Neutrino transport in core-collapse supernovae
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Citation for published version (APA):

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Appendix A

Neutrino-Matter Interactions

A summary is given in this appendix of the interactions that were taken into account in neutrino transport calculations in this thesis work. In the first section the general form of the collision kernel for binary collisions is given and absorption/emission and scattering processes are reviewed. Then, in Sect A.2, numerical expressions are given for particular interactions. Natural units $c = k = \hbar = 1$ are adopted in the text below.

A.1 Collision kernel

The neutrino-matter interactions considered here are binary collisions:

$$\nu + j = k + l \; , \tag{A.1}$$

i.e. two particles in $(\nu, j)$, and two out $(k, l)$. Absorption/emission processes have $j \neq k \neq l \neq \nu$, while for scattering $k = \nu, l = j$.

In the transport equation, Eq.(1.2), the collision kernel for (A.1) can be written as (cf. van den Horn & van Weert 1984)

$$\frac{\omega_{\nu}}{(2\pi)^6} B[F_{\nu}] = \frac{g_j}{(2\pi)^6} \int \frac{d^3p_j}{E_j} \frac{d^3p_k}{E_k} \frac{d^3p_l}{E_l}$$

$$[F_k F_l(1 - F_{\nu})(1 + \eta_j F_{j}) - F_{\nu} F_j(1 + \eta_k F_{k})(1 + \eta_l F_{l})] W_{\nu j \rightarrow kl} \; , \tag{A.2}$$

in which $W_{\nu j \rightarrow kl}$ is the transition probability for the process (A.1), $E_i$ is the energy of particle species $i$ (N.B.: $\omega_{\nu} = E_{\nu}$), $g_j$ is the statistical weight, and

$$\eta_i = \begin{cases} 
1 & \text{for bosons} \\
-1 & \text{for fermions} 
\end{cases} \tag{A.3}$$

Detailed balance, $g_k g_l W_{kl \rightarrow \nu j} = g_{\nu} g_j W_{\nu j \rightarrow kl}$ has been used in writing Eq.(A.2).

When the process (A.1) is in equilibrium, the collision kernel must vanish. With the equilibrium distributions of each particle species $i = \nu, j, k, l$ given by

$$F_i^0 = \frac{1}{e^{(E_i - \mu_i)/T} - \eta_i} \; , \tag{A.4}$$

we have, in Eq.(A.2),

$$F_k^0 F_l^0 (1 - F_{\nu}^0)(1 + \eta_j F_j^0) = F_{\nu}^0 F_j^0(1 + \eta_k F_k^0)(1 + \eta_l F_l^0) \; . \tag{A.5}$$

This ascertains that $B = 0$, because we have conservation of energy

$$E_{\nu} + E_j = E_k + E_l \; . \tag{A.6}$$
and, for the chemical potentials,
\[ \mu_\nu + \mu_j = \mu_k + \mu_l \, , \]  
(A.7)
where the last equation expresses (see van den Horn 1982) the conservation of all elementary charges (lepton number, baryon number, and electric charge). It is assumed that the matter-fluid is in a state of thermal equilibrium, characterised by the matter temperature \( T \), density \( \rho \), and one of the particle chemical potentials, usually \( \mu_e \), of the electron component. The chemical potentials of the other matter species are fixed by the equation of state.

### A.1.1 Absorption/Emission

In neutrino absorption and its inverse process, neutrino emission from electron capture, all the non-neutrino collider species \( i = j, k, l \) have \( F_i = F_i^0 \) given by Eq.(A.4). The integrations in Eq.(A.2) can, in principle, be done, because all the arguments are known functions (except for \( F_\nu \), but this term may be taken out of the integral). The collision kernel of emission/absorption processes can consequently be written as
\[ B_{\text{se}}[F_\nu] = j_e(1 - F_\nu) - \chi_\nu F_\nu \, , \]
(A.8)
in which \( j_e \) is the emissivity and \( \chi_\nu \) the absorption function. Using Eq.(A.5) in Eq.(A.2), Kirchhoff’s LTE relation for fermions is retrieved:
\[ j_e(\omega) = \frac{F_\nu^0}{1 - F_\nu^0} \chi_\nu(\omega) \equiv \kappa_\nu(\omega) F_\nu \, , \]
(A.9)
where the last equivalence defines the absorption opacity \( \kappa_\nu \) (including Fermi-blocking). The kernel is finally written as
\[ B_{\text{se}}[F_\nu] = \kappa_\nu(F_\nu^0 - F_\nu) \, . \]
(A.10)

