Essays on mathematical and computational finance: With a view towards applied probability
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Chapter 5

A Correlated Overflow Model
With Applications in Credit Risk

Doof uw inspiratie en verbeeldingskracht niet uit,
wordt geen slaaf van ’t model.

Vincent van Gogh (1853 – 1890)
Letter to his brother Theo, Sunday, November 5, 1882

In this chapter we present and explicitly solve a specific multi-dimensional, correlated overflow problem. Due to the ordering of the underlying components in our model, explicit results are obtained for the probabilities under consideration; importantly, the results are not in terms of Laplace transforms. In our setting, each component behaves as a compound Poisson process with unit-sized upward jumps, decreased by a linear drift. The approach relies on a Beneš-type argumentation [12], that is, the idea of partitioning the overflow event with respect to the last “exceedance epoch”. This type of problems arises naturally in various branches of applied probability, and therefore has several applications. In this chapter we point out two specific application areas: one in mathematical finance, and one in queueing. In the former, several “obligors” (whose “distances-to-default” are correlated, as is the case in practice) are considered, and it is quantified how likely it is that there are defaults before a given time $T$. In the latter, one is often interested in the workload or storage process; the results of this chapter provide expressions for overflow probabilities in the context of specific coupled queuing systems.

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1 This chapter is based on joint work with Michel Mandjes and will appear as [58].
2 Don’t snuff out your inspiration and power of imagination, don’t become a slave to the model.
5.1 Introduction

In this chapter we are interested in the class of multivariate overflow problems by considering probabilities of the type

\[ P \left( \sup_{t \in [0, T]} A_1(t) \geq D_1, \ldots, \sup_{t \in [0, T]} A_d(t) \geq D_d \right). \]

Here the parameter \( T > 0 \) is the time horizon (which is possibly \( \infty \)), \( d \in \mathbb{N} \) the dimension of the problem, and

\[ A(t) \equiv (A_1(t), \ldots, A_d(t)), \]

with \( t \geq 0 \), and \( A(0) = 0 \), is a \( d \)-dimensional stochastic process.

There is a huge body of works that are devoted to the one-dimensional case, that is, \( d = 1 \). The probability

\[ P \left( \sup_{t \in [0, T]} A(t) \geq D \right) \]  

(5.1)

can be interpreted in several ways, and as a consequence, results can be found in various branches of the applied probability literature.

In the first place, (5.1) can be interpreted in financial terms. It can be viewed as a ruin probability; see e.g. [6]. Let \( Y(t) \) be the reserve of, say, an insurance firm, with initial reserve \( Y(0) = D \), then, with \( A(t) := D - Y(t) \), probability (5.1) represents the probability that the firm goes bankrupt before time \( T \).

In the second place, (5.1) can be interpreted in queueing terms. As pointed out in, e.g., [5], the reflected process \( Q(t) \) (to be thought of as a workload process or storage process) of a stochastic process \( Y(t) \) can be represented as

\[ Q(t) = Y(t) - \inf_{s \in [0, t]} Y(s), \]

in case \( Q(0) = 0 \). Now defining \( A(s) := Y(T) - Y(s) \), we observe that the transient workload distribution can be expressed in terms of overflow probabilities, in the sense that \( P(Q(T) \geq D) \) equals (5.1).

For the one-dimensional case results are available for various driving processes \( A(t) \). Two standard models are
(i) $A(t)$ representing Brownian motion, for which (5.1) can be computed explicitly, see e.g. [60, p. 49];

(ii) $A(t)$ representing a compound Poisson process with independent and identically distributed upward jumps from a general distribution, and negative drift, in the queueing literature referred to as the $M/G/1$ queue [5].

Model (ii) is contained in a more general class, viz. the class of Lévy processes (processes with independent and identically distributed increments) with no negative jumps [38]; for this general model the probability (5.1) can be uniquely determined through a double transform. That is, an explicit formula [5, 38] for

$$
\int_0^\infty e^{-\vartheta T} \int_0^\infty e^{-\alpha D} \mathbb{P} \left( \sup_{t \in [0,T]} A(t) \geq D \right) dD dT
$$

has been derived. Also for the case of no positive jumps, such a result can be derived, while for the case of two-sided jumps the results, derived relying on Wiener-Hopf theory, are considerably more implicit [38, 75]. A general remark is that the class of models for which (5.1) is available in explicit terms, is rather limited; this class includes the model in which upward unit-sized jumps arrive according to a Poisson process, and in which there is a negative drift, also known as the $M/D/1$ queue [101, Section 15.1], and a similar model in which $N$ unit-sized upward jumps arrive at epochs that are homogeneously distributed on $[0, T]$, also known as the $N \cdot D/D/1$ queue [101, Section 15.2].

