Chapter 1

1 Introduction

Neutron star formation is one of the most energetic phenomena in Nature, and occurs during the early stages of type II Supernova (SN) explosions. In our present-day universe, these are among the Biggest Bangs after the Big One. A type II SN is powered by the gravitational collapse of a 1–2 $M_\odot$ heavy-element, $Fe$ or $Ne-O-Mg$, central core of a 8–25 $M_\odot$ star, which has exceeded the Chandrasekhar mass limit. ($M_\odot = solar mass =1.989 \times 10^{33}g.$) In contrast, a type I SN is powered by a runaway thermonuclear explosion in $C$ or $O$-shells, a 'carbon-bomb' for short. A more detailed botany of various subclasses of supernovae (SNe) exists, but may not be of great physical importance.\(^\text{(1)}\)

Supernovae are Nature's way of recycling building material in grand fashion. In particular, heavy elements (heavier than $He$) that were gradually synthesized by the nuclear fusion reactions during a star's life and evolution, are in this way dispersed and fed into the interstellar medium. In the wild energetic binge of the explosion itself, elements heavier than $Fe$, which are not made in stellar interiors, are formed. The energy source for the explosion is the gravitational binding energy of the forming neutron star, which is of the order

$$E_{\text{bind}} \approx 3 \times 10^{52} \left( \frac{M_*}{M_\odot} \right)^2 \left( \frac{R}{10 \text{ km}} \right)^{-1} \text{erg},$$

with $M_*$ and $R$ the neutron star mass and radius.

This way the interstellar medium becomes enriched with heavy elements, and these contaminations are re-used in the formation of subsequent generations of stars, often triggered by SNe, with possibly accompanying planets.

We ourselves, as $C$-based life forms, living on a $Si$-based planet circling a $n^{th}$-generation ($n > 2$) star are in fact such contaminations of the interstellar medium, and hence owe our existence in no small way to the occurrence of SNe.

The formation of a neutron star implies a successful supernova explosion. If the explosion fails, matter outside the collapsed core will accrete onto it, until its critical mass is exceeded, and the collapse continues with a black hole as final product. As SNe can only be observed, and not experimented with, the theoretical investigation of their working is mainly done by computer simulations.

The supernova phenomenon and neutron star formation involve all four fundamental forces of nature, and require ingredients from nearly all fields of physics, ranging from transport physics and hydrodynamics to Special and General Relativity and particle physics. All volumes of Landau and Lifshits can be fruitfully employed. The inclusion of all the relevant known physics in all detail would be computationally overwhelming. Unavoidably, approximations must be made, and aspects of the problem thought of as less important or beyond technical feasibility must be neglected.

Today, the main problem in supernova modelling is how to turn an implosion into a successful explosion. A mechanism is required which transfers a small part of the binding energy released in the core collapse to the outer layers, forming an explosion that carries some $O(10^{51})$ erg in kinetic energy, and emits about $O(10^{49})$ erg in electromagnetic radiation. The remaining 99% of the energy balance is carried away by neutrinos. Neutrinos are elementary particles without electrical charge or known mass, that interact only through the weak force. Their main characteristic is that under 'normal' circumstances they participate in next to nothing.
The neutrinos, and hence the overwhelming part of the total energy that is released, are emitted during the neutron star formation phase, which is the subject of this thesis. Energetically, the visible SN explosion is a mere afterthought to neutron star formation.

Nature has obviously found a way to solve the problem, as SNe are not infrequent phenomena. In our Galaxy the SN-rate is estimated at about 1/50–1/100 per year, and many SNe a year are observed in distant galaxies. Also the great number of pulsars, (\sim 500 known to date), all of them neutron stars, indicates that successful explosions are not extraordinary.

