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On the chemical and spectro-photometric evolution of nearby galaxies
It is the task of a scientist, guided by the knowledge of his time, to propose a theory that takes into account what is known but which over and above this forecasts what future experiments should show

Karl Popper
On the chemical and spectro-photometric evolution of nearby galaxies

Academisch Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit van Amsterdam, op gezag van de Rector Magnificus prof. dr. J.J.M. Franse, ten overstaan van een door het college van dekanen ingestelde commissie, in het openbaar te verdedigen in de Aula der Universiteit op 10 April 1997 te 10.00 uur

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Cover illustration: Cube with the core of the spiral galaxy M100. The CCD image of M100 was taken with the Hubble Space Telescope after the STS–61 first Hubble Servicing Mission. During this mission, the Wide Field Planetary Camera II was installed to compensate for the effects of optical aberration in the 2.4 m primary mirror. Hubble's improved optics probed the core of M100 at a distance of about 56 million light years from the Earth and revealed for the first time details as small as 30 light years across (about 100 pc or 3 \(10^{14}\) km). The composite image of M100 shows two majestic spiral arms of bright stars and several fainter arms in which individual stars can be resolved. M100 is one of the brightest members of the Virgo cluster of \(\sim2500\) galaxies (about 75% of these systems are spiral galaxies, the remainder being mainly ellipticals with a few irregulars). Photo courtesy: NASA/STScI and JPL.
voor Karel Brink

voor Hester, Ellen, en Olga
fremant omnes licet dicam quod sentio
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1

General introduction and thesis outline

If you stare at the deeply lined face of an old, grey man and you have a close look at his eyes, expression, eyebrows, cheeks, skin, mouth, chin, beard, and pose, you may get a tiny impression of the kind of person he is and/or used to be. If you watch his movements, gestures, pliancy, the way he walks, eats, breaths, turns, and holds his walking-stick, you may deduce some of his character and way of life. If you see his hands, fingers, muscles, and knees, you may infer details about his occupations, habits, and daily pursuits. For instance, you may find out that the old man smokes (or did smoke), plays the guitar, is patient, does not sport frequently, works with his hands, reads a lot, and is a gentleman. If you compare the characteristics and behaviour of the old man with that of other aged persons, you may find that the old man is rather tall and active for people of his profession. You may deduce where the old man comes from and what is his age, provided that you can determine the other persons’ ages and birthplaces in an independent way. Then, from striking similarities, you may be able to conclude that old men who don’t smoke are more patient than people who do smoke with an age of 25 years and above, if they were born in western Europe. If you take a closer look at the interaction of the old man with that of other persons in his neighbourhood, you may observe that the old man predominantly interacts with men and women born in Japan so that he probably speaks Japanese. If the old man was born in Helsinki then he probably travelled a lot around the world. Combined with his habits, oversized hands, and strong muscles, this may indicate that he used to be a constructional engineer. In turn, you may conclude that most people in Japan become interested in architecture during their lives and that there must be a connection with their cultural education when they are approximately 18 years old. Thus, from the characteristics of many different people observed – at the same instant in time – you may be able to deduce what the old man among these people has experienced – during his life – as well as to forecast what he will be doing – in the next twentyfive years. This principle essentially forms the basis of the research on the chemical and spectro-photometric evolution of nearby galaxies presented in this thesis.

In this thesis, we focus on the star formation history and chemical evolution of the Galactic disk. Using a wide range of observational constraints, most of which have become available during the last few years, we aim to reconstruct the Galactic star formation history by modelling simultaneously various aspects of Galactic chemical evolution. In the second part of this thesis, we investigate the spectro-photometric and chemical evolution of nearby galaxies by means of a photometric evolution model. This model is applied to the stellar populations of Low Surface Brightness galaxies, a class of very faint galaxies for which a wealth of observational data have recently become available.

The chemical enrichment of the interstellar medium (ISM) by successive generations of stars is a key issue in understanding the chemical evolution of galaxies in general, and the formation history and abundance distributions of the stellar populations in our Galaxy in particular. The goal of galactic chemical evolution modelling is to predict reliable abundances in our Galaxy as well as in other galaxies both as a function of time and location. From such modelling, we can learn what processes essentially determine the chemical enrichment history of different galactic regions (e.g. disk, bulge, halo) and deduce how the formation and evolution of galaxies in general may have proceeded according to their chemical properties.
Both the chemical and spectro-photometric properties of galaxies are directed by the complex interplay between stars and interstellar matter. In order to unravel the underlying physics that can explain adequately the integrated properties of stellar populations observed in galaxies, detailed knowledge is required of how these populations formed, evolved, and interacted with the ambient ISM. In this thesis, we will test several basic concepts of galactic evolution and examine the ability of these concepts to reproduce various observational constraints simultaneously. From a comparison of such predictions for many individual galaxies, we hope to converge to a consistent and uniform description of how galaxies evolve and to understand the processes that can explain the origin of both the diversity and the striking similarities of the galaxies in our neighbourhood.

This thesis is organized as follows:

In Chapter 2, we deal with several observational and theoretical aspects of local and global star formation processes in galaxies. The chapter is meant as a brief introduction to the subject and highlights some of the basic assumptions and ingredients involved in the theoretical description of the process of star formation in galaxies.

In Chapter 3, we describe a general model for the chemical evolution of a galactic region such as the Galactic disk and discuss the basic equations, assumptions, and stellar input data used. The region is considered as an open, non closed-box system and is allowed to experience inflow and outflow of both gas and stars. In particular, we assume that the region is homogeneous at any time in its evolution. Up-to-date metallicity dependent stellar lifetimes, remnant masses, and nucleosynthesis yields are taken into account. We present the complete set of stellar yields of elements up to Zn used, both for Asymptotic Giant Branch (AGB) stars, Supernovae Type Ia (SNIa), SNIb/c, and SNII and discuss the uncertainties involved. An overall comparison of the yields of intermediate and massive stars is made by means of the cumulative initial mass function (IMF) weighted yields and the net yield function. These quantities are well suited for comparison with other galactic chemical evolution studies and may have direct implications for the chemical evolution of the Galactic disk.

The ultimate goal of modelling various observational characteristics of the chemical evolution of the local disk ISM is to obtain information about the principal processes that have directed the star formation history and chemical enrichment of the Galactic disk. In particular, the abundances of stars observed in the local Galactic disk and halo probably provide the most stringent test for Galactic chemical evolution models.

In Chapter 4, we concentrate on the star formation history and chemical evolution of the local Galactic disk. We consider several basic issues concerning the dynamical and chemical evolution of the Galaxy as a whole. We investigate the sensitivity of the age-metallicity relation (AMR) to specific model assumptions, and we select a set of models which can explain adequately the mean [Fe/H] vs. age relation observed in the local Galactic disk. The models selected are confronted with a wide range of observational constraints related to the chemical enrichment and stellar content of the disk. Our study has several important improvements over previous investigations which include: 1) simultaneous modelling of various independent observational constraints which were not available until recently (e.g. the stellar abundance-abundance variations for disk and halo stars, present-day element abundances in the disk ISM, the luminosity functions of white dwarfs and AGB stars, the age- and metallicity distributions of main-sequence F, G, and K dwarfs, the abundances in planetary nebulae, the gas consumption rate and gas ejection rate by evolved stars, the present-day mass fraction, the remnant mass distribution) using one and the same galactic chemical evolution model, and 2) the application of a uniform, up-to-date, and comprehensive metallicity dependent set of stellar evolution data. In particular, we investigate what kind of models are able to explain adequately the chemical evolution of the Galaxy as traced by the abundance-abundance variations observed among long-living stars and we discuss our results with respect to the validity of the various assumptions made.

While exploiting numerous improvements, both in theoretical and observational fields of Galactic evolution, we aim to reconstruct the star formation history of the Galactic disk and to derive essential quantities such as the current gas infall and star formation rate, the typical enrichment time scale for different elements, and the upper mass limit for SNII. Apart from studies that concern the evolution of our Galaxy, knowledge of these quantities are of particular interest for research on the evolution of nearby systems such as the Magellanic Clouds and M31. In turn, these Local Group galaxies provide an important reference frame of
observations to which the evolution of more distant galaxies in the universe can be compared. We summarize the type of chemical evolution models that are in best overall agreement with the observations and we discuss what our results may imply for the chemical evolution of the Galaxy as a whole.

Observational studies related to the heavy element enrichment of the local Galactic disk have long shown that stars of similar age exhibit large abundance variations (e.g., Mayor 1976; Twarog 1980a; Twarog & Wheeler 1982; Carlberg et al. 1985; Gilmore 1989; Klochkova et al. 1989; Schuster & Nissen 1989; Meusinger et al. 1991). Recently, Edvardsson et al. (1993) presented accurate abundance data for nearly 200 F and G main-sequence dwarfs in the solar neighbourhood (SNBH). Their spectroscopic data, analysed with up-to-date input physics, confirms abundance variations as large as $\sim 0.6$ dex in e.g. $\Delta [\text{Fe}/\text{H}]$ among similarly aged stars. In contrast to previous understanding, these variations are much in excess of experimental uncertainties and seem to suggest that the abundance spread for stars born at roughly the same galactocentric distance is similar in magnitude to the overall increase in metallicity during the lifetime of the disk.

In Chapter 5, we investigate the origin of the abundance variations observed among similarly aged F and G dwarfs in the local Galactic disk. We briefly review recent observations related to the inhomogeneous heavy element enrichment of the local disk ISM and we present arguments in support of combined infall of metal-deficient gas and sequential enrichment by successive stellar generations in the SNBH. A model for the inhomogeneous chemical evolution of a star forming gas cloud is presented which incorporates sequential stellar enrichment and mixing processes (including infall) and which allows for temporal and/or spatial inhomogeneities in the ISM. With this model, we investigate in detail the combined effect of metal-deficient gas infall and sequential stellar enrichment by successive stellar generations on the chemical evolution of multiple gas clouds in the Galactic disk. We show that Galactic chemical evolution models which take into account these processes simultaneously are consistent with the observed abundance variations among similarly aged F and G dwarfs in the SNBH as well as with the abundances observed nowadays in the local disk ISM. We discuss these results in the more general context of the chemical evolution of the Galactic disk and adduce observational arguments in support of both sequential star formation and metal-deficient gas infall in the local ISM.

In Chapter 6, we investigate the star formation history and chemical evolution of low surface brightness (LSB) disk galaxies by means of their observed spectro-photometric and chemical properties. The goal of this investigation is to address several plausible scenarios for the evolution of LSB galaxies by detailed modelling of their spectro-photometric and chemical properties. To this end, we developed a galactic spectro-photometric evolution model incorporating a detailed metallicity dependent set of up-to-date stellar input data covering all relevant stages of stellar evolution. We compare and calibrate the model and describe the basic set of star formation histories studied. Model results are compared directly with the observed colors, gas phase abundances, gas contents, and current star formation rates of LSB galaxies to constrain the global star formation history and chemical evolution of these systems. In particular, the impact of small amplitude star formation bursts in LSB galaxies is investigated and the star formation rates predicted are compared with the observations. We discuss our results in the context of the star formation history and dynamical evolution of LSB galaxies and we address the important question whether LSB spirals do have an evolutionary history fundamentally different from that of HSB spirals (such as the Galaxy) and dwarf galaxies.

We like to note that the research of galactic chemical and spectro-photometric evolution comprises many different areas which by far cannot be dealt with in one single thesis. In particular, the kinematical and hydrodynamical aspects of galactic evolution are not taken into account here. Other possibly important aspects include the influence on galactic chemical and spectro-photometric evolution of: the compact galaxy nucleus and spiral arms, the merging and interaction of galaxies in clusters, a distinction between molecular and atomic gas components, dust grains, galactic chimneys and superbubble blowouts, primordial nucleosynthesis, the early evolution of galaxies after the big bang, and dark matter. Certainly, these aspects will be considered in future studies of galactic evolution.
Naarmate het getij verloopt worden de bakens verzet
Star formation in galaxies: local and global processes in the ISM

Abstract
We briefly consider several observational and theoretical concepts of local and global star formation in galaxies. This chapter is meant as a brief introduction to the subject and highlights some of the basic assumptions and ingredients in the theoretical description of the process of star formation in galaxies.

2.1 Introduction
The underlying physics and mechanisms that determine the process of star formation in galaxies comprise one of the most fundamental problems involved in galactic evolution studies. We here briefly discuss several observational and theoretical aspects of local and global star formation processes. In particular, we consider the complex nature of the formation of protostars in molecular clouds and make a connection with some of the basic ideas concerning the overall process of star formation in galaxies.

2.2 Star formation in the local disk ISM
Molecular clouds are observed to be the birth sites of all stars nowadays formed in the Galactic disk as well as in other nearby galaxies. Individual molecular clouds (MCs) are thought to form by contraction and accumulation of atomic hydrogen clouds. Molecular cloud complexes may then be built from the coalescence of several giant MCs. The largest MC complexes in the Galactic disk (kpc scales, $\sim10^7 - 10^8 \, M_\odot$) are observed to contain several giant MCs ($10^2$ pc, $10^4 \, M_\odot$). These giant MCs often include smaller clouds ($\sim1-10$ pc, $10^2 - 10^3 \, M_\odot$) which in turn may contain dense and compact cloud cores ($\sim0.1$ pc, $1-10 \, M_\odot$). Stars are observed to form only in these compact molecular cloud cores (e.g. Myers et al. 1987).

A young radiating star may be observed as a far-infrared source when still embedded in its opaque parent MC. If the star's ionizing flux is sufficiently high, the parent MC will be associated with an extended HII region surrounding the star. Extended far-infrared (FIR) and CO surveys (e.g. Scoville et al. 1987; Myers et al. 1987) have shown that the Galactic disk contains at least two distinct populations of MCs (Lo et al. 1987): 1) warm, giant MCs (T$\sim$50–200 K; e.g. OMC1 in Orion) which are mainly observed in the spiral arm regions and wherein preferentially massive O and B stars are formed, and 2) cold, small MCs (T$\sim$20–40 K; B227 cloud) which are found throughout the Galactic disk and usually do not contain stars earlier in spectral type than late type B. The Orion Nebula is one of the nearest regions of recent star formation in the Galaxy. The youngest stars in this giant MC at a distance of about 450 pc are about $3 \times 10^5$ yr old (e.g. Zinnecker 1993).

Nearly all HII regions present in the CO and FIR studied parts of the Galaxy are found to be associated with giant, warm MCs (e.g. Myers 1986). Exceptionally, there are giant MCs which do not show evidence for the formation of OB stars (Myers et al. 1987). This suggests that the star formation process in MCs is stimulated under specific local conditions both in time and space. For instance, CO-observations of MCs in the Galactic disk reveal that massive stars form preferentially in the cores of MCs (e.g. Grabelsky et al. 1987; Larson 1991). In most HII regions, massive stars do not appear alone but are usually accompanied by a cluster of low-mass pre-main-sequence stars. Evidence for high-mass star formation without accompanying low-mass star formation are very rare (Zinnecker 1993). In contrast, there are many sites of low-mass star formation without associated high-mass star formation. The preponderance of small, low-mass MCs without HII regions indicates that massive stars tend to be absent in such MCs (Myers 1977; Waller et al. 1987).
The observations above suggest that: 1) the mechanism underlying the formation of the cores and large-scale structures in MCs must play a fundamental role in the physics of star formation (Lada et al. 1993), 2) the conditions of MCs which form solely low-mass stars are very different from MCs that form high-mass stars (e.g. Cernicharo 1992), and 3) star formation is probably a self-regulated process of which the efficiency depends on the efficiencies of cloud formation, of processes initiating star formation, and of cloud destruction mechanisms (Lada et al. 1993).

- **Bimodal star formation and other concepts**

The concept of bimodal star formation is based on the observed duality in the birth sites of stars of different mass (see review Shu et al. 1993). The decoupling of low and high-mass star formation originates from Herbig (1962) who proposed that low-mass stars form continuously over an extended period, interrupted only by the relatively rapid formation of high-mass stars (see below). These massive stars presumably heat, ionize, and disrupt the parental MC, preventing further star formation (e.g. Grabelsky et al. 1987; see also Larson 1988).

Observational studies from Leisawitz (1985; see also Larson 1988) show that remnant molecular cloud fragments are detected mainly around clusters of stars older than $\sim 5 \times 10^6$ yr. These results are consistent with that of Garmany, Conti & Chiosi (1982) who found cloud dipersal times of a few $10^6$ yr based on a comparison of evolutionary tracks for massive stars ($m \gtrsim 20$ $M_\odot$) that just appear as visible objects. The violence of massive star formation which may lead to dissipation of the remaining molecular material (e.g. by means of supersonic winds, ionization driven shock fronts, and/or supernova explosions), supports the idea of temporal variations in the star formation process. In case of bimodal star formation, such variations may differ for stars differing in mass.

In contrast to the suppression of star formation, massive OB-stars from a preceding star formation event nearby may also trigger the formation of a new generation of massive stars (i.e. sequential star formation). Although the majority of clusters of pre-main-sequence stars associated with OB stars appear to form from the dense gas concentrations in the cores of MCs, it appears that some clusters owe their origin to activity in the ISM associated with pre-existing generations of massive stars (e.g. Elmegreen 1989; Blaauw 1991; Massey et al. 1995). In several cases, multiple daughter star formation sites appear to originate from one parent site of star formation (cf. Zinnecker 1993; Goldsmith 1995). This addresses the concept of spontaneous vs. induced high-mass star formation in the Galactic disk ISM. Other examples of induced star formation include star clusters which are formed during the impact of a high-velocity cloud with the Galactic disk ISM (e.g. in case of the majority of the open clusters in the Perseus spiral arm, cf. Phelbs 1993; see also Sect. 5.5.2). The process of infall induced star formation is common in the Galactic disk (e.g. Lépine & Duvârt 1994; see Chap. 5) and may be the dominant mode of star formation in galaxies in which low surface densities inhibit global star formation (see Chap. 6).

