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# MEASURING UNDER- AND OVERREACTION IN EXPECTATION FORMATION

Simas Kučinskas and Florian S. Peters*

*Abstract*—We develop a framework for measuring under- and overreaction in expectation formation. The basic insight is that under- and overreaction to new information is identified (up to sign) by the impulse response function of forecast errors. Our measurement procedure yields estimates of under- and overreaction to different shocks at various horizons. In an application to inflation expectations, we find that forecasters underreact to aggregate shocks but overreact to idiosyncratic shocks. We illustrate how our approach can be used to (i) quantify the importance of different biases, (ii) estimate theoretical models, and (iii) shed light on existing empirical approaches and puzzles.

## I. Introduction

### A. Motivation

THERE is strong evidence that subjective expectations deviate from simple forms of rational expectations.[1] The precise direction of these deviations, however, is hotly debated. For instance, much research in macroeconomics focuses on models of expectations featuring *underreaction* to news.[2] At the same time, many prominent models in finance exhibit *overreaction*.[3] What is the reason for this apparent lack of consensus?

One potential answer is that measuring under- and overreaction is challenging. To measure under- and overreaction empirically, economists typically employ various reduced-form regressions. Unfortunately, these reduced-form regressions suffer from two shortcomings. First, these regressions are not tightly linked to an underlying theoretical definition of under- and overreaction. Hence, interpreting reduced-form regression estimates is not straightforward. Second, different reduced-form regressions can yield conflicting results, even when applied to the *same data*.

We illustrate these challenges in table 1. The table provides estimates of three commonly used reduced-form regressions in a sample of one-quarter-ahead inflation forecasts (see section IV for the data description). All regressions are estimated on identical samples. We perform the exercise with both individual- and consensus-level forecasts. For all regressions, positive (negative) estimates are typically taken to indicate underreaction (overreaction).[4]

For individual-level regressions, we find that depending on the reduced-form regression chosen, a researcher may conclude that expectations exhibit overreaction, underreaction, or neither. Consensus-level regressions more consistently suggest underreaction. Similarly inconsistent results have also been documented in other empirical contexts.[5]

### B. What We Do

We develop a general framework for measuring under- and overreaction in subjective expectations that is not subject to these challenges. We make two contributions.

First, we provide a theoretical framework that nests existing models of expectation formation as special cases. Within this framework, we explicitly define under- and overreaction. We say that an agent *underreacts* to a shock when forming expectations about some target variable $x_t$ if the agent perceives the impact of a shock to $x_t$ to be smaller, in absolute terms, than it actually is (vice versa for overreaction). Under- and overreaction can be quantified via a measure we term a *bias coefficient*. Intuitively, a bias coefficient measures the difference between the perceived and actual response of the target variable to some shock. Our main theoretical result is that bias coefficients are identified (up to sign) from the impulse-response function (IRF) of forecast errors. The sign of the bias coefficient is determined by the sign of the response of the target variable to that particular shock. Hence, given knowledge of this sign, under- and overreaction can be measured by estimating the IRF of forecast errors. We also show how bias coefficients can be used to decompose the observed mean-squared forecast error (MSE) into components due to (i) unconditional bias, (ii) under- or overreaction to aggregate shocks, (iii) overreaction to idiosyncratic shocks, and (iv) irreducible error (unpredictable shocks). Finally, we demonstrate that existing reduced-form approaches (i) estimate a weighted average of the underlying bias coefficients, conflating responses to various shocks, and (ii) are vulnerable to certain econometric problems. Put together, these results show that the IRF of forecast errors is

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[1]For surveys, see Pesaran and Weale (2006), Coibion, Gorodnichenko, and Kamdar (2018), and Manski (2018).

[2]Examples include sticky information (Mankiw & Reis, 2002), rational inattention (Sims, 2003; Maćkowiak & Wiederholt, 2009), imperfect information (e.g., Lucas, 1973; Woodford, 2003), and sparsity-based models of limited attention (Gabaix, 2014).

[3]Examples include diagnostic expectations (Bordalo et al., 2018), extrapolative expectations (e.g., Barberis et al., 2015), and overconfidence (e.g., Daniel et al., 1998).

[4]See section IIE for discussion and references.

[5]For example, see Bordalo et al. (2019) and Bouchaud et al. (2019). Similar results as those in table 1 have also been found by Kohlhas and Walther (2021).

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TABLE 1.—REDUCED-FORM ESTIMATES OF UNDER- AND OVERREACTION: ONE-QUARTER-AHEAD INFLATION FORECASTS

| | Individual | Consensus |
|---|---|---|
| Coibion-Gorodnichenko | −0.38 | 0.40 |
| | (0.04) | (0.40) |
| Extrapolation | 0.12 | 0.11 |
| | (0.09) | (0.08) |
| Autocorrelation | 0.23 | 0.37 |
| | (0.07) | (0.11) |
| *N* | 6030 | 209 |

Positive (negative) coefficients indicate underreaction (overreaction). Each coefficient is obtained from a separate regression, using either individual-level (first column) or consensus-level (second column) data. Forecast errors are defined as $e_{t+1} = \text{realization}_{t+1} - \text{forecast}_t$. *Coibion-Gorodnichenko:* $e_{t+1} = \alpha + \beta r_t + u_{t+1}$, where $r_t$ is the forecast revision at time $t$ (Coibion & Gorodnichenko, 2015). *Extrapolation:* $e_{t+1} = \alpha + \beta x_t + u_{t+1}$, where $x_t$ is realized inflation. *Autocorrelation:* $e_{t+1} = \alpha + \beta e_t + u_{t+1}$. Standard errors shown in parentheses; Newey-West standard errors with five lags for regressions estimated using consensus-level forecasts, and clustered standard errors (clustered by forecaster and time) for regressions estimated using individual-level forecasts. Forecaster fixed effects are included in the individual-level regressions. See section IV for the description of data and variable construction.

the key object for understanding under- and overreaction in expectation formation.

Second, we develop a set of measurement tools that can be used to consistently estimate under- and overreaction empirically. These tools reveal how expectations react to specific types of information at various horizons. We illustrate these methods in an empirical application to inflation expectations from the Survey of Professional Forecasters. We find that expectations underreact to aggregate shocks but overreact to idiosyncratic shocks. Underreaction to aggregate shocks is observed with both individual- and consensus-level forecasts. Our MSE decomposition reveals that underreaction to aggregate shocks explains around one fifth of the observed MSE at the consensus level. For individual forecasters, however, overreaction to idiosyncratic shocks is quantitatively more important, accounting for more than 40% of the variation. We further demonstrate that this feature of the data is key for understanding the diverging signs of the Coibion and Gorodnichenko (2015) estimates at the individual and consensus levels (Broer & Kohlhas, 2019; Bordalo et al., 2020). Finally, bias coefficients can be used as target moments for estimating theoretical models of expectation formation. We find that the data appear to be most consistent with the noisy-diagnostic model of Bordalo et al. (2020) and the under-extrapolation model of Ryngaert (forthcoming).

### C. Related Literature

The existing literature on expectations is voluminous, and we refer to the surveys cited in footnote 1 for comprehensive reviews. In his pioneering work, Muth (1961, pp. 321–322) already considered a model with potential under- and overreaction that is a special case of our theoretical framework. However, Muth did not study how such biases may be estimated empirically. The paper that is most closely related to our work is Coibion and Gorodnichenko (2012). Their main empirical specification is, in fact, nested in our instrumental-variables framework (see section IIIC). However, Coibion and Gorodnichenko used the IRF of forecast errors primarily as a way to study information rigidities and did not fully explore the close connection between the IRF of forecast errors and under- and overreaction—the key point of our paper. In addition, we develop tools to measure how expectations respond to idiosyncratic shocks using individual-level data, whereas Coibion and Gorodnichenko focused on consensus forecasts.

In a related recent paper, Angeletos et al. (2021) also analyze expectations via IRFs. Their focus lies in using IRFs to establish a new empirical finding—initial underreaction and delayed overreaction—that helps reconcile conflicting prior evidence, and studying its implications in a general-equilibrium model. The goal of our paper, in contrast, is to develop a comprehensive framework for measuring under- and overreaction in expectations. Although our application mostly serves to illustrate the theoretical framework, we also come to slightly different empirical conclusions, finding little evidence for delayed overreaction in our data set.

We discuss other existing approaches (including Coibion & Gorodnichenko (2015) and extrapolation regressions) in section IIE after presenting our framework.

## II. Methodology

### A. Framework

Our goal is to measure under- and overreaction in subjective expectations about some *target variable* $x_t$. Throughout we assume that any deterministic component in $x_t$ is removed, and $x_t$ is demeaned.

Suppose that the target variable follows a linear stationary process

$$x_{t+1} = \sum_{j=1}^{J} \sum_{\ell=0}^{+\infty} \alpha_{j\ell} \varepsilon_{j,t+1-\ell} \tag{1}$$

for some square-summable coefficients $\alpha_{j\ell}$ with $\alpha_{j0} = 1$. There are $J \geq 1$ *aggregate shocks* $\varepsilon_{jt}$ driving $x_t$. Each aggregate shock $\varepsilon_{jt}$ is a martingale difference sequence with unconditional variance $\sigma_j^2 > 0$. To capture the notion that these shocks are structural (Ramey, 2016), we assume that aggregate shocks are mutually uncorrelated at all leads and lags: $\mathbb{E}[\varepsilon_{jt}\varepsilon_{k,t-\ell}] = 0$ for all $j \neq k$ and all $\ell$. This specification is general, and it allows $x_t$ to be generated by any stationary VARMA process. However, the data-generating process can be generalized further to allow for state and time dependence (see section IIF).

We observe a number of agents making one-step-ahead forecasts. We assume that one-step-ahead forecasts of agent $i$ made at time $t$ are given by

$$\mathbb{F}_{it}[x_{t+1}] = b_{i0} + \sum_{j=1}^{J} \sum_{\ell=1}^{+\infty} a_{j\ell} \varepsilon_{j,t+1-\ell} + \nu_{it}, \tag{2}$$

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where $v_{it} = \sum_{\ell=1}^{\infty} \gamma_\ell \omega_{i,t+1-\ell}$ denotes the *idiosyncratic component* of expectations, and $\omega_{it}$ is an *idiosyncratic shock*. Idiosyncratic shocks are mutually uncorrelated across forecasters and orthogonal to aggregate shocks. Formally, idiosyncratic shocks satisfy $\mathbb{E}[\omega_{it}\varepsilon_{j,t-\ell}] = 0$ for all $i$, $j$, and $\ell$, and $\mathbb{E}[\omega_{it}\omega_{k,t-\ell}] = 0$ for all $i \neq k$ and all $\ell$. The coefficients $a_{j\ell}$ and $\gamma_\ell$ are assumed to be square summable, and $\omega_{it}$ is a martingale difference sequence with unconditional variance $\sigma_\omega^2 > 0$.

The specification of subjective expectations in equation (2) has the following economic features. First, we allow for a time-invariant bias $b_{i0}$. Second, the coefficients $a_{j\ell}$ capture how subjective expectations respond to aggregate shocks. If $a_{j\ell} \neq \alpha_{j\ell}$, the subjective reaction to aggregate shocks is different from the reaction of the true process. Finally, there may be idiosyncratic shocks to expectations. Such idiosyncratic shocks may capture, for example, noise in idiosyncratic signals, as in the large literature on noisy information (Angeletos & Lian, 2016). We note that the framework accommodates feedback effects from expectations to the target variable. Specifically, equations (1) and (2) can be interpreted as (possibly approximate) equilibrium solutions, with $x_{t+1}$ and $\mathbb{F}_{it}[x_{t+1}]$ written in terms of the underlying structural shocks (Ramey, 2016; Stock & Watson, 2018). Our formulation of expectations generalizes an early approach of Muth (1961). Muth studied a special case of the present model (with $J = 1$ and no idiosyncratic shocks) in which he allowed agents to under- or overreact to the most recent shock.