However, in numerical simulations of SNe the most frequent simulation outcome is failure. Certainly for progenitor stars with main–sequence masses above \sim 12 M_\odot, and given reasonable input physics, no successful SN explosions are obtained. With SN 1987 A, one of only two SNe observed whose progenitor was identified (the other one being SN 1961v), Nature has provided the proof of existence that a star with a main–sequence mass close to 20 M_\odot collapses, and can successfully explode. The failure so far to duplicate this numerically, indicates that the explosion mechanism(s) in heavy stars, and perhaps in all, is as yet not quite understood.

The first ingredient to supernova simulation is the stellar evolution calculation of the progenitor star. It simulates the 10^6–10^8 years of the progenitor's evolution, which consists of continued contraction of its core, interrupted by phases of nuclear energy generation. A succession of nuclear fuels is burned, including H, \,^4He, \,^{12}C, \,^{16}O and \,^{28}Si. The last stage of nuclear burning produces \,^{56}Fe, and lasts only about one day. At this point all available sources of nuclear energy have been exhausted, and once the growing Fe core has reached its Chandrasekhar stability limit, catastrophic gravitational collapse is inevitable. (Actually, only stars heavier than \sim 11 M_\odot undergo this full sequence and form Fe cores.)

The single most important input from the stellar evolution for the collapse calculation is the size of the iron core. There are considerable uncertainties in calculating presupernova progenitors. Prominent among these are the inclusion of convection, details of the equation of state (EOS), the nuclear reaction chains and the electron-capture rates. Details of the simulations can greatly influence the nuclear evolution and the final pre-collapse structure and mass of the iron core.

During the stellar evolution of the progenitor, the star is in hydrostatic equilibrium, and the structure of the codes used to simulate the evolution is not entirely unlike the neutron star evolution code that is presented in chapter 5, and used to simulate the neutron star formation.

Photodissociation of the iron-peak nuclei in the core into \alpha-particles or electron capture, destabilize the core which as a result starts to collapse under its own weight. The rising density enhances further electron capture, which in turn accelerates the collapse. The collapse time scale of the core is \sim 0.1 sec, its dynamical (free fall) time scale is of the order of 1 msec. The collapsing core can be treated as an ideal fluid, because except for energy transport by neutrinos and shock waves, no dissipative transport processes are important for its dynamics.

As the density of the collapsing core exceeds \sim 3 \times 10^{12} g cm^{-3}, the material becomes opaque to neutrinos, as their diffusion time scale becomes longer than the collapse time scale. During the remainder of the collapse the neutrinos are trapped, and chemical $\beta$-equilibrium through the reaction

\[ e^- + p \rightarrow n + \nu_e \]

rapidly establishes. As a result, the total lepton number of the core remains constant.
and it is only depleted during the long, \( > 10 \text{sec} \), neutron star formation process. After \( \beta \)-equilibrium is attained, also the entropy remains constant during the remainder of the collapse. The opacity during collapse is mainly due to weak neutral–current coherent scattering off nuclei\(^8,10\) and free nucleons.

Throughout the collapse, the core is divided into an inner core (IC), which collapses homologously (infall velocity proportional to radius), and an outer mantle which collapses supersonically, the boundary being formed by the point where the Mach number approximately equals 1 (the sonic point). Nothing that happens to the inner core can therefore be communicated to the outer mantle. The collapse is halted when the core reaches nuclear densities, and the strong force cuts in. At these densities the nuclei that have so far remained intact due to the low entropy of the core,\(^{11}\) start to overlap and undergo a phase transition into a degenerate nucleon Fermi liquid, with strong repulsive forces at small distances. Suddenly the EOS stiffens, and within milliseconds the subsonic inner core rebounds as a whole. In the meantime the supersonically falling mantle is totally oblivious of the obstacle that is forming. It becomes aware suddenly, as it crashes into it, and adjusts by piling up pressure waves which propagate from the centre outward and accumulate into a shock wave at the sonic point. The energy of the shock is in first instance determined by the expansion work performed by the rebounding inner core, and is of the order of \(4-10 \times 10^{51} \text{erg} \).\(^1,3\)

The bounce is followed by the shock propagation phase. Within milliseconds, the collapsed inner core and the material accreted onto it settle into hydrostatic equilibrium, forming the hot, lepton rich proto-neutron star, and signalling the beginning of neutron star formation.

