{MSb}}$, equal to the number of stars within $\pm 0.2$ of the assumed value of $M_{V,\text{cut}}$, corresponding to the uncertainty in $M_{V,\text{TO}}$. In addition, we give the number of giants, $n_G$; the number of ‘red’ giants, $n_{RG}$ (i.e., the giants with $B-V > 0.8$); and the number of blue stragglers, $n_{BS}$, derived from the list of Mermilliod (1982, Table 1).

A comparison of the morphologies of our synthetic CMDs and the observed diagrams implies higher ages than those determined by Mermilliod (1981b). In order to be able to compare simulated and observed CMDs with the same morphology (i.e., position of the turn-off, etc.) we have recalibrated the ages of the age groups. Following Mermilliod (1981b), we have used the observationally determined values of $(B-V)_{\text{TO}}$ and $(U-B)_{\text{TO}}$ (as published in Table 7 in his paper; see § 5.5.2) as the prime criterion in determining the age of a cluster. These values were compared with the theoretical isochrones constructed from Maeder & Meynet's grids. The observational value of $M_{V,\text{TO}}$ was compared to $M_{V,TAMS}$ as a secondary criterion, and was found in all cases to correspond very well within the error estimate. The resulting ages are also given
Table 5.3: Star counts and age estimates of Mermilliod’s composite colour-magnitude diagrams

<table>
<thead>
<tr>
<th>age group</th>
<th>( M_{V,\text{tot}}^1 )</th>
<th>( \log t ) (Myr)</th>
<th>( n_{\text{G}} )</th>
<th>( n_{\text{RG}} )</th>
<th>( n_{\text{BS}}^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hyades</td>
<td>0.9</td>
<td>3.14 ±0.06</td>
<td>4</td>
<td>102</td>
<td>27</td>
</tr>
<tr>
<td>2 NGC 2281</td>
<td>0.15</td>
<td>2.88 ±0.06</td>
<td>9</td>
<td>92</td>
<td>28</td>
</tr>
<tr>
<td>3 NGC 3532</td>
<td>-0.5</td>
<td>2.72 ±0.08</td>
<td>3</td>
<td>41</td>
<td>11</td>
</tr>
<tr>
<td>4 NGC 6475</td>
<td>-0.7</td>
<td>2.54 ±0.08</td>
<td>4</td>
<td>82</td>
<td>20</td>
</tr>
<tr>
<td>5 NGC 2287</td>
<td>-0.8</td>
<td>2.43 ±0.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6 NGC 2516</td>
<td>-1.4</td>
<td>2.19 ±0.07</td>
<td>7</td>
<td>69</td>
<td>13</td>
</tr>
<tr>
<td>7 Pleiades</td>
<td>-1.7</td>
<td>2.08 ±0.07</td>
<td>8</td>
<td>55</td>
<td>17</td>
</tr>
<tr>
<td>8 α Per</td>
<td>-2.2</td>
<td>1.93 ±0.08</td>
<td>4</td>
<td>27</td>
<td>5</td>
</tr>
<tr>
<td>9 IC 4665</td>
<td>-2.5</td>
<td>1.78 ±0.10</td>
<td>5</td>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>10 NGC 3766</td>
<td>-3.0</td>
<td>1.55 ±0.12</td>
<td>6</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td>11 NGC 457</td>
<td>-3.7</td>
<td>1.35 ±0.14</td>
<td>6</td>
<td>53</td>
<td>9</td>
</tr>
<tr>
<td>12 NGC 884</td>
<td>-4.5</td>
<td>1.16 ±0.15</td>
<td>4</td>
<td>48</td>
<td>13</td>
</tr>
<tr>
<td>13 NGC 2362</td>
<td></td>
<td>0.89 ±0.15</td>
<td>5</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>14 NGC 6231</td>
<td></td>
<td>0.63 ±0.20</td>
<td>3</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^1\)For age groups 13 and 14 the turn-off magnitude could not determined, and a value of \( M_{V,\text{tot}} = -3.0 \) was assumed.

