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Being tolerant about identity?

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Abstract

Are some identity statements vague for ontic reasons? Are some contingent-identity statements true? Is there a notion of relative identity such that two things are identical relative to one sortal, but not with respect to another? I don't know. What I do know, though, is that all these claims *can* be made true, and are all consistent with Leibniz' law being a validity. I show how this is possible by making use of recent ideas for the modeling of vagueness; one that accepts the tolerance principle as a validity. I will show that being tolerant about the notion of consequence allows (though not forces) one to be tolerant about identity as well.

1 Introduction

Identity and identification are very important concepts in philosophy and logic. They are crucial for the analysis of quantification and for counting. As already remarked by Frege, one cannot even start counting before one knows how to identify objects. For all its importance, identity still seems like a very straightforward relation. It is the relation that an object has, and only has, towards itself. The relation can be characterized as the *unique* relation that is reflexive and satisfies Leibniz' law, or *the principle of indiscernibility of identicals*: $\forall x, y, \forall P((Px \wedge x = y) \rightarrow Py)$.¹ Indeed, this so-called *absolutist*' view seems to be the dominant view among philosophers, and is prominently defended, for instance, by Kripke (1971).

Despite its naturalness and simplicity, the absolutist' view has not gone unchallenged. First of all, at first sight Leibniz' law seems to be obviously false. For can't I imagine this table to have been somewhere else? Also, it may happen that you meet someone at two occasions, and that at the first occasion he is healthy, but that at the second occasion he is not. The latter problem can easily be dissolved, however, by stipulating that properties of objects include an index of time, e.g. healthy at t , sick at t' . In a similar way we can account for the fact that it is *contingent* that the first man was not sick at time t , for we could imagine *the same man* to have been sick at time t as well. Notice that once we acknowledge the latter, we admit individuals to have

*I would like to thank Pablo Cobreros, Paul Egré and Dave Ripley. In our joint work we developed our non-transitive analysis of vagueness and truth. Together we also published a paper on ontic vagueness (Cobreros et al, 2013), of which a slightly different version of current paper was actually a (too long) early draft. Especially discussions with Pablo Cobreros helped me to (hopefully) better represent some of the ideas in this paper. I would also like to thank Manfred Kupffer for his useful comments on this paper. Finally, I would like to thank Ede Zimmermann for the many discussions we had during the time I was his PhD student on issues related to the ones dealt with in this paper.

¹It can easily be shown that this is equivalent to $\forall x, y(x = y \rightarrow \forall P(Px \leftrightarrow Py))$.

not just temporal, but also modal properties. We could start saying that although this table occupies this space-time-region, it is *not identical* with it, because the table could have occupied a different space-time region. The existence of such so-called *de re* modalities was disputed by Quine (according to whom we should identify this table with the space-time region it occupies), but even he had to admit that we attribute *de re* beliefs to individuals. He even constructed a well-known example—involving Ralph’s beliefs about Ortcutt—that either seems to be incompatible with the Law of Non-Contradiction, or is straightforwardly in conflict with Leibniz’ law. Soon, others argued that similar examples of so-called *contingent identity* can be constructed involving other modalities. What these examples seem to show is that we don’t simply talk about individuals *per se*, but only about individuals *under a certain guise*.

According to some philosophers, many examples that are supposed to show that identity is contingent, in fact show that the notion of identity is *relative*. But the idea behind this view is very similar to the earlier one: Identity depends on the method of conceptualisation, and—or so it is argued—different methods might be incompatible with each other. A well-known proponent of the latter view is Geach. He argued, for instance, that although the cat on the mat *with*, and the cat on the mat *without* hair h_i might be different lumps of feline tissue, they are still the same cat. If we would make the assumption—or so it is argued—that if being the same cat made two objects thereby also *absolutely* identical, this same object would have as a result two properties that are evidently incompatible with each other, even without considering temporal or modal properties. Thus, it seems one has to give up on the notion of absolute identity, on pain of being in conflict with Leibniz’ law. But Geach’s cat on the mat has also been used as a prominent example to ‘prove’ the existence of *vague identity*: to answer the same threat, perhaps one can still assume that there is only one notion of identity, but it is just that this one notion is vague.

Of course, proponents of absolute identity want to hear nothing like it. The so-called examples of contingent-, relative-, and vague-identity *have* to be wrong. And they can even *prove* this, making use only of rules and principles (like abstraction and Leibniz’ law) that everybody will (or should) accept (or so they argue). From these proofs one can conclude that identity is a once-and-for-all *absolute* relation, and thus that proponents of contingent, vague, and relative identity must have made a *logical* mistake. So this should have settled the matter, shouldn’t it?