In addition to the concepts of bimodal star formation and spontaneous/induced star formation, recent observations indicate two main star formation modes (Lada, Strom, and Myers 1993): 1) an isolated star formation mode in which single stars (or binaries) form from the individual gravitational collapse of small, isolated cores distributed throughout a MC (typically 0.3 stars pc$^{-2}$), and 2) a clustered mode of star formation in which rich clusters of stars form more or less simultaneously from a single massive concentration of dense gas, not in an isolated environment but in a densely packed one (e.g. 50 stars pc$^{-2}$). Observations show that the clustered mode is the dominant mode of star formation in several giant MCs in the Galactic disk. Clearly, the physical conditions in the massive MC cores producing rich clusters of stars are expected to be different from those in isolated MC cores forming single stars, e.g. in terms of star formation efficiency, stellar mass function at birth (see below), and star formation rate. A discussion of these conditions is beyond our scope. Instead, we consider in some detail the basic idea of star formation by fragmentation of MC clumps.

- **Fragmentation**

Fragmentation seems a plausible mechanism for the formation of stars in MCs (Spitzer 1978; Downes et al. 1987; Larson 1988). Gravitational instabilities in a homogeneous gas in pressure equilibrium will occur for cloud masses larger than the critical mass for gravitational collapse according to the Jeans criterion (cf. Spitzer 1978):

$$M_{\text{cl}} > M_3 \equiv \left( \frac{\pi kT}{\mu G} \right)^{3/2} \frac{1}{\rho_0^{1/2}} \approx 0.03 T^{1/4} \left( \frac{\mu m_{\text{H}}}{M_\odot} \right)^{3/4}$$

(2.1)

where $T$ is the cloud kinetic temperature, $\mu$ the mean mass per particle, and $\rho_0$ the unperturbed, homogeneous cloud density ($k$ and $G$ are the Boltzmann and the gravitational constant, respectively, and $m_{\text{H}}$...
2.3 Global star formation in galaxies

denotes the mass of a hydrogen atom). Due to the self gravitation of the denser gas regions, gravitational instabilities may grow and protostars are likely to form. With \( t_{ff} \) the free-fall time of a homogeneous sphere (defined as the time for a particle to travel halfway to the gravitational center of a spherical system) with mean density \( \rho_0 \) (and \( n_H \) the hydrogen number density in the cloud):

\[
t_{ff} = \left( \frac{3\pi}{32G\rho_0} \right)^{1/2} = \frac{4.3 \times 10^7}{n_H(0)} \quad [\text{yr}]
\]

one expects the densest regions to collapse at the shortest free-fall times. Usually, the free-fall time in a homogeneous, spherical MC is longer than the time scale for thermal adjustment (see Kippenhahn & Weigert 1994) so that the collapse proceeds almost isothermal. The collapse of the homogeneous central part of the cloud resembles free-fall as long as the matter can release gravitational energy via radiation. For isothermal collapse, the Jeans mass decreases with \( \rho^{-1/2} \) (Eq. 2.1). During the collapse, the free-fall time becomes shorter as the density increases (Eq. 2.2). When the gas becomes opaque and \( T \) increases, pressure becomes important and the cloud collapse is halted.

Only cloud masses large compared to stellar masses can become gravitationally unstable. A gas cloud exceeding the Jeans mass, collapses and undergoes fragmentation: cloud fragments become unstable and collapse faster than the cloud as a whole. The result is that for a collapsing cloud with mass \( M_d > M_J \), increasingly smaller and less massive gravitational condensations become the precursors of protostars with subsequently smaller masses. This is the basic idea of opacity-limited fragmentation. As discussed above, this kind of fragmentation can go on as long as the collapse remains roughly isothermal. Fragmentation stops approximately when the gas becomes so opaque that the radiative cooling time becomes longer than the free-fall time (Downes 1987).

The fragmentation scenario roughly explains the observed mass limits of stars (i.e. between \(~0.1\) and \(60-120 \, M_\odot\); e.g. Scalo 1986) according to the gas densities and temperatures in MCs (e.g. Spitzer 1978). Stars with masses as large as \(~10^3 \, M_\odot\) do not form, because fragmentation will occur during the extremely slow collapse of such low-density, high-mass concentrations. From estimates of the thermal adjustment time scale, fragmentation ends when the Jeans mass of the fragments are of the order of \(0.1-0.5 \, M_\odot\) (see Kippenhahn and Weigert 1994). Fragmentation appears the most probable, although incomplete, concept for star formation in MCs. The reverse process whereby massive stars would form by coalescence of low-mass protostars is (as is the case for the formation of entire galaxies) rather unlikely since in that case the narrow range in stellar mass observed would be hard to explain.

The observational fact that massive stars are formed predominantly in giant MCs while the formation of low-mass stars in such clouds appears inhibited is in the view of fragmentation explained by heating, ionization, and radiation pressure associated with the first generations of high-mass stars formed. However, we recall that opacity-limited fragmentation may be an oversimplified picture since e.g. magnetic fields (support of MCs against their self-gravity), cloud rotation, and turbulent velocities may affect the star formation process in MCs as well (e.g. Shu et al. 1993).

2.3 Global star formation in galaxies

A first step towards understanding the gas-star cycle in galaxies is to consider the impact of the different formation and evolutionary histories of low and high-mass stars (as well as their element yields and stellar remnants) on galactic chemical evolution (Dopita 1990). High and low-mass (\(m \lesssim 1-2 \, M_\odot\)) stars may be formed in different environments (see above). This may constrain theories of the history of the star formation rate (SFR) in galaxies. The formation of low-mass stars (e.g. \(m \lesssim 1 \, M_\odot\)) in galaxies may be primarily related to the gas surface density \(\sigma_g\) according to a Schmidt (1959) like variation of the SFR: \(\propto \sigma_g^n\) (with \(n = 1-2\); see also Caimmi 1995). The formation of high-mass stars in galaxies may be related to the total (both gas and stellar) surface density (see e.g. Dopita 1990; Dopita & Ryder 1994) which results in a SFR depending both on the total surface density and surface density of the gas.

Since most stellar mass is contained in low-mass stars (\(m \lesssim 1 \, M_\odot\); see below) one may expect that the formation of such stars is associated with the dominant mode of star formation in galaxies. The concept of a critical minimum gas density, i.e. a ‘star formation threshold’, below which star formation cannot take place may play a key-role in the onset of star formation in galaxies (e.g. Kennicutt 1983). This may apply both to the low-mass and relatively high-mass star formation modes in galaxies although the threshold for massive star formation may require e.g. considerably higher gas densities and larger MC masses. In principle, the star formation threshold is implicitly included in models in which the SFR strongly depends on the local gas density. To explain an important high-mass mode of star formation, additional conditions may be required such as the presence of a deep gravitational well associated with a pre-existing stellar population, e.g.
concentrated in the galactic nucleus and spiral arms. This would trigger efficient star formation of massive stars in the densest regions of galaxies until the gas is exhausted. Such gravitational instabilities stimulating massive star formation may be associated also with local gas inflow and/or tidal interactions/merging between galaxies.

A star formation law in disk galaxies based on the assumption that the pressure in the ISM is determined by the energetic processes associated both with star formation and the older stellar populations has been derived by Dopita (1985). This is the smaller component of a double star system with an orbital separation of about 3 10^8 km. Hubble observations suggest that these low-mass stars are surprisingly rare and certainly do not belong to the most luminous stars at birth have very important implications for Galactic chemical evolution.

For example, the IMF provides detailed information about the relative formation probability of stars of a given mass (averaged over the lifetime of the star forming system that is considered). We will deal with the IMF in more detail in Sect. 4.3.1. Here we will restrict ourselves to several relevant observations concerning the IMF of stars formed in the local Galactic disk and emphasize the importance of the IMF for galactic evolution studies.

2.4 The stellar mass function at birth

The stellar mass function at birth is a fundamental ingredient of theories of star formation: it is the essential link between the small-scale processes leading to star formation and the overall properties of entire stellar populations in galaxies. The so-called initial mass function (IMF) provides detailed information about the relative formation probability of stars of a given mass (averaged over the lifetime of the star forming system that is considered). We will deal with the IMF in more detail in Sect. 4.3.1. Here we will restrict ourselves to several relevant observations concerning the IMF of stars formed in the local Galactic disk and emphasize the importance of the IMF for galactic evolution studies.

The empirical IMF peaks at both 1.2 and ~3 M⊙ (Scalo 1986) and roughly follows a power law with slope 2.35 (i.e. the Salpeter (1955) IMF) for stars more massive than ~2–3 M⊙. The detailed variation of the IMF at low masses (m ≤ 2–3 M⊙) is not very well known. It may either continue to increase or it may flatten towards low-mass stars. Observations over the past decade suggest that the slope of the mass spectrum for less massive stars down to m ~0.08 M⊙ is much flatter, with slopes γ ≤ 1.3 (e.g. Scalo 1986). For convenience, it is usually assumed in galactic evolution studies that the masses of stars at birth are simply distributed according to the Salpeter IMF over the entire mass range. As we will show in Sects. 4.3.1–4, the slope of the mass function at low masses (i.e. m ≤ 1 M⊙) and the lower mass limit of stars at birth have very important implications for Galactic chemical evolution.

Recently, its luminosity is about 6 10^−4 times fainter than the Sun and its estimated mass is about 0.1 M⊙. G1623b is the smaller component of a double star system with an orbital separation of about 3 10^8 km. Hubble observations suggest that these low-mass stars are surprisingly rare and certainly do not belong to the most numerous stars in the Galaxy (e.g. Barbieri 1994; see also Tinney 1993). Objects in the mass range 0.04–0.1 M⊙ have been suggested to exist in significant numbers in regions associated with the Pleiades (Hambly & Jameson 1991; Simons & Becklin 1992), although large uncertainties are involved (see e.g. Zinnecker 1993).

There are several indications that the stellar lower mass limit at birth may vary in different galactic environments (Silk 1987). When the origin of the IMF and stellar mass limits at birth are described in terms of the distribution of suprathermal linewidths in MCs (Silk 1995; line widths are generated by protostellar...
2.4 The stellar mass function at birth

Outflows which are responsible also for limiting the mass that is accreted by a forming star; see also Adams & Fatuzzo 1996) such variations can be explained naturally. For instance, there is accumulating evidence that starburst galaxies are forming exclusively massive stars and that very few low-mass stars are formed in such systems (e.g. Rieke et al. 1985; Larson 1986; Silk 1987; Doane & Mathews 1993). This may be due to a shift of the lower mass limit towards masses as large as $\sim 2\text{–}3 \, M_\odot$ (note that at these masses the IMF of stars in the Galactic disk peaks). In particular, the lower mass limit will be extremely sensitive to the gas temperature according to the Jeans criterion (Eq. 2.1).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{mass_function.png}
\caption{Empirical mass function of M dwarfs in the Galaxy obtained with HST images. Stellar mass decreases from left to right. Figure adopted from Gould, Bahcall, and Flynn (1996).}
\end{figure}

Recently, the IMF of M dwarfs in the Galactic disk has been derived by Gould, Bahcall, and Flynn (1996; see Fig. 2.1) using observational data for 257 M dwarfs ($8 \lesssim M_V [\text{mag}] \lesssim 18.3$) detected in HST images after its first service. These observations reveal that the disk luminosity function of M dwarfs drops strongly for $M_V > 12 \, \text{mag}$ ($m \lesssim 0.25 \, M_\odot$) and decreases by a factor $\gtrsim 3$ by $M_V \sim 14 \, \text{mag}$ ($m \sim 0.14 \, M_\odot$). The derived mass function appears to peak at $\sim 0.23 \, M_\odot$ for a linear mass function $dN/dm$ (cf. Fig. 2.1). The slope of the observed mass function at $1 \, M_\odot$ is about -2.33 which is close to Salpeter. The rise of the mass function of the M dwarfs with the lowest masses detected is suggestive but without statistical significance (see Gould et al. 1996). The M dwarfs with $0.1 \lesssim m \lesssim 1.6$ appear to have characteristics intermediate to that of thin disk and spheroid populations while the estimated scale length of the M stars detected is $3\pm0.4 \, \text{kpc}$.

Gould et al. state that a Salpeter IMF for M stars with $M_V \gtrsim 8 \, \text{mag}$ is ruled out at the $12\sigma$ level by the HST images. Furthermore, they emphasize that the behaviour of the observed decrease in the luminosity function of M dwarfs is in good agreement with the ground-based photometry studies of nearby stars by Stobie et al. (1989). If the observed mass function of M dwarfs is not affected by selection effects, statistical incompleteness, or uncertainties in the mass-luminosity relation for these low-mass stars (which may e.g. depend on initial metallicity, stellar age, or other stellar quantities) and if all low-mass stars ever formed in the Galactic disk have been formed according to the same mass function, these observations exclude a Salpeter-like IMF or Scalo IMF. However, it is in particular unclear if and how corrections for vertical stellar orbital diffusion (escape of stars to larger scale heights) have been applied.

We will argue in Sect. 4.3.4 that if the IMF decreases for stars with masses less than $\sim 0.25 \, M_\odot$, as is suggested by the HST observations reported by Gould et al., there would be a severe overproduction of heavy elements in the Galactic disk ISM due to the enhanced formation probability of stars with masses $\gtrsim 1 \, M_\odot$ as compared to the Salpeter IMF. To convert $\sim 90\%$ of the disk ISM into stars by star formation over the lifetime of the Galactic disk (as suggested by the observations), low-mass stars which lock up ISM for times long compared to the lifetime of the Galaxy must be formed in sufficiently high numbers. For the IMF shown in Fig. 2.1, combined with the Salpeter IMF for more massive stars, the gas consumption argument would imply that too many massive stars are formed over the lifetime of the Galactic disk. Consequently, substantial over-enrichment of the disk ISM would be the result (according to the stellar yields described in Sect. 3.3). Thus, there would be no way to explain the present-day abundances observed in the ISM by means of such strongly decreasing IMFs towards low-mass stars.
Although a detailed investigation of the present-day ISM abundances as constraint to the IMF for low-mass stars is beyond the scope of this section, the above example illustrates that there is not much freedom for a strongly decreasing IMF towards low-mass stars. This may imply that: 1) not all long-living stars are formed according to the mass function implied by the M dwarf observations, i.e. a distinct IMF for stars of a given mass differing in properties other than initial mass (e.g. metallicity, galactic age at which the star is born, ISM conditions of the parent MC, etc.), 2) not all stars are formed according to the same star formation history (i.e. a bimodal SFR: stars are formed with rates depending on e.g. initial mass; see above). A stellar mass function varying with galactic age would be an convenient theoretical description of these possibilities. For instance, when the upper stellar mass limit at birth $m_u$ at early epochs in the evolution of the Galaxy would be considerably smaller ($m_u \sim 8 \, M_\odot$) than indicated by the observations ($m_u \sim 60-120 \, M_\odot$; e.g. Scalo 1986; Kroupa et al. 1993), this would avoid the overproduction of heavy elements discussed earlier. The above example illustrates that variations in the stellar mass limits at birth (or in the slope of the IMF) with galactic age may solve the inconsistencies implied by the HST observations.

Cameron (1962) was among the first ones to suggest that the IMF of the stellar generations formed at the time of onset of star formation in the Galaxy may have been very different from the present-day mass function of stars at birth (e.g. Salpeter’s IMF). Since stars with masses $m < 0.9 \, M_\odot$ are the only ones still around from the very metal-poor stars formed at the early epoch of star formation in the Galaxy, no detailed information is available on the formation history of more massive stars during these epochs. In particular, present-day observations cannot exclude the possibility that the formation of massive stars was suppressed at early Galactic evolution times. As we will discuss in Sect. 4.3.4, the abundance-abundance relations observed among Galactic halo stars cannot provide detailed tests for the IMF of stars more massive than $\sim 1 \, M_\odot$. Although massive stars must have been formed in substantial numbers to enrich the star forming ISM during early Galactic evolution, a small population of massive stars would be sufficient to enrich the disk ISM up to the level of enrichment observed in the most metal-poor disk stars (e.g. $[\text{Fe/H}] \lesssim -1.2$). This is true in particular if the disk built up gradually (by accretion and accumulation of material e.g. from the Galactic halo) so that the injection of relatively small amounts of heavy elements would enrich the disk ISM efficiently.

If the SFR is mainly regulated by the local gas density, one may expect that the mass spectrum of stars is sensitive to the gas density as well. For instance, the formation of massive stars could be suppressed in relatively low-density regions in the ISM. In this scenario, the formation of massive stars would be favoured towards later epochs in the evolution of the disk provided that the gas density in the plane of the disk increases with age (i.e. gas infall/accretion exceeds the amount of gas consumed by star formation). Clearly, this would have important consequences for the chemical enrichment of the disk ISM. Instead of variations in the stellar mass function at birth with Galactic age, similar effects on Galactic chemical evolution could be achieved e.g. by variations in the upper mass limit of stars that eject a substantial fraction of the enriched material exterior of their iron core (i.e. by supernova explosions). Such variations may be expected if the detailed explosion mechanism of massive stars depends on e.g. the initial stellar metallicity (see Sect. 3.3.3). We note that the effects of an age-dependent IMF on Galactic chemical evolution heavily depends on the manner in which infall regulates the gas reservoir in the disk as well as the rate of star formation therein.

