



UvA-DARE (Digital Academic Repository)

Socio-dynamic discrete choice: Theory and application

Dugundji, E.R.

[Link to publication](#)

Citation for published version (APA):

Dugundji, E. R. (2013). *Socio-dynamic discrete choice: Theory and application*.

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

INTRODUCTION

1.1 CONTEXT

A wide spectrum of policy measures have been put forward over the past decade to try to address the infamous rush hour road congestion in the "Randstad," the western region of the Netherlands marked by the ring of cities Amsterdam - Utrecht - The Hague - Rotterdam. These measures range from flexible work hours to congestion pricing to light rail to facilitation of park-and-ride to road construction. The research reported here is a small part of a larger work aimed at understanding, measuring and modeling the combined residential choice and travel behavior of households in the north wing of the Randstad, that is, the Amsterdam-Utrecht greater region. The focus of the larger work is on the promotion and facilitation of multi-modal transportation as a land use transportation planning policy instrument for reducing road congestion (Timmermans et al, 2002; Joh, 2004; Krygsman, 2004; Maat et al, 2004). Here is understood for example the use of so-called park-and-ride "transferia" (typically transferring modes between private vehicle and multiple passenger transit vehicle). The contribution of this particular subproject is the treatment of social and spatial interactions between households and generated feedback dynamics in the adoption of transportation mode alternatives.

Pioneered in the domain of travel demand by Warner (1962), Lisco (1967), Stopher (1969), Ben-Akiva (1973), Domencich and McFadden (1975) and others, discrete choice analysis has become an industry standard in land use and transportation planning models. Some subsequent elegant and elaborate operational examples of the development of this methodology are due to Wegener and Spiekermann (1996), Hensher and Ton (2001), Waddell (2002), Miller et al (2004), Martinez and Aguila (2004), Almeida et al (2009), to cite just a few. Meanwhile, the field itself has flourished in the past 40 years¹ extending the basic random utility model to incorporate cognitive and behavioral processes, flexible error structures and different types of data in so-called hybrid choice models (Ben-Akiva et al, 1999, 2002;

¹ The importance of discrete choice theory in the field of economics was recognized in the Nobel Prize awarded to Daniel McFadden in 2000. In his Nobel lecture, Economic Choices, he paid particular tribute to nine other individuals who played a major role in channeling microeconometrics and choice theory toward their modern focus, Zvi Griliches, L.L. Thurstone, Jacob Marschak, Duncan Luce, Amos Tversky, Danny Kahneman, Moshe Ben-Akiva, Charles Manski and Kenneth Train. Interestingly, the home disciplines of these influential scholars include not only economics, but range from psychology to civil engineering.

Walker, 2001). However as discrete choice theory is fundamentally grounded in individual choice, an outstanding methodological challenge remains in the treatment of the interdependence of various decision-makers' choices, be that via global or local interactions. The formulation of the nature of the interaction in turn raises the issues of networks, and network evolution. When considering the domain of land use and transportation, both social networks and spatial networks may be relevant (Dugundji et al, 2001).

There is growing awareness and interest in the influence that social factors have on transportation and land use behaviors (Dugundji, Páez and Arentze, 2008; Dugundji, Páez, Arentze and Walker, 2011; Dugundji, Scott, Carrasco and Páez, 2012). In this context, a distinction can be made between identifiable versus aggregate interactions. We speak of interaction between "identifiable" decision-makers when the links in the network are well-known and explicitly defined on an individual decision-maker by decision-maker basis. We speak of interaction between "aggregate" decision-makers when interdependence is assumed to take place only at an aggregate level with links being defined by implicit networks, for example, more generally based on decision-maker characteristics, by latent constructs or by studying behavior of abstract classes of networks (Dugundji and Gulyás, 2003a, 2003b, 2008; Dugundji and Walker, 2005).

