The Metaphysical Basis of Logic
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Rede

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door

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1. Cases

I sometimes tell people that I do two things: logic and metaphysics. What I actually think is that I do one thing under two labels. When I applied for the position in Metaphysics and History of Philosophy advertised by this University, connections between Amsterdam, the Netherlands, and what I do, freely popped into my head. A metaphysician associates the Netherlands with the great Amsterdammer, Baruch Spinoza. A logician associates the Netherlands with the great logician – actually from Overschie, but a graduate of the University of Amsterdam – L.E.J. Brouwer. Then I began to wonder about the connection between the two strands, the logical and the metaphysical, in present-day Dutch philosophy. Now that I have started to look at the amazingly rich variety of philosophical activities going on at the Department of Philosophy of the University of Amsterdam, I have the impression of a gap.

Not that this is peculiar: many metaphysicians and theoretical philosophers around the world refuse to bother with logic, while logicians often see metaphysical issues like smoke in the eyes. There may be sociological explanations for this, having to do with the history of contemporary philosophy: the Great Analytic-Continental Divide, to begin with – another gap I am uncomfortable with. I am given the opportunity to conduct philosophical research in this great place and, though I cannot fill these gaps all on my own – that would be pretentious –, I hope to help; or at least, I’d like to avoid sitting in the gaps alone.

Brouwer himself may not have been happy with the idea of logic and metaphysics being disconnected from each other. Logic was for Brouwer subordinate to mathematics just as language is to thought, and his critique of classical mathematics was grounded in his deep metaphysical convictions: his rejection of the idea that mathematics deals with a realm of abstract, mind-independent objects; his conviction that mathematics is the product of free exact thinking, grounded in the Kantian pure intuition of inner time. Intuitionistic logic, with its unflagging attack to the Law of Excluded Middle, originated from such metaphysical convictions (one could call such convictions anti-metaphysical, but that’s no worry: metaphysics is such a peculiar subject that anti-metaphysical thoughts often end up being themselves metaphysics of some kind).

But I should say something on how ‘logic’ and ‘metaphysics’ are understood in this oratie. Here another association brings me from intuitionistic logic to somebody who took intuitionism very seriously: Sir Michael Dummett. Out of his William James Lectures, Dummett wrote a justly celebrated book called The Logical Basis of Metaphysics, in which he wanted to set up ‘a
[logical] base camp for an assault on the metaphysical peaks.’ (Dummett, 1991, p. 16). I reversed his title because I’d like do go the other way around. Not that I want to go downhill: rather, I think that the grounding relation should be turned upside down. I hope to set up a metaphysical base camp to climb the logical peaks (‘assault’ may sound too bellicose), because I think that logic is based on metaphysics in a variety of ways.

What does this mean? Well, what is metaphysics, to begin with? Thus opens Crane and Farkas’ textbook *Metaphysics*, which I use in my Bachelors courses:

Metaphysics asks the most general questions about the most general nature of reality. [...] Metaphysical enquiries deal with the nature of what there is, abstracting away from the particular details of goings on in our world, in an attempt to uncover the underlying structure of reality. (Crane and Farkas, 2004, p. vii)

And what is logic? Here is page one of Jeffrey’s textbook *Formal Logic*:

Logic aims to provide systematic means for telling whether or not given conclusions follow from given premises, i.e., whether arguments are valid or invalid. [...] Validity is easily defined: a valid argument is one whose conclusion is true in every case in which all its premises are true. (Jeffrey, 1991, p. 1)

(This is a model-theoretic characterisation, which may make proof theorists unhappy; of course, I cannot get into this debate here: bear with me).

Talk of cases brings yet another association; this time, with the very first words of one of the greatest philosophical books ever written. You – at least, the philosophers among you – have heard the sentence so many times:

The world is everything that is the case. (Wittgenstein, 1922, p. 25)

Of course (Wittgenstein knew that) our world, what actually is the case, may have been otherwise. But in the expression ‘every case’, in Jeffrey’s definition above, ‘every’ really means every: as he says, ‘the mark of validity is the absence of counterexamples.’ (Ibid). Logical validity is understood via unrestricted quantification over all cases, not via restricted quantification over all cases of a special kind. Don’t worry about how the world turns out to be for, in any case, logic will hold. It will hold come what may. This is the hallmark
of logical modality: of the kind of necessity that, since the origins of Western thought, has been attached to logic.