In the following, we will restrict ourselves mainly to the assumption of a Salpeter or Scalo mass function for stars at birth independent of Galactic age. Nevertheless, we found it worth to consider the possibility that the IMF at low masses may have varied as indicated by the HST observations as well as to address the possible implications this may have for the chemical enrichment and star formation history of the Galaxy.
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Abstract

We describe the set of equations used to model the chemical evolution of the Galactic disk and discuss in detail the initial conditions, basic assumptions, and stellar evolution data involved.

Several important improvements over previous galactic chemical evolution models (see e.g. Arnett 1973; Audouze & Tinsley 1977; Twarog 1980; Tinsley 1980; Chiosi 1982; Matteucci 1990; Timmes et al. 1995) are made here which include: 1) the use of an up-to-date and comprehensive library of metallicity dependent stellar evolution data (e.g. stellar lifetimes, remnant masses, and nucleosynthesis yields) which cover all relevant phases up to the latest stages of stellar evolution, 2) a new, iterative method for solving the galactic chemical evolution equations taking into account the dependence of stellar evolution and chemical enrichment on initial metallicity, 3) the inclusion in the galactic chemical evolution equations of terms associated with the inflow and outflow of both gas and stars, 4) a new, detailed treatment of the chemical evolution of Asymptotic Giant Branch stars and their element yields (see van den Hoek and Groenewegen (1997) for a more extended description of the AGB yields than is given below), and 5) the wide range of plausible star formation histories (SFR) and stellar initial mass functions (IMF) that can be used in the model (e.g. IMF depending on the SFR; see Chap. 4).

Considerable effort has been made to discuss and to emphasize the assumptions and uncertainties involved with the current generation of galactic chemical evolution models. In particular, a thorough comparison is made between the two state-of-the-art models for the core-collapse and chemical evolution of massive stars, i.e. the model presented by the group of Woosley and Weaver (1996) on one hand, and that presented by the group of Nomoto, Thielemann, and Hashimoto (1996) on the other. This comparison clearly reveals for the first time the magnitude and origin of the uncertainties and differences between the yields of massive stars predicted by the two groups and allows for a more reliable interpretation of several discrepancies between the models and specific observational constraints to the chemical evolution of the Galactic disk.

Introduction

The aim of galactic chemical evolution modelling is to predict reliable abundances in our Galaxy as well as in other galaxies both as a function of time and location. Basic ingredients involved with galactic chemical evolution models are the star formation history and accretion or outflow history of the region of interest (e.g. galactic disk, halo, or bulge), and the nucleosynthesis yields of the stars enriching this region. In principle, both the star formation and accretion/outflow history are determined by a complex set of processes including the kinematical and dynamical evolution of the galaxy, the hydrodynamical evolution of the ISM (e.g. density, temperature, and pressure), and the interaction between gas and stars (mixing, radial flows, etc.). We deal with the assumptions related to these processes in Chapter 4. Here we describe a general model for the chemical evolution of a galactic region such as the Galactic disk and discuss the basic equations and stellar input data used. The region is considered as an open, non closed-box system and is allowed to experience inflow and outflow of both gas and stars. We assume that the region is homogeneous at any time in its evolution. Furthermore, we relax the instantaneous recycling approximation (e.g. Searle & Sargent 1972) and take into account metallicity dependent stellar lifetimes, remnant masses, and nucleosynthesis yields.
In Sect. 3.1, we describe the basic equations, assumptions, and main input parameters used. In Sect. 3.2, we discuss the adopted set of metallicity dependent stellar lifetimes and remnant masses. In Sect. 3.3, we present the complete set of metallicity dependent stellar yields of elements up to Zn, both for AGB stars, SNII, SNIIb/c, and SNIa. A discussion of the uncertainties involved in the yields is given in this section as well. Finally, in Sect. 3.4, we make an overall comparison of the yields of intermediate and massive stars by means of both the cumulative IMF-weighed yields and the net yield function, and discuss their possible implications for galactic chemical evolution.

### 3.1 Basic equations and model assumptions

Defining $C$, $E$, $F$, and $T$ to be, respectively, the star formation rate by mass (SFR), the ejection rate of matter by evolved stars, the infall rate of gas, and the transport rate of stars, the equations governing the evolution of the total masses of stars, gas, and metals can be written as (for details see e.g. Audouze and Tinsley 1976; Twarog 1980; Tinsley 1980; Marshall et al. 1982; Güsten 1986):

\[
\frac{dM_*}{dt} = C - E + T 
\]

\[
\frac{dM_g}{dt} = -C + E + F + T_g 
\]

\[
\frac{d[ZM_j]}{dt} = -CZ + E_z + F_z + T_z 
\]

where $E_z$ is the metal ejection-rate of stars formed within the region under consideration, $F_z$ the infall (or accretion) rate of metals onto this region, and $T_g$ and $T_z$ are the amounts of gas and metals returned by stars that moved into (or out of) this region. In general, all quantities in the right hand side of Eqs. (3.1–3.3) are age dependent. For a composite galactic system of multiple regions, these quantities depend also on spatial position (e.g. galactocentric radius). Terms associated with gas infall ($F$) and transport of stars ($T$) can be either positive (addition) or negative (reduction).

The evolution of the stellar and gas content is described by Eqs. (3.1) and (3.2) with corresponding boundary conditions $M_g(t = 0) = M_{\text{rem}}(0)$ and $M_g(t = 0) = 0$. The total mass of metals present in the ISM is denoted by $ZM_g$ where $Z(t) \equiv M_g(t)/M_g(0)$ is defined as the interstellar metal-abundance integrated over elements heavier than helium (including elements contained within dust). The chemical evolution of element $j$ can be described in a similar way when replacing the $Z$-dependent terms in Eq. (3.3) by corresponding quantities related to the abundance of element $j$. The resulting age-metallicity relations (AMR) are obtained by solving iteratively the above system of non-linear, coupled integro-differential equations while accounting for the dependence of the stellar lifetimes, remnants, and stellar yields on initial metallicity (see below). An approximate analytical solution method is given in the appendix to this chapter (Appendix A).

#### 3.1.1 Star formation rate and initial mass function

The star formation rate by mass $C$ and returned matter rate $E$ are defined as follows:

\[
C(t) \equiv S(t) \int_{m_1}^{m_*} m M(m) \, dm = S(t) < m_* > 
\]

\[
E(t) \equiv \int_{m_1}^{m_*} (m - m_{\text{rem}}(m, Z_*)) M(m) S(t - \tau(m, Z_*)) \, dm 
\]

where $<m_*>$ is the mean stellar mass formed, $m_1$ and $m_*$ are the stellar mass limits at birth, $M(m)$ is the IMF, $S(t)$ is the SFR by number, $\tau(m, Z_*)$ is the lifetime of a star with initial mass $m$ born with metallicity $Z_*$, and $m_{\text{rem}}(m, Z_*)$ is the stellar remnant mass. In this notation, $S(t - \tau(m, Z_*))$ denotes the star formation rate by number at the time a star of initial mass $m$ (dying at galactic age $t$) was formed. The turnoff mass $m_*(t)$ is defined as the mass of a star formed at ($t = 0$) which ends its life at galactic age $t$. We emphasize that the turnoff mass $m_*(t)$ is very sensitive to the initial stellar metallicity $Z_*$ which has important implications for galactic chemical evolution (see Sect. 3.2). For convenience, we will exclude this metallicity dependence in the notation of $m_*(t)$.

The star formation rate $C(t)$ is defined as the product of the number of stars formed per unit time and the mean stellar mass formed. For the IMF, we will adopt a power law, i.e. $M(m) \equiv dN(m)/dm \propto m^{-\gamma}$ normalised as $\int_{m_1}^{m_*} M(m) \, dm = 1$. In general, the conventional assumptions of a separable SFR (i.e. the
age and mass-dependent parts of the SFR are separated) and of an age-independent IMF are convenient approximations when solving the galactic chemical evolution equations. More general expressions for the SFR and IMF will be considered in Sect. 4.2.

The ejection rate \( E(t) \) includes all matter ejected by stars ending their lives at a galactic age \( t \). Since the remnant mass of a star with initial mass \( m \) depends on initial metallicity \( Z_* \), direct computation of \( E(t) \) using Eq. (3.5) is impossible without detailed knowledge of \( Z(t) \). Therefore, an iterative solution method is required in order to compute \( E(t) \) and \( Z(t) \) simultaneously. To facilitate this procedure, Eq. (3.5) has been transformed into an integral over stellar birth time \( t^b \):

\[
E(t) = \int_{t^b=0}^{t} (m_t - m_{\text{rem}}(m_t, Z_*)) \, M(m_t)S(t^b) \, \frac{dm}{dt^b} \, dt^b
\]

where \( m_t \equiv m_0(t - t^b, Z_*(t^b)) \). This integral can be calculated directly by incorporating the stellar birth time and the initial metallicity \( Z_* = Z(m, t^b) \) of a star born with mass \( m \). Analogue transformations are used below when solving integrals with similar terms related to the ejection of material by evolved stars. In this manner, the galactic chemical evolution equations (Eqs. 3.1–3.3) can be solved iteratively.

### 3.1.2 Newly synthesized and unprocessed metals

We consider the total ejection rate of metals \( E_z(t) \) (cf. Eq. 3.3) in more detail. We define the rate of newly synthesized and ejected metals by evolved stars at galactic age \( t \) as:

\[
E_{z,\text{syn}}(t) = \int_{m_0(t)}^{m_\text{tot}} mp_\text{z}(m, Z_*) M(m)S(t - \tau(m, Z_*)) \, dm
\]

where \( p_\text{z}(m, Z_*) \) is the heavy element integrated stellar yield which is defined relative to the initial metal-abundance of the star (cf. Sect. 3.3). In a similar way, we define the ejection rate of unprocessed metals (which originate from the material out of which a star was formed) as:

\[
E_{z,\text{old}}(t) = \int_{m_0(t)}^{m_\text{tot}} \Delta m^\text{tot} Z(t - \tau(m, Z_*)) M(m)S(t - \tau(m, Z_*)) \, dm
\]

where \( \Delta m^\text{tot} \equiv (m - m_{\text{rem}}(m, Z_*)) \) refers to the total amount of material ejected by a star of initial mass \( m \), and \( Z_* = Z(t - \tau(m, Z_*)) \) is the initial metal-abundance of stars that evolve off the RGB at galactic age \( t \). Note that Eqs. (3.7) and (3.8) are valid also for the ejection rates of elements \( j \) by replacing \( p_\text{z} \) (and \( Z \)) with \( p_j \) (and \( Z_j \)). In this manner, the total ejection rate of metals can be written as:

\[
E_z(t) = E_{z,\text{old}}(t) + E_{z,\text{syn}}(t)
\]

This expression is consistent with the definitions of \( \Delta m^\text{tot} \) and \( p_\text{z} \) above (see Eq. 3.12). In contrast, as has been pointed out by Maeder (1992), the inclusion of a term \(-mp_\text{z}\) in the definition of \( \Delta m^\text{tot} \) (e.g. Tinsley 1980; Chiosi 1986; many others) is inconsistent with the above definitions and results in large errors for elements with high ISM abundances such as helium.

From Eqs. (3.3) and (3.8) it is evident that the present-day metal-abundance is very sensitive to the solution of the metallicity equation in the past. Apart from the retardation term \( Z(t - \tau(m, Z_*)) \) in Eq. (3.8), both the metallicity dependent stellar yields \( p_\text{z}(m, Z_*) \), remnant masses \( m_{\text{rem}}(m, Z_*), \) and stellar lifetimes \( \tau(m, Z_*), \) determine the stellar ejection rates \( E(t) \) and \( E_z(t) \). In turn, the stellar ejection rate \( E(t) \) affects the present-day gas-to-total mass ratio \( \mu_1 \equiv M_g/M_{\text{tot}} \) which, thereby, is a strong function of the enrichment history of the ISM (and so are all quantities that are related to \( \mu \) such as the SFR). Conversely, the gas fraction \( \mu(t) \) plays an important role for the abundances in the ISM and an iterative solution method of the galactic chemical evolution equations is required.

In Sect. 3.3 we will describe the yields \( p_\text{z}(m, Z_*) \) of Asymptotic Giant Branch (AGB) stars, SNII, SNIa, and SNIIb/c. These yields are used in Eq. (3.7) and are linearly interpolated in \( m \) and \( Z_* \), if necessary. We will assume that the enriched material is returned instantaneously at the moment a star ends its life, even though a substantial fraction of the metals ejected is returned during the lifetime of the star. This assumption seems justified as model calculations by Guzik & Struck-Marcell (1988), which incorporate the effects of continuous stellar mass loss, show that the resulting gas fraction \( \mu(t) \) as well as age-metallicity relation \( Z(t) \) do not differ by more than \( \sim 10\% \) compared to models with instantaneous loss of the stellar envelope at the end of a star’s lifetime. Note that this assumption is rather distinct from the instantaneous recycling approximation in which the stellar lifetimes are assumed to be zero.
As discussed above, we assume that the interstellar gas is well mixed, i.e. homogeneous, at all evolution times. This assumption is valid as long as the mixing time scale of the ISM ($\sim 10^7 - 10^8$ yr; e.g. Edmunds 1975; Tinsley 1980; Cioffi & Shull 1991; Roy & Kunth 1995) is comparable to (or shorter than) the enrichment time scale. Inhomogeneous chemical evolution of the ISM will be considered in Chapter 5.

### 3.1.3 Initial conditions, model parameters, and solution method

- **Initial conditions**

The main initial conditions to the galactic chemical evolution equations, Eqs. (3.1–3.1), are the galaxy initial mass, the present-day gas mass and total mass, the initial abundances, the galactic age, and the normalisation of the SFR. In this section, we discuss these conditions for the Galactic disk (MW) and the Magellanic Clouds (LMC and SMC).

In Table 3.1 we summarize the values adopted for the total mass and gas content in these systems. The present-day gas content is given by the sum of the masses contained in molecular and atomic hydrogen, and in helium. We neglect the mass contained in ionized species as well as that contained in elements other than hydrogen and helium. The corresponding present-day gas-to-total mass ratio $\mu_1$ is also listed in Table 3.1. Unless stated otherwise, we will assume that the initial galaxy is void of stars, i.e. $M_*(0) = 0$ and has total mass $M_*(0) = M_{tot}$ (i.e. $\mu_1(0) = 1$) as listed in Table 3.1. We note that absolute errors in the data included in Table 3.1 may be as large as 50%.

The value of $\mu_1 = 0.05 - 0.15$ in the Galactic disk is based on observations in the solar neighbourhood (e.g. Kulkarni & Heiles 1987; Fich & Tremaine 1991). In the following, we will adopt $\mu_1 = 0.1$ as the most probable value. For the LMC, a somewhat larger value of $\mu_1 = 0.15$ is adopted. In the SMC, nearly half of the total galaxy mass has been converted into long-living stars (and remnants), i.e. $\mu_1 > 0.5$.

Initial element abundances are taken to be primordial (unless stated otherwise). Primordial hydrogen and helium abundances are adopted as $X_p = 0.768$ and $Y_p = 0.232$ (cf. Pagel 1992). Primordial abundances of elements heavier than helium are set to zero.

<table>
<thead>
<tr>
<th></th>
<th>$M_{HI}$</th>
<th>$M_{H_2}$</th>
<th>$M_{He}$</th>
<th>$M_g$</th>
<th>$M_{tot}$</th>
<th>$\mu_1$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW*</td>
<td>$2.2 \times 10^8$</td>
<td>$2.3 \times 10^9$</td>
<td>$1.4 \times 10^9$</td>
<td>$6.0 \times 10^6$</td>
<td>$1.8 \times 10^9$</td>
<td>$0.10 \pm 0.05$</td>
<td>1, 2, 3, 4, 5, 6</td>
</tr>
<tr>
<td>LMC</td>
<td>$3.6 \times 10^8$</td>
<td>$1.8 \times 10^9$</td>
<td>$1.6 \times 10^9$</td>
<td>$5.6 \times 10^6$</td>
<td>$6.1 \times 10^9$</td>
<td>$0.15 \pm 0.05$</td>
<td>7, 8, 9, 10, 11–13</td>
</tr>
<tr>
<td>SMC</td>
<td>$4.8 \times 10^8$</td>
<td>$1.6 \times 10^8$</td>
<td>$1.4 \times 10^8$</td>
<td>$6.5 \times 10^6$</td>
<td>$1.5 \times 10^9$</td>
<td>$0.50 \pm 0.10$</td>
<td>14, 15, 16, 10, 14</td>
</tr>
</tbody>
</table>

* disk component for $R < 15$ kpc; $M_{tot}$ includes bulge-halo mass contained within $R < 15$ kpc.


In principle, the ages of the oldest open disk clusters observed in the solar neighbourhood define a minimum age of the Galactic disk (i.e. the gaseous disk may have been considerably older). Similarly, ages of the oldest globular clusters present in the Galactic halo provide a lower limit to the age of the Galaxy as a whole. There are, however, several caveats with such age determinations for the Galaxy. First, the present-day cluster distribution in the Galaxy, as well as the distinction observed between disk, halo, and bulge components, may be different from that in the past. For instance, star clusters may have been tidally disrupted by and/or accreted from nearby galaxies such as the Magellanic Clouds. Second, the onset of main star formation in our Galaxy may not have coincided with the epoch of formation of our Galaxy. Since star formation probably propagated outwards from the bulge to the outer regions of the Galactic disk and halo, age determinations based on the stellar contents of these regions are likely to differ substantially (see Chapter 2).