Well, it didn’t. As the saying goes: one’s modus ponens is the other’s modus tollens. And indeed, oponents of the absolutist’ view can say that the only thing the proofs show—given the additional empirical evidence—is that it doesn’t make sense to speak of individuals or objects independent of how they are conceptualized. But on this view the examples mentioned above challenging the absolute identity view, are now examples that challenge Leibniz’ law. But Leibniz’ law should be upheld, or so almost everybody would say.

In order to make sense of objects identified over times and otherworldly possibilities, Lewis and others have argued that *similarity* plays a major role: in order to identify x at one possibility with y at another, it seems natural to demand that they must at least be similar to each other in one or more crucial aspects. But similarity gives rise to its own problem: the Sorites paradox. The Sorites paradox is based—among others—on the acceptance of the *Tolerance Principle*: if x has property P and x is similar (with respect to P) to y , then y should have property P as well. It is well-known, however, that the acceptance of this principle in addition to other seemingly uncontroversial assumptions leads to absurdities: every object has property P

and not property P . Many theorists about vagueness have concluded that it is the tolerance principle that is to blame, and the assumption that should be given up. Notice, however, that the Tolerance principle $\forall x, y((Px \wedge x \sim_P y) \rightarrow Py)$ is in fact almost identical with Leibniz' law, $\forall x, y((Px \wedge x = y) \rightarrow Py)$!² Thus, the standard reactions to puzzles of identity and vagueness brings us in the rather peculiar and unsatisfactory situation that although Leibniz' law is taken to be unsuspicious, the almost identical Tolerance principle is almost unanimously abandoned.

Isn't it more natural that we should either give up both the Tolerance principle and Leibniz' law—even though both seem to express obvious truths—or keep both principles, and blame other assumptions to be responsible for the derived absurdities in reasoning involving identity and similarity? Indeed, it is. And it is the latter line that I will advocate in this paper.³

2 Contingent identity

Leibniz' principle is closely related to the *principle of substitution*. According to the substitution principle, if two expressions have actually the same denotation, they should be substitutable for each other without change of truth value. In contrast to Leibniz' principle, it is almost uncontroversial that once we allow for modal and temporal contexts, the substitution principle has to be given up. And there are well known examples showing this. Why is the falsity of this principle so uncontroversial? The reason is that the substitution principle involves *expressions*, and it is uncontroversial that expressions that have a *descriptive meaning* like 'the number of planets' and 'the Morning star' can denote different objects in different possible worlds. Leibniz' principle, in contrast, is not about the meaning of expressions, but is only supposed to be about the *objects* themselves. According to the absolutists, once we talk about the objects themselves it only makes sense to assume Leibniz' principle. But once this is done, one can prove that identity is a rigid relation. Here is a famous proof due to Kripke (and Barcan Marcus), showing that once ' $x = y$ ' is true, one has to admit that this is necessarily so (where ' $\Box$ ' means 'necessarily'):

(1) $x = y \vdash \Box(x = x)$	(Truism)
(2) $x = y \vdash \widehat{\Box}(x = z)(x)$	(Abstraction)
(3) $x = y \vdash \Box(x = y)$	(by (1) and (2), and Leibniz' law)
(4) $\vdash x = y \rightarrow \Box(x = y)$	(conditional proof)

If one assumes that ' $\Box$ ' is interpreted in terms of an accessibility relation that is *symmetric*, and we assume that $\Diamond$ is the dual of $\Box$ such that $\Diamond\phi \equiv \neg\Box\neg\phi$, the following so-called Brouwersche principle is valid: $\neg\phi \rightarrow \Box(\neg\Box\phi)$. If we assume that ' $\Box$ ' gives rise to the rule of necessitation, i.e., that if ϕ is provable, it follows that $\Box\phi$ holds, we can follow Wiggins (1980) and derive from the necessity of identity also the necessity of non-identity, i.e., that if $x \neq y$, then $\Box(x \neq y)$.⁴

²If $x \sim y$ means ' x appears identical to y ', ' $\sim$ ' can even be turned into ' $=$ ', if x and y are turned into their appearances.

³This paper can be thought of as a follow-up of Cobreros et al (2013), although it was written earlier, and sometimes takes a different position.