Important avenues of research in the area of identifiable interactions include capturing influences from tight social networks such as among household members (Golob and McNally, 1997; Fujii, Kitamura and Kishizawa, 1999; Simma and Axhausen, 2001; Meka, Pendyala and Kumara, 2002; Gliebe and Koppelman, 2002; Scott and Kanaroglou, 2002; Borgers, Hofman and Timmermans 2002; Zhang, Timmermans and Borgers 2002,2005; Zhang and Fujiwara 2004; Srinivasan and Bhat 2005; Bradley and Vovsha 2005; Timmermans and Zhang 2009; Arentze and Timmermans 2009; Kang and Scott 2010) as well as research to understand the explicit structure of loose social networks of extended family, friends and colleagues (Carrasco and Miller, 2006; Carrasco et al, 2008; Schwanen, 2008; Axhausen, 2008). While there exists a growing stream of research in identifiable intra-household interactions and explicit inter-household interactions in travel demand modeling such as coordination of individual daily activity patterns, joint participation in activities and travel, mechanisms for allocation of maintenance activities, and activity location choice behavior as cited above, the topic of aggregate social interactions between individuals in different households at a market level in travel demand has only recently begun to attract attention.

Some examples of research questions we might like to answer related to aggregate interactions in implicit inter-household networks at a market level include spatial coordination/feasibility and social awareness/acceptance in the take-up of various transportation mode

choices. If a certain critical mass of households is willing to choose public transit in a particular region or at a particular park-and-ride location on an intercity travel trip, it can become economically viable to provide a high level of public transit service to that region or from that park-and-ride location. Being able to guarantee a high level of service might then in turn attract additional households. On the other hand, lack of sufficient transit ridership base can lead to a poor level of service, which in turn might discourage transit use by segments of the population that have other reasonable transportation mode alternatives at their disposal, which in turn could lead to further cut-backs in the level of service. Such social feedback thus can have very important implications for the prediction of (system-wide) results over the course of time. If such feedback exists, it can propel or hinder the adoption of a mode over time.

Some examples of the empirical estimation of a discrete choice model with application to transportation include Goetzke (2008), Goetzke and Andrade (2010), Goetzke and Rave (2011) and Goetzke and Weinberger (2012). Goetzke's work is particularly notable due to the attention paid on specific policy implications of social network effects in the empirical cases studied. Some explorations of the dynamical behavior of such a model with application to transportation include Fukuda and Morichi (2007), Páez and Scott (2007), Páez, Scott and Volz (2008).

Drawing on concepts from statistical physics and developments² in the mathematical theories of Markov random fields and interacting particle systems (Kendall and Snell 1980; Liggett 2005), economists over the past two decades have made notable progress with respect to both a theoretical framework for discrete binary and multinomial choice with social interactions as well as identification issues for empirical models. Our starting point in considering interdependence of various decision-makers' choices is a trio of papers by economists Aoki (1995), Brock and Durlauf (2001a) and Blume and Durlauf (2003). They introduce social interactions in binary discrete choice models by allowing a given decision-maker's choice for a particular alternative to be dependent on the overall share of decision-makers that choose that alternative. Such a specification is interesting because of the above-described inherent dynamic that could arise if the choice model were to be applied repeatedly in successive time steps with the shares of decision-makers continuously updated as a result of the choice in the previous time step. The specification namely captures feedback between decision-makers that can potentially be reinforcing over the course of time depending on parameters. In diverse literature this socio-dynamically reinforcing behavior is referred to as

² The theory of interacting particle systems was motivated by attempts to put the Ising model of ferromagnetism (1920,1925) into a general probabilistic setting and represents a natural departure from the established theory of Markov processes.

a social multiplier, a cascade, a bandwagon effect, imitation, contagion, herd behavior, etc (Manski, 1995). Brock and Durlauf (2001b) give an excellent and extensive literature review. Aoki (1995) gives a prescient example of a rigorous theoretical stochastic dynamic analysis of mean first passage time³ between local equilibria in a binary choice model with global mean-field interaction. Aoki uses the backward Chapman-Kolmogorov equation, ie. "master equation,"⁴ to keep track of probability flows in Markov processes. Brock and Durlauf (2001a) on the other hand use the principle of self-consistent beliefs to directly derive static equilibrium results, ie. each decision-maker has beliefs about the choices made by others and these beliefs are correct. They draw on laws of large numbers to consider sample average population choices. They note that their model bears mathematical resemblance to the Curie-Weiss model of ferromagnetism in statistical mechanics (Ellis 1985). Blume and Durlauf (2003) show that the equilibria in Brock and Durlauf's model can also be seen as the steady states of a mean-field differential equation. The solution path of this differential equation approximates the sample path behavior of a strategic adjustment population game model studied by Blume (2003) for large populations.