From studies of the Galactic open cluster system, the age of the Galactic disk is estimated to be older than 9–10 Gyr (e.g. Grenon 1989; Twarog & Anthony-Twarog 1989). Recent age determinations of galactic disk open clusters (e.g. Meynet, Mermilliod, and Maeder 1993), based on up-to-date theoretical isochrones for stars born with $Z = Z_\odot$, reveal the presence of open disk clusters older than 9 Gyr (e.g. NGC 2682, NGC 188, and NGC 6791) from colour-magnitude diagram fits. The mean age of globular clusters is estimated to be 14–18 Gyr, both in the MW (e.g. Demarque 1980; Carney 1980; Sandage 1986; Armandroff 1989; Hesser & Bolte 1990) and in the LMC (e.g. Gascoigne 1980; Da Costa 1991). Still many uncertainties are involved...
with the ages of the low-metallicity, low-mass stars present in globulars, in particular concerning the detailed chemical structure and initial abundances of such stars.

We will assume a galactic age \( t_{\text{ev}} = 14 \) Gyr both for the Galaxy and Magellanic Clouds. The onset of star formation in these systems as a whole is assumed to coincide with galactic evolution time \( t = 0 \). However, we can also simulate e.g. a present-day age of the stellar disk in our Galaxy of \( \sim 10 \) Gyr by assuming that the onset of main star formation in the disk occurred at a galactic evolution time \( t \sim 4 \) Gyr. In other words, for a given value of \( t_{\text{ev}} \), the present-day age of the majority of stars in e.g. the Galactic disk can be fixed in our models by adjusting the specific variation of the SFR with galactic evolution time. The sensitivity of e.g. the resulting age-metallicity relations to the adopted value of \( t_{\text{ev}} \) will be discussed in Sect. 4.1.

Once the galactic age \( t_{\text{ev}} \), the initial galaxy mass \( M_{\text{tot}} \), and the present-day gas fraction \( \mu_1 \) are specified, the net gas consumption rate (NGCR) averaged over the galaxy lifetime can be derived from:

\[
< \text{NGCR} > = \frac{(1 - \mu) M_{\text{tot}}}{t_{\text{ev}}} \quad [\text{M}_\odot \text{ yr}^{-1}] \tag{3.10}
\]

and the average past SFR by:

\[
< \text{SFR} > = \frac{1}{t_{\text{ev}}} \int_{0}^{t_{\text{ev}}} C(t) \, dt = \frac{< \text{NGCR} >}{1 - < R >} \quad [\text{M}_\odot \text{ yr}^{-1}] \tag{3.11}
\]

where \( R(t) \equiv E(t)/C(t) \) denotes the returned fraction which depends on the SFR, IMF, and the set of metallicity dependent remnant masses assumed (see Appendix B). In this manner, the normalisation constant of the SFR, i.e. \( C_0 = S_0 < m_* \) (cf. Eq. 3.4), is constrained by the gas fraction \( \mu_1 \) reached at a galactic evolution time \( t_{\text{ev}} \).

<table>
<thead>
<tr>
<th></th>
<th>SFR\text{OB}</th>
<th>SFR\text{H\alpha}</th>
<th>SFR\text{WIR}</th>
<th>SFR\text{TIR}</th>
<th>&lt;SFR\text{1}&gt;</th>
<th>&lt;SFR&gt;</th>
<th>\kappa</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW</td>
<td>3.6</td>
<td>-</td>
<td>2.5</td>
<td>5.0</td>
<td>3.6 ± 1.5</td>
<td>16.5 ± 1.5</td>
<td>~4.5</td>
</tr>
<tr>
<td>LMC</td>
<td>0.14</td>
<td>0.18</td>
<td>0.23</td>
<td>0.30</td>
<td>0.2 ± 0.1</td>
<td>0.6 ± 0.05</td>
<td>~3.</td>
</tr>
<tr>
<td>SMC</td>
<td>0.038</td>
<td>0.048</td>
<td>0.023</td>
<td>0.026</td>
<td>0.035 ± 0.02</td>
<td>0.08 ± 0.02</td>
<td>~2.5</td>
</tr>
</tbody>
</table>


Apart from constraints on the normalisation constant, the present-day SFR\text{1} can be constrained by independent observations of the Galactic disk and Magellanic Clouds. In Table 3.2 we include present-day SFRs as determined from the observed number of O and B stars (label OB), the integrated H\alpha emission, and the warm and total infrared luminosities (labelled WIR and TIR, respectively). Present-day SFRs based on the IR luminosities were obtained by applying the method described by Walterbos (1991, as applied to M31) with the IR luminosities adopted from Schwering (1988). The present-day SFR based on the total IR luminosity provides an upper limit to the actual SFR (e.g. Parravano 1989; Walterbos 1991).

Mean present-day SFRs <SFR\text{1}> in the Galactic disk and Magellanic Clouds were estimated by averaging the values based on these individual methods. Average past SFRs <SFR> in these systems were derived using Eq. (3.11), <\,R\,> = 0.3 (see Sect. 3.4), and the data in Table 3.1. The ratio \( \kappa \) of the average past and present-day SFR is given in the last column of Table 3.2. We find upper limits of \( \kappa = 4.5 \), ~3 and 2.5, for the Galactic disk, LMC, and SMC, respectively. The upper limit obtained for the Galactic disk is consistent with values of \( \kappa = 2.5–4.5 \) derived by Twarog (1980) from star counts of F and G dwarfs. We conclude that the mean SFR in the past has been considerably higher than at present, both in the Galactic disk and Magellanic Clouds. Other constraints to the normalisation constant and variation of the SFR with galactic age will be discussed in Chapter 4.
• Model parameters

In addition to the initial conditions discussed above, the galactic chemical evolution equations incorporate input parameters related to the assumed SFR, IMF, infall, and transport functions, as well as parameters related to the stellar evolution data used. We here concentrate on the latter kind of input parameters while parameters associated with the SFR, etc. will be discussed in Sect. 4.1.

In Table 3.3 we list for the standard chemical evolution model of the Galactic disk, the adopted IMF-slope, stellar mass limits at birth, and progenitor mass ranges for stars ending their lives as AGB stars, SNIa, SNIb/c, and SNII.

IMF slope: The stellar visual luminosity function provides severe constraints on the Galactic IMF and SFR (e.g. Scalo 1986, 1988). These observations indicate that $\gamma \approx -2.35$ to $-2.7$ for stars more massive than $\sim 1 \, M_{\odot}$ in the solar neighbourhood. The shape of the IMF in the Magellanic Clouds is found to decrease more steeply towards larger masses ($\gamma \approx -2.7$; cf. Scalo 1986). In our models, the slope (or detailed shape) of the IMF, in particular at stellar masses less than $1 \, M_{\odot}$, is an input parameter which is further constrained by independent observations (cf. Sect. 4.3).

| $\gamma$, $m_u$ | -2.35 | slope of power-law IMF $m^\gamma$
| $m_{AGB}$, $m_{AGB}$ | [0.1, 60] $M_{\odot}$ | stellar mass limits at birth
| $m_{SNII}$, $m_{SNII}$ | [0.8, 8] $M_{\odot}$ | progenitor mass range for AGB stars
| $m_{SNII}$, $m_{SNII}$ | [8, 30] $M_{\odot}$ | progenitor mass range for SNII and SNIb/c
| $m_{SNII}$, $m_{SNII}$ | [2.5, 8] $M_{\odot}$ | progenitor mass range for SNIa
| $\phi_{SNII}$ | 0.005 | fraction of progenitors ending as SNIa
| $\phi_{SNIb/c}$ | 0.33 | fraction of SNI progenitors ending as SNIb/c

We assume stellar mass limits at birth of $m_1 = 0.1$ and $m_u = 60 \, M_{\odot}$. Observations are inconclusive about the precise value of $m_1$ due to the uncertain mass-luminosity relation for low-mass stars (i.e. $m \lesssim 0.2 \, M_{\odot}$; cf. Sect. 2.3). The precise value of $m_u = 60 - 120 \, M_{\odot}$ is unimportant for the results presented here as long as the IMF decreases strongly towards massive stars. The formation probability of high-mass stars ($m \gtrsim 60 \, M_{\odot}$) is extremely low according to the local IMF (cf. Scalo 1986; Rana 1991), i.e. less than $10^{-5}$ times the formation probability of a $1 \, M_{\odot}$ star (assuming a Schmidt-like SFR; cf. Sect. 4.1). In addition, the yields of such stars do not strongly increase with initial mass (e.g. Maeder 1992).

AGB: We assume that stars with initial masses between $\sim 0.8$ and $8 \, M_{\odot}$ pass through the AGB phase at the end of their lives (e.g. Groenewegen 1993). The lower mass limit for AGB stars is relatively uncertain and probably depends on initial metallicity (cf. Sect. 3.2). The exact value assumed for $m_u^\text{AGB}$ is relatively unimportant for the results presented here.

SNIa: Accreting white dwarfs (WD) in binary systems are thought to be the immediate progenitors of SNIa (e.g. Nomoto et al. 1991). To ensure that the accreting WD eventually reaches the Chandrasekhar limit and ends as SNIa, a minimum progenitor mass of $m^\text{SNIa} \sim 2.5 \, M_{\odot}$ is required (e.g. Nomoto et al. 1984). Similarly, the maximum mass of the primary that can produce a carbon-oxygen WD is $m^\text{SNIa} \sim 8 \, M_{\odot}$. Both mass limits are somewhat uncertain due to the uncertainties involved with the detailed evolution scenario assumed for SNIa. In addition, these limits depend on e.g. the mass-ratio and the orbital separation of the binary members (e.g. Greggio & Renzini 1983), as well as on initial metallicity. In the standard evolution model, we assume that a fraction $\phi^\text{SNIa} = 0.005$ of all WD progenitors, with initial masses between $\sim 2.5$ and $8 \, M_{\odot}$, will ultimately end as SNIa (e.g. Ishimaru & Arimoto 1995). In fact, $\phi^\text{SNIa}$ is the main input parameter used to vary the contribution by SNIa to the enrichment of the ISM.

SNIII: In our models, the main parameter related to the enrichment by SNI II is the upper mass limit for SNI. We assume that stars more massive than $m^\text{SNII}$ do not explode as supernova but end as black hole (cf. Maeder 1992; Nomoto et al. 1994; Prantzos 1994; Tsujimoto et al. 1995; Timmes et al. 1995). Consequently, such stars contribute to the enrichment of the ISM only during their wind phases. Recently, Woosley and Weaver (1995) argued that when the shock energy (available from neutrino deposition) does not increase substantially as one moves towards higher mass stars, there is a critical mass $m^\text{crit} \sim 30 \, M_{\odot}$
above which stars leave a black hole remnant (instead of a neutron star). However, black hole formation may be suppressed (e.g. Maeder 1992) when massive stars experience intense mass loss, especially if they are formed with high metallicities (see Sect. 3.2). 

Another argument often used for black hole formation by very massive stars is based on the fact that galactic evolution models, which use up-to-date SNII yields, predict abundances that are considerably too large compared to the abundances observed in the local ISM and the Sun, in particular for helium and oxygen (e.g. Twarog & Wheeler 1982; Maeder 1992, 1993; Timmes et al. 1995). However, there is no consensus on this issue due to the uncertainties involved with the pre-SN and SN yields of massive stars which allow for $m_{\text{crit}} \lesssim 50 \ M_\odot$ (cf. Prantzos 1994).

A similar argument is based on the observed ratio of $\Delta Y/\Delta Z \sim 4$ which would require that stars with $m \geq 25 - 30 \ M_\odot$ collapse as a black hole (e.g. Maeder 1992; Brown & Bethe 1995). This argument relies on theoretical models which predict $\Delta Y/\Delta Z$ to decrease towards more massive stars (e.g. Maeder 1992). However, when a massive star experiences efficient mass loss during its wind phase, the mean $\Delta Y/\Delta Z$ in the stellar ejecta can be much larger than for the inefficient mass-loss case (Woosley et al. 1995). Therefore, the critical mass above which stars end as black hole is still very uncertain.

For now, we like to emphasize that the value of $m_{\text{SNII}}^0 = 30 \ M_\odot$ assumed, either motivated by black hole formation or by intense mass loss, has a decisive impact on the enrichment by SNII and is of crucial importance for the outcome of galactic chemical evolution models (e.g. Maeder 1992; Tsujimoto et al. 1994; Timmes et al. 1995). We will address this important issue in more detail in Sect. 3.3.

**SNII:** Since SNII are thought to originate from stars that lost their hydrogen-rich envelopes during the pre-SN stage, the upper mass limit for SNII may be considerably larger than in the case of SNII (e.g. Woosley et al. 1995a). However, because of the uncertainties involved with the observations we here simply assume $n_{\text{SNII}}^{\text{SNII}} = n_{\text{SNII}}^{\text{SNII}}$. In our models, the contribution by SNII to the enrichment of the ISM is primarily determined by the fraction $\phi_{\text{SNII}}$ of stars with initial masses in the range $[m_{\text{SNII}}, m_{\text{SNII}}]$ that ultimately end as SNII (e.g. when they experience intense mass loss in a close-binary system or during the Wolf-Rayet stage). In the standard evolution model, we assume $\phi_{\text{SNII}} \equiv [\text{SNII} / (\text{SNII} + \text{SNIb/c})] \sim 0.33$ which is based on the observed ratio of the present-day formation rates of SNII and SNII in the Galaxy (e.g. van den Bergh & Tammann 1991; Tutukov, Yungelson & Iben 1992; Cappellaro et al. 1993). This value is somewhat larger than $\phi_{\text{SNII}} \sim 0.13 - 0.26$ derived by Podsiadlowsky et al. (1992) assuming a Salpeter IMF.

**Solution method**

After specifying the SFR, IMF, infall functions ($F(t), F_2(t)$), and transport functions ($T_g, T_l$), the galactic chemical evolution equations (Eqs. 3.3–3.3) are solved for a given set of boundary conditions and stellar input data.

We start from a zero order estimate of the enrichment history of the galactic ISM (e.g. linearly increasing with age) according to the star formation history (SFH) assumed. Thereafter, we compute the resulting age-metallicity relation $Z(t)$ as well as the revised star formation rate $C(t)$ (which e.g. depends on $\mu(t)$ by means of the metallicity dependent stellar ejection rate $E(t)$; see Eq. 3.5). Subsequently, the resulting relations for $Z(t)$ and $C(t)$ are used as input in Eqs. (3.1–3.3) and are recomputed. This procedure is repeated until both relations converge and do not differ by more than a few percent from the corresponding relations derived in the previous iteration. Usually, between 5–10 iterations are sufficient to find stable solutions, depending on e.g. the initial conditions and whether or not infall is involved, etc.

The integro-differential equations are solved using a fourth order Runge-Kutta integration method with variable step-size. In general, each integral is calculated up to an accuracy of a few percent. Quantities of interest are computed with a time resolution of $\Delta t = 5 \times 10^7$ yr. This corresponds to $\sim 250$ grid points for a galactic age of $t_{\text{age}} = 15$ Gyr. In particular cases, e.g. burst models in which the SFR (or the gas-fraction $\mu$) varies strongly with age over time scales $\lesssim 10^7$ yr, the number of grid points is increased accordingly.

We verified that the solutions of $Z(t)$ and $C(t)$ are independent of the initial conditions assumed and are insensitive to the adopted age resolution $\Delta t \lesssim 10^8$ yr as well as the minimum step-size used in the integration routines. After convergence of $Z(t)$, the entire set of galactic chemical evolution equations is solved in a self-consistent manner for usually $\sim 15$ elements (e.g. H, He, O–Fe). In this manner, the stellar yields are self-consistently coupled to the variation of the overall metallicity $Z(t)$ according to the assumed star formation history.
3.2 Stellar lifetimes and remnant masses

We describe the metallicity dependent set of stellar lifetimes and remnant masses that are used as input data to the galactic chemical evolution model described in the previous section.

3.2.1 Stellar lifetimes

We use the stellar lifetimes from the metallicity dependent stellar evolution tracks presented by the Geneva group (i.e. Schaller et al. 1992; Schaerer et al. 1993, 1995; Charbonnel et al. 1993; Meynet et al. 1994). Fig. 3.1 illustrates the stellar lifetimes adopted as a function of initial mass (between 0.1 and 60 M$_\odot$) and initial metallicity (between $Z = 0.001$ and 0.04) for the following evolutionary phases: main sequence (MS: core hydrogen burning), red giant branch (RGB: shell hydrogen burning), horizontal branch (HB: core helium burning), and (early) asymptotic giant branch (EAGB, AGB: double shell burning).

The Geneva tracks adopted here have been checked against a very wide range of independent observations (e.g. Meynet & Maeder 1992). Nevertheless, these tracks are rather sensitive to e.g. the detailed treatment of convection, mixing, overshooting, mass-loss, and nuclear reaction rates used (cf. Schaller et al. 1992; Bertelli et al. 1994). In particular, post-RGB lifetimes are uncertain but this has negligible effect on the ISM enrichment and primarily affects the integrated properties of post-RGB stars (e.g. total number and luminosity; cf. Sect. 4.3). Overall, we expect that the uncertainties involved with the stellar lifetimes are unimportant for the (qualitative) results presented below.

For stars with $m < \sim 1.7$ M$_\odot$, the Geneva tracks were computed up to the He flash. For these stars, we use the metallicity dependent HB and EAGB lifetimes from Lattanzio (1991). The Geneva tracks do not extend beyond the HB for stars with $2 < m[M_\odot] < 5$. For stars in this mass range, EAGB lifetimes are taken from Lattanzio (1991) as well. For low and intermediate mass stars (i.e. $m = 0.8-8$ M$_\odot$), we adopt the AGB lifetimes from Groenewegen & de Jong (1993). Note that the EAGB lifetimes for stars with $m > 8$ M$_\odot$ refer to the time spent by these stars after the HB, i.e. these stars usually do not become AGB stars but become Wolf-Rayet stars.
3.2 Stellar lifetimes and remnant masses

In general, stellar lifetimes are dominated by the times spent on the main-sequence and increase with initial metallicity for low- and intermediate mass stars (i.e. \( m < \sim 10 \, M_\odot \); see below). For more massive stars, main-sequence lifetimes decrease with increasing metallicity. This is primarily due to the large hydrogen content of stars born with relatively low metal abundances (cf. Schaller et al. 1992). Metallicity effects on the stellar MS lifetimes are generally small for massive stars (i.e. \( < \sim 15\% \)) but can be important for low-mass stars (i.e. up to \( \sim 65\% \)). Post-MS lifetimes are relatively insensitive to initial metallicity except for low-mass stars (i.e. \( m < \sim 8 \, M_\odot \); i.e. variations less than \( \sim 25\% \)).

