Constraints on the annihilation cross section of dark matter particles from anisotropies in the diffuse gamma-ray background measured with Fermi-LAT

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Annihilation of dark matter particles in cosmological halos (including the halo of the Milky Way) contributes to the diffuse gamma-ray background (DGRB). As this contribution will appear anisotropic in the sky, one can use the angular power spectrum of anisotropies in the DGRB to constrain the properties of dark matter particles. By comparing the updated analytic model of the angular power spectrum of the DGRB from dark matter annihilation with the power spectrum recently measured from the 22-month data of the Fermi Large Area Telescope (LAT), we place upper limits on the annihilation cross section of dark matter particles as a function of dark matter masses. We find that the current data exclude \( \sigma_{\alpha\nu} \approx 10^{-25} \text{ cm}^3 \text{s}^{-1} \) for annihilation into \( bb \) at the dark matter mass of 10 GeV, which is a factor of 3 times larger than the canonical cross section. The limits are weaker for larger dark matter masses. The limits can be improved further with more Fermi-LAT data as well as by using the power spectrum at lower multipoles (\( \ell \approx 150 \)), which are currently not used due to a potential Galactic foreground contamination.

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I. INTRODUCTION

Understanding the identity and nature of dark matter, which makes up more than 80% of the total matter density in the Universe, is a major goal of modern physics and cosmology. The most promising candidate for dark matter is weakly interacting massive particles (WIMPs), with which one can naturally explain the observed dark matter density using a simple thermal freeze-out argument [1]. If dark matter particles annihilate into standard model particles, as expected for most WIMP scenarios, one can indirectly detect and constrain properties of dark matter particles [1–3]. In this paper, we shall focus on high-energy (1–50 GeV) gamma-ray photons produced by the cascade of annihilation products.

Dark matter annihilation occurs in all cosmological halos, including the halo of the Milky Way, and thus contributes to the diffuse gamma-ray background (DGRB) [4–8]. Due to the large-scale structure of the Universe, the observed gamma-ray emission appears anisotropic in the sky in a predictable manner, which makes it easy to identify the dark matter origin of high-energy gamma rays in the sky [9] (also see Refs. [10–26]).

Recently, the Fermi-LAT Collaboration has measured the power spectrum of DGRB anisotropy from 22 months of data [27]. They have detected significant excesses of the angular power spectrum over the shot noise of photons for a multipole range between \( \ell = 155 \) and 504 and for multiple energy bins. A further study shows that most of these excesses come from unresolved blazars [28]. Subtracting the estimate of the blazar contribution, we have upper bounds on the residual anisotropy of the DGRB.

In this paper, we use the upper bounds on the power spectrum to constrain the annihilation cross section of dark matter particles. For this purpose, we update our theoretical framework for computing the angular power spectrum presented in Refs. [9,10,20] as follows:

1. We use the results from recent numerical simulations (e.g., Ref. [29]) to model the mass function and spatial distribution of subhalos within a given host halo.

2. We include contributions from both the extragalactic dark matter halos and the Galactic dark matter subhalos.

3. We also include the cross correlation between dark matter annihilation signals and blazars. Although this term was often ignored in the literature (except for Ref. [10]), one should include this term for self-consistency: the same halo hosting a blazar also contains annihilating dark matter particles, and there is a spatial correlation between halos hosting blazars and those not hosting blazars but containing annihilating dark matter particles.

This paper is organized as follows. In Sec. II, we present the predicted angular power spectrum of the DGRB from
extragalactic dark matter halos and compare this to the data to find constraints on the annihilation cross section. In Sec. III, we discuss the contribution from the cross-correlation term with blazars. In Sec. IV, we present the predicted angular power spectrum of the DGRB from Galactic subhalos and find combined constraints on the annihilation cross section using extragalactic and Galactic contributions. We conclude in Sec. V. Throughout the paper, we adopt a flat cold dark matter model with a cosmological constant ($\Lambda$CDM) with the following cosmological parameters: $\Omega_m = 0.277$, $\Omega_\Lambda = 0.723$, $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, $h = 0.7$, $n_s = 0.96$, and $\sigma_8 = 0.81$.

II. EXTRAGALACTIC CONTRIBUTION

In this section, we discuss the mean intensity and anisotropy of the DGRB from dark matter annihilation in extragalactic halos. Much of the calculations are based on our earlier work [9,10], but with extensions of the framework and significant updates on input models as explained below.