We emphasize that the specification in equation (2) is designed to be agnostic and nest various existing models of expectations as special cases, as we demonstrate in section IID. For that reason, we leave the coefficients $a_{j\ell}$ and $\gamma_\ell$ completely unrestricted (beyond requiring square summability). Given that currently no consensus model of expectation formation is found, that is an important feature of our framework.

We now formally define *under- and overreaction* in expectation formation.

**Definition 1.** *Suppose that* $\text{sgn}(\alpha_{j\ell}) = \text{sgn}(a_{j\ell})$.[6] *Then expectations are said to* overreact *to aggregate shock $j$ at lag $\ell$ if $|a_{j\ell}| > |\alpha_{j\ell}|$ and* underreact *if $|a_{j\ell}| < |\alpha_{j\ell}|$.*

**Definition 2.** *Expectations are said to overreact to idiosyncratic shocks at lag $\ell$ if $|\gamma_\ell| > 0$.*

When subjective expectations coincide with the true conditional expectations with probability 1 (i.e., $\mathbb{F}_{it}[x_{t+1}] = \mathbb{E}_t[x_{t+1}]$ a.s., where the expectation is conditional on all current and past values of $\varepsilon_{jt}$), we say that the expectation formation process is *unbiased*. Our definition of unbiased expectations is sometimes referred to as full-information rational expectations in the existing literature (see, e.g., Coibion, Gorodnichenko, & Kamdar, 2018).

Intuitively, we define under- and overreaction relative to full-information rational expectations. Consider aggregate shocks first. Suppose that an oil price shock increases inflation one year from now by one percentage point (pp) on average. If an agent thinks inflation will only increase by 0.5pp, the agent is underreacting according to our definition (vice versa if the agent thinks inflation will increase by 1.5pp). Idiosyncratic shocks do not enter the equation for the target variable, and so any reaction of expectations to idiosyncratic shocks constitutes overreaction.

For aggregate shocks, we define under- and overreaction only when the actual and perceived responses have the same sign. With differing signs, expectations are clearly no longer unbiased. However, such biases do not naturally correspond to basic notions of under- and overreaction.[7] For this reason, we leave these cases out of our definition. Although our empirical apparatus can be used even when the signs differ, the results need to be interpreted with more care.

To measure under- and overreaction empirically, we define a quantity we term a *bias coefficient.*

**Definition 3.** *Aggregate bias coefficients* are defined as $b_{j\ell}^{(\varepsilon)} \equiv \text{sgn}(\alpha_{j\ell})(a_{j\ell} - \alpha_{j\ell})$, whereas *idiosyncratic bias coefficients* are defined as $b_\ell^{(\omega)} \equiv |\gamma_\ell|$.

Bias coefficients measure the difference between the *actual* response of the target variable to some shock at a given lag and the *perceived* response. To account for the possibility that the response of the target variable to some aggregate shocks may be negative, we multiply the difference between the two responses by $\text{sgn}(\alpha_{j\ell})$.

The following basic lemma connects bias coefficients to under- and overreaction.

**Lemma 1.** *Bias coefficients are related to under- and overreaction as follows:*

1. *Agents overreact to idiosyncratic shocks at lag $\ell$ if and only if $b_\ell^{(\omega)} > 0$.*
2. *Assume that* $\text{sgn}(\alpha_{j\ell}) = \text{sgn}(a_{j\ell})$. *Then agents overreact to aggregate shock $j$ at lag $\ell$ if and only if $b_{j\ell}^{(\varepsilon)} > 0$ (and underreact if and only if $b_{j\ell}^{(\varepsilon)} < 0$).*

Bias coefficients reveal whether expectations under- or overreact to new information. Negative bias coefficients indicate underreaction, whereas positive bias coefficients indicate overreaction. In terms of timing, $b_{j\ell}^{(\varepsilon)}$ measures under- or overreaction of time-$t$ expectations to the shock realized at

[6] The sign function, $\text{sgn}(\alpha_{j\ell})$, is equal to $-1$ if $\alpha_{j\ell} < 0$ and 1 otherwise.

[7] For example, suppose that the true process exhibits reversals, in the sense that the true IRF is initially positive but turns negative at some point, while the perceived IRF is always positive. An economist may classify this bias as "underreaction to a reversal," but it is also possible to view such behavior as a type of overreaction.

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time $(t-\ell+1)$. For example, $b_{j1}^{(\varepsilon)}$ measures under- or overreaction to the current shock.

Combining equations (1) and (2) and using the definitions of bias coefficients, we obtain that *forecast errors* $e_{i,t+1} \equiv x_{t+1} - \mathbb{F}_{it}[x_{t+1}]$ can be written as

$$e_{i,t+1} = -b_{i0} - \sum_{j=1}^{J}\sum_{\ell=1}^{+\infty} \mathrm{sgn}(\alpha_{j\ell}) b_{j\ell}^{(\varepsilon)} \varepsilon_{j,t+1-\ell} - \sum_{\ell=1}^{+\infty} \mathrm{sgn}(\gamma_\ell) b_\ell^{(\omega)} \omega_{i,t+1-\ell} + \sum_{j=1}^{J} \varepsilon_{j,t+1}. \quad (3)$$

The right-hand side of the equation above is just the impulse-response function (IRF) of forecast errors. In particular, writing the MA($\infty$) representation of forecast errors as

$$e_{i,t+1} = -b_{i0} + \sum_{j=1}^{J}\sum_{\ell=0}^{+\infty} \delta_{j\ell}^{(\varepsilon)} \varepsilon_{j,t+1-\ell} + \sum_{\ell=1}^{+\infty} \delta_\ell^{(\omega)} \omega_{j,t+1-\ell}, \quad (4)$$

for some coefficients $\delta_{j\ell}^{(\varepsilon)}$ and $\delta_\ell^{(\omega)}$, we obtain the following result.

**Proposition 1.** *Aggregate bias coefficients are identified (up to sign) from the impulse response function of forecast errors as* $b_{j\ell}^{(\varepsilon)} = -\mathrm{sgn}(\alpha_{j\ell})\delta_{j\ell}^{(\varepsilon)}$. *Idiosyncratic bias coefficients are identified by the impulse response function of forecast errors as* $b_\ell^{(\omega)} = |\delta_\ell^{(\omega)}|$.

We wish to stress that some knowledge of the true data-generating process is necessary to conclude whether the agent under- or overreacts, namely, the sign of $\alpha_{j\ell}$. Simply knowing the IRF of forecast errors is *not* enough for measuring under- and overreaction.[8] However, in many economic settings, even if the precise value of $\alpha_{j\ell}$ is not known, we have some prior knowledge about its sign. For example, we may be uncertain about the precise quantitative impact of a monetary policy shock on future inflation, but there is a broad consensus on its sign (see Coibion, 2012 or Ramey, 2016, section 3).

### B. Quantifying the Relative Importance of Biases

Bias coefficients can be used to quantify the relative importance of different biases. For example, suppose that an economist finds that inflation expectations underreact to monetary policy shocks but overreact to idiosyncratic shocks. Which of these biases is more important quantitatively? The following proposition—which is a direct consequence of equation (3)—demonstrates that bias coefficients are a key metric for answering such questions.

[8] As discussed in section IIE, the same caveat applies to existing reduced-form approaches.

**Proposition 2.** *The mean-squared forecast error,* $MSE_i \equiv \mathbb{E}[e_{i,t+1}^2]$, *can be decomposed as*

$$MSE_i = \underbrace{b_{i,0}^2}_{\text{uncond. bias}} + \underbrace{\sum_{j=1}^{J}\sigma_j^2\left(\sum_{\ell=1}^{+\infty}[b_{j\ell}^{(\varepsilon)}]^2\right)}_{\text{under/overreaction to agg. shocks}} + \underbrace{\sigma_\omega^2\sum_{\ell=1}^{+\infty}[b_\ell^{(\omega)}]^2}_{\text{overreaction to idio. shocks}} + \underbrace{\sum_{j=1}^{J}\sigma_j^2}_{\text{irreducible error}}. \quad (5)$$

Proposition 2 decomposes mean-squared forecast error into components due to (i) unconditional bias, $b_{i,0}$, (ii) under- or overreaction to aggregate shocks, driven by aggregate bias coefficients $b_{j\ell}^{(\varepsilon)}$, (iii) overreaction to idiosyncratic shocks, driven by idiosyncratic bias coefficients $b_\ell^{(\omega)}$, and (iv) irreducible error. Intuitively, the relative importance of various biases is driven by two factors: (i) the size of the bias, as measured by the squared bias coefficients and (ii) the size of the shock, as captured by the variance of the shock. Hence, even small under- and overreaction can be important quantitatively if the relevant shock is large.

### C. Composite Bias Coefficients

We now define another type of a bias coefficient—termed *composite bias coefficient*—that can be identified from data on forecasts and realizations alone. Composite bias coefficients are determined by the univariate IRF of forecast errors to their Wold innovations. Wold innovations do not correspond to specific structural shocks, and so composite bias coefficients provide a summary measure of under- and overreaction. Although less theoretically appealing, composite bias coefficients have the advantage of being straightforward to estimate empirically.

Given the assumptions in section IIA, forecast errors are weakly stationary, and hence by Wold's Theorem we can write

$$e_{it} = -b_{i0} + \sum_{\ell=0}^{+\infty} \lambda_\ell \zeta_{t-\ell}, \quad (6)$$

for some square-summable coefficients $\lambda_\ell$ and a white noise series $\zeta_t$. Loosely speaking, the Wold innovations, $\zeta_t$, are residuals from a projection of forecast errors on all their past values. Hence, Wold innovations are distinct from aggregate and idiosyncratic shocks in equations (1) and (2). We can similarly represent the target variable as

$$x_t = \sum_{\ell=0}^{+\infty} \alpha_\ell \xi_{t-\ell},$$

with a slight abuse of notation (i.e., $\alpha_\ell$'s are not equal to the $\alpha_{j\ell}$'s in equation [1]).

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With this notation, we define *composite bias coefficients* as $b_\ell = -\text{sgn}(\alpha_\ell)\lambda_\ell$. We say that (for $\alpha_\ell \neq 0$) the agent exhibits *overall overreaction* if $b_\ell > 0$, whereas the agent exhibits *overall underreaction* if $b_\ell < 0$. (If $\alpha_\ell = 0$ and $\lambda_\ell \neq 0$, the agent is said to exhibit overall overreaction.) We jointly call aggregate and idiosyncratic bias coefficients *shock specific* to distinguish them from composite bias coefficients.

The next proposition summarizes the key properties of composite bias coefficients.

**Proposition 3.** *For the expectation-formation process to be unbiased, it is necessary (but not sufficient) for all composite bias coefficients to be zero. Composite bias coefficients,* $b_\ell = -\text{sgn}(\alpha_\ell)\lambda_\ell$*, can be calculated from shock-specific bias coefficients* $\{b_{j\ell}^{(\varepsilon)}\}$ *and* $\{b_\ell^{(\omega)}\}$ *by solving*

$$\sum_{\ell=0}^{+\infty} \lambda_\ell \lambda_{\ell+k} = \frac{g_k}{g_0} \sum_{\ell=0}^{+\infty} \lambda_\ell^2 \quad \text{for } k = 1, 2, \ldots, \tag{7}$$

*where* $g_k = g_k(\{b_{j\ell}^{(\varepsilon)}\}, \{b_\ell^{(\omega)}\})$ *is the univariate autocovariance function of forecast errors (explicit expression given in equation [19]).*

**Proof.** In the appendix. □

Given values for the aggregate and idiosyncratic bias coefficients, one can calculate the univariate autocovariance function of forecast errors. The univariate autocovariance function can then be inverted to find the implied composite bias coefficients. In this way, composite bias coefficients provide a summary measure of shock-specific bias coefficients.[9]

**Remark.** Composite bias coefficients may misleadingly suggest underreaction if expectations persistently overreact to idiosyncratic shocks (when using forecaster-level data). For this reason, we recommend estimating shock-specific bias coefficients whenever possible empirically. When that is not feasible, we suggest estimating idiosyncratic bias coefficients to check whether overall underreaction at the forecaster level may be due to persistent overreaction to idiosyncratic shocks (see section IIID).