Jeffrey then adds, still on p. 1 of his textbook, that ‘Difficulties in applying this definition arise from difficulties in canvassing the cases mentioned in it.’ What solving these difficulties – of which we will see examples below – calls for is, thus, a most general characterization of *case*. Most general, because the question, ‘What is a case?’, asks for a most general answer, as logic is to hold no matter what, that is, in all cases. I claim that such a question is utterly metaphysical in the sense of the textbook definition above: it deals with the most general and structural features of reality, as Crane and Farkas say, ‘abstracting away from the particular details of goings on in our world’, that is, of what actually is the case. The question deals with *any* way the world could be – with the nature and structure common to any case whatsoever.

Logicians who don’t want to be bothered by metaphysical considerations may object that such talk is way too loaded with metaphysical modalities. The most common way of understanding Jeffrey’s cases nowadays is as Tarskian models, and these, one may say, are just set-theoretic constructions: pieces of mathematics. But even if cases are taken as Tarskian models, facts about logical validity depend not only on what models there are, but also on what they themselves model.

One can easily build a Lindenbaum algebra out of the syntax of any logic satisfying minimal requirements, and then prove completeness with respect to such algebra. To the accusation that this is a cheap result, one may reply by resorting to an instrumentalist conception of logic – in a slogan: what matters is just that the model works. Still, instrumentalism is unsatisfactory for many philosophically inclined minds. If a mathematical black box happens to deliver the desired results, one may ask to know why this is so. Then we need to know how to understand the model itself, that is to say, how it itself should be interpreted.

Many textbooks, from Haack’s *Philosophy of Logics* to Kirwan’s *Logic and Argument*, distinguish between *pure* and *applied* logical semantics (Dummett, 1978, p. 204, talked of ‘merely algebraic notion of logical consequence’ as opposed to ‘semantic notion of logical consequence properly so called’). ‘Purity’ here has to do with the mere construction of the formal model. Talk of ‘application’ sounds a bit misleading, for it makes us think of the applications of a logic to specific problems and domains. But applied semantics, instead, deals with the interpretation of the models themselves.

I think that when we move to applied semantics, logic (as characterized above) cannot but defer to some extent to metaphysics (ditto). We want a characterization of the things, which the set-theoretic structures are to repre-
sent. And this characterization has to be most general and, in this sense, meta-
pysical – for to be logically valid is to hold in all cases, none excluded, if
‘the mark of [logical] validity is the absence of counterexamples.’

One well-known example of this situation obtains in a field in which Am-
sterdam has a great tradition: the model-theoretic semantics of modal logic.
Possible worlds semantics taught us how to understand modal concepts fol-
lowing Leibniz’s insight: we take them as quantifiers on worlds, restricted by
accessibility relations from the standpoint of a given world. By imposing sim-
ple algebraic conditions on the relevant accessibility, we validate various in-
ferences characteristic of different modal systems. This result, due to Kanger,
Kripke, Hintikka, and others, is perhaps the most celebrated of Twentieth
Century philosophical logic.

But although they don’t refrain from calling the points in their frames ‘pos-
sible worlds’, many logicians claim neutrality on what those points at which
formulas are evaluated stand for. When we move towards applied semantics,
though, we want to know what is it that is represented by those models, and
then we are going to ask questions on the metaphysical status of these things.
David Lewis, who notoriously pressed the point at length, puts it vividly:

For [the model-theoretic analysis of modal logic], we need no possible
worlds. We need sets of entities which, for heuristic guidance, ‘may be
regarded as’ possible worlds, but which in truth may be anything you
please. We are doing mathematics, not metaphysics. Where we need pos-
sible worlds, rather, is in applying the results of these metalogical investi-
gations. Metalogical results, by themselves, answer no questions about the
logic of modality. They give us conditional answers only: if modal opera-
tors can be correctly analysed in so-and-so way, then they obey so-and-so
system of modal logic. We must consider whether they may indeed be so
analysed; and then we are doing metaphysics, not mathematics. […] If
modal operators were quantifiers over towns restricted by the relation of
being connected by rail, that would validate some system or other of mod-
al logic. – So what, since modal operators are nothing of the sort? What
good is it to know which misinterpretations would validate a system? (Le-
wis, 1986, pp. 19-20)

