The universe on edge: Limits of the effective field theory approach in the very early universe

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CHAPTER 3

INFLATIONARY COSMOLOGY IN SUPERGRAVITY

3.1 PROBLEMS OF FRW COSMOLOGY AND COSMIC INFLATION AS THEIR SOLUTION

Whilst in chapter 2 we have dealt with the beginning of the universe, we are now turning our attention to its further development. Already in chapter 2 I have established that the universe is expanding as seen from the red-shifting of galaxies around us. Now we are going to look at which form this expansion has taken, which turns out to be not at all uniform. In fact, the latest physics Nobel prize at the time of writing was awarded for the precision observation at distant supernovae that the universe is currently expanding at an increasing rate [130–132], for which there has since also been further evidence [133,134]. These observations mean that, today, there is a non-zero vacuum density or “dark energy”. Here, we are interested in the form of the acceleration in the early universe, shortly after the big bang.

There are a number of theoretical and observational problems with the standard FRW cosmology as described by equation (2.1). An in-depth treatment of the following material can be found e.g. in [135–138]. It is already clear from the second Friedman equation (2.5) that the Hubble parameter is constant only for specific combinations of the cosmological constant $\Lambda$ and the equation of state parameter $w$. In general, the rate of expansion will change, $\ddot{a} \neq 0$. We will now introduce models that interpret the cosmological constant as some vacuum energy, which changes in time. It is given by a potential, which has to obey certain restrictions.

The problems of FRW cosmology are

- the flatness problem,
• the horizon problem,
• the missing monopole problem
• and the explanation of the power spectrum of the cosmic background radiation (CMB).

In the following I will explain these problems before introducing cosmic inflation as a solution to them.

THE FLATNESS PROBLEM

One of the parameters of the first Friedman equation (2.4) is the curvature \( k \) of the universe. The spatial curvature of the universe is a quantity, measured to be extremely small. For the universe to be flat, its energy density must have a critical value for a given Hubble parameter. If we take the cosmological constant to be (approximately) zero, we can read off the critical density for a given Hubble parameter

\[
\rho_c = \frac{3H^2}{8\pi G}.
\]  

(3.1)

We now introduce the ratio between the actual and the critical energy density

\[
\Omega = \frac{\rho}{\rho_c}.
\]  

(3.2)

In terms of this, the Friedman equation can be rewritten as

\[
\frac{1 - \Omega}{\Omega} \rho a^2 = \frac{3kc^2}{8\pi G},
\]  

(3.3)

where the right hand side is constant. The scale factor \( a \) increases with the expansion, while the energy density \( \rho \) decreases. For matter and radiation dominated universes this decrease is quicker than the increase of the scale factor squared \( a^2 \) (cf. equation (2.9)). This means that the left hand side of equation (3.3) decreases rapidly. The order of magnitude for this decrease within one Planck time is \( 10^{60} \). Figure 3.1 depicts how quickly the universe deviates from a flat initial configuration.

The CMB provides a wealth of information about the early universe. Its anisotropies can for example be used to measure the flatness of the universe. The typical angular distance between a cold and a hot spot, i.e. the first peak in the angular power spectrum, depends on the curvature of the universe. Besides this, comparing the distance of type 1a supernovae, which act as standard candles, to their redshift can be used to measure the expansion rate of the universe at different times. From those measurements combined, we arrive at the current value of \( \Omega \) to be within 1% of unity [133, 134], or \(|1 - \Omega| \leq 0.01\). This implies that it was fine-tuned to \( 10^{-62} \) during the Planck era. This fine-tuning problem is called the flatness problem.
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Figure 3.1: A flat universe is not stable. Small perturbations quickly drive the universe away from the critical mass density, unless the initial configuration is fine tuned to be precisely unity.

Figure 3.2: The seven year WMAP scan of the CMB. The relative size of anisotropies is only $10^{-5}$. Figure courtesy of the WMAP science team from the LAMBDA archive.

Horizon Problem

Although the CMB has much sought-after anisotropies, it is remarkably homogeneous. In fact, the temperature fluctuations have a relative size of only $\frac{\Delta T}{T} \approx 10^{-5}$, and the average temperature of the CMB is uniform over the whole sky. A plasma would only be so homogeneous in a region, which has been in causal contact for a long enough time to equilibrate. The CMB is conjectured to have formed 360,000 years after the big bang, when electrons and ions in the early plasma combined to form neutral hydrogen and photons were no longer scattered such that they decoupled from matter. Therefore, the Hubble horizon at this time is the maximum distance at which two points in the sky could still have been in causal contact. Assuming a standard expansion, this patch would have blown up to what now appears under one degree at the sky (see figure 3.3). A fluid dynamical equilibration process therefore cannot account for the isotropy of the CMB, which must have been pre-imposed by another mechanism. Unless there is a natural explanation of that, this is a fine-tuning problem known as the *horizon problem*.
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MISSING MONOPOLE PROBLEM

In the very early universe at very high temperatures, the electro-magnetic, weak and strong nuclear forces are supposed to be described by one unified gauge theory. Those grand unified theories (GUTs) \[139,140\] are a generalization of the Glashow-Salam-Weinberg (GSW) model for the electro-weak unification \[141–143\]. In such models, the different forces become unified above a critical temperature, which is about \(3 \cdot 10^{15} \text{K}\) for the GSW-model and about \(10^{27} \text{K}\) for GUTs. While cooling down with the expansion, the temperature drops below this critical value and the unified symmetry gets broken. This phase transition leads to the production of topological defects such as magnetic monopoles and domain walls. Since one domain must have been around the size of the Hubble radius at the time of the freeze-out, the number of defects can be estimated \[144–146\]. The fact that we have not yet been able to observe them is referred to as the missing monopole problem.

INFLATION AS A SOLUTION

It seems as if all those problems could be solved simultaneously if there was a mechanism which made the universe expand at a very large rate very early on, such that a causally connected region would be stretched out to match the size of the nowadays observable universe, space-time would be flattened out and the monopole remnants would be diluted. Such a mechanism was proposed by Alan Guth in 1980 under the name of inflation \[147\] and later improved by Andrei Linde \[148\], Andreas Albrecht and Paul Steinhard \[149\]. It should be noted that since the above problems concern the fine tuning of initial conditions, any mechanism solving them is only a valid improvement if it is somehow more natural and requires less tuning. One of the advantages of the inflationary paradigm is that it erases the dependence on the initial state.

The way inflation can solve the cosmological conundrums is that a contribution to the energy density with an equation of state parameter \(w \leq -\frac{1}{3}\), such as a cosmological constant, makes the causal radius grow faster than the Hubble radius. The region which is causally connected to a point can be determined from the comoving particle horizon, which is the distance a null
Figure 3.4: During the inflationary phase, the Hubble horizon grows much faster than the physical distance between two points. After the end of inflation, the scales re-enter the horizon. Inflation magnifies a formerly causally connected patch to fill the visible universe.

particle can have travelled since the big bang with \( a(0) = 0 \). It is equal to the comoving time

\[
\tau = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a' a' H(a')}. \tag{3.4}
\]

For a spacially flat universe, the scale factor behaves as

\[
a(t) = a_0 t^{rac{2}{3(w+1)}}. \tag{3.5}
\]

With the definition of the Hubble parameter \( H = \dot{a}/a \), we see that the comoving time depends on the equation of state parameter as

\[
\tau \sim a^{rac{1}{2}(1+3w)}. \tag{3.6}
\]

This means, that the horizon indeed shrinks for \( w < -\frac{1}{3} \), and the horizon problem is solved (cf. fig. 3.4). Such a configuration will lead to an accelerated expansion

\[
\frac{d}{dt} \frac{1}{aH} < 0 \Rightarrow \ddot{a} > 0. \tag{3.7}
\]

Hence the name inflation.

The easiest incarnation of inflation is single-field slow-roll inflation, in which inflation is driven by a single scalar field \( \phi \), the so-called inflaton. This is added to the gravitational action

\[
S_{\text{inflation}} = \int d^4x \sqrt{g} \left( \frac{M_{\text{pl}}^2}{2} R + \frac{1}{2} \left( \partial \phi \right)^2 - V(\phi) \right). \tag{3.8}
\]

The equations of motion and the Friedman equations derived from that action are

\[
0 = \ddot{\phi} + 3H \dot{\phi} + V'(\phi), \tag{3.9}
\]

\[
H^2 = \frac{1}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right), \tag{3.10}
\]

\[
\frac{\ddot{a}}{a} = (\rho + 3p), \tag{3.11}
\]
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Figure 3.5: The energy density of a universe which is dominated by a cosmological constant or some potential energy is driven towards the critical value. Therefore, inflation erases the initial condition and yields a flat universe.

where

\[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) , \]  

(3.12)

\[ p = \frac{1}{2} \dot{\phi}^2 - V(\phi) . \]  

(3.13)

We see that to get \( w < -\frac{1}{3} \), the system needs to be dominated by potential energy. For the “no-roll”-case \( \dot{\phi}^2 = 0 \), we recover a cosmological constant.

To be a solution, inflation needs to last long enough to extend space-time by a sufficient amount. To test this for a given model the two slow-roll parameters can be used. They are defined to be

\[ \epsilon = \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2 , \]  

(3.14)

\[ \eta = M_{pl}^2 \frac{V''}{V} \]  

(3.15)

and are required to be small \( \epsilon \ll 1, \eta \ll 1 \) to have a good model of slow-roll inflation. In that limit, the Friedman equations can be easily solved to give

\[ H^2 = \frac{1}{3} V(\phi) \sim \text{constant} , \]  

(3.16)

\[ \dot{\phi} = -\frac{V'}{3H} , \]  

(3.17)

\[ a(t) \sim e^{Ht} . \]  

(3.18)

and the expansion is exponential, indeed.

An accelerated expansion also solves the flatness problem. Since the scale factor is exponential, it now wins against the scaling of the matter density, and the critical density naturally becomes an attractor for a variety of initial values of \( \Omega \) (see figure 3.5). Potentially present monopoles...
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![Inflaton Potential Diagram](image)

**Figure 3.6:** A possible form of an inflaton potential. In the flat piece, the universe will inflate. The CMB is formed at $\phi_{\text{CMB}}$. Its anisotropies reflect the fluctuations $\delta \phi$ of the inflaton. After inflation has come to an end at $\phi_{\text{end}}$, the energy of the inflaton field must be transferred to the standard model particles, which is called reheating.

and topological defects are diluted by the vast expansion such that they are hardly visible at present.

The shape of a possible slow-roll inflationary potential is depicted in figure 3.6. Inflation will end if $\epsilon \sim 1$. It is customary to express the amount of inflation in so-called e-foldings, which is the powers of $e$ with which the scale factor $a$ has grown. It can be expressed in terms of the first slow-roll parameter as

$$N \equiv \ln \left( \frac{a_{\text{final}}}{a_{\text{initial}}} \right) = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \approx \int_{\phi_i}^{\phi_f} \frac{V'}{V} d\phi \approx \int_{\phi_i}^{\phi_f} \frac{d\phi}{\sqrt{2\epsilon}},$$

(3.19)

where we have employed the slow-roll approximation. To solve the cosmological problems, $\epsilon$ needs to be such that inflation lasts for at least $N > 60$ e-foldings.

I do not want to leave unmentioned that there are alternative explanations for the conundrums of the early universe. Amongst such models are string gas cosmology [150,151], the ekpyrotic or cyclic universe [152,154] or, taking the holographic lessons from chapter 1 more seriously, a holographic model [155,156].

### 3.2 Inflation in String Theory and Supergravity

So far, we have got to know inflation as a merely phenomenological model which can reduce the fine-tuning problems in the early universe. Nothing has been said so far about what the inflaton should be and where its potential would come from. These ingredients, if at all real, must finally come from a fundamental, UV-complete theory of gravity.

One of the reasons why we would like to see inflation be backed up by a UV-safe theory is the objective of this chapter, namely the $\eta$-problem. The second slow-roll parameter, which is
basically the inflaton mass measured in Hubble units

\[ \eta = M_{\text{pl}}^2 \frac{V''}{V} \approx \frac{m_{\phi}^2}{3H^2} \]  

needs to be much smaller than unity to ensure successful inflation. However, when going to higher energies, this parameter will receive UV-corrections. In fact, integrating out Planck scale degrees of freedom will add a generic dimension six operator as a correction to the Lagrangian

\[ \frac{\mathcal{O}_6}{M_{\text{pl}}^2} = \frac{\mathcal{O}_4}{M_{\text{pl}}^2} \phi^2. \]  

If the dimension four operator has a vacuum expectation value comparable to the scale of inflation

\[ \langle \mathcal{O}_4 \rangle \sim V, \]  

this term corrects the inflaton mass by an order \( \frac{\sqrt{V}}{M_{\text{pl}}} \sim H \), which would correspond to correcting \( \eta \) by order unity. This reflects the fact that generically, in an effective theory the mass would run all the way to the cut-off. Therefore, to ensure that inflation remains valid, we need to have good control over the UV physics.

