Neutrino transport in core-collapse supernovae
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Introduction

Half a century ago, Baade & Zwicky (1934) recognised that supernovae might be connected with the formation of neutron stars. It is now generally accepted that this is indeed the case, although not all supernovae make neutron stars. There exist at least two mechanisms that power supernovae. One way to blow a star to bits (in this case literally), is by a thermonuclear explosion of an accreting white dwarf in a binary, and this produces a Type Ia display. This kind of supernova leaves no stellar remnant and is not the subject of this thesis. The type of supernovae considered here, leaving behind a neutron star (or black hole) are collapse-driven, and give rise to a Type II (and possibly Type Ib/c) phenomenon (see Harkness & Wheeler 1990, for a discussion of these different observational supernova types).

In the formation of a neutron star, a binding energy of $GM^2/R \sim 10^{53}$ erg must be released. The actual supernova explosion involves the ejection of several solar masses into space with a kinetic energy of $\sim 10^{51}$ erg, and the optical display of $10^{46-49}$ erg that is observed as a supernova. The bulk of the energy ($\sim 10^{58}$ erg) is set free as neutrinos of all types ($\nu_e, \nu_\mu, \nu_\tau$, and their antiparticles). Only a fraction of the binding energy is sufficient to power the explosion. But despite the fact that supernova theorists have worked on the problem for decades and still do, the basic explosion mechanism still eludes us. The cause of the problem is the delicacy of the interplay between matter and neutrinos. Neutrinos can both be harmful and helpful in the explosion. But whatever the role of neutrinos may be, it is obvious from the energy budget that neutron star formation is mainly a neutrino affair. This was confirmed in 1987, when, for the first time ever, neutrinos from a supernova were detected with the Kamiokande II (Hirata et al. 1987) and IMB (Bionta et al. 1987). On Feb 23.23 UT, these detectors captured a total of 19 neutrinos (11 in the Kamiokande, 8 in IMB) of the $\sim 10^{11}$ neutrinos that passed each cm$^2$, and this was followed several hours later by the optical firework of supernova SN1987A in the Large Magellanic Cloud.

The explosion of Sanduleak -$69^\circ 202$, a blue supergiant at a distance of $50 \pm 5$ kiloparsecs (Andreani et al. 1987), was a gift from heaven for supernova modelers, who saw their basic ideas, developed in several decades, confirmed by the 19 observed neutrinos. The duration of the signal and the average energy of the neutrinos (see Fig.I) evidenced that a gravitational collapse had taken place at the core of Sanduleak, and that a proto neutron star had formed which shed its heat and potential energy in a burst of neutrinos lasting several seconds. It is not certain that in the end a neutron star was formed (a pulsar is not yet seen). Possibly, the proto neutron star collapsed to a black hole at a later stage due to fallback of matter (Burrows 1988, Gourgoulihon & Haensel 1993) or due to hyperonic phase transitions at supranuclear densities (Keil & Janka 1995, Baumgarte et al. 1996).

The overview given here of core-collapse theory is by necessity limited. The same holds for the description of the current core-collapse scenario given below. Extended reviews that cover also the observational aspects of supernovae are given by Bethe (1990), Burrows (1990), Müller (1990), and Trimble (1982, 1983). Further references can be found therein.
Figure I: Neutrino detections from SN1987A detected with Kamiokande II (circles) and IMB (stars). The energy (in MeV) refers to the secondary positron or electron that generated the Cerenkov light detected by phototubes in the detectors. Probably all detections resulted from $\bar{\nu}_e$ absorptions (rather than electron scattering), which are followed by positron emission. Data and error bars from Hirata et al. (1987) and Bionta et al. (1987).

The core-collapse scenario

Numerical simulations have been an important tool for the construction of the current supernova scenario. There is simply too much physics to deal with for the supernova problem to be solved on the back of an envelope. Hydrodynamics, transport theory, both in a general relativistic setting, the energy dependence of the neutrino cross sections, and the complexity of the equation of state, all of this together makes core-collapse a very non-linear problem which requires numerical methods. The first hydrodynamical numerical calculations (the first ever in astronomy) of core-collapse were performed by Colgate & White (1966), and many others have followed in their footsteps: Arnett (1977), Bludman et al. (1982), Baron et al. (1985), Baron & Cooperstein (1990), Bowers & Wilson (1982b), Bruenn (1975, 1985, 1986a,b, 1989a, 1989b), Bruenn et al. (1977), Burrows (1988), Burrows & Lattimer (1985,1986), Burrows et al. (1995), Hillebrandt & Müller (1981), Hillebrandt (1982), Hillebrandt et al. (1984), Lichtenstadt & Bludman (1984), Mezzacappa & Bruenn (1993a, 1993b, 1993c), Müller & Hillebrandt (1981), Myra et al. (1987), Myra & Burrows (1990), van Riper (1978, 1979, 1982), van Riper & Lattimer (1981), Swesty et al. (1994), Wilson (1985), Wilson et al. (1986). As mentioned earlier, the “supernova problem” has not been solved. Nevertheless, from all this work a scenario has emerged in which there are several parts that we can be quite certain of.
**Introduction**

