Rydberg atoms on a chip and in a cell
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SINGLE-ATOM ABSORPTION IMAGING

In this chapter we show that the sensitivity of absorption imaging of ultracold atoms can be significantly improved by imaging in a standing-wave configuration. We present simulations of single-atom absorption imaging both for a travelling-wave and a standing-wave imaging setup, based on a scattering approach to calculate the optical density of a single atom. We find that the optical density of a single atom is determined only by the numerical aperture of the imaging system. We determine optimum imaging parameters, taking all relevant sources of noise into account. For reflective imaging we find that an improvement of 1.7 in the maximum signal-to-noise ratio can be achieved. This is particularly useful for imaging in the vicinity of an atom chip, where a reflective surface is naturally present.

This chapter is based on the paper Sensitive Absorption Imaging of Single Atoms in Front of a Mirror, Optics Express 21, 10188 (2013) [99]
5.1 INTRODUCTION

The detection of single atoms with high fidelity is an important requirement for many modern atomic physics experiments, and in particular for quantum information science. Single particle imaging has been achieved using fluorescence imaging of ions [100] and neutral atoms [101–105], but is difficult in atom-chip experiments due to a scattering background from the chip [103]. A different approach increasing the detection sensitivity is to enhance the atom-photon interaction using resonant cavities [106–108]. However, free-space absorption imaging of single particles [81, 109, 110] is still a daunting task.

In this chapter we propose absorption imaging in a standing wave as a simple and effective way to improve the Signal-to-Noise Ratio (SNR) of (single) atom detection. This is particularly useful in the context of atom-chip experiments [80] where a standing wave naturally forms if the imaging is done perpendicular to the chip surface. We take the setup discussed in this thesis and currently in use in our group as a reference [86, 87, 98] for realistic experimental parameters.

In the first section we will calculate the absorption signal of a single atom, both for traditional one-pass imaging and for reflection imaging as employed in our experiment. In the second half of the chapter we present numerical simulations of the expected absorption images and SNR under realistic imaging conditions, taking into account all common sources of noise, for in-trap atoms as well as for untrapped atoms. To do so we use a hybrid approach, first calculating the movement of the atom in the imaging beam based on the recoil caused by scattering individual photons, and subsequently investigating the absorption signal due to this atom, now taking the imaging beam as a classical wave. We show that using reflection imaging significantly increases the expected SNR and thus the single-atom detection accuracy.

5.2 ABSORPTION IMAGING OF SINGLE ATOMS

Our starting point is the absorption signal of a single atom at rest, illuminated by a resonant laser beam with homogeneous intensity distribution (see Figure 5.2). As is shown in Appendix B the intensity in the imaging plane is then given by

\[ \frac{I_{\text{abs}}(\rho)}{I_{\text{in}}(\rho)} = \frac{\sigma}{1 + s} \left[ \text{Re}(p(\rho)) - a \chi |p(\rho)|^2 \right] \]  

(5.1)

where \( I_{\text{abs}}(\rho) \) is the absorbed intensity, i.e. the missing intensity in the image plane with image-plane coordinates \( \rho \) and \( I_{\text{in}} \) is the intensity of the imaging beam before absorption. Furthermore, \( \sigma \) is the absorption cross section of the atom and \( s \) the saturation parameter in the object plane. The response of the imaging system to a point source is described by the Point-Spread Function (PSF) \( p(\rho) \) discussed below in more detail, and \( a = \left[ \int |p(\rho)|^2 \, d^2 \rho \right]^{-1} \) is the effective area over which the signal is distributed in the detection plane. Finally \( \chi \) is the fraction of the scattered light collected by the imaging system related to the numerical aperture of the objective lens by \( \chi = \frac{3}{8} s \text{NA}^2 \).
In an experiment the quantities \( a \) and \( I_{\text{abs}}(\rho) \) are not directly accessible due to the finite size of any detector and the finite time resolution. Instead one would observe the quantities

\[
N_{\text{det}}(x) = \int_A d\rho \int_0^\tau dt \left( I_{\text{in}} - I_{\text{abs}} \right)
\]

\[
N_{\text{ref}}(x) = \int_A d\rho \int_0^\tau dt I_{\text{in}}
\]