An effective way of removing an irrelevant operator is by employing some kind of symmetry which forbids it. In the case at hand, this could be a shift symmetry. If the inflaton was taken to be e.g. the phase \( \varphi \) of a complex field \( \phi \) with a U(1) symmetry \( \phi \rightarrow e^{i\alpha} \phi \), shifting the inflaton \( \varphi \rightarrow \varphi + \alpha \) doesn’t change the theory. A flat potential breaks this symmetry only weakly during the inflationary era and the symmetry could remove the dimension six coupling. Yet, quite general arguments seem to suggest that global continuous symmetries are not allowed in a generic theory of quantum gravity \[157\]. Therefore a shift symmetry is not a natural thing to assume, and how to deal with the \( \eta \)-problem depends crucially on how the details of the fundamental theory are reflected in the low energy action. On top, the natural scale of inflation is some \( 10^{14} \) GeV such that we expect new physics to enter the picture.

For a long time, string theory has been building up hopes of providing such a UV complete understanding of gravity. Examining the observational and theoretical constraints of inflationary models built from string theory is therefore of extreme interest. In addition, string theory comes with a lot of new degrees of freedom, like moduli fields, branes, extra dimensions and warp factors, that can be used as an inflaton and for building inflationary models. It is widely known that a consistent formulation of string theory requires ten space-time dimensions, whereas our observations determine the number of extended dimensions to be only four. This means that the extra dimensions would be accessible only at high energies but need to be compactified at low energies. Then, if the energy scale is low as compared to the string tension, four-dimensional supergravity is the appropriate effective theory to describe cosmology because only string zero modes enter the description and the effects of the extra dimensions can be integrated out. For an introduction to string theory, I refer the reader to one of the many textbooks \[158–161\].

When compactifying the extra dimensions one inevitably tampers with the symmetries of the theory. The more internal symmetries the compactification manifold has, the more symmetries
are preserved by the compactification. For instance, compactifying supergravity on a torus leads to maximal supergravity, whereas compactifying on a Calabi-Yau manifold will lead to an $\mathcal{N} = 2$ theory. Adding even more ingredients like fluxes, branes or orientifolds, the effective theory can be down to $\mathcal{N} = 1$ [162–167]. Calabi-Yau manifolds are widely used to construct low dimensional models. They come with a lot of symmetries [168] which manifest themselves as massless scalar degrees of freedom, so-called moduli in the effective theory. The proliferation of extra degrees of freedom is a vice as much as a virtue of the theory. Whereas some of them come in handy as inflaton fields, most of these massless fields are not observed and hence must be made heavy by creating some potential. This procedure is known as moduli stabilization. Another problem when looking at cosmology is the fact that in de Sitter space, which describes our universe, supersymmetry is broken. This makes it much more difficult to find string theory solutions. A first attempt to solve both problems was done in the famous KKLT paper [169]. It builds on the insight that three-form fluxes can stabilize the complex structure modulus and the dilaton [170–173]. On top, when including non-perturbative corrections, the volume modulus is also stabilized, and adding the potential of a small number of anti-D3 branes lifts the vacuum to being de Sitter. Large volume scenarios provide another way of stabilizing the moduli [174,175].

The KKLT procedure has been extended also to allow for inflation with the potential of a $D3 − \overline{D3}$-brane pair [176] and a large class of models inspired thereby. These models only allow for a small scale for inflation as determined by the Lyth bound, which constrains the variation of a field to be sub-Planckian [177]. A higher scale for inflation is allowed by axion monodromy inflation [178–180]. With a large supply of degrees of freedom, it is also possible to drive inflation with multiple fields as e.g. in [181–184]. Multiple fields will generically interact with each other, which will lead to observable signatures in the CMB, the most important of which are non-Gaussianities and isocurvature modes [185,186]. A comprehensive re- and overview of string theoretic inflationary models is given in [187,188] and references therein.

For the purpose of this chapter, the important common property of all such models is the following. To construct an inflationary potential, some of the degrees of freedom are chosen, while the others are supposed to remain silent at the minimum of their stabilizing potential. This is not only assumed at some point in field space but along the whole inflationary trajectory, such that the physics is assumed to be only described by the fields that have been picked to play a rôle in inflation. I group the fields participating in inflationary dynamics to form the inflationary sector. From the pint of view of inflation, the dynamics of the other fields is not visible and thus I call them the hidden sector. In a realistic model, they comprise in particular the standard model fields. They become dynamical only during reheating, when the inflation's energy is transferred to them (cf. figure 3.7). This truncation of the theory to a sub-sector is justified by the observation that the coupling is only of gravitational strength. Revisiting the $\eta$-problem for a simple setup with multiple fields in supergravity is one example study, which shows that the gravitational strength coupling must not be underestimated.
3.3 Inflationary theories with multiple sectors

The construction of realistic models of slow-roll inflation in supergravity is a longstanding puzzle. Supersymmetry can alleviate the fine-tuning necessary to obtain slow-roll inflation — if one assumes that the inflaton is a modulus of the supersymmetric ground state — but cannot solve it completely. This is most clearly seen in the supergravity \( \eta \)-problem: if the inflaton is a lifted modulus, then its mass in the inflationary background is proportional to the supersymmetry breaking scale. Therefore, the slow-roll parameter \( \eta \simeq V''/V \) generically equals unity rather than a small number [189].

We will show here, however, that the \( \eta \)-problem is more serious than a simple hierarchy problem. In the conventional mode of study, the inflaton sector is always a sub-sector of the full supergravity theory presumed to describe our Universe. When the inflationary sub-sector of the supergravity is studied \textit{an sich}, tuning a few parameters of the Lagrangian to order \( 10^{-2} \) will generically solve the problem. We will clarify that this split of the supergravity sector into an inflationary sector and other hidden sectors implicitly makes the assumption that all the other sectors are in a ‘supersymmetric’ ground state: i.e. if the inflaton sector which must break supersymmetry is decoupled, the ground state of the remaining sectors is supersymmetric. If this is not the case, the effect on the \( \eta \)-parameter or on the inflationary dynamics in general can be large, even if the supersymmetry breaking scale in the hidden sector is small. Blind truncation in supergravities to the inflaton sector alone, if one does not know whether other sectors preserve supersymmetry, is therefore an inconsistent approach towards slow-roll supergravity inflation. Coupling the truncated sector back in completely spoils the naïve solution found. This result, together with recent qualitatively similar findings for sequestered supergravities (where only the potential has a two-sector structure) [190], provides strong evidence that to find true slow-roll inflation in supergravity one needs to know the global ground state of the system. The one obvious class of models where sector-mixing is not yet considered is the
newly discovered manifest embedding of single field inflationary models in supergravity [191]. If these models are also sensitive to hidden sectors, it would arguably certify the necessity of a global analysis for cosmological solutions in supergravity and string theory.

We will obtain our results on two-sector supergravities by an explicit calculation. The gravitational coupling between the hidden and the inflaton sectors is universal, which can be described by a simple $F$-term scalar supergravity theory. As in most discussions on inflationary supergravity theories, we will ignore $D$-terms as one expects its VEV to be zero throughout the early Universe [192]. Including $D$-terms (which themselves always need to be accompanied by $F$-terms) only complicates the $F$-term analysis, which is where the $\eta$-problem resides. Furthermore, although true inflationary dynamics ought to be described in a fully kinetic description [193], we can already make our point by simply considering the mass eigenmodes of the system. In a strict slow-roll and slow-turn approximation the mass eigenmodes of the system determine the dynamics of the full system.

Specifically we shall show the following for two-sector supergravities where the sectors are distinguished by independent R-symmetry invariant Kähler functions:

- Given a naïve supergravity solution to the $\eta$-problem, this solution is only consistent if the other sector is in its supersymmetric ground state.

- If it is not in its ground state, then the scalar fields of that sector cannot be static but must evolve cosmologically as well.

- In order for the naïve solution to still control the cosmological evolution these fields must move very slowly. This translates in the requirement that the contribution to the first slow-roll parameter of the hidden sector must be much smaller than the contribution from the naïve inflaton sector, $\epsilon_{\text{hidden}} \ll \epsilon_{\text{naïve}}$.

- There are two ways to ensure that $\epsilon_{\text{hidden}}$ is small: Either the supersymmetry breaking scale in the hidden sector is very small or a particular linear combination of first and second derivatives of the generalized Kähler function is small.
  
  - In the latter case, one finds that the second slow-roll parameter $\eta_{\text{naïve}}$ receives a very large correction $\eta_{\text{true}} - \eta_{\text{naïve}} \gg \eta_{\text{naïve}}$, unless the supersymmetry breaking scale in the hidden sector is small. This returns us to the first case.

  - In the first case, one finds that the hidden sector always contains a light mode, because in a supersymmetry breaking (almost) stabilized supergravity sector there is always a mode that scales with the scale of supersymmetry breaking. This light mode will overrule the naïve single field inflationary dynamics.

Thus for any nonzero supersymmetry breaking scale in the hidden sector — even when this scale is very small — the true mass eigenmodes of the system are linear combinations of the hidden sector fields and the inflaton sector fields. We compute these eigenmodes. By assumption, the true value of the slow-roll parameter $\eta$ is the smallest of these eigenmodes. Depending
on the values of the supersymmetry breaking scale and the naïve lowest mass eigenstate in the
hidden sector, we find that

1. The new set of mass eigenmodes can have closely spaced eigenvalues, and thus the
initial assumption of single field inflation is incorrect. Then a full multi-field re-analysis
is required.

2. The relative change of the value of \( \eta \) from the naïve to the true solution can be quantified
and shows that for a supersymmetry breaking hidden sector, the naïve model is only
reliable if the naïve lowest mass eigenstate in the hidden sector is much larger than
the square of the scale of hidden sector supersymmetry breaking divided by the inflaton
mass. This effectively excludes all models where the hidden sector has (nearly) massless
modes.

3. The smallest eigenmode can be dominantly determined by the hidden sector, and thus
the initial assumption that the cosmological dynamics is constrained to the inflaton sector
is incorrect. Again a full multi-field re-analysis is required.

One concludes that in general one needs to know/assume the ground states and the lowest
mass eigenstates of all the hidden sectors to reliably find a slow-roll inflationary supergravity.

The structure of the rest of this chapter is the following. Section 3.4 reviews some definitions
in supergravity and explains how sectors are coupled in supergravity. This leads directly to the
first result that in a stabilized supergravity sector there always is a mode that scales with the
scale of supersymmetry breaking. In section 3.5 we discuss the \( \eta \)-problem in a single sector
theory and then consider the effect of a hidden sector qualitatively and quantitatively. The
quantitative result is analyzed in section 3.6 both in terms of effective parameters and direct
supergravity parameters. As a notable example of our result, we show that if the hidden sector
is the Standard Model, where its supersymmetry breaking is not caused by the inflaton sector
but otherwise, spoils the naïve slow-roll solution in the putative inflaton sector. The chapter is
supplemented with two appendices in which some of the longer formulae are given.

### 3.4 A STABILIZED SECTOR IN A SUPERGRAVITY TWO-SECTOR
SYSTEM

We shall start by recalling how two sectors are gravitationally coupled in supergravity. Although
this coupling is universal, the definition differs from regular gravity in an important way: the
superpotentials multiply rather than add.

We will then consider one of the two sectors to be a stable hidden sector. We show that a light
mode develops, which indicates that the hidden sector obtains a flat direction and is not stable
any more. This extends the result of [194], in which it is shown that non-supersymmetric
Minkowski minima always develop at least one light mass mode, to de Sitter and Anti-de Sitter vacua.

### 3.4.1 The Supergravity Action

The action for the scalar sector of $N = 1$ supergravity is

$$S = M_{pl}^2 \int d^4x \sqrt{g} \left[ \frac{1}{2} R - g^{\mu\nu} G_{\alpha\beta} \partial_\mu X^\alpha \partial_\nu \overline{X}^\beta - V M_{pl}^2 \right],$$  \hspace{1cm} (3.23)

in which $G_{\alpha\beta}$ is the field space metric and $g_{\mu\nu}$ is the spacetime metric with associated Riemann scalar $R$. The Greek indices run over all fields $\{\alpha, \beta\}$ or over spacetime coordinates $\{\mu, \nu\}$. For calculational convenience we have defined the scalar fields $X$ and functions $V$, $K$ and $W$ to be dimensionless. The ($F$-term) potential $V$ of the scalar sector is defined as

$$V = e^G (G_\alpha G^\alpha - 3).$$  \hspace{1cm} (3.24)

Through $G_\alpha = \partial_\alpha G$, $G_{\alpha\beta} = \partial_\alpha \partial_\beta G$, the action (3.23) is completely specified by the real Kähler function $G(X, \overline{X})$, which is related to global supersymmetry quantities through

$$G(X, \overline{X}) = K(X, \overline{X}) + \log (W(X)) + \log (\overline{W}(\overline{X}))$$  \hspace{1cm} (3.25)

in terms of the real Kähler potential $K(X, \overline{X})$ and the holomorphic (dimensionless) superpotential $W(X)$. The definition for $G$ is convenient as it is invariant under Kähler transformations, i.e. it is invariant under the simultaneous transformation of $K(X, \overline{X}) \to K(X, \overline{X}) + f(X) + \overline{f}(\overline{X})$ and $W(X) \to e^{-f(X)}W(X)$ for an arbitrary holomorphic function $f(X)$.