### *D. Example*

We now provide a concrete illustration of the general framework. The model of expectations used in this section combines the most prominent deviations from full-information rational expectations proposed in the recent literature.

The target variable $x_t$ is specialized to follow an AR(1) process with persistence $\rho \in (0, 1)$:

$$x_t = \rho x_{t-1} + \varepsilon_t, \varepsilon_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma_\varepsilon^2). \tag{8}$$

[9] If the autocovariance function of forecast errors vanishes at some finite lag, then the Wold representation in equation (6) is finite, and equation (7) boils down to a system of nonlinear equations with a unique solution (see, e.g., Ansley et al., 1977).

Agents do not observe the target variable directly but obtain a noisy signal given by

$$s_{it} = x_t + \omega_{it}, \omega_{it} \sim \text{i.i.d. } \mathcal{N}(0, \sigma_\omega^2), \tag{9}$$

where $i$ denotes a particular agent. The idiosyncratic shocks $\omega_{it}$ are i.i.d. over time and across agents, and independent of the aggregate shock $\varepsilon_t$ at all leads and lags.

In addition to the information friction, we allow for two additional features of expectations. First, agents may misperceive the parameters of the data-generating process. Second, we allow for diagnosticity (Bordalo et al., 2018). Formally, for any parameter $\vartheta$ of the data-generating process in equations (8) and (9), we denote its perceived value by $\hat{\vartheta}$. To ensure stationarity, we impose $\hat{\rho} \in (0, 1)$.

We first define the *nondiagnostic component* of expectations by

$$\mathbb{F}_{it}^*[x_{t+1}] = \hat{\rho}\{\hat{G}s_{it} + (1 - \hat{G})\mathbb{F}_{i,t-1}^*[x_t]\}. \tag{10}$$

Here $\hat{G}$ is the steady-state Kalman gain calculated using the perceived parameter values. Then forecasts are given by

$$\mathbb{F}_{it}[x_{t+1}] = \mathbb{F}_{it}^*[x_{t+1}] + \theta\hat{\rho}\hat{G}\{s_{it} - \mathbb{F}_{i,t-1}^*[x_t]\}, \tag{11}$$

where $\theta \geq 0$ is the *diagnosticity parameter*. Intuitively, diagnosticity distorts expectations in the direction of the time-$t$ surprise, $s_{it} - \mathbb{F}_{i,t-1}^*[x_t]$.[10]

The setup nests the following well-known models of expectations:

- Full-information rational expectations (when $\sigma_\omega^2 = \theta = 0$ and all parameters are perceived correctly)
- Noisy information (when $\theta = 0$, $\sigma_\omega^2 > 0$, and all parameters are perceived correctly) as in Sims (2003), Woodford (2003), or Maćkowiak and Wiederholt (2009)
- Diagnostic expectations (when $\theta > 0$, $\sigma_\omega^2 = 0$, and all parameters are correctly perceived) as in Bordalo et al. (2018), or noisy diagnostic expectations as in Bordalo et al. (2020) (when $\theta > 0$, $\sigma_\omega^2 > 0$, and all parameters are correctly perceived)
- Overconfidence (when $\hat{\sigma}_\omega^2 < \sigma_\omega^2$) as in Broer and Kohlhas (2019)
- Over-extrapolation (when $\hat{\rho} > \rho$) as in Angeletos et al. (2021) and under-extrapolation (when $\hat{\rho} < \rho$) as in Ryngaert (forthcoming)
- Strategic behavior among forecasters (with $\hat{\sigma}_\omega^2 \neq \sigma_\omega^2$) as in Huo and Pedroni (2020).

[10] In appendix D.1 we show that Bordalo et al. (2020)'s proposition 1, which motivates equation (11), remains valid in the presence of misperception.

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It is straightforward to check that the model in the present section is a special case of the general framework of section IIA. The next proposition fully characterizes the behavior of bias coefficients in this model.

**Proposition 4.** *For the model in equations (8), (9), and (11) forecast errors have a univariate ARMA(2,2) representation. Aggregate bias coefficients are given by*

$$b_\ell^{(\varepsilon)} = \frac{\left(\begin{array}{l}\hat{\rho}\hat{G}[\hat{\rho}(1-\hat{G})]^\ell + \rho^\ell(\rho-\hat{\rho}) \\ \quad + \theta\hat{\rho}\hat{G}[\rho^{\ell-1}(\hat{\rho}-\rho) - \hat{\rho}^\ell\hat{G}(1-\hat{G})^{\ell-1}]\end{array}\right)}{\hat{\rho}(1-\hat{G})-\rho},$$

*and idiosyncratic bias coefficients are equal to*

$$b_1^{(\omega)} = \hat{\rho}\hat{G}(1+\theta) \text{ and}$$

$$b_\ell^{(\omega)} = \hat{\rho}^2\hat{G}[\hat{\rho}(1-\hat{G})]^{\ell-2}|1-(1+\theta)\hat{G}| \text{ for } \ell \geq 2.$$

**Proof.** In the appendix. □

*Intuition.* To obtain intuition behind proposition 4, it is helpful to start with a single deviation from full-information rational expectations. First, consider *aggregate bias coefficients*:

- *Noisy information* (information friction but no other biases). Agents are Bayesian but observe only the target variable with noise. Because of this information friction, the response of expectations to aggregate shocks is muted. Aggregate bias coefficients simplify to $b_\ell^{(\varepsilon)} = -[\rho(1-G)]^\ell$. The coefficients are always negative, correctly suggesting underreaction, and decay geometrically with persistence $\rho(1-G)$.

- *Misperceived variance* ($\hat{\sigma}_\varepsilon^2 \neq \sigma_\varepsilon^2$ and/or $\hat{\sigma}_\omega^2 \neq \sigma_\omega^2$ but no other biases). Expectations depend on perceived variances only through the perceived Kalman gain, $\hat{G}$. The model therefore makes the same qualitative predictions as the noisy-information model. Quantitatively, the amount of underreaction is determined by the perceived signal-to-noise ratio, with higher perceived signal-to-noise ratios (e.g., due to overconfidence) leading to less underreaction.

- *Diagnostic expectations* ($\theta > 0$ but no information friction, and all parameters are perceived correctly). Forecasters overreact in the direction of the time-$t$ shock, $\varepsilon_t$, generating short-lived reversals. Aggregate bias coefficients are given by $b_1^{(\varepsilon)} = \rho\theta$ and $b_\ell^{(\varepsilon)} = 0$ for $\ell \geq 2$. The first coefficient is positive, correctly indicating overreaction.

- *Misperceived persistence* ($\hat{\rho} \neq \rho$ but no information friction, and all other parameters are perceived correctly). Aggregate bias coefficients equal $b_\ell^{(\varepsilon)} = \rho^{\ell-1}(\hat{\rho}-\rho)$. Hence, if forecasters over-extrapolate ($\hat{\rho} > \rho$), the model yields overreaction, and underreaction is obtained with under-extrapolation ($\hat{\rho} < \rho$). Since bias coefficients have persistence $\rho$, the model can produce longer-lasting under- and overreaction than the noisy-information model.

With *multiple deviations* from full-information rational expectations, the model can generate both under- and overreaction to aggregate shocks. For example, consider the noisy-diagnostic model. On the one hand, diagnosticity alone would yield overreaction. On the other hand, the information friction generates underreaction. The combined effect therefore depends on the relative strength of the two forces. In fact, aggregate bias coefficients for this model simplify to $b_\ell^{(\varepsilon)} = -\rho^\ell(1-G)^{\ell-1}[1-(1+\theta)G)]$. Hence, agents underreact to aggregate shocks if and only if diagnosticity is not too strong, namely, $\theta < (1-G)/G$.[11]

With misperceived persistence and an information friction, the predictions hinge on whether forecasters under- or over-extrapolate. Intuitively, we find two sources of under- and overreaction to aggregate shocks: (i) information friction generates underreaction with persistence $\hat{\rho}(1-G)$ and (ii) under-extrapolation (over-extrapolation) generates underreaction (overreaction) with persistence $\rho$. With under-extrapolation, both factors push in the direction of underreaction, and the model unambiguously yields underreaction. With mild over-extrapolation, the information friction dominates initially, leading to initial underreaction. However, because the information friction is less persistent, over-extrapolation dominates at deeper lags, giving rise to delayed overreaction. With strong over-extrapolation, the model produces overreaction at all lags.

Now consider *idiosyncratic bias coefficients*. Since we define under- and overreaction relative to full-information rational expectations, all models predict overreaction to idiosyncratic shocks as long as an information friction is present. To better understand the term structure of such overreaction, it is again instructive to study special cases. (In all cases below, we assume that an information friction is present.)

- *Noisy information*. Idiosyncratic bias coefficients equal $b_\ell^{(\omega)} = \rho G[\rho(1-G)]^{\ell-1}$. Idiosyncratic and aggregate bias coefficients therefore both decay at the same rate.

- *Misperceived variance*. The predictions are again qualitatively the same as in the baseline noisy-information model.

[11] This condition is required by Bordalo et al. (2020) to generate a positive Coibion and Gorodnichenko (2015) coefficient at the consensus level (see their proposition 2). Hence, the noisy-diagnostic model can generate a positive consensus-level Coibion-Gorodnichenko coefficient if and only if expectations underreact to the aggregate shock.

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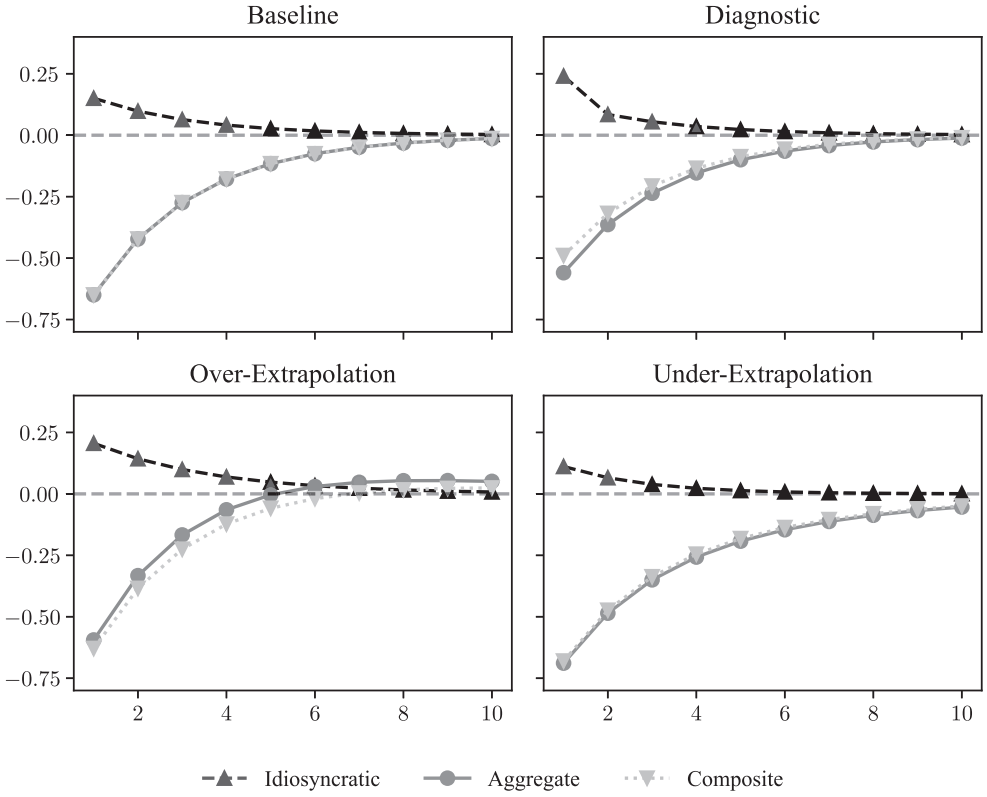
FIGURE 1.—BIAS COEFFICIENTS FOR SELECTED MODELS OF EXPECTATION FORMATION




Parameters common for all models: $\rho = 0.80$, $\sigma_\varepsilon = 1.0$ (normalization), and $\sigma_\omega = 3.0$. *Baseline*: Correctly perceived parameters and an information friction. *Diagnostic*: Correctly perceived parameters, an information friction, and diagnostic expectations ($\theta = 0.60$). *Over-Extrapolation*: Overestimated ($\hat{\rho} = 0.90$) persistence and an information friction. *Under-Extrapolation*: Underestimated persistence ($\hat{\rho} = 0.70$) and an information friction.