2. The Neutrality of Logic

The general moral I would like to draw has been nicely expressed in Timothy
Williamson’s most recent book, Modal Logic as Metaphysics. For Williamson,
we should reject the picture in which ‘logic and metaphysics appear disjoint’, with logic playing the role of

a neutral referee of disputes between scientific theories, including metaphysical theories, blowing the whistle when the rules are broken […]. This book is written in the contrary conviction that, just as metaphysics is much more like the rest of science than was once thought, so too is logic. (Williamson, 2013, p. x)

The conception of logic as a neutral referee or, as we may say, as unsubstancial, has a venerable pedigree. Wittgenstein himself, in the *Tractatus*, thought logical laws to be unsubstancial: because they hold come what may, that is, in all cases, they impose no constraint on the world. They are not quite Un-sinnig, but they are Sinnlos. ‘Tautology and contradiction are not pictures of reality’ (4.462): they exhibit the logical form of the world, but give no description of how the world is like. Because they impose no constraint on the world, logical laws are utterly uninformative. A student asks me whether she passed the exam. I answer with Excluded Middle: ‘Either you passed the exam, or not.’ She won’t be happy. There is a theory of information (Bar-Hillel and Carnap, 1968) that makes sense of this: to become informed of something is to manage to rule out some previously entertained case as a candidate for actuality. But, if classical logic is right, there is no case where Excluded Middle fails. So by asserting an instance, I rule out no case for my hearer: ‘There can never be surprises in logic.’ (6.1251)

Still, all of this may depend on what the later Wittgenstein would have called an unbalanced diet of examples. Consider one claim dear to him in the second phase of his thought: ‘Either there are four consecutive 7’s in the decimal expansion of π, or not.’ A constructivist may decline to assert this until a construction is provided, that proves one disjunct or the other (by the way the four 7’s were proved to be there in the Sixties, when Daniel Shanks and John Wrench Jr. developed π for 100,000 decimals). Serious ways of doubting the validity of Excluded Middle advance considerations on how the world could or could not be. According to constructivists, mathematical constructions are legitimate cases: consideration of such cases calls for a revision of classical logic into one which limits Excluded Middle.

We are now venturing into disputes about alternative logics: situations in which different parties disagree on what the correct logic is. These debates have something in common with debates on the basic notions of metaphysics: such disputes are extremely difficult to handle. Metaphysicians can disagree on the absolutely most basic concepts involved in our understanding of the
world: identity, existence, instantiation, causation, etc. Logicians can disagree on the validity of basic principles of inference, such as Contraposition, *reductio ad absurdum*, or Disjunctive Syllogism. In both venues, discussions often face methodological impasses, or turn into hard clashes of intuitions. We cannot inspect such notions as predication, negation, etc., without resorting to them. We are digging as deep in our concepts as one can get. It is then hard to decide when some party starts to beg the question, or who carries the burden of proof. It is not easy to tell whether a non-standard explanation of a basic notion involves a real disagreement with a mainstream account of that notion, or its principles just characterize something else under the same name or symbol.

Now the unsubstantial conception of logic as a neutral referee tends to view these debates as people talking past each other. Doesn’t the claim that ‘Either A or not-A’ holds for any A just show what makes for the meaning of ‘not’ (and of ‘or’)? Someone who rejects Excluded Middle must be talking of something else. Quine made the point a long ago, in *Philosophical Logic*. He mentions the Law of Non-Contradiction, which will be my case study below, so I quote him extensively:

To turn to a popular extravaganza, what if someone were to reject the law of non-contradiction and so accept an occasional sentence and its negation as both true? An answer one hears is that this would vitiate science. Any conjunction of the form ‘p . ~p’ logically implies every sentence whatever; therefore acceptance of one sentence and its negation as true would commit us to accepting every sentence as true, and thus as forfeiting all distinction between true and false. […]