### 3.4.2 Canonical Coupling

To describe a two-sector system we consider a class of minimally coupled scenarios

$$G(\phi, \overline{\phi}, q, \overline{q}) = G^{(1)}(\phi, \overline{\phi}) + G^{(2)}(q, \overline{q}),$$  \hspace{1cm} (3.26)

with $\phi, q$ denoting the fields in the two sectors respectively. In the following, we will take the indices $\{i, \overline{j}\}$ to run over the $\phi$-fields, while $\{a, \overline{b}\}$ denote the fields in the $q$-sector. Later in this chapter we will take the $\phi$-fields to drive inflation, while the $q$-fields reside in another sector which is naively assumed not to take part in the inflationary dynamics and is hence called the hidden sector. This split of the Kähler function $G(\phi, \overline{\phi}, q, \overline{q})$ (3.26) is invariant under Kähler transformations in each sector separately and thus defines a sensible way of splitting up the action in multiple sectors. Amongst other properties, this split guarantees that a BPS solution in one particular sector is a BPS solution of the full theory. In terms of $K$ and $W$,

\footnote{Note that this definition requires $W \neq 0$. For $W = 0$ a Kähler function $G$ cannot be defined. In this paper we will assume that $W \neq 0$.}
this definition has a conventional separation of the Kähler potential, but the superpotentials in each sector combine multiplicatively rather than add

\[ K(\phi, \bar{\phi}, q, \bar{q}) + \log |W(\phi, q)|^2 = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(q, \bar{q}) + \log |W^{(1)}(\phi)W^{(2)}(q)|^2 . \quad (3.27) \]

Let us illustrate the importance of this multiplicative superpotential in the situation in which the hidden sector resides in a supersymmetric vacuum, i.e. \( \partial_\phi V(q_0) = 0 \) and \( \partial_\phi G^{(2)}(q_0) = 0 \). We write the superpotential of the hidden sector as

\[ W = W^{(2)}(q) = W^{(2)}_0 + W^{(2)}_{\text{global}}(q - q_0) \]

The second term in this expression is what determines the potential for fluctuations around the minimum of the hidden sector, while the first constant term is just an overall contribution and hence not interesting for the internal hidden sector dynamics at energies much less than the Planck scale. However, for the gravitational dynamics and the remaining \( \phi \)-sector this ‘vacuum energy contribution’ \( W^{(2)}_0 \) is of crucial importance as it sets the scale of the potential

\[ V = e^{K^{(2)}(q)} |W^{(2)}_0|^2 e^{G^{(1)}(q)} \left( G^{(1)}_i G^{(1)i} - 3 \right) , \quad (3.28) \]

which is evaluated at \( q = q_0 \) such that all terms depending on \( W^{(2)}_{\text{global}} \) vanish. The normal practice of setting \( W^{(2)}_0 \) to zero as an overall contribution to the hidden sector is neglecting the fact that gravity also feels the constant part of the potential energy, as opposed to field theory. The inflationary sector feels the presence of the hidden sector through this coupling and as such it may be more intuitive to regard \( W^{(2)}_0 \) to contain information about the inflationary sector rather than the hidden sector. Making a similar split in \( W^{(1)} \), the constant part \( W^{(1)}_0 \) is the overall contribution to the hidden sector due to the inflaton sector.

The multiplicative superpotential also means that the zero-gravity limit to a global supersymmetry is more subtle than just taking \( M_{\text{pl}} \to \infty \), as is usually done [2]. One must first determine a ground state which sets \( W^{(1)}_0 \) and \( W^{(2)}_0 \), and then send both \( W^{(1)}_0 \to 0 \) and \( W^{(2)}_0 \to 0 \) in such a way that the combinations \( W^{(1)}_0 M_{\text{pl}} \) and \( W^{(2)}_0 M_{\text{pl}} \) remain constant. Instating the canonical dimensions for the fields and the Kähler potential and rescaling the couplings such that \( W^{\text{eff}}_i = W^{(1)}_0 W^{(2)}_{\text{global}} \) and \( W^{\text{eff}}_i = W^{(2)}_0 W^{(1)}_{\text{global}} \) scale as \( M_{\text{pl}}^{-3} \), the total superpotential

\[ W = W^{(1)}_0 W^{(2)}_0 + W^{(1)}_0 W^{(2)}_{\text{global}} + W^{(2)}_0 W^{(1)}_{\text{global}} + W^{(1)}_{\text{global}} W^{(2)}_{\text{global}} , \quad (3.29) \]

then consists of a constant term which scales as \( W^{(1)}_0 W^{(2)}_0 \sim M_{\text{pl}}^{-2} \), cross-terms which scale as \( W^{(1)}_0 W^{(2)}_{\text{global}} + W^{(2)}_0 W^{(1)}_{\text{global}} \sim M_{\text{pl}}^{-3} \) and a multiplicative term which scales as \( W^{(1)}_{\text{global}} W^{(2)}_{\text{global}} \sim M_{\text{pl}}^{-4} \). Considering the dimensionful superpotential this results in an overall infinite contribution, a finite sum of two terms and a vanishing product. In this decoupling limit one recovers the two independent global supersymmetry sectors with the naïve additive behavior in both the superpotential and the Kähler potential,

\[ K(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(q, \bar{q}) , \]
\[ W(\phi, q) = W^{\text{eff}}_i (\phi) + W^{(2)}_i (q) . \quad (3.30) \]

However, one cannot use this split [3,30] and couple gravity back in [203]. As explained, in supergravity the definition [3.30] is not invariant under Kähler transformations in each sector.
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separately and is valid only in a specific Kähler frame or, say, gauge dependent [201]. Another way to understand the result is to realize that the definition (3.30) does not lead to a Kähler metric and mass matrix that can be made block diagonal in the same basis [202], and thus there is no sense of ‘independent’ sectors.

Insisting on the separate Kähler invariance of (3.26), the two-sector action (3.23) reads

\[ S = M^2_{\text{pl}} \int d^4 x \sqrt{g} \left[ \frac{1}{2} R - g^{\mu\nu} (G^{(1)}_i \partial_\mu \phi^i \partial_\nu \phi^i + G^{(2)}_{ab} q^a \partial_\mu q^b) - VM^2_{\text{pl}} \right], \tag{3.31} \]

with

\[ V(\phi, \bar{\phi}, q, \bar{q}) = e^{G^{(1)} + G^{(2)}} \left( G^{(1)}_i G^{(1)}_i + G^{(2)}_{a} G^{(2)}_{a} - 3 \right). \tag{3.32} \]

We will allow ourselves to drop the sector label from \( G \) in the remainder, since \( G^{(1)}_\phi = G_\phi \) and similarly for \( q \). For a short overview of relevant conventions and identities in supergravity, we refer the reader to appendix D.

### 3.4.3 Zero mass mode for a stabilized sector

Anticipating the situation for an inflationary scenario we will analyze the mass spectrum of a stabilized \( q \)-sector in a de Sitter background. For Minkowski spaces it is known that the lightest mass in a stabilized sector scales with the supersymmetry breaking VEV [194]. Here we extend the analysis to de Sitter vacua as the zeroth order approximation of slow-roll inflation. Already in this zeroth order approach we will show that a similar light mode develops in the stabilized sector. Throughout this discussion we assume that the potential \( V \) is kept positive by the presence of the ‘inflationary’ sector. In the next section we show that this result can be translated directly into an inflationary setting, where this light mode will affect the slow-roll dynamics.

Given that we insist the \( q \)-sector to be stabilized, we have \( \partial_a V = 0 \). In terms of the Kähler function \( G(\phi, \bar{\phi}, q, \bar{q}) \) this means

\[ (\nabla_a G_b) G^b = -G_a (1 + e^{-G} V). \tag{3.33} \]

If the \( q \)-ground state breaks supersymmetry, i.e. \( G_a \neq 0 \), we may rewrite it in terms of the supersymmetry breaking direction \( f_a = G_a / \sqrt{G^b G_b} \),

\[ (\nabla_a G_b) f^b = -f_a (1 + e^{-G} V). \tag{3.34} \]

For simplicity we will assume that the \( q \)-sector consists of only a single complex scalar field \( q \), in which case we may write this equation as

\[ \nabla_q G_q = -G_q (1 + e^{-G} V) \hat{G}_q. \tag{3.35} \]

A hat \( \hat{q} \) on a complex number denotes the ‘phase’-part of the number, \( z = |z| e^{i \arg(\cdot)} \). As such \( \hat{G}_q = \sqrt{G^q} f_q \). Note that in an arbitrary supersymmetric configuration \( G_a = 0 \) there
are no restrictions on $\nabla_a G_b$, but on a supersymmetry broken configuration this is no longer true. Were one to turn on supersymmetry breaking, one would first have to reach a surface in parameter space where this restriction can be imposed at the onset of supersymmetry breaking.

We will now compute the mass spectrum for the two modes of the complex scalar field $q$, at the hyper-surface defined by (3.35). The mass modes are given by the eigenvalues of the matrix

$$M^2 = \begin{pmatrix} V^q_q & V^q_\bar{q} \\ V^\bar{q}_q & V^\bar{q}_\bar{q} \end{pmatrix},$$

which in our case means

$$m_{\pm} = (V^q_q \pm |V^q_\bar{q}|) = G^{\alpha\beta} (V^{\alpha\bar{q}}_q \pm |V^{\alpha\bar{q}}_q|).$$

Expanding the second derivatives of the potential (cf. appendix E) to first order in $|G^q_q|$, these eigenvalues are

$$m_\pm = e^{G^{\alpha\beta}} \text{Re}\left\{(\nabla_q \nabla_q G_q) \tilde{G}^{q3} \right\} |G^\alpha| + \mathcal{O}(|G_q|^2),$$

$$m^+_q = e^{G} \left[2(2 + e^{-G}V)(1 + e^{-G}V) - G^{\alpha\beta} \text{Re}\left\{(\nabla_q \nabla_q G_q) \tilde{G}^{q3} \right\} |G^\alpha| \right] + \mathcal{O}(|G_q|^2).$$

We see from (3.38) that in the limit of vanishing supersymmetry breaking the lightest mass mode becomes massless, just as in the case of Minkowski space. It is important to note that this result depends crucially on taking the limit $G_q$ to zero in the supersymmetry breaking direction. When supersymmetry is restored and both $G_q = 0$ and $G_\bar{q} = 0$, the phases of these vectors have no meaning. In fact, we see that then a new degree of freedom arises: $\nabla_q G_q$ becomes unrestricted which allows one to choose the masses freely.

The geometrical picture is that there is a whole plane of supersymmetric solutions where arbitrary masses are allowed. However, when supersymmetry is broken, the supersymmetry breaking direction has to align with its complex conjugate fixing one point on this plane where supersymmetry can be broken. In this point, the lightest mode becomes massless.

### 3.5 Two-sector Inflation in Supergravity

Generally, when inflation is described in supergravity, realistic matter resides in a hidden sector. Supergravities descending from string theory often have additional hidden sectors as well. These sectors are always gravitationally coupled. In the previous section we have seen that for de Sitter vacua the hidden sector develops a light direction. In this section we will consider how

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2The result can also be extended to hold for anti-de Sitter vacua. However, for $-2 < e^{-G}V < -1$, also a tachyonic mode develops.

3The supersymmetric partners of the Standard Model are not good inflaton candidates, as these partners are charged under the Standard Model gauge group and gauge fields taking part in inflation would lead to topological defects, e.g. [204, 205]. The exception could be a gravitationally non-minimally coupled Higgs field, e.g. [206, 207].
this light mode of the hidden sector can affect the naïve dynamics of the inflationary sector. We will show that despite the weakness of gravity, these effects can be large. Realistic slow-roll inflation is characterized by small numbers, the slow-roll parameters $\epsilon$ and $\eta$, and even small absolute changes to these numbers can be of the order of 100% in relative terms.

We will first briefly review the $\eta$-problem in the context of single field inflation in supergravity. Then we will explain what effects are to be expected when including an additional (hidden) sector. The section ends with calculating the relevant objects to determine the true dynamics of the full system.

### 3.5.1 Inflation and the $\eta$-Problem in Supergravity

In single scalar field models of inflation the spectrum of density perturbations is characterized by the two slow-roll parameters $\epsilon$ and $\eta$. To ensure that this spectrum matches the observed near scale invariance, both $\epsilon \ll 1$ and $\eta \ll 1$. Inflationary supergravity in its simplest form consists of a single complex scalar field, the inflaton, whose potential is generated by $F$-terms (3.24). The definition of $\eta$ may be phrased as the lightest direction of the mass matrix in units of the Hubble rate $3H^2 = V$, i.e. $\eta$ is the smallest eigenvalue of the matrix

$$\tilde{N}_{IJ} = \frac{1}{V} \begin{pmatrix} \nabla^i \nabla_j V & \nabla^i \nabla_\tau V \\ \nabla^\tau \nabla_j V & \nabla^\tau \nabla_\tau V \end{pmatrix},$$

(3.40)

where the tilde on $\tilde{N}$ indicates that this value of $\eta$ is defined with respect to the inflaton sector only and $I \in \{i, \tau\}$, $J \in \{j, \bar{\tau}\}$, respectively.

From the second $\phi$-derivative of $V$,

$$V_{ij} = G_{ij}V + G_iV_j + G_jV_i - G_iG_jV + e^G \left[ R_{ijk\bar{l}}G^kG^\bar{l} + G^k\nabla_iG_k\nabla_jG_\tau + G_{ij} \right],$$

(3.41)

we see that a natural value for $\eta$ is $V_{ij}V \sim \nabla^i \nabla_j V \sim 1$ is unity. Therefore, we must tune $G_i$, $\nabla_iG_j$ and $R_{ijk\bar{l}}$ so that $V_{ij} = O(10^{-3})V$. The necessity of this tuning is known as the $\eta$-problem.