(5.2)

where \( x \) is an index over camera pixels, \( A \) is the size of a pixel and \( \tau \) is the exposure time, i.e.
the duration of the imaging pulse. Both \( \sigma \) and \( s \) now depend on time, as the atom acquires a finite velocity during the imaging pulse, leading to Doppler shifts. In addition the local saturation parameter in the object plane varies in the case of standing wave imaging, as will be discussed below. We can define the apparent column density of atoms per pixel in the image plane as

\[
n(x) = \frac{1 + s_0 N_{\text{det}}(x)}{\sigma_0 N_{\text{ref}}(x)}
\]

(5.3)

where \( s_0 \) is the saturation parameter for the intensity of the incoming beam and \( \sigma_0 \) is the absorption cross section of an atom at rest for the incoming intensity. The apparent atom number per pixel would be given by \( N_{\text{app}}(x) = A n(x) \).

As the atoms will move out of the focal plane during the imaging pulse, simulating this signal will require a point-spread function that is a function not only of the image plane coordinates, but also takes defocusing into account. This PSF can be numerically determined as

\[
p(\rho, f; t) = \frac{2\pi}{\rho_0^2} \int_0^1 r \exp\left(ifr^2\right) \times J_0\left(\frac{2\pi r}{\rho_0}\right) dr
\]

(5.4)

for any given defocus \( f = \frac{2\pi}{\lambda} Z(1 - \sqrt{1 - \frac{\lambda}{2\pi Z}}) \) where \( Z \) is a real-space coordinate in the imaging direction, \( \lambda \) is the numerical aperture of the imaging system and both \( \rho \) and \( Z \) might depend on time. The parameter \( \rho_0 = \lambda /\text{NA} \) is proportional to the resolution of the imaging system, determined by the wavelength and the numerical aperture. Aberrations in the imaging system can be taken into account in a similar manner. We assume these to be negligible near the optical axis. In the simulations the numerical point-spread function given above is used to calculate the intensity in the image plane. The real part of the point-spread function is shown in Figure 5.1 for our experimental parameters.

5.2.1 ANALYTICAL ESTIMATES

For a better intuition of the above results we consider some simple analytic estimates. Assuming a stationary atom in perfect focus of a two-lens imaging system with unit magnification \((f - 2f - f)\) we can use a simple point-spread function of the form \( p(\rho) = \frac{1}{\rho \rho_0} J_1(2\pi \rho /\rho_0) \) where \( p(0) = \pi /\rho_0^2 = a^{-1} \). Using \( \chi = (3/8)\lambda^2 \text{NA}^2 \) and assuming a two-level system where \( \sigma_0 = 3\lambda^2 /2\pi \)
the maximum signal at the center of the image simplifies to

\[ \frac{I_{\text{abs}}(0)}{I_{\text{in}}(0)} = \frac{1}{1 + s_0} \left( \frac{3}{2} \text{NA}^2 - \frac{9}{16} \text{NA}^4 \right) \]

(5.5)

which is thus determined purely by the numerical aperture of the imaging system and the saturation parameter \( s_0 \). For a numerical aperture \( \text{NA} = 0.4 \) as is the case in our experiment we then find an absorption in the centre of the image of \( I_{\text{abs}}(0)/I_{\text{in}}(0) = 0.23 \) in the low saturation limit \( s_0 \ll 1 \) for a single atom.

We can further consider the total apparent number of atoms one would extract from this signal as the peak signal times the area over which the signal is distributed,

\[ N_{\text{app}} = a \frac{1 + s_0}{\sigma_0} \frac{I_{\text{abs}}(0)}{I_{\text{in}}(0)} = 1 - \frac{3}{8} \text{NA}^2 \]

(5.6)

This shows that the expected number of atoms is reduced by the second, NA-dependent term. This term is due to light which is scattered by the atom, but scattered into the solid angle of the lens, thus reducing the apparent amount of light that is absorbed. For a numerical aperture of 0.4 we therefore expect to find an apparent atom number \( N_{\text{app}} \) of 0.94 rather than one.