- *Diagnostic expectations*. Relative to the noisy-information model, initial overreaction is stronger, and it dissipates more rapidly. Idiosyncratic bias coefficients also exhibit sharper initial decay than aggregate bias coefficients.[12] This latter result is mostly driven by the specified functional form. Diagnosticity enters expectations additively, and so the initial overreaction is amplified by a factor of $(1 + \theta)$. However, diagnosticity has no "memory" in the sense that, each period, diagnostic expectations are obtained by distorting Bayesian forecasts. As a result, the initial diagnostic overreaction is "forgotten" in subsequent periods, and idiosyncratic bias coefficients mirror aggregate bias coefficients thereafter.

- *Misperceived persistence*. Idiosyncratic bias coefficients again decay geometrically but at a different rate than aggregate bias coefficients. With under-extrapolation, aggregate bias coefficients are more persistent than idiosyncratic bias coefficients. With mild over-extrapolation, aggregate bias coefficients initially decay more rapidly than idiosyncratic bias coefficients (as over-extrapolation pushes in the direction of overreaction to aggregate shocks), whereas at deeper lags aggregate bias coefficients are more persistent (as persistence approaches $\rho$).

[12] Formally, $1 - b_2^{(\omega)}/b_1^{(\omega)} > 1 - |b_1^{(\varepsilon)}|$. Note that aggregate bias coefficients measure under- or overreaction to a shock that increases forecast errors by one unit upon impact. The amount of initial overreaction may vary across models, and so we do not impose an equivalent normalization for the idiosyncratic bias coefficients. Hence, we divide $b_2^{(\omega)}$ by $b_1^{(\omega)}$ to correctly compare the rates of decay.

*Numerical example.* Figure 1 illustrates the bias coefficients for selected parameter values. In each subplot of figure 1, we plot aggregate, idiosyncratic, and composite bias coefficients. Parameter values are chosen to approximate the behavior of quarterly GDP deflator inflation in the United States, with fairly high persistence ($\rho = 0.80$). We normalize $\sigma_\varepsilon = 1.0$, and we set $\theta = 0.6$ and $\sigma_\omega = 3.0$, informed by the structural estimation results in Bordalo et al. (2020). To calculate composite bias coefficients, we use the univariate ARMA(2,2) representation of forecast errors given in the proof of proposition 4.

For the baseline model (top-left panel) that contains only an information friction, expectations underreact to aggregate shocks and overreact to idiosyncratic shocks. Aggregate bias coefficients decay geometrically. Agents respond identically to aggregate and idiosyncratic shocks in this model, and so composite bias coefficients coincide with aggregate bias coefficients. With diagnostic expectations (top-right panel), agents exhibit stronger overreaction to idiosyncratic shocks and weaker underreaction to aggregate shocks than in the

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baseline model. Under the chosen parameter values, diagnosticity is dominated by the information friction (technically, $\theta < (1-\hat{G})/\hat{G}$), and hence expectations underreact to the aggregate shock. Relative to aggregate bias coefficients, composite bias coefficients are attenuated toward zero due to the faster decay of idiosyncratic bias coefficients.

Next, we plot bias coefficients for two models with misperceived persistence. With over-extrapolation (bottom-left panel), similar to diagnostic expectations, the model generates stronger overreaction to idiosyncratic shocks and milder underreaction to aggregate shocks. Importantly, the model exhibits delayed overreaction to aggregate shocks. An interesting feature that differentiates this model from the other cases we consider is that composite bias coefficients are larger (in absolute value) than aggregate bias coefficients. This result is driven by overreaction to idiosyncratic shocks being more persistent in this model.

Finally, in the model with under-extrapolation (bottom-right panel), as in Ryngaert (forthcoming), expectations exhibit weaker overreaction to idiosyncratic shocks and stronger underreaction to aggregate shocks relative to the baseline. In this model, similarly to diagnostic expectations, composite bias coefficients are also attenuated toward zero.

### *E. Existing Methods*

We believe that we are the first to provide a general framework for measuring under- and overreaction in subjective expectations. However, past empirical work has used various reduced-form regressions to obtain proxies for under- and overreaction. The theory of section IIA is useful for understanding under what conditions these empirical approaches yield valid measures of under- and overreaction.

*Relation to bias coefficients.* We consider three commonly used regressions:

$$e_{i,t+1} = \alpha_i + \beta_{\text{CG}} r_{it} + u_{it}, \tag{12}$$

$$e_{i,t+1} = \alpha_i + \beta_{\text{extr.}} x_t + u_{it}, \tag{13}$$

$$e_{i,t+1} = \alpha_i + \beta_{\text{auto.}} e_{it} + u_{it}. \tag{14}$$

Here $r_{it} \equiv \mathbb{F}_{it}[x_{t+1}] - \mathbb{F}_{i,t-1}[x_{t+1}]$ denotes *forecast revisions*. For all these regressions, positive estimates of $\beta$ are taken to indicate underreaction, and negative estimates are interpreted as suggesting overreaction.

The first approach in equation (12) regresses forecast errors on lagged forecast revisions. This specification was first proposed by Nordhaus (1987). Coibion and Gorodnichenko (2015) derived the structural interpretation of the coefficient as a measure of information rigidities when applied to consensus-level forecasts. However, subsequent literature has also applied the same specification to forecaster-level data (Broer & Kohlhas, 2019; Bordalo et al., 2020). The second specification, equation (13), regresses forecast errors on the lagged realization of the target variable; we call this regression an *extrapolation regression*. This regression has been used, for example, by Kohlhas and Walther (2021) and Barrero (2022). Finally, equation (14) regresses forecast errors on their lagged values; we term this regression an *autocorrelation regression*. This regression has been employed by Abarbanell and Bernard (1992) and Ropele et al. (2020), among others.

In online appendix A, we provide a formal analysis of these reduced-form regressions. We show that these regressions estimate a weighted average of the underlying bias coefficients (multiplied by the sign of the true IRF of the target variable), with weights that differ across regressions. We note the following economic implications of our formal results:

- Multiple combinations of bias coefficients can yield the same reduced-form estimates. Hence, reduced-form regressions are strictly less informative than bias coefficients.
- Reduced-form regressions conflate under- and overreaction to different shocks and across time horizons.
- For extrapolation and autocorrelation regressions, a researcher may estimate a negative slope coefficient (suggesting overreaction) even when expectations only exhibit underreaction. The reason is that these reduced-form regressions estimate a weighted average of bias coefficients *multiplied by the sign* of the true impulse response. This result echoes proposition 1, further underscoring the fact that the sign of the true impulse response is essential for identifying under- and overreaction.

*Econometric issues.* The discussion above highlights the theoretical limitations of reduced-form approaches. In addition, as we now discuss, the extrapolation and Coibion-Gorodnichenko regressions are vulnerable to econometric biases in certain settings.

The *extrapolation regression* is subject to a well-known finite-sample bias. Suppose that $x_t$ follows a univariate AR(1) process with persistence $\rho \in (-1, 1)$, and consider the extrapolation regression with consensus forecasts. Furthermore, assume that the true value of the extrapolation coefficient is zero. Then, using the results in Nelson and Kim (1993), we have that $\mathbb{E}[\hat{\beta}_{\text{extr.}}] = -T^{-1}(1+3\rho) + \mathcal{O}_p(T^{-2})$, where $\hat{\beta}_{\text{extr.}}$ denotes the OLS estimate, and $T$ is the sample size. Hence, the expected value of the extrapolation coefficient is negative even when subjective expectations exhibit no under- or overreaction. The reason for this finite-sample bias is that the regression error term is correlated with future values of $x_t$, violating strict exogeneity.

The *Coibion-Gorodnichenko regression* may be sensitive to measurement error in expectations. As Coibion and Gorodnichenko (2015) noted, measurement error biases the CG coefficient downwards since time-$t$ forecasts enter both the left- and right-hand side of the regression, with

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opposite signs. Measurement error is unlikely to be problematic for consensus-level forecasts, as in Coibion and Gorodnichenko (2015). However, measurement error can be substantial in forecaster-level data. In such settings, the Coibion-Gorodnichenko regression can misleadingly suggest overreaction even when expectations exhibit no overreaction.

### F. Extensions

In online appendix B, we show that the framework in section IIA can be extended naturally to account for (i) multiple-step-ahead forecasts, (ii) heterogeneity in expectation formation, and (iii) state and time dependence. We also discuss (iv) how our framework may be used to measure deviations from Bayesian updating. Overall, we conclude that the framework in section IIA is applicable in a wide range of empirical settings.

## III. Estimation

Measuring bias coefficients empirically boils down to estimating the IRF of forecast errors. The best way to estimate the IRF depends on the application at hand, and many existing approaches may be employed. Although our paper has nothing new to say about the estimation of IRFs, we outline how bias coefficients can be estimated in the most empirically relevant scenarios.

In the discussion below, we assume that the IRFs are estimated using local projections (Jordà, 2005). We have many ways to estimate an IRF, but practical benefits can be found to using local projections:[13] They can be easily used with both consensus- and forecaster-level data. In addition, it is straightforward to adjust standard errors for clustering when individual forecasts are used (Keane & Runkle, 1990) and allow for state and time dependence.

### A. Known Shocks

If the true shocks are observed, bias coefficients can be estimated by directly regressing forecast errors, $e_{i,t+1} = x_{t+1} - \mathbb{F}_{it}[x_{t+1}]$, on past shocks:

$$e_{i,t+\ell} = \alpha_i + \beta \varepsilon_{jt} + u_{i,t+\ell} \tag{15}$$

for some shock $j$ at horizon $\ell$. The estimated bias coefficient is then $\hat{b}^{(\varepsilon)}_{j\ell} = -\text{sgn}(\alpha_{j\ell})\hat{\beta}$, where $\alpha_{j\ell}$ is the impulse response of the target variable. This approach is likely to be especially relevant for experimental work; see He and Kucinskas (2024) for an example.

[13] For example, we may fit a vector autoregression. As shown by Plagborg-Møller and Wolf (2021) local projections and VARs estimate the same IRF. However, the two methods implicitly make different choices on how to trade off bias and variance in finite samples. Hence, the choice between the two options necessarily depends on the application.

### B. Composite Bias Coefficients

To estimate composite bias coefficients, only data on forecasts and realizations are required. With local projections, we estimate the univariate IRF of forecast errors by

$$e_{i,t+\ell} = \alpha_i + \beta e_{it} + \sum_{k=1}^{K-1} \pi_k e_{i,t-k} + u_{i,t+\ell} \tag{16}$$

for each horizon $\ell = 1, \ldots, L$. Here $K$ is the number of lagged forecast errors included in the regression as controls. The estimated composite bias coefficient is then $\hat{b}_\ell = -\text{sgn}(\alpha_\ell)\hat{\beta}$, where $\alpha_\ell$'s are the coefficients of the univariate IRF of $x_t$ (see section IIC).

### C. Aggregate Bias Coefficients

Underlying aggregate shocks are generally unobserved. Hence, we typically cannot regress forecast errors on past shocks as in section IIIA. However, in some empirical settings we may have an *instrument* for these shocks. In this case we can use the *method of external instruments* (Stock, 2008; Mertens & Ravn, 2013; Stock & Watson, 2018) to estimate aggregate bias coefficients consistently. For example, we may wish to learn how the agent reacts to a monetary policy shock (say, $\varepsilon_{1t}$) when forming expectations about inflation ($x_t$). In this case the instrument, $z_t$, may be a monetary surprise as measured by Romer and Romer (2004).