Parallel to the development of economists drawing on statistical physics and stochastic processes, physicists too have applied their tools to social and economic systems. Weidlich and Haag (1983), and Weidlich and Braun (1992) are forerunners to Aoki in the application of the master equation. Also notable is Helbing's work on the development of a statistical theory of binary social interactions (1992, 1993). Weidlich (2000) and Helbing (2010) both provide comprehensive introduction to methods of physics for modeling "sociodynamics" in general. Throughout this thesis we will use the terms "with social feedback," "sociodynamic," "with social interactions" and "with state dependence" interchangeably in the context of discrete choice models.

Since the early theoretical work by Aoki, Brock, Durlauf and Blume on binary discrete choice models, there have been a few extensions addressing the complexity of the discrete choice model as well as the complexity of the feedback effect. For example, Brock and Durlauf (2002, 2006) have extended their theoretical results on the behavior of binary logit models to multinomial logit models. Brock and Durlauf (2006) have also proposed a variant of the nested logit model with social interactions, noting that, "*There has yet to be any analysis of (such) models ... when self-consistency is imposed on the expected group choice percentages. Such an analysis should provide a number of interesting results.*" It is one of the aims of this thesis to fill this gap. The consideration

³ For details on mean first passage time and the renewal equation approach, see for example van Kampen (2007), Chapter 12.

⁴ See for example Reif (1965), Chapter 15, or van Kampen (2007), Chapter 5.

of the nested logit model with social interactions can be useful as the nested logit formulation provides a relatively simple closed form expression for individual choice probabilities that accommodates unobserved heterogeneity between choice alternatives. Also, while the behaviour over time derived in early work assumed each decision-maker to be influenced by all other decision-makers (so-called *global* interactions), Ioannides (2006) derives more general behavior for binary choice in the case where each decision-maker is influenced by only a subset of decision-makers (so-called *local* interactions).

Our main research goal in this dissertation is a systematic, step-by-step exploration of discrete choice with social and spatial interactions, where the conditions in the seminal work of Aoki (1995), Brock and Durlauf (2001a, 2002, 2006) and Blume and Durlauf (2003), such as assuming homogeneous decision makers, global interactions and laws of large numbers, are incrementally relaxed. Hereby relevant in the incremental flexibilization of the model will be attention to unobserved heterogeneity between alternatives, but ultimately also unobserved heterogeneity between decision-makers. In doing so, we will employ different techniques from different disciplinary fields, but in all cases using readily available software with the intent to facilitate the adoption of these techniques by others in further research.

After a brief review of fundamentals Part I, in Part II of the dissertation, we begin by reviewing binary choice with social interactions where the concept of correlation between alternatives is simply irrelevant. We then proceed to derive theoretical results for mean field, steady state corner solutions in parameter space for multinomial choice where correlation between alternatives can indeed be relevant, and we allow for unobserved preference heterogeneity between choice alternatives by studying the nested logit model. These analytical results are subsequently used later in the dissertation as a benchmark for empirical work. In the derivation of *theoretical* results in Part II, we draw on methodology from the mathematics of dynamical systems and bifurcation theory. In addition to analytical derivations, we also employ simple graphical approaches to solutions of (systems of) differential equations. This is the "Matlab" ([mathworks.com/products/matlab](https://www.mathworks.com/products/matlab)) part of the dissertation. Technically speaking, the latent utility U_{in} that an individual n is presumed to associate with a particular choice alternative i will be given by the expression

$$U_{in} = \beta p_i + \eta_{in} \quad (1.1)$$

where: p_i represents the global segment of the population making each choice; β is a generic utility parameter specifying the importance of the social influence; and η_{in} is an error term for alternative i for individual n . As we will see, even such a seemingly simple expression can yield complex non-linear dynamics and bifurcation behavior.