The commonly used quantity \textit{optical density} (OD) can thus be defined for a single particle by the point-spread function of the imaging system by \( \text{OD} = -\ln \left( 1 - \frac{I_{\text{abs}}}{I_{\text{in}}} \right) \). At the center of the image (for vanishing pixel size) we find for the peak optical density a value of \( \text{OD}(0) \approx \frac{3}{2} \text{NA}^2 + \frac{9}{16} \text{NA}^4 \) in the low-saturation limit (where the sign change is due to a series expansion of the logarithm). For our numerical aperture this equals a peak optical density of 0.26. It is worth noting here that the atom number extracted in the usual way from optical density, in
this case

\[ N_{OD} = -a \frac{1 + s_0}{s_0} \ln \left( 1 - \frac{I_{abs}(0)}{I_{in}(0)} \right) \]  \hspace{1cm} (5.7)

would yield 1.07 atoms rather than 0.94. The atom number is slightly overestimated, as Lambert-Beers law is valid only for a continuously absorbing medium, not for a single absorber. In the following we always use the spatially dependent equivalent of (5.6) rather than (5.7) to determine atom numbers.

5.2.2 REFLECTION IMAGING

![Figure 5.2: Sketch of the imaging system considered for this chapter. On the left the situation of a travelling wave without reflection is depicted. On the right the situation of reflective imaging (with the atom chip on top) is shown.](image)

The above discussion is for an atom imaged by a travelling wave, as depicted in Figure 5.2 a). In reflection imaging, such as in Figure 5.2 b) the situation is slightly different. In this case the probe light forms a standing wave at the position of the atoms. For Rb atoms in our magnetic lattice \([86, 87, 98]\) we expect a ground-state size of the atomic wavefunction of approximately 40 nm, about a factor of 10 smaller than the period of the standing wave. We therefore approximate the atom as a point-particle which can initially be positioned in an anti-node of the standing wave. In our atom chip experiment the trap position can be adjusted by means of an externally controlled magnetic field to achieve this.

The intensity of the probe beam then varies sinusoidally along the probe direction and the maximum intensity in the anti-nodes is a factor four greater than the intensity of the incoming travelling wave. The coherent scattering amplitude \(A\) scales with the the local field amplitude (at low saturation). Thus the scattered wave is two times as strong as for a travelling wave,
leading to an accordingly higher observed signal. Furthermore one also has to take the changing saturation parameter into account.

It should be noted here that while in the limit of low saturation and short exposure times the observed signal will indeed be two times larger than for a travelling wave, for realistic imaging with finite exposure time the atom starts to probe the spatial variation of the light field, leading to an observed signal which depends on the exact imaging parameters. The expected signal can then only be predicted by simulations such as described below. This is further discussed in 5.3.3.

A further important difference to travelling wave imaging is that the atom is not pushed out of the focus of the imaging system by the probe beam, but performs a random walk in all dimensions. This point will also be further discussed in the simulations below.

Another factor of two could in principle be gained because the scattered wave is emitted towards and reflected by the mirror. This again doubles the amplitude of the scattered wave in the detection plane. A practical consideration here is that for single-atom imaging one will typically use high-NA imaging, with a Rayleigh length \( z_R = \frac{\lambda}{\pi NA^2} \lesssim 10 \mu m \). For the typical situation in our experiment the atom is at about 10 \( \mu m \) from the surface. This implies that the atom and its mirror image are not simultaneously in focus in the image plane. In practice this reflected wave hardly contributes to the optical density for our experimental parameters. If the imaging is done extremely close to a reflecting surface, or if the Rayleigh length of the imaging system is significantly larger this can however be a significant effect.

### 5.3 SIMULATIONS

After having calculated the absorption signal of a single atom at rest in the previous section, we would now like to determine the accuracy with which we can detect a single atom in a more realistic setup. To do so we have to take two important effects into account: recoil blurring and imaging noise. The first describes the process where the atom starts to move due to scattering photons, leading to a blurring of the image. The second describes the uncertainty with which one can determine the local intensity of the imaging beam, both for fundamental reasons, i.e. shot-noise, and technical reasons such as read-out noise of the camera. These two cases impose counteracting constraints: recoil blurring is minimal for short exposure times and weak probe intensities, shot-noise is reduced for large photon counts, i.e. long exposure times and high probe intensities. In addition, at high intensities the signal saturates. Therefore there exists an optimum SNR for a certain exposure time and probe intensity. We use a hybrid approach in treating these effects: recoil blurring is calculated based on scattering individual photons off the atom (see below), while the absorption signal is calculated based on a classical-wave approach as described above.

