We show experimentally that the sliding friction on sand is greatly reduced by the addition of some—but not too much—water. The formation of capillary water bridges increases the shear modulus of the sand, which facilitates the sliding. Too much water, on the other hand, makes the capillary bridges coalesce, resulting in a decrease of the modulus; in this case, we observe that the friction coefficient increases again. Our results, therefore, show that the friction coefficient is directly related to the shear modulus; this has important repercussions for the transport of granular materials. In addition, the polydispersity of the sand is shown to also have a large effect on the friction coefficient.
formed at the contact points of grains; this is the “pendular regime.” For 5% liquid [Fig. 3(b)], liquid bridges around the contact points and liquid-filled pores coexist. Both give rise to cohesion between particles: this is usually referred to as the “funicular regime.” Finally, for 10% liquid [Fig. 3(c)], more pores are filled with the liquid. The liquid surface forms large pockets within the material; this is the “capillary regime” [6,15,16].

The mechanical behavior of the sand upon addition of small amounts of water is fully understood, and has been tested for different grain materials and different liquids [15]. The basic physics is that the modulus starts to increase when capillary bridges form between the grains. However, for too much liquid, the capillary bridges start to merge (as is shown in Fig. 3), and they eventually disappear altogether when the sand is fully saturated. Therefore, there must be an optimum strength at a finite amount of added water.

This turns out to have large repercussions for the friction coefficient. The force as a function of the sled displacement (Fig. 2) shows that, especially for the dry sand, a high peak force has to be exceeded before a steady state can be reached. In steady state, we find that the pulling force is independent of pulling speed $v$ over the range of our measurements ($10 < v < 800 \text{ mm/s}$), but depends roughly linearly on the weight that is on the sled [Fig. 4(a)]. Defining an overall dynamic friction coefficient $\mu_d$ as the plateau value of the tangential force divided by the normal (gravitational) force given by the total weight of the sled, the friction coefficient is found to decrease if a small amount of water is added to the sand (Fig. 2). One of the reasons for this is rather simple, and hence was perhaps also observed by the Egyptians: in the dry case, a heap of sand forms in front of the sled before it can really start to move. This is also the reason for the peak in the force-displacement curve observed for the dry sand (Fig. 2), which shows that the static friction coefficient is

FIG. 2 (color online). Force-displacement curves for wet and dry Iranian sand. Inset: Picture of the setup. The picture on the left was taken while sliding over dry normalized sand. The picture on the right was taken while sliding over normalized sand wetted with 5% water. In the dry sand, a heap clearly builds up in front of the sled. The $11 \times 7.5$ cm sled is made out of PVC with rounded edges (as the Egyptian sled) and a roughness of 35 $\mu$m with sandpaper on its bottom; the results were qualitatively similar but less reproducible with a smooth bottom.

FIG. 3. Sections through 3D x-ray microtomograms of 500 $\mu$m polystyrene beads mixed with different amounts of liquid.

FIG. 4 (color online). (a) Macroscopic dynamic friction coefficient for different water contents (Iranian sand). (b) Friction coefficient and shear elastic modulus (right axis) as a function of the water content in Iranian sand. The blue horizontal line indicates the optimum shear modulus according to the model in [15] using a grain radius of 100 $\mu$m, a Young’s modulus of the grains of 60 GPa, and a water surface tension of 70 mN/m. The latter measurements were done on a commercial rheometer using a plate-in-cup geometry where the bottom of the cup, as well as the plate, was covered with sandpaper and the sand was compacted as for the sled experiments.
significantly higher for the dry sand. The peak, and hence the static friction, progressively decreases in amplitude when more water is added to the system; visual observation confirms that, indeed, the amount of sand that heaps up in front of the sled also decreases with increasing water content. We checked that our conclusion is not affected by the roughness of the bottom of the sled: similar results were obtained with and without sandpaper glued to the bottom.

Surprisingly, we find that for water contents in excess of 5%, the pulling becomes more difficult again: the friction coefficient increases [Figs. 2 and 4(b)]. We also verified this conclusion for two other types of sand: more polydisperse (ISO 679 standard) and more monodisperse (Nemours) sand (Figs. 5 and 6). On all three sand types, there is a minimum in the friction vs water content curve. The reason for this behavior follows from our measurement of the shear modulus [Fig. 4(b)]; for too high water contents, the stiffness of wet sand decreases again. In [15], a detailed description of the behavior of the shear modulus of wet granular material is given. We use the model from [15] to successfully predict (without adjustable parameters) the correct order of magnitude of the maximal shear modulus of the wet sand [blue horizontal line in Fig. 4(b)]. The nonmonotonic behavior of the shear modulus with water content is also known from building sand castles [6]; for too high water contents, the capillary bridges start to merge [16], the capillary pressure in the bridges decreases, and the elastic modulus decreases also.