For the multi-dimensional case, considerably fewer results are available (unless the $d$ components are independent, obviously). Without aiming to give a complete account of the literature, we mention a number of relevant contributions. Roughly speaking, the results can be divided into two categories. In the first place there are papers that do not impose strong conditions on the process $A(\cdot)$, but that settle for finding just asymptotics, for instance by just identifying a function $\varphi(u)$ such that

$$
\log \mathbb{P} \left( \sup_{t \in [0,T]} A_1(t) \geq uD_1, \ldots, \sup_{t \in [0,T]} A_d(t) \geq uD_d \right) \cdot \varphi(u) \rightarrow 1
$$

as $u \rightarrow \infty$; see e.g. [2, 32, 37]. In the second place there are models in which the components have a special structure; for instance mapping on the workloads in the various queues of a queueing network, or modeling market wide and company specific shocks in a financial model. The results are in terms of Laplace
transforms; see e.g. [36, 44, 71].

The contribution of the present chapter is that we present a framework in which the $d$ components are dependent, but in which the multi-dimensional overflow probability can be computed explicitly (that is, not in terms of a Laplace transform). The special structure that we impose is such that the components of $A(\cdot)$ are ordered, in the sense that $A_i(t) \geq A_{i-1}(t)$. Each process $A_i(t)$ corresponds to unit-sized upward jumps arriving according to a Poisson process of rate $\Lambda_i$, minus a linear drift. Due to the specific construction chosen, a Beneš-type argument [101, Section 14.2] can be exploited, cf. an analysis for tandem and priority queues in [90].

This chapter is organized as follows. We start by providing a brief account of the literature in Section 5.2. Section 5.3 introduces our model, and specifies in more detail how are model can be used in a credit risk setting. In Section 5.4 we present and prove our results. A discussion, concluding remarks, as well as directions for further research are presented in Section 5.5.

## 5.2 Literature

Queueing theory is one of the major branches in (stochastic) operations research (OR), see e.g. the textbook [5], or the more recent contributions [15, 23, 59, 67, 72]. Recently, it was increasingly recognized that OR techniques often provide useful tools that facilitate solving problems that originate from mathematical finance, see e.g. [26, 31]; the present chapter can be seen in this context.

In the mathematical finance literature dependent multivariate overflow probabilities play an important role. One could think of the setting of multiple obligors whose “distances-to-default” $\Delta(t)$ follow a given stochastic process; obligor $i$ defaults if $\Delta_i(t) \leq 0$ for some $t \in [0, T]$. As argued above, such a credit risk model fits in the setup of this chapter; one could be interested in, for instance, the distribution of the number of defaults before time $T$. In some models the obligors are assumed to be independent [79], or the dependence is incorporated in a somewhat crude way [55]. In reality, the distances-to-default are obviously quite strongly correlated, explaining the need for models that incorporate the dependence in a sound way. An advanced model, in which the components follows specific Lévy processes, has been proposed in [44]; the components are made dependent by composing them from an obligor-specific part and a common part.
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There is a vast body of other papers that propose methods to describe correlations in a mathematical context. We here mention the copula-based approach \[83\] and the somewhat crude approach proposed by \[55\], but for a more complete account of the state of the art in this field, see also the introduction of \[44\].

5.3 Model and Preliminary

In this section we first introduce our model, and then point out how our model can be used in studying credit risk.