Fig. 3.2 shows the turnoff mass as a function of age of a stellar population formed with \( Z = 0.001, 0.005, 0.01, \) and \( 0.02 \). This figure corresponds to the main-sequence panel from Fig. 3.1 and illustrates that at a galactic lifetime of \( t_{ev} = 15 \) Gyr, the main-sequence turnoff mass \( m_o(t_{ev}) \) is rather sensitive to initial metallicity, i.e. \( m_o(t_{ev}) = 0.8 \) and \( 0.95 \, M_\odot \) for \( Z = 0.001 \) and \( 0.02 \), respectively. Conversely, the stellar lifetime \( \tau(m, Z) \) for a given turnoff mass of \( m = 0.9 \, M_\odot \) increases from 11 to \( 18 \) Gyr for stars born with \( Z = 0.001 \) and \( 0.02 \), respectively. This has important implications for galactic chemical evolution models since, at a galactic age \( t \), only stars with \( \tau(m, Z) \lesssim t \) are able to contribute substantially to the enrichment of the ISM.

\[ \text{Figure 3.2 Stellar main-sequence turnoff mass vs. galactic evolution time} \]

The primary effect of taking metallicity dependent (main-sequence) lifetimes into account is that intermediate and low-mass stars with low metallicities contribute their nucleosynthesis products at relatively early epochs in galactic chemical evolution as compared to conventional (solar metallicity) models (e.g. Bazan & Mathews 1990).

3.2.2 Stellar remnant masses

- Asymptotic giant branch stars

Fig. 3.3 shows the initial-final mass relations adopted for low and intermediate mass stars born with metallicities between \( Z = 0.001 \) and \( 0.04 \) (see Groenewegen & de Jong 1993; van den Hoek & Groenewegen 1997). These relations were computed by taking implicitly into account the dependence of mass-loss on the AGB on initial stellar composition (standard AGB model, cf. Sect. 3.3). At high metallicities (e.g. \( Z \sim 0.04 \)), the initial-final mass relation predicts the smallest WD remnant masses.

The initial-final mass relations \( m_{rem}(m) \) at different initial compositions are roughly consistent with the maximum limit of remnant-masses allowed for by observations of C+O white dwarfs (WD) in the solar neighbourhood (see Weidemann & Koester 1983; Weidemann 1990). Furthermore, these relations agree well with the theoretical predictions by Vassiliadis & Wood (1993).

Observations of intermediate mass AGB stars (\( 3.5 \lesssim m[M_\odot] \lesssim 8 \)) are dominated by WDs born at high metallicity (\( Z \approx 0.02 \)). This is probably due to the short cooling times of massive WDs (\( m_{WD} \gtrsim 0.8 \, M_\odot \); cf. Wood 1992). Also, the initial-final mass relation observed for stars with \( m \lesssim 1 \, M_\odot \) appears to include predominantly stars born with \( Z \gtrsim 0.02 \). This is probably due to the combined effects of: 1) stellar main-sequence lifetimes which decrease with metallicity for low and intermediate mass stars, and 2) the rapid early enrichment of the Galactic disk.
For AGB stars in the Magellanic Clouds, having metallicities substantially below that observed in the Galactic disk, we will use initial-final mass relations similar to those shown in Fig. 3.3. In particular, AGB stars in the SMC probably experience mass-loss that is less efficient than predicted by the standard model for AGB stars in the Galactic disk. Thus, the WD remnants of low and intermediate stars in the SMC on average will have substantially larger masses than those in the Galactic disk (see Sect. 6.2).

- Supernova progenitors

SNIa progenitors presumably originate from accreting and/or coagulating WDs in a binary system. SNIa leave no remnant as the WD is believed to be disrupted completely during the explosion. In contrast, SNII progenitors usually leave a neutron star remnant (or a black hole if the progenitor mass is larger than a certain critical stellar mass; see above).

Fig. 3.4 shows the adopted sets of metallicity dependent initial-final mass relations for stars with \( m = 8 - 60 \, M_\odot \). Remnants of stars with \( m \lesssim 8 \, M_\odot \) are shown for comparison. We use two distinct data sets: the first data set is from the Geneva group which includes metallicity dependent stellar mass loss, the second one is from Woosley and Weaver (1995, hereafter WW) and does not include mass loss. The remnant masses shown are the final masses after core collapse. For stars with masses \( m \lesssim 30 \, M_\odot \), the core collapse is accompanied by a SNII (or SNIb/c) explosion. For more massive stars, core collapse may be so rapid that an explosive shock wave cannot escape from the stellar surface. Such stars presumably end as black hole.

As can be seen from Fig. 3.4, large differences are present in the remnant masses predicted by the two groups, especially for stars more massive than \( \sim 30 \, M_\odot \). These differences are primarily due to the different treatment of stellar mass loss as we will discuss below.

In case of the Geneva group data, a star with \( m = 10 \, M_\odot \) born with solar metallicity leaves a neutron star remnant of \( \sim 1.4 \, M_\odot \). Similarly, a \( 25 \, M_\odot \) star is predicted to leave a \( \sim 3.2 \, M_\odot \) neutron star whose mass is relatively insensitive to initial metallicity. Stars with \( 10 \lesssim m[\, M_\odot] \lesssim 30 \) usually experience iron core collapse inside a helium and hydrogen-rich envelope while those with \( m \gtrsim 30 \, M_\odot \) presumably undergo iron core collapse inside an envelope which predominantly contains helium (e.g. helium-rich Wolf-Rayet stars; e.g. Branch & Nomoto 1991). This has important consequences for the remnant mass as can be seen from Fig. 3.4.

The Geneva group explicitly takes into account the dependence of stellar mass loss on initial metallicity. In particular, the large opacity in high-metallicity stars results in relatively high mass-loss rates and small remnant masses. In fact, stars with \( m \gtrsim 25 - 30 \, M_\odot \) born with high metallicities may loose most of their envelope, during the stellar wind phase and/or the Wolf-Rayet stage. Alternatively, stars with \( m \gtrsim 25 - 30 \, M_\odot \) may end their lives as a black hole depending on the part of the mantle layer that is accreted onto the core (e.g. Maeder 1992). For this reason, remnant masses of stars with \( m \gtrsim 25 \, M_\odot \) are relatively uncertain, especially at low metallicities. Remnant masses for stars with \( Z = 0.02 \) (cf. Maeder 1992) are similar to those given by Hashimoto & Nomoto (1992) and Thielemann et al. (1993), except for small differences for stars in the mass range \( 20 - 25 \, M_\odot \).
3.3 Stellar Nucleosynthesis Prescriptions

We describe the enrichment of the Galactic ISM in terms of the characteristic element contributions by Asymptotic Giant Branch (AGB) stars, SNII, SNIa, and SNIb/c. This distinction is based on the specific abundance patterns observed within the ejecta of stars that pass through these evolutionary stages (e.g. Trimble 1992; Matteucci 1992; Russell & Dopita 1992). AGB stars usually end their lives by gradual ejection of their outer envelope during double-shell burning (H + He). SNIa are probably related to exploding white dwarfs (WD) in binary systems although variations in the explosion mechanism and progenitor history are likely to exist. SNII and SNIb/c are believed to be linked to the gravitational collapse of massive stars ($m \gtrsim 8 \, M_\odot$) at the end of their evolution. The possible impact of binary stars on galactic chemical evolution will be briefly discussed at the end of this section.

A detailed comparison of the stellar yields is made in order to provide the reader with some affinity with the adopted stellar input data as well as with its uncertainties. This will be particularly useful when discussing model results for the chemical evolution of the Galactic disk which incorporate the detailed star formation and enrichment history of both AGB stars, SNII, SNIa, and SNIb/c, and which account for the

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**Figure 3.4** Left panel: Remnant masses of stars in the mass range 0.8–60 $M_\odot$ formed with initial metallicities $Z = 0.04, 0.02, 0.008, 0.004, 0.001$ (see legend). Data from the Geneva group (1995) and van den Hoek & Groenewegen (1997) for stars with $m \lesssim 8 \, M_\odot$. Right: Same as left panel but data from Woosley & Weaver (1995). Data for SNIb/c are taken from Woosley, Langer, and Weaver (1995). Remnants of SNII and SNIb/c progenitors with $m = 8 - 12 \, M_\odot$ and with $m > 40 \, M_\odot$ were extrapolated.

In case of the WW data, the remnants of stars with $m \lesssim 25 - 30 \, M_\odot$ are substantially less massive than those given by the Geneva group. For instance, the WW models predict a remnant of $m_{\text{rem}} \sim 1.9 \, M_\odot$ for a 25 $M_\odot$ progenitor, compared to $m_{\text{rem}} \sim 3.2$ for the Geneva tracks. In contrast to the Geneva group, WW followed the iron collapse and the subsequent explosion of the stars in detail. This in part explains the large differences observed between the two sets of remnant masses, in particular for stars with $m \gtrsim 25 - 30 \, M_\odot$ since the WW models suggest that a considerable amount of envelope matter is accreted onto the collapsing core after a reverse shock (see below). For the more massive stars, the large differences are primarily due to the exclusion of mass loss in the WW models. This is also the reason why the metallicity dependence of the initial-final mass relation for stars with $m \gtrsim 30 \, M_\odot$ is weak in the WW models compared to the Geneva models.

For comparison, we show in Fig. 3.4 (right panel) the remnant masses of SN progenitors that have lost their hydrogen-rich envelope. Such stars may be associated with the progenitors of SNIb/c for which we adopt the models from Woosley, Langer, and Weaver (1995; hereafter WLW). The WLW models follow the collapse of SNIb/c in a fashion similar to that for the SNII models of WW. Due to substantial mass loss prior to collapse, both the helium star progenitors and the collapsed remnant masses of SNIb/c are usually much smaller than for SNII (cf. Fig. 3.4). We note that the initial-final mass relation for SNIb/c is very uncertain. We will adopt an ad hoc relation which will be discussed further in Sect. 3.3 and is unimportant for the results presented below.
dependence of stellar lifetimes, remnant masses, and nucleosynthesis yields, on initial stellar metallicity.

We first recall some definitions related to the stellar yields which depend on both the initial stellar mass and initial abundances of a star. Thereafter, we describe in detail the adopted yields for each of the stellar groups mentioned above and discuss their relative roles in enriching the Galactic ISM.

### 3.3.1 Definition of stellar yields

Stellar yields are measures of the lifetime-averaged element abundances within the ejecta of a star. In principle, abundances within the stellar envelope are determined by the initial chemical structure of the star, the distinct nuclear and envelope burning phases the star experiences (hydrostatic and/or explosive), the amount of material that is dredged-up and mixed by convection to the stellar surface and/or accreted to the compact core, and by the mechanisms driving the stellar mass-loss (thermal expansion, radiation pressure, Roche-lobe overflow in binary systems, etc.).

The element yield \( p_j \) of a star of initial mass \( m \) is defined as the newly formed and ejected mass of element \( j \) integrated over the stellar lifetime \( \tau(m) \) (normalised to the initial stellar mass):

\[
mp_j(m) = \int_0^{\tau(m)} E^*(m,t) \cdot (Z_j^*(t) - Z_j^*(0)) \, dt
\]

where \( E_j(m,t) \) denotes the stellar mass-loss rate and \( Z_j^*(t) \) the abundance by mass of element \( j \) in the stellar ejecta at age \( t \). Negative yields may occur e.g. in the case of consumption of hydrogen.

In general, stellar yields depend on the initial stellar abundances in various ways. First, the abundances in the stellar envelope \( Z_j^*(t) \) are related in a complex manner to the initial abundances of distinct elements (e.g. helium and/or oxygen). To first order, we take this important effect into account by the dependence of stellar lifetimes, remnant masses \( m_{\text{rem}}(m) \), and mass-loss rates \( E^*(m,t) \) depend strongly on the initial stellar metallicity (see below). Third, stellar yields are defined with respect to the initial element abundances \( Z_j^*(0) \) (cf. Eq. 3.12).

We here adopt the initial abundances as used in the stellar evolution tracks presented by the Geneva group (see above). In brief, the Geneva group derived the initial helium abundance from:

\[
Y = Y_0 + \frac{\Delta Y}{\Delta Z} Z
\]

assuming a primordial helium abundance \( Y_0 \) of 0.232 (e.g. Audouze 1987; Steigman et al. 1989) and \( \Delta Y/\Delta Z = 3 \) (e.g. Pagel et al. 1986; Pagel 1992) for stars in the Galactic disk. This implies a revised solar metallicity of \( Z_\odot = 0.0188 \) with \( Y_\odot = 0.299 \) (see Schaller et al. 1992). Initial abundances of C, N, and O were taken according to the relative ratios (cf. Anders & Grevesse 1989) used in the opacity tables by Rogers & Iglesias (1991). The hydrogen content was calculated from \( X = 1 - Y - Z \). Table 3.4 lists the adopted initial abundances of H, \(^4\)He, \(^{12}\)C, \(^{13}\)C, \(^{14}\)N, and \(^{16}\)O at initial stellar metallicities \( Z^*(0) = 0.001, 0.004, 0.008, 0.02, \) and 0.04. Unless stated otherwise, abundances will be given by mass throughout this thesis.

### Table 3.4 Initial element abundances adopted

<table>
<thead>
<tr>
<th>Element</th>
<th>( Z=0.001 )</th>
<th>( 0.004 )</th>
<th>( 0.008 )</th>
<th>( 0.02 )</th>
<th>( 0.04 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.756</td>
<td>0.744</td>
<td>0.728</td>
<td>0.68</td>
<td>0.62</td>
</tr>
<tr>
<td>(^4)He</td>
<td>0.243</td>
<td>0.252</td>
<td>0.264</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>(^{12})C</td>
<td>2.24(-4)</td>
<td>9.73(-4)</td>
<td>1.79(-3)</td>
<td>4.47(-3)</td>
<td>9.73(-3)</td>
</tr>
<tr>
<td>(^{13})C</td>
<td>0.04(-4)</td>
<td>0.16(-4)</td>
<td>0.29(-4)</td>
<td>0.72(-4)</td>
<td>1.56(-4)</td>
</tr>
<tr>
<td>(^{14})N</td>
<td>0.70(-4)</td>
<td>2.47(-4)</td>
<td>5.51(-4)</td>
<td>1.40(-3)</td>
<td>2.47(-3)</td>
</tr>
<tr>
<td>(^{16})O</td>
<td>5.31(-4)</td>
<td>2.11(-3)</td>
<td>4.24(-3)</td>
<td>1.06(-2)</td>
<td>2.11(-2)</td>
</tr>
</tbody>
</table>

It is convenient to compare stellar yields for distinct stellar evolutionary phases such as the RGB and AGB. In this case, the total mass of element \( j \) ejected during mass-loss phase \( i \) (with age boundaries \( t_i(m) \) and \( t_{(i+1)}(m) \) in Eq. 3.12) is defined as:

\[
\Delta m_j^i = \Delta m^i Z_j^*(0) + mp_j^i (m)
\]

where \( \Delta m^i = \Sigma_j \Delta m_j^i \) denotes the total stellar mass lost during phase \( i \). Similarly, the mean abundance of element \( j \) in the ejecta returned to the ISM during phase \( i \), can be written as:

\[
\langle Z_j^i \rangle = \frac{mp_j^i}{\Delta m^i} + Z_j^*(0)
\]
The lifetime-integrated stellar yield of element $j$ in terms of the stellar yields for distinct mass-loss phases $i$ is given by:

$$m_{p_j}(m) = \Sigma_i m_{p_j}^i(m) = \Sigma_i (\Delta m_j^i - \Delta m_j^i Z_j^i(0))$$  \hspace{1cm} (3.16)

According to Eq. \((3.12)\), we have $\Sigma_i p_j = 0$ and $\Sigma_j Z_j = 1$. Hence, the total stellar mass ejected can be expressed as: $\Delta m_j^i = \Sigma_i \Delta m_j^i = \Sigma_i \Sigma_j \Delta m_j^i = m - m_{rem}(m)$ where $m_{rem}(m)$ is the stellar remnant mass. In this manner, Eq. \((3.12)\) also can be written as:

$$m_{p_j}(m) = \Delta m_j - (m - m_{rem}) Z_j^*(0)$$  \hspace{1cm} (3.17)

We will refer to the terms $m_{p_j}^i$ (i.e. newly formed and ejected mass of element $j$) and $\Delta m_j$ (i.e. total returned; cf. Eq. \((3.14)\)) by using indices new and tot, respectively.

We like to emphasize that in the galactic chemical evolution models presented below, the stellar yields of all elements considered are corrected according to the initial abundances of the progenitor star at time of its formation (cf. Sect. 3.2). For the moment, by lack of a detailed chemical evolution model, we approximate these corrections by using the initial abundances of H, He, C, N, and O from the Geneva group (as described above). For elements heavier than oxygen, we simply ignore such corrections which may result in a small overestimate of the newly synthesized yields of these elements.

### 3.3.2 Asymptotic Giant Branch stars

We discuss the yields of intermediate mass AGB stars for appropriate ranges in initial mass, composition, mass loss parameter $\eta_{\text{AGB}}$, and effects of second dredge-up and HBB. We show that the yields of intermediate mass stars are determined by their final stages and play an important role for the carbon and nitrogen enrichment of the Galactic disk ISM.