My view of the dialogue is that neither party knows what he is talking about. They think that they are talking about negation, ‘~’, ‘not’; but surely the notion ceased to be recognisable as negation when they took to regarding some conjunctions of the form ‘p . ~p’ as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician’s predicament: when he tries to deny the doctrine he only changes the subject. (Quine, 1970, p. 81)

I very much agree with Williamson’s view (this time from *The Philosophy of Philosophy*: Williamson, 2007) that the opposite of Quine’s last claim is usually the case. Very rarely do people debating on alternative logics talk past each other. What these people mostly have is alternative theories of some concepts, rather than talk about different concepts. Even if there is only One True Lo-
logic, giving a mistaken theory of the meaning of a word is quite different from
using the word in an idiosyncratic way (linguists often do the former but not
the latter, Williamson remarks: see Ibid, p. 89).

MIT logician Vann McGee claims that there are counterexamples to modus
ponens. CUNY logician Graham Priest, that there are counterexamples to the
Law of Non-Contradiction. Do they misunderstand or misuse ‘if’ and ‘not’?
Qua logicians, they have been thinking about these words and what they
mean at least as much as any of us. Qua speakers of English, they use such
words as words of a public language, full participation in which entails un-
derstanding. Williamson concludes:

No given argument or statement is immune from rejection by a linguisti-
cally competent speaker. Quine’s epistemological holism in ‘Two Dogmas’
dermines his notorious later claim about the deviant logician’s predic-
ament: ‘when he tries to deny the doctrine, he only changes the subject.’
(Ibid: 97)

For Williamson, logic does impose constraints on the world and thus, won-
dering about the laws of logic is wondering about the world. Because such
constraints achieve the maximal level of generality – they hold come what
may, that is, in all cases – for Williamson there is a lot more in common
between logical necessity and metaphysical necessity than is often thought by
supporters of the neutrality of logic.

3. The Law of Non-Contradiction

Here is my favourite case study on how metaphysics, that is, the investigation
of the most general and fundamental features of reality as such, allows us to
tackle logical issues. It is about a law, of which Aristotle (Met. 1005b 24)
claimed that it is the βεβαιωτάτη πασῶν ἄρχη, ‘the most certain of all princi-
pies’:

The most indisputable of all beliefs is that contradictory statements are not
at the same time true. (Met. 1011b 13-4)

‘Contradictory statements’ (ἀντικειμένως φάσεις) according to Aristotle are
a sentence (or its avowal, κατάφωσης) and its negation (ἀπόφωσης). There-
fore, to say that contradictory statements cannot both be true amounts to a
claim to the effect that a sentence and its negation cannot both be true or, equivalently – and to use the principle’s modern name:

The Law of Contradiction [claims]: ‘No statement is both true and false.’
(van Benthem, 1979, p. 335)

The Law of Non-Contradiction is nowadays taken as a principle of minimal logic. In spite of occasional attacks from vociferous minorities (think of Hegel and Marx – if they can be so interpreted, which is dubious), it is possibly the most venerable principle in the history of Western thought. The Medievals called it *fimissimum omnium principiorum*. Thomas Reid, a popular philosopher where I worked before coming to Amsterdam, put the Law among the dictates of common sense (together with other alleged self-evident truths, such as that every complete sentence must have a verb, or that those things did really happen which I distinctly remember).

However, the Law came under assault in the late Twentieth Century. The hardest attack came not from chatty postmodernists, but from within logic itself. Its precondition was the development of so-called paraconsistent logics, in which the principle called *ex contradictione quodlibet*, mentioned by Quine above, is rejected: \( A \) and \( \sim A \) do not entail an arbitrary \( B \), and the derivation of a contradiction within a formal theory having an underlying paraconsistent logic does not lead to Frege’s Mourning (the technical term for one’s reaction when one finds that one’s favourite system is inconsistent). Contradictions can be kept under control.