As shown in [209], successful inflation is achievable if one tunes the Kähler function $G$ such that

$$R_{ijk\bar{l}}f^i f^j f^k f^\bar{l} \lesssim \frac{2}{3} \frac{1}{1 + \gamma},$$

(3.42)

where $\gamma = e^{-G}V/3$ is inversely proportional to an overall mass scale $m_{3/2} = e^{G/2}$, which is related to the gravitino mass and $R_{ijk\bar{l}}$ is the Riemann tensor of the inflaton sector. As $f^i f_i = 1$ the above equation defines the normalized sectional curvature along the direction of supersymmetry breaking. The constraint becomes stronger as $\gamma \gg 1$, thus as $H \gg m_{3/2}$. When the bound is met, one can always tune $\eta$ to be small by tuning $G_i$, $\nabla_iG_j$ and $R_{ijk\bar{l}}$.

Finding a suitably tuned supergravity potential from a (UV-complete) string theoretical setup has proven to be incredibly difficult [210, 211], but possible [169, 173, 212]. Currently, in
models with correctly tuned slow-roll parameters it is typically assumed that the ‘hidden sector’
does not affect the fine-tuning of parameters. The subject of this work is to examine whether
such an assumption is justified and hence how relevant tuned models are that only consider
the inflationary sector.

3.5.2 Stability of the hidden sector during inflation

Having reviewed the $\eta$-problem in single sector supergravity theories, we will now consider
if and how the fields in the hidden sector can affect the inflationary evolution. From the
diagonalization of the kinetic terms in (3.23) the distinction between $\phi$-fields and $q$-fields is
explicit, leading naturally to an inflationary and a hidden sector. We will again assume these
sectors to both consist of only one complex scalar field, $\phi$ and $q$ respectively. The argument
we shall present can already be made in a two-field system. It carries through to multi-field
models because the field $\phi$ is viewed as the inflaton in an effective single field inflationary
model, while the field $q$ can be seen as the lightest mode in the hidden sector. Following the
usual practice [187, 188, and references therein], we assume that inflation is solved by tuning
the inflationary sector only, including obtaining satisfactory values for the slow-roll parameters
from a phenomenological viewpoint. As a result all data in the inflationary sector are fixed and
known. Contrarily, the hidden sector is left unspecified and the restrictions we find on it are a
function of model specific parameters of the inflaton sector only.

To ensure that the hidden sector does not take part in the inflationary dynamics, one generally
assumes that the fields in the hidden sector are stabilized in a ground state at a constant field
value $q = q_0$ throughout inflation

$$\partial_q V|_{q_0} = 0 \quad (3.43)$$

and, hence, are not dynamical. Clearly this is true if $G_q = 0$, i.e. when the ground state of the
hidden sector preserves supersymmetry. As was shown in detail in [191, 199, 202, 213, 215],
when $G_q = 0$ the ground state of the hidden sector decouples gravitationally from the infla-
tionary sector and the inflationary sector truly determines the inflationary evolution without
any contributions from the hidden sector.

The case we examine here is when supersymmetry is broken in the hidden sector, $G_q \neq 0$.
The first thing to note is that the stability assumption (3.43) cannot be met anymore. In
supergravity the position $q = q_0$ of the minimum of the potential is given by

$$V_q = G_q V(\phi, \bar{\phi}, q, \bar{q}) + e^{G(\phi, \bar{\phi}, q, \bar{q})} \left( (\nabla_q G_q) G^q + G_q \right) = 0 \quad (3.44)$$

which shows that for $G_q \neq 0$ the ground state $q_0$ depends on the inflaton field $\phi$, through
$V(\phi, \bar{\phi}, q, \bar{q})$ and $G(\phi, \bar{\phi}, q, \bar{q})$. In the situation of unbroken supersymmetry, $G_q = 0$, all $\phi$-
dependence drops out, but for $G_q \neq 0$ we see that it is impossible to keep the position of the
minimum constant during inflation. As the inflaton $\phi$ rolls down the inflaton direction, the
‘stabilized’ hidden scalar $q$ will change its value. It is clear that the assumption of a vanishing
$V_q = 0$ for all $q$ is incompatible with $G_q \neq 0$ and we should therefore abandon it. This in turn
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means that the hidden sector field \( q \) must be dynamical, through its equation of motion. Since we still want to identify the field \( \phi \) as the inflaton in the sense that it drives the cosmological dynamics, we have to assume that \( q \) moves very little. We must therefore also assume a slow-roll, slow-turn approximation to the solution of the \( q \) equation of motion

\[
\dot{q} = \frac{G_{q\bar{q}} V_{\bar{q}}}{3H}.
\] (3.45)

The statement that the cosmological dynamics is driven by the \( \phi \)-sector means that \( \|\dot{q}\| \ll \|\dot{\phi}\| \), where \( \|\dot{q}\| \equiv \sqrt{G_{q\bar{q}} \dot{q}^2} \), etc. Through both slow-roll equations of motion this equates to \( \|V_{\phi}\| \ll \|V_q\| \) or \( \epsilon_q \ll \epsilon_\phi \), etc.

As the hidden sector has now become dynamical, we have to treat the system as a multi-field inflationary model. Since it is impossible to diagonalize the Kähler transformations and mass matrix simultaneously, the fields will mix in the case of a hidden sector with broken supersymmetry [201]. In the next section we will study the consequences of this mixing by explicitly diagonalizing the mass matrix of the full two-field system. From the result we shall find three possible effects on the inflationary dynamics.

First, the lightest masses of fields from the different sectors can be too close together. It is obvious that one cannot consider an effective single field model if this is the case, since for the dynamics to be independent of initial conditions, the lightest field needs to be much lighter than the other fields. When the masses of the two fields are similar, both of them contribute to the dynamics, resulting into a multi-field rather than a single field inflationary scenario. As is known from the literature, a multi-field inflationary model will produce effects such as isocurvature modes, e.g. [216–230], features in the power spectrum, e.g. [183, 186, 193, 231] and non-Gaussianities, e.g. [232–241], pointing to a qualitatively different model.

Second, a change of the true value of \( \eta \) can occur. We have assumed the inflaton sector to be tuned in such a way that it agrees with observed values for the slow-roll parameters. If the effects of the hidden sector on the total dynamics are such that \( \eta \) will change significantly, the initial naïve tuning would be of no meaning and one would have to start the tuning process all over again after the hidden sector has been added. Again we note that there is no contribution in the case of unbroken supersymmetry in the hidden sector, since we shall show that the contribution to \( \eta \) from the hidden sector is mostly determined by the cross terms in the mass matrix,

\[
V_{\phi q} = G_{\phi} V_q + G_q V_\phi - G_\phi G_q V,
\] (3.46)

which vanish when \( G_q = 0 \).

Third, a complete change of the sector that determines \( \eta \) is possible. It is possible that the eventual \( \eta \)-parameter is still within the limits of its naïve tuned value, satisfying the second bound, but instead it is determined by the hidden sector rather than the inflationary sector. Any initial control obtained by tuning the inflationary sector is superseded by the sheer coincidental configuration of the hidden sector.
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3.5.3 The Mass Matrix of a Two-Sector System

To investigate when effects from the hidden sector are to be expected, we need to calculate the eigenvalues of the mass matrix of the full two-field system. Since we assume the inflationary evolution to be in the slow-roll, slow-turn regime, the dynamics is completely potential energy dominated. The mass matrix of the full two-field system determines which directions are stable or steep, as characterized by the eigenvalues of this matrix. Normalizing by $1/V$ to obtain the value of $\eta$, directly, the matrix we want to diagonalize is the $4 \times 4$-matrix

$$N_{AB} = \frac{1}{V} \begin{pmatrix} \nabla^\alpha \nabla^\beta V & \nabla^\alpha \nabla^\eta V \\ \nabla^\alpha \nabla^\beta V & \nabla^\alpha \nabla^\eta V \end{pmatrix},$$

where $A \in \{\alpha, \bar{\alpha}\}$ and $B \in \{\beta, \bar{\beta}\}$ run over both fields $\phi$ and $q$ and their complex conjugates. Equation (3.47) is to be evaluated at a point near $q_0 = q_0(\phi_0)$, where $q_0$ is such that $\partial_q V(q_0) = 0$, with $\phi_0$ indicating the beginning of inflation. As is clear from the discussion of section 3.5.2, we cannot truly expect the hidden sector to be stabilized throughout the inflationary evolution. Nevertheless, we may consider $\partial_q V(q_0) = 0$ at a certain point $q_0 = q_0(\phi_0)$, with $\|\partial_q V\| \ll \|\partial_\phi V\|$ around $q_0$ in accordance with the restriction $\epsilon_q \ll \epsilon_\phi$.

The mass matrix is Hermitian and, considering again a two-field system, can be put in the form

$$N_{AB} = \frac{1}{V} \begin{pmatrix} \nabla^\phi V_{\phi} & \nabla^\phi V_{\bar{\phi}} & \nabla^\phi V_{\bar{q}} & \nabla^\phi V_{\bar{q}} \\ \nabla^{\bar{\phi}} V_{\phi} & \nabla^{\bar{\phi}} V_{\bar{\phi}} & \nabla^{\bar{\phi}} V_{\bar{q}} & \nabla^{\bar{\phi}} V_{\bar{q}} \\ \nabla^q V_{\phi} & \nabla^q V_{\bar{\phi}} & \nabla^q V_{\bar{q}} & \nabla^q V_{\bar{q}} \\ \nabla^\bar{q} V_{\phi} & \nabla^\bar{q} V_{\bar{\phi}} & \nabla^\bar{q} V_{\bar{q}} & \nabla^\bar{q} V_{\bar{q}} \end{pmatrix},$$

by a coordinate transformation. Diagonalizing the full matrix in general is involved. Therefore, we adopt the strategy to diagonalize the two sectors separately and then pick the lightest modes only. The first step yields

$$N_{AB} = \frac{1}{V} \begin{pmatrix} \frac{1}{V}(V_{\phi} - |V_{\bar{\phi}}|) & 0 & A_{11} & A_{12} \\ 0 & \frac{1}{V}(V_{\phi} + |V_{\bar{\phi}}|) & A_{21} & A_{22} \\ \bar{A}_{11} & \bar{A}_{21} & \frac{1}{V}(V_{\bar{q}} - |V_{\bar{q}}|) & 0 \\ \bar{A}_{12} & \bar{A}_{22} & 0 & \frac{1}{V}(V_{\bar{q}} + |V_{\bar{q}}|) \end{pmatrix},$$

with

$$A = \frac{1}{2V} \begin{pmatrix} -\tilde{V}_{\phi\phi} & \tilde{V}_{\phi\bar{\phi}} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} V_{\phi} & V_{\bar{q}} \\ V_{\bar{\phi}} & V_{\bar{q}} \end{pmatrix} \begin{pmatrix} -\tilde{V}_{\bar{\phi}\bar{\phi}} & \tilde{V}_{\bar{\phi}\bar{q}} \\ 1 & 1 \end{pmatrix}.$$ (3.50)

Here, the first matrix is the inverse of the similarity transformation of the $\phi$-sector and the last matrix diagonalizes the $q$-sector.

In general the eigenmodes in the individual sectors will be different, one always being smaller than the other. Dynamically the most relevant direction is the lightest mode of each sector, but by restricting to these light directions, one assumes a hierarchy already within the sectors. For the inflationary sector this is phenomenologically justified if we assume that inflation is
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described by a single field, where we know that $V^\phi\phi$ and $V^\phi\phi$ combine such that a light mode appears with mass $\eta V$, much lighter than the other mass modes. For the hidden sector we will simply assume that a large enough hierarchy between mass modes exists. This will simplify matters without weakening our result. By including only the lightest mode of the hidden sector, we can already show that the true dynamics is in many cases not correctly described by the naïve inflaton sector. Our case would only be more strongly supported if we would include the heavy mode of the hidden sector, but this is technically more involved. Projecting on the light directions, we get a sub-matrix of light mass modes

$$N_{\text{light}} = \begin{pmatrix} \lambda_\phi & A_{11} \\ A_{11} & \lambda_q \end{pmatrix} ,$$

(3.51)

with

$$\lambda_\phi = \frac{1}{V} \left( V^\phi - |V^\phi| \right) = \frac{G^\phi\phi}{V} (V^\phi - |V^\phi|) ,$$

(3.52)

$$\lambda_q = \frac{1}{V} \left( V^q - |V^q| \right) = \frac{G^q\eta}{V} (V^q - |V^q|) ,$$

(3.53)

$$A_{11} = \frac{G^\phi\phi V}{2V} \left( \tilde{V}^\phi\eta V^\eta\phi - \tilde{V}^\eta\eta V^\phi\eta + V^\phi\phi - \tilde{V}^\phi\phi V^\eta\eta \right) .$$

(3.54)

The eigenvalues of this two-field system are given by

$$\mu_\pm = \frac{1}{2} (\lambda_\phi + \lambda_q) \pm \frac{1}{2} \sqrt{ (\lambda_\phi - \lambda_q)^2 + 4|A_{11}|^2} .$$

(3.55)

Since $\mu_- < \mu_+$ the second slow-roll parameter for the full system is given by $\eta = \mu_-$.

