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Citation for published version (APA):
A Note on Taxation and Human Capital Accumulation*

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April 15, 2000

Abstract
Taxes on labor income reduce human capital accumulation due to: i) tax progression; ii) a positive uncompensated wage elasticity of labor supply; and iii) non-tax deductible direct costs of education. Taxes do not affect learning decisions if the following conditions hold: i) taxes are flat; ii) the uncompensated wage elasticity of labor supply is zero, or if the rate of substitution between consumption and effective leisure is not affected by the level human capital; iii) direct costs of education are either absent or fully deductible.

Keywords: human capital, (progressive) taxation.
JEL-codes: H20, J24

*I gratefully thank Lans Bovenberg and Hessel Oosterbeek for comments and suggestions. Financial support from the NWO priority program 'Scholar' funded by the Netherlands Organization for Sciences is gratefully acknowledged. Any remaining errors are mine.

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1 Introduction

Previous studies have focussed on three effects of taxing labor income on the incentives to invest in human capital. First, progressive taxation of labor income harms human capital accumulation if agents face higher marginal tax rates for future earnings than for current (forgone) earnings. The reason is that the marginal rate at which costs are deductible are lower than rates at which future benefits of learning are taxed. Second, taxes can reduce labor supply as a consequence of a dominant substitution effect and thereby lower the 'utilization' of human capital investments. Accordingly, this lowers returns and investments in human capital. This effect holds only when human capital is not used in leisure time. Third, if costs of direct expenditures are not deductible from the income tax, human capital investments are reduced as a consequence of (flat) taxes. The reason is that the costs - direct expenditures and forgone earnings - are less affected by taxation than the returns - future earnings.

This note synthesizes these various insights on the effects of labor income taxation on human capital accumulation in a single consistent framework. We further show under what conditions taxation does not affect human capital investment decisions. The setup of this paper is as follows. Section 2 describes the model for analyzing the effects of taxation. Section 3 discusses three special cases of the model. Section 4 concludes.

2 Model

We consider a two period life-cycle model of human capital accumulation. A representative agent is assumed. In the first period the agent works and learns. The second period is spend working and enjoying leisure. In the first period, a fraction \( X \) of total time - normalized at unity - is devoted to learning and \( 1 - X \) of total time is spend working. Labor supply in the second period is endogenous. A fraction \( L \) of total time, normalized at unity, is leisure, while the rest \( 1 - L \) is spend working. The agent derives utility from consumption in the second period \( C \) and leisure \( L \). We assume that there is no consumption in period one. Hence, all labor income earned in the first period is saved and consumed in the second period. The utility function \( U(C, L) \) is concave and homothetic. Households receive wage income in the first period \((1 - T_1)WH_1(1 - X)\), and wage income in the second period
\[ (1 - T_2)WH_2(1 - L) \]

\( W \) is the exogenously given wage rate per unit of human capital which is assumed to be equal in both periods. \( T_i \) denotes the flat income tax rate in period \( i = 1, 2 \). Therefore, average tax and the marginal tax rate on income coincide. The tax schedule is defined to be progressive if the tax on second period income is higher than the tax on first period income, i.e. \( T_2 > T_1 \). \( H \) denotes the amount of human capital in both periods. In the remainder we normalize \( H_1 \) to unity. In addition to time \( X \), the agent has to invest market goods \( Y \) in accumulating human capital.

Production of human capital is of the Ben-Porath (1967) form:

\[ H_2 = G(X, Y ; H_1) - \delta H_1 = F(X, Y) = X^{\omega_x}Y^{\omega_y} \]  

We assume that the depreciation rate \( \delta \) of human capital is zero. \( F(X, Y) \) is the production function of human capital with positive but diminishing partials. We assume that \( F \) is homogeneous of degree \( \xi = \omega_x + \omega_y < 1 \). I.e., there are decreasing returns to scale. In this note we specialize to the case of a Cobb-Douglas learning technology where the elasticity of substitution between goods and time in the production of human capital equals unity.\(^6\)

The lifetime budget constraint is:

\[ M \equiv (1 + R)(1 - \chi T_1)Y + C \\
= (1 + R)(1 - T_1)W(1 - X) + (1 - T_2)WH_2(1 - L) \]  

where \( R \) is the exogenously given real interest rate. The present value of consumption and expenditures on goods invested in education should be equal to the present value of life-time income.\(^5\) \( \chi \) denotes the fraction of direct costs that can be deducted from the income tax. \( \chi = 1 \) corresponds to full tax-deductibility, while \( \chi = 0 \) corresponds to non-tax deductibility.