\(^2\)Number of clusters included in the composite CMD. For a few of age groups, several member clusters were excluded by Mermilliod because of poor photometric data: NGCs 6529 and 6705 in group 4, and the entire group 5 (3 clusters). In the CMD of the Hyades group (1), only the giants of NGC 6633 were included by Mermilliod. We have added the estimated number of bright MS stars in NGC 6633 (28 ± 3), from the CMD by Hagen (1970, no. 142), to the number \( n_{\text{MSB}} \) from Mermilliod’s CMD.

\(^3\)Number of blue stragglers derived from the list of Mermilliod (1982, Table 1). Excluded from this list are those stars which either (1) belong to clusters not included in the CMD, or (2) fall outside the selection criterion \( \epsilon_{\text{col}} = 0.0^{10}, \) or (3) have only poor evidence for membership (no proper motion or radial velocity data, and a position outside one cluster radius). The total number of stragglers in the list, as far as their parent clusters are included in the CMD, is given in parentheses.

in Table 5.3, with an error estimate based on the assumption that the determination of \((U - B)_{\text{bol}}\) is uncertain by 0.005 and \((B - V)_{\text{bol}}\) by 0.002.

The re-determined ages are higher than those determined by Mermilliod (see his Table 7), by about a factor 1.5 for most age groups, and as much as 2 for the oldest three groups. The main reason is the fact that we have used evolutionary grids with convective core overshooting, whereas Mermilliod used the grids of Maeder & Mermilliod (1981) with much less overshooting. We do not claim that our ages are necessarily more trustworthy than Mermilliod’s, given the uncertainties involved with overshooting and the errors in the lifetimes of Maeder & Meynet’s models for low-mass stars (Bertelli et al. 1992).

Since the observed numbers are small, better statistics can be achieved by grouping several age groups together. We have collected the age groups from Table 5.3 into three ‘supergroups’, as follows: group I includes age groups 1–4 (intermediate-age clusters, with well-defined giant ‘clumps’) with 2.35 < \( \log t \) (Myr) < 3.2; group II includes age groups 6–9 (young clusters showing less concentrated giant branches) with 1.65 < \( \log t \) (Myr) < 2.35; and group III includes age groups 10–12 (very young clusters with red and blue supergiants) with 1.0 < \( \log t \) (Myr) < 1.65. Groups 13 and 14 have been left out because they have neither giants nor blue stragglers. In
### Table 5.4: Comparison of observations with model predictions

<table>
<thead>
<tr>
<th>Group</th>
<th>$\log t$ (Myr)</th>
<th>$n_G/n_{MSB}$</th>
<th>$n_{RG}/n_G$</th>
<th>$n_{BS}/n_{MSB}$</th>
<th>$n_{BS}/n_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (1–4)</td>
<td>2.35–3.2</td>
<td>0.209</td>
<td>0.895</td>
<td>0.0158</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>2.35–3.2</td>
<td>0.199</td>
<td>0.940</td>
<td>0.0101</td>
<td>0.050</td>
</tr>
<tr>
<td>II (6–9)</td>
<td>1.65–2.35</td>
<td>0.201</td>
<td>0.0489</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.65–2.35</td>
<td>0.194</td>
<td>0.0267</td>
<td>0.142</td>
<td></td>
</tr>
<tr>
<td>III (10–12)</td>
<td>1.0–1.65</td>
<td>0.141</td>
<td>0.0231</td>
<td>0.161</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0–1.65</td>
<td>0.131</td>
<td>0.0146</td>
<td>0.112</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4 we give the values of $n_G/n_{MSB}$, $n_{RG}/n_G$, $n_{BS}/n_{MSB}$, and $n_{BS}/n_G$ for these groups, and the predicted values from the simulations. The errors correspond to 1σ uncertainty limits, assuming a Poisson distribution of counting noise, to which the additional errors in $n_{MSB}$ as given in Table 5.3 have been quadratically added. The predicted values are averaged over the age intervals as given above. Since the clusters in Table 5.3 are distributed homogeneously in $\log t$ rather than in $t$, in calculating the averages we have weighted each simulated system with the interval $\Delta \log t$ during which it is a blue straggler.