### A.1.2 Scattering

With \( k = \nu, l = j \) in Eq.(A.2), the kernel of scattering processes can be written as
\[ B_s[F_\nu] = \int d^3p_\nu \left[ R_{\text{in}}(p_\nu, p_\nu) F_\nu(1 - F_\nu) - R_{\text{out}}(p_\nu, p_\nu) F_\nu(1 - F_\nu) \right] \, , \]
(A.11)
with the scattering rates
\[ R_{\text{in}}(\omega, \omega', \hat{\Omega}, \hat{\Omega}') = g_j(2\pi)^{-3} \int d^3p_j d^3p_j' \left\{ \frac{F_\nu^0(1 + \eta_j F_\nu^0)}{F_j^0(1 + \eta_j F_j^0)} \right\} \frac{W_{\nu_j \rightarrow \nu_j'}}{\omega_j \omega_{\nu'} E_j E_{\nu'}} \]  
(A.12)
Note that the angular dependence of the scattering rates is restricted to the combination \( \hat{\Omega} \cdot \hat{\Omega}' \) as a result of local isotropy of the matter-fluid.

Two symmetries relate the \( \text{in} \) and \( \text{out} \) scattering rates. The former results from internal symmetries of the transition probability (see de Groot et al. 1980)
\[ W_{\nu_j \rightarrow \nu_j'} = W_{j \nu \rightarrow j' \nu'} = W_{\nu_{j'} \rightarrow \nu_j} \, , \]
(A.13)
A.2 Opacities

from which

\[ R^{\text{in}}(p_{\nu}, p_{\nu'}) = R^{\text{out}}(p_{\nu'}, p_{\nu}) \quad (A.14) \]

Secondly, the LTE requirement, \( B_s[\mathcal{F}^0] = 0 \), gives

\[ R^{\text{in}}(p_{\nu}, p_{\nu'}) = e^{(\omega' - \omega)/T} R^{\text{out}}(p_{\nu}, p_{\nu'}) \quad (A.15) \]

In conservative (iso-energetic) scattering processes the neutrino energy \( \omega \) is unchanged in the collision, \( \omega' = \omega \). The scattering rate for conservative scattering contains a delta-function:

\[ R^{\text{in}} = R^{\text{out}} = R_{\text{aa}}(\omega, \hat{\Omega} \cdot \hat{\Omega}') \delta^1(\omega' - \omega)\omega^{-2} \quad (A.16) \]

in which the first identity is the result of \( \omega' = \omega \) in Eq.(A.15). The conservative scattering kernel becomes

\[ B_{\text{aa}}[\mathcal{F}^i] = \int d\hat{\Omega}' R_{\text{aa}}(\omega, \hat{\Omega} \cdot \hat{\Omega}') (\mathcal{F}^i(\omega, \hat{\Omega}') - \mathcal{F}^i(\omega, \hat{\Omega})) \quad (A.17) \]

Expanding \( R_{\text{aa}} \) in a Legendre series of the cosine of the scattering angle, \( \cos \theta \equiv \hat{\Omega} \cdot \hat{\Omega}' \),

\[ R_{\text{aa}}(\omega, \hat{\Omega} \cdot \hat{\Omega}') = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l + 1) \Phi_l(\omega) P_l(\cos \theta) \quad (A.18) \]

the kernel becomes

\[ B_{\text{aa}}[\mathcal{F}^i] = \kappa_s \left( \frac{E^0}{4\pi} - \mathcal{F}^i \right) + \kappa_s \frac{F^0}{4\pi} \hat{\Omega} \cdot \hat{\Omega}' + \sum_{l=2}^{\infty} (2l + 1) \Phi_l \int \frac{d\hat{\Omega}'}{4\pi} P_l(\hat{\Omega} \cdot \hat{\Omega}') \mathcal{F}^i(\omega, \hat{\Omega}') \quad (A.19) \]

with \( \kappa_s = \Phi_0, \kappa_s = 3\Phi_1 \), and \( E^0 \) and \( F^0 \) the first two angular moments of \( \mathcal{F}^i \), defined in Eq.(1.17). For the conservative neutrino scattering processes considered below in Sect.A.2.3 and A.2.4, it is sufficient to retain only the first two terms of the Legendre expansion.