### 3.6 Dynamics due to the hidden sector

In slow-roll and slow-turn approximation, the mass modes $\mu_\pm$ from (3.55) determine the dynamics of the full system. In general the true dynamics will deviate from the naïve single sector evolution. As explained in section [3.5.2](#), it is necessary to put constraints on the full system for the true dynamics to still (largely) agree with the initial naïve dynamics. We will quantify these constraints in terms of the hidden sector light mode $\lambda_q$ and the dynamical cross coupling $|A_{11}|$ between sectors. The results are graphically summarized in figures [3.8](#) and [3.9](#). In section [3.6.2](#) and figure [3.10](#) we will discuss the result again but then interpreted from the viewpoint of supergravity. Finally we will explain that a simple application of these bounds implies that the Standard Model cannot be ignored during cosmological inflation, if Standard Model supersymmetry breaking is independent of the inflaton sector.

#### 3.6.1 Conditions on the hidden sector data

From (3.55) we see that the light modes $\lambda_\phi, \lambda_q$ from the two separate sectors mix through a cross coupling $|A_{11}|$ and combine to the true eigenvalues $\mu_\pm$ of the full two-sector system. As
explained in [3.5.2] for the inflaton sector to still describe the cosmological evolution and the \( \eta \)-parameter reliably, the three constraints it must obey are (1) the bound arising from demanding a hierarchy between \( \mu_\pm \) to prevent multi-field effects, (2) the bound arising from demanding the second slow-roll parameter \( \mu_- = \eta \) to not change its value too much and (3) the bound from demanding that \( \eta \) is mostly determined by the \( \phi \)-sector rather than the \( q \)-sector.

To prevent multi-field effects from setting in we take as a minimum hierarchy that \( \mu_+ \) is at least five times as heavy as \( \mu_- \) in units of the scale of the problem, \(|\mu_-|\),

\[
\frac{\mu_+ - \mu_-}{|\mu_-|} > 5. \tag{3.56}
\]

This bound is rather arbitrary, but clearly a hierarchy between \( \mu_+ \) and \( \mu_- \) must exist. Calculations in [183] show that for a mass hierarchy \( \lesssim 5 \) multifold effects are typically important.

The second bound is given by the \( A_{11} \)-dependence of \( \mu_- \). The value of the second slow-roll parameter from the single field inflationary model only is \( \eta_{\text{naive}} = \lambda_\phi \). In the full two-sector system, \( \mu_- \) takes over the role as the true second slow-roll parameter \( \eta_{\text{true}} = \mu_- \). The contribution to the actual \( \eta \)-parameter from the presence of the hidden sector is therefore

\[
\Delta \eta = \mu_- - \lambda_\phi = \frac{1}{2} \left[ \left( \lambda_q - \lambda_\phi \right) - \sqrt{\left( \lambda_q - \lambda_\phi \right)^2 + 4 |A_{11}|^2} \right], \tag{3.57}
\]

which is always negative. We argue that this difference should stay within \( |\Delta \eta/\lambda_\phi| < 0.1 \), i.e. \( \eta \) should not change by more than 10%. This choice for the range of \( \eta \) is given by current experimental accuracy. Current experiments can only determine \( n_s = 1 - 6 \epsilon + 2 \eta \). WMAP has a 1\( \sigma \) error of 6.53% [134], Planck will have an error of 0.70% [242]. For \( n_s - 1 \), assuming 0.96, this gives a 17.5% error on the combination of \( -6 \epsilon + 2 \eta \), which means an uncertainty of about 10% on the value of \( \eta \).

We will examine \( \lambda_q, A_{11} \) in units of \( |\lambda_\phi| \) and exclude regions in which the hidden sector affects the tuned inflationary sector too much. The analysis is best done separately for the cases \( \lambda_\phi = \eta_{\text{naive}} > 0 \) and \( \lambda_\phi = \eta_{\text{naive}} < 0 \) because of the qualitative differences between these cases.

**The Case \( \eta_{\text{naive}} > 0 \)**

We first examine the hierarchy bound as explained above and focus first on the situation where \( \mu_- > 0 \). In this case (3.56) means that we demand

\[
\frac{\mu_+ - 6 \mu_-}{\lambda_\phi} = \frac{1}{2} \left[ -5 \left( \frac{\lambda_q}{\lambda_\phi} + 1 \right) + 7 \sqrt{\left( \frac{\lambda_q}{\lambda_\phi} - 1 \right)^2 + 4 \left( \frac{|A_{11}|}{\lambda_\phi} \right)^2} \right] > 0, \tag{3.58}
\]

which allows us to solve \( \lambda_q/\lambda_\phi \) as a function of \( |A_{11}|/\lambda_\phi \),

\[
\left( \frac{12}{35} \right)^2 \left( \frac{\lambda_q}{\lambda_\phi} - \frac{37}{12} \right)^2 + \left( \frac{2\sqrt{6}}{5} \right)^2 \left( \frac{|A_{11}|}{\lambda_\phi} \right)^2 = 1. \tag{3.59}
\]
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Figure 3.8: Bounds from a dynamical hidden sector for $\eta_{\text{naive}} > 0$. The multi-field constraint excludes an ellipse near the $\lambda_q$-axis (shaded in green). The bound from having too much effect on $\eta$ excludes large $|A_{11}|$ (shaded with increasing intensities of blue for larger deviations). Around $\lambda_q = A_{11} = 0$ the hidden sector mode $\lambda_q$ rather than $\lambda_\phi$ determines $\eta$, excluding that region as well (shaded in purple).

This excludes everything inside the ellipse demarcating the green region in figure 3.8. The case $\mu_- < 0$ is not relevant as it is already excluded by the second bound.

For this second bound, to be somewhat more general than the observationally inspired constraint $\Delta \eta/\lambda_\phi > -0.1$, we give the bound $\Delta \eta/\lambda_\phi > -f$. Solving for $\lambda_q$ this gives

$$\frac{\lambda_q}{\lambda_\phi} > 1 - f + \frac{1}{f} \left( \frac{|A_{11}|}{\lambda_\phi} \right)^2,$$

as is indicated in blue in figure 3.8. Note that since the true value of $\eta$ is always lower than $\eta_{\text{naive}}$ (see [180] for some specific examples), a change in $\eta$ of 100% means that $\eta$ changes sign from its naive value. This shows that we were justified to only consider positive $\mu_- $ in the hierarchy bound earlier.

The third bound is given by a $\lambda_q$-dominance in $\mu_-$. Since $\lambda_\phi$ and $\lambda_q$ are treated on equal footing in $\mu_-$, the true $\eta$ is dominantly determined by the smallest eigenvalue, which is not necessarily $\lambda_\phi$. When $\lambda_\phi \gg \lambda_q$ and $\lambda_\phi \gg |A_{11}|$ we see immediately that the true $\eta = \mu_-$ is determined by $\lambda_q$ and is independent of $\lambda_\phi$,

$$\mu_- = \frac{1}{2} \left[ (\lambda_q + \lambda_\phi) - \lambda_\phi \left( 1 - \frac{\lambda_q}{\lambda_\phi} + \mathcal{O} \left( \frac{\lambda_q^2}{\lambda_\phi^2}, \frac{|A_{11}|^2}{\lambda_\phi^2} \right) \right) \right].$$

(3.61)

It is clear that this arguments excludes the lower left corner of parameter space. We will take the bound to be $1/\sqrt{2}$ such that $(\lambda_q/\lambda_\phi)^2, (|A_{11}|/\lambda_\phi)^2 < 1/2 \ll 1$, the radius of convergence
Figure 3.9: Bounds from a dynamical hidden sector for $\eta_{\text{naive}} < 0$. The multi-field bound excludes a hyperbola starting at $\lambda_q = 4|\lambda_\phi|$ and, in particularly small $\lambda_q$ (shaded in green). The bound from having too much effect on $\eta$ excludes the large $|A_{11}|$-region (shaded with increasing intensities of blue for larger deviations), but leaves open in particular the full range of $\lambda_q$.

of this Taylor expansion. Contrarily to the somewhat debatable bounds imposed by $\Delta\eta/\lambda_\phi$, the points within this circle are truly excluded because they violate one of the core assumptions in the approach, viz. that the $\phi$-sector is responsible for all cosmological dynamics including determining the value of $\eta$. The circle

$$\left(\frac{\lambda_q}{\lambda_\phi}\right)^2 + \left(\frac{|A_{11}|}{\lambda_\phi}\right)^2 = \frac{1}{2},$$

(3.62)

is indicated as the purple region in the figure.

In figure 3.8 we have indicated in which regions of $\lambda_q/\lambda_\phi$ and $|A_{11}|/\lambda_\phi$-parameter space the effects of a hidden sector can be rightfully ignored. We have shown that all negative values of $\lambda_q$ are excluded and only in the region with large $\lambda_q/\lambda_\phi$ and small $|A_{11}|/\lambda_\phi$ there are no large effects from the hidden sector. This result is qualitatively easily understood, as the hidden sector with broken supersymmetry will still decouple if the masses in the hidden sector are truly large. We argue that this possibility is too easily assumed to be the case in the literature without considering the actual hidden constraints it imposes on the hidden sector. These hidden assumptions should be mentioned explicitly and one should show that they can be obtained.

**The case $\eta_{\text{naive}} < 0$**

In the case that $\lambda_\phi = \eta_{\text{naive}}$ is negative, the last bound of section 3.6.1 does not impose any condition on $\lambda_q/|\lambda_\phi|, |A_{11}|/|\lambda_\phi|$-parameter space. When $\lambda_\phi < 0$, i.e. when $\lambda_\phi = -|\lambda_\phi|$, the
eigenvalues can be written as
\[ \mu_{\pm} = \frac{|\lambda_\phi|}{2} \left[ \left( \frac{\lambda_q}{|\lambda_\phi|} - 1 \right) \pm \sqrt{\left( \frac{\lambda_q}{|\lambda_\phi|} + 1 \right)^2 + 4 \frac{\Lambda_{11}}{|\lambda_\phi|^2}} \right], \] (3.63)
which means that \( \mu_- \) is not determined by \( \lambda_q \) to first order in \( \lambda_q/|\lambda_\phi| \) but by \( \lambda_\phi \) as should be,
\[ \mu_- = \frac{|\lambda_\phi|}{2} \left[ \left( \frac{\lambda_q}{|\lambda_\phi|} - 1 \right) - \left( 1 + \frac{\lambda_q}{|\lambda_\phi|} + \ldots \right) \right]. \] (3.64)
However, by the hierarchy bound the small \( \lambda_q/|\lambda_\phi| \)-regime does get excluded. Since \( \mu_- \) is always negative in this case,
\[ \mu_- \leq \frac{|\lambda_\phi|}{2} \left[ \left( \frac{\lambda_q}{|\lambda_\phi|} - 1 \right) - \frac{\lambda_q}{|\lambda_\phi|} + 1 \right] = -|\lambda_\phi|, \] (3.65)
equation (3.56) translates into
\[ \frac{\mu_+ + 4\mu_-}{|\lambda_\phi|} = \frac{1}{2} \left[ 5 \left( \frac{\lambda_q}{|\lambda_\phi|} - 1 \right) - 3 \sqrt{\left( \frac{\lambda_q}{|\lambda_\phi|} + 1 \right)^2 + 4 \frac{\Lambda_{11}}{|\lambda_\phi|^2}} \right] > 0. \] (3.66)
This excludes everything beneath the upper branch of the hyperbola given by the line
\[ \frac{\lambda_q}{|\lambda_\phi|} > \frac{17}{8} + \frac{1}{8} \sqrt{15^2 + 28 \frac{\Lambda_{11}}{|\lambda_\phi|^2}}, \] (3.67)
which is shaded green region in figure 3.9.
The final constraint on the parameter space comes from the bound on the change in \( \eta \), see the previous paragraph on the \( \eta_{\text{naive}} > 0 \)-case for a discussion. In the blue region in figure 3.9 we have indicated the bound \(|\Delta \eta/\lambda_\phi| < f\), which means
\[ \frac{\lambda_q}{|\lambda_\phi|} > -1 - f + \frac{1}{f} \frac{\Lambda_{11}}{|\lambda_\phi|^2}, \] (3.68)
for different fractions of \( f \).
In figure 3.9 we have indicated in which regions of \( \lambda_q/|\lambda_\phi| \)- and \( |\Lambda_{11}|/|\lambda_\phi|\)-parameter space the effects of a hidden sector can be rightfully ignored after imposing both constraints. As in the case for \( \eta_{\text{naive}} > 0 \), the only allowed region is for large \( \lambda_q/|\lambda_\phi| \) and small \( |\Lambda_{11}|/|\lambda_\phi| \). Note that all values of \( \lambda_q < 4 \) are explicitly excluded by the imposed bounds.