$$n=3 \text{ polytrope. } M=1.2 \, M_{\odot}. $$

![Velocity profiles](image)

Figure II: Velocity profiles at a number of snapshots (at unequal time intervals) representing infall, core bounce, shock formation and propagation. Results from a hydrodynamical calculation (Smit 1996, unpublished). Initial model was a $n = 3$ polytrope with central density $\rho_c = 10^{10}$ g cm$^{-3}$. Collapse was initiated by lowering the electron fraction by 5% in the entire star.

Hoyle & Fowler (1960) showed that heavy stars ($M \gtrsim 8 \, M_{\odot}$) undergo the nuclear fusion sequence all the way up to Si burning, producing a core of iron-peak elements, after which collapse is inevitable. When the iron core has formed, pressure is provided predominantly by degenerate relativistic electrons. Such a configuration can support only a maximum mass, which is approximately the Chandrasekhar mass$^1$ (Chandrasekhar 1939)

$$M_{_{\text{Ch}}} = 5.76 \, Y_e^2 \, M_{\odot} \quad \text{(0.1)}$$

of a zero temperature gas of electron fraction $Y_e$. Anything larger than the Chandrasekhar mass is unstable and must collapse under its own weight.

Collapse of the iron core is initiated by neutrino losses that result from electron captures and by photodissociation of iron-peak elements. Both undermine the pressure support, or, equivalently, reduce the adiabatic index $\Gamma = \partial \ln P / \partial \ln \rho |_{n,Y_e}$ to less than the critical value $\Gamma = 4/3$. The collapse starts at a central density of $\rho_c \sim 10^9 - 10^{10}$ g cm$^{-3}$ and proceeds unhindered until nuclear densities $\sim 2 \times 10^{14}$ g cm$^{-3}$ are reached. Collapse takes about 0.5 seconds. Above nuclear density, nuclei "touch" and abruptly contribute to the pressure (the equation of state stiffens; $\Gamma > 4/3$), halting the collapse almost instantaneously in a large part of the iron core. The inner $\sim 0.6-0.7 \, M_{\odot}$ of the iron core bounces like a spring, the rebound smashing into the still infalling outer layers. A shockwave is formed at the edge of the inner core that starts to travel through the outer iron mantle. This stage of the collapse is depicted in Fig.II, as a time

$^1$The effects of non-zero temperature, pressure from the non-degenerate outer layers, Coulomb screening, etc. (Timmes et al. 1996) modify the "cold" Chandrasekhar mass given in Eq.0.1.
series of matter velocity $v$ versus radius $r$, showing how implosion (negative velocities), turns into an explosion (positive velocities) with shockfront moving outwards.

For some time, it was thought that this "prompt" shock wave would rush through the entire iron core, and emerge from it, expelling the much more tenuous and loosely bound outer layers of the star. However, the shock must deal with several obstacles. Energy is taken from the shock by the dissociation of iron material into alpha particles, which costs about $2 \times 10^{51}$ erg per $0.1 \, M_\odot$ mass of iron that the shock meets on its way out (Mazurek et al. 1980, Hillebrandt & Müller 1981), a loss comparable to the mechanical explosion energy. Neutrinos take away shock energy when the shock passes beyond the neutrino sphere (less than a millisecond after bounce). Matter is heated to high temperatures by the shock, boosting neutrino emission of all flavours as a result of mainly pair-processes. The shock is hindered further by the gravitational pull of the matter behind it.

In most numerical simulations, the shock wave cannot overcome the energy losses suffered along the way, and it stalls at a radius of $r \sim 200$ km, becoming an accretion shock a few milliseconds after bounce. Matter keeps accreting through the shock onto the proto neutron star behind it. If nothing else would happen, matter would keep accreting onto the proto neutron star until it would become too massive even to be supported by degenerate neutrons. A second collapse would then follow, forming a black hole. To produce a neutron star rather than a black hole, something else should happen. Wilson (1985) left his numerical code running for several days, and found that on the time scale of a second, the shock was re-energised by neutrino emission from the proto neutron star and able to expel the outer layers with just about the right energy (see also Bethe & Wilson 1985, and Bethe 1990). This "delayed" explosion mechanism has not proved to be sufficiently robust to find itself reproduced with the same success in other numerical simulations.