In our simulations we distinguish six cases:

A) free atom in travelling wave

B) trapped atom in travelling wave, quantisation axis \( \parallel \) chip plane

C) trapped atom in travelling wave, quantisation axis \( \perp \) chip plane
D) free atom in standing wave

E) trapped atom in standing wave, quantisation axis $\parallel$ chip plane

F) trapped atom in standing wave, quantisation axis $\perp$ chip plane

where the cases a) – c) are for traditional one-pass imaging, and the cases d) – f) are for our reflective atom-chip setup (see Figure 5.2). While case a) and d) assume a freely moving atom, case b), c) e) and f) assume the atom to be held in a magnetic trap.

In the free atom case we assume to be free to adjust the quantisation axis to be optimal for imaging, as we can always apply a homogeneous external field in any direction. However, the quantisation axis of trapped atoms is fixed in the plane of the atom chip in our experimental setup. To drive $\sigma^+$ transitions for the atoms we therefore have to decompose the incoming light which is linearly polarised in the chip plane into $\sigma^+$ and $\sigma^-$ components for the quantisation axis of the atoms, losing half of the intensity, and hence half the signal for cases b) and e). We do assume a sufficiently strong quantisation field in the traps that off-resonant absorption of the $\sigma^-$ radiation is strongly suppressed in these cases. By only driving $\sigma^+$ transitions we can then avoid decay into untrapped or anti-trapped states.

In case c) and f) we assume the quantisation axis to be rotated perpendicular to the chip surface. While this is not the case in our setup, it is useful to treat these cases as well, both for comparison purposes and to make the results of this chapter more generally applicable.

For the magnetic traps we assume the potential calculated from the magnetic pattern of microtraps present on our atom chip (see chapter 4) which is approximately equivalent to trap frequencies of $(\omega_x, \omega_y, \omega_z) = 2\pi \times (38.1, 36.5, 14.0)\ kHz$ and a trap depth of 8.8 G for the square lattice section. In the simulations the calculated potential is used without harmonic approximations. The atom is treated as free as soon as it acquires sufficient energy to escape the trap. We assume optical pumping to non-trapped states is negligible for an appropriate choice of polarisation for the imaging beam.

### 5.3.1 RECOIL BLURRING

As the atom is illuminated by the imaging beam it scatters photons at a rate determined by the local intensity at the position of the atom as well as any detuning due to e.g. Doppler shifts as the atom acquires momentum with each of these scattering events. In the case of a travelling wave the photon scattering drives the atom strongly along the direction of the imaging beam, as the absorption of a photon always happens in this direction. This results in the atom performing a biased random walk in momentum space, resulting in a rapid movement of the atom out of the focal plane of the imaging lens. The atom further performs an unbiased random walk in momentum space in the plane perpendicular to the imaging direction due to the spontaneous emission of a scattered photon, leading to a blurring of the image. In the case of a standing wave the atom can absorb a photon from both directions along the imaging axis, removing the strong bias present in travelling wave imaging and leading to an unbiased random walk both in the imaging direction as well as in the plane perpendicular to it. The atom therefore leaves the

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1The values here are different from those in the experiment, as here a thickness of the magnetic film of 300 nm was assumed. In all other aspects the design is the same as that in chapter 4.
Table 5.1: Optimum exposure parameters, and resulting SNR for the six cases described in the text. SNR_{px} is the SNR obtainable from evaluating a single pixel, while SNR_{CRB} is the SNR for an estimator achieving the Cramér-Rao bound (without taking fringe-removal into account). Finally we give the probability of true positive measurements p_{1,1} and false positive measurements due to the presence of zero atoms p_{1,0} (see 5.3.2)

<table>
<thead>
<tr>
<th>Case</th>
<th>τ (µs)</th>
<th>I/I_{sat}</th>
<th>SNR_{px}</th>
<th>SNR_{CRB}</th>
<th>p_{1,1}</th>
<th>p_{1,0}</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>12.4</td>
<td>0.59</td>
<td>1.55</td>
<td>1.57</td>
<td>57%</td>
<td>21%</td>
</tr>
<tr>
<td>b)</td>
<td>12.6</td>
<td>1.09</td>
<td>1.13</td>
<td>1.15</td>
<td>45%</td>
<td>24%</td>
</tr>
<tr>
<td>c)</td>
<td>12.0</td>
<td>0.65</td>
<td>1.57</td>
<td>1.59</td>
<td>57%</td>
<td>20%</td>
</tr>
<tr>
<td>d)</td>
<td>17.0</td>
<td>0.56</td>
<td>1.73</td>
<td>1.76</td>
<td>62%</td>
<td>19%</td>
</tr>
<tr>
<td>e)</td>
<td>49.2</td>
<td>0.72</td>
<td>1.81</td>
<td>1.85</td>
<td>65%</td>
<td>18%</td>
</tr>
<tr>
<td>f)</td>
<td>42.5</td>
<td>0.56</td>
<td>2.63</td>
<td>2.68</td>
<td>82%</td>
<td>9.5%</td>
</tr>
</tbody>
</table>