So-called strong paraconsistency, also called *dialetheism*, also claims that there are true contradictions, against the Law – a *di-aletheia*, ‘double truth’, being a truth-bearer \( A \), such that both it and its negation, \( \sim A \), are true. Motivations for dialetheid range from the semantic paradoxes of self-reference (think of the famous Liar: ‘This sentence is untrue’), to the set-theoretic paradoxes (think of the universal set, which is, by Cantor’s Theorem, bigger – and, of course, also not bigger – than itself), to legal inconsistencies and paradoxes involving change, motion, and the mereotopological boundaries of objects. We don’t need to enter into the motivations someone may have to challenge the Law, anyway, for this is not our issue here (see Berto and Priest, 2013, for an overview). Dialetheists have mounted such a serious case that dozens of papers on the topic are published every year in the world’s top philosophy journals, ranging from the *Journal of Philosophy* to *Mind* and the *Journal of Philosophical Logic*. What matters for us is, again, how methodologically difficult such a debate is.
The unsubstantial conception of logic may come into play here, in the form of Quinean change-of-subject allegations. Aristotle himself had stated (Met. 1005b 25-6) that, when someone claims that the same thing can both be and not be at the same time (the main villain was, in that case, Heraclitus), we should wonder if one actually means what one says. Epigones also suspect that dialetheists just play tricks with the meaning of ‘not’:

The fact that a logical system tolerates $A$ and $\neg A$ is only significant if there is reason to think that the tilde means ‘not’. Don’t we say ‘In Australia, the winter is in the summer’, ‘In Australia, people who stand upright have their heads pointing downwards’, ‘In Australia, mammals lay eggs’, ‘In Australia, swans are black’? If ‘In Australia’ can thus behave like ‘not’ [...], perhaps the tilde means ‘In Australia’? (Smiley, 1993, p. 17)

But things are even worse here. In his 1969 criticism of Hegel and Marx’s ‘dialectical logic’, Popper observed that arguing against someone who accepts contradictions (as, according to him, Hegel and Marx did) is methodologically baffling. Let $T = \{A_1, \ldots, A_n\}$ be a theory or belief set. One criticizes the $T$-theorist, or $T$-believer, by inferring from premises in $\{A_1, \ldots, A_n\}$, via rules of inference she accepts, some $B$ she rejects. A standard reductio move takes $B = \neg A_i$, $1 \leq i \leq n$. But the Marxist who accepts ‘dialectical contradictions’ in reality, such as the contradictions of capitalism, can be steadfast: she can maintain her $T$ without dumping $A_i$, and take $\neg A_i$ on board too. One who finds contradictions acceptable may not revise beliefs on pain of contradiction. Popper saw in this the death of rational criticism, freedom, and democracy.

Even masterful arguers like Lewis have surrendered the possibility of rational debate in the face of challenges to such basic logical principles as the LNC:

Nothing is, and nothing could be, literally both true and false. [...] That may seem dogmatic. And it is: I am affirming the very thesis that [the dialetheists] have called into question and – contrary to the rules of debate – I decline to defend it. Further, I concede that it is indefensible against their challenge. They have called so much into question that I have no foothold on undisputed ground. So much the worse for the demand that philosophers always must be ready to defend their theses under the rules of debate. (Lewis, 1982, p. 434)
4. Exclusion in the World

I have been interested for some time in how to do better. I believe that the way is via a metaphysical grounding for (a version of) the Law of Non-Contradiction. Here is a sketchy résumé.

Many have noticed (Batens, 1990; Parsons, 1990; Shapiro, 2004; Berto, 2008) that the dialetheist’s aforesaid ability to swallow contradictions cuts both ways. Dialetheists may not manage to rule out things they don’t want and to express their aversion. In their mouth, ‘~A’ may not exclude that A is the case, since for them both A and ~A may obtain: as they themselves admit, ‘we cannot use content-exclusion as a way of defining the sense, or content, of negation’ (Priest, Routley and Norman, 1989, p. 513). Nor are ‘A is false’, or ‘A is not true’, going to help: they may not rule out A’s being true either. In general, it seems that no operator $ that, applied to A, outputs a $A which has a designated truth value only if A hasn’t, can be a dialetheic exclusion-expressing device, on pain of devastating ‘revenge paradoxes’: logical paradoxes which breed disaster also for a contradiction-swaller.