The measurement of the shear elastic modulus vs water fraction of water in fact shows a trend that is exactly opposite to that of the friction coefficient, showing that there is an inverse relation between the two: the softer the sand, the higher the friction coefficient [Fig. 4(b)].

We further investigate this relation by plotting the friction coefficient as a function of shear modulus for the three different sand types. Figure 7 not only shows that the friction coefficient goes down as the sand becomes more rigid, but also that the decrease in friction coefficient is proportional to the increase in modulus. In fact, the data for three different sand types collapse onto a single line, indicating that all three frictional systems follow the exact same relation between shear modulus and friction coefficient.

Considering the three types of sand, we see that the drop in friction coefficient with the addition of small amounts of water becomes larger as the sand is more polydisperse: Nemours sand, which is the most monodisperse sand type, gives a 10% decrease, Iranian sand a 26% decrease, and the polydisperse standard sand a 40% decrease in the dynamic friction coefficient (Fig. 5). The Egyptians were pulling their sled through desert sand, which is very polydisperse [17] (Fig. 6). On such polydisperse sand the addition of a...
small amount of water reduces the pulling force by almost a factor of 2, according to our measurements.

Our measurements in fact span a similar range of stresses as the Egyptians; an estimate of the maximum load they pulled is one ton per square meter or 10,000 Pa. We put up to 20 N on roughly 80 cm², so we get to 2500 Pa, of the same order of magnitude. As for the archeologists, some have interpreted the pouring of the water in front of the sled as being purely ceremonial [1,2], which does not seem a correct interpretation in view of the results presented here. There is also evidence that in some cases the Egyptians built roads for the sleds out of wooden sleepers [3–5]. The possibility of dragging the sled through desert sand is often precluded because it is believed to be too difficult [3,5]. However, in view of our results, it seems very possible to drag the sleds over wet sand with the manpower available to the Egyptians [5]. In fact, the value of the friction coefficient of wood on wood is in the range of 0.25 < μₙ < 0.7 [18]; especially for the polydisperse sand here, which is closest to the Egyptian desert sand [17], we arrive at friction coefficients as low as 0.3; thus, the dragging can be just as easy over sand as over the wooden sleepers. In addition, the “optimal” friction coefficient of 0.3 that we find here coincides remarkably well with estimates that have been made on the basis of the tomb drawings. A friction coefficient of 0.33 was estimated, on the basis of the coefficient of maximum pulling strength that the ropes were able to sustain [19].

Summarizing, we find that there is a pronounced effect of the addition of small amounts of water to sand. The force necessary to move the sled at constant speed with a given weight on top of it can be reduced by as much as 40%, and the force necessary to get the sled to move by up to 70% on standard sand. This happens because the addition of water makes the sand more rigid, which prevents the heaping up of sand in front of the sled that makes the pulling difficult. This result strongly contrasts earlier experiments, where the pulling in fact became more difficult upon the formation of capillary bridges between the grains [10,13]. Interestingly, the measured friction coefficients for the highest water contents measured here are again larger than that of dry sand; perhaps the proposed mechanism of friction increase due to breaking of capillary bridges applies here [10,13].

One of the most striking results is that the friction coefficients measured for polydisperse sand are significantly lower than those for monodisperse sand. Perhaps the modulus of wet polydisperse sand can exceed that of wet monodisperse sand because the grain size distribution allows for a denser packing (which is more rigid). In view of the large amount of energy consumed worldwide for the transport of granular materials, this merits a more detailed study. It has been suggested for dry sand that the polydisperse grains can form the sand’s own ball-bearing system, in which friction is minimized by a size segregation that allows the grains to roll over each other with little friction [20]; perhaps a similar mechanism is at play here.

On the other hand, Fig. 7 shows that in fact all the measured friction coefficients decrease roughly linearly with increasing modulus. The conclusion must be that the more polydisperse sand has a lower friction coefficient simply because it has a higher modulus. The reason for the higher modulus is likely to be that the more polydisperse sand can be more densely packed, leading to a larger number of capillary bridges per unit volume, and hence a higher modulus. More generally, the frictional drag for transporting sand is still an issue of debate [6], and our results show that the presence of even very small quantities of water and polydispersity can change the friction, and hence the flow behavior, profoundly.

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