⁴See Williamson (1999) for more discussion.

Still, even when we are talking about the objects themselves, Leibniz' principle can be challenged on the basis of so-called 'modal examples'. Lewis (1976) argues, for instance, that a person is identified with his body, although this identity is contingent. Or consider Gibbard's (1975) famous example of a statue, named 'Goliath', and the clay out of which it is composed, named 'Lumpl'. It is natural to assume that the statement 'Lumpl = Goliath' is true. Still, the identity seems to be only contingently true, because the piece of clay might have been rolled into a ball and turned into a new, very different statue, named 'David'. Thus, afterwards the truth of the original identity statement is replaced by the truth of a new identity statement: 'Lumpl = David'. Of course, an absolutist will deny that this new and the old identity statements are, or were, true, and they will appeal to Leibniz' law: Lumpl cannot be identified with Goliath, because only Lumpl could be transformed into David. Although defensible, this view is not unproblematic either: does it really make sense to claim that objects that at one time occupy the same space and are made of exactly the same material can nevertheless be distinct? It is easy to account for the intuitions that contingent identity is possible by assuming that the names denote individual concepts, but that the truth of identity statements only involves extensions. If one does not want to do that (by assuming with Kripke that proper names denote rigidly), one could still make use of individual concepts by assuming that quantifiers range over them. The contingent identity of 'Lumpl = Goliath' can then be analyzed as $\exists x, y(x = l \wedge y = g \wedge x = y \wedge \Diamond(x \neq y))$. More generally, contingent identity is then characterized by the truth of $\exists x, y(x = y \wedge \Diamond(x \neq y))$.

Unfortunately, quantifying over individual concepts is well-known to be problematic, both for an empirical and for a conceptual reason. To start with the former, notice that to assume that quantifiers quantify over *all* individual concepts gives rise to the prediction that $\forall x\phi \rightarrow \phi[\iota yPy/x]$ is valid (where ιyPy denotes the unique individual that satisfies atomic or complex predicate P). The reason for this is that a definite description denotes something equivalent to an individual concept. Unfortunately, this principle seems to be false in many modal contexts. From the truth of 'All numbers larger than 9 are necessarily larger than 7' one cannot conclude to 'The number of planets is necessarily larger than 7', although the principle predicts this to be ok (given that 'the number of planets' denotes a number larger than 7). Perhaps the conceptual worry of quantifying over all individual concepts is even more problematic: the worry is that *de re* modal statements are secretly turning into *de dicto* ones. As a result, $\forall x, y(x = y \rightarrow \Box(x = y))$ is no longer predicted to be valid.

Kripke's proof of the necessity of identity is appealing, but so are the examples involving contingent identity. This gives rise to the question whether it is even possible to have a semantics for quantified modal logic which predicts that ' $\forall x\forall y(x = y \rightarrow \Box(x = y))$ ' is valid, even though ' $\exists x\exists y(x = y \wedge \Diamond(x \neq y))$ ' can still be true. Using classical logic, it is not, because if the first is valid, the second cannot be satisfiable. However, I will argue in favor of a modification of the classical notion of logical consequence which makes this possible.

3 Relative identity

Geach has argued that identity is always *relative identity*. Geach argues for the following three theses: (i) There does not exist a notion of absolute identity; (ii) complete identity statements are always of the form ' x is the same F as y ', ' $x =_F y$ ', where F stands for a count-noun; and (iii) it is possible that ' $x =_F y$ ' is true and

' $x =_G y$ ' is false, when F and G are distinct count nouns. Counting presupposes the notion of identity, and Geach motivates theses (i) and (ii) by what Frege says about counting: one cannot count apart from counting some kind of thing. Whether there exists an absolute notion of identity is highly controversial, and so is Geach's motivation against it by appealing to Fregean arguments. We will only be concerned with thesis (iii) (assuming that a notion of relative identity is at least useful): that there can be individuals x and y , and sortals F and G for which it holds that ' $x =_F y$ ' is true, but ' $x =_G y$ ' is false. One proposed example that shows this pattern we have seen already, and involves Gibbard's (1975) piece of clay. Recall that according to this example, the statue Goliath is identified with the piece of clay it is composed of, even though this piece of clay might turn into another statue, David. According to identity-relativists one should analyze this example in terms of just two objects, David and Goliath, which are identical in terms of constitution (both are made of the same piece of clay), $d =_C g$, but not identical when thinking of the objects as statues, $d \neq_S g$. Many other examples have been proposed in the literature as well. Wiggins (1980), however, has argued that existence of the pattern $\exists x, y (x =_F y \wedge x \neq_G y)$ is impossible if F and G are sortals, because from the assumption that ' $=_G$ ' is reflexive and the truth of ' $x =_F y$ ' one can derive ' $x =_G y$ ' for any G , when one also assumes the following version of Leibniz' law: $\forall x, y, G ((Gx \wedge \exists F (x =_F y)) \rightarrow Gy)$.

