



**UvA-DARE (Digital Academic Repository)**

**A comment on "A note on distance transformation in digital images"**

Beckers, A.L.D.; Smeulders, A.W.M.

*Published in:*  
Computer Vision, Graphics, and Image Processing

*DOI:*  
[10.1016/0734-189X\(89\)90056-X](https://doi.org/10.1016/0734-189X(89)90056-X)

[Link to publication](#)

*Citation for published version (APA):*  
Beckers, A. L. D., & Smeulders, A. W. M. (1989). A comment on "A note on distance transformation in digital images". *Computer Vision, Graphics, and Image Processing*, 47, 89-91. DOI: 10.1016/0734-189X(89)90056-X

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <http://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

NOTE

A Comment on "A Note on 'Distance Transformations in Digital Images'"

A. L. D. BECKERS AND A. W. M. SMEULDERS

*Department of Medical Informatics, Erasmus University Rotterdam,  
Dr. Molewaterplein 50, 3000 DR Rotterdam, The Netherlands*

Received October 13, 1988; accepted November 28, 1988

Recently, in a communication by Vossepoel [2] on optimal coefficients  $a$ ,  $b$  (and  $c$ ) for distance transformations [1], distance weights are assigned according to the following schemes:

$b$	$a$	$b$	$2b$	$c$	$2a$	$c$	$2b$
$a$	$0$	$a$	$c$	$b$	$a$	$b$	$c$
$b$	$a$	$b$	$2a$	$a$	$0$	$a$	$2a$
			$c$	$b$	$a$	$b$	$c$
			$2b$	$c$	$2a$	$c$	$2b$

The optimal parameter values for distance transformations are equal to the optimal parameter values for linear length estimates of type  $l = a \cdot \Delta x + (b - a) \cdot \Delta y$ . Therefore, to compute optimal values for  $(a, b)$  it is sufficient to evaluate the integrals for line length estimation [3, 4]:

$$\text{Bias}(l) = \int \int_D (l - l_e(r, \varphi)) \cdot p(r, \varphi) \, dr \, d\varphi \quad (1)$$

and

$$\text{MSE}(l) = \int \int_D (l - l_e(r, \varphi))^2 \cdot p(r, \varphi) \, dr \, d\varphi \quad (2)$$

where  $l_e$  is the Euclidean distance,  $p(r, \varphi)$  is the probability of straight lines parameterized as indicated in Fig. 1, and  $D$  is the ensemble of all continuous straight lines in 2-dimensional space.

For the estimate  $l = n \cdot (a + (b - a) \cdot \tan(\varphi))$ ,  $0 < \varphi < \pi/4$ , in [2] the author effectively evaluates the following set of integrals:

$$\text{Bias}/n = \int_0^{\pi/4} (a + (b - a) \cdot \tan(\varphi) - \sec(\varphi)) \, d\varphi \quad (3)$$

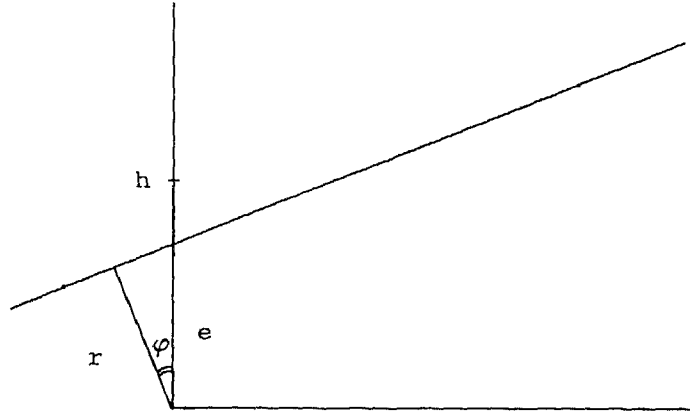


FIG. 1. The normal representation of a straight line.