focus of the lens much more slowly in the case of standing wave rather than travelling wave imaging, greatly reducing recoil blurring in this configuration.

We simulate the movement of the atoms by calculating stochastic trajectories where random scattering events occur at the local scattering rate, taking the effect of the trap into account as a classical potential. In the case of an untrapped travelling wave we recover the expected behaviour, i.e. the RMS position along the imaging beam growing as \( t^2 \) and the RMS position in a plane perpendicular to the imaging direction growing as \( t^{3/2} \), as is expected for uniform acceleration and a random walk in momentum space respectively. The effect of the trapping potential is almost negligible in the case of travelling wave imaging, as the force of the light field is much greater than the restoring force of the potential. In the case of reflective imaging the behaviour proportional to \( t^{3/2} \) is also expected in the probe direction. This is however modified in a non-trivial fashion by the periodic intensity of the standing wave leading to a low (high) number of scattering events near nodes (anti-nodes) of the standing wave. Furthermore, since the net force on the atoms is much lower than in the case of a travelling wave the influence of the trapping field is also more important, particularly at low intensities. The RMS trajectories calculated here are then used in the integrations of Equation 5.2 to determine the signal after a given exposure time for a certain probe intensity.

5.3.2 DETECTION NOISE

To determine the feasibility of absorption imaging for single atoms one needs to consider various sources of noise in addition to the atomic trajectories to determine the signal-to-noise ratio. In the following we assume near shot-noise limited imaging, with further contributions from camera read-out noise and dark counts, as well as a finite quantum efficiency of the imaging system. Equations 5.2 determine the number of photons per pixel in the signal and light fields, and can be trivially modified to take a finite quantum efficiency of the imaging system into
account. With these values we can calculate the expected signal \( n(x) \) as atom density per pixel (Equation 5.3) for any combination of exposure time and intensity. The variance of this signal is given by

\[
\sigma_n(x)^2 = \left( \frac{1 + s_0}{\sigma_0} \right)^2 \left( \frac{\sigma_{\text{det}}^2}{N_{\text{ref}}^2} + \frac{N_{\text{det}}^2 \sigma_{\text{ref}}^2}{N_{\text{ref}}^4} \right)
\]  

(5.8)

where \( N_{\text{det}} \) and \( N_{\text{ref}} \) are the number of electrons per pixel in the signal and reference images respectively, also integrated over pixel size and exposure time. \( \sigma_{\text{det}}^2 \) and \( \sigma_{\text{ref}}^2 \) are the variances of these quantities, mostly determined by photon shot noise (\( \propto N \)) and camera readout noise.

The signal to noise ratio per pixel is then given by \( n(x)/\sigma_n(x) \); one can further improve the signal to noise ratio by not only evaluating the central pixel, but using an optimal estimator achieving the Cramér-Rao bound, such as

\[
N = \frac{\sum n(x) q(x)}{\sum q(x)^2}
\]

(5.9)

where \( q(x) \) is the spatial mode function of the signal [112]. In our results we list the SNR both for evaluating only the central pixel and for an estimator achieving the Cramér-Rao bound.

We define the accuracy of the measurement as the probability of finding true positives \( p_{1,1} \), i.e. finding one atom if there is one atom present. Here we assume all measurement outcomes for which \( 0.5 < N < 1.5 \) to indicate the presence of exactly one atom. We have determined this value by simulations of \( 10^6 \) individual absorption images for each case. The results are in excellent agreement with those expected for a normal distribution, for which the accuracy is given by \( F = \text{Erf}(z/\sqrt{2}) \) with \( z = N/\sigma \).