### Giants

In Fig. 5.14 we compare, as a function of $\log t$ of the age group, the observed ratio of the number of giants to bright MS stars from Table 5.3, $n_G/n_{MSB}$, with the ratios resulting from our simulations A17 and B17. We have also plotted the expected ratio for a population of only single stars. This figure can be used for two purposes. As we have argued in § 5.5.1, the number of giants is depleted as a result of mass exchange, so the ratio of giants to bright MS stars should depend on the fraction of close binaries in the zero-age population. Therefore the comparison can in principle be used to constrain this fraction, but unfortunately the errors in the observed numbers are too large to draw any conclusions. On the other hand, the comparison provides a test of the incompleteness of the observed sample. We can reasonably assume that the giant observations are complete, since giants are the brightest stars in the cluster. Hence, Fig. 5.14 mainly serves to show that the incompleteness of the MS stars up to $1^{m}5$ below the turn-off is not very large, since the observed ratios compare well with the models.

In § 5.5.1 it was also argued that the number of yellow straddler giants for $t > 200$ Myr could provide information about the initial mass-ratio distribution, or at least about the fraction of systems with $q_0 > 0.7$. In Fig. 5.15 we have plotted the observed and simulated fractions $n_{RG}/n_G$ of ‘red’ giants (with $B-V > 0.8$) among all giants. Indeed, model A17 yields a larger fraction of ‘yellow’ giants for $\log t > 2.0$, by about a factor two compared to model B17. The much smaller fraction of red giants for ages less than 200 Myr is not due to yellow straddlers, but caused by the fact that He-burning giants spend part of the time in a blue loop. A comparison of $n_{RG}/n_G$ with observed values for the purpose of finding constraints on the $q$-distribution is therefore only useful for group I (age groups 1–4). The observed fractions (cf. Table 5.4) seem to favour model A17, i.e., a rather flat $q$-distribution, but the significance is only marginal.
Figure 5.14: Ratio of the number of giants \( n_G \) to the number of bright MS stars \( n_{\text{MSB}} \) (i.e., MS stars with \( M_V < M_{\text{Vcut}} \) defined by eq. 5.20), from observations (Table 5.3, solid dots and error bars) and resulting from the simulations of models A17 (solid line) and B17 (dashed line). The vertical error bars correspond to 1\( \sigma \) uncertainty limits, assuming a Poisson distribution of counting noise and taking into account the additional uncertainty in \( n_{\text{MSB}} \), as given in Table 5.3, by quadratic adding.

Figure 5.15: Fraction \( n_{RG}/n_G \) of 'red' giants, with \( B-V < 0.8 \), to all giants, from observations and resulting from simulations (see Fig. 5.14).
5.5 Results

Figure 5.16: Fraction $n_{BS}/n_{MSb}$ of blue stragglers to all bright MS stars, from observations and resulting from simulations (see Fig. 5.14). The simulated MS+NS systems with $v_{sys} > 5$ km/s have been excluded.

Blue stragglers

The last column $n_{BS}$ of Table 5.3 represents the number of ‘true’ blue stragglers in the list of Mermilliod (1982). Mermilliod’s list contains a few stars which do not follow our selection criterion $\delta_{col} = 0^\circ10$ (e.g., VB56 in the Hyades and Alcyone in the Pleiades), although we have chosen the criterion to match as closely as possible the observational one. (The two blue stragglers in group 14 are included by Mermilliod on the basis of their early spectral type, which is a better criterion than $U-B$ for the very young clusters.) Furthermore, the list contains a number of stars for which the evidence for membership is poor (mainly stars in a few intermediate-age clusters, group 1). These ‘stragglers’ have been excluded from our statistics.