This has a counterpart in terms of cases. In the framework of many paraconsistent logics, beginning with Graham Priest’s favourite one, LP (the Logic of Paradox: Priest 1979), the very notion of logical possibility is empty – or, maybe, completely filled. This happens because of the so-called trivial model of LP. Let all atomic sentences be both true and false (a truth value allowed in LP’s semantics, and which accommodates dialetheias). Then all sentences, truth-functionally, are. In particular, everything is true. So there is an admissible case in the semantics of LP, where everything obtains. Of course, we know that such a case is not a candidate for actuality, for the actual world is not like that – even for dialetheists, dialetheias occur only in specific and circumscribed contexts. However, we don’t know this by logic alone. In a nutshell, nothing is ruled out on logical grounds only in the dialetheic framework.

I think that we can address these issues by appealing to a primitive notion of metaphysical exclusion, or incompatibility (on why it should be taken as primitive, that is, not reducible to other concepts via definition, see Berto 2014). We can resort to such a notion to characterize an exclusion-expressing device, which works both for dialetheists and for supporters of the Law of Non-Contradiction. We can then define via such an operator a special metaphysical version of the Law, which may be immune from dialetheic challenges. I have pursued the strategy for some time now, but I won’t bore you with the technical details (and my open problems, which are many and difficult). I am just interested in the metaphysical foundations of the framework.
One begins with the plain remark that we have acquaintance with worldly incompatibility or exclusion. I cannot think of a more basic and more general feature of our experience of the world, than the experience of differences. Not everything is the same out there – or inside of us for that matter. More phenomenologically: differences manifest themselves. In particular, some things are incompatibly different. Incompatibility or exclusion shows up in the most rudimentary things new-borns learn to do: distinguishing objects, recognizing a border between something and something else; acknowledging that this thing’s being here rules out its simultaneously being there. We know that if an ordinary material object is uniformly green, it cannot simultaneously be uniformly red; that if it’s shorter than one inch then it cannot be longer than a mile.

How do we know? This is a challenging epistemological issue, but it seems safe to say at least: not by way of logic. I speak of material exclusion or, equivalently, of material incompatibility, to stress that it is not a merely logical, in the narrow sense of formal and a priori, notion: it is based on the content of the non-logical concepts involved. Neil Tennant called such concepts antonyms, observing that

Here the antonyms A and B are so simple and primitive that there cannot be any question of their ‘dialetheically’ holding simultaneously. Such antonyms A and B are antonymic not on the basis of their logical form, but on the basis of their primitive non-logical contents. The tension between them – their mutual exclusivity – is a matter of deep metaphysical necessity. (Tennant, 2004, p. 362)

There is something in material exclusion being characterized as a primitive completely general and, in this sense, metaphysical feature of the world. We have seen that there is no purely logically warranted exclusion for the dialetheist: given any A, in LP there is a model – if not others, the trivial one – both for A and for any other sentence. So ‘there is no logical guarantee against a person being a trivialist’ (Priest, 2006, p. 107). The dialetheist may have a vacuous notion of logical, formal incompatibility, at least in the sense that logic alone cannot exclude that the world is trivial. But she does have a notion of material incompatibility, for she knows that there are incompatible differences out there – things that just cannot obtain together. We have some from the dialetheist’s own mouth:

Someone who rejects A cannot simultaneously accept it any more than a person can simultaneously catch a bus and miss it, or win a game of chess
and lose it. If a person is asked whether or not A, he can of course say ‘Yes and no’. However this does not show that he both accepts and rejects A. It means that he accepts both A and its negation. Moreover a person can alternate between accepting and rejecting a claim. He can also be undecided as to which to do. But do both he can not. (Priest, 1989, p. 618)

Now we can exploit this primitive concept of exclusion, shared by both parties in the debate, by attempting to define via it a dialetheic-friendly exclusion-expressing device (call it ‘NOT’ – though it may end up being quite unlike any negation on the logical market), not yielding dialetheically intractable revenge paradoxes. One can phrase it, for instance, in terms of predicates and the corresponding properties: taken as input property $P$, NOT outputs its minimal incompatible, that is, that which is entailed by any $Q$ incompatible with $P$. Roughly, that $x$ is NOT-$P$ means: there is a $Q$ such that $x$ is $Q$, and $P$ and $Q$ are incompatible.