First-order conditions for the optimal choices of \( C, L, X \) and \( Y \), are as follows. First, marginal utility of consumption should be equal to the marginal utility of income \( \lambda \): \( U_C = \lambda \). Second, marginal utility of leisure equals the price of leisure as measured by the value of forgone earnings: \( U_L = \lambda (1 - T_2)WH_2 \). This yields the marginal rate of substitution between consumption and leisure:

\[ \frac{U_L}{U_C} = (1 - T_2)WH_2 \]  

Third, marginal benefits of learning in the form of higher future wages should be equal to costs of learning in terms of forgone income: \( (1 - T_2)F_X(1 - L) = \)
\( (1 + R)(1 - T_1) \). Fourth, marginal benefits of investing an additional unit of goods in human capital should be equal to the marginal costs in terms of forgone interest income: \((1 - T_2)WF_Y(1 - L) = (1 + R)(1 - \chi T_1)\). The marginal rate of technical substitution between time and goods invested in human capital is given by:

\[
\frac{F_X}{F_Y} = \left( \frac{1 - T_1}{1 - \chi T_1} \right) W \tag{4}
\]

To find the effects of taxation on learning we linearize the first order conditions - where lowercase variables denote log-linear deviations from an initial equilibrium, e.g. \(y \equiv dY/Y\) - except for \(t_i \equiv \frac{dT_i}{1+R}\), \(i = 1, 2\), and \(r \equiv \frac{dR}{1+R}r\):

\[
c - l = \sigma(w - t_2 + \omega_x x + \omega_y y) \tag{5}
\]

\[
y - x = w - (1 - \alpha_x) t_1 \tag{6}
\]

\[
(\omega_x - 1)x + \omega_y y - \alpha_x l = t_2 - t_1 + r \tag{7}
\]

We define Allen’s elasticity of substitution between consumption and leisure \(\sigma \equiv \frac{d \ln(C/L)}{d \ln(U_L/U_c)}\), the fraction of income spend on consumption \(\gamma_c \equiv C/M\), the fraction of income spend on goods invested in education \(\gamma_y \equiv (1 + R)(1 - \chi T_1)Y/M\). Note that \(\gamma_c + \gamma_y = 1\). Further, we define the shares of first and second period earnings in income \(\eta_1 \equiv (1 + R)(1 - T_1)W_1H_1(1 - X)/M\) and \(\eta_2 \equiv (1 - T_2)W_2H_2(1 - L)/M\), \(\eta_1 + \eta_2 = 1\). Finally, we define \(\alpha_x \equiv X/(1 - X)\), \(\alpha_l \equiv L/(1 - L)\), \(\alpha_x \equiv \chi(1 - T_1)/(1 - \chi T_1)\).

We use the envelope theorem to derive that \(\eta_1 \alpha_x - \eta_2 \omega_x = 0\) and \(\gamma_y - \eta_2 \omega_y = 0\), since a marginal change in a parameter affecting choices of \(X\) and \(Y\) does not affect life-time income. The linearized budget constraint can thus be written as:

\[
\gamma_c c = w - \eta_1 t_1 - \eta_2 t_2 + (\eta_1 - \gamma_y) r - \eta_2 \alpha_x l \tag{8}
\]

The change in time spend learning follows from solving the last four equations for \(x\) - see the appendix:

\[
x = \Delta (\gamma_c \sigma - 1 + \omega_y (\gamma_c \sigma + \psi)) w - \Delta ((\omega_y (\gamma_c \sigma + \psi) - \psi) (1 - \alpha_x) - \eta_1) t_1 - \Delta (\eta_2 - \gamma_c \sigma - \psi) t_2 - \Delta (\psi - \gamma_y + \eta_1) r \tag{9}
\]
where $\Delta \equiv -[(1 - \xi)\psi - \gamma c \sigma \xi]^{-1}$ is assumed to be positive in order to ensure stability\(^7\), and $\psi \equiv \gamma c / \alpha l + \eta > 0$. Constant returns in learning ($\xi = 1$) should be excluded in order to avoid a 'bang-bang' solution. In that case, either all time is invested in learning or all income is invested in saving depending on the returns to savings and learning. The last equation implies that taxes affect investments in human capital through a number of mechanisms that are potentially mutually reinforcing. In order to isolate the different effects of taxation we now discuss three simplified cases.