A. Mean intensity

The mean intensity of gamma rays from dark matter annihilation is given by

$$I(E) = \int d\chi W([1+z]E, \chi)\langle \delta^2 \rangle,$$

(1)

where we define as the number of photons received per unit area, unit time, unit energy range, and unit solid angle, i.e., $I(E) = dN/d(Adt/dEd\Omega)$. $E$ is the energy of the photons; $E$ is the comoving distance to a source at redshift $z$ [$\chi$ and $z$ are used interchangeably through the relation $d\chi/dz = c/H(z)$]; and $\langle \delta^2 \rangle$ is the variance of the overdensity field, $\delta = (\rho - \langle \rho \rangle)/\langle \rho \rangle$, and is often referred to as an “intensity multiplier.” $W(E, z)$ is the window function that contains particle physics information such as a velocity-averaged annihilation cross section times relative velocity, $\langle \sigma v \rangle$, a dark matter mass, $m_{dm}$, and a gamma-ray spectrum per annihilation, $dN_{\gamma,ann}/dE$:

$$W(E, z) = \frac{\langle \sigma v \rangle}{8\pi} \left(\frac{\Omega_{dm} \rho_c}{m_{dm}}\right)^2(1+z)^3dN_{\gamma,ann}/dE e^{-\tau(E, z)},$$

(2)

where $\Omega_{dm} = 0.23$ is the density parameter of dark matter, and $\rho_c$ is the critical density of the present Universe. Here, $\tau(E, z)$ is the optical depth for a gamma ray emitted at energy $E$, for which we adopt the model of Ref. [30]. Note that the annihilation cross section required to produce dark matter at the right relic density by the thermal freeze-out mechanism with S-wave annihilation is $\langle \sigma v \rangle \approx 3 \times 10^{-26}$ cm$^3$ s$^{-1}$, which is largely independent of the dark matter mass [1].

The intensity multiplier is

$$\langle \delta^2 \rangle = \left(\frac{1}{\Omega_m \rho_c}\right)^2 \int dM \frac{dn(M, z)}{dM} [1 + b_{sh}(M)]$$

$$\times \int dV \rho_{host}^2(r|M),$$

(3)

where $M$ is the virial mass, $dn/dM$ is the halo mass function, for which we adopt an ellipsoidal collapse model [32,33], $\rho_{host}(r|M)$ is the density profile of a host halo of mass $M$, $r$ is the comoving radius from the halo center, and $b_{sh}(M)$ is a boost factor due to annihilation in subhalos.

We adopt a Navarro-Frenk-White (NFW) [34] profile for host halos,

$$\rho_{host}(r) = \frac{\rho_s}{(r/r_s)(r/r_s + 1)^2},$$

(4)

where $\rho_s$ and $r_s$ are the scale density and the scale radius, respectively, and this relation holds out to the virial radius, $r_{vir}$, which, in turn, is given as a function of $M$ and $z$ through the relation $M = 4\pi r_{vir}^3 \Delta_{vir}(z) \rho_c(z)/3$, with $\Delta_{vir}(z) = 18\pi^2 + 82d - 39d^2$ and $d = \Omega_m(1+z)^3/\left[\Omega_m(1+z)^3 + \Omega_\Lambda\right] - 1$ [35].

The scale radius is defined as $r_s = r_{vir}/c_{vir}$, where $c_{vir}(M, z)$ is the concentration parameter, for which we adopt the model of Ref. [36] for masses below $2.5 \times 10^{14} M_\odot$ and that of Ref. [37] otherwise. By taking the volume integral of the density profile, $\rho_{host}(r)$, out to $r_{vir}$ and equating it to $M$, we obtain the scale density as

$$\rho_s = \frac{M}{4\pi r_s^3} \left[\ln(1 + c_{vir}) - \frac{c_{vir}}{1 + c_{vir}}\right]^{-1}.$$

(5)

The volume integral of the density squared has an analytic form:

$$\int dV \rho_{host}^2 = \frac{4\pi r_s^3 \rho_s^2}{3} \left[1 - \frac{1}{(1 + c_{vir})^3}\right].$$

(6)

The gamma-ray intensity is further boosted by annihilation in subhalos, which is represented by the boost factor, $b_{sh}(M)$, for which we adopt a fitting formula based on results of recent numerical simulations [29]: $b_{sh} = 110(M_{200}/10^{12} M_\odot)^{0.39}$, where $M_{200}$ is an enclosed mass within a radius $r_{200}$ in which the average density is 200 times the critical density; there is a simple relation between $M_{200}$ and the virial mass $M$ [38]. This boost is realized if the subhalo mass function extends down to Earth-mass scales, $M_{sh, min} = 10^{-6} M_\odot$, which is a typical cutoff scale for the neutralino dark matter [39–42], and we adopt this value throughout the paper unless otherwise stated. We note, however, that the boost factor strongly depends on the minimum subhalo mass chosen, $b_{sh} \propto M_{sh, min}^{-0.2}$ [29]. Given that a wide range of minimum subhalo mass is still allowed, as small as $\sim 10^{-12} M_\odot$ [43], the annihilation rate may be boosted even further.