With a valid instrument at hand, aggregate bias coefficients with respect to a particular aggregate shock (say, $\varepsilon_{1t}$) can be estimated by the instrumental-variables (IV) regression

$$e_{i,t+\ell} = \alpha_i + \beta w_t + u_{i,t+\ell}, \tag{17}$$

for each horizon $\ell = 1, \ldots, L$, with $w_t$ instrumented by $z_t$. In the example above, $w_t$ could be the Federal Funds Rate. The aggregate bias coefficients w.r.t. $\varepsilon_{1t}$ are then given by $\hat{b}_{1\ell} = -\text{sgn}(\alpha_{1\ell})\hat{\beta}$. As before, $\alpha_{1\ell}$ denotes the impulse response of the target variable w.r.t. $\varepsilon_{1t}$.

We now show that, under certain assumptions, equation (17) consistently estimates aggregate bias coefficients. Our setup and proof closely follows the ideas in Stock and Watson (2018).

Let $w_t = \sum_{j=1}^{J} \sum_{\ell=0}^{+\infty} \phi_{j\ell} \varepsilon_{j,t-\ell}$ be another observed variable that will be instrumented by $z_t$, where $\phi_{j\ell}$ are square-summable coefficients. Further, let the external instrument, $z_t$, satisfy

**Assumption 1.** *External instrument $z_t$ satisfies the following conditions:*

- *Relevance:* $\mathbb{E}[z_t \varepsilon_{1t}] = \psi \neq 0$
- *Contemporaneous exogeneity:* $\mathbb{E}[z_t \varepsilon_{jt}] = 0$ *for* $j \neq 1$

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- *Lead-lag exogeneity:* $\mathbb{E}[z_t \varepsilon_{j,t+s}] = 0$ *for all* $j$ *and* $s \neq 0$
- *Normalization:* $\phi_{10} = 1$.

The first three assumptions are familiar from standard IV estimation. The last assumption is necessary because, given the shocks are unobservable, the scale of $\varepsilon_{1t}$ is indeterminate. In the example above, the normalization corresponds to a choice of units for the monetary policy shock under which the monetary policy shock equals the change in the Federal Funds Rate. In cases in which the normalization is not appropriate, the external instrument identifies the aggregate bias coefficients up to the scale parameter $(1/\phi_{10})$. Finally, to simplify notation, we assume that $z_t$ has been demeaned so that $\mathbb{E}[z_t] = 0$.

Stock and Watson (2018) show that under assumption 1 the instrument $z_t$ identifies the IRF of $x_t$ to a shock in $\varepsilon_{1t}$. Our key result is that the same instrument also identifies aggregate bias coefficients. Hence, the informational requirements for measuring under- and overreaction in expectations are the same as estimating the IRF of the target variable.

**Proposition 5.** *Suppose that an instrument* $z_t$ *satisfies assumption 1. Then aggregate bias coefficients w.r.t.* $\varepsilon_{1t}$, $b_{1\ell}^{(\varepsilon)}$, *are consistently estimated by* $\hat{b}_{1\ell} = -\text{sgn}(\alpha_{1\ell})\hat{\beta}$ *where* $\hat{\beta}$ *is the instrumental-variables estimate of* $\beta$ *in equation (17).*

**Proof.** In the appendix. □

An attractive feature of the IV procedure is its robustness to measurement error (Mertens & Ravn, 2013). This feature of our proposed approach differentiates us from the foundational work of Coibion and Gorodnichenko (2012). In their main specification, Coibion and Gorodnichenko regress forecast errors on past shocks, as in section IIIA. However, Coibion and Gorodnichenko do not observe these shocks directly but instead estimate them from observational data. As argued by Mertens and Ravn (2013) and Stock and Watson (2018) such estimates of shocks likely contain some measurement error and therefore attenuate Hence, estimates of under- and overreaction obtained by the original Coibion-Gorodnichenko procedure may be attenuated toward zero. In contrast, our IV approach is robust to structural shocks being measured with error.[14]

### D. Idiosyncratic Bias Coefficients

We now consider the estimation of idiosyncratic bias coefficients. We show that similar ideas as in section IIIC can be applied to estimate idiosyncratic bias coefficients. Specifically, under certain assumptions, the deviation from the consensus forecast can be used as an instrument in a method-of-external-instruments regression.

[14]The reduced-form equation associated with equation (17) is $e_{i,t+\ell} = \beta_{i0} + \beta z_t + u_{i,t+\ell}$, where $z_t$ denotes the instrument, which is precisely the original Coibion-Gorodnichenko specification.

Let $d_{it} \equiv \mathbb{F}_{it}[x_{t+1}] - \bar{\mathbb{F}}_t[x_{t+1}]$ denote the *deviation from the consensus forecast*, where $\bar{\mathbb{F}}_t[x_{t+1}]$ denotes the consensus (average) forecast. To estimate idiosyncratic bias coefficients, we use the following IV regression:

$$e_{i,t+\ell} = \alpha_i + \beta \mathbb{F}_{it}[x_{t+1}] + \sum_{k=1}^{K} \pi_k d_{i,t-k} + u_{i,t+\ell}, \tag{18}$$

for each horizon $\ell = 1, \ldots, L$, instrumenting $\mathbb{F}_{it}[x_{t+1}]$ with $d_{it}$, and lagged deviations, $d_{i,t-k}$, serving as their own instruments. The idiosyncratic bias coefficients are estimated by $\hat{b}_\ell^{(\omega)} = |\hat{\beta}|$.

The following proposition provides conditions under which the regression in equation (18) consistently estimates idiosyncratic bias coefficients (up to a scaling factor).

**Proposition 6.** *Consider the DGP in equations (1) and (2). Suppose furthermore that (i) the idiosyncratic component of expectations,* $\nu_{it}$, *follows a stationary* AR($K$) *process with* $\nu_{it} = \sum_{k=1}^{K} \phi_k \nu_{i,t-k} + \gamma_1 \omega_{it}$*; (ii) there is a continuum of forecasters* $i \in [0, 1]$*; and (iii)* $\gamma_1 \neq 0$. *Then the probability limit of the IV estimate of* $\beta$ *in equation (18) is given by* $\text{plim}_{T\to\infty} \hat{\beta} = -\gamma_1^{-1} \text{sgn}(\gamma_\ell) b_\ell^{(\omega)}$.

**Proof.** In the appendix. □

The takeaway is that, under the conditions of proposition 6, idiosyncratic bias coefficients can be estimated consistently via equation (18). Since idiosyncratic shocks are unobserved, bias coefficients are identified only up to a scaling factor of $\gamma_1^{-1}$. To ensure that controlling for a finite number of past deviations allows for consistent estimation, we make the technical assumption that the idiosyncratic component follows an AR($K$) process. For analytical tractability, we also assume that there is a continuum of forecasters, as is standard in the existing literature (see, e.g., Coibion & Gorodnichenko, 2015; Angeletos et al., 2021; Kohlhas & Walther, 2021).

The key identifying assumption is that forecasters react to aggregate shocks homogeneously. This assumption is implicit in the notation of the $a_{j\ell}$ coefficients in equation (2), which do not depend on $i$. Intuitively, given the specification in equation (2), expectations may deviate from the consensus forecast only because of the idiosyncratic component. The idiosyncratic component may be persistent, and so individual expectations at time $t$ may deviate from the average forecast because of idiosyncratic shocks realized at any time up to time $t$. However, conditional on past deviations, the only reason why an individual forecaster may deviate from the consensus is the realization of a contemporaneous idiosyncratic shock. If the homogeneity assumption is violated, then factor-based methods for estimating idiosyncratic shocks may be used (Juodis & Kucinskas, 2023), as we illustrate in our empirical application below.

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### E. Measurement Error

In this section we assumed that expectations are observed without measurement error. Online appendix B.5 relaxes this assumption. We show that classical measurement error attenuates composite and idiosyncratic bias coefficients toward zero; measurement error does not affect the estimated aggregate bias coefficients. However, and differently from some existing reduced-form approaches, such bias is conservative in the sense that measurement error makes it *less* likely that we find under- or overreaction.

## IV. Empirical Application: Inflation Expectations

We now apply our methodology to inflation forecasts from the Survey of Professional Forecasters (SPF). This data set has been used extensively in prior work and provides a natural testing ground for our method. Data construction is explained in online appendix C.1, where we also provide summary statistics.

We focus on one-quarter-ahead GDP deflator inflation forecasts (1968Q4 to 2021Q2). Both consensus- and individual-level forecasts are considered. We use median forecasts for the consensus and first-release data for realized inflation to be consistent with prior work.

### A. Composite Bias Coefficients

The estimated composite bias coefficients are shown in figure 2. As discussed in section IIC, composite bias coefficients provide a summary measure of under- or overreaction and do not disentangle the responses to specific shocks. We estimate the univariate IRF of forecast errors using local projections.[15]

Figure 2a plots the bias coefficients estimated using consensus forecasts. We observe statistically significant negative bias coefficients for lag 1 ($p = 0.004$),[16] lag 3 ($p = 0.010$), and lag 4 ($p = 0.032$). The evidence therefore suggests that participants in the SPF underreact to information that arrived up to one year ago.[17] The magnitude of underreaction is substantial. The point estimates indicate that a positive $1\sigma$ shock to inflation in the current quarter leads the forecasters to underpredict inflation by $0.22\sigma$ four quarters from now.

Figure 2b performs the same exercise using individual forecasts. To account for the fact that all respondents are forecasting the same variable, and forecast errors may be correlated over time for a given respondent, we cluster standard errors by both forecaster and quarter. The pattern of the estimated bias coefficients is very similar to that obtained using consensus forecasts. The key difference is that the coefficients appear to be attenuated toward zero. As a result, bias coefficients at lags 3 and 4 are no longer statistically significant at the 5% level ($p = 0.101$ and $p = 0.189$, respectively). In the online appendix (figure A6), we show that the estimates are very similar when forecast errors are winsorized at 5% and 95% levels.[18]

### B. Shock-Specific Bias Coefficients

First, we consider the effects of monetary policy on inflation expectations. We use Romer and Romer (2004) monetary surprises, as extended to a longer sample period by Wieland and Yang (2020). Because the Romer-Romer surprises are not available for the full sample period, our sample is reduced and stops at 2007Q4. Our specification regresses forecast errors on lagged forecast errors and lagged values of the Federal Funds Rate, instrumenting the Federal Funds Rate with the Romer-Romer monetary surprise.[19]

Figure 2c indicates statistically significant underreaction to monetary policy shocks that arrived eight to sixteen quarters ago. The timing of underreaction is consistent with the effects of monetary policy shocks on realized inflation, which typically kicks in around two years after the initial shock (see figure A8 in the online appendix and figures 4 and 5 in Romer & Romer, 2004). The magnitude of underreaction is again substantial. Bias coefficients are equal to roughly half of the actual response of inflation (see figure A8 in the online appendix). Overall, these results are consistent with both our findings for the composite bias coefficients above, as well as those of Coibion and Gorodnichenko (2012) who document underreaction to other aggregate shocks, including technology, news, and oil price shocks.

Next, we investigate how agents react to idiosyncratic shocks. By "idiosyncratic," we refer to expectational shocks that are specific to each individual forecaster. These shocks could reflect noise in forecaster-specific signals as well as behavioral phenomena such as overconfidence or diagnosticity. As shown in section IIID, idiosyncratic bias coefficients can be identified using deviations from the consensus as an instrument.

The results of this exercise are shown in figure 2d. Forecasters overreact to idiosyncratic shocks. However, overreaction is short lived, lasting up to one quarter, and fairly

[15] As shown in the online appendix (figure A5), the univariate IRF of inflation is positive at all the relevant lags. Hence, we obtain the bias coefficients by multiplying the estimated univariate IRF of forecast errors by $(-1)$.

[16] At the time of the survey, participants know the advance estimate of inflation in the previous quarter but not inflation in the current quarter (Federal Reserve Bank of Philadelphia, 2017, p. 21). As a result, interpretation of the bias coefficient at the first lag requires some care (Keane & Runkle, 1990).