In Part III, we present an initial application of a socio-dynamic binary logit model to transportation mode choice using survey data collected by the Hague Consulting Group on intercity travel in the Netherlands over a sweep of parameter space with abstract classes of networks. Then, we present an empirical application of a socio-dynamic nested logit model to transportation mode choice using pseudo-panel microdata collected by the Municipality of Amsterdam Agency for Traffic, Transport and Infrastructure in the greater Amsterdam region with various hypothesized social-spatial networks. In the derivation of *computational* results in Part III we draw on the possibilities permitted through social simulation of multi-agent systems (MAS). This is the "Repast" (repast.sourceforge.net) part of the dissertation, using software developed at University of Chicago Argonne National Laboratory (Macal and North2010). Technically speaking, the latent utility U_{in} that an individual n is presumed to associate with a particular choice alternative i will be given by the expression

$$U_{in} = \beta p_{ig_n} + \vartheta_i' \mathbf{q}_{in} + \eta_{in} \quad (1.2)$$

where: p_{ig_n} now represents the choices made by the specific members of individual n 's reference group g_n ; q_{in} is a vector-valued function of length K_i of observable individual characteristics for decision making agent n and observable individual-specific attributes of choice alternative i for decision-making agent n (whereby the length of the vector is allowed to vary across alternatives); and ϑ_i is a corresponding vector of K_i unknown alternative-specific parameters. Discrete choice estimation results controlling these heterogeneous individual preferences are embedded in a multi-agent based simulation model in order to observe the evolution of choice behavior over time. This approach also gives us an additional advantage in the possibility to test size effects, and thus relax the assumption of large numbers, as well as test the effect of different initial conditions. Finally an extra benefit is gained in that we have the immediate possibility to observe the time-varying trajectories of the choice behavior.

In Part IV, we further explore econometric issues, estimating more complex discrete choice models. We revisit the empirical application to transportation mode choice using pseudo-panel microdata collected by the Municipality of Amsterdam Agency for Traffic, Transport and Infrastructure in the greater Amsterdam region considering the same hypothesized social-spatial networks as in Part III. In the derivation of *econometric* results in Part IV we draw on the possibilities permitted through the estimation of mixed generalized extreme value panel models. This is the "Biogeme" (biogeme.epfl.ch) part of the dissertation, using software developed at Ecole Polytechnique Fédérale de Lausanne (Bierlaire2003). Technically speaking, the latent

utility U_{in} that an individual n is presumed to associate with a particular choice alternative i will given by the expression

$$U_{in} = \gamma_{in} p_{ig_n} + \vartheta_i' \mathbf{q}_{in} + \xi_{ig_n} + \eta_{in} \quad (1.3)$$

where: γ_{in} is now unknown alternative-specific random parameter; and ξ_{ig_n} is a group-specific error term for alternative i for each individual's reference group g_n . While in general, the parameter vector ϑ_i may also be expressed with taste variation, we will be specifically interested in taste variation on the parameter for the social feedback effect, which we can express as

$$\gamma_{in} = \beta_i + \zeta_{ig_n} + \psi_{in} \quad (1.4)$$

where: β_i is an unknown alternative-specific scalar parameter; ζ_{ig_n} is the portion of an alternative-specific deviation defining the difference between individual n 's parameters and the average for the population, that is specific to each individual's reference group g_n ; ψ_{in} is the portion of an alternative-specific deviation defining the difference between individual n 's parameters and the average for the population, that varies across all decision-makers.

We conclude highlighting limitations of the present study and provide recommendations for future work. The appendices provide supplementary theoretical results and a selective overview of various modeling considerations that could be necessary to take into account in for an extension to residential choice behavior, when studying social-spatial influence with endogenous networks.

1.2 OVERVIEW

This section provides a more detailed overview of the organization of this interdisciplinary thesis. First in the remainder of Part I: *Setting the Stage*, Chapter 2 reviews the necessary notation and convention for the classic multinomial logit model and the nested logit model, and draws mathematical parallels in statistical physics.