Figure 1 shows the integrand of Eq. (3) as a function of the virial mass, $M$, for various redshifts, $z$. Namely, this

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shows what fraction of $\langle \delta^2 \rangle$ is contributed from which mass ranges. As can be seen from this figure, the dominant contribution comes from cluster-sized halos ($M \sim 10^{14}M_\odot$) at $z \sim 0$ and galaxy-sized halos ($M \sim 10^{12}M_\odot$) at $z \sim 2$. This is mainly because of the boost due to annihilation in subhalos. In Fig. 2, we compare the contribution from the host halos (dashed line) and that from the host halos and subhalos (solid line) at $z = 0$. If there are no gamma rays from subhalos, then the mean intensity would be dominated by the smallest dark matter halos.

Figure 3 shows the intensity multiplier $\langle \delta^2 \rangle$ as a function of redshifts for the cases with and without subhalo contributions. Here we multiply $\langle \delta^2 \rangle$ by $(1 + z)^3/\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ to show contributions to the mean intensity from different redshift ranges. One can see that the presence of subhalos boosts the intensity by a factor of 10 at low redshifts, and by a factor of $\sim 2$ even at $z = 5$. Since cluster-sized halos host most subhalos and they form later, the contribution from the low-redshift regime increases relatively more compared with the host-only case. The dotted lines are further multiplied by the absorption factor, $e^{-\gamma}$, for observed energies of $E = 10, 20, 50,$ and $100$ GeV. There is little absorption for photons received below 10 GeV, but this effect is significant for energies above tens of GeV and should be taken into account.

Figure 4 shows the predicted mean intensity of the DGRB from dark matter annihilation with $m_{dm} = 100$ GeV, $\langle \sigma v \rangle = 3 \times 10^{-26}$ cm$^3$s$^{-1}$, and the $b\bar{b}$ annihilation channel. This model gives the dark matter contribution that is as large as 10% of the mean intensity measured by Fermi-LAT [44] at $E \sim 10$ GeV. This contribution is quite significant, given that even the most dominant contributors known to date, i.e., unresolved blazars, contribute to the DGRB at around the same level [45].

B. Angular power spectrum

The angular power spectrum at a given multipole, $\ell$, is given by

$$C_\ell(E) = \int \frac{dX}{X^2} W^2([1 + z]E, z) P_{\theta}(k = \ell, z)$$

(7)
Following conventions of recent publications (e.g., Ref. [27]), our definition of the angular power spectrum [Eq. (7)] has the units of \((\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1})^2\) sr, which is referred to as the intensity angular power spectrum.\(^2\) In order to obtain the fluctuation angular power spectrum that has units of sr and is adopted in the earlier papers (e.g., Ref. [9]), one simply divides the intensity power spectrum by the mean intensity squared, \(\bar{I}^2(E)\).

Here, \(P_{\delta^2}(k, z)\) is the power spectrum of the overdensity squared, \(\delta^2\), which can be divided into one- and two-halo terms [9]:

\[
P_{\delta^2}(k, z) = P_{\delta^2}^{1h}(k, z) + P_{\delta^2}^{2h}(k, z).
\]

The one-halo term correlates two points in one identical halo, whereas the two-halo term does the same in two distinct halos. Correspondingly, the angular power spectrum is also divided into two terms:

\[
C_\ell(E) = C_{\ell}^{1h}(E) + C_{\ell}^{2h}(E).
\]

These two terms of the power spectrum can be explicitly written as

\[
P_{\delta^2}^{1h}(k, z) = \frac{1}{(\Omega_m \rho_c)^2} \int dm \frac{d^2n(M, z)}{dM} |\tilde{u}(k|M)|^2 \times \left[ (1 + b_{\text{sh}}(M)) \int dV \rho_{\text{host}}^2(r|M) \right]^2,
\]

\[
P_{\delta^2}^{2h}(k, z) = \left[ \left( \frac{1}{\Omega_m \rho_c} \right)^2 \int dm \frac{d^2n(M, z)}{dM} \tilde{u}(k|M) b_1(M, z) \right] \times (1 + b_{\text{sh}}(M)) \int dV \rho_{\text{host}}^2(r|M) \right]^2 P_{\text{lin}}(k, z),
\]

where \(P_{\text{lin}}(k, z)\) is the linear power spectrum of the matter density field \(\delta\), and \(b_1(M, z)\) is the linear halo bias [33].