### 3 Effects of taxes on human capital formation

#### 3.1 Case 1 - The progression effect

We assume that the production of human capital requires no market goods $Y$ so that $F(X, Y) = X^{\xi}$. Further we assume that there is no labor-leisure decision: $U(C, L) = u(C)$. In that case investments in human capital are independent of consumption decisions and chosen such that $X$ maximizes lifetime income. Taxes are progressive in the sense that second period income is taxed at a higher rate than first period income, i.e. $T_1 < T_2$. The solution for learning time is:

$$x = -\Delta' \pi - \Delta' r$$

where $\Delta' \equiv (1 - \xi)^{-1} > 0$, $\pi \equiv t_2 - t_1$.\(^8\)

**Proposition 1** If there is no labor-leisure decision and learning requires no direct costs, more (less) progressive taxes, i.e. higher (lower) $\pi$, reduce (increase) investments in human capital because the benefits of learning are subject to relatively higher (lower) taxes than the costs of learning.

If the tax rates in both periods are equally affected, $\pi = 0$, learning time is unchanged.

The progression effect is driving the adverse effects on human capital accumulation in Sorensen and Nielsen (1993) and Bovenberg and Van Ewijk (1997). Boskin (1977) finds no effect of taxing human capital, because he assumes that taxes are flat.
3.2 Case 2 - The utilization effect

We assume that taxes are flat $T_1 = T_2 = T$, and production of human capital requires no market goods $F(X, Y) = X^\xi$. Solving for learning time yields:

$$x = \Delta''(\sigma - 1)(w - t) - \frac{\Delta''}{L}r$$ (11)

where $\Delta'' \equiv -[\xi \sigma - (1 - \xi)\psi]^{-1} > 0$ is assumed to get stability.

The wage elasticity of labor supply is positive (negative) if the elasticity of substitution between consumption and leisure $\sigma$ is larger (smaller) than 1. The uncompensated wage elasticity is zero if $\sigma = 1$.

**Proposition 2** If learning requires no direct costs, and the tax-system is flat, taxes reduce (increase) investments in human capital if the substitution effect in labor supply dominates (is being dominated by) the income effect in labor supply, $\sigma > 1$ ($\sigma < 1$). In that case taxes induce agents to work less, thereby lowering the returns on their human capital investments. If the substitution and income effects cancel out, $\sigma = 1$, taxes have no effect on learning decisions.

In growth models with human capital accumulation and leisure it has to be assumed that substitution and income effects of consuming more leisure should cancel out in order to arrive at a feasible steady state with positive growth rates of consumption and human capital in which the amount of leisure is constant. There are two solutions to address this problem.

First, utility functions of the following (weakly) separable form can be applied: $U(C, L) = C \exp[Z(L)]$ where $Z(L)$ is a leisure sub-utility function with $Z' > 0$, and $Z'' < 0$, see also King, Plosser and Rebelo (1988). In models with this utility function, human capital investments are only affected by taxation of labor income in the transition towards the steady state. The amount of leisure will increase (decrease) during the transition towards the steady state value in an economy with positively growing human capital if the income effect dominates (being dominated by) the substitution effect. A dominant income effect is being regarded as the empirically relevant case, as more leisure is consumed at later stages of economic development, see Barro and Sala-i-Martin (1995, ch9). It is conjectured that taxation will then positively affect investments in human capital during the transition. However, substitution and income effects cancel out in the steady state, so
that in the steady state human capital investments are not affected by taxes. In most cases, an even simpler utility function is used with a unitary elasticity of substitution: $U(C, L) = CL^\phi$, see Pecorino (1993), Trostel (1993), Jones, Manuelli and Rossi (1997). In this the leisure sub-utility function is $Z(L) = \phi \ln L$. Using this utility function in the model of this paper gives: $U_L/U_C = \phi C/L = (1 - T)WH$. It is clear that if $H/C$ is constant, so will be the amount leisure time chosen.

Second, leisure time can be rescaled with the stock of human capital to get effective leisure. This is based on Becker’s (1965) notion that a higher level of human capital increases the value of leisure time, see also Heckman (1976), Lucas (1990), Stokey and Rebelo (1995), and Milesi-Ferretti and Roubini (1998). In this case taxes affect the labor-leisure decision, but not the human capital investment decision, as the leisure choice is not affected by the level of human capital. This will be the case as long as the marginal rate of substitution (MRS) between consumption and effective leisure is not changing if consumption and stocks of human capital grow at the same rates. Therefore, preferences must satisfy additional restrictions to render this result.