### 3.6.2 Conditions on Supergravity Models

In principle, figures 3.8 and 3.9 provide all the information needed to verify whether the hidden sector of a given model may be neglected while studying the inflationary dynamics. Through equations (3.53–3.54) and the expressions for \( V_{\alpha\beta} \) as summarized in appendix D, one can...
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Figure 3.10: Excluded regions for the supergravity parameter range for $|G_q|$ and $\beta$, which contains in particular $\nabla_q \nabla_q G_q$, in units of $|\eta_{\text{naive}}|$ and $|\alpha|$, which contains $\epsilon_\phi$ and $G_\phi$. The indicated regions come from the multi-field bound (shaded in green), the correct identification of sectors (shaded in purple) and allowing only for small deviations of $\eta$ (shaded in higher intensities of blue for larger deviations). The left (right) picture describes the case $\eta_{\text{naive}} > 0$ ($\eta_{\text{naive}} < 0$).

explicitly calculate the corresponding $\lambda_q$ and $A_{11}$ for a given model and compare them with the figures. However, we would like to have some direct intuition about the dependence of the excluded regions on the supergravity data. In this section we will investigate how much we can say about this in general without having to specify a model. The main question to answer is whether the fact that $\lambda_q$ and $A_{11}$ are determined by a supergravity theory, provides any additional constraint on which regions are obtainable to begin with. The answer to this question turns out to be that a priori supergravity is not restrictive enough to exclude any of the regions in $\lambda_q, A_{11}$-parameter space.

The easiest way to translate figures 3.8 and 3.9 in terms of supergravity data would be to simply map the regions into supergravity parameter space. Unfortunately the expressions (3.53) and (3.54) are highly nonlinear and depend on too many supergravity variables to conveniently represent figures 3.8 and 3.9 in terms of supergravity data. However, for small $|G_q|$ this does turn out to be possible.

Using the expressions for $V_{\alpha\beta}$ in (3.54), yields

$$A_{11} = \alpha(\phi, \bar{\phi}, q, \bar{q})|G_q|, \quad \text{with}$$

$$\alpha(\phi, \bar{\phi}, q, \bar{q}) = \frac{G_{\phi\bar{\phi}}}{2} \left( \bar{\nabla}_q - \bar{V}_{\phi\bar{\phi}} \bar{G}_q \right) \left( \left( \frac{V_\phi}{V} - G_\phi \right) - \bar{V}_{\phi\phi} \left( \frac{V_{\bar{\phi}}}{V} - G_{\bar{\phi}} \right) \right).$$

From this equation we learn that $A_{11}$ vanishes in the limit $G_q \to 0$, which makes sense as we know that the two sectors should decouple in the limit of restored supersymmetry. It is difficult to retrieve more information from this explicit expression of $A_{11}$ in terms of supergravity data.
In principle $A_{11}(|G_q|,\ldots)$ may be inverted to give some function $|G_q|(A_{11},\ldots)$, but this is more tricky than (3.69) suggests. Although we have managed to extract one factor of $G_q$, the function $\alpha(\phi, \bar{\phi}, q, \bar{q})$ still depends on $G_q$ through the phases of $\hat{V}_{q\bar{q}}$ and $\hat{V}_{\phi\bar{\phi}}$, making it hard to perform the inversion explicitly.

The expression for $\lambda_q$ looks even worse,

$$\lambda_q = \frac{G_q}{V} \left( V_{q\bar{q}} - \sqrt{V_{qq}V_{\bar{q}\bar{q}}} \right).$$

(3.70)

At this stage we have even refrained from substituting in the expressions for $V_{q\bar{q}}$, $V_{qq}$ and its complex conjugate. The square root clearly shows that the dependence of $\lambda_q$ on $|G_q|$ and the other supergravity data is extremely involved and difficult to invert. To get a useful expression we revert to the result of section 3.4.3 and consider $\lambda_q$ in the small $|G_q|$-regime by performing a Taylor expansion. Copying from (3.38), we find

$$\beta(\phi, \bar{\phi}, q, \bar{q}) = \frac{G_q}{e^{-c\sqrt{V}}} \Re \{ (\nabla_q \nabla_{\bar{q}} G_q) \hat{G}_q^3 \}.$$ (3.71)

Having obtained the relations (3.69) and (3.71) we can now accommodate the reader with a graph of the allowed and excluded regions directly in terms of the supergravity data. For small $G_q \ll 1$ both $\lambda_q$ and $|A_{11}|$ scale linearly with $G_q$, making it relatively easy to rewrite the bounds we found $\lambda_q/|\lambda_{\phi}| = \lambda_q/|\lambda_{\phi}| (|A_{11}|/|\lambda_{\phi}|)$ in terms of $G_q$, $\alpha$ and $\beta$ as $\beta/|\alpha| = \beta/|\alpha| (|\alpha G_q|/|\lambda_{\phi}|)$.

The resulting figure is depicted in 3.10. Note that $\alpha$ and $\beta$ are still underdetermined — depending on $R_{q\bar{q}q\bar{q}}$ and $\nabla_q \nabla_{\bar{q}} G_q$ at higher orders in $|G_q|$ — and are naturally of order 1. It is these numbers that determine where in figure 3.10 the model under investigation lies.

### 3.6.3 Inflation and the Standard Model

As a simple application of the previous section, we can consider to what extent the Standard Model ought to be included in any reliable supergravity model for cosmological inflation. Our current understanding of Nature includes a present-day supersymmetrically broken Standard Model after an inflationary evolution right after the big bang. As such the combined model is exactly that of a two-sector supergravity theory with an inflationary and a hidden sector whose ground state breaks supersymmetry in which it resides throughout the inflationary era.

Supersymmetry in the Standard Model sector can either have been broken by gravity mediation of the inflaton sector or by a mechanism in the Standard Model sector itself. The first situation would be consistent approach as far as our analysis goes: as $G_q = 0$ the sector decouples from the inflationary dynamics, can be stabilized and the slow-roll parameters are reliably determined from the inflaton sector alone. Nevertheless, from the point of view of our understanding of the Standard Model it would be unsatisfactory to not know the precise mechanism behind its supersymmetry breaking and (complete) models describing such mechanisms would still have to be analyzed to shed light on the situation.
Figure 3.11: The effects of the multi-field bound (shaded in green), the identification of the correct inflaton sector (shaded in purple) and the small deviations of $\eta$ (shaded in blue) on a doubly logarithmic scale for $\eta_{\text{naïve}} > 0$ (left) and $\eta_{\text{naïve}} < 0$ (right). The approximate location of the Standard Model supergravity data is indicated with a red bar, showing that a large range of parameters is excluded. In this plot $\alpha = 1$ and $\lambda = \eta_{\text{naïve}} = 10^{-3}$.

In the second situation $G_q \neq 0$ and we should apply the results of the previous sections. The field $q$ may be seen as some light scalar degree of freedom in the (supersymmetrically broken) Standard Model. We assume the standard lore, that supersymmetry is broken in the Standard Model at a scale of about 1 TeV. In the $F$-term scalar potential, this scale enters via $G_q$. To determine the correct numerical value, we relate our dimensionless definition of the Kähler function to the standard dimensionful definition. Dimensionful quantities are denoted with a tilde in the following. We recall from section 3.4.2 that in order to have a non-vanishing vacuum energy, the superpotential in both sectors must have a non-zero constant term $W^{(1)}_0 = m^{(1)}/M_{\text{pl}}$, $W^{(2)}_0 = m^{(2)}/M_{\text{pl}}$, which accounts for the always present gravitational coupling between the sectors. Hence, the dimensionful constant term in the total superpotential has value $\tilde{W}_{\text{tot}}^{(1)} W^{(2)}_0 m^{(2)} M_{\text{pl}}$. In contrast, the supergravity quantities $\tilde{K}^{(2)}$ and $\tilde{W}^{(2)}_{\text{eff}} = \tilde{W}^{(1)}_0 \tilde{W}^{(2)}_{\text{global}}$ describing the Standard Model are naturally of the order of the TeV-scale, $[\tilde{W}^{(2)}_{\text{eff}}] = \text{TeV}^3$, $[\partial_q \tilde{K}^{(2)}] = \text{TeV}$. We relate the scale of supersymmetry breaking $\tilde{G}_{\tilde{q}}$ to the superpotential via

$$
\tilde{G}_{\tilde{q}} = \frac{M_{\text{pl}}^2}{W} \left( \partial_q \tilde{W} + \frac{\partial_q \tilde{K}^{(2)}}{M_{\text{pl}}^2} \tilde{W} \right),
$$

(3.72)

\footnotetext{E.g. in dimensionful units $[\tilde{G}] = \text{mass}^2$ and $[\tilde{q}] = \text{mass}$, while our conventions are $[G] = [q] = 0$. To relate $G_q$ to $\tilde{G}_{\tilde{q}}$ we can use the expression $[G_q] = \frac{[\tilde{G}_{\tilde{q}}]}{M_{\text{pl}}^2}$.
which is naturally of order

\[
\left(\tilde{G}_q\right) = \frac{M^2_{\text{pl}}}{m^{(1)}_\Lambda m^{(2)}_\Lambda M_{\text{pl}}} \left(\text{TeV}^2 + \frac{\text{TeV}}{M_{\text{pl}}(m^{(1)}_\Lambda m^{(2)}_\Lambda M_{\text{pl}} + \ldots)}\right) = \frac{M_{\text{pl}}\text{TeV}^2}{m^{(1)}_\Lambda m^{(2)}_\Lambda} + \text{TeV} + \ldots ,
\]

(3.73)

where the \ldots are of sub-leading order. We expect that \(m^{(1)}_\Lambda\), the constant term of the inflaton sector, is of order \([H] = 10^{-5}M_{\text{pl}}\), while \([m^{(2)}_\Lambda] = \text{TeV}\). Hence, translating back to dimensionless units, we find \(G_q \sim 10^{-11}\).

Taking the kinetic gauge, i.e. a canonical Kähler metric \(G_{\phi\phi} = 1\), we can easily find the natural value of \(\alpha\). From (3.69) we see that \(\alpha\) depends on \(\epsilon_\phi\) and \(G_\phi\) via

\[
\alpha \propto \sqrt{\epsilon_\phi} - G_\phi ,
\]

modulo some unknown but negligible phase factors. \(G_\phi\) is of order \(\sqrt{3}\) in order to have a potential \(V > 0\). Since \(\epsilon_\phi\) is of order \(O(10^{-3})\), the value of \(|\alpha|\) is of order unity. For a rough estimate for \(\eta_{\text{naive}} \sim 10^{-3}\) we can therefore pinpoint the Standard Model as indicated in figure [3.11]. In both cases, \(\eta_{\text{naive}} > 0\) as well as \(\eta_{\text{naive}} < 0\), the lightest supersymmetric particle is too light for the single sector inflationary dynamics to truly describe the full system. Any tuned and working inflationary supergravity model in which the Standard Model is assumed to not take part considerably in the cosmic evolution, requires implicit assumptions on the Standard Model that either the inflaton sector is responsible for Standard Model supersymmetry breaking through gravity mediation or the masses of its scalar multiplets are unnaturally large in terms of the now independent Standard Model supersymmetry breaking scale.

### 3.7 Clarification of the Rigid Limit to Supersymmetry

Multiple sectors are not only a common feature in supergravity cosmology but also in phenomenology. These sectors are necessary to either incorporate inflation or supersymmetry breaking or are a consequence of string model-building. It is therefore of general importance to understand the restrictions of combining several sectors with their individual actions to one theory. In particular to study inflation, it is desirable to separate the dynamics of all fields that do not contribute to the exponential expansion of the Universe from the inflaton fields that do. Since gravity is the weakest possible interaction, the inflationary sector is assumed to only couple gravitationally to an unknown “hidden” sector that may also break supersymmetry by itself. Whereas it is natural for a rigid supersymmetric theory to be separated into several sectors, the restrictive structure of supergravity forces the different sectors to couple not only non-locally through graviton exchange but also directly. For this reason embedding supersymmetric theories as sectors into a supergravity can be notoriously difficult, see e.g. [243],[250].

Though multiple sector supergravities are a long studied subject, the context of cosmology has seriously sharpened the question. In supergravity models of inflation it is commonly noted
that one seeks a consistent truncation of the scalar sector. This is necessary but not sufficient. Even with a consistent truncation one may have dominating instabilities towards the naïvely non-dynamical sectors, that can move them away from their supersymmetric critical points. One needs either a symmetry constraint or an energy barrier to constrain the dynamics to the putative inflaton sector.

During inflation, supersymmetry is broken and although it is frugal to consider scenarios where the inflaton sector is also responsible for phenomenological supersymmetry breaking (see e.g. [251–253]), this need not be so. For instance, in a generic gauge-mediation scenario, the mechanism responsible for supersymmetry breaking need not involve the fields that drive inflation. This example immediately shows that the generic cosmological set-up must be able to account for a sector that breaks supersymmetry independently of the inflationary dynamics.