and

$$\text{MSE}/n^2 = \int_0^{\pi/4} (a + (b - a) \cdot \tan(\varphi) - \sec(\varphi))^2 d\varphi, \quad (4)$$

integrating over two columns of the grid separated by  $n$  pixels (see Eqs. (2)–(8) in [2]). To establish isotropy in the ensemble  $D$  of straight lines, a uniform distribution is assumed for  $\varphi$ . Minimizing (4) under the constraint that (3) equals zero, the author then finds the following optimal values for  $a$  and  $b$ :

$$a = \frac{(4 - \pi) \cdot \ln(1 + \sqrt{2}) - (\sqrt{2} - 1) \cdot \ln 4}{\pi \cdot (1 - \pi/4) - \ln^2 2} = 0.9413 \quad (5)$$

and

$$b = \frac{(4 - \pi - \ln 4) \cdot \ln(1 + \sqrt{2}) + (\sqrt{2} - 1) \cdot (\pi - \ln 4)}{\pi \cdot (1 - \pi/4) - \ln^2 2} = 1.3513.$$

Similarly for the  $(5 \times 5)$  distance transformation, are found:

$$a = 0.9813, \quad b = 1.4031, \quad \text{and} \quad c = 2.1953. \quad (6)$$

Both sets of values (5) and (6), however, are *not* optimal values for *isotropic* distance transformations. To appreciate this consider in Fig. 1 the entrance height  $e$  in the first column. Implicitly, when integrating over  $n$  columns in [2] it is assumed that  $p(e, \varphi) = 4/\pi$  is uniform. The isotropic distribution of random lines, however, is equivalent with a uniform distribution of  $p(r, \varphi)$ . Hence  $p(e, \varphi)$  is not uniform, but  $p(e, \varphi) = p(r, \varphi) \cdot |J|$  is, where  $J$  is the Jacobian of the coordinate transition from  $(r, \varphi)$  to  $(e, \varphi)$ . From Fig. 1 it is seen that  $r = e \cdot \cos(\varphi)$ , hence  $|J| = |d(r, \varphi)/d(e, \varphi)| = \cos(\varphi)$  and  $p(e, \varphi) = \sqrt{2} \cdot \cos(\varphi)$  over the first octant.

In [3, 4] the integrals (1) and (2) are calculated for a variety of length estimators (and distance transforms), using the proper  $p(r, \varphi)$  to establish isotropy. For the two parameter length estimator the following optimal coefficients are found:

$$a = \frac{\pi/2 \cdot \{\sqrt{2} \cdot \ln(\sqrt{2} + 1) - 1\} - (\sqrt{2} - 1) \cdot \ln 2}{2 \cdot \ln(\sqrt{2} + 1) - 4 \cdot (\sqrt{2} - 1)} = 0.9445$$

$$b = \frac{\pi/\sqrt{2} \cdot \{\ln(\sqrt{2} + 1) - 1\} + (2 - \sqrt{2}) \cdot \ln 2}{2 \cdot \ln(\sqrt{2} + 1) - 4 \cdot (\sqrt{2} - 1)} = 1.3459$$
(7)

and for the three parameter estimates:

$$a = 0.980, \quad b = 1.406, \quad c = 2.204. \quad (8)$$

The differences between (7) and (5) and between (8) and (6) are small. Since it is claimed in [2] that (5) and (6) are optimal values they have a theoretical meaning. For practical purposes the conclusions in [2] still hold.

#### REFERENCES

1. G. Borgefors, Distance transformations in digital images, *Comput. Vision Graphics Image Process.* **34**, 1986, 344-371.
2. A. M. Vossepoel, A note on "Distance Transformations in Digital Images," *Comput. Vision Graphics Image Process.* **43**, 1988, 88-97.
3. L. Dorst, *Discrete Straight Line Segments: Parameters, Primitives and Properties*, Ph.D. thesis, Delft University of Technology, Delft, The Netherlands, June 10, 1986.
4. L. Dorst and A. W. M. Smeulders, Length estimators for digitized contours, *Comput. Vision Graphics Image Process.* **40**, 1987, 311-333.