[17] In the online appendix (figure A7), we show that a similar pattern is also observed for two-, three-, and four-quarter-ahead forecasts.

[18] In unreported results, we also find that the estimates remain largely unchanged when, following Zarnowitz and Lambros (1987) and Lahiri and Sheng (2010), we restrict the sample to "regular respondents" who participate in at least thirty survey waves.

[19] Monetary policy shocks are deflationary, and so we do not multiply the estimated IRF of forecast errors by $(-1)$.

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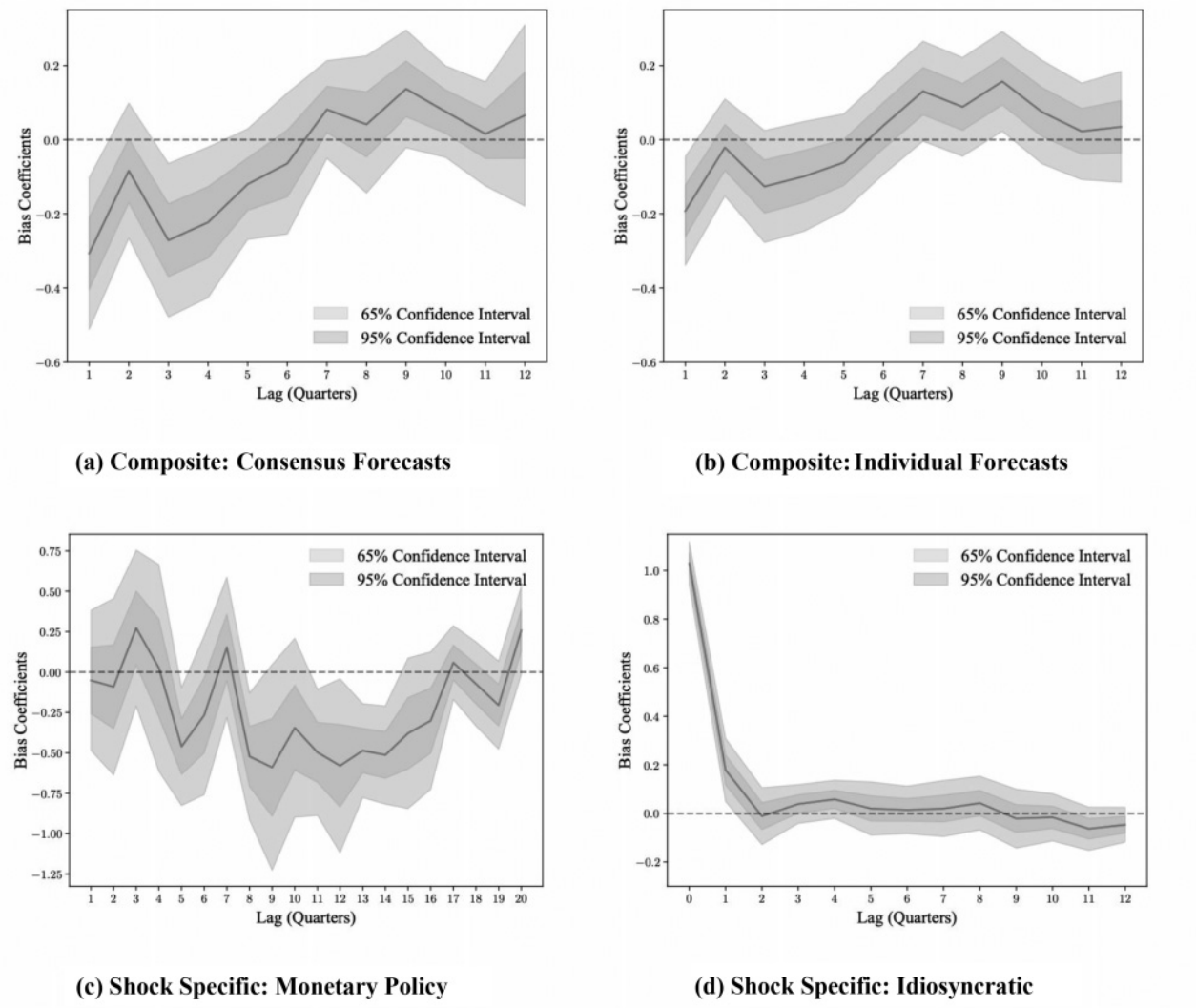
FIGURE 2.—ESTIMATED BIAS COEFFICIENTS FOR ONE-QUARTER-AHEAD INFLATION FORECASTS




*Top-left panel*: Composite bias coefficients estimated using consensus (median) forecasts; Newey-West standard errors with $\max\{5, \ell - 1\}$ lags used to calculate the confidence intervals. *Top-right panel*: Composite bias coefficients estimated using individual-level data (with forecaster fixed effects); standard errors clustered by both forecaster and quarter. *Bottom-left panel*: Bias coefficients w.r.t. monetary policy shocks. Consensus-level forecasts are used in the IV procedure of section IIIC, instrumenting the Federal Funds Rate with the Romer and Romer (2004) monetary policy shock. Newey-West standard errors with $\max\{5, \ell - 1\}$ lags used to calculate the confidence intervals. *Bottom-right panel*: Bias coefficients w.r.t. idiosyncratic shocks. Individual-level forecasts are used in the IV procedure of section IIID, instrumenting $\mathbb{F}_{it}[x_{t+1}]$ with the deviation from consensus, $\mathbb{F}_{it}[x_{t+1}] - \bar{\mathbb{F}}_t[x_{t+1}]$; standard errors clustered by both forecaster and quarter.

small quantitatively.[20] The key identifying assumption is that forecasters react to aggregate shocks homogeneously. In the online appendix (figure A9), we investigate the robustness of this assumption. We estimate idiosyncratic shocks using the procedure proposed by Juodis and Kucinskas (2023), which allows for heterogeneous responses to aggregate shocks. We find a similar pattern for the idiosyncratic bias coefficients, with short-lived overreaction. Finally, also in the online appendix (figure A10), we demonstrate that very similar results are obtained when all variables are winsorized at 5% and 95% levels. This latter result is reassuring since some reduced-form measures of under- and overreaction can be sensitive to outliers (Kohlhas & Walther, 2021).

[20]Note that, by construction, the initial idiosyncratic bias coefficient is statistically indistinguishable from one.

### C. *Taking Stock: Lessons for Theory*

Overall we find that inflation expectations persistently underreact to aggregate shocks and exhibit short-lived overreaction to idiosyncratic shocks. Hence, the data appear inconsistent with full-information rational expectations (FIRE).[21] We can also reject certain deviations from FIRE that predict overreaction to aggregate shocks, such as the basic diagnostic-expectations model (Bordalo et al., 2018), or a model in which forecasters overestimate the persistence of inflation but otherwise exhibit no biases (e.g., Gabaix, 2019,

[21]Underreaction to aggregate shocks may be generated by full-information rational expectations if forecasters have an asymmetric loss function, and the target variable exhibits time-varying volatility (Patton & Timmermann, 2007). However, such models also predict nonzero average forecast errors, something that we do not observe in our data set.

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section 2.3.5). More generally, the fact that we find underreaction to aggregate shocks suggests that such features of expectations (diagnosticity and/or over-extrapolation) cannot be too large relative to other biases, as otherwise the models would counterfactually predict overreaction to aggregate shocks. The data are also (trivially) inconsistent with models in which expectations exhibit no cross-sectional dispersion, as such models cannot generate the type of overreaction to idiosyncratic shocks that we observe.

The basic noisy-information model (Sims, 2003; Woodford, 2003; Maćkowiak & Wiederholt, 2009) also appears inconsistent with the data but for a more subtle reason. The baseline noisy-information model predicts that underreaction to aggregate shocks and overreaction to idiosyncratic shocks should exhibit the same amount of persistence (see section IID). Our estimates, however, suggest that underreaction to aggregate shocks is substantially more persistent. Versions of the noisy-information model with overconfidence (Broer & Kohlhas, 2019) or certain forms of strategic behavior among forecasters (Huo & Pedroni, 2020) make the same counterfactual prediction.

We also document that composite bias coefficients estimated using forecaster-level data are attenuated toward zero (relative to consensus-level estimates). Two possible reasons may explain this finding. If expectations are measured with error, estimated composite bias coefficients are biased toward zero (see section IIIE). Alternatively, the pattern may be generated by nonrational expectations. As shown in section IID, such attenuation is predicted by the noisy-diagnostic model of Bordalo et al. (2020), and the noisy-underextrapolation model of Ryngaert (forthcoming). The observed pattern, however, is inconsistent with the basic noisy-information model (with or without overconfidence) or the noisy-overextrapolation model of Angeletos et al. (2021). At the very least, these models would require some amount of measurement error to be consistent with the data.

To conclude, the noisy-diagnostic model of Bordalo et al. (2020) and the noisy-underextrapolation model of Ryngaert (forthcoming) appear to fit the data best. These models both successfully predict underreaction to aggregate shocks, overreaction to idiosyncratic shocks, and attenuation of individual-level composite bias coefficients. However, the predictions of the two models are similar, and the data do not allow us to clearly differentiate between them. This identification problem resurfaces in the quantitative model estimation exercise that we turn to in section IVD.

### D. Bias Coefficients: Further Applications and Results

We now illustrate how bias coefficients may be further applied to (i) quantify the relative importance of various biases, (ii) estimate theoretical models of expectation formation, and (iii) study state and time dependence in under- and overreaction.

TABLE 2.—DECOMPOSITION OF MEAN-SQUARED FORECAST ERROR AT THE CONSENSUS AND INDIVIDUAL LEVELS

| | | Bias | | | |
|---|---|---|---|---|---|
| | MSE | Uncond. | Aggreg. shocks | Idiosyn. shocks | Irred. error |
| Consensus | 2.05 | 0.04% | 21.89% | | 78.07% |
| Individual | 3.86 | 4.47% | 11.64% | 42.39% | 41.50% |

The first column gives the mean-squared forecast error, and the other columns provide the decomposition of mean-squared error to components due to (i) unconditional bias, (ii) under- or overreaction to aggregate shocks, (iii) overreaction to idiosyncratic shocks, and (iv) irreducible error.

*How quantitatively important is under- and overreaction?* In this section we use the estimated bias coefficients to quantify the importance of under- and overreaction. To do so, we empirically implement the mean-squared forecast error decomposition in proposition 2. The full details behind the empirical implementation are provided in online appendix C.2.

The results of this decomposition are shown in table 2. At the consensus level, approximately 20% of forecast errors are due to underreaction to aggregate shocks. Unconditional bias contributes very little to consensus forecast errors. At the individual level, overreaction to idiosyncratic shocks plays a dominant role, contributing approximately 40% of the mean-squared forecasts error. This result is driven by the fact that the mean-squared forecast error is almost twice as large at the individual level—which the decomposition attributes to idiosyncratic shocks. Unconditional bias again plays only a minor role, explaining less than 5% of the mean-squared forecast error. We conclude that although underreaction to aggregate shocks is quantitatively important at the consensus level, overreaction to idiosyncratic shocks appears to be the key driver of forecast errors for individual forecasters.

The key assumption required for the decomposition in this section to be valid is that forecasters respond to aggregate shocks homogeneously. However, this assumption can be relaxed using factor-based methods proposed by Juodis and Kucinskas (2023). Reassuringly, Juodis and Kucinskas (2023) obtain similar results, with a leading role for idiosyncratic shocks at the individual level.

*Implications for Coibion-Gorodnichenko estimates:* Idiosyncratic shocks appear to be a key driver of forecast errors at the forecaster level. This feature of expectations is key for understanding the diverging estimates of the Coibion and Gorodnichenko (2015, CG for short) regression at the consensus and individual levels (Bordalo et al., 2020). In online appendix C.3, we theoretically decompose individual-level CG estimates into components due to (i) under- or overreaction to aggregate shocks and (ii) overreaction to idiosyncratic shocks. Our empirical results show that the individual-level CG regression puts more than 80% weight on the idiosyncratic shocks, explaining its negative sign.