In Fig. 5.16 we have plotted, similar to the previous two figures, the observed and simulated fractions $n_{BS}/n_{MSb}$ of blue stragglers among the ‘bright’ MS stars. The MS+NS binaries with system velocities $v_{sys} > 5$ km/s have not been included in the simulated fractions, because these systems are expected to have left their parent cluster. Unfortunately, the numbers of blue stragglers in each age group is so small that a detailed comparison is hampered by the large error bars. Although, within these limits, the fraction of blue stragglers produced in the simulations corresponds with the observed fraction, it appears that different trends are present in the observed and simulated fractions. While the observed fraction is rather constant at about 0.04, the predicted fractions first increase and then decrease with age, with a maximum at $t \approx 100$ Myr. For the oldest two age groups, the simulations seem to be unable to account for all the observed blue stragglers. This conclusion is confirmed when looking at the numbers in Table 5.4. For young clusters (groups II and III) the observed fractions agree well with the predictions of model A17, but for intermediate-age clusters (group I) the predicted value is too small by about $2\sigma$. The lack of agreement is worse for model B17.
An additional comparison comes from the ratio of blue stragglers to giants, \( n_{BS}/n_{G} \). This ratio should be less affected by incompleteness than \( n_{BS}/n_{MSB} \). Wheeler (1979b) found that in old open clusters this ratio lies in the range \( \frac{1}{4} \) to \( \frac{1}{3} \). From Table 5.4 it follows that the ratio is smaller for young and intermediate-age clusters, about 0.15 to 0.2. Perhaps Wheeler's determination is somewhat high because his estimate is based on clusters selected on the basis of their having blue stragglers. Comparing this ratio with the model predictions for group I, the correspondence is somewhat better but still off by more than 1σ even for model A17.

The decrease of the fraction of blue stragglers with age in the simulated clusters is mainly caused by the decreasing fraction of progenitors. These progenitors are binaries with orbital periods such that the primaries have radiative envelopes when mass transfer starts (i.e., case A and early case B mass transfer), and thus evolve conservatively for initial mass ratios \( q_0 \geq 0.5 \). Systems with larger orbital periods have convective envelopes and evolve non-conservatively. The range in orbital periods for radiative case B decreases with decreasing primary mass, and becomes very small for \( m < 3M_\odot \).

Collier & Jenkins (1984) found that binary mass exchange could also explain the observed number of blue stragglers in clusters older than the Hyades. They only considered case B evolution and assumed the mass transfer to be conservative for \( q_0 > 0.4 \), even when the primary is on the giant branch and has a convective envelope. In our models (although we have not applied them to primaries below \( 2M_\odot \)) these systems would go through dynamical instability and evolve non-conservatively. In this case, the actual numbers of blue stragglers produced should be considerably smaller than predicted by Collier & Jenkins' calculation.

A different test of our model as an explanation for the blue-straggler origin may come from the observed blue-straggler properties, as discussed in § 5.2. As we have seen, the evidence for binarity of blue stragglers in young open clusters from radial-velocity variations is poor. These young blue stragglers are often Be stars and rapid rotators. From our model, we expect that a large fraction should indeed be single, since they are mergers. Of the fraction that is binary, the majority is expected to show only small radial-velocity variations (< 10 km/s) and orbital periods over 100 days. Such small variations will be extremely hard to detect in the spectrum of an early-type star, especially if it has a rotationally broadened line or emission lines. It should be noted that rapid rotation is exactly what one would expect from stars that are the remnants of mass exchange (see also Chapter 3).