The power spectrum of \(\delta^2\), \(P_{\delta^2}\), depends on profiles of density squared in a halo of mass \(M\), \(u(r|\mathcal{M})\), where \(u(r|\mathcal{M})\) is normalized such that its volume integration becomes unity; \(\tilde{u}(k|M)\) is the Fourier transform of \(u(r|\mathcal{M})\).

Fourier transform of the density-squared profile, \(\tilde{u}(k|M)\), is the sum of the density-squared profiles of the host halo and subhalos weighted by a fractional luminosity of each component:

\[
\tilde{u}(k|M) = \frac{\bar{u}_{\text{host}}(k|M) + b_{\text{sh}}(M) \tilde{u}_{\text{sh}}(k|M)}{1 + b_{\text{sh}}(M)}.
\]

Here, we ignore a contribution from the cross term, \(2\rho_{\text{host}}(r)\rho_{\text{sh}}(r)\), which is important only when \(\rho_{\text{host}}(r) \approx \rho_{\text{sh}}(r)\) at the same radius, \(r\). Given that spatial distributions of the host halo and subhalo contributions are quite different (the host halo being important inside the scale radius and the subhalos being important outside), this approximation is very good. (See the Appendix for the contribution of the cross term.)

Fourier transform of the host halo profile, \(u_{\text{host}}(r) \approx \rho_{\text{host}}^2(r)\), has an analytic form [9], but here we use an even simpler fitting formula,

\[
\bar{u}_{\text{host}}(k|M) = \frac{1}{1 + a(kr_0)^{2/b}}.
\]

with \(a = 0.13\) and \(b = 0.7\), which is largely independent of \(c_{\text{vir}}\) [20]. On the other hand, we obtain the density-squared profile of subhalos, \(u_{\text{sh}}(r|\mathcal{M})\), by deprojecting the surface brightness profiles of numerical simulations [29,46], assuming spherical symmetry as follows:

\[
u_{\text{sh}}(r) \approx \begin{cases} 
\left( \frac{r}{r_{200}} \right)^2 + \frac{1}{16} r^{-3/2}, & \text{for } r \leq r_{200}, \\
\left( \frac{r}{r_{200}} \right)^{3/2} \left( \frac{r}{r_{200}} \right)^{-1} e^{-\eta(r/r_{200})^{-1}}, & \text{for } r > r_{200},
\end{cases}
\]

with \(\eta = 2.78\). Note that the distribution of subhalos is typically more extended than the density profile of the host halo. Its Fourier transform is
where $\kappa = kr_{200}$ and $A = 0.64$ is the normalization constant such that $\bar{u}_h(0) = 1$.

Figure 5 shows $\bar{u}(k|M)$ for various host-halo masses, $M$. When $\bar{u}(k)$ is close to unity (for small $k$), the halo can be regarded as a point source. On the other hand, when $\bar{u}(k)$ deviates significantly from unity at a given wave number $k$, the source extension cannot be ignored at that wave number, and the power spectrum (especially the one-halo term) is suppressed. Figure 5 shows that $\bar{u}(k)$ is larger for smaller host halos, which are less extended. It also shows that the contributions from subhalos are more important for larger host halos. As the distribution of subhalos is more extended than the density profile of the host halo, the subhalo contribution dominates at small $k$ and the host halo dominates at large $k$. This makes a hump at scales corresponding to the scale radius, $r_s$. In other words, annihilation from the smooth host-halo component dominates inside the scale radius, where subhalos are tidally disrupted.

Figure 6 shows the integrand of the one-halo power spectrum of $\delta^2$, $dP^{1h}_{\delta^2}(k, <M)/d\ln M$ [Eq. (10)], at $z = 0$ for various wave numbers, $k$. (Note that the lines are normalized at $M = 1 M_\odot$, and thus it shows relative contributions rather than absolute.) The bulk of the contributions come from large-mass halos, and halos smaller than a typical dwarf size ($M < 10^6 M_\odot$) do not make any sizable contributions to any relevant ranges of $k$. This is particularly true for scales larger than a typical cluster size (i.e., $k \lesssim 1$ Mpc$^{-1}$). For smaller scales, on the other hand, the relative importance of large-mass halos is smaller, as large-mass halos are more extended, and thus the power from them is suppressed (as also shown in Fig. 5).