*Estimating theoretical models of expectation formation.* Bias coefficients encapsulate how expectations respond to

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TABLE 3.—MODEL ESTIMATES OBTAINED BY MATCHING EMPIRICAL BIAS COEFFICIENTS

| Model | $\theta$ | $\hat{\rho}$ | $G$ | $SSR_{agg.}$ | $SSR_{idio.}$ | $SSR_{tot.}$ |
|---|---|---|---|---|---|---|
| Noisy diagnostic | 2.12 | 0.82 | 0.19 | 0.10 | 0.02 | 0.13 |
| | (2.25) | | (0.26) | (0.04) | (0.04) | (0.07) |
| Noisy misperception | 0.00 | 0.68 | 0.76 | 0.12 | 0.02 | 0.14 |
| | | (0.07) | (0.05) | (0.05) | (0.04) | (0.07) |
| Misperception | 0.00 | 0.60 | 1.00 | 0.13 | 0.05 | 0.18 |
| | | (0.07) | | (0.06) | (0.04) | (0.07) |
| Baseline noisy | 0.00 | 0.82 | 0.68 | 0.17 | 0.03 | 0.20 |
| | | | (0.06) | (0.07) | (0.05) | (0.10) |
| Full-info rational | 0.00 | 0.82 | 1.00 | 0.28 | 0.05 | 0.33 |
| | | | | (0.11) | (0.04) | (0.11) |

The total sum of squared residuals ($SSR_{tot.}$) is calculated as $SSR_{tot.} = SSR_{agg.} + SSR_{idio.}$, where $SSR_{agg.} = \sum_{\ell=1}^{12}[\hat{b}_\ell^{agg.} - b_\ell^{agg.}(\vartheta^*)]^2$ and $SSR_{idio.} = \sum_{\ell=1}^{12}[\hat{b}_\ell^{idio.} - b_\ell^{idio.}(\vartheta^*)]^2$ denote the sum of squared residuals due to aggregate and idiosyncratic bias coefficients, respectively. Empirically estimated bias coefficients are denoted by $\hat{b}_\ell$, and theoretical bias coefficients are given by $b_\ell(\vartheta^*)$; $\vartheta^* = (\theta, \hat{\rho}, G)^\top$ denotes the parameter vector that minimizes the total sum of squared residuals. The algebraic expressions for the theoretical bias coefficients are given in proposition 4. Realized inflation is assumed to follow an AR(1) process with an estimated persistence of 0.82. Parameters: $\theta$ (diagnosticity), $\hat{\rho}$ (perceived persistence), and $G$ (Kalman gain). Standard errors calculated using the moving-blocks bootstrap with 2,000 replications shown in the parentheses.

new information. For this reason, bias coefficients are natural "target moments" for estimating models of expectation formation. For example, we may wish to know how large information frictions need to be to generate the observed level of underreaction in figure 2. In this section we use our empirical estimates to perform such an exercise.

For our exercise, we estimate the models introduced in section IID by targeting empirical bias coefficients. The target moments are (i) composite bias coefficients (estimated using consensus-level forecasts; figure 2a) and (ii) idiosyncratic bias coefficients (figure 2d). We estimate standard errors using a moving-blocks bootstrap adapted to the present panel-data setting; see online appendix C.5 for details.

The results are shown in table 3. The noisy-diagnostic and noisy-misperception models fit the data best. Although the noisy-diagnostic model has a slightly better empirical fit, the difference is quantitatively small and not statistically significant (s.e.$[\Delta SSR] = 0.015$). This outcome is not an artifact of our methodology but, rather, a reflection of the fact that the two models make very similar predictions for under- and overreaction. For the noisy-misperception model, we find that a perceived persistence of 0.68 (s.e. $= 0.07$) best fits the estimated bias coefficients. Hence, our results provide support for the noisy under-extrapolation model of Ryngaert (forthcoming) and stand in contrast to the predictions of Angeletos et al. (2021). The amount of under-extrapolation we estimate (around 17%) is quantitatively similar to the empirical results in Ryngaert (forthcoming).

Models with only a single deviation from full-information rational expectations struggle to fit the estimated bias coefficients. The model with misperceived persistence performs best in this class of models. Again, we find that under-extrapolation is necessary to fit the evidence. The basic noisy information model predicts that aggregate and idiosyncratic bias coefficients should decay at the same rate. Our empirical results, in contrast, indicate that idiosyncratic bias coefficients decay much more rapidly. The baseline noisy-information model struggles to fit this pattern, explaining its poorer performance. However, the difference in goodness of fit between the misperception and baseline-noisy models is small quantitatively and not statistically significant (s.e.$(\Delta SSR) = 0.067$). For the baseline noisy-information model, we estimate a Kalman gain of 0.68 (s.e. $= 0.06$), close to Coibion and Gorodnichenko (2015) who find $\hat{G} = 0.54$ (s.e. $= 0.10$).[22]

With the exception of the noisy-diagnostic model, parameter estimates appear to be fairly precise. The estimates of the noisy-diagnostic model, however, have relatively wide standard errors. The reason for the lack of statistical precision is that the noisy-diagnostic model is poorly identified by the empirical bias coefficients. Intuitively, the estimation procedure struggles to distinguish between (i) high diagnosticity with strong information frictions (high $\theta$, low $G$) and (ii) low diagnosticity with weak information frictions (low $\theta$, high $G$). Higher diagnosticity means stronger reaction to aggregate shocks, and greater information frictions imply weaker reaction, and so the effects partially cancel out, and multiple values of $\theta$ and $G$ yield a similar level of fit.

*State and time dependence.* In online appendix C.4, we illustrate how our methodology can be used to study whether biases in expectation formation exhibit time and/or state dependence. Allowing for state dependence is important empirically. For example, firms report being more likely to seek new information after receiving bad news about the economy (Coibion, Gorodnichenko, & Kumar, 2018). We modify the estimation procedure in section IIC to allow composite bias coefficients to vary with the state of the economy. Overall, we find evidence of state dependence, with stronger underreaction in high-inflation periods and milder underreaction during recessions. However, persistence of realized inflation also varies over time, with greater persistence in high-inflation periods and lower persistence during recessions, suggesting that forecasters underreact to time and state dependence in realized inflation.

[22] We use the Delta Method to calculate the standard error for $\hat{\lambda}$ from the estimates provided by Coibion and Gorodnichenko, that is, s.e.$(\hat{\lambda}) =$ s.e.$(\hat{\beta})/(1+\hat{\beta})^2$.

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## V. Conclusions

Expectations play a key role in economics and finance. Although ample evidence suggests that expectations deviate from simple forms of rational expectations, we find a lack of consensus about the precise direction of these deviations. The lack of consensus stems in part from challenges in measuring under- and overreaction empirically. Existing reduced-form regressions are not tightly linked to theoretical definitions of under- and overreaction. In addition, different reduced-form regressions can yield conflicting results, even when applied to the same data.

This paper provides a measurement framework that is not subject to these challenges. The key insight is that under- and overreaction to news is identified (up to sign) by the IRF of forecast errors. The estimated bias coefficients measure under- and overreaction to different shocks at various lags. In our empirical application to inflation expectations, we find that forecasters exhibit persistent underreaction to aggregate shocks and short-lived overreaction to idiosyncratic shocks.

### A. Suggestions for Empirical Research

Existing reduced-form approaches, although easy to use, conflate the response of expectations to various shocks at different horizons. Furthermore, some reduced-form approaches are subject to econometric problems due to measurement error and small-sample bias. We therefore recommend estimating shock-specific bias coefficients whenever empirically feasible. When data limitations preclude the use of our method, we recommend estimating multiple reduced-form regressions (e.g., both autocorrelation and Coibion-Gorodnichenko regressions). If some reduced-form regressions indicate underreaction, while others suggest overreaction, care should be taken when interpreting the estimates.

### B. Limitations

We finish by listing certain limitations of our work: (i) Estimating shock-specific bias coefficients requires a valid instrument. Obtaining a valid instrument may not always be feasible. (ii) Composite bias coefficients are straightforward to estimate empirically. However, they subsume responses to multiple shocks, similarly to existing reduced-form approaches. (iii) Our approach requires estimating multiple bias coefficients. This requirement comes at the cost of potentially lower statistical precision. In data sets with a limited time-series dimension, estimating a single reduced-form coefficient can be preferable. (iv) In our calibration exercise (section IVD), we found that models with multiple deviations from full-information rational expectations are sometimes poorly identified. (v) Without data on actions, idiosyncratic shocks to expectations are observationally equivalent to measurement error (see, e.g., Juodis & Kucinskas, 2023). Our empirical finding of short-lived overreaction to idiosyncratic shocks may be explained by serially correlated measurement error. (vi) We define under- and overreaction relative to full-information rational expectations. Expectations may deviate from this benchmark for both rational and behavioral reasons. Distinguishing between competing explanations requires additional structure. Addressing these limitations may be a fruitful avenue for future research.

## A. Proofs

*Proof of Proposition 3*

For necessity, suppose that some composite bias coefficients are not zero but (to obtain a contradiction) all shock-specific bias coefficients and the time-invariant bias term are zero. In that case, equation (3) implies that $e_{it} = \sum_{j=1}^{J} \varepsilon_{jt}$. But then all composite bias coefficients are zero, a contradiction. To see that zero composite bias coefficients are not sufficient for the expectation formation process to be unbiased, suppose that $x_t = 0$ with probability 1 but $\mathbb{F}_{it}[x_{t+1}] = \omega_{it}$ where $\omega_{it}$ is a nondegenerate i.i.d. idiosyncratic shock. Then the agent overreacts to $\omega_{it}$ at the individual level, but the composite bias coefficients are all 0 since $\omega_{it}$ is an i.i.d. shock.

To see that equation (7) holds, note that

$$g_0 = \mathbb{V}\text{ar}[e_{it}] = \mathbb{V}\text{ar}[\zeta_t] \sum_{\ell=0}^{+\infty} \lambda_\ell^2,$$

$$g_k = \mathbb{C}\text{ov}[e_{it}, e_{i,t-k}] = \mathbb{V}\text{ar}[\zeta_t] \sum_{\ell=0}^{+\infty} \lambda_\ell \lambda_{\ell+k}.$$

Dividing the second equation by the first and rearranging yields equation (7). The explicit expressions for $g_k$ in terms of shock-specific bias coefficients are

$$g_0 = \sum_{j=1}^{J} \sigma_j^2 \left(1 + \sum_{\ell=1}^{+\infty} [b_{j\ell}^{(\varepsilon)}]^2\right) + \sigma_\omega^2 \sum_{\ell=1}^{+\infty} [b_\ell^{(\omega)}]^2,$$

$$g_k = \sum_{j=1}^{J} \sigma_j^2 \Bigg( - \text{sgn}(\alpha_{jk}) b_{jk}^{(\varepsilon)} + \sum_{\ell=1}^{+\infty} \text{sgn}(\alpha_{j\ell}) \text{sgn}(\alpha_{j,\ell+k}) b_{j\ell}^{(\varepsilon)} b_{j,\ell+k}^{(\varepsilon)} \Bigg) + \sigma_\omega^2 \sum_{\ell=1}^{\infty} \text{sgn}(\gamma_\ell) \text{sgn}(\gamma_{\ell+k}) b_\ell^{(\omega)} b_{\ell+k}^{(\omega)} \text{ for } k \geq 1. \quad (19)$$

*Proof of Proposition 4*

From standard results, the steady-state Kalman gain is given by

$$\hat{G} = \frac{\hat{\Sigma}}{\hat{\Sigma} + \hat{\sigma}_\omega^2}, \text{ where}$$

$$\hat{\Sigma} = \frac{-(1 - \hat{\rho}^2)\hat{\sigma}_\omega^2 + \hat{\sigma}_\varepsilon^2 + \sqrt{[(1 - \hat{\rho}^2)\hat{\sigma}_\omega^2 - \hat{\sigma}_\varepsilon^2]^2 + 4\hat{\sigma}_\varepsilon^2 \hat{\sigma}_\omega^2}}{2};$$

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see, for example, (Hamilton, 1994, chapter 13). Our notation here follows Bordalo et al. (2020).