A.2 Opacities

The particular neutrino-matter interactions used in calculations in the preceding chapters are listed in Table A.1. The transition probabilities \( W_{\nu_j \rightarrow kl} \) for these have been by computed by Tubbs & Schramm (1975), and Yueh & Buchler (1976a, 1976b) in the framework of the theory of electroweak interactions. The notation of Bruenn (1985) has been adopted in writing the opacities below.

A.2.1 Neutrino absorption on nucleons

The opacity for neutrino absorption on non-degenerate free neutrons is given by

\[ \kappa_{\text{aa}} = n_n \frac{G^2}{\pi} \frac{1 - \mathcal{F}^0(\omega + Q)}{1 - \mathcal{F}^0(\omega)} \left( 3g_\alpha^2 + g_\beta^2 \right)(\omega + Q)^2 \left( 1 - \left[ \frac{m_e}{\omega + Q} \right]^2 \right)^{1/2} \quad (A.20) \]
Neutrino-Matter Interactions

<table>
<thead>
<tr>
<th>absorption/emission</th>
<th>$\nu + n = e + p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scattering (conservative)</td>
<td>$\nu + A'(N + 1, Z - 1) = e + A(N, Z)$</td>
</tr>
<tr>
<td>scattering (non-con.)</td>
<td>$\nu + n = \nu + n$</td>
</tr>
<tr>
<td></td>
<td>$\nu + p = \nu + p$</td>
</tr>
<tr>
<td></td>
<td>$\nu + (A, Z) = \nu + (A, Z)$</td>
</tr>
<tr>
<td></td>
<td>$\nu + e = \nu + e$</td>
</tr>
</tbody>
</table>

Table A.1: Neutrino-matter interactions incorporated in $\nu$-transport.

The factor $Q$ is the neutron-proton mass difference, $m_n - m_p = 1.2935$ MeV, $m_e$ is the electron restmass, and $n_n$ is the free neutron number density in cm$^{-3}$. The coupling constants are $g_{\nu} = 1$, $g_{\alpha} = 1.23$. The Fermi constant $G$ has in standard units the value (Particle Data Group, 1996):

$$ G = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2} $$

In Eq.(A.20) it must be multiplied by a power of $\hbar c$ to give the rhs the dimension of inverse length,

$$ G^2 \rightarrow (\hbar c)^2 G^2 = 5.2973 \times 10^{-44} \text{ MeV}^{-2} \text{ cm}^2 . $$

A.2.2 Neutrino absorption on nuclei

The reaction

$$ \nu + A' = e + A $$

involves a nucleus $A(N, Z)$ and a nucleus $A' = A(N + 1, Z - 1)$ in an excited state after electron capture. The opacity is given by

$$ \kappa_{a,A} = e^{-\Delta/kT} \frac{1 - F^0_{\nu}(\omega + Q')} {1 - F^0_{\nu}(\omega)} \frac{1} {n_A} \sigma_{a,A} , $$

with the cross section

$$ \sigma_{a,A} = \frac{G^2} {\pi} \frac{2} {7} \frac{2}{N_p(Z) N_h(N)} \left( \frac{m_e} {\omega + Q'} \right)^2 \frac{1} {1 - \left( \frac{m_e} {\omega + Q'} \right)^2}^{\frac{1} {2}} . $$

The $Q$-value for electron capture on a nucleus is the mass difference of the excited and ground state nucleus:

$$ Q' = M_{A'} - M_A + \Delta \approx \mu_n - \mu_p + \Delta $$

The excitation energy $\Delta$ is set to 3 MeV (Bethe et al. 1979, Fuller et al. 1982). The functions $N_p(Z)$ and $N_h(N)$ are estimated from a zero-shell model (see Bruenn 1985)

$$ N_p(Z) = \begin{cases} 
0 & Z < 20 \\
Z - 20 & 20 < Z < 28 \\
8 & 28 < Z 
\end{cases} , $$

$$ N_h(N) = \begin{cases} 
6 & N < 34 \\
40 - N & 34 < N < 40 \\
8 & 40 < N 
\end{cases} . $$