1. $x =_F y \vdash x =_G x$ (Truism)
2. $x =_F y \vdash \widehat{z}(x =_G z)(x)$ (Abstraction)
3. $x =_F y \vdash x =_G y$ ((1), (2), and Leibniz' law)
4. $\vdash x =_F y \rightarrow x =_G y$ (Conditional proof)

But still, isn't it natural to say that in Gibbard's example David and Goliath are *the same* in one sense (constitution) but *different* in another (statue)? Notice that as far as Gibbard's example is concerned, it doesn't really seem to matter much whether we analyze it in terms of relative identity or in terms of contingent identity. In both ways we account for the intuition Gibbard pointed to. The reason that such cases of claimed 'relative identity' can be analyzed as contingent identity is that David and Goliath are objects that 'live' in different possibilities (times), and that David and the piece of clay it is composed of have different *modal properties*. Unfortunately, other examples of claimed 'relative identity' exist that cannot be (naturally) analyzed in terms of contingent identity. Such examples—e.g., Geach' (1980) example of the 1001 cats—are cases where modalities don't play any essential role.

4 Vague identity

Geach's example of the 1001 cats is seen by some not as an example of relative identity, but of *vague identity*, instead. There can be little doubt that some identity statements are vague. But what is the source of this vagueness? Standardly, it is assumed that a sentence like 'John is tall' is vague, because the predicate 'tall' is vague. But does it make sense to say that ' $s = t$ ' is vague, because in different languages/models, identity means something different? The natural idea would be that if an identity statement is vague, the vagueness concerns what is denoted by the terms ' s ' and ' t '. But this still leaves open two possibilities: either one of the two *terms* is vague, or the terms are not vague, but what is denoted by (at least one of) the terms is vague. The latter

would be a case of *ontic vagueness*. Yet another possibility is that identity statements are vague, because identity itself is vague. Also this would fall under the heading of ontic vagueness. Many philosophers find ontic vagueness absurd. Moreover, vague identity due to ontic vagueness cannot exist, or so it is argued. That is, one can prove that if $x = y$ holds, then it cannot hold vaguely so. To illustrate this, let us say that an identity statement is vaguely true iff ' $\nabla x = y$ ' holds. We assume that ∇ is the dual of Δ such that $\nabla\phi \equiv \neg\Delta\neg\phi$, where ' $\Delta\phi$ ' means that it is determinate whether ϕ . Then one can easily show that $\forall x, y(x = y \rightarrow \Delta(x = y))$ is valid, using Leibniz' law.

- (1) $x = y \vdash \Delta(x = x)$ (Truism)
- (2) $x = y \vdash \widehat{\Delta}(x = z)(x)$ (Abstraction)
- (3) $x = y \vdash \Delta(x = y)$ (by (1) and (2), and Leibniz' law)
- (4) $\vdash x = y \rightarrow \Delta(x = y)$ (conditional proof)

Perhaps the proof shows that to make sense of vague identity, one has to give up on Leibniz' law (Parsons, 2000). Of course, this was not the standard reaction to Evans' (1980) proof, of which the above argument is a close relative. The standard reaction was to conclude that Δ creates a modal context for which abstraction is not valid, because singular terms might have different denotations in different possibilities (Lewis, 1988). We take the above reasoning to be sound, and so we accept the assumption in the proof that variables are 'rigid' expressions (or better, perhaps, that Δ does not create a modal context). We also accept Leibniz' law, thus we accept the validity of $\forall x, y(x = y \rightarrow \Delta(x = y))$. Still, we want vague identity to be possible, such as in the well-known example of Theseus ship. How to account for that?