To show this, we allow for effective leisure. Let utility be designated by $U(C, V(H, L))$ where $V$ denotes effective leisure. $V$ is a sub-utility function or the ‘leisure technology’. It consists of ‘raw’ leisure $L$, and human capital $H$. In the case of this paper, we can derive the following MRS between consumption and effective leisure: $U_VV_L/U_C = (1 - T)WH$. If utility is homothetic, we get that $U_V/U_L$ is constant if $H$ and $C$ are growing at constant and common rates, if $L$ is fixed, and if $\partial V/\partial L = H$. This is the case only if $V$ is linear in $H$, i.e. $V = HL$. Exactly this specification is used in the literature.

The feature that productivity of leisure time increases one for one with the stock of human capital is therefore rather special. This mechanism is essential for the neutrality of income taxes in Heckman’s (1976) analysis. It is not necessary to make this assumption in a life-cycle model, see e.g. Weiss (1986). Studies of life-cycle models have therefore found negative effects of taxing human capital income on learning, see e.g. Kotlikoff and Summers (1979), Drifil and Rosen (1983), and Hendricks (1999).
3.3 Case 3 - The deduction effect

We assume that taxes are flat $T_1 = T_2 = T$ and there is no labor-leisure decision, $U(C, L) = u(C)$. Again, investments in human capital are independent of consumption decisions and chosen such that $X$ and $Y$ maximize life-time income. Solving for learning time yields:

$$x = \Delta' \omega_y w - \Delta' \omega_y (1 - \alpha_x) t - \Delta' r$$

(12)

$\Delta' \equiv (1 - \xi)^{-1} > 0$. From the last equation it can be seen that $\omega_y$, the share of direct costs in the production function of human capital determines to what extent the non-tax deductibility reduces investments in human capital. If direct costs are fully tax-deductible, $\alpha_x = 1$, taxes do not distort the human capital decision.

Proposition 3 If there is no labor-leisure decision, and the tax-system is flat, taxes reduce investments in human capital if costs of education are not fully tax-deductible.

Lord (1989), Trostel (1993), Nerlove et al. (1993) formulate models with non tax-deductibility of goods invested in human capital. This discourages human capital formation. Heckman (1976) and Hendricks (1999) assume that goods invested in education are deductible from the income tax, so that there is no effect through this mechanism.

In growth models with human capital accumulation, taxes can affect human capital investments if the capital stock enters the production function of human capital. If capital income is taxed at a lower rate than labor income, or not taxed at all, the rental payments to capital owners makes that costs of learning are lowered less, than the benefits as a result of higher taxes on human capital income. The mechanism is thus similar as when goods enter the production function of human capital. See e.g. King and Rebelo (1990), Rebelo (1991), Pecorino (1993), Jones, Manuelli and Rossi (1993, 1997), Stokey and Rebelo (1995), and Milesi-Ferretti and Rubini (1998).

4 Conclusion

Taxes distort the human capital decision through three channels. First, higher future marginal tax rates compared with current marginal tax rates
makes that benefits future incomes are subject to higher marginal tax rates than costs (forgone earnings and direct expenditures). Therefore investment in human capital will be depressed. We call this effect the ‘progression effect’. Second, (flat) taxes may reduce labor supply if the uncompensated wage elasticity of labor supply is positive. In that case, higher taxes reduce labor supply and thus reduce the ‘utilization rate’ of human capital. Since less time will be spend working, returns on learning fall, so that less time will be invested in learning. This is called the ‘utilization effect’. Third, if direct expenditures cannot be fully deducted, (flat) taxes on human capital depress investment because benefits are subject to higher effective tax-rates than costs. This is called the ‘non-deduction effect’.

Some studies have found no effect of taxing labor income on human capital accumulation. This paper has shown that in these cases the following special conditions must be met: i) taxes are flat; ii) the uncompensated wage elasticity of labor supply is zero, or if the rate of substitution between consumption and effective leisure is not affected by the level human capital; iii) direct costs of education are either absent or fully deductible. These conditions eliminate all the potential distortionary effects of taxation on human capital accumulation.