The absorption opacities due to nuclei, Eq. (A.23), and free nucleons, Eq.(A.20), are added to give the total absorption opacity $\kappa_a = \kappa_{a,n} + \kappa_{a,A}$ in the absorption/emission kernel, Eq.(A.10).
A.2 Opacities

A.2.3 Scattering on free nucleons

Scattering on free neutrons and protons is collectively described by

$$\nu + N \rightleftharpoons \nu + N$$ \hspace{1cm} (A.28)

with the target \(N = (p, n)\) either protons or neutrons.

The thermal energy of the target particles and also the neutrino energy are taken to be small compared with the rest mass \(m_w\). As a consequence, the scattering is iso-energetic and takes the form Eq.(A.16). Furthermore, from neglecting the target motion, \(|p_k| \ll E_k\), the scattering rate becomes a linear function of the cosine of the scattering angle, i.e., the first order Legendre expansion, Eq.(A.18), is exact, and the interaction is completely described by the two scattering opacities

$$\kappa_{\nu,N} (\omega) = \frac{G^2}{\pi} n_w \omega^2 \left[ 3(h^N_\lambda)^2 + (h^N_\sigma)^2 \right]$$ \hspace{1cm} (A.29)

$$\tilde{\kappa}_{\nu,N} (\omega) = \frac{G^2}{\pi} n_w \omega^2 \left[ (h^N_\lambda)^2 - (h^N_\sigma)^2 \right].$$ \hspace{1cm} (A.30)

The neutral nucleon current form factors are

$$h^N_\lambda = \frac{1}{2} - 2 \sin^2 \theta_W, \quad h^N_\sigma = \frac{1}{2} g_A, \quad h^N_\pi = -\frac{1}{2} h^N_\lambda = -\frac{1}{2} g_A.$$ \hspace{1cm} (A.31)

The current value (Particle Data Group, 1996) for the Weinberg mixing angle is \(\sin^2 \theta_W = 0.2315(4)\).

A.2.4 Scattering on nuclei

An iso-energetic and zero momentum transfer approximation of neutrino scattering on nuclei

$$\nu + A \rightleftharpoons \nu + A$$ \hspace{1cm} (A.32)

brings the scattering rate in the form

$$R_{\nu,A} \propto (1 + \cos \theta)e^{-y(1-\cos \theta)},$$ \hspace{1cm} (A.33)

with \(y = 4b\omega^2\), and \(b = 4.8 \times 10^{-6} A^{2/3} \text{MeV}^{-2}\). Due to the presence of the exponential, the angular dependence is not linear, as was the case with scattering on nucleons. The first two terms of a Legendre expansion, Eq.(A.18) give rise to the scattering opacities

$$\left\{ \begin{array}{c}
\kappa_{\nu,A} \\
\tilde{\kappa}_{\nu,A}
\end{array} \right\} = n_A \sigma_{\nu,A} \left\{ \begin{array}{c}
(2y - 1 - e^{-2y})y^{-2} \\
(2 - 3y + 2y^2 - (2 + y)e^{-2y})y^{-3}
\end{array} \right\},$$ \hspace{1cm} (A.34)

in which

$$\sigma_{\nu,A} = \frac{G^2}{\pi} n_A \omega^2 A^2 \left[ C_{V0} - \frac{1}{2} \frac{N - Z}{A} C_{V1} \right]^2,$$ \hspace{1cm} (A.35)

with \(C_{V0} = -\sin^2 \theta_W\), and \(C_{V1} = 1 - \sin^2 \theta_W\).

A.2.5 Neutrino-electron scattering

Neutrino-electron scattering is discussed in Chapter 4, where expressions for the scattering rates \(R_{\nu e,S}(\omega, \omega', \hat{\Omega}, \hat{\Omega}')\) are given.