#### Introduction

Presumably all main sequence stars with initial masses between $\sim 0.9$ and $\sim 8$ $M_\odot$ pass through a double-shell burning phase at the end of their lifetime, also referred to as the asymptotic giant branch (AGB) phase. During this phase, intermediate mass stars lose most of their envelope mass while they contribute substantially to the interstellar abundances of He, C, N, and s-process elements (e.g. Renzini & Voli 1981, hereafter RV; Iben & Renzini 1983; Dopita & Meatheringham 1991).

Before reaching the AGB phase, the main sequence stellar composition is changed during the first dredge-up (experienced by all stars on the red giant branch (RGB)) and during the second dredge-up (experienced by stars with initial masses larger than some certain critical mass). The first dredge up occurs when the convective envelope moves inwards as a star becomes a red giant for the first time so that helium and CNO processed material are brought to the surface. Several tenths of solar masses can be lost in this phase for intermediate mass stars (e.g. Schaller et al. 1992; Sweigart et al. 1990; Rood 1973).

The second dredge-up is associated with the formation of the electron-degenerate CO core after central helium exhaustion and occurs on the early-AGB (hereafter EAGB). The base of the convective envelope moves inward through matter pushed outwards by the He-burning shell. In this case, helium and nitrogen may be dredged up towards the stellar surface. No mass loss is assumed during the second dredge-up.

During the third dredge up, carbon is dredged up to the stellar surface by convection of the carbon-rich pocket formed after each helium shell flash (or thermal pulse (TP)). By mixing additional carbon to the envelope, the star may undergo a transition from M-star (oxygen-rich), to S-star (carbon roughly equal to oxygen), and C-star (carbon outnumbering oxygen). For stars with $m \gtrsim 3 - 4$ $M_\odot$, this transition is affected by hot bottom burning when both carbon already present and newly dredged-up carbon are processed at the base of the convective envelope according to the CNO cycle. During the thermal pulsing AGB, stars lose most of their mass; typically $\sim 0.4$ $M_\odot$ for a 1 $M_\odot$ star and $\sim 4.8$ $M_\odot$ for a 6 $M_\odot$ star (at solar initial metallicity; see below). The stellar yields of intermediate mass stars are dominated by the mass loss and chemical evolution during the third dredge up.

After gradual ejection of their outer envelope, most AGB stars leave a white dwarf remnant usually accompanied by the formation of a planetary nebula (PN).

#### Synthetic evolution model used

We use the evolutionary tracks of the Geneva group up to the early AGB, in combination with the synthetic thermal-pulsing AGB evolution model presented by Groenewegen & de Jong (1993, hereafter GJ) to follow in detail the chemical evolution and mass loss up to the end of the AGB including the first, second, and third
dredge-up phases. The adopted model has been applied to various observational aspects of AGB evolution, both for AGB stars in the Galactic disk and Magellanic Clouds (GJ; Groenewegen, van den Hoek & de Jong 1995, hereafter GHJ). Resulting yields for models in good agreement with observations of AGB stars both in the Galactic disk and Magellanic Clouds have been presented by van den Hoek & Groenewegen (1997, hereafter HG).

An important aspect of AGB evolution largely neglected in previous studies is the metallicity dependence of the evolutionary algorithms used. In the adopted model, a nearly complete metallicity dependent treatment of the evolution of AGB stars is used covering the first, second, and third dredge up. In addition, several new physical ingredients have been accounted for including the variation of the luminosity during the interpulse period, the fact that the first few pulses are not yet at full amplitude, and the detailed inclusion of mass loss and chemical evolution prior to the AGB (see GJ).

We here briefly repeat that part of the synthetic evolution model that is related to the chemical evolution of stars on the AGB and we discuss the impact of the basic assumptions and parameter values used on the resulting yields of intermediate mass stars.

• Pre-AGB evolution

Pre-AGB evolution is based on the comprehensive set of metallicity dependent stellar evolution tracks recently provided by the Geneva group (see above). These uniform grids of stellar models are based on up-to-date physical input (e.g. opacities, nuclear reaction rates, mixing schemes, etc.) and cover the relevant initial stellar mass range from 0.8 to 8 $M_\odot$ as well as initial metallicity from $Z = 0.001 – 0.04$. For stars with $m < 1.7$ $M_\odot$ these tracks have been computed up to the He flash, for $2 < m[M_\odot] < 5$ up to the EAGB, and for $m > 7$ $M_\odot$ until the end of central C-burning.

For stars with initial mass above 1.25 $M_\odot$, the Geneva tracks used are with overshooting and standard mass loss rates (see e.g. Schaller et al. 1992). For stars below 1.25 $M_\odot$, the tracks used are without overshooting (for $m = 1.25$ $M_\odot$ we include yields both for tracks with and without overshooting). We ignored the fact that the Geneva tracks for stars with $m < 1.7$ $M_\odot$ and $Z = 0.004, 0.008$, and 0.04 end at the helium flash and do not extent to the end of the EAGB. However, these low-mass stars do not experience the second dredge-up and are expected to loose little mass on the horizontal branch and EAGB, so that the influence on their yields is negligible (see HG).

• Thermal-pulsing AGB

The evolutionary model of GJ is started at the first TP, taking into account the changes in mass and abundances prior to the first TP, and is terminated when the envelope mass has been lost due to mass loss or if the core reaches the Chandrasekhar mass. The latter situation never occurs in the best fitting models for the Galaxy and the Large Magellanic Cloud (see GJ, GHJ).

In brief, GJ account for the dependence of core mass on initial stellar metallicity and assume that third dredge-up occurs only if the core mass is larger than a critical value $M_c^{\text{min}}$. They argue that a value of $M_c^{\text{min}} \sim 0.58$ $M_\odot$ is required to fit the low-luminosity tail of the observed carbon star luminosity function in the LMC (see below).

The time scale on which thermal pulses occur is a function of core mass as discovered by Paczynski (1975). GJ use the core-mass-interpulse relation presented in Boothroyd & Sackmann (1988) where the increase in core mass during the interpulse period ($t_{\text{ip}}$) is given by:

$$\Delta M_c = \int_{t_{\text{ip}}}^{t_{\text{p}}} \frac{dM_c}{dt} \, dt$$  \hspace{1cm} (3.18)$$

A certain fraction of this amount is assumed to be dredged up:

$$\Delta M_{\text{dredge}} = \lambda \Delta M_c$$  \hspace{1cm} (3.19)$$

The free dredge-up parameter $\lambda$ is assumed to be a constant. In GJ it was found that a value of $\lambda = 0.75$ is required to fit the peak of the observed carbon stars LF in the LMC (see below).

In principle, the composition of the dredged-up material is determined by the detailed chemical evolution of the core. For simplicity, GJ assume that the composition of the material dredged-up after a TP is: $^4\text{He} = 0.76$, $^{12}\text{C} = 0.22$, and $^{16}\text{O} = 0.02$ (cf. Boothroyd & Sackmann 1988). The carbon is formed through incomplete helium burning in the triple $\alpha$ process and the oxygen through the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction.

Newly dredged-up material can be processed at the base of the convective envelope in the CNO-cycle, a process referred to as hot bottom burning (HBB) and extensively discussed by RV. To a large extent, HBB determines the composition of the material in the stellar envelope of thermal-pulsing AGB stars. The process of HBB is able to slow down or even prevent the formation of carbon stars (e.g. Groenewegen & de
Dependence on mass loss neglected except for the case where the WD remnant becomes a Type Ia supernova (see below). Furthermore, post-AGB yields (i.e. possible synthesis during and after the PN stage) have been not considered here since the abundance variations during the AGB of these elements are relatively small.

The other parameters are as for the standard model (unless stated otherwise). Low-mass AGB stars (usually referred to as J-type carbon stars) and $14$N-rich objects (e.g. Richer et al. 1979).

RV treated HBB in considerable detail as a function of the mixing length parameter (e.g. $\alpha = 0, 1.0, 1.5, 2$). In GJ (see for details their Appendix A) it was decided to approximate in a semi-analytical way the results of RV for their $\alpha = 2$ case as it gave the largest effect of HBB. Since then new results regarding HBB have been obtained, both theoretically (Boothroyd et al. 1993, 1995) and observationally for AGB stars in the Magellanic Clouds (Plez et al. 1993; Smith et al. 1995). These results suggest that HBB is a common phenomenon in AGB stars which occurs at a level roughly consistent with that predicted by RV in case $\alpha = 2$. In particular, Boothroyd et al. (1995) estimate that the initial stellar mass above which HBB takes place is $\sim 4.5 \, M_\odot$ which is similar to the value of $\sim 3.3 \, M_\odot$ predicted by RV ($\alpha = 2$). Observations indicate that virtually all stars brighter than $M_{bol} \approx -6$ mag undergo envelope burning (Smith et al. 1995). These luminosities are reached for stars with initial masses slightly below $4 \, M_\odot$ and larger (Boothroyd et al. 1993).

The free parameters in the synthetic evolution model are the mass loss scaling parameter $\eta_{AGB}$ for stars on the AGB (using a Reimers law), the minimum core mass for dredge up $M_{AGB}^{min}$, the third dredge-up efficiency $\lambda$, and the core mass $m_{HBB}$ at which HBB is assumed to operate (according to the recipes outlined in the Appendix in GJ). As derived from previous extensive modeling, $\eta_{AGB} = 4$, $M_{AGB}^{min} = 0.58 \, M_\odot$, $\lambda = 0.75$, and $m_{HBB} = 0.8 \, M_\odot$, are in best agreement with observations of AGB stars both in the Galactic disk and Magellanic Clouds (see GJ, GHJ). In the following, we will refer to this set of parameters as the standard model.

**Chemical enrichment by AGB stars**

We consider the resulting yields for the standard model and discuss the dependence of the stellar yields on the values adopted for the Reimers mass loss coefficient $\eta_{AGB} = 1–5$, the third dredge-up efficiency $\lambda = 0.6–0.9$, the critical core mass for dredge up $M_{AGB}^{min} = 0.56–0.62 \, M_\odot$, and the minimum core mass for HBB $m_{HBB} = 0.8–1.3 \, M_\odot$. A detailed discussion of the assumptions and uncertainties involved with the AGB yields as predicted by the synthetic evolution model can be found in HG.

For the standard model, we present in Tables 3.5–3.8 metallicity dependent theoretical stellar yields of $H$, $^4He$, $^{12}C$, $^{13}C$, $^{14}N$ and $^{16}O$ for AGB stars with initial masses $m = 0.8–8 \, M_\odot$ and initial metallicities $Z = 0.001, 0.008, 0.02$, and $0.04$. In these tables, we list subsequently the initial mass $m_{ini}$, the yields $p_j$ of the above elements, total element yield $Y_{tot}$ (elements heavier than helium), the total amount of mass returned $\Delta m_{ej}$, and the stellar mass $m_{end}$ at the end of the AGB.

Elements heavier than oxygen are not considered here since the abundance variations during the AGB of these elements are relatively small (see HG).

**Dependence on mass loss**

Fig. 3.5 shows the resulting AGB yields for various values of the Reimers mass-loss parameter $\eta_{AGB} = 1–5$. The other parameters are as for the standard model (unless stated otherwise). Low-mass AGB stars ($m \lesssim 4 \, M_\odot$) predominantly contribute to helium and carbon. High-mass AGB stars are important contributors to helium and nitrogen (see below). For the standard model (i.e. $\eta_{AGB} = 4$), element yields are smaller by factors typically $2–3$ compared to the $\eta_{AGB} \sim 1$ case. Theoretical yields increase with decreasing values of $\eta_{AGB}$ (i.e. smaller mass-loss rates) since a lower value of $\eta$ results in longer AGB lifetimes and therefore more thermal pulses (assuming that the amount of matter dredged-up is roughly constant for each thermal pulse). For high values of $\eta_{AGB} \gtrsim 5$, the effect of increasing $\eta_{AGB}$ on both the AGB lifetimes and number of thermal pulses becomes negligible and the predicted yields remain approximately constant.

**Dependence on initial metallicity**

In general, carbon and oxygen yields are found to increase with decreasing initial metallicity $Z$ (cf. Fig. 3.5). This is due to the fact that dredge-up with subsequent CNO-burning affects more strongly the composition of envelopes with relatively low initial abundances. In addition, the core mass at the first thermal pulse is larger for low metallicities (see GJ). Therefore, the amount of material dredged-up from the core to the envelope is substantially larger in initially low $Z$ AGB stars. In contrast, nitrogen yields slightly increase with metallicity since nitrogen is formed during CNO-burning by consumption of $C$ and $O$. For hydrogen and helium, the sensitivity of the yields to initial metallicity is mainly due to the effect of dredge-up, i.e. post dredge-up processing of $H$ and $He$ is usually low.

**3.3 Stellar nucleosynthesis prescriptions**

Jong 1994a). Since $^{12}C$ is converted into $^{16}O$ and $^{14}N$, it also gives rise to the formation of $^{13}C$-rich carbon stars (usually referred to as J-type carbon stars) and $^{14}N$-rich objects (e.g. Richer et al. 1979).
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3 Basics of modelling the chemical evolution of the Galactic disk

Table 3.5 Total yields for Zini = 0.001, ηAGB =4, and mHBB =0.8
mini
.9
1.0
1.3
1.3
1.5
2.0
3.0
4.0
5.0
7.0
8.0

H
-.140E-01
-.148E-01
-.244E-01
-.247E-01
-.245E-01
-.295E-01
-.203E-01
-.268E-01
-.311E-01
-.412E-01
-.382E-01

4

He
.140E-01
.148E-01
.213E-01
.215E-01
.210E-01
.242E-01
.149E-01
.202E-01
.234E-01
.316E-01
.295E-01

12

C
-.226E-04
-.260E-04
.281E-02
.281E-02
.321E-02
.463E-02
.472E-02
.109E-02
.580E-03
.734E-03
.603E-03

13

C
-.135E-04
-.149E-04
-.175E-04
-.178E-04
-.195E-04
-.220E-04
-.234E-04
.223E-03
.525E-04
.910E-04
.858E-04

14

N
.264E-04
.304E-04
.364E-04
.371E-04
.424E-04
.811E-04
.802E-04
.472E-02
.657E-02
.869E-02
.794E-02

16

O
-.197E-05
-.223E-05
.249E-03
.249E-03
.284E-03
.400E-03
.417E-03
.431E-03
.412E-03
.629E-04
.960E-04

YTOT
-.108E-05
-.847E-06
.309E-02
.309E-02
.353E-02
.529E-02
.535E-02
.656E-02
.772E-02
.961E-02
.878E-02

∆mej
.38
.46
.67
.69
.89
1.32
2.18
3.09
4.02
5.87
6.17

mend
.56
.57
.59
.59
.62
.68
.82
.91
.98
1.13
1.84

Table 3.6 Total yields for Zini = 0.008, ηAGB =4, and mHBB =0.8
mini
.9
1.0
1.3
1.3
1.5
2.0
3.0
4.0
5.0
7.0
8.0

H
-.121E-01
-.136E-01
-.191E-01
-.192E-01
-.222E-01
-.303E-01
-.422E-01
-.316E-01
-.331E-01
-.395E-01
-.508E-01

4

He
.120E-01
.134E-01
.174E-01
.174E-01
.191E-01
.249E-01
.342E-01
.262E-01
.265E-01
.311E-01
.406E-01

12

C
-.147E-03
-.169E-03
.128E-02
.128E-02
.253E-02
.439E-02
.681E-02
.658E-03
.104E-05
.458E-04
-.520E-04

13

C
.124E-04
.144E-04
.170E-04
.171E-04
.183E-04
.215E-04
.245E-04
.300E-03
.971E-04
.130E-03
.158E-03

14

N
.198E-03
.241E-03
.306E-03
.308E-03
.356E-03
.576E-03
.784E-03
.443E-02
.643E-02
.881E-02
.111E-01

16

O
.285E-04
.536E-04
.139E-03
.148E-03
.216E-03
.254E-03
.305E-03
.572E-04
.660E-04
-.632E-03
-.992E-03

YTOT
.683E-04
.112E-03
.170E-02
.171E-02
.306E-02
.543E-02
.798E-02
.544E-02
.659E-02
.835E-02
.102E-01

16

O
-.387E-04
.981E-03
.187E-02
.416E-03
.371E-03
.777E-04
-.147E-05
-.216E-03
-.291E-03
-.128E-02
-.160E-02

YTOT
-.150E-07
.161E-02
.303E-02
.744E-03
.208E-02
.394E-02
.692E-02
.624E-02
.628E-02
.774E-02
.841E-02

16

YTOT
.360E-05
.321E-02
.298E-02
.158E-04
.247E-03
.180E-02
.529E-02
.739E-02
.867E-02
.845E-02
.924E-02

∆mej
.33
.44
.68
.68
.92
1.39
2.36
3.10
4.03
5.89
6.81

mend
.58
.58
.59
.59
.59
.61
.64
.90
.97
1.11
1.20

Table 3.7 Total yields for Zini = 0.020, ηAGB =4, and mHBB =0.8
mini
.9
1.0
1.3
1.3
1.5
2.0
3.0
4.0
5.0
7.0
8.0

H
-.992E-02
-.208E-01
-.294E-01
-.157E-01
-.182E-01
-.240E-01
-.399E-01
-.376E-01
-.377E-01
-.436E-01
-.474E-01

4

He
.992E-02
.186E-01
.257E-01
.148E-01
.160E-01
.200E-01
.329E-01
.313E-01
.314E-01
.358E-01
.390E-01

12

C
-.415E-03
-.123E-03
-.334E-04
-.494E-03
.804E-03
.235E-02
.511E-02
.462E-02
-.741E-03
-.786E-03
-.923E-03

13

C
.309E-04
.480E-04
.670E-04
.491E-04
.513E-04
.597E-04
.668E-04
.698E-04
.141E-03
.170E-03
.176E-03

14

N
.485E-03
.773E-03
.122E-02
.867E-03
.986E-03
.127E-02
.173E-02
.182E-02
.729E-02
.976E-02
.108E-01

∆mej
.33
.53
.90
.72
.97
1.40
2.38
3.21
4.08
5.93
6.85

mend
.57
.58
.59
.59
.59
.60
.62
.79
.92
1.07
1.15

Table 3.8 Total yields for Zini = 0.040, ηAGB =4, and mHBB =0.8
mini
.9
1.0
1.3
1.3
1.5
2.0
3.0
4.0
5.0
7.0
8.0

H
-.858E-02
-.243E-01
-.229E-01
-.921E-02
-.903E-02
-.142E-01
-.318E-01
-.388E-01
-.452E-01
-.460E-01
-.513E-01

4

He
.858E-02
.204E-01
.193E-01
.919E-02
.874E-02
.123E-01
.262E-01
.314E-01
.364E-01
.372E-01
.416E-01

12

C
-.974E-03
-.347E-03
-.804E-03
-.146E-02
-.167E-02
-.108E-02
.189E-02
.454E-02
.468E-02
-.188E-02
-.195E-02

13

C
.729E-04
.113E-03
.127E-03
.982E-04
.109E-03
.142E-03
.158E-03
.161E-03
.168E-03
.276E-03
.277E-03

14

N
.114E-02
.171E-02
.214E-02
.171E-02
.205E-02
.244E-02
.317E-02
.330E-02
.359E-02
.116E-01
.124E-01

O
-.805E-04
.191E-02
.174E-02
-.107E-03
.228E-04
.878E-04
.964E-04
-.417E-03
-.590E-03
-.125E-02
-.130E-02

∆mej
.35
.55
.80
.69
.94
1.41
2.39
3.32
4.23
6.02
6.91

mend
.55
.55
.56
.56
.57
.59
.61
.68
.77
.98
1.09


We like to emphasize that the AGB yields of intermediate mass stars are strongly dependent on the abundances of distinct elements (e.g. C, N, and O) in the galactic ISM from which these stars formed. In turn, even small changes in the yields of AGB stars due to variations in initial metallicity can significantly affect the enrichment of the ISM (after weighing by the initial mass function and star formation rate at the time these stars were formed). Consequently, the yields of intermediate mass stars and the enrichment of the ISM wherein these stars formed are mutually dependent. This is an important property which should be adequately taken into account when modeling e.g. the chemical evolution of the Galactic disk (see Sect. 3.1).

