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NOTE

A Comment on "A Note on 'Distance Transformations in Digital Images'"

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Recently, in a communication by Vossepoel [2] on optimal coefficients a , b (and c) for distance transformations [1], distance weights are assigned according to the following schemes:

b	a	b	$2b$	c	$2a$	c	$2b$
a	0	a	c	b	a	b	c
b	a	b	$2a$	a	0	a	$2a$
			c	b	a	b	c
			$2b$	c	$2a$	c	$2b$

The optimal parameter values for distance transformations are equal to the optimal parameter values for linear length estimates of type $l = a \cdot \Delta x + (b - a) \cdot \Delta y$. Therefore, to compute optimal values for (a, b) it is sufficient to evaluate the integrals for line length estimation [3, 4]:

$$\text{Bias}(l) = \int \int_D (l - l_e(r, \varphi)) \cdot p(r, \varphi) \, dr \, d\varphi \tag{1}$$

and

$$\text{MSE}(l) = \int \int_D (l - l_e(r, \varphi))^2 \cdot p(r, \varphi) \, dr \, d\varphi \tag{2}$$

where l_e is the Euclidean distance, $p(r, \varphi)$ is the probability of straight lines parameterized as indicated in Fig. 1, and D is the ensemble of all continuous straight lines in 2-dimensional space.

For the estimate $l = n \cdot (a + (b - a) \cdot \tan(\varphi))$, $0 < \varphi < \pi/4$, in [2] the author effectively evaluates the following set of integrals:

$$\text{Bias}/n = \int_0^{\pi/4} (a + (b - a) \cdot \tan(\varphi) - \sec(\varphi)) \, d\varphi \tag{3}$$

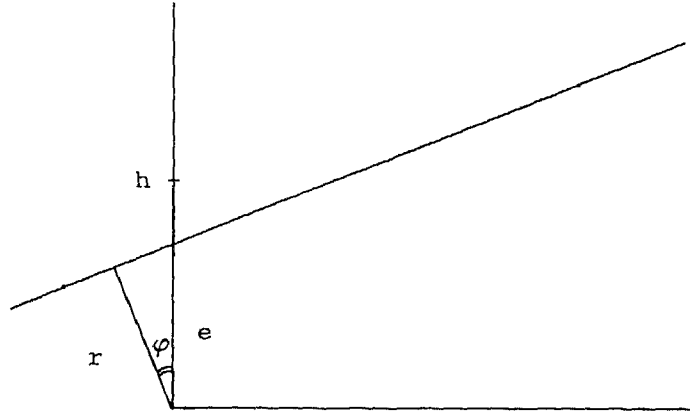


FIG. 1. The normal representation of a straight line.

and

$$\text{MSE}/n^2 = \int_0^{\pi/4} (a + (b - a) \cdot \tan(\varphi) - \sec(\varphi))^2 d\varphi, \quad (4)$$

integrating over two columns of the grid separated by n pixels (see Eqs. (2)–(8) in [2]). To establish isotropy in the ensemble D of straight lines, a uniform distribution is assumed for φ . Minimizing (4) under the constraint that (3) equals zero, the author then finds the following optimal values for a and b :

$$a = \frac{(4 - \pi) \cdot \ln(1 + \sqrt{2}) - (\sqrt{2} - 1) \cdot \ln 4}{\pi \cdot (1 - \pi/4) - \ln^2 2} = 0.9413 \quad (5)$$

and

$$b = \frac{(4 - \pi - \ln 4) \cdot \ln(1 + \sqrt{2}) + (\sqrt{2} - 1) \cdot (\pi - \ln 4)}{\pi \cdot (1 - \pi/4) - \ln^2 2} = 1.3513.$$

Similarly for the (5×5) distance transformation, are found:

$$a = 0.9813, \quad b = 1.4031, \quad \text{and} \quad c = 2.1953. \quad (6)$$

Both sets of values (5) and (6), however, are *not* optimal values for *isotropic* distance transformations. To appreciate this consider in Fig. 1 the entrance height e in the first column. Implicitly, when integrating over n columns in [2] it is assumed that $p(e, \varphi) = 4/\pi$ is uniform. The isotropic distribution of random lines, however, is equivalent with a uniform distribution of $p(r, \varphi)$. Hence $p(e, \varphi)$ is not uniform, but $p(e, \varphi) = p(r, \varphi) \cdot |J|$ is, where J is the Jacobian of the coordinate transition from (r, φ) to (e, φ) . From Fig. 1 it is seen that $r = e \cdot \cos(\varphi)$, hence $|J| = |d(r, \varphi)/d(e, \varphi)| = \cos(\varphi)$ and $p(e, \varphi) = \sqrt{2} \cdot \cos(\varphi)$ over the first octant.

In [3, 4] the integrals (1) and (2) are calculated for a variety of length estimators (and distance transforms), using the proper $p(r, \varphi)$ to establish isotropy. For the two parameter length estimator the following optimal coefficients are found:

$$a = \frac{\pi/2 \cdot \{\sqrt{2} \cdot \ln(\sqrt{2} + 1) - 1\} - (\sqrt{2} - 1) \cdot \ln 2}{2 \cdot \ln(\sqrt{2} + 1) - 4 \cdot (\sqrt{2} - 1)} = 0.9445$$

$$b = \frac{\pi/\sqrt{2} \cdot \{\ln(\sqrt{2} + 1) - 1\} + (2 - \sqrt{2}) \cdot \ln 2}{2 \cdot \ln(\sqrt{2} + 1) - 4 \cdot (\sqrt{2} - 1)} = 1.3459$$
(7)

and for the three parameter estimates:

$$a = 0.980, \quad b = 1.406, \quad c = 2.204. \quad (8)$$

The differences between (7) and (5) and between (8) and (6) are small. Since it is claimed in [2] that (5) and (6) are optimal values they have a theoretical meaning. For practical purposes the conclusions in [2] still hold.

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