We can also determine the probability of finding false positives in the absence of any atoms \( p_{1,0} \), i.e. finding one atom if zero atoms are present. False positives due to the presence of more than one atom can not easily be determined, as an accurate calculating of the absorption signal of two atoms is non-trivial. We expect these to be of similar magnitude as those due to the absence of any atoms.

### 5.3.3 Atom Number Estimation

Here also the expected atom number for all cases should be discussed in more detail: For the running wave cases a)–c) this is quite straightforward, as Equations 5.3 or (5.9) are directly applicable in an appropriate form. The only consideration necessary here is that in case b) the resulting atom number needs to be multiplied by a factor of 2 as due to the wrong alignment of the quantisation axis only half of the incident light contributes to the signal. Nonetheless, as Figure 5.3 shows these considerations are only correct in the limit of short exposure times. For longer exposure times the apparent atom number drops as the atom moves out of the focus of the imaging system.

The matter is slightly more involved in the standing-wave cases d)–f). The approach we take here is to double the saturation parameter in Equations 5.3 and 5.9, to take the locally increased intensity at the beginning of an imaging pulse into account, but to half the resulting
atom number as the signal is twice that of a single atom in single-pass imaging, as discussed in subsection 5.2.2. With these adjustments and treating case e) in the same manner as case b) above, we again find the expected number of 0.94 atoms in the limit of very short exposure times for all cases. The time evolution of this atom number (see Figure 5.3) initially shows a sharp drop as the atom moves out of the first anti-node followed by oscillating behaviour as the atom moves through multiple further nodes and anti-nodes of the standing wave. Not surprisingly the oscillation frequency is higher at higher intensities, and very low in the low-saturation limit, as scattering events here occur much less frequently. The signal approaches half of the initial value for an intermediate time around 30...40 µs, when it has been averaged over multiple periods of the standing wave, but before recoil blurring has a strong effect. Finally the atom moves out of focus, just like in the running wave cases, but at much longer timescales.

5.3.4 SIMULATION RESULTS

We simulate the expected absorption image and SNR assuming a numerical aperture of the objective lens of 0.4 (Edmund Optics NT47-727 with custom AR coating), a total magnification of the imaging system of 10, a pixel size of the camera of 13 × 13 µm² (Andor iKon-M 934), a quantum efficiency of 0.9 and a readout noise of 13 counts per pixel [112]. We consider resonant
Figure 5.4: Simulated SNR as function of intensity of the incoming beam (in units of saturation intensity) and exposure time. The four subplots correspond to the six cases described in the text as indicated near the top and right axes, the color scale is normalised to the maximum SNR of case f). The black dot marks the position of the optimum.

\(\sigma^+\)-transitions on the D2-line of rubidium 87, where \(\sigma_0 = 2.907 \times 10^{-9}\) cm\(^2\) and the saturation intensity \(I_{sat} = 1.669\) mW cm\(^{-2}\) [113]. We have confirmed that in the limit of low saturation our simulations yield the expected number of atoms in all six cases as discussed in subsection 5.3.3 above.

We find optimum imaging parameters by numerically maximising the signal-to-noise ratio of the central pixel (assuming the atom to be located at the center of this pixel) as a function of exposure time and intensity, i.e. maximising \(\text{SNR}(0)\) where

\[
\text{SNR}(x) = \frac{n(x)}{\sigma_n(x)}
\]

(5.10)

using the pre-calculated trajectories of an atom in the integrations of Equations 5.2. To avoid confusion in the case of reflective trajectories imaging where the intensity is spatially varying, the intensity is here always given in units of saturation intensity for the incoming beam (cf. Figure 5.2). The local intensity at the initial position of the atoms is therefore higher by a factor of four for the standing wave. We also determine the SNR of an estimator achieving the Cramér-Rao bound by weighting the signal by the mode function of the atomic distribution as described above. SNR as a function of exposure time and intensity is depicted in Figure 5.4. Table 5.1 lists the
Figure 5.5: Simulated single-shot absorption images for optimum exposure parameters, including fringe removal. The six subplots correspond to the six cases described in section 5.3, giving an indication of the different SNR for the different cases. Bright colors correspond to high optical density. Each subplot shows $5 \times 5$ pixels, with the atom initially located at the center. At larger distances from the initial position the PSF becomes negligibly small. Using Equation 5.9 we extract $N=(1.01, 1.04, 0.99, 1.01, 0.96, 1.04)$ atoms for subplots a)-f) respectively, where we have selected the individual images to be no more than 5% off the real value to make them more comparable. These atom numbers are normalised by $N_{app}$ for each imaging setup.

signal-to noise ratio for optimum imaging parameters. Note that this can be further improved by a factor $\approx \sqrt{2}$ by using the fringe-removal algorithm [112] which reduces the shot noise in the light field by optimal averaging of reference images even in the absence of any fringes. It is also important to point out that the optima in the standing-wave cases are much less sensitive to imaging parameters than in the running wave cases, so an excellent SNR can be expected even if control over the imaging parameters is imperfect.