The case of \( \theta \) Car (HR 4199) in IC 2602 deserves special attention, since it appears to present a perfect case for a mass-exchange history. It is a spectroscopic binary and has peculiar CNO abundances and a higher He abundance than other cluster members. The mass function indicates a present mass ratio smaller than 0.1, in accordance with the predictions from our simulations (§ 5.5.2). Its mass should be close to twice the turn-off mass, which implies that the mass transfer must have been quite conservative and the initial mass ratio close to unity. However, the interpretation of this system as a product of mass exchange is not without problems. Firstly, the short orbital period implies that considerable angular momentum losses must have occurred, even though the fraction of mass that was lost from the system is very small (cf. also Eggen & Iben 1988). Secondly, one would not expect an eccentric orbit after a mass transfer phase. It is unlikely to be caused by a supernova explosion, since the system should then have been kicked out of the cluster and the companion should then be a neutron star.\footnote{\( \theta \) Car was detected as an X-ray source in the ROSAT all-sky survey, with an X-ray luminosity of about \( 2 \times 10^{31} \) erg/s (Peters 1993). However, the ratio of X-ray to bolometric luminosity, \( \log(L_X/L_{bol}) \approx -6.7 \), is normal for early B-type and O-type stars (Meurs et al. 1992, Chapter 4 of this thesis).} An eccentricity might be produced by a disk of ejected mass around the system after the mass exchange (Artymowicz et al. 1991).

Interestingly, the blue stragglers in the intermediate-age clusters considered here (group I)
are not rapidly rotating and do not show emission lines (see § 5.2). If these stars are the remnants of conservative mass transfer, one would expect them to have been spun up significantly in the mass-transfer process. Therefore, if these stars are produced by mass transfer, they must have slowed down again afterwards. Since the lifetimes of these systems are relatively long compared to more massive stars (in younger clusters), there is more time available for slowing down. Magnetic fields, that may be left over from large-scale circulations during the mass transfer process, could possibly play a role in braking the rotation, like in cool stars. Indeed, these late-type blue stragglers usually are Bp or Ap stars, which often have strong magnetic fields (cf. Abt 1985).

Blue interlopers

From our simulations it follows that ‘blue interlopers’ are produced in a ratio of about 1 to 50 with respect to the number of bright MS stars, $n_{MS}^b$, or about 1 to 10 with respect to the number of giants, $n_G$, for both models A17 and B17. However, the number produced is very sensitive to the model assumptions about non-conservative mass transfer. In a test calculation with a value for the spiral-in efficiency parameter $\alpha_{CE} = 0.1$, and spiral-in assumed for all dynamical time scale mass transfer processes, almost none of these systems were produced.

These systems are interesting because in a few of the older clusters stars are present at positions corresponding to those of the blue interlopers in our simulations ($M_V \sim 5$ and $B - V \sim 0.4 - 0.5$; see, e.g., the diagrams of Praesepe and the Hyades by Hagen 1970). These stars have not been included by Mermilliod in his composite CMDs on the suspicion of being non-members, although they can not be distinguished on the basis of proper motion or radial velocity. Especially in the Hyades, a whole sequence of such ‘subdwarfs’ is observed parallel to the main sequence. However, their total number in Hagen’s diagrams is much larger than expected from our simulations (it exceeds that of the giants).

Our special attention was drawn to the star KW425 in Praesepe, which lies about $1^{\circ}0$ below the main sequence (or $0^{\circ}2$ to the left in $B - V$), but has a central position and a very high membership probability (Jones & Stauffer 1991). Dr. R.G.M. Rutten (private communication) has been able to take high-resolution spectra of this star around H$\alpha$ and H$\beta$ wavelengths at La Palma Observatory. Its spectrum appears to be normal for its colour ($B - V = 0.53, V = 11.42$), and no evidence was found for either a later (G-type) main-sequence star or a helium star, and we must conclude that this star is most probably a non-member.

However, we suggest that more of these ‘interlopers’ should be observed spectroscopically in order to determine their nature. The discovery of a composite He star + late-type MS star spectral signature could serve as an interesting test of evolutionary scenarios.