5 Vagueness

The case of Theseus ship (TS) is like that of sorites paradoxes but this time for identity. Vague predicates seem to be *tolerant* in the sense that a small enough difference in the relevant properties of two objects cannot make a difference in the applicability of the predicate. Suppose we've got a first-order language with similarity relations, $\sim_P$, for each predicate ' P ' (similarity relations are reflexive, symmetric but possibly non-transitive). Should we take that, as vague predicates, identity is tolerant? For vague predicates one can formulate the idea of tolerance in at least two ways ('tolerance principle' and 'tolerant reasoning'):

$$\mathbf{TP} \quad \forall x \forall y ((Px \wedge x \sim_P y) \supset Py) \qquad \mathbf{TR} \quad a \sim_P b, Pa \models Pb$$

The following two principles correspond to this for identity (for a richer language with '=')

$$\mathbf{TP_I} \quad \forall x \forall y ((TS = x \wedge x \sim y) \supset TS = y) \qquad \mathbf{TR_I} \quad a \sim b, a = TS \models b = TS$$

For a Gap theorist endorsing a K3 Kleene logic, the tolerance principle TP(I) is untrue. For an Epistemicist, endorsing classical semantics and logic, the tolerance principle is false. Also supervaluation theory predicts TP(I) to be false. Seemingly, the only roughly tolerant solution is the Dialetheist (Priest, 1979), but she endorses TR(I)

at the price of sacrificing Modus Ponens and the deduction theorem (for material implication). The case of identity is, however, peculiar and there is a deeper reason for intolerance. The “received view” on Sorites reasoning with identity statements takes it that TPI is untrue, *but only due to semantic indeterminacy*. If there were no issue of reference indeterminacy (suppose that for each t in the sequence, it is semantically determinate whether $t = TS$), TPI would not just be untrue, but *false*. The received view, then, is that vagueness with identity is *intolerant*. Either because identity is not tolerant for the same reason as vague predicates are not tolerant: accepting tolerance requires dubious logical commitments. Or because there cannot be vague objects due to the fact that identity is assumed to be a transitive relation. In this paper I try to dispute this way of seeing things. I show how to extend our account of tolerance to identity, showing this way that identity can be made tolerant avoiding the logical commitments of dialetheism.

Tolerant Logic In (TCS, Cobreros et al (2012, 2013b, 2014)) a Tolerance Logic was formulated to deal with vagueness (and truth). We might formulate the logic making use of three truth-values and Kleene’s strong valuation schema:

$$\mathbb{I}(Pa) \in \{1, \frac{1}{2}, 0\} \quad \mathbb{I}(A \wedge B) = \min(\mathbb{I}(A), \mathbb{I}(B)) \quad \mathbb{I}(\neg A) = (1 - \mathbb{I}(A))$$

The universal quantifier generalizes conjunction and the other connectives are defined in a standard way. We say that A is *strictly satisfied* in a model when $\mathbb{I}(A) = 1$ and *tolerantly satisfied* when $\mathbb{I}(A) > 0$. Logical consequence is the absence of a countermodel, but in this semantics there are several ways of defining the notion of a countermodel. If a countermodel for an argument is a model in which all premises take value 1 and no conclusion takes value 1 this leads to the Kleene logic K3. If a countermodel is a model in which every premise takes value more than 0 and no conclusion takes value more than 0, this leads to the logic LP (Priest, 1979). We consider, however, a further alternative in which both strict and tolerant satisfaction play a role. A countermodel is a model where all the premises take value 1 and no conclusion takes value more than 0 (that is, a countermodel is a model where all the premises are strictly true and no conclusion is even tolerantly true). This leads to the logic we call ST. A remarkable feature of ST is that it is classical logic for the classical vocabulary (that is, over the classical vocabulary, $\Gamma \models^{st} \Delta$ iff $\Gamma \models^{cl} \Delta$). Our accommodation of tolerance deals with the interplay of vague predicates with similarity relations. So we restrict our attention to models in which $\sim$ is interpreted as a similarity relation (models where $\mathbb{I}(a \sim_P b) = 1$ iff $|\mathbb{I}(Pa) - \mathbb{I}(Pb)| < 1$). Call the restriction of $\models^{st}$ to these models ‘ $\models_{\sim}^{st}$ ’. An immediate consequence of this is that $\models_{\sim}^{st}$ is an extension of $\models^{st}$ (thus, classically valid inferences, such as MP, are preserved). Now TR is valid in $\models_{\sim}^{st}$ and TP is tolerantly true in every model (and so $\models_{\sim}^{st}$ -valid). This shows that, contrary to the common opinion, one need not deviate too much from classical logic in order to accommodate tolerance: $\models_{\sim}^{st}$ is fully classical, except that... it is *not* a *transitive* relation. (In TCS a complete proof-systems is provided for $\models_{\sim}^{st}$.)