As the bounce shock propagates outward it rapidly loses energy by photodisintegration of heavy nuclei into free nucleons. This way it loses\(^{12,13}\) \(\simeq (1.6-1.8) \times 10^{51} \text{erg} \) per disassociated \(0.1 M_\odot \). After it reaches regions of the star increasingly transparent to neutrinos (\(\rho < 10^{12} \text{g cm}^{-3}\)), neutrino emission adds to the energy loss. The shock gains energy from the dissipated kinetic energy of the material falling through it, although most of this gain is used to put the infalling material into hydrostatic equilibrium as it piles up on top of the proto-neutron star.\(^5\) The total photodisintegration losses are proportional to the difference in mass between the total mass of the heavy–element iron core, and the mass of the inner core inward of the sonic point, where the shock originates. This is why the total mass of the iron core as obtained from the progenitor evolution is such an important input parameter. There exists a tentative consensus that (prompt) explosion can only occur if the iron core mass \(M_{\text{core}} < 1.35 M_\odot \), see also below.

The inner core mass is equally vital. It decreases during the collapse due to lepton loss, and depends mainly on the IC's initial entropy, and the neutrino transport which determines the lepton fraction at trapping.\(^{14-18}\) A low initial entropy implies fewer free protons, which allows less electron capture and leaves the trapped lepton fraction \(Y_L\) higher, which in turn gives rise to a heavier IC, according to \(M_{\text{IC}} \propto Y_L^2\).

The initial strength of the shock depends on the stiffness of the EOS. For a stiff EOS the core overshoots less, and the maximum density at bounce reaches only about twice nuclear saturation density.\(^{12,19}\) For a soft EOS the maximum density reached may be as high as ten times nuclear saturation density.\(^{20,22}\) Because in the latter case the collapse proceeds to higher density, the binding energy of the proto-neutron star is higher, increasing the initial energy of the shock. General Relativity (GR) enhances this effect.\(^{20,22}\) Details of the EOS above nuclear density are uncertain, and consist almost uniquely of theoretical models based on effective nucleon–nucleon interactions.
In the prompt-explosion scenario the shock wave survives the assaults on it, and manages to plough through the heavy-element shells of the outer mantle and surrounding layers. It then continues its way outward through the less dense outer $He-H$ layers of the star. Within a few hours it reaches the edge of the star and bursts out in the optical display of a visible, successful SN explosion.

If the mass difference between the total iron core and the IC exceeds about 0.45 $M_\odot$, the shock stalls\[13, 14, 17, 12, 22, 23, 24\] within tens of milliseconds after bounce. To date, there is general agreement that with somewhat detailed neutrino transport taken into account, currently available progenitor cores, particularly those heavier than 1.35 $M_\odot$, cannot explode by the prompt mechanism.\[16, 18, 25\] This excludes progenitors with main sequence masses above roughly 12 $M_\odot$.

With the demise of the prompt scenario the so called\[26, 27, 28, 29\] 'delayed-explosion mechanism' has gained popularity. In this scenario the stalled shock is revived by energy deposited from the passing neutrino flow. The neutrino flow needs to deposit only about 0.1% of its energy to resuscitate the shock and produce a successful explosion. Numerical simulations of the hydrodynamic evolution of the SN carried on until about 1 sec after core bounce,\[26, 28, 30, 31\] have produced successful but rather weak explosions through this mechanism. However, as it turned out, small changes in the neutrino transport can easily turn success into failure.\[3\] Other results\[32\] indicate that the neutrino heating which proceeds on a time scale of hundreds of milliseconds, (see also chapter 5), which is much longer than the dynamical time scale of the shock propagation, simply causes the absorbing regions to expand, lowering the absorption opacity, and thus turning the heating off.