More recently, core collapse supernovae have attained more dimensions. It was already shown by Epstein (1979) that the thermodynamical profile of the hot proto neutron star plus shocked region is unstable against convection, but spherical symmetry was not abandoned until the 90s when Herant et al. (1992) investigated the effect of convection in two dimensions and found it to power a delayed explosion through enhanced neutrino luminosities. For a while, it seemed that the holy grail was found, but as more groups tuned in on multi-D hydro (Burrows & Fryxell, 1992, 1993; Burrows et al. 1995, Colgate et al. 1993, Herant et al. 1994, Janka & Müller 1993, 1995, 1996; Keil et al. 1996) the details and importance of convection became disputed, and now calculations with convection are reported that again fail to explode (Mezzacappa 1997, Mezzacappa et al. 1998).

The different outcome of numerical core collapse simulations can be attributed to differences in the equation of state used above nuclear densities, the particular progenitor model taken as the $t = 0$ model, details of the numerical scheme, the neutrino transport algorithm, or a combination of these factors. But more than being different, these investigations agree to a large extent in the basic ingredients, and what we learn from the fact that some do and some don't find an explosion is that with the current input physics, the explosion mechanism is not robust. At a recent (1997) supernova workshop in Santa Barbara, Wilson revealed that his delayed explosion mechanism did not work if his 12-bin neutrino spectrum was chosen to lie at slightly different energies. Whether that reflects the intrinsic nature of the problem, and that some heavy stars undergoing core collapse produce a neutron star, while others, with a slightly different structure go into black hole, is as yet unclear. It may well be that only stars with small iron cores ($M < 1.4 \, M_\odot$) explode and leave neutron stars, while in stars with fatter cores, the energy drain from dissociation of the iron material is too severe so that these end up as black
holes. Still, there may also be missing physics that would make the explosion more robust than it is now. It remains to be seen how neutrino oscillations, or rotation (Leblanc & Wilson 1970, Hillebrandt 1985, Mönchmeyer & Müller 1989, Mönchmeyer 1990) affect the explosion. And neutrino opacities are still being scrutinised (Reddy et al. 1997, Burrows & Sawyer 1998).

There is, however, an intrinsic marginality present in the collapse-driven supernova problem. During the formation of a neutron star, $10^{53}$ erg of binding energy is released. The bulk of this energy, 99%, is set free as neutrinos of all flavours. The kinematical explosion itself is a tiny tidbit of the main course. In a delayed mechanism, this bit results from subtraction of large numbers, and as such requires high accuracy to compute its precise magnitude. From this point of view, it is neutrino transport that deserves the most care, if only because neutrinos represent the largest part of the energy budget.

**Neutrino transport**

Early in the collapse, neutrinos freely escape from the infalling iron core, and neutrino “transport” is a pure emission process. Midway in the collapse, as densities increase beyond $\rho \sim 10^{11}$ g cm$^{-3}$, the iron core becomes opaque to neutrinos. Neutrinos become trapped in the flow when diffusion is slower than the infall velocity (In a Lagrangean scheme, they still move outwards with respect to the mass coordinate, but their average drift is directed inwards in an inertial frame at rest with respect to the center of the iron core). When trapped, neutrinos can attain thermal and chemical equilibrium with the matter. Because neutrinos have a nonzero chemical potential (Mazurek 1974, 1975, 1977; Sato, 1975), they can and do build up degeneracy when trapped.

During the cooling phase of the proto neutron star, there is a region where the neutrino radiation field decouples from the matter environment. This region is called the neutrinosphere, in analogy with a photosphere. Around this neutrinosphere and beyond, the neutrino radiation field is highly non-LTE. To describe the transition in time and space from neutrino-thin to neutrino-thick correctly, one needs in principle to turn to the Boltzmann equation. The Boltzmann equation describes the flow of particles not necessarily in thermal and/or chemical equilibrium with their surroundings.