Notes

1Essentially the same holds for growth models where the capital stock is an input in learning, rather than a flow of expenditures on goods.
2There is also a fourth effect, from which we will abstract in this paper. Eaton and Rosen (1980) analyze optimal taxation of human capital in a model with uncertainty. They show that taxation might increase learning because taxation partially insures agents against uninsurable risks. This gain is balanced against disincentives on learning caused by taxation.
3Nothing substantial changes by allowing for an endogenous first period labor-leisure choice. The presence of a well functioning capital market would ascertain that the results in this case are not affected by a first period labor-leisure choice.
4This excludes the possibility that demand for leisure is affected by tax progression as it is commonly used: an increasing average tax rate in income. One may denote this ‘within’ tax progression as this marginal tax rate only applies to a particular period of the life-cycle. Unless, of course, the govern-
ment taxes life-time income. In the model used in this paper, it suffices to use flat tax rates on income so there is no 'within' tax progression. However, allowing for 'within' tax progression does not affect our qualitative results. Given an average second period income tax rate, higher marginal tax rates induce substitution towards the consumption of leisure so that the utilization effect applies, see below.

5 This is done for analytical tractability. Qualitative results are not affected by this assumption. This specification is assumed almost everywhere in the literature - see e.g. Heckman (1976), Weiss (1986), Pecorino (1993), Trostel (1993).

6 We abstract from taxes on consumption goods. In the absence of initial income endowments, a consumption tax is equivalent to a flat labor income tax in the current model (if costs of education are deductible).

7 If $\sigma > 1$, an increase in e.g. wages leads to an increase in labor supply, this induces agents to learn more. Therefore, at higher levels of human capital investments, forgone earnings increase, so even less time will be spend on leisure. This would continue *ad infinitum*, if there were not diminishing returns in learning. The reverse reasoning holds for $\sigma < 1$. Therefore, diminishing returns in human capital accumulation should guarantee that there is stability. The same reasoning holds for the elasticity of substitution between consumption and leisure for given returns to investment in human capital. A too high elasticity $\sigma$ may induce the same vicious circle.

8 Note that $\xi = \omega_x \equiv \frac{F_{xx}}{F_x}$, and $\xi - 1 = \frac{F_{xxx}X}{F_{XX}X}$. The last expression follows from the fact that if $F$ is homogeneous of degree $\xi$, $F_X$ is homogeneous of degree $\xi - 1$.

9 This follows from labour supply:

$$ l = -\left(\frac{1-\xi}{\alpha_l}\right) \Delta''(\sigma - 1)(w - t) - \frac{1}{\alpha_l} \left(1 - \frac{(1 - \xi)\Delta''}{L}\right) r $$

We can sign the uncompensated wage elasticity of labor supply as: $(d(1 - L)/dW)(W/(1 - L) = -\alpha_l(dL/dW)(w/L) \equiv -\alpha_l dl/dw = (1 - \xi)\Delta''(\sigma - 1)$.

10 Note that this is equivalent to: $\ln U = \ln C + Z(L)$ which is also used in the literature.


12 $H$ in the utility function can also be interpreted as denoting the consumption benefits of human capital.
References


Appendix

The linearized model is:

\[ c - l = \sigma(w - t_2 + \omega_x x + \omega_y y) \]

\[ y - x = w - (1 - \alpha_x)t_1 \]

\[ (\omega_x - 1)x + \omega_y y - \alpha_x l = t_2 - t_1 + r \]

\[ \gamma_c c = w - \eta_1 t_1 - \eta_2 t_2 + (\eta_1 - \gamma_y) r - \eta_2 \alpha_x l \]

Multiply the first equation with \( \gamma_c \) and substitute for \( \gamma_c c \) in the last equation to get:

\[ \psi l = -\gamma_c \sigma \omega_x x - \gamma_c \sigma \omega_y y + (1 - \gamma_c \sigma) w - \eta_1 t_1 - (\eta_2 - \gamma_c \sigma) t_2 + (\eta_1 - \gamma_y) r \]
Use the third equation to eliminate $t$:

$$ (\psi \omega_x + \gamma_c \sigma \omega_x) x = -(\psi (\omega_y - 1) + \gamma_c \sigma \omega_y) y + (1 - \gamma_c \sigma - \psi) w $$

$$ -\eta_1 t_1 - (\eta_2 - \gamma_c \sigma - \psi) t_2 + (\eta_1 - \gamma_y + \psi) r $$

Now, use the second equation to substitute out $y$ in order to arrive at the equation in the text.