*ARMA representation.* We proceed in two steps. In the first step, we derive the ARMA representation of forecast errors for the nondiagnostic component of expectations. In the second step, we add diagnostic expectations.

Let $e^*_{it} \equiv x_t - \mathbb{F}^*_{i,t-1}[x_t]$ denote the forecast error due to the nondiagnostic component of expectations. Use equations (8), (9), and (10) to deduce that

$$e^*_{i,t+1} = \hat{\rho}(1-\hat{G})e^*_{it} + (\rho - \hat{\rho})x_t + \varepsilon_{t+1} - \hat{\rho}\hat{G}\omega_{it}.$$

Now define $v_{i,t+1} \equiv e^*_{i,t+1} - \hat{\rho}(1-\hat{G})e^*_{it}$. We have that $v_{i,t+1} = (\rho - \hat{\rho})x_t + \varepsilon_{t+1} - \hat{\rho}\hat{G}\omega_{it}$. Subtracting $\rho v_{it}$ from both sides we obtain

$$v_{i,t+1} - \rho v_{it} = \varepsilon_{t+1} - \hat{\rho}\varepsilon_t - \hat{\rho}\hat{G}(\omega_{it} - \rho\omega_{i,t-1}).$$

The right-hand side of the equation is the sum of two independent MA(1) processes, hence, also an MA(1) process. Therefore, we can write

$$v_{i,t+1} = \rho v_{it} + \eta_{i,t+1} + \kappa\eta_{it},\ \eta_{it} \sim \text{i.i.d. } \mathcal{N}(0, \sigma^2_\eta)$$

for some parameters $\kappa$ and $\sigma^2_\eta$ that can be calculated given $\{\sigma^2_\varepsilon, \sigma^2_\omega, \rho, \hat{\rho}, \hat{G}\}$ (Hamilton, 1994, pp. 106–107). We now use the definition of $v_{it}$ to find

$$\begin{aligned} e^*_{i,t+1} = {} & [\hat{\rho}(1-\hat{G}) + \rho]e^*_{i,t} - \hat{\rho}\rho(1-\hat{G})e^*_{i,t-1} \\ & + \eta_{i,t+1} + \kappa\eta_{i,t}. \end{aligned} \tag{20}$$

Hence, $e^*_{it}$ follows an ARMA(2, 1) process.

We now derive the ARMA representation for the full model. First, note that from equation (11) forecast errors can be written as

$$e_{i,t+1} = e^*_{i,t+1} - \hat{\rho}\theta\hat{G}e^*_{it} - \hat{\rho}\theta\hat{G}\omega_{it}.$$

Using this relationship and equation (20), we obtain

$$\begin{aligned} & e_{i,t+1} - [\hat{\rho}(1-\hat{G}) + \rho]e_{it} + \hat{\rho}\rho(1-\hat{G})e_{i,t-1} \\ & \quad = \varepsilon_{t+1} - \hat{\rho}(1+\theta\hat{G})\varepsilon_t + \hat{\rho}^2\theta\hat{G}\varepsilon_{t-1} \\ & \qquad - \hat{\rho}\hat{G}(1+\theta)\left[\omega_{it} - \frac{\rho + \theta(\hat{\rho}+\rho)}{1+\theta}\omega_{i,t-1}\right. \\ & \qquad\qquad \left. + \frac{\theta\hat{\rho}\rho}{1+\theta}\omega_{i,t-2}\right]. \end{aligned} \tag{21}$$

The right-hand side is the sum of two independent MA(2) processes. Hence, it can be written as an MA(2) process with some parameters. Hence, forecast errors follow an ARMA(2,2) process.

*Shock-specific bias coefficients.* We now calculate idiosyncratic and aggregate bias coefficients. We again first obtain the MA($\infty$) representation of the nondiagnostic component of expectations. Since $\hat{\rho}(1-\hat{G}) \in (0, 1)$, by repeated substitution we have that

$$\mathbb{F}^*_{it}[x_{t+1}] = \hat{\rho}\hat{G}\sum_{\ell=0}^{+\infty}[\hat{\rho}(1-\hat{G})]^\ell s_{i,t-\ell}.$$

The signal can be written as $s_{i,t-\ell} = \sum_{m=0}^{+\infty}\rho^m\varepsilon_{t-\ell-m} + \omega_{i,t-\ell}$, and hence

$$\begin{aligned} \mathbb{F}^*_{it}[x_{t+1}] = {} & \hat{\rho}\hat{G}\sum_{\ell=0}^{+\infty}\sum_{m=0}^{+\infty}[\hat{\rho}(1-\hat{G})]^\ell\rho^m\varepsilon_{t-\ell-m} \\ & + \hat{\rho}\hat{G}\sum_{\ell=0}^{+\infty}[\hat{\rho}(1-\hat{G})]^\ell\omega_{i,t-\ell} \\ = {} & \hat{\rho}\hat{G}\sum_{\ell=0}^{+\infty}\left(\frac{[\hat{\rho}(1-\hat{G})]^{\ell+1} - \rho^{\ell+1}}{\hat{\rho}(1-\hat{G}) - \rho}\right)\varepsilon_{t-\ell} \\ & + \hat{\rho}\hat{G}\sum_{\ell=0}^{+\infty}[\hat{\rho}(1-\hat{G})]^\ell\omega_{i,t-\ell}. \end{aligned} \tag{22}$$

The MA($\infty$) representation of the surprise, $s_{it} - \mathbb{F}^*_{i,t-1}[x_t]$, is therefore

$$\begin{aligned} s_{it} - \mathbb{F}^*_{i,t-1}[x_t] = {} & \sum_{\ell=0}^{\infty}\left(\frac{\rho^\ell(\hat{\rho}-\rho) - \hat{\rho}^{\ell+1}\hat{G}(1-\hat{G})^\ell}{\hat{\rho}(1-\hat{G}) - \rho}\right)\varepsilon_{t-\ell} \\ & + \omega_{it} - \hat{\rho}\hat{G}\sum_{\ell=1}^{+\infty}[\hat{\rho}(1-\hat{G})]^{\ell-1}\omega_{i,t-\ell}. \end{aligned} \tag{23}$$

Combining the results in equations (22) and (23), the process for the target variable in equation (8), and using the definitions of bias coefficients yields the expressions stated in the proposition.

*Proof of Proposition 5*

Calculate that

$$\begin{aligned} \mathbb{C}\text{ov}[z_t, e_{i,t+\ell}] &= -\text{sgn}(\alpha_{1\ell})b^{(\varepsilon)}_{1\ell}\psi \text{ for } \ell \geq 1, \\ \mathbb{C}\text{ov}[z_t, w_t] &= \underbrace{\phi_{10}\psi = \psi}_{\text{normalization } \phi_{10}=1}. \end{aligned}$$

As a result, since $\psi \neq 0$ by assumption 1, we have that

$$\frac{\mathbb{C}\text{ov}[z_t, e_{i,t+\ell}]}{\mathbb{C}\text{ov}[z_t, w_t]} = -\text{sgn}(\alpha_{1\ell})b^{(\varepsilon)}_{1\ell}.$$

But the instrumental-variables estimator of $\beta$ is given by

$$\begin{aligned} \hat{\beta} = {} & \left(\frac{1}{T-\ell}\sum_{t=1}^{T-\ell}(z_t - \bar{z}_t)(w_t - \bar{w}_t)\right)^{-1} \\ & \times \left(\frac{1}{T-\ell}\sum_{t=1}^{T-\ell}(z_t - \bar{z}_t)(e_{i,t+\ell} - \bar{e}_{i,t+\ell})\right) \\ & \xrightarrow{p} \frac{\mathbb{C}\text{ov}[z_t, e_{i,t+\ell}]}{\mathbb{C}\text{ov}[z_t, w_t]}, \end{aligned}$$

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under standard regularity conditions under which a Law of Large Number applies to the terms in the parentheses. Since $\text{sgn}(x)^2 = 1$, the result follows.

*Proof of Proposition 6*

The consensus forecast is given by

$$\mathbb{F}_t[x_{t+1}] = \int_0^1 \mathbb{F}_{it}[x_{t+1}]di = \bar{b}_0 + \sum_{j=1}^{J}\sum_{\ell=1}^{+\infty} a_{j\ell}\varepsilon_{j,t+1-\ell},$$

where $\bar{b}_0 = \int_0^1 b_{i0}di$. Hence, $d_{it} = (b_{i0} - \bar{b}_0) + \nu_{it}$.

For some variable $w_{it}$, let $w_{it}^{\perp}$ denote the residuals from a projection of $w_{it}$ on $\{d_{i,t-1}, \ldots, d_{i,t-K}\}$ and a constant. That is, $w_{it}^{\perp} = \hat{u}_{it}$ in a regression estimated by least squares:

$$w_{it} = \delta_0 + \sum_{k=1}^{K}\delta_k d_{i,t-k} + u_{it}.$$

Because $\nu_{it}$ follows an AR($K$) process, and $d_{it} = (b_{i0} - \bar{b}_0) + \nu_{it}$, we deduce that $d_{it}^{\perp} = \gamma_1\omega_{it}^{\perp}$.

From equation (3) and definition of $e_{i,t+\ell}^{\perp}$,

$$e_{i,t+\ell}^{\perp} = -\sum_{j=1}^{J}\sum_{s=1}^{+\infty}\text{sgn}(\alpha_{j\ell})b_{js}^{(\varepsilon)}\varepsilon_{j,t+\ell-s}^{\perp} - d_{i,t+\ell-1}^{\perp} + \sum_{j=1}^{J}\varepsilon_{j,t+\ell}^{\perp}.$$

Given that $\varepsilon_{j\ell}$ is uncorrelated with $\omega_{it}$ at all leads and lags, $\mathbb{E}[\varepsilon_{j,t+\ell-s}^{\perp}d_{it}^{\perp}] = 0$ for all $s$ by standard properties of orthogonal projections. Because the idiosyncratic component, $\nu_{it}$, follows a stationary AR($K$) process, we can write

$$d_{i,t+\ell} = (b_{i0} - \bar{b}_0) + \gamma_1\omega_{i,t+\ell} + \gamma_2\omega_{i,t+\ell-1} + \cdots + \gamma_{\ell+1}\omega_{it} + \sum_{k=1}^{K}\psi_k\nu_{i,t-k}$$

for some coefficients $\psi_k$ (see, e.g., Hamilton, 1994, section 1.2). Therefore, $\mathbb{E}[d_{i,t+\ell}^{\perp}d_{it}^{\perp}] = \gamma_1\gamma_{\ell+1}\mathbb{E}[(\omega_{it}^{\perp})^2]$ and $\mathbb{E}[e_{i,t+\ell}^{\perp}d_{it}^{\perp}] = -\gamma_1\gamma_{\ell}\mathbb{E}[(\omega_{it}^{\perp})^2]$ for $\ell \geq 1$.

Let $f_{it} \equiv \mathbb{F}_{it}[x_{t+1}]$. Following the same steps as above, we calculate that $\mathbb{E}[f_{it}^{\perp}d_{it}^{\perp}] = \gamma_1^2\mathbb{E}[(\omega_{it}^{\perp})^2]$. But from the Frisch-Waugh-Lovell Theorem for IV estimation (e.g., Giles, 1984, theorem 1), we know that

$$\hat{\beta} = \left(\frac{1}{T}\sum_{t=P+1}^{T-h} d_{it}^{\perp}f_{it}^{\perp}\right)^{-1}\left(\frac{1}{T}\sum_{t=P+1}^{T-h} d_{it}^{\perp}e_{i,t+\ell}^{\perp}\right) \xrightarrow{p} \frac{\mathbb{E}[e_{i,t+\ell}^{\perp}d_{it}^{\perp}]}{\mathbb{E}[f_{it}^{\perp}d_{it}^{\perp}]} = -\gamma_1^{-1}\gamma_{\ell},$$

and the result follows.

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