This consideration is our starting point. We consider a multiple-sector supergravity that decouples in the strictest sense in the limit $M_{\text{pl}} \to \infty$. In this limit the action must then be the sum of two independent functions:

$$S[\phi, \bar{\phi}, q, \bar{q}] = S[\phi, \bar{\phi}] + S[q, \bar{q}] ,$$

such that the path integral factorizes. For a globally supersymmetric field theory with a standard kinetic term this can be achieved by demanding that the independent Kähler and superpotentials sum

$$K_{\text{susy}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(q, \bar{q}) , \quad W_{\text{susy}}(\phi, q) = W^{(1)}(\phi) + W^{(2)}(q) .$$

The issue we address here is that in supergravity complete decoupling in the sense of (3.75) appears to be impossible, even in principle. Even with block diagonal kinetic terms from a sum of Kähler potentials, the more complicated form of the supergravity potential

$$V_{\text{sugra}} = e^{K/M_{\text{pl}}^2} \left( K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - \frac{3}{4} |W|^2 M_{\text{pl}}^2 \right) , \quad D W_{\text{sugra}} = \partial W_{\text{sugra}} + \partial K_{\text{sugra}} \frac{W_{\text{sugra}}}{M_{\text{pl}}^2} ,$$

implies that there are many direct couplings between the two sectors. It raises the immediate question: if the low-energy $M_{\text{pl}} \to \infty$ globally supersymmetric model must consist of decoupled sectors, what is the relation between $K_{\text{sugra}}, W_{\text{sugra}}$ and $K_{\text{susy}}, W_{\text{susy}}$, or vice versa given a globally supersymmetric model described by $K_{\text{susy}}, W_{\text{susy}}$, what is the best choice for $K_{\text{sugra}}, W_{\text{sugra}}$ such that the original theory can be recovered in the limit $M_{\text{pl}} \to \infty$?

In this section we shall show that the scaling implied by the explicit factors of $M_{\text{pl}}$ in the supergravity potential (3.77) is an incomplete answer to this question. The direct communication between the sectors, controlled by $M_{\text{pl}}$, has serious consequences for both the ground state

\[\text{As example we consider the simplest case, a model with two uncharged scalar supermultiplets } X^a = (\phi, q) \text{ that are singlets under all symmetries. Gauge interactions and global symmetries will not change this general argument provided the two sectors are not mixed by symmetries or coupled by gauge fields. Therefore, we will also ignore } D\text{-terms in the supergravity potential below.}\]
structure (solutions to the equation of motion, i.e. the cosmological dynamics) and the interactions between the two sectors. To be explicit, the first guess at how the rigid supersymmetry and supergravity Kähler potentials and superpotentials are related

\[ K_{\text{sugra}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}_{\text{susy}}(\phi, \bar{\phi}) + K^{(2)}_{\text{susy}}(q, \bar{q}) + \ldots, \]

\[ W_{\text{sugra}}(\phi, q) = W^{(1)}_{\text{susy}}(\phi) + W^{(2)}_{\text{susy}}(q) + \ldots, \]

with \ldots indicating Planck-suppressed terms and possibly a constant term, suffers from the drawback that the ground states of the full theory are no longer the product of the ground states of the individual sectors, except when both (rather than only one) ground states are supersymmetric [199, 200] (see also [201, 202, 213]). This directly follows from considering the extrema of the supergravity potential

\[ \nabla_i V = \frac{D_i W}{W} V + e^{K/M^2_{\text{pl}}} |W|^2 \left( \nabla_i \left( \frac{D_j W}{W} \right) \frac{D^j W}{W} + 1 \right) \frac{D_i W}{W} + \nabla_i \left( \frac{D_\beta W}{W} \right) \frac{D^\beta W}{W}, \]

\[ \nabla_i \nabla_\alpha V = \frac{D_\alpha W}{W} \nabla_i V + \frac{D_i W}{W} \nabla_\alpha V - \frac{D_i W}{W} \frac{D_\alpha W}{W} V + D_i \left( \frac{D_\alpha W}{W} \right) (V + 2 \frac{M^2_{\text{pl}} e^{K/M^2_{\text{pl}}} |W|^2}) \]

\[ + e^{K/M^2_{\text{pl}}} |W|^2 \left( \nabla_i \nabla_\alpha \left( \frac{D_\beta W}{W} \right) \frac{D^\beta W}{W} + \nabla_\alpha \nabla_i \left( \frac{D_j W}{W} \right) \frac{D^j W}{W} \right). \]

Supersymmetric ground states, for which the covariant derivatives of \( W \) vanish on the solution, \( D_i W = 0 \) and \( D_\alpha W = 0 \), are still product solutions. But for Kähler - and superpotentials that sum (3.78), even if only one sector is in a non-supersymmetric ground state, by which we mean \( D_i W = 0, D_\alpha W \neq 0 \), we can neither conclude that sector 2, labeled by \( i \), is in a minimum, for which \( \nabla_i V \) would vanish, nor that the condition for sector 1, labeled by \( \alpha \), to be in a local ground state is independent of the sector 2 fields \( q_i \), which would mean that \( \nabla_i \nabla_\alpha V = 0 \). The former is only true when

\[ \nabla_i \left( \frac{D_\beta W}{W} \right) \frac{D^\beta W}{W} = 0. \]  

(3.81)

The second requires, in addition,

\[ \nabla_i \nabla_\alpha \left( \frac{D_\beta W}{W} \right) \frac{D^\beta W}{W} + \nabla_\alpha \nabla_i \left( \frac{D_j W}{W} \right) \frac{D^j W}{W} = 0, \]  

(3.82)

and also sharpens the first condition (3.81) to

\[ D_i \frac{D_\alpha W}{W} = 0. \]  

(3.83)

\[ ^7 \text{To derive (3.80) note that, since } DW/W \text{ is Kähler invariant and since the Levi-Civita connection } \nabla \text{ of the field space manifold does not get cross-contributions in a product manifold,} \]

\[ \nabla_i \frac{D_\alpha W}{W} = \partial_i \frac{D_\alpha W}{W} = D_i \frac{D_\alpha W}{W}. \]

\[ ^8 \text{These conditions are merely sufficient not necessary. However, it is clear that the restrictive nature of supergravity enforces conditions on the unknown sectors for the system to be separate.} \]
Equations (3.81–3.83) are conditions for decoupling which apply not only to the ground state of the full system but also to other critical points of the potential, for instance along an inflationary valley. Generically these conditions are not met on the solution (the second derivative need not vanish at an extremum; recall that $D_i W$ does not vanish identically but only on the solution).

Hence, generically the ground states of hidden sectors mix and this spoils many cosmological supergravity scenarios that truncate the action to one or the other sector (see e.g. [254] and references therein). It is this issue that is particularly relevant for inflationary model building, where a very weak coupling between the inflaton sector and all other sectors has to persist over an entire trajectory in field space where the expectation values of the fields are changing with time (see e.g. [1, 193, 197, 203, 214]). At the same time, one is interested in the generic situation in which both sectors may contribute to supersymmetry breaking.

### 3.8 Natural multi-sector supergravities

There is a well-known natural way to construct supergravity potentials for which the ground states (and critical points) do separate better. This obvious combination of superpotentials automatically satisfies (3.81–3.83) and hence does ensure that if one of the ground states is supersymmetric, the ground state of the other sector is a decoupled field theory ground state whether it breaks supersymmetry or not. This is if we choose a product of superpotentials, keeping the sum of Kähler potentials as before,

$$K_{\text{sugra}}(\phi, \bar{\phi}, q, \bar{q}) = K^{(1)}_{\text{sugra}}(\phi, \bar{\phi}) + K^{(2)}_{\text{sugra}}(q, \bar{q}) , \quad W_{\text{sugra}}(\phi, q) = \frac{1}{M^2_{\text{pl}}} W^{(1)}_{\text{sugra}}(\phi) W^{(2)}_{\text{sugra}}(q) .$$

(3.84)

This is well-known [195,196,257] and has recently been emphasized in the context of cosmology [1,201–203,213,214,254,258,259]. This ansatz conforms to the more natural description

This situation has to be contrasted to phenomenological models appropriate for studying gravity mediated supersymmetry breaking, such as an ansatz [255]

$$K(\phi, \bar{\phi}, q, \bar{q}) = K^{(0)}(\phi, \bar{\phi}) + q^a \bar{q}^b K^{(1)}_{ab}(\phi, \bar{\phi}) + (q^2 + \bar{q}^2) K^{(2)}(\phi, \bar{\phi}) ,$$

$$W(\phi, q) = W_0(\phi) + q^i q^j W_{ij}(\phi) .$$

or equivalently, if $W \neq 0$,

$$G(\phi, \bar{\phi}, q, \bar{q}) = G^{(0)}(\phi, \bar{\phi}) + q^i \bar{q}^j G_{ij}^{(1)}(\phi, \bar{\phi}) + q^i q^j G_{ij}^{(2)}(\phi, \bar{\phi}) + \bar{q}^i \bar{q}^j G_{ij}^{(0,2)}(\phi, \bar{\phi}) + \ldots .$$

In models like these, it is understood that $\dot{q} = 0$ and the $q$-sector can remain in its supersymmetric critical point throughout the evolution of the supersymmetry breaking fields. For inflation, such an expectation is unrealistic, as the supersymmetry-preserving sector can become unstable during the inflationary dynamics, see e.g. a recent discussion of the case in which the inflaton field $\phi$ is solely responsible for supersymmetry breaking during inflation (253 and references therein). In this relatively simple case, and except for very fine-tuned situations, the generic scenario appears to be that one or more of the $q$-fields are destabilized somewhere along the inflationary trajectory and they trigger an exit from inflation (in other words, they become “waterfall” fields, and inflation is of the hybrid kind [256]). This implies that the pattern of supersymmetry breaking today is not related to the one during inflation, and also, since the waterfall fields are forced away from their supersymmetric critical points, that supersymmetry is broken by both sectors as the Universe evolves towards the current vacuum.
of supergravities in terms of the Kähler invariant function

\[ G(X, \bar{X}) = \frac{1}{M_{pl}^2} K_{\text{sugra}}(X, \bar{X}) + \log \left( \frac{W_{\text{sugra}}(X)}{M_{pl}^2} \right) + \log \left( \frac{W_{\text{sugra}}(\bar{X})}{M_{pl}^2} \right), \]  

(3.85)

which can be defined if \( W \) is non-zero in the region of interest.\(^\text{10}\) This Kähler function in turn underlies a better description of multiple sectors in supergravity where \( G \) is a sum of independent functions, as we have already been advocating in equation (3.26)

\[ G(\phi, \bar{\phi}, q, \bar{q}) = G^{(1)}(\phi, \bar{\phi}) + G^{(2)}(q, \bar{q}), \]  

(3.86)

such that the two sectors are separately Kähler invariant. The sum implies the product superpotential put forward above. This is the simplest ansatz that still allows some degree of calculational control when both sectors break supersymmetry — as well as optimizing decoupling along the inflationary trajectory. One of the simplest models of hybrid inflation in supergravity, \( F \)-term inflation [260, 261], is in this class.

### 3.9 Decoupling

Given that we have just argued that a product of superpotentials is a more natural framework to discuss multiple sector supergravities, the obvious question arises how to recover a decoupled sum of potentials for a globally supersymmetric theory in the limit where gravity decouples, i.e. in which

\[ V_{\text{sugra}} = e^{K/M_{pl}^2} \left( |D\phi|^2 - \frac{3}{M_{pl}^2} |W|^2 \right) \rightarrow V_{\text{susy}} = \sum_j |\partial_j W^{(j)}|^2 . \]  

(3.87)

For a two-sector supergravity defined by equations (3.84) one would not find this answer, if one takes the standard decoupling limit \( M_{pl} \rightarrow \infty \) with both \( K = K^{(1)} + K^{(2)} \) and \( W = M_{pl}^3 W^{(1)} W^{(2)} \) fixed.\(^\text{11}\) Instead, the product structure of the superpotential introduces a cross-coupling between sectors,

\[ V_{\text{eff}} = \frac{1}{M_{pl}^3} \left( |W^{(2)}|^2 |\partial_\alpha W^{(1)}|^2 + |W^{(1)}|^2 |\partial_\beta W^{(2)}|^2 \right) \neq V_{\text{susy}} , \]  

(3.88)

\(^\text{10}\)We expect this condition to hold around a supersymmetry breaking vacuum with almost vanishing cosmological constant. It also holds in many models of supergravity inflation, although a notable exception is [191, 215].

\(^\text{11}\)Strictly speaking the decoupling limit sends \( M_{pl} \rightarrow \infty \) while keeping the fields \( \phi, q \) fixed with \( W^{(j)}/M_{pl}^3 \) a holomorphic function of \( \phi/M_{pl} \) or \( q/M_{pl} \) and \( K^{(j)}/M_{pl}^2 \) a real function of \( \phi/M_{pl}, \bar{\phi}/M_{pl} \) or \( q/M_{pl}, \bar{q}/M_{pl} \). The limit zooms in to the origin so \( K \) must be assumed to be non-singular there. Formally the decoupling limit does not exist otherwise. Physically it means that one is taking the decoupling limit w.r.t. an a priori determined ground state, around which \( K \) and \( W \) are expanded. If \( K \) is non-singular at the origin, the overall factor \( e^{K/M_{pl}^2} \) yields an overall constant as \( M_{pl} \rightarrow \infty \), which may be set to unity, i.e. the constant part of \( K \) vanishes. In the decoupling limit, both \( K \) and \( W \) may then be written as polynomials. Letting the coefficients in \( W \) and \( K \) scale as their canonical scaling dimension such that \( W \) has mass dimension three and \( K \) has mass dimension two, then gives the rule of thumb that both \( K \) and \( W \) are held fixed as \( M_{pl} \rightarrow \infty \).
whose behavior under the limit $M_{\text{pl}} \to \infty$ is best examined at the level of the superpotential.