Collapse calculation codes, which simulate the events as sketched so far, are explicit hydrodynamical codes that solve for the hydrodynamical velocity. A typical collapse calculation simulates the $\sim 0.5$ sec real time that the collapse lasts at the cost of tens to hundreds of supercomputer hours. These codes are unsuited for the simulation of the hydrostatic evolution of the dense collapse residue, which takes place on a time scale dictated by the neutrino diffusion time scale due to the weak interaction, and takes tens of seconds.

From the onset of shock propagation, the deleptonization of the collapse residue takes place, and the neutron star is forming. The neutrinos that diffuse out of the dense IC drive the evolution of the proto-neutron star. During the neutron star formation the binding energy which is stored in the trapped neutrinos, heat and degeneracy energy, is released on the neutrino diffusion time scale. The evolving neutron star is the (energy) source of the neutrino flow which is crucial to the working of the delayed-explosion scenario. Therefore the neutron star formation process in its early stages, during the first tens to hundreds of milliseconds, is likely to have some bearing on the delayed-explosion mechanism. As all cross sections of the processes that are responsible for the energy deposition, mainly absorption on free nucleons, depend on the neutrino particle energy, the details and efficiency of the deposition mechanism will depend on the non–LTE neutrino spectrum as it is produced by the evolving neutron star, see chapters 4 and 5. The success or failure of the delayed-explosion mechanism may therefore well hinge on the details of a spectral neutrino transport scheme. The approaches that have so far been employed in the various simulations are all rather approximate, and nearly without exception bulk schemes, that do not retain information about the spectrum. Not until the neutrino transport in these simulations is improved will it be possible to give the final verdict on the feasibility of the delayed-explosion scenario.

The subject of this thesis is the simulation of the neutrino driven neutron star for-
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Such calculations were done previously by Burrows and Lattimer,\(^{133}\) and by Suzuki.\(^{134}\) We have put the emphasis heavily on the neutrino transport, as neutrinos are clearly the star players in this phenomenon, and perhaps in the SN as a whole. It is fascinating to consider an object of such high density, that neutrinos, which in ordinary 'everyday' matter \((P6)\) have a mean free path of \(O(10^{10}-10^{11}) \text{ km}\) play a dynamical, and even pivotal role.

During the collapse phase, neutrino transport plays an important role in the determination of the trapped lepton fraction, which is a major quantity determining the size of the IC, and hence the possible success of the explosion. The forming neutron star represents a state of matter which cannot be studied in the laboratory. The neutrino radiation emanating from the object is the sole carrier of information available that could teach something about this exotic state of condensed matter. To be able to decipher the message, accurate neutrino transport simulations are crucial. Massive neutrino emission is the unmistakable signature of gravitational collapse. Until SN 1987 A, this was purely a theoretical prediction. The 19 events of neutrinos detected on earth and identified with SN 1987 A, have provided the first and so far only observational evidence that the theoretical picture is basically correct. The events were, however, too few to teach us anything about either the collapse mechanism, details of the state of (neutron) matter at and above nuclear density or neutron star formation. For this, we will have to wait for the next galactic SN to occur. When that happens, the detectors on earth will be able to resolve the neutrino spectra in detail, and these will educate us on these questions.\(^{4}\)

Clearly neutrino transport is central to further understanding of SN and neutron star formation. There are many ways to treat neutrino transport in this context. These range from the conceptually very nice and rigorous, to extremely ad hoc. The 'cleanest' way to go about it is Monte Carlo (MC) simulations.\(^{35}\) In this approach not even the assumption that the Boltzmann equation is the correct transport equation is required. One simply throws dice, and follows the test particles on their journey through the dense stellar object. However, the computational cost is overwhelming. With present day hardware it is out of the question that MC could be coupled to any dynamical simulation, either in hydrodynamical collapse calculations or hydrostatic neutron star formation. However, MC calculations on static backgrounds can be quite useful to calibrate other transport methods, as MC calculations define the standard of physical realism.