Figure 7 is the same as Fig. 6 for $k = 1$ Mpc$^{-1}$, but in comparison with the host-halo contribution. We find that the impact of subhalos on the power spectrum is much greater than that on the mean intensity (as shown in Fig. 2), and it
can boost the power spectrum by almost 4 orders of magnitude at this particular wave number.

After being projected on the sky, the three-dimensional wave number, \( k \), and the angular multipole, \( \ell \), are related by \( k = \ell / \chi(z) \) for a given redshift, \( z \) [see Eq. (7)]. Note, however, that this simple relation is valid only for small angular scales, \( \ell \gg 1 \) (e.g., Refs. [9,10]), on which we mainly focus in this paper.

Figures 8 and 9 show the contributions to the angular power spectra at \( \ell = 10, 100, \) and 1000 from one-halo and two-halo terms, respectively, as a function of redshifts. To calculate a contribution to \( C_\ell \) from a given \( z \), we multiply \( P_\ell^{\gamma} \theta(z) \) by some combination of functions of redshift [see the integrand of Eq. (7) and also the redshift dependence in Fig. (2)]. We find that lower multipoles are dominated by nearby sources: one-halo terms at \( \ell = 10, 100, \) and 1000 are dominated by sources at \( z \approx 0.002, 0.02, \) and 0.2, respectively, whereas the dominant contributions to the two-halo term come from somewhat higher redshifts.

One should not, however, include contributions from arbitrarily small redshifts in the integral of Eq. (7), as cosmic variance in such small redshifts is so large that taking the ensemble average (as we do here) no longer makes sense. In addition, when a source is sufficiently close, it should give enough gamma-ray fluxes to be identified as an individual source which we can remove from the map. In the following discussion, we use three different minimum redshifts in the integration of Eq. (7): \( z_{\text{min}} = 0.001, 0.003, \) and 0.01. The rms overdensity within radii corresponding to these three redshifts are 1.4, 0.74, and 0.28, respectively. Since none of these are much larger than 1, we can argue that our results by setting these lower cutoffs are not subject to strong cosmic variance. We also note that, as we show below, this choice does not make any significant difference for the multipoles we consider (\( \ell > 150 \)) in this paper.

In Fig. 10, we show both the one-halo and two-halo terms of the angular power spectrum, \( C_\ell \), integrated over the 5–10 GeV energy range, for three different values of \( z_{\text{min}} = 0.001, 0.003, \) and 0.01. The particle physics parameters are \( \langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) and \( m_{\text{dm}} = 100 \text{ GeV} \), and we assume the \( b \bar{b} \) annihilation channel. Note that \( C_\ell \) scales as \( \langle \sigma v \rangle^2 \). Taking smaller \( z_{\text{min}} \) increases the power at large angular scales, in particular for the one-halo term, because of the contributions from closer, more extended halos. For the rest of this paper, we shall use \( z_{\text{min}} = 0.003 \) for definiteness.

The thick solid line in Fig. 10 shows the upper limits on the angular power spectrum [27] (with the blazar contribution subtracted [28]) between \( \ell = 155 \) and 504 for 5–10.4 GeV, \( C_{155 \leq \ell \leq 504} \approx 8 \times 10^{-20} \text{ (cm}^2 \text{ s}^{-1} \text{ sr}^{-1})^2 \text{ sr} \). The current upper limits in 5–10.4 GeV, whose energy region is most sensitive to dark matter particles with \( m_{\text{dm}} = 100 \text{ GeV} \) annihilating into \( b \bar{b} \) (see Fig. 4), are 3 orders of magnitude larger than the prediction with the canonical particle physics parameters. Recalling \( C_\ell \propto \langle \sigma v \rangle^2 \), we find an upper limit on the cross section of \( \langle \sigma v \rangle \approx 8 \times 10^{-25} \text{ cm}^2 \text{ s}^{-1} \) for the 100-GeV dark matter annihilating into \( b \bar{b} \).