5.3.1 Model

In this chapter the following model is considered. Let \( N_i(\cdot) \) be a compound Poisson process in which unit-sized jobs arrive at a Poisson rate of \( \lambda_i \); in other words, \( N_i(t) \) denotes the number of jumps up to time \( t \) in dimension \( i \), for \( i = 1, \ldots, d \) (with \( d \in \mathbb{N} \)). It is throughout assumed that the processes \( N_1(\cdot), \ldots, N_d(\cdot) \) are independent. Define \( A_0(t) = -t \), and in addition, for \( i = 1, \ldots, d \), recursively

\[
A_i(t) := A_{i-1}(t) + N_i(t).
\]

Obviously, by construction, the \( A_i(\cdot) \) processes are ordered, in the sense that \( A_i(t) \geq A_{i-1}(t) \), for \( i = 1, \ldots, d \) and \( t \geq 0 \).

Let \( T > 0 \) be a given time horizon. The primary objective of our study is to compute the probability of the bivariate supremum over a finite horizon: we wish to quantify the likelihood that in the \((i-1)\)-st dimension a specific level \( x > 0 \) has not been exceeded up to time \( T \), while in dimension \( i \) the level \( y \) was not exceeded up to time \( T \). This means that, in formal terms, we focus on, for \( i = 1, \ldots, d \) and \( x, y > 0 \),

\[
p_i(x,y) := \mathbb{P}((\forall t \in [0,T] : A_{i-1}(t) < x, \forall t \in [0,T] : A_i(t) < y) = \mathbb{P}
\]

\[
\left( \sup_{t \in [0,T]} A_{i-1}(t) < x, \sup_{t \in [0,T]} A_i(t) < y \right);
\]

to rule out trivial situations we require \( x \leq y \). We also denote

\[
q_i(y) := \mathbb{P}((\forall t \in [0,T] : A_i(t) < y) = \mathbb{P}
\]

\[
\left( \sup_{t \in [0,T]} A_i(t) < y \right).
\]

Fix a vector \((D_1, \ldots, D_d)\) (componentwise positive), with \( D_1 \leq D_2 \leq \cdots \leq D_d \). Let \( N \) be defined as the number of thresholds \( D_i \) exceeded:

\[
N := \# \{ i : \exists t \in [0,T] : A_i(t) \geq D_i \}.
\]
Clearly, for $i = 1, \ldots, d - 1$,

$$\{N = i\} = \{\exists t \in [0, T] : A_i(t) \geq D_i, \forall t \in [0, T] : A_{i+1}(t) < D_{i+1}\},$$

while

$$\{N = 0\} = \{\forall t \in [0, T] : A_1(t) < D_1\}, \quad \{N = d\} = \{\exists t \in [0, T] : A_d(t) \geq D_d\}.$$

It is straightforward that the probability distribution of $\pi_i := \mathbb{P}(N = i)$ can be written in terms of the probabilities $p_i(x, y)$ and $q_i(y)$. Indeed, for $i = 1, \ldots, d - 1$,

$$\pi_0 = q_1(D_1), \quad \pi_i = q_{i+1}(D_{i+1}) - p_{i+1}(D_i, D_{i+1}), \quad \pi_d = 1 - q_d(D_d).$$

We note that the probability of multivariate suprema (that is, of dimension $d > 2$) over a finite horizon can also be computed explicitly. However, for expository purposes, we only consider the bivariate case in this chapter.

### 5.3.2 Credit Risk Interpretation

The above setting can be interpreted in terms of an, albeit stylized, model for credit risk. Let (at time $t$) the distance-to-default of obligor $i$ be given by

$$\Delta_i(t) = D_i + t - \sum_{j=1}^{i} N_i(t).$$

It means that the initial distance-to-default $\Delta_i(0)$ of obligor $i$ is $D_i$, that “under normal circumstances” $\Delta_i(t)$ grows linearly in time (one can think of the risk-free interest rate), but that there are “negative shocks”. The negative shocks for obligor $i$ arrive according to a Poisson process with rate $\Lambda_i := \sum_{j=1}^{i} \lambda_i$. Observe that $\Delta_i(\cdot)$ and $\Delta_j(\cdot)$ are correlated, in the sense that they have negative shocks arriving at a rate $\Lambda_{\min\{i,j\}}$ in common. A default of obligor $i$, with respect to time horizon $T$, can be represented by the event

$$\{\exists t \in [0, T] : \Delta_i(t) < 0\}.$$

Without loss of generality we can assume that $D_j > D_i$ if $j < i$; as a consequence a default of obligor $j$ implies a default of obligor $i$ (but not vice versa).