As noted before, the tolerance principle ($\forall x, y((Px \wedge x \sim_P y) \rightarrow Py)$) is in fact almost identical with Leibniz’ law ($\forall x, y((Px \wedge x = y) \rightarrow Py)$). We have seen in this section that by making use of 3-valued logic and our notion of tolerant entailment, we are on the one hand able to preserve the tolerance principle, while on the other hand still account for vagueness: the fact that there exists a sorites sequence of pairwise P -similar objects such that the first element has property P while the last one does not. This gives rise to the question whether we can do something similar with respect

to Leibniz' law: On the one hand preserve the validity of Leibniz' law, and as a consequence validate all of (i) $\forall x, y(x = y \rightarrow \Box(x = y))$, (ii) $\forall x, y, \forall P, Q(x =_P y \rightarrow x =_Q y)$, and (iii) $\forall x, y(x = y \rightarrow \Delta(x = y))$. But on the other hand, still account for (i) contingent identity, (ii) relative identity, and (iii) vague identity due to ontic vagueness. The purpose of the rest of the paper is to show that this is indeed possible.

6 Vague identity revisited

Consider now $\mathcal{L}^=$ languages. These are $\mathcal{L}^\sim$ -languages extended with the identity symbol '='. In the same way in which we allowed two different readings of predicates, a strict and a tolerant reading, we will now also allow for two different readings of identity. $a = b$ is strictly satisfied in a model for $\mathcal{L}^=$, written $\mathbb{I}(a = b) = 1$, iff $\mathbb{I}(a) = \mathbb{I}(b)$. $a = b$ is tolerantly satisfied in a model for $\mathcal{L}^=$, written $\mathbb{I}(a = b) > 0$, iff $\mathbb{I}(a) \sim \mathbb{I}(b)$ (for now it is assumed that ' $\sim$ ' is a primitive similarity relation that it is reflexive, symmetric but need not be transitive). As usual, the definitions of satisfaction and logical consequence carry over from previous models to models for $\mathcal{L}^=$.

We propose the following. First, we say that for complex predicate, $\hat{x}A$ it holds that $\mathbb{I}(\hat{x}A(\underline{d})) = \mathbb{I}(A[\underline{x}/\underline{d}])$.⁵ Second, $\mathbb{I}(\Delta A) = 1$ iff $\mathbb{I}(A) \in \{0, 1\}$, 0 otherwise. ∇A abbreviates $\neg\Delta\neg A$.

Let us now see how our analysis avoids the fatal contradiction in Evans' (1980) proof (Evans' argument only goes from 1 to 5, assuming them already to be problematic enough):

Table 1: Evans' argument

1	$\nabla(x = y)$	(Assumption)
2	$\hat{z}[\nabla(x = z)](y)$	(From 1, by abstraction)
3	$\neg\nabla(x = x)$	(Assumption, since $a = a$ is a logical truth!)
4	$\neg\hat{z}[\nabla(x = z)](x)$	(From 3, by abstraction)
5	$\neg(x = y)$	(From 2, 4, by Leibniz' law and Contraposition)
6	$\neg\nabla(x = y)$	(From 5 and meaning of ' ∇ ')

Notice that according to our analysis, abstraction preserves truth value, and Leibniz' Law is *st*-valid, but it is not *K3*-valid. As a result, we predict that if 1 and 3 are strictly true, 5 has to be tolerantly true. Thus, the inference from 1 and 3 to 5 is *st*-valid. Likewise for the inference from 5 to 6. However, these inferences cannot be joined together, as the following counterexamples illustrates: $M = \langle D, \mathbb{I}, =, \sim \rangle$ with $D = \{a, b\}$, $a \neq b$, but $a \sim b$.

7 Contingent identity again

In this section we will account for contingent identity by making use of *sortal* variables with a constraint on their interpretation. That x is a lump of clay, we will express as x_l , and that y and z are statues, we will express by y_s and z_s . We will assume that during time (or possible worlds) the lump of clay stays the same, but that in

⁵We use the substitution analysis for convenience, and assume that $\underline{d}$ names d .