Part II: *Mean Field Analysis*

Chapters 3, 4 and 5 review existing theory for binary discrete choice models with global interactions, and focuses on extending the complexity of the discrete choice model, making Brock and Durlauf's multinomial results precise for sociodynamic trinary multinomial choice with mean-field interaction and extending the results for the more general case of sociodynamic trinary nested logit.

In Chapter 3, we review Aoki's formulation (1995) of his groundbreaking sociodynamic binary logit model via his use of a "field variable" to represent social interactions in a jump Markov process, as

well as Aoki's rigorous derivation of equilibrium solutions via the "master equation" approach, and the condition for stability. These results will be subsequently used as a benchmark for the application in Chapter 6.

In Chapter 4, we apply techniques from the mathematics of dynamical systems and bifurcation theory to re-visit the sociodynamic multinomial logit model originally studied by Brock and Durlauf (2002, 2006). Hereby we reveal an intuitively logical but previously unnoticed hysteresis regime in midrange parameter space when there are more than two choice alternatives.

In Chapter 5, by considering the nested logit model, a possibility to account for unobserved heterogeneity between choice alternatives is allowed via the nesting of alternatives that are assumed to be correlated. The analysis of the nested logit model with global social interactions yields rich bifurcation diagrams demonstrating several major additional new emergent steady-state regimes where symmetry is broken by the scale parameter for the level of correlation between alternatives. These results will be subsequently used as a benchmark for the application in Chapter 7.

A key feature in the theoretical results, however, is the assumption that the only explanatory variable in the model is the feedback effect. While such a specification may be plausible for a fad, it is much less intuitive for transportation mode choice where other explanatory variables would be assumed to be significant, including both attributes of the alternatives such as travel time, as well as characteristics of the decision-making agents such as gender, age and income. Analytic results for such a case with, for example, several thousand distinct and individually-specified decision-makers would become prohibitive. Thus, a stylized multi-agent based simulation model is also presented which gives straightforward possibility to test more realistic empirical cases. The computational model at the same time gives a possibility to test the effect of various forms of local interactions. The multi-agent based model is docked against the analytical results for the mean-field, steady-state corner solutions in parameter space for the purpose of verification of the programming implementation of the model as well as for the purpose of guiding the interpretation of more complex cases.

Part III: *Decision in Networks*

Chapters 6 and 7 illustrate the multi-agent based simulation of a discrete choice model with global and local interactions using microdata on transportation mode choice of households in the Netherlands as a testbed, highlighting some hypothesized network interaction effects on the basis of abstract classes of networks in a sociodynamic binary choice model, and on the basis of socioeconomic peer group, spatial

proximity of residential location and spatial proximity of work location in a sociodynamic trinary nested logit model.

In Chapter 6, a binary mode choice model is explored in an example of intercity travel demand using survey data collected by the Hague Consultancy Group. Heterogeneity is easily included through different mechanisms, such as individual-specific socio-demographic characteristics of the agents and individual-specific attributes of the choice alternatives. We find that the model's characteristic phase transition is dependent on network density and clustering in examples with Erdős-Rényi graphs and Watts-Strogatz graphs, and on the importance of the estimated value of the coefficient for the local interaction variable relative to other coefficients in the model.

In Chapter 7, we depart from abstract network classes and present a framework for conceptualizing interdependence of decision-makers' choices, making a distinction between social versus spatial network interdependencies and between identifiable versus aggregate agent interdependencies. We again present an empirical application of the model to transportation mode choice, now using pseudo-panel data collected by the Municipality of Amsterdam Agency for Infrastructure, Traffic and Transport in the greater Amsterdam region. Due to the nature of data available at the time of embarking on the thesis, we consider aggregate agent interdependencies, with various hypothesized socio-geographic networks. The modeling principles are however extendable to the case of identifiable agent interactions, given suitable empirical data. We introduce unobserved heterogeneity by accounting for common unobserved attributes of the choice alternatives in the error structure. We observe that these extensions generate dramatically different dynamics and thus cannot be ignored in any empirical application. In order to understand better the contributions to emergent outcomes, we furthermore study incrementally the effects of initial conditions, network size effects, the effects of local interactions in isolated clusters and overlapping clusters, and finally the role of the utility parameters in relation to the network connectivity in emergent modal split outcomes.