We end this subsection by a brief account of the practical relevance in terms of credit risk. Evidently, the system presented above provides us with a stylized representation of a credit risk model, as some features are not-realistic. In the first place, obligor $j$ is more “default-prone” than obligor $i$ if $i < j$, in that obligor
1 is the first to default (if at all), then obligor 2, etc. This strict ordering may be a good approximation (as some stocks are more stable than others, as indicated by e.g. credit ratings), but it would be desirable to lift this assumption in future work. A second item one might consider dropping concerns the unit-sized shocks. It would obviously be better to draw the shocks from some (perhaps obligor-specific) distribution, possibly calibrated to the corresponding market. In the third place one could opt for more realistic shapes of the trend (rather than our linear trend).

5.4 Main Result

In this section we derive expressions for the probabilities $q_i(y)$ and $p_i(x, y)$.

5.4.1 Computation of $q_i(y)$

We start by recalling how $q_i(y)$ can be computed; it essentially follows from the line of reasoning in e.g. Mandjes [90]. The number of jumps at time $T$ is obviously Poisson distributed, with mean $\Lambda_i T$; let $\bar{N}_i(\cdot)$ be a Poisson process with rate $\Lambda_i$. We obtain

$$q_i(y) = \sum_{j=0}^{\lfloor y+T \rfloor} e^{\Lambda_i T} \frac{(\Lambda_i T)^j}{j!} \times P(\forall t \in [0, T] : \bar{N}_i(t) < y + t \mid \bar{N}_i(T) = j).$$

Denote by $\Bin(n \mid N, p)$ the probability that a binomial random variable with parameters $N$ and $p$ attains the value $n$ (with $n \in \{0, \ldots, N\}$). As can be found in e.g. Humblet et al. [64, Section III.C] and Roberts and Virtamo [102], we have the following ballot result:

$$P(\forall t \in [0, T] : \bar{N}_i(t) < y+t \mid \bar{N}_i(T) = j) = 1 - \sum_{n \in \mathbb{N} : n \in (y, j]} \Bin(n \mid j, \frac{n-y}{T}) \frac{T-j+y}{T-n+y};$$

observe that this probability equals 1 if $y \leq j$, as desired. The proof of this result heavily relies on the fact that the positions of the $j$ jumps are uniformly distributed on $[0, T]$.

We arrive at the following result.

Lemma 1. For $y > 0$,

$$q_i(y) = \sum_{j=0}^{\lfloor y+T \rfloor} e^{\Lambda_i T} \frac{(\Lambda_i T)^j}{j!} \left(1 - \sum_{n \in \mathbb{N} : n \in (y, j]} \Bin(n \mid j, \frac{n-y}{T}) \frac{T-j+y}{T-n+y}\right).$$
5.4. MAIN RESULT

5.4.2 Computation of \( p_i(x,y) \)

We now point out how to compute the probability

\[
\bar{p}_i(x,y) := \mathbb{P}(\forall t \in [0,T] : A_{i-1}(t) < x, \exists t \in [0,T] : A_i(t) \geq y) = q_{i-1}(x) - p_i(x,y),
\]

which obviously enables us to compute \( p_i(x,y) \).

The main idea of the computation is that we partition the event under consideration, alternatively written as

\[
E := \{ \forall t \in [0,T] : \tilde{N}_{i-1}(t) < x + t, \exists t \in [0,T] : \tilde{N}_i(t) \geq y + t \},
\]

with respect to the last epoch in the interval \([0,T]\) at which \( A_i(\cdot) \) is larger than \( y \). To this end, we first define \( t_{n,y} := n - y \); observe that if \( \tilde{N}_i(t_{n,y}) = n \), then \( A_i(t_{n,y}) = y \). We then have that

\[
E = \bigcup_{n \in \mathbb{N} : n \in (y,y+T]} E(n),
\]

where the events \( E(n) \) are disjoint and given by

\[
E(n) := \left\{ \forall t \in [0,T] : A_{i-1}(t) < x \right. \\
\left. \tilde{N}_i(t_{n,y}) = n, \forall t \in (t_{n,y},T) : A_i(t) < y \right\}.
\]

Due to the fact that the \( E(n) \) are disjoint, we are left with the task of computing the probabilities \( \mathbb{P}(E(n)) \). We remark that the idea of partitioning the overflow event with respect to the last “exceedance epoch” is due to Beneš, see e.g. [12] and [96, Eqn. (2.4)], and has been exploited in a variety of models [101].