Figure 5.5 finally shows simulated single-shot absorption images of a single atom for the parameters given above, including the use of fringe removal. The number of atoms extracted from these images using the optimum estimator is given in the figure caption. The optimum case f) stands out not so much because the signal would be much larger, but because the noise is much lower, as visible by the pixel-to-pixel fluctuations.

Depending on the imaging method the effect of recoil blurring on the optical resolution of the imaging system can be significant. For our optimum exposure parameters we find the strongest recoil blurring in case c), where the RMS position of the atoms in the object plane increases by $0.9 \, \mu m$ during the imaging pulse. For all other cases considered here recoil blurring is smaller by at least a factor of 3. This blurring should be compared to the base resolution of
the imaging system of 1.2 µm (Rayleigh criterion). Furthermore the images are discretised by the effective pixel size of 1.3 µm in object space.

![Histograms of atom number distribution](image)

**Figure 5.6:** Atom number distribution extracted from one million simulated absorption images for all cases, for the absorption signal of a single atom (solid lines) as well as for zero and two atoms (dashed). The signal for two atoms is based on a doubling of the absorption of a single atom (neglecting coherent effects) and should therefore be only taken as an indication of the true signal.

Finally we present histograms of the atom number extracted from one million simulated absorption images for each case in Figure 5.6. As expected these are in excellent agreement with the values of Table 5.1. Noteworthy are in particular case b), significantly worse than all of the others and case f), significantly better than all of the others. The differences between case a) and c) are negligible, and the standing wave cases d) and e) are only slightly better than those two running wave cases. While it is straightforward to simulate results for zero atoms based on images containing only noise, the expected signal in the presence of two atoms is non-trivial, as coherent effects between the two atoms, each moving along independent trajectories play a role. The resulting atom number distribution for two atoms in Figure 5.6 should therefore only be taken as a rough indication. The width of the distributions is almost identical for the cases discussed here.

These results can be used to determine the probability of true positive and false positive measurements given in Table 5.1. For simulated absorption images of zero atoms, the fraction of measurements yielding an atom number > 0.5, i.e. integrating the tail of the leftmost distribution between 0.5 and 1.5, normalised by the total area of the peak, determines \( p_{1,0} \). Similarly for the simulated absorption images of one atom, integrating with the same limits and normalising by the total area of the peak determines the fraction of true positives \( p_{1,1} \), with the results given in Table 5.1.
5.4 DISCUSSION & CONCLUSION

In this chapter we have outlined the theoretical prospects of detecting single rubidium atoms using absorption imaging. We find the optical density of a single atom in single-pass imaging to be ultimately determined only by the numerical aperture of the imaging system with a peak optical density of $OD = \frac{3}{2} NA^2 \left( 1 + \frac{3}{8} NA^2 \right)$ in the low saturation limit. We have calculated optimum imaging parameters for a number of different cases and could show that the use of a reflective imaging setup can significantly improve the signal-to-noise ratio of absorption imaging. We expect an accuracy of about 82% for the detection of single atoms for the optimum parameters given above in case f), which can be improved to 94% (with a false-positive rate due to zero atoms of 4%) using the reduction in shot noise achieved by the fringe removal algorithm [114]. This compares to a detection accuracy of only 74% (with a false-positive rate due to zero atoms of 15%) for single-pass imaging with the same imaging system. Unfortunately the situation in our experiment is not as favourable, due to the alignment of the quantisation axis. There we can only expect a detection accuracy of 81% including fringe-removal. This is however still better than any of the running-wave cases. Furthermore the broader optimum in the standing wave cases requires less precise control over imaging parameters.

For measuring statistical averages in a regular array of traps further improvements can be made using correlation analysis [114].