The universe on edge: Limits of the effective field theory approach in the very early universe
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We first reproduce the one- and two-loop results of scalar $\phi^4$ theory. In order to use the background field method, we decompose the scalar field

$$\Phi = \phi + \frac{R + iC}{\sqrt{2}}, \quad (C.1)$$

where $\phi$ is the background field and $R, C$ are the real and complex perturbations around it. We can choose $C = 0$ to examine the case of a real scalar field first.

Expanding the potential term of the Lagrangian

$$\mathcal{L}_{\text{pot.}} = \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4} \Phi^4 \quad (C.2)$$

we read of the effective mass of the real scalar $R$ with the value $\phi$ of the background field

$$m^2_R = m^2 - 9\lambda \phi^2. \quad (C.3)$$

We find then for the effective one-loop potential

$$\nu^{(1)} = \frac{(m^2 - 9\lambda \phi^2)^2}{8} \left(-3 + 2 \ln \frac{m^2 - 9\lambda \phi^2}{Q^2}\right). \quad (C.4)$$

### C.1 The IR Cutoff

We observe that the effective potential calculated in the previous subsection acquires an imaginary part above a certain field value. In general, this corresponds to information loss and is not expected for an effective potential. However, since we are indeed expanding the potential
around an unstable point, the apparent loss of information is to be expected (see \cite{106,129}). When expanded around the turnaround point, the reality of the effective potential should be recovered.

Technically, the imaginary part comes from integrating out also tachyonic modes, i.e. such modes for which the effective mass is negative. Since we are only interested in the UV behavior of the theory, we can use an IR cutoff such that only non-tachyonic modes are integrated over and a real effective potential is obtained. In our case, we see that for \( \phi^2 \leq \frac{m^2}{\pi \lambda} \), the field becomes tachyonic. Hence, we apply an IR cutoff, which prevents the effective mass from becoming negative. We see, that this cutoff must depend on the field value and we can choose \( \mu_{IR} = 3\lambda \phi^2 + \epsilon^2 \).

To see how the IR cutoff takes effect in the background field method, we compare the treatment of the cutoff in appendix B of \cite{106} (equation B.26 onwards) with \ref{eq:C.4}. We have

\[
V^{(1)}_{\text{CHT}} = \frac{1}{32\pi^2} \left( -\frac{9\lambda^2 \phi^4}{4} + \frac{9\lambda^2 \phi^4}{2} \ln \frac{-3\lambda \phi^2}{\Lambda^2} \right) \quad \text{(C.5)}
\]

\[
V^{(1)}_{\text{Martin}} = \frac{1}{32\pi^2} \left( -\frac{3m_{\text{eff}}^4}{4} + \frac{m_{\text{eff}}^4}{2} \ln \frac{m_{\text{eff}}^2}{Q^2} \right). \quad \text{(C.6)}
\]

Comparing the two expressions, we see that the effective mass in \cite{106} is \( m_{\text{eff}}^2 = -3\lambda \phi^2 \), which is consistent with the fact, that their bare mass is zero. We see a mismatch of a factor 3 in the term quartic in the effective mass, but this is merely a scheme dependence.

In the background field method, we see that we need to replace \ref{eq:C.4} with

\[
-\frac{m_{\text{eff}}^2}{4} - \frac{\mu_{IR}^4}{2} \ln \frac{\mu_{IR}^2 + m_{\text{eff}}^2}{\mu_{IR}^2} - \frac{m_{\text{eff}}^2 \mu_{IR}^2}{2} + \frac{m_{\text{eff}}^4}{2} \ln \frac{\mu_{IR}^2 + m_{\text{eff}}^2}{\Lambda^2}. \quad \text{(C.7)}
\]

We can check that for \( \mu_{IR} \to 0 \) this reproduces \ref{eq:C.4}.

Note that the IR cutoff is still ok due to perturbation theory, because for that we need

\[
\mu_{IR}^2 \ll Q^2 \ll \Lambda^2, \quad \text{(C.8)}
\]

which with the above IR cutoff and after the replacement \( Q \to \phi \) reads

\[
\lambda \phi^2 \ll \phi^2, \quad \text{(C.9)}
\]

which is fulfilled automatically as long as \( \lambda \ll 1 \). 

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