We now make the following observations:

1. First notice that if \( \tilde{N}_i(t_{n,y}) \) equals \( n \), then the event \( \{ \forall t \in (t_{n,y},T) : A_i(t) < y \} \) is equivalent to the event \( \{ \forall t \in (t_{n,y},T) : \tilde{N}_i(t) - \tilde{N}_i(t_{n,y}) \leq t - t_{n,y} \} \).

2. It is easily verified that the event \( \{ \tilde{N}_i(t) - \tilde{N}_i(t_{n,y}) \leq t - t_{n,y} \} \) implies the event \( \{ \tilde{N}_{i-1}(t) - \tilde{N}_{i-1}(t_{n,y}) \leq t - t_{n,y} \} \). Combining this with the first observation, the event \( E(n) \) can alternatively be written as

\[
E(n) = \left\{ \forall t \in [0,t_{n,y}) : \tilde{N}_{i-1}(t) < x + t \\
\tilde{N}_i(t_{n,y}) = n, \forall t \in (t_{n,y},T) : \tilde{N}_i(t) - \tilde{N}_i(t_{n,y}) \leq t - t_{n,y} \right\}.
\]

3. Using standard distributional properties, with \( \text{Poiss}(n \mid \lambda) \) denoting the probability that a Poisson random variable with mean \( \lambda \) attains the value \( n \),

\[
\mathbb{P}(\tilde{N}_i(t_{n,y}) = n) = \text{Poiss}(n \mid \Lambda_i t_{n,y}).
\]
5. A CORRELATED OVERFLOW MODEL

In addition, with $j \in \{0, \ldots, n\}$,

$$\mathbb{P}(\bar{N}_{i-1}(t_{n,y}) = j \mid \bar{N}_i(t_{n,y}) = n) = \text{Bin}\left(j \mid n, \frac{\Lambda_{i-1}}{\Lambda_i}\right).$$

It is now readily checked that

$$\rho_i(y, n, j) := \mathbb{P}(\bar{N}_{i-1}(t_{n,y}) = j, \bar{N}_i(t_{n,y}) = n) = \text{Pois}(j \mid \Lambda_{i-1}t_{n,y}) \text{Pois}(n - j \mid \lambda_it_{n,y}).$$

4. Conditional on $\{\bar{N}_i(t_{n,y}) = n\}$ intersected with $\{\bar{N}_{i-1}(t_{n,y}) = j\}$, the events

$$\{\forall t \in [0, t_{n,y}) : \bar{N}_{i-1}(t) < x + t\}$$

and

$$\{\forall t \in (t_{n,y}, T) : \bar{N}_i(t) - \bar{N}_i(t_{n,y}) \leq t - t_{n,y}\}$$

are independent. Obviously the former event depends on $\{\bar{N}_{i-1}(t_{n,y}) = j\}$ only, whereas the latter depends on $\{\bar{N}_i(t_{n,y}) = n\}$ only.

5. Again using the ballot result of e.g. [64, 102],

$$\mathbb{P}(\forall t \in [0, t_{n,y}) : \bar{N}_{i-1}(t) < x + t \mid \bar{N}_i(t_{n,y}) = n, \bar{N}_{i-1}(t_{n,y}) = j) = \mathbb{P}(\forall t \in [0, t_{n,y}) : \bar{N}_{i-1}(t) < x + t \mid \bar{N}_{i-1}(t_{n,y}) = j)$$

can be written as

$$\xi(x, y, n, j) := 1 - \sum_{m \in \mathbb{N} : m \in (x, j]} \text{Bin}\left(m \mid j, \frac{m - x}{t_{n,y}}\right) \frac{t_{n,y} - j + x}{t_{n,y} - m + x}.$$