Table 2: The impossibility of contingent identity

1	$\Diamond(x \neq y)$	(Assumption)
2	$\widehat{z}[\Diamond(x \neq z)](y)$	(From 1, by abstraction)
3	$\neg\Diamond(x \neq x)$	(Assumption, since $x = x$ is a logical truth!)
4	$\neg\widehat{z}[\Diamond(x \neq z)](x)$	(From 3, by abstraction)
5	$\neg(x = y)$	(From 2, 4, by Leibniz' law and Contraposition)
6	$\neg\Diamond(x = y)$	(From 5 and Wiggins' proof)

Again, 2 and 4 follow from 1 and 3, and from this 5 can be derived. 6, in turn, can be derived from 5. But the two arguments cannot be joined together.

Because '=' is reflexive and satisfies Leibniz' law, it is also symmetric and transitive, i.e. both $\forall x_k, y_l (x_k = y_l \rightarrow y_l = x_k)$ and $\forall x_k, y_l, z_m ((x_k = y_l \wedge y_l = z_m) \rightarrow x_k = z_m)$ are *st*-valid. In fact, in this case we predict that it is not possible that $x_k = y_l \wedge y_l = z_m \wedge x_k \neq z_m$ can still be tolerantly true. We will see that things are different in the case of relative identity.

8 ... and relative identity

We propose the following analysis of relative identity:

- $\mathbb{I}_{s,g,C}(x_k =_P y_l) = 1$ iff $\mathbb{I}_{s,g,w}(P(x_k) \wedge P(y_l) \wedge x_k = y_l) = 1$ and $k = p = l$
 > 0 iff $\mathbb{I}_{s,g,w}(\exists z_p (P(z_p) \wedge x_k = z_p \wedge y_l = z_p)) > 0$.⁷

Of course, this analysis is not really in the spirit of Geach's argument at that relative identity is the best we can have, because we have interpreted relative identity in terms of its non-relative counterpart. But still, it is easy to see that in terms of this definition we can account for the relative identity concerning Lump, David, and Goliath. So what goes wrong in the following argument?

Table 3: The impossibility of relative identity

1	$(d_S =_C g_S)$	(Assumption)
2	$(d_S =_S g_S)$	(From 1 and (relative version of) Leibniz' law)
3	$(d_S \neq_S g_S)$	(Assumption)
4	$(d_S =_C g_S) \wedge (d_S \neq_S g_S)$	(Contradiction from 2 and 3)
5	ϕ	(From 4 and explosion)

Again, all steps are valid, but we cannot combine them. David and Goliath would be identical pieces of clay, $d_S =_C g_S$, but not identical statues, $d_S \neq_S g_S$. On the other hand, because d_S and g_S are not of the sort C , but are the same piece of clay, both statements $d_S =_C g_S$ and $d_S \neq_S g_S$ will receive value $\frac{1}{2}$. If these were the only

⁷We assume that $x_k = z_p$ can have value $\frac{1}{2}$ with respect to s, g and C for two reasons. First—and most obviously—because of contingent identity, i.e. $[[x_k]]^{s,g,C} = [[z_p]]^{s,g,C}$, but $k \neq p$. Second—and more important later in this section—because the identity between $[[x_k]]^{s,g,C}$ and $[[z_p]]^{s,g,C}$ is vague, as discussed in section 6.

three objects in our domain, the formula $\exists x_s, y_s (x_s =_C y_s \wedge x_s \neq_S y_s)$ would indeed be counted as (tolerantly) true.

Notice that although $\exists x_s, y_s (x_s =_C y_s \wedge x_s \neq_S y_s)$ is tolerantly true in our imagined situation, it still holds that $\forall P, Q, \forall x, y (x =_P y \rightarrow \forall Q (x =_Q y))$ will be *st*-valid, and thus also $(x_s =_C y_s) \rightarrow (x_s =_S y_s)$. How can that be? The reason is that in our situation $x_s =_C y_s$ is only tolerantly true, not strictly. As a result, the relative version of Leibniz' law can be *st*-valid even though $x_s =_S y_s$ is not even tolerantly true.