The blazar-subtracted upper limits on the angular power spectrum are available in several energy bands, 1.04–1.99 GeV, 1.99–5 GeV, 5–10.4 GeV, and 10.4–50 GeV [28]. In Fig. 11, we show the combined upper limits on the annihilation cross section for the \( b \bar{b} \) channel, \( \langle \sigma v \rangle_{bb} \), as a function of the dark matter masses, \( m_{\text{dm}} \), using all the available data on the power spectrum. (Note that the model is still based only on the extragalactic contribution.)
We calculate the upper limit on the cross section such that the predicted $C_\ell$ at $\ell = 155$ (the lowest multipole at which the measurement is reported) is equal to the $2\sigma$ upper limit reported by Ref. [28]. The limits from each energy range are shown separately as the dotted lines. The combined limits shown as the solid line are simply the best of the four limits at a given dark matter mass. A more optimal analysis would improve these limits.

These results depend on assumed subhalo models. In particular, a different number of subhalos will change them significantly. Therefore, we also investigated with another value of the assumed minimum subhalo mass, $M_{\text{sh,min}} = 10^{-10} M_\odot$, which is well within the currently preferred WIMP framework [43]. In this case, the rate of annihilation and the mean intensity are boosted by a factor of $\sim 6$. Numerical simulations suggest that the radial profile of the subhalo distribution may not change (e.g., Ref. [29]), and hence we use the same formula as Eq. (14). The upper limits on the annihilation cross section with the angular power spectrum are then improved by a factor of $\sim 6.3$, almost independently of the dark matter mass. The canonical cross section for thermal WIMPs is now excluded for almost independently of the dark matter mass. The canoni-
\[ C^\chi(E) = \int \frac{d\chi}{\chi^2} W_B([1 + z]E, z)W_{dm}([1 + z]E, z) \times P_\chi(k = \frac{\ell}{\lambda}, z). \]  
\hfill (17)

where the subscripts “B” and “dm” denote blazars and dark matter, respectively. Here, \( W_B \) is the same as Eq. (2), while \( W_{dm} \) is the window function for the mean intensity of blazars [similarly defined as Eq. (1) but replacing \( \delta^2 \) with 1]:

\[ W_B([1 + z]E, z) = \chi^2 \int_0^E dL \Phi_E(L, z)F_E(L, z), \]  
\hfill (18)

where \( \chi \) is the differential luminosity (i.e., luminosity per unit energy) at a given energy \( E \), \( F_E \) is the differential flux, and \( \Phi_E(L, z) \) is the gamma-ray luminosity function of blazars. The upper limit of the integral, \( L \), is the luminosity giving the flux corresponding to the point-source sensitivity of Fermi-LAT at a given redshift \( z \), for which we adopt \( 10^{-8} \) \( \text{cm}^{-2} \text{s}^{-1} \) above 100 MeV. More formal definitions of these quantities can be found in Ref. [10].

The three-dimensional cross power spectrum is

\[ P_\chi(k, z) = \langle b_B(z) \rangle \left[ \frac{1}{\Omega \rho_c} \right]^2 \int dM dm \rho_c(M, z) \tilde{a}(k|M) \times b_1(M)(1 + b_{dh}(M)) \times \int dV \rho_{host}^2(r|M) \]  
\[ \times \left[ \Phi_E(L, z)F_E(L, z)b_B(L, z) \right] P_{lin}(k, z) \]
\hfill (19)

where \( \langle b_B(z) \rangle \) is the blazar bias averaged over the luminosity function and the flux as follows:

\[ \langle b_B(z) \rangle = \frac{\int dL \Phi_E(L, z)F_E(L, z)b_B(L, z)}{\int dL \Phi_E(L, z)F_E(L, z)}. \]  
\hfill (20)

with the luminosity-dependent bias, \( b_B(L, z) \), and the same upper and lower limits of integration as those in Eq. (18).

Note that this power spectrum [Eq. (19)] includes the two-halo term only. While the one-halo term, where dark matter annihilation happens in the same halo that hosts a blazar, also exists, the previous study [10] shows that the one-halo term of the cross correlation is much smaller than the two-halo term; thus, we shall ignore the one-halo term of the cross power spectrum.

For the luminosity function, \( \Phi_E(L, z) \), we adopt the luminosity-dependent density-evolution model [11,51] with the gamma-ray spectra assumed to be a power law with an index of 2.4, which is in agreement with the spectrum of resolved blazars as well as that of the DGRB.\(^4\) Compared with the earlier study [11], where the luminosity function was based on pre-Fermi data, we here adopt different values for parameters of the luminosity function (\( \kappa = 10^{-4}, q = 3.5, \) and \( \gamma_L = 1.05 \)) such that the model reproduces the flux distribution of blazars resolved by Fermi [45].