**Figure 3.5** Stellar yields of H, $^4$He, $^{12}$C, $^{14}$N, $^{16}$O, and total CNO vs. initial stellar mass for $\eta_{\text{AGB}} = 1-5$ and initial compositions $(Z, Y) = (0.02, 0.32; \text{ solid line})$ and $(0.001, 0.24; \text{ em dashed})$. Parameters values are further as for the standard model (i.e. $\eta_{\text{AGB}} = 4$)

- Dependence on the amount of HBB

As discussed before, HBB may prevent or slow down the formation of carbon stars by the destruction of newly dredged up carbon at the base of the convective envelope. Fig. 3.6 illustrates that for low mass AGB
stars (\(m < 4 \, M_\odot\)), the effect of HBB is negligible. This is due to the low temperature at the bottom of their envelopes (GJ). For high mass AGB stars, the effect of HBB depends on the amount of matter exposed to the high temperatures at the bottom of their envelopes, the net result being the conversion of carbon and oxygen to nitrogen. Yields of H, He, and total CNO are not affected by HBB since basically two reaction chains of the CNO-cycle are involved, i.e. the CN and the ON-cycle.

![Figure 3.6](image)

**Figure 3.6** Stellar yields of H, \(^3\)He, \(^{12}\)C, \(^{14}\)N, \(^{16}\)O, and total CNO vs. initial stellar mass for the standard model: effect of varying 1) the amount of HBB (first two columns), 2) the dredge-up efficiency (center columns), and 3) the critical core mass for dredge-up (last two columns). Dashed and dotted curves have the same meaning as in Fig. 3.5.

We compare in the first two columns of Fig. 3.6 the resulting yields in case \(m_{\text{HBB}} = 0.9\) and 1.3 \(M_\odot\), respectively. A choice of \(m_{\text{HBB}} \approx 1.3 \, M_\odot\) or larger results in no HBB as none of the AGB stars in our model reach such high core masses. In the case of HBB, the strong decrease of the carbon and strong increase in the nitrogen yield can be seen at masses at \(m \gtrsim 4 - 5 \, M_\odot\). In the no HBB case, the stellar yields of
carbon are seen to dominate the total CNO-yields over the entire mass range.

In GHJ we argued that \( m_{\text{HBB}} = 0.9 \, M_\odot \) provides a reasonable upper limit to the minimum stellar mass experiencing HBB (as indicated by the masses of carbon stars observed in the Galactic disk). The effect of changing \( m_{\text{HBB}} \) from 0.9 to 0.8 \( M_\odot \) is that HBB operates in stars of initial mass \( > 4 \, M_\odot \) instead of \( > 5 \, M_\odot \). Also, in case \( m_{\text{HBB}} = 0.9 \, M_\odot \) the yields are somewhat smaller than those given by the standard model (i.e. \( m_{\text{HBB}} = 0.8 \, M_\odot \)). We emphasize that substantial HBB in AGB stars is required to explain the observations (see GJ, GHJ).

We note that many uncertainties are still involved with HBB which may affect the resulting AGB yields. In particular, the temperature structure of the envelope, the fraction of dredged up material processed in the CNO cycle, and the amount of envelope matter mixed down and processed at the bottom of the envelope may vary among AGB stars differing in initial mass, composition, and age (see HG). Notwithstanding, the default choice of \( m_{\text{HBB}} = 0.8 \, M_\odot \) for the standard model, based on our implementation of the RV \( \alpha = 2 \) model, appears consistent with recent observations of HBB in AGB stars both in the SMC and LMC as well as with recent model calculations on massive AGB stars (see above).

- **Dependence on third dredge-up efficiency**

  Fig. 3.6 shows the resulting AGB yields for third dredge-up efficiencies \( \lambda = 0.6 \) and 0.9 (i.e. the range allowed for by the observations; GJ). Predicted yields increase substantially when \( \lambda \) is increased, i.e. enhancing the amount of carbon and helium that is dredged-up and added to the stellar envelope after each thermal pulse. In addition, the composition of dredged-up material may be strongly affected by HBB, in particular for high mass stars. In other words, increasing \( \lambda \) leads to an increase in the carbon yields for low mass stars and to an increase in nitrogen for high mass stars. Furthermore, helium yields increase for all stars with initial masses above \( \approx 1.5 \, M_\odot \) corresponding to the limit of \( M_{\text{c min}}^\lambda = 0.58 \, M_\odot \).

- **Dependence on critical core mass for dredge up**

  Yields for extreme values of the minimal core mass for third dredge-up \( M_{\text{c min}}^\lambda = 0.56 \) and 0.60 \( M_\odot \), respectively, are shown in last two columns of Fig. 3.6. The effects of varying \( M_{\text{c min}}^\lambda \) are limited to relatively low mass AGB stars \(( \leq 2 \, M_\odot )\). A larger value of \( M_{\text{c min}}^\lambda \) implies a higher initial mass for stars that can turn into carbon stars. This results in negative carbon yields (corresponding to the depletion of carbon during first dredge-up) over a larger range in initial mass (helium yields decrease over this mass range as well). A value of \( M_{\text{c min}}^\lambda \) as small as \( \approx 0.56 \, M_\odot \) would imply that all AGB stars end as carbon stars while \( M_{\text{c min}}^\lambda > 0.61 \, M_\odot \) would inhibit carbon star formation. Clearly, the third dredge-up and the precise values of \( M_{\text{c min}}^\lambda \) and \( \lambda \) are of crucial importance for the formation of carbon stars. Note that the parameter value ranges consistent with the observations are rather narrow and are mutually correlated (e.g. in case of \( M_{\text{c min}}^\lambda \) and \( \lambda \)).

- **Dependence on pre-AGB evolution**

  In GJ and GHJ the description of the pre-thermal pulsing AGB evolution was taken from recipes in the literature or fits made to published results. An alternative approach is to directly use metallicity dependent stellar evolution tracks as has been done here. In both cases, the stellar evolution prior to the AGB is coupled consistently to the thermal-pulsing AGB phase.

  In HG we found that differences between the two sets of yields are very small except for hydrogen and helium where the GJ/GHJ approach predicts higher yields for massive stars. This is traced back to differences in the treatment of the second dredge-up process. The larger yields imply higher helium abundances which has interesting consequences for the helium abundances predicted in PNe. Abundances observed in PNe in the galactic disk suggest that the effect of second dredge-up is more pronounced than predicted by the Geneva tracks for \( \sim 30\% \) of the AGB stars (see Sect. 4.3). We list in Table 3.9 the AGB yields of H and He for pre-AGB evolution according to the GJ/GHJ recipes. These yields will be used as an alternative to the standard model yields in the galactic chemical evolution models discussed below (cf. HG).

- **Application to AGB stars in nearby galaxies**

  From the results presented in GJ and GHJ, it was argued that the standard model with \( \eta_{\text{AGB}} \sim 4 \) provides a reasonable approximation of the yields of intermediate mass AGB stars in the Galactic disk and the Large Magellanic Cloud (LMC). These systems have a metallicity that differ by only a factor of \( \approx 2 \) (e.g. Russell & Dopita 1992). In galaxies with a substantial lower metallicity such as the SMC, a lower value of \( \eta_{\text{AGB}} \) may be more appropriate. Note that using a fixed value of \( \eta_{\text{AGB}} \) does not necessarily mean identical mass-loss rates since two stars of the same initial mass evolve differently in the synthetic model due to the explicit metallicity dependence of the recipes used.
Table 3.9 Total AGB yields for H, He for synthetic evolution model

<table>
<thead>
<tr>
<th>$m_{\text{ini}}$</th>
<th>$Z_{\text{ini}}=0.001$</th>
<th>$Z_{\text{ini}}=0.008$</th>
<th>$Z_{\text{ini}}=0.020$</th>
<th>$Z_{\text{ini}}=0.040$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{\text{He}}$</td>
<td>$Z_{\text{H}}$</td>
<td>$Z_{\text{He}}$</td>
<td>$Z_{\text{H}}$</td>
<td>$Z_{\text{He}}$</td>
</tr>
<tr>
<td>.8</td>
<td>-12E-01</td>
<td>.12E-01</td>
<td>-11E-01</td>
<td>-99E-02</td>
</tr>
<tr>
<td>.9</td>
<td>-13E-01</td>
<td>.13E-01</td>
<td>-12E-01</td>
<td>-99E-02</td>
</tr>
<tr>
<td>1.0</td>
<td>-14E-01</td>
<td>.14E-01</td>
<td>-13E-01</td>
<td>-11E-01</td>
</tr>
<tr>
<td>1.3</td>
<td>-24E-01</td>
<td>.21E-01</td>
<td>-18E-01</td>
<td>-17E-01</td>
</tr>
<tr>
<td>1.5</td>
<td>-24E-01</td>
<td>.21E-01</td>
<td>-22E-01</td>
<td>-19E-01</td>
</tr>
<tr>
<td>1.7</td>
<td>-23E-01</td>
<td>.20E-01</td>
<td>-22E-01</td>
<td>-18E-01</td>
</tr>
<tr>
<td>2.0</td>
<td>-23E-01</td>
<td>.18E-01</td>
<td>-21E-01</td>
<td>-17E-01</td>
</tr>
<tr>
<td>2.5</td>
<td>-22E-01</td>
<td>.17E-01</td>
<td>-24E-01</td>
<td>-18E-01</td>
</tr>
<tr>
<td>3.5</td>
<td>-19E-01</td>
<td>.13E-01</td>
<td>-18E-01</td>
<td>-12E-01</td>
</tr>
<tr>
<td>4.0</td>
<td>-36E-01</td>
<td>.30E-01</td>
<td>-27E-01</td>
<td>-22E-01</td>
</tr>
<tr>
<td>4.5</td>
<td>-53E-01</td>
<td>.46E-01</td>
<td>-46E-01</td>
<td>-41E-01</td>
</tr>
<tr>
<td>5.0</td>
<td>-68E-01</td>
<td>.60E-01</td>
<td>-61E-01</td>
<td>-55E-01</td>
</tr>
<tr>
<td>6.0</td>
<td>-89E-01</td>
<td>.80E-01</td>
<td>-84E-01</td>
<td>-77E-01</td>
</tr>
<tr>
<td>7.0</td>
<td>-10E+00</td>
<td>.95E-01</td>
<td>-10E+00</td>
<td>-92E-01</td>
</tr>
<tr>
<td>8.0</td>
<td>-10E+00</td>
<td>.94E-01</td>
<td>-11E+00</td>
<td>-10E+00</td>
</tr>
</tbody>
</table>

Direct observational information on the metallicity dependence of the mass loss and element yields in AGB stars is rare. In Groenewegen et al. (1995), the spectral energy distributions and 8-13 µm spectra of three long-period variables (one each in the SMC, LMC and Galaxy) with roughly the same period were fitted. From the derived ratios of the dust optical depths in these stars, it was argued that the mass loss rates of AGB stars in the Galaxy, LMC, and SMC are roughly in the ratio of 4:3:1. This corroborates that $\eta_{\text{AGB}}$ could be similar for AGB stars in the Galaxy and LMC. Furthermore, this suggests that for AGB stars in low metallicity systems like the SMC ($Z \leq 0.004$), values of $\eta_{\text{AGB}} \approx 1-2$ may be more appropriate. We will study this effect by using yields for models with $\eta_{\text{AGB}} = 2$ for $Z = 0.004$ and $\eta_{\text{AGB}} = 1$ for $Z = 0.001$ in case of AGB stars in the SMC (see Chap. 6; HG).

Concluding remarks

The resulting AGB yields are most sensitive to the mass loss parameter $\eta_{\text{AGB}}$, the effect of HBB, and the initial stellar abundances. Variations in other model parameters result in stellar yields not substantially different from those predicted by the standard model with $\eta_{\text{AGB}} = 4$, $M_{\text{c min}} = 0.58 \, M_{\odot}$, $\lambda = 0.75$, $m_{\text{HBB}} = 0.8 \, M_{\odot}$, and second dredge-up according to the synthetic evolution model. We will show in Sect. 4.3 that the standard model predictions are in good agreement with the observed abundances in planetary nebulae (PNe) in the Galactic disk and Magellanic Clouds (see also GHJ; Groenewegen & de Jong 1994c).

When modelling the chemical evolution of the Galactic disk, we will use the standard model yields for an equidistant grid of about 100 initial masses between 0.8 and 8 $M_{\odot}$ at 10 metallicities in the range $Z = 10^{-4}$ to 0.04 (unless stated otherwise). At intermediate compositions and masses, stellar yields will be linearly interpolated.

The dependence of the AGB yields on the integrated metal-abundance $Z$ as well as on the individual H, He, C, N, and O abundances, will be taken into account in a self-consistent manner by using an iterative solution method of the galactic chemical evolution equations (cf. Sect. 3.1). As an example, we show in Fig. 3.7 the resulting AGB yields when taking into account the abundance variations of individual elements during the chemical evolution of the disk as predicted by the reference model. The strong metallicity dependence of the yields of intermediate mass stars in the Galactic disk is illustrated in Fig. 3.7. Note that small differences in the predicted AGB yields may be important when integrating over the initial mass function (and star formation rate) at the time these AGB stars were formed according to the galactic chemical evolution model adopted (see Chapter 4). In the following, we will refer to the standard model yields computed with the Geneva group initial abundances (as listed in Table 3.4) when discussing the cumulative yields of low and intermediate mass.
3.3 Stellar nucleosynthesis prescriptions

Figure 3.7 Theoretical yields of intermediate mass stars at metallicities $Z = 0.04, 0.02, 0.008, 0.004,$ and $0.001$ according to the standard AGB model from Groenewegen & de Jong (1993). Initial abundances of individual elements are as predicted by the reference model for the chemical evolution of the Galactic disk (see Chapter 4). Shown are the newly formed and ejected masses of He, total $Z$ (heavier than He), C, N, and O. In case of H, the hydrogen mass consumed is plotted.

3.3.3 Supernovae Type II

Supernovae Type-II (SNII) presumably originate from the gravitational collapse of stars more massive than $m > \sim 8 - 10 \ M_\odot$ at the end of their hydrostatic evolution. SNII all show hydrogen lines in their optical spectra (e.g. Branch & Nomoto 1991) which implies that the immediate progenitors of SNII are stars still surrounded by their hydrogen-rich envelopes (e.g. Hashimoto & Nomoto 1988; Woosley & Weaver 1992). In contrast, SNIb/c seem to be events related to the core collapse of massive stars that have lost their hydrogen-rich envelope (SNIb) and even lost (most of) their helium-rich envelope (SNIc).

In this section, we concentrate on the yields of massive stars that ultimately end as SNII. For these stars, we consider the pre-SN and SNII stages separately. In sections 3.3.4+5, we will discuss the yields of stars that end as SNIa and SNIb/c.

Pre-SN evolution of massive stars

Hydrostatic burnings of subsequently H, He, C, Ne, O, and Si, in the interiors of massive stars are important for abundance changes in the stellar core and envelope before the eventual supernova explosion occurs. This sequence ends with the formation of $^{56}$Fe since later nuclear fusion reactions are endoenergetic. Only stars with initial masses larger than $\sim 10 - 12 \ M_\odot$ undergo all six hydrostatic burnings in their cores while most of the products of hydrostatic Si burning usually end up in the compact remnant. However, stars slightly differing in initial mass and composition may show considerably different final structures and abundances owing to the complex interplay of convective shells that occurs during the late stages of their evolution (e.g. Woosley and Weaver 1995).
The initial abundances of a star determine the conditions under which these hydrostatic burnings occur, the mass of the resulting helium core $M_\alpha$, and the final compound structure of the star at the time of core collapse (e.g. Maeder 1992, 1993). In turn, the resulting helium-core masses and the chemical structure of the stellar envelope determine the sensitivity of the yields of massive stars to the initial abundances. In addition to the hydrostatic burnings, the materials in the deepest layers of the stellar envelope e.g. may undergo CNO cycling thereby being enriched in helium and nitrogen.

During pre-SN evolution, the yields of massive stars change strongly with initial metallicity (e.g. Maeder 1992; Weaver & Woosley 1995). This is primarily due to the fact that metallicity affects the opacities in the outer stellar envelope, and therefore, the mass-loss rates in the winds of massive stars (Abott 1982; Kudritzki et al. 1987, 1991). In turn, the envelope abundances and mass loss strongly affect the stellar evolution and nucleosynthesis (e.g. Chiosi & Maeder 1986).

For the pre-SN yields of massive stars, we consider two distinct sets of theoretical data which differ by means of their physical ingredients (e.g. nuclear reaction network, treatment of convection, mixing, overshooting, and mass loss). The first set contains the pre-SN yields from the Geneva group (the higher mass-loss rates were used) for stars with $m \gtrsim 8 \, M_\odot$ formed with initial metallicities $Z = 0.001, 0.004, 0.008, 0.02,$ and $0.04$, for the elements H, He, C, N, O, and Ne. We use this particular set of pre-SN yields because: 1) it is compatible with the tracks used for low and intermediate mass stars describe above, 2) this set is used in many studies related to the chemical evolution of the Galactic disk, and 3) until very recently other extensive metallicity dependent data sets were unavailable. The second set contains the pre-SN yields as part of the final SN yields presented by Woosley & Weaver (1995) for stars with $11 \leq m[M_\odot] \leq 40$ at metallicities $Z = 0, 10^{-6},$ $10^{-4},$ $0.002,$ and $0.02$ (see below). These data include the most extensive set of metallicity dependent evolution tracks currently available for massive stars and follow in detail the chemical evolution from the main-sequence up to the end of the core collapse and SNII explosion for $\sim 200$ isotopes.

For the first set of SN yields, we assume that the initial abundances of elements heavier than Ne are unaltered during pre-SN evolution as these elements were not incorporated in the Geneva models. This introduces unavoidable errors because the pre-SN abundance changes and ejection of elements like Mg, Si, S, and Ca can be substantial and even can be larger than the corresponding amounts ejected during the explosive nucleosynthesis (e.g. Woosley and Weaver 1995).

The last observable stage in massive star evolution before neutron star (or black hole) formation is the Wolf-Rayet (WR) stage which is followed in detail by the Geneva tracks. We couple the pre-SN tracks from the Geneva group to the evolution of exploding helium stars as presented by Hashimoto & Nomoto (1988; see for a recent review Thielemann et al. 1995). This is done by using the metallicity dependent helium core masses $M_\alpha$ predicted by the Geneva group (see e.g. Maeder 1992) as input to the models of Hashimoto & Nomoto (1992; hereafter HN). This procedure may be somewhat unsafe because: 1) the HN models were computed for solar metallicity only, 2) the Geneva tracks for massive stars do not extend beyond carbon burning (i.e. ignoring later hydrostatic burnings), and 3) the pre-SN evolution and chemical structure of the helium stars in the HN models may be different from that predicted by the Geneva group.

Notwithstanding, the major uncertainty is that the metallicity dependent helium core masses $M_\alpha$ at the end of the C-burning phase predicted by the Geneva group may not correspond to the immediate progenitors of SNII (as considered by HN). In fact, this is a more general problem since mass loss during the WR stage critically affects the initial mass vs. helium core mass relation (e.g. Maeder 1992; Woosley, Langer, & Weaver 1993, hereafter WLW):

- First, the final helium core masses $M_\alpha$ of massive stars may be drastically reduced by efficient mass loss during and/or prior to the WR stage. In fact, such stars resemble the evolution of low mass stars (e.g. WLW). In this case, the metallicity dependence of the yields becomes evident as mass loss by massive stars (i.e. $m \gtrsim 30 \, M_\odot$) is argued to increase with initial metallicity (e.g. Kudritzki et al. 1987, 1991; Maeder 1992). In particular, the high-Z models of the Geneva group predict relatively high mass-loss rates during the pre-WR stages (see Sect. 3.2). This means that high-Z stars enter the WR phase relatively early. Recent results suggest that mass-loss during the WR stage is proportional to the actual stellar mass (cf. Langer 1989). Consequently, for high metallicity stars, high mass-loss rates may continue for long times during the WR phase and can strongly reduce the resulting helium core masses (e.g. Maeder 1992).

- Second, $M_\alpha$ may increase or decrease during the helium burning phase depending on the size of the helium core and the amount of fresh helium mixed into the core by convection. In this case, the metallicity dependence of the pre-SN yields enters because $M_\alpha$ increases with decreasing metallicity. This is due to the fact that the central temperatures of stars during main-sequence evolution are higher at lower metallicity. Consequently, the convective cores of low metallicity stars
are relatively large so that fresh helium can be mixed into the core by convection. In addition to the treatment of overshooting and convection, the growth of the helium burning core is determined by the $^{12}\text{C} (\alpha, \gamma)^{16}\text{O}$ reaction rate which is still somewhat uncertain (e.g. Thielemann et al. 1996).

Consequently, the largest uncertainties in the above procedure are probably associated with the metallicity dependent $m$ vs. $M_\alpha(m)$ relations (i.e. the adopted parametrisation of the mass-loss rates, treatment of convection, and nuclear reaction rates). Since the Geneva tracks have been successfully checked with many independent observations (e.g. Maeder 1992; Maeder & Meynet 1994; Meynet et al. 1992), we use their $M_\alpha(m)$ relations for the first set of pre-SN yields, in spite of the uncertainties and problems involved with the chemical structure of the precollapsing star discussed above.

For the second set of SN yields computed from the models of Woosley & Weaver (1995, hereafter WW), the evolution of massive stars is followed in a consistent manner, i.e. from the main-sequence up to the end of the core collapse, and part of the problems discussed previously are not encountered. Nevertheless, the metallicity dependent $M_\alpha(m)$ relations predicted by WW are uncertain for the same reasons as discussed before. Since mass loss has not been included in the WW models, the helium core masses predicted are probably overestimated (independent of initial metallicity). For massive stars which do experience considerable mass loss during the pre-SN stage, we will use the models of SNII from WLW (1993, 1995) discussed in Sect. 3.3.5.

It is tempting to see how the above two sets of nucleosynthesis yields of massive stars, based on the most extensive and up-to-date stellar evolution tracks available today, compare to each other and to the observational constraints imposed by the chemical evolution of the Galactic disk. This will be subject of Sect. 3.4 (see also Sect. 4.3).

Core collapse of SNII

Core collapse of SNII progenitors occurs as soon as the mass of the iron core, formed at the end of a star’s lifetime, reaches the Chandrasekhar mass. During the collapse, the proto-neutron star (or black hole) grows by accretion and releases binding energy in the form of neutrinos while it contracts to neutron star densities. After core collapse, the bounce at nuclear densities generates a delayed shock wave which propagates through all the material exterior to the iron core. Simultaneously, the temperature in the stellar mantle rises rapidly, both due to the propagation of the shock wave and the heating by neutrinos escaping from the proto-neutron star (or proto black hole). In principle, this process may lead to the explosive burnings of Si, O, Ne, C, He, and H, during the supernova outburst (e.g. Thielemann et al. 1996; WW). The shock breaks through the stellar surface only a few minutes after core collapse.

Depending on the the initial energy of the shock and the density structure of the star, material may fall back onto the collapsed remnant (WW). The shock may bounce at the interface between the helium core and the hydrogen-rich envelope and its deceleration can lead to significant amounts of material falling back to the collapsed remnant, under the influence of gravity. This has important implications for the nucleosynthesis and the nature of the remnant (i.e. depending on the size of the iron core either a neutron star or a black hole; cf. Bethe 1990). Core collapse models from WW suggest that stars with initial masses larger than about 30 $M_\odot$ will experience considerable reimplosion of heavy elements, provided that the explosion energy does not exceed some critical value ($\approx 1.2 \times 10^{51}$ ergs).

The nucleosynthesis products by SNII are determined by the complex set of processes that occur after core collapse. In brief, essential parameters describing the explosion mechanism are: 1) the delay time between the core bounce at nuclear densities and the explosion caused by neutrino heating (usually on a time scale of seconds or less depending on the neutrino transport and core structure), 2) the location of the mass cut between the neutron star and the stellar envelope ejected, and 3) the energy of the shock wave (a reduction of the explosion energy results in less heavy elements to be ejected). Clearly, theoretical nucleosynthesis yields of SNII depend strongly on these parameters (apart from their dependence on progenitor mass and compound structure) which cannot be constrained very tightly yet by the observations. Nevertheless, we expect that the qualitative conclusions presented below are insensitive to the uncertainties involved with the present generation of comprehensive SNII models.

Chemical enrichment by SNII

Enrichment of the ISM in elements such as O, Ne and the $\alpha$-elements (Mg, Si, S, Ca) is accepted to be mainly due to SNII. As discussed above, the SNII abundance spectrum of elements heavier than helium depends on many details of the nucleosynthesis during the explosion (e.g. Woosley & Weaver 1986; Nomoto 1992, Thielemann et al. 1993; Hashimoto et al. 1993; Woosley, Langer & Weaver 1993, hereafter WLW;
WW; Thielemann et al. 1996). Intermediate mass nuclei (i.e. more massive than Si) are found to originate predominantly from explosive O and Si burning which supply similar amounts of heavy nuclei for progenitor masses up to \( \lesssim 25 \, M_\odot \). The amount of lighter elements in the range C-Si (e.g. O, Ne and Mg) has a dominant contribution from hydrostatic burning of the C/Ne core (or explosive Ne burning) and strongly depends on progenitor mass. The ejected amount of iron-group nuclei (V, Cr, Zn, Mn, Ni) is directly related to the explosion mechanism and to the mass cut between the ejecta and the central object. In particular, the innermost layers experience complete Si burning and are the source of the iron group elements and some intermediate-mass elements like Ti. For the two sets of pre-SN+SN yields described above, we discuss the dependence of the nucleosynthesis yields on the SNII progenitor mass and initial stellar metallicity.

- **Pre-SN and SNII yields from the Geneva group and Nomoto et al.**

Explosive nucleosynthesis yields of SNII have been described in detail by Nomoto & Hashimoto 1988, Nomoto et al. 1991, Hashimoto & Nomoto (1992; hereafter HN), and Thielemann et al. (1993, 1996). We have used a complete set of numerical models for SNII progenitors born with \( Z = 0.02 \) presented by HN for initial stellar masses of 13, 15, 18, 20, 25, 40 and 70 \( M_\odot \) (corresponding to helium core masses \( M_\alpha = 3.3, 4, 5, 6, 8, 16 \) and 36 \( M_\odot \), respectively). The 20 \( M_\odot \) progenitor mass model of this set has been very successful in explaining the observed element abundances in SN1987A (see further Nomoto et al. 1991). In the following, we will refer to this data set as the Nomoto et al. data set.

Nomoto et al. have computed SNII yields of He up to Zn by following the envelope abundances from the start of gravitational contraction of the helium core to the shock wave propagation through the stellar mantle. The critical initial mass above which stars become SNII is assumed to be 8 \( M_\odot \) while stars with \( m \geq 13 \, M_\odot \) are argued to form an iron core after central ignition of both Ne and Si. In principle, the remaining iron core mass is determined by the carbon abundance both after helium burning and during the overshooting of convective elements at the edge of the helium burning core. The fate of stars in the mass range 6 \( \lesssim m[M_\odot] \lesssim 8 \) is still matter of debate. These stars may explode by carbon deflagration (subspherical explosion) or detonation (supersonically) completely disrupting the star (e.g. Nomoto, Thielemann, and Yokoi 1984). Alternatively, these stars may end their lives as a C+O white dwarf. For stars with \( m \geq 8 \, M_\odot \), we discuss the explosive nucleosynthesis yields from Nomoto et al.

Electron degeneracy becomes significant during and/or after carbon burning in the C+O core for stars with masses between 8 and \( \sim 10 \, M_\odot \) (\( M_\alpha = 2.5-2.7 \, M_\odot \)). In general, the mass of the O/Ne/Mg core does not exceed the critical mass of 1.37 \( M_\odot \) needed for neon ignition (Nomoto & Hashimoto 1988; for comparison, the critical mass for carbon ignition is \( \sim 1.06 \, M_\odot \)) and the compact dwarf residual consists of a degenerate O/Ne/Mg core surrounded by a thin outer layer of C+O containing small amounts of elements up to Ne. As a consequence these stars do not contribute substantially to the ISM enrichment of elements heavier than Ne and, in general, produce small amounts of heavy elements (see e.g. Nomoto, Shigeyama & Tsujimoto 1991). Although these stars presumably leave a degenerate O/Ne/Mg core, the abundances in the ejecta may be very similar to those in massive AGB stars (cf. Hashimoto, Iwamoto & Nomoto 1993).

Stars in the mass range 10 to \( \sim 13 \, M_\odot \) presumably form a semi-degenerate O/Ne/Mg core which is massive enough to allow for (off-center) neon ignition (Nomoto & Hashimoto 1988). An important question is whether or not the inwards propagating neon-burning shell reaches the center. This depends on details of the core structure. If not, a degenerate O/Ne/Mg core is left. Alternatively, explosive neon-flashes may lead to the ejection of a helium layer. We will assume that stars in the mass range 8 to \( \sim 13 \, M_\odot \) eject a helium-layer moderately enriched in heavy elements up to Si. For these masses, we linearly extrapolate the Nomoto et al. yields from the 13 \( M_\odot \) and 15 \( M_\odot \) SNII models for elements up to Si. These stars do not contribute to the interstellar iron content since iron is predominantly produced by explosive O or Si burning (e.g. Woosley & Weaver 1986; Nomoto, Shigeyama & Tsujimoto 1991). We emphasize that the yields of stars in the mass range 8 – 13 \( M_\odot \) are relatively uncertain and depend strongly on the detailed evolution scenario adopted. Unfortunately, the IMF-weighted contributions by stars in this mass range are important, in particular for elements like O, Ne, Mg, Al, and Si.

Stars above \( \sim 13 \, M_\odot \) (\( M_\alpha = 3.3 \, M_\odot \)) develop core masses sufficiently large for central Ne ignition. These stars undergo nuclear burning under non-degenerate conditions and eventually form iron cores more massive than 1.4 \( M_\odot \), while ejecting large amounts of metals (e.g. Hashimoto et al. 1993). The detailed explosion mechanism for these stars is determined both by the final mass of the iron core and by the density gradient near this core.

Fig. 3.8 displays the yields of the 20 and 40 \( M_\odot \) SNII progenitor masses for the Nomoto et al. data set. In general, the main elements returned by SNII progenitors are He, O, and Fe. Comparison of the 20 and 40 \( M_\odot \) SNII yields reveals that the latter yields are larger for all elements up to the iron peak except for the \( \alpha \)-elements which are relatively unimportant.
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Figure 3.8 Yields of SNII progenitors with initial masses of $20\ M_\odot$ (left) and $40\ M_\odot$ (right), formed with solar metallicity (data from Nomoto et al.)

The iron mass ejected by SN 1987A, which probably has a progenitor of $\sim 20\ M_\odot$, is estimated to be $\Delta m(\text{Fe}) = 0.075 \pm 0.1\ M_\odot$ from its optical light curve (e.g. Kumagai et al. 1991). This is in good agreement with the ejected amount of iron of a $20-25\ M_\odot$ progenitor star (assuming a helium core mass of $\sim 6\ M_\odot$) as predicted by the theoretical SNII models from Nomoto et al. A $20\ M_\odot$ star further returns $\sim 0.2\ M_\odot$ of Mg and $\sim 0.1\ M_\odot$ of Si. In case of Al and Ca, these amounts are 0.001 and 0.005 $M_\odot$, respectively. Stars with main-sequence masses in the range of $20-25\ M_\odot$ are probably the dominant sites of nucleosynthesis products since they produce most species from oxygen to the iron-peak elements in about the solar abundance ratios (Woosley & Weaver 1986; Nomoto 1992). In addition, the IMF weighted heavy element yields of these stars are relatively important (see below).

Fig. 3.9 shows the total synthesized and ejected yields of H, He, C, N, O, α-elements, Al, Fe, and total Z, as a function of initial stellar mass and metallicity (data from the Geneva group for the higher mass-loss rates and from Nomoto et al. described below). In order to be consistent with the stellar yields presented by the Geneva group for phases preceding the SNII explosion, we adopted the SNII yields for H, He, C, N, and O from the Geneva group as well (see Maeder 1992). For stars with $m \lesssim 8\ M_\odot$, the AGB yields discussed in Sect. 3.3.2 are shown for comparison.

We briefly discuss the burning stages during which these elements are synthesized as well as their qualitative dependence on initial stellar mass and metallicity. This discussion also refers to the data from Woosley and Weaver described below.

The elements H, He, C, and N are ejected mostly during the wind (i.e. pre-SN) phase of massive stars (see below). Carbon is formed during helium burning, its production depends on the details of convection at the end