In addition, due to a straightforward application of the Markov property,

$$\mathbb{P}(\forall t \in (t_{n,y}, T) : \bar{N}_i(t) - \bar{N}_i(t_{n,y}) \leq t - t_{n,y} \mid \bar{N}_i(t_{n,y}) = n, \bar{N}_{i-1}(t_{n,y}) = j) = \mathbb{P}(\forall t \in (0, T - t_{n,y}) : \bar{N}_i(t) \leq t),$$

which can be explicitly computed, by conditioning on the number of jumps in an interval of length $T - t_{n,y}$:

$$\zeta_i(y, n) := \sum_{k=0}^{T-t_{n,y}} e^{\Lambda_i(T-t_{n,y})} \frac{(\Lambda_i(T-t_{n,y}))^k}{k!} \times \left(1 - \sum_{m \in \mathbb{N} : m \in (0, k]} \text{Bin}\left(m \mid k, \frac{m}{T-t_{n,y}}\right) \frac{T-t_{n,y} - k}{T-t_{n,y} - m}\right).$$
Upon combining the above, we arrive at the following result.

**Theorem 1.** For $x, y > 0$ with $x \leq y$,

\[
\bar{p}_i(x, y) = \sum_{n \in \mathbb{N} : n \in (y, y+T]} \sum_{j=0}^{n} q_i(y, n, j) \cdot \xi(x, y, n, j) \cdot \zeta_i(y, n).
\]

In this section we have focused on two-dimensional overflow probabilities $p_i(x, y)$. As mentioned earlier, the same ideas can be applied for more-dimensional overflow probabilities as well. However, the computations become tedious, and the resulting formulas become extremely lengthy and are hence omitted.

### 5.5 Discussion, Concluding Remarks and Future Work

In this chapter we presented a model in which overflow probabilities in a multi-dimensional setting, with dependent components, are explicitly computed. There are very few models for which this has been done, a notable exception being two-dimensional Brownian motion (with correlated components), see the paper by Iyengar [65], corrected in [44, 94], and also the paper by Zhou [108]. These models are typically hard to analyze, but, due to the explicit *a priori* ordering of the underlying components in our model, we have succeeded in obtaining explicit results. This is the main contribution of this chapter.

Our model setup is such that each component behaves as a compound Poisson process with unit-sized upward jumps, decreased by a linear drift. The approach essentially relies on a Beneš-type argumentation [12], that is, we partition the overflow event with respect to the last “exceedance epoch”. Of course, in the context of various applications a Brownian motion setting has an obvious advantage over our model; whereas in our setting the components are a-priori ordered (in the sense that $A_i(t) \geq A_{i-1}(t)$ for all $i$), two-dimensional Brownian motion does not have this (restrictive) feature. The expressions relate to the bivariate case, but we can generalize our setup to higher dimensions (the expressions become cumbersome, though). For the setting of multivariate Brownian motion we are not aware of results in dimension 3 or higher.

Other models for computing multivariate, correlated default probabilities can be (roughly) divided into two categories. In the first place there are papers that do not impose strong conditions on the underlying processes (i.e., the class of driving stochastic processes is relative large), but that cover a specific a specific
asymptotic regime only; see e.g. [2, 32, 37] for papers that relate to the regime in which the initial distance-to-default is large. Secondly, there are papers that consider models in which the components have a special probabilistic structure; these components for instance describe the workloads in the various queues of a queueing network, or they describe market-wide and company-specific shocks in a financial model. The results of this second category tend to be in terms of Laplace transforms, see e.g. [36, 44, 71], and therefore require the application of a numerical inversion technique.

In terms of applications in credit risk, one would obviously prefer a framework in which there is no \textit{a priori} ordering of the components (describing the distance-to-default of the obligors). There is a clear need for “clean” models in which multi-dimensional overflow probabilities can be computed explicitly. This chapter covers an example which facilitates explicit computation of overflow probabilities in a correlated multi-dimensional setting, but one wonders whether there are more general models that can be analyzed in a similar way. In our study we heavily relied on the specific features of our model. Apart from the ordering we mentioned above, the fact that the jumps are unit-sized plays a crucial role. This assumption can be relaxed, though: an extension to a model in which the jumps are sampled from a distribution of finite support are possible (but this leads to rather complicated formulas and are hence omitted). Another extension could relate to the shape of the trend; whereas we now assume a linear drift, one could investigate whether other shapes lead to explicit results as well.