In Fig. 4, we show the blazar contribution to the mean intensity. It is difficult to explain the DGRB intensity measured by Fermi with blazars alone, in agreement with the previous study [45,54].

Figure 12 shows the angular power spectra from the cross correlation with \( \langle b_B \rangle = 1 \) and 5, for the energy range between 5 GeV and 10 GeV. The particle physics parameters are the same as those in Fig. 10. We find that, if the annihilation cross section is around the canonical value required to produce dark matter particles at the right abundance by the thermal freeze-out mechanism, then the cross-correlation term cannot be ignored. This is particularly important when the bias of blazars is as high as 5.

As the dark matter term \( (C_{\ell}^{1h} + C_{\ell}^{2h}) \propto \langle \sigma v \rangle \) and the cross-correlation term \( (C_{\ell}^{\chi} \propto \langle \sigma v \rangle \) scale with \( \langle \sigma v \rangle \) differently, one may ask, “At which value of the annihilation cross section does the cross-correlation term become important?” Figure 13 shows \( C_{\ell=155}^{\chi} \) divided by the upper

\(^4\)More elaborated spectra in combination with the luminosity function and the DGRB intensity are studied in Refs. [52,53].
FIG. 13 (color online). Predicted angular power spectrum at $\ell = 155$ in 5–10 GeV divided by the current upper limit, $C_{155 < \ell \leq 504}^{UL}$, as a function of the annihilation cross section, $\langle \sigma v \rangle$. The solid lines include the dark matter–blazar cross correlation with $(b_B) = 1$ and 5, while the dashed line does not. The vertical dotted line shows the canonical cross section for thermal WIMPs.

The angle-averaged mean intensity from dark matter annihilation in Galactic subhalos can be computed from

$$I_{sh} = \int dL \int_{s_{\ell}(L)}^{r_{vir,MW}} ds \frac{d\pi_{sh}(L, s)}{dL} L,$$

where $L$ is the gamma-ray luminosity of a subhalo, $s$ is the line-of-sight coordinates, and $d\pi_{sh}/dL$ is the angle-averaged luminosity function of subhalos. The lower limit of the line-of-sight integral, $s_{\ell}(L)$, corresponds to the flux sensitivity of Fermi-LAT, i.e., $L = 4\pi s^2 F_{sens}$.

The subhalo luminosity, $L$, is related to the subhalo mass, $M$, via

$$L = B_{sh} \langle \sigma v \rangle_{X,ann} \int dV_{sh} \rho_{sh}^2(r_{sh}|M),$$

where $r_{vir,MW}$ is the virial radius of the Galactic halo, the subscripts “sh” denote subhalos, and $B_{sh}$ is a boost factor due to the presence of substructure in subhalos (sub-subhalos).

We assume that the density profile of subhalos, $\rho_{sh}$, is well described by the NFW function [Eq. (4)]. Then, the volume integral of the density squared has the analytic form given by Eq. (6) with the concentration parameter, $c_{vir}$, replaced with $c_{cut}$, which corresponds to the cutoff radius of subhalos due to the tidal disruption, i.e., $c_{cut} \equiv r_{cut}/r_s$. With this mass-luminosity relation [Eq. (22)] and the subhalo mass function $dn_{sh}/dM$, one can compute the luminosity function. Most model inputs such as the...
subhalo mass function, spatial distribution, and mass-concentration relation \([c_{\text{vir}}(M)]\) are adopted from recent numerical simulations of the Galactic halo, Aquarius [55]. More details on how to apply these models to gamma-ray computations are described in Ref. [20].

The intensity angular power spectrum is

\[
C^{\text{sh}}_\ell = \frac{1}{16\pi} \int dL \int ds L^2 \frac{d\bar{n}_{\text{sh}}(L, s)}{dL} \left| \tilde{u}_{\text{sh}}(\ell, s, M) \right|^2,
\]

where \(\tilde{u}_{\text{sh}}(k, M)\) is the Fourier transform of the density-squared profile of the subhalo distribution, which is given by Eq. (13) if the density distribution of subhalos follows a NFW profile. Note that Eq. (23) only includes a “one-subhalo” term, where one correlates two points in one identical subhalo. There is, however, the two-subhalo term that correlates two points in two distinct subhalos, but this term is much smaller than the one-subhalo term at small angular scales [20].