Supergravity is sensitive to the expectation value $W_0 = \langle W \rangle$ of $W$, which relates the scale of supersymmetry breaking to the expectation value of the potential, i.e. the cosmological constant

$$\Lambda^2 M_{\text{pl}}^2 = \langle V \rangle \sim \langle DW^2 \rangle - \frac{3}{M_{\text{pl}}^2} \langle W^2 \rangle = m_{\text{susy}}^4 - \frac{3 W_0^2}{M_{\text{pl}}^2}. \quad (3.89)$$

The vacuum expectation value cannot vanish in a supersymmetry breaking vacuum with (nearly) zero cosmological constant, such as our Universe. Therefore, in the following we assume $\langle W \rangle \neq 0$ in the region of interest. Instead of the usual way to incorporate it, $W_{\text{sugra}} = W_0 + W_{\text{dyn}}$ with $W_{\text{dyn}} = W_{\text{susy}} + \ldots$, we include the vacuum expectation value for a two-sector product superpotential by writing

$$W(\phi, q) = \frac{1}{M_{\text{pl}}^3} W^{(1)} W^{(2)} = \frac{1}{M_{\text{pl}}^3} \left( W^{(1)}_0 + W^{(1)}_{\text{dyn}}(\phi) \right) \left( W^{(2)}_0 + W^{(2)}_{\text{dyn}}(q) \right)$$

$$= \frac{1}{M_{\text{pl}}^3} \left( W^{(1)}_0 W^{(2)}_0 + W^{(2)}_0 W^{(1)}_{\text{dyn}}(\phi) + W^{(1)}_0 W^{(2)}_{\text{dyn}}(q) + W^{(1)}_{\text{dyn}}(\phi) W^{(2)}_{\text{dyn}}(q) \right). \quad (3.90)$$

This is physically equivalent to a sum of superpotentials except for the last term. Note again, that if one uses the standard scaling, $\frac{\phi}{M_{\text{pl}}} \to 0; \frac{q}{M_{\text{pl}}} \to 0$ with all couplings in $W^{(\text{total})}$ having the canonical scaling dimensions, this last term contains renormalizable couplings involving the scalar partner of the goldstino, and these are not Planck-suppressed: if supersymmetry is broken by the $\phi$ sector, terms of the form $\phi q^2$ are renormalizable and would survive the $M_{\text{pl}} \to \infty$ limit, leading to a direct coupling between the two sectors. If both sectors break supersymmetry then mass-mixing terms $\phi q$ also survive. All such (relevant) terms are of course absent if none of the two sectors break supersymmetry, but this is not the case we are interested in. One would have expected that these cross-couplings naturally vanish in the decoupling limit.

The point of this note is simply to remark that the realization that each of the superpotentials $W^{(j)} = W^{(j)}_0 + W^{(j)}_{\text{dyn}}$ contains a constant term can resolve this conundrum by assuming a non-standard scaling for the constituent parts $W^{(j)}_0$, $W^{(j)}_{\text{dyn}}$. To achieve a decoupling we need that the cross term $W^{(1)}_{\text{dyn}} W^{(2)}_{\text{dyn}}$, which contains the coupling between the two sectors, scales away in the limit $M_{\text{pl}} \to \infty$. As a result the first term in (3.90) has to diverge, because its product with the cross term should remain finite. In particular we can choose an overall scaling

$$W = \frac{1}{M_{\text{pl}}^3} \left( W^{(1)}_0 W^{(2)}_0 \right)_{\sim M_{\text{pl}}^{3+r}} + W^{(1)}_{\text{dyn}} W^{(2)}_{\text{dyn}} + W^{(1)}_{\text{dyn}} W^{(1)}_{\text{dyn}} + W^{(1)}_{\text{dyn}} W^{(2)}_{\text{dyn}} \right), \quad (3.91)$$

For a product of superpotentials we can always choose a Kähler gauge at every point with $\langle K \rangle = \langle \partial_\phi K \rangle = \langle \partial_\phi K \rangle$ = 0 without mixing the superpotentials. In that case $F$-term supersymmetry breaking is given by the linear terms in the expansion of $W^{(1)}$ and $W^{(2)}$: $\langle D_\phi W \rangle \sim \langle \partial_\phi W^{(1)} \rangle, \langle D_2 W \rangle \sim \langle \partial_\phi W^{(2)} \rangle$.  

\[12\]
with \( r > 0 \). Let us account for dimensions by introducing an extra scale \( m_\Lambda \) such that

\[
W^{(1)}_0 = m_\Lambda^{3-r} M_{\text{pl}}^{2-A}, \quad W^{(1)}_{\text{dyn}} = m_\Lambda^{3-r} M_{\text{pl}}^{2-A}, \\
W^{(2)}_0 = m_\Lambda^{3-r} M_{\text{pl}}^{2-A}, \quad W^{(2)}_{\text{dyn}} = m_\Lambda^{3-r} M_{\text{pl}}^{2-A},
\]

(3.92)

with \( W^{(j)}_{\text{susy}} \) fixed as \( M_{\text{pl}} \rightarrow \infty \). Formally one can choose an inhomogeneous scaling with \( A \neq 0 \), but as we shall see it has no real consequences. For any \( A \) it is easily seen that with this scaling,

\[
D_\alpha W = \partial_\alpha W^{(1)}_{\text{susy}} + \frac{m_\Lambda^{r-3}}{M_{\text{pl}}} W^{(2)}_{\text{susy}} \partial_\alpha W^{(1)}_{\text{susy}} \\
+ \frac{\partial_\alpha K^{(1)}}{M_{\text{pl}}} \left( m_\Lambda^{3-r} M_{\text{pl}}^{2-A} + W^{(1)}_{\text{susy}} + \frac{m_\Lambda^{r-3}}{M_{\text{pl}}} W^{(1)}_{\text{susy}} W^{(2)}_{\text{susy}} \right) \rightarrow \partial_\alpha W^{(1)}_{\text{susy}},
\]

(3.93)

in the limit \( M_{\text{pl}} \rightarrow \infty \) if and only if \( 0 < r < 2 \) and thus

\[
V_{\text{sugra}} = e^{K/M_{\text{pl}}^2} \left( |DW|^2 - \frac{3|W|^2}{M_{\text{pl}}^2} \right) \rightarrow \sum_j |\partial_j W^{(j)}_{\text{susy}}|^2 - 3m_\Lambda^{2(3-r)} M_{\text{pl}}^{2(r-1)} + O \left( \frac{1}{M_{\text{pl}}} \right).
\]

(3.94)

For \( r < 1 \) the manifestly constant term in the potential vanishes as well and we recover the strict decoupled field theory result, with the gravitino mass going to zero as \( m_{3/2} = \langle W \rangle M_{\text{pl}}^{-2} = m_\Lambda^{3-r} M_{\text{pl}}^{-2} = \frac{m_{\text{susy}}}{\sqrt{3} M_{\text{pl}}} \). We see that the gravitino mass is independent of \( r \) in physical scales.

The parameter \( r \) should not be larger than unity for the new decoupling limit to be well defined. For the special case \( r = 1 \) \([195]\), the potential has an additional overall “cosmological” constant. For a generic non-gravitational field theory in which \( M_{\text{pl}} \rightarrow \infty \) this is just an overall shift of the potential, which we can arbitrarily remove since it does not change the physics. Nevertheless from a formal point of view, we know that absolute ground state energy of a globally supersymmetric field theory equals zero, as a result of the supersymmetry algebra \( \{ Q, Q \} = H \). For this reason it is more natural to restrict the value of \( r \) to the range \( 0 < r < 1 \).

Finally, the novel scaling in (3.92) can be readily generalized to an arbitrary number of sectors. For \( s \) sectors, writing \( W^{(j)} = W^{(j)}_0 + W^{(j)}_{\text{dyn}} \),

\[
W = \frac{1}{M_{\text{pl}}^{3(s-1)}} \prod_{j=1}^s W^{(j)}_0 = \frac{1}{M_{\text{pl}}^{3(s-1)}} \left[ \prod_{j=1}^s W^{(j)}_0 + \sum_{k=1}^s \left( \prod_{j \neq k} W^{(j)}_0 \right) \prod_{j \neq k} W^{(j)}_0 + \sum_{l > k} \left( \prod_{j \neq k} W^{(j)}_0 \right) \prod_{j \neq k,l} W^{(j)}_0 \right] + \ldots
\]

(3.95)

We want the last and all further terms to scale away as \( M_{\text{pl}}^{-r} \) and higher with \( r > 0 \), while the second term(s) should be constant. As a consequence the first term will scale as \( M_{\text{pl}}^r \). Assuming a scaling that is homogeneous across sectors, this implies

\[
W^{(j)}_0 \sim M_{\text{pl}}^{\frac{3(s-1)+r}{s}}, \quad W^{(j)}_{\text{dyn}} \sim M_{\text{pl}}^{\frac{(3-r)(s-1)}{s}},
\]

(3.96)
for each of the \( j \in \{1,\ldots,s\} \). With this scaling, a general term consisting of \( l \) dynamical superpotentials and \( s - l \) constant parts, scales as

\[
\frac{W_{0}^{l}W_{s-l}^{(s)}M_{3\text{pl}}}{M_{3\text{pl}}^{3(s-1)}} \sim M_{\text{pl}}^{3(1-l)},
\]

and as constructed any term containing dynamical interactions between sectors, \( l > 2 \), is Planck-suppressed. To ensure a vanishing constant term as in eq. (3.94), \( r \) is again limited to the range \( 0 < r < 1 \).

Let us conclude with a comment on the physical meaning behind the scaling (3.92). It may appear that we have changed the canonical RG-scaling of the theory. This is not quite true. For the interacting terms in the potential, it is the coefficients in the product \( W_{0}^{(2)}W_{\text{dyn}}^{(1)} = W_{\text{susy}}^{(1)} \) that ought to obey canonical RG-scaling. This precisely corresponds to holding \( W_{\text{susy}}^{(j)} \) fixed as \( M_{\text{pl}} \to \infty \) (see footnote 11). On the other hand, the scaling of the constant term in the potential has changed from its canonical value. However, this is very natural in a supersymmetric theory. The constant term, \( \prod_{j} W_{0}^{(j)} \), equals the ground state energy. Precisely supersymmetric theories can “naturally” explain non-canonical scaling of the cosmological constant (at the loop level; the scaling of the bare ground state energy can be different in every model). A non-integer power is strange but \( r = 1 \) is certainly a viable option in a supersymmetry-breaking ground state: it is the natural scaling in theories with higher supersymmetry when combined with a subleading \( \log(M_{\text{pl}}/m_{\text{susy}}) \) breaking. Our engineering analysis only focuses on power-law scaling and these can always have subleading logarithms. (\( r = 2 \) would correspond to the cosmological constant for a spontaneously broken \( \mathcal{N} = 1 \) theory due to mass splitting).

### 3.10 Conclusions

In this chapter we have studied the effect of hidden sectors on the fine-tuning of \( F \)-term inflation in supergravity, identifying a number of issues in the current methodology of fine-tuning inflation in supergravity. Fine-tuning inflationary models is only valid when the neglected physics does not affect this fine-tuning, in which case the inflationary physics can be studied independently. As shown in figures 3.8 and 3.9, this assumption holds only under very special circumstances. The reason is that the everpresent gravitational couplings will always lead to a mixing of the hidden sectors with the inflationary sector.

First, we have argued in which way the action represents the two sectors as minimally coupled as possible. Rather than adding the superpotentials, the correct action is obtained by adding the Kähler functions, which preserves Kähler invariance in each sector independently. This leads to the action (3.31), where the superpotentials are multiplied.

Although this ansatz is extremely useful in the context of cosmology, it demands due diligence in a number of aspects. We have argued in section 3.9 that a (cosmological) constant term must be included to prevent the superpotentials from vanishing and rendering the Kähler function...
infinite on the solution. This constant and its cross-terms call for a second scale when taking
the limit in which the Planck mass goes to infinity and gravity is turned off. This limit is delicate
and restricts the scaling of the individual terms in the superpotential.

For a hidden sector vacuum that preserves supersymmetry, the sectors decouple consistently
\([198-202]\). However, for a supersymmetry breaking vacuum the inflationary dynamics is gener-
ically altered, where the nature and the size of the change depends on the scale of supersym-
metry breaking. For a hidden sector with a low scale of supersymmetry breaking, like the
Standard Model, the cross coupling scales with the scale of supersymmetry breaking, and is
therefore typically small. Yet, as shown in section \([3.4.3]\) also the lightest mass of the hidden
sector scales with the scale of supersymmetry breaking within that sector. This light mode is
strongly affected by the inflationary physics and thus evolves during inflation. Therefore, any
single field analysis is completely spoiled as discussed in section \([3.6.3]\).

For massive hidden sectors, the problem is more traditional. For a small hidden sector super-
symmetry breaking scale, one has a conventional decoupling as long as the lightest mass of the
hidden sector is much larger than the inflaton mass. However, for large hidden sector super-
symmetry breaking, this intuition fails. Then, the off-diagonal terms in the mass matrix \((3.47)\)
will lead to a large correction of the \(\eta\)-parameter.

To conclude, any theory that is working by only tuning the inflaton sector has made severe hid-
den assumptions about the hidden sector, which typically will not be easily met. Methodologi-
cally the only sensible approach is to search for inflation in a full theory, including knowledge
of all hidden sectors.