5.6 Summary and conclusions

Our simulations demonstrate that the inclusion of close-binary evolution in the study of the stellar population of young open clusters explains several observed features. Especially, blue stragglers are produced in quantities that correspond well to the observed numbers in clusters younger than about 300 Myr. The synthetic blue stragglers can be either single, merged main-sequence stars or binaries with a low-mass, evolved companion. However, the observed numbers are too small to constrain the model parameters in our simulations. The lack of observed radial velocity variations in blue stragglers is consistent with our models: a large fraction of the synthetic blue stragglers are truly single, coalesced stars, while the majority of the binary systems have long orbital periods and orbital velocities below the detection limit. The fact that the observed early-type blue stragglers are rapid rotators and often Be stars is in accordance with a mass-exchange origin.
For the clusters of intermediate age considered in this study (between 300 and 1500 Myr), our model systematically produces lower numbers of blue stragglers than observed, by a factor two to three. Furthermore, the observed blue stragglers in these clusters are slow rotators, contrary to what one might expect after mass transfer. This suggests that formation processes other than mass transfer play an increasing role at these ages. However, we cannot exclude the possibility that the parameter space we have assumed for the formation of blue stragglers might be too small, i.e., that a larger fraction of binaries might evolve conservatively. This would imply that a significant fraction of stars with deep convective envelopes at the onset of mass transfer can avoid dynamically unstable mass transfer. The loss of a significant fraction of the envelope mass before Roche-lobe overflow occurs, e.g., due to enhanced stellar-wind mass loss in a close binary, would stabilize the mass transfer (Tout & Eggleton 1988). This possibility deserves further investigation.

Our calculations confirm the finding by Haffner & Heckmann (1937) that the occurrence of a second main sequence is result of unresolved binaries, even with a mass-ratio distribution that decreases towards equal component masses. The number of giants in open clusters, and the distribution of ‘yellow straddler’ giants, are found to depend on the input distributions for the mass ratio and orbital separation, but it is difficult to find observational constraints to these distributions because of the small number of giants in young clusters.

Acknowledgements

We are much indebted to Jacqueline Coté for collecting data on colour conversions and preparing the temperature-to-colour conversion tables, to René Rutten for using part of his valuable observing time, to Ankie Piter for discussions about errors and Poisson distributions, to Ed van den Heuvel for many valuable comments and discussions, and to André Maeder for providing an electronic version of his evolution grids.

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Appendix: Orbital evolution with mass loss and mass exchange

Consider a binary in a circular orbit, with orbital separation $a$ and masses $m_1$ and $m_2$. The total mass of the system is $M = m_1 + m_2$, and the total orbital angular momentum is $J = m_1 m_2 (Ga/M)^{1/2}$. The rotational angular momenta of both stars are assumed to be zero. Then the evolution of the orbit with respect to slow changes (with respect to the orbital period) in $m_1$, $m_2$ or $J$ follows from:

$$ \frac{da}{a} = -2 \left( \frac{dm_1}{m_1} + \frac{dm_2}{m_2} \right) + 2 \frac{dJ}{J} + \frac{dM}{M}. \quad (5.21) $$

Suppose now that $m_1 = m_d$ is the mass of the donor (i.e., $dm_d < 0$), and $m_2 = m_a$ is the accretor mass. A fraction $\alpha$ of this mass is ejected from the system, i.e., $dM = \alpha dm_d$. Hence, a fraction $1 - \alpha$ is accreted by the companion: $dm_a = -(1 - \alpha) dm_d$. Furthermore, suppose that the mass ejected from the binary has a specific angular momentum $j = dJ/dM = \beta J/M$. If $\alpha$ and $\beta$ are known as a function of time, then the orbital evolution can be calculated by integrating eq. (5.21). For a number of idealized cases simple formulae can be derived. We will describe these below, as far as they are of interest to our model.
Appendix

Constant $\beta$

If the value of $\beta$ is constant in time, eq. (5.21) can be easily integrated to give the change of $a$ from its initial value $a_0$ when the donor mass has decreased from $m_{d,0}$ to $m_d$:

$$\frac{a}{a_0} = \left( \frac{m_d}{m_{d,0}} \frac{m_a}{m_{a,0}} \right)^{-2} \left( \frac{M}{M_0} \right)^{2\beta+1}.$$  

(5.22)

where, of course, $m_a = m_{a,0} + (1 - \alpha)(m_{d,0} - m_d)$ and $M = M_0 - \alpha(m_{d,0} - m_d)$. Note that $\alpha$ is here an average over time, i.e., $\alpha$ need not be constant.