Encouraging as the analysis might be, it still seems to give rise to *counting problems*. Although Gibbard's case seems to involve only one piece of clay, it seems we have to admit that there are two such pieces: $\exists x_k, y_l (C(x_k) \wedge C(y_l) \wedge x_k \neq y_l)$ will now be strictly true, because of David and Goliath. But, then, recall that one of the motivations for the existence of relative identity is that counting presupposes prior conceptualization. As a consequence, it seems only natural to claim that the above formula should not be used to express that there are at least two pieces of clay. To express this, it is much more natural to make use of relative identity as follows: $\exists x_k, y_l (C(x_k) \wedge C(y_l) \wedge x_k \neq_C y_l)$. This sentence is now counted as tolerantly true, but not strictly true. Moreover, $\forall x_k, y_l ((C(x_k) \wedge C(y_l)) \rightarrow x_k =_C y_l)$ will not be strictly true, but still tolerantly true. Thus, 'There is exactly one piece of clay' will be (tolerantly) true. Unfortunately, this is also the case for 'There are exactly two pieces of clay' and even for 'There are exactly three pieces of clay'. In a sense, the present analysis would only worsen Geach's problem of the 1001 cats. Fortunately, there is yet another natural possibility to represent counting sentences: we can represent that there are at least two pieces of clay, or cats, by the following formula: $\exists x_c, y_c (C(x_c) \wedge C(y_c) \wedge x_c \neq_C y_c)$.⁸ But how does this help? Indeed, so far it doesn't: David and Goliath would still make this sentence tolerantly true. The reason is that both are counted as pieces of clay, and we have not yet ruled out the possibility that Goliath might be the interpretation of x_c . But our sortal variables allow us to do exactly that. We can make the following extra assumptions:⁹

- For all sorts k matching predicate K and for all states s : $c_s^k(g(x_k)) \in \mathbb{I}_s(K)$. This validates $\forall x_k \Box K(x_k)$.¹⁰
- For all sortals k, l s.t. $k \neq l$, there is an $s \in S$: $c_s^k(g(x_k)) \neq c_s^l(g(x_k))$. This validates $\forall x_k \Diamond \neg L(x_k)$.

Thus, Goliath and David, as being statues, no longer can be the values of the sorted variable x_c ; only our one lump of clay can. Now we correctly predict that 'There is exactly one piece of clay' is strictly true, just as we want it to be.

Now, how does this analysis help us to account for the problem of the 1001 cats on the mat? Think of lumps of feline tissues t_1 and t_9 . They are not the same lumps of feline tissue. But we still want to say that they are the same cat: $t_1 \neq_T t_9$, but $t_1 =_C t_9$. This is immediately accounted for once we think of the lumps of feline tissues and the cat as separate objects, just like we thought of David, Goliath, and Lump1.¹¹ But now it seems crucial that ' $t_1 =_C t_9$ ' will have a value higher than 0 because the cat c is now vaguely identified with both t_1 and with t_9 .

⁸Similarly for 'There is at most one piece of clay': $\forall x_c, y_c ((C(x_c) \wedge C(y_c)) \rightarrow x_c =_C y_c)$.

⁹It is natural to think of sortals now as something like Aristotelian secondary substances.

¹⁰Of course, it doesn't validate the more general $\forall x_l (K(x_l) \rightarrow \Box K(x_l))$, even if K is a sortal predicate. Indeed, Lump1 is a statue now, though it doesn't have to be one.

¹¹Indeed, this can be immediately accounted for if we think of the 'objects' of state s not simply as elements of $D(s)$, but rather as elements of O , which is the following set of individual concepts: $O = \{\lambda s. c_s^k(d) : d \in D(s) \text{ and } k \text{ a sortal}\}$.

Notice that because relative identity is reflexive and satisfies Leibniz' law, it is also symmetric and transitive, i.e. both $\forall x_k, y_l (x_k =_F y_l \rightarrow y_l =_F x_k)$ and $\forall x_k, y_l, z_m ((x_k =_F y_l \wedge y_l =_F z_m) \rightarrow x_k =_F z_m)$ are *st*-valid. Even $\forall x_k, y_l (x_k =_F y_l \rightarrow y_l =_G x_k)$ and $\forall x_k, y_l, z_m ((x_k =_F y_l \wedge y_l =_G z_m) \rightarrow x_k =_Z z_m)$ are predicted to be *st*-valid.¹² In contrast to the case of identity discussed in the previous section it is now possible, however, that $x_k =_F y_l \wedge y_l =_F z_m \wedge x_k \neq_F z_m$ can still be tolerantly true.

9 Conclusion

Are some identity-statements vague for ontic reasons? Are some contingent-identity statements true? Is there a notion of relative identity such that two things are identical relative to one sortal, but not with respect to another? I don't know. What I do know, though, is that all these claims *can* be made true, and are all consistent with Leibniz' law being a validity. Being tolerant about the notion of consequence allows (though not forces) one to be tolerant about identity as well.

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¹²Although transitivity for a very simple reason.

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