An important econometric issue arises however in empirical estimation of such a feedback effect in discrete choice models using standard multinomial logit or nested logit models, in that the error terms are assumed to be identically and independently distributed across decision-makers (Ben-Akiva and Lerman, 1985). It is not obvious that this is in fact a valid assumption when we are specifically considering interdependence between decision-makers' choices. We might reason that if there is a systematic dependence of each decision-maker's choice on an explanatory variable that captures the choices of other decision-makers who are in some way related to that decision-maker, as considered in the literature referenced above, then there might be an analogous dependence in the error structure. Otherwise said, the

same unobserved effects might be likely to influence the choice made by a given decision-maker as well as the choices made by those in the decision-maker's reference group. In terms of transportation mode choice, for example accessibility measures for residents in the neighborhood could play such a role to the extent that these were unable to be directly captured through explanatory variables in the utility specification. In this case, the use of transportation mode shares of neighbors living in the same zone as an explanatory variable will be correlated with the unobserved error of the given decision-maker, which is a classic case of endogeneity. The results and coefficients of such a model are likely to be biased. To try to separate out effects, it is therefore first and foremost critically important to begin with an as well-specified model as possible, making use of relevant available explanatory variables.

Part IV: *Challenges*

Chapters 8 and 9 explore issues in the empirical estimation of discrete choice models with feedback effects, by specifically testing for correlation among agents in the error structure in a case study of mode choice to work, through the use of mixed Generalized Extreme Value family models. We conclude highlighting recommendations for future work in moving discrete choice models with aggregation social interactions in transportation into practice.

In Chapter 8, we re-visit the econometric estimation of the trinary choice model in the application studied in Chapter 7. We discuss five strategies for introducing social and spatial network interdependencies into choice models, focusing on feedback effects and on correlated effects. We apply the model strategies for nine variations on three network treatments, one of these defined by socioeconomic group and two defined spatially at different levels of geographic scale. Due to constraints in the structure of the available pseudo-panel data, we consider aggregate interactions. The modeling principles are however extendable to the case of identifiable agent interactions, given suitable empirical data.

In Chapter 9, we highlight a few of the basic theoretical concepts that we seen throughout the thesis in policy context as speculative avenues for future research. We also suggest research directions related to methodological extensions and innovative data collection.

Part V: *Appendix*

Appendix A, B, C and D present supplementary theoretical analysis for the sociodynamic binary logit and sociodynamic trinary multinomial logit with alternative specific constant bias. It also provides

a reflection on challenges related to modeling social interactions in residential choice.

In Appendix A, we re-visit the sociodynamic binary logit model originally reviewed in Chapter 3 at the outset of Part II. Now we apply the approach from the mathematics of dynamical systems and bifurcation theory, used in Chapters 4 and 5. We derive the bifurcation diagram for the sociodynamic binary logit model as well as the potential function.

In Appendix B, we consider a two-parameter bifurcation of the sociodynamic binary logit model with alternative-specific constant bias. We show that this leads to a bifurcation cusp in parameter space separating the regime where there is one unique solution and where there are two solutions, thereby making Brock and Durlauf's general results for the binary discrete choice model with social interactions precise for this case.

In Appendix C, we consider a two-parameter bifurcation of the sociodynamic trinary multinomial logit model with alternative-specific constant bias. We show that this leads to eight distinct regimes in parameter space where symmetry of the sociodynamic trinary multinomial logit model discussed in Chapter 4 is now broken by a constant bias for one of the alternatives. Seven of these regimes are qualitatively similar to those that we have seen in Chapter 5, plus there is an additional new regime that emerges. It is quite interesting to compare and contrast the analytical bifurcation curves, the solution trajectories and the general bifurcation diagrams derived in this appendix with those derived earlier in Chapter 5.

In Appendix D, we provide a selective overview of some considerations to be addressed when modeling residential location choice behavior with endogenous networks. These issues are presented and framed in terms of extracts from the literature. This is by no means intended to be comprehensive, but rather merely provocative in pointing a few specific directions for further research related to the geography of family networks.

Enjoy! Let us start our journey!