In the case of conservation of total mass and angular momentum ($dM = dJ = 0$) $\alpha = 0$ and $M = M_0$, and the expression becomes:

$$\frac{a}{a_0} = \left( \frac{m_d}{m_{d,0}} \frac{m_a}{m_{a,0}} \right)^{-2}.$$  

(5.23)

Isotropic mass loss

If the ejected matter leaves the system with the specific angular momentum of the orbit of the accretor, we have $\beta = m_d/m_a$. This is the case when the matter leaves the system in the form of an isotropic ‘wind’ or in the form of a jet from $m_a$, e.g., when the accretor is a neutron star accreting at a super-Eddington rate. If furthermore $\alpha$ is constant throughout the mass transfer, the following expression can be derived, when $\alpha < 1$:

$$\frac{a}{a_0} = \left( \frac{m_d}{m_{d,0}} \left[ \frac{m_a}{m_{a,0}} \right]^{1/(1-\alpha)} \right)^{-2} \left( \frac{M}{M_0} \right)^{-1}.$$  

(5.24)

In the limit $\alpha = 1$, we have $m_a = \text{constant}$, and the expression becomes:

$$\frac{a}{a_0} = \left( \frac{m_d}{m_{d,0}} \right)^{-2} \left( \frac{M}{M_0} \right)^{-1} \exp \left( 2 \frac{m_d - m_{d,0}}{m_a} \right).$$  

(5.25)

In the case of an isotropic wind from the donor, of which a fraction $1 - \alpha$ is accreted by $m_a$, we derive:

$$\frac{a}{a_0} = \left( \frac{m_d}{m_{d,0}} \right)^{1-\alpha} \left[ \frac{m_a}{m_{a,0}} \right]^{-2} \left( \frac{M}{M_0} \right)^{-1},$$  

(5.26)

which becomes in the limit $\alpha = 1$:

$$\frac{a}{a_0} = \left( \frac{M}{M_0} \right)^{-1}.$$  

(5.27)

Explosive mass loss

In the case of sudden, explosive mass loss, eq. (5.21) does not apply. Instead, if the mass loss is isotropic (which applies to symmetric supernova explosions) an eccentricity is induced into the orbit as follows:

$$e = \frac{1 - M/M_0}{M/M_0}.$$  

(5.28)

Hence, if $M < \frac{1}{2} M_0$ the orbit becomes unbound and the two stars become single with velocities $v_d = v_{\text{orb},d,0}$ and

$$v_a = v_{\text{orb},a,0} \left( 1 + \left[ 1 - 2 \frac{M}{M_0} \left[ \frac{M_0}{m_{a,0}} \right]^2 \right]^{1/2} \right).$$  

(5.29)
If $M > \frac{1}{2}M_0$ the orbit remains bound and the separation becomes

$$\frac{a}{a_0} = \frac{M/M_0}{2M/M_0 - 1}$$

and furthermore the system receives a recoil velocity of magnitude

$$v = e \cdot v_{\text{orb,d.o.}}.$$  \hspace{1cm} (5.31)

Tidal interaction can eventually cause the orbit to circularize, after which the final separation becomes:

$$a' = a(1 - e^2).$$ \hspace{1cm} (5.32)

(I.e., the relation between the circularized separation and the pre-explosion separation as just as in the slow mass loss case, $a'/a_0 = (M/M_0)^{-1}$).