I believe that via such a NOT we may express in a non-question-begging fashion exactly what the divergence between dialetheists and their rivals on the concept of truth consists in. Those who deny that anything could be both true and false (or untrue) take truth and falsity (ditto) as reciprocally exclusive features of truth-bearers, in our primitive sense of exclusion. On the contrary, ‘Dialetheists hold that truth and falsity overlap’ (Priest, 1993, p. 40), that is, they are compatible: some truth-bearers can bear both. Then the disagreement between dialetheists and supporters of consistency has to do with the extension of a notion (whose intension) they both grasp and share: the notion of material exclusion. I also believe we can use such a NOT to define a notion of metaphysical contradiction making any such contradiction unacceptable for dialetheists as well as for their rivals.

Again, these are just hints. Even if the strategy works, such a result will not constitute a cheap victory for the friends of consistency. We may just learn that different things have been historically conflated under the label of ‘Law of Non-Contradiction’; that dialetheists rightly attack some formulations of the Law, and orthodox logicians and philosophers have been mistaken in assimilating them to the indisputable one. What matters for us is how such metaphysical considerations may allow us to have that ‘foothold on undisputed ground’ Lewis felt he lacked in order to have a rational debate on the topic.

Now take one of Aristotle’s so-called ‘metaphysical formulations’ of the Law, from Book Gamma of the Metaphysics, and just stick in it our putative operator, NOT. The formulation can be simply taken as a definition of ὀδὐνατον, ‘the impossible’:
For the same thing to hold good and NOT hold good simultaneously of the same thing and in the same respect is impossible [ἀδύνατον]. (Met. 1005b 18-21)

The ἀδύνατον is that which has no chance, no power (δύναμις) to be. It is what fails no matter what: what obtains in no case whatsoever. ‘P does NOT hold good of x’ should be a short form for ‘to x belongs some feature, Q, which is materially incompatible with P’. This does not seem to be question-able by the dialetheist anymore, provided she has understood NOT – and to understand NOT is to understand exclusion (which the dialetheist does understand, as we have seen).

That a version of the Law of Non-Contradiction has a metaphysical ground may say something on why, although Aristotle formulates variants of the Law both in his Organon, that is, in his writings on the subject of logic, and in the Metaphysics, only in the latter does he attempt a defence. More, he states that it is only up to the ‘first philosopher’, what we nowadays call the metaphysician, as opposed to the logician, to undertake the task of saying why the ‘axioms’ (and the ‘axiom’ par excellence is the Law of Non-Contradiction) are true, rather than false. He takes the Law to express a fundamental, structural, and most general – in this sense, again, metaphysical – feature of reality as a whole: ‘a principle which every one must have who knows anything about being.’ (Met. 1005b 14-15)

5. Thank You

Time to thank some persons. I am grateful to the Executive Board of this University, to the Dean of the Faculty of Humanities, to the Chairperson of Philosophy, and to all the people of the University of Amsterdam who wanted me.

During these early months here I have sometimes been feeling like a kid – this little Italian surrounded by tall Dutch people – thrown in the middle of something big, important, and challenging. As I do my best to understand your culture, your institutions, the workings of this University, and your language, I feel belittled by the importance of what goes on around me. I am grateful for your trust and at the same time I ask for your help, for I have a lot to learn from you all.

Something philosophical reassures me in this new adventure. I have been travelling a lot before getting here. I come from Venice, where I first learned philosophy from Vero Tarca, Luca Illetterati, and – though he may not want
to be called this way – the great Italian metaphysician of our time, Emanuele Severino. I moved to do ontology at the Sorbonne and the Ecole Normale Supérieure in Paris; then to the US, lost in the middle of Indiana at the University of Notre Dame with German virtuoso Vittorio Hösle. Then back to this side of the Ocean and up to Scotland, working with Crispin Wright at the edge of the world. Along the road, great people like Graham Priest, Greg Restall, Diego Marconi, and Achille Varzi shaped my philosophical mind. By the time this beautiful city, Amsterdam – the closest thing to my Venice I managed to find on the market! –, became my new home, I had acquired the belief that at least good philosophy is the same all over the world.

But the price of moving is that one leaves good friends behind. I mention none for they are far too many to be listed here; and also because listening to myself pronouncing their names again would resuscitate the pain of separation. But three persons I cannot but mention, who are forever, wherever I go, under my skin: mamma, papa, e Valeria. I love you with all my heart.

Ik heb gezegd.
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