Figure 15 shows the predicted angular power spectra from Galactic subhalos and extragalactic halos (but not including the cross correlation). We have used the canonical model of the Galactic subhalos given in Ref. [55], which has the mass resolution of about \(4 \times 10^4 M_\odot\). We have extrapolated their result down to the Earth-mass scale (model A1 of Ref. [20]). The intensity power spectrum is about the same for both the extragalactic and Galactic components, with the latter slightly larger in the angular scales constrained by Fermi-LAT.

In Fig. 16, we show the limits on \(\langle \sigma v \rangle\) from the Fermi-LAT data, taking into account both the extragalactic and Galactic terms. As expected, the limits from either alone are similar, and the combined limits improve by a factor of 2. In particular, for low-mass dark matter particles, the combined limits are only a factor of 3 larger than the canonical cross section. The limits are weaker for larger masses.

While our limits are not yet as stringent as those obtained from analyses of dwarf galaxies [56,57] or galaxy clusters [46,58], where the canonical cross section is already excluded for low-mass (~10 GeV) dark matter particles, they are not so far away (i.e., only a factor of 3 to 4 worse). Also, our limits are derived in a completely different way: they are based on the diffuse emission rather than on individual objects, and they are based on anisotropy rather than on the mean intensity. It is certainly encouraging that the first limits using the DGRB anisotropy are already not so far away from the best limits.

V. CONCLUSIONS

In this paper, we have used the angular power spectrum of DGRB recently detected in the 22-month data of Fermi-LAT [27] to place limits on the annihilation cross section of dark matter particles as a function of dark matter masses. As dark matter annihilation occurs in all cosmological halos and subhalos, our model includes all the contributing terms in the extragalactic halos, the Galactic subhalos, and the cross correlation between dark matter annihilation and blazars. The smooth Galactic component is predicted to be...
subdominant in the high Galactic region ($|b| > 30$ deg) and is ignored.

We have revised our earlier model of the extragalactic contribution by including the results from recent numerical simulations of the subhalo distribution [29]. Combined with the model of the Galactic subhalos of Ref. [20], we find that the Galactic and extragalactic contributions are comparable to each other. The cross correlation with blazars is important for annihilation cross sections smaller than the canonical value ($\langle \sigma v \rangle \lesssim 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}$).

By comparing our model with the upper limit on the nonblazar contribution to the angular power spectrum of the DGRB [28], we find upper limits on the annihilation cross section as a function of dark matter masses as shown in Fig. 16. The current limit from anisotropy excludes regions of $\langle \sigma v \rangle \gtrsim 10^{-25} \text{ cm}^3 \text{s}^{-1}$ at the dark matter mass of 10 GeV, which is only a factor of 3 larger than the canonical value. The limits are weaker for larger dark matter masses. The first limits from DGRB anisotropy that we find in this paper are already competitive with the best limits in the literature.

Our limits will improve as Fermi collects more data. At the same time, an improvement in the analysis can significantly improve our limits. Currently, the angular power spectrum on large angular scales, $\ell < 155$, is not used because of a potential contamination by the Galactic foreground emission (such as pion decay). As the angular power spectrum of the DGRB from dark matter annihilation, $C_{\ell}$ (without multiplying by $\ell^2$), rises towards low multipoles, including the low-multipole data will significantly improve the limits. This line of investigation (i.e., a better characterization and removal of the Galactic foreground) should be pursued.

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APPENDIX: EFFECT OF HOST-SUBHALO CROSS TERM

The dark matter annihilation signal is proportional to the density squared. When both host halo and subhalo contributions are present, one has the host-density-squared term, $\rho^2_{\text{host}}$, the subhalo-density-squared term, $\rho^2_{\text{sh}}$, and the host-subhalo cross term, $2\rho_{\text{host}}\rho_{\text{sh}}$. In our analysis, we have ignored the cross term [see Eq. (12)], as spatial distributions of the host halo and subhalo contributions are quite different (the host halo being important inside the scale radius and the subhalos being important outside). In this appendix, we quantify the importance of the cross term.

Figure 17 shows the density-squared profiles of a host halo, subhalos, and the cross term. As expected, the cross term becomes comparable to the other terms only within a narrow window in radii. For the host halo mass of $M = 10^{14} M_\odot$ and the redshift of $z = 0$, the cross term becomes comparable to the other terms at $r \sim 20$ kpc, which is 1/10 of the scale radius, $r_s = 210$ kpc. Fig. 18 shows the Fourier transform.