The second most rigorous way is to solve the Boltzmann transport equation, resolved into energy and angular groups. Apart from being unaffordable as well, there are apparently problems with the inherent stability of this approach.\(^{36}\)

At the moment, the approaches where the level of approximation and computational feasibility meet, are the flux–limited diffusion (FLD) methods. They solve the energy balance equation, which is the angular integral of the Boltzmann equation, and in addition give a prescription on how to treat the anisotropy of the non–LTE distribution function. On top of that, they are formulated such that the neutrino flux cannot become superluminal, which it would in pure diffusion. In most FLD schemes,\(^{14, 17, 24, 37, 38}\) the neutrino distribution function is expanded in a Legendre series of which only the first two terms are retained. This unavoidably leads to a pure diffusion equation. The flux limiter must in that case be added by hand.

The Flux–limited Neutrino Diffusion Theory (FNDT) is the FLD approach developed in chapters 2 and 3 of this thesis. It is conceptually superior in the sense that it can be systematically derived from the Boltzmann equation, with total control over all necessary assumptions made in its derivation. In particular, the expansion of the distribution func-
tion from the start is not necessary. In this approach the flux-limited behaviour follows from the derivation rather than being added by hand a posteriori as in other FLD approaches. It is adapted for Fermi particles from Levermore and Pomraning's Flux-limited Diffusion Theory\textsuperscript{39} (FDT) which was formulated for radiative photon transport. This is the subject of chapter 2, in which we in addition show how under certain quantitative conditions FNDT can be anchored in the maximum entropy principle. In chapter 3 a 'gauge' freedom in the formulation of F(N)DT is used to systematically incorporate the anisotropic neutrino scattering, and to enhance the conceptual self-consistency of the formalism. The FNDT method is spectral, and with the exclusion of explicit energy-bin coupling processes, monochromatic. This allows the energy bins to be transported independently, rather than as a coupled set. Although FNDT is only a diffusion method, one should not think too lowly of diffusion. The correct form of the simple diffusion equation for particles obeying quantum (Fermi) statistics, like neutrinos, diffusing in inhomogeneous media cannot be determined from first principles.\textsuperscript{40}

In chapter 4, FNDT is implemented in a computer code, which is used to simulate the non-LTE $\nu_e$ and $\bar{\nu}_e$ transport in dense static stellar backgrounds. In these calculations it is among other things found that substantial energy deposition in density regions of the models relevant to the delayed-explosion mechanism is common.

In chapter 5, the stellar background is made dynamic, and two model neutron star formation simulations employing FNDT are presented. It is found that a model that contains only free nucleons in its baryonic component, and no nuclei, initially expands due to the energy deposition by the combined neutrino flows. In the second model nuclei are added by hand below a certain density, to study the effect of the reduced absorption opacity on the deposition and the subsequent dynamical behaviour. We found that the inclusion of nuclei impedes the initial expansion. Nevertheless, in both models the energy deposition in the atmospheres considerably slows down the contraction. Also in both models a density inversion which is probably unstable against convection develops near $\rho \approx 10^{13} \text{ g cm}^{-3}$.

The formation of a neutron star obviously depends on the success of the explosion. Conversely, the proto-neutron star evolution may be helpful in the creation of an explosion in some version of the delayed-explosion mechanism.

The transport method developed and numerically implemented in this thesis is a tool for the exploration of these problems. The stellar–evolution code developed for the simulation of neutron star formation, with which the results presented in chapter 5 were obtained, can be further refined to include more details and aspects of the physical phenomenon. The transport method and results that are presented in the following chapters are to be seen as the foundation and the bricks for further steps in a contribution to the understanding of the supernova and neutron star formation.

References