However, to determine the neutrino distribution function in a fully dynamic setting is numerically a heavy task. Even in the case of spherical symmetry, a four-dimensional problem must be solved; the spherically symmetric distribution function, $F_\nu(t, r, \omega, \mu)$, depends on time $t$, radius $r$, neutrino energy $\omega$ and one directional angle $\theta = \arccos \mu$. In the past, this presented a major obstacle in numerical calculations, and approximative methods were developed and applied.

In the most simple approach the neutrino transport is ignored altogether (van Riper 1979; Lichtenstadt & Bludman, 1984), or described in an ad hoc manner where above a certain density neutrinos are fully trapped and in LTE with the matter, while allowed to freely escape at lower densities (Hillebrandt & Müller, 1981). Slightly more advanced is the leakage scheme (van Riper 1981 & Lattimer; Bludman et al. 1982) where the average neutrino mean free path determines the amount of neutrino energy flow. In these methods, the energy dependence of the neutrinos is not taken into account. However, the energy dependence of the neutrino cross-sections does not everywhere allow for a simple energy-averaged treatment. Moreover, in the end we want to know the emergent neutrino spectra, since in principle we are able to measure these, along with the neutrino light curves.
Knowledge of the local angular dependence of the neutrino radiation field is not strictly necessary as far as the hydrodynamics of neutrinos and matter is concerned. Taking this point of view, approximate methods have been designed which derive from angular averages of the radiation field. The most widely applied method is (multi-group) flux-limited diffusion (FLD), explained in more detail in the next chapter. FLD describes the neutrino flow in terms of the spectral energy density and the flux. It has been used in neutrino transport (Arnett 1977; Mazurek et al. 1982, Bowers & Wilson 1982a,b; Brue净水 1985; Myra et al. 1987; Mayle et al. 1987; Swesty et al. 1994; Burrows et al. 1995) as well as in dynamical photon transport. FLD describes the neutrino flow in terms of the spectral energy density and the flux. Shortcomings of FLD were outlined by Cernohorský & van den Horn (1990). An improvement over FLD is two-moment transport (TMT), which is the method used and analysed in this thesis. Two-moment transport was developed and applied by Dganí & Cernohorský (1991) and Cernohorský & van Weert (1992). In numerical calculations, spectral FLD or TMT are at this moment the best alternative to solving the Boltzmann equation. In the next two chapters it is argued that TMT should have preference over FLD.

Demanding accurate neutrino transport is more than of academic interest. When neutrinos of the next galactic collapse-driven supernova reach Earth, the successors of Kamiokande and IMB will detect many more neutrinos than the handful from SN1987A. Totani et al. (1998) estimate that Superkamiokande will detect 5000 to 10000 neutrinos if the event occurs at 10 kpc (or 200-400 at 50 kpc). The neutrino light curve and the spectral evolution of such a supernova will be able to tell us if a black hole formed rather than a neutron star, whether the explosion was direct or delayed, or may contain in another way the final clue of the explosion mechanism. So when it blows, we should be ready for it with accurate neutrino transport in our numerical codes.

Summary of this thesis

This thesis deals with several aspects of neutrino transport phenomena in core-collapse supernovae. The main theme is the Boltzmann equation, which describes the nature and evolution of the neutrino radiation field in dynamical or static situations present in neutron star formation.

Chapter 2 discusses in general terms the Boltzmann equation and its interplay with the fluid equations describing the dynamic environment. It is explained how the approximate scheme, two-moment transport, is obtained from the Boltzmann equation, and it is argued that two-moment transport provides an improvement over flux-limited diffusion. The role of the "closure" in two-moment transport is scrutinised in Chapter 3, with a prime focus on a particular closure which takes into account the fermionic nature of neutrinos. In that chapter, numerical solutions of both the Boltzmann equation and the two-moment transport equations in a typical proto neutron star environment are presented to allow for a direct comparison. Chapter 4 deals mostly with the mathematical structure of the two-moment equations. It is shown that the equations should contain a critical point, and the nature of this critical point is discussed. We argue that the critical point is not an obstacle that precludes numerical solutions of the two-moment equations, as had been suggested in the literature. An extra condition on the closure is derived from the requirement that the two-moment equations must be a hyperbolic set.

In the last two chapters two-moment transport is actually applied to a specific problem: the role of neutrino electron scattering. This interaction plays a crucial role during the infall phase (the actual collapse, prior to core-bounce), and it determines to a large extent the fate of
the shock that forms after core-bounce. From stationary state neutrino transport calculations against a "frozen" infall model, the detailed effects of neutrino-electron scattering are distilled in Chapter 4. Chapter 5, concluding this thesis, investigates the angular dependence of the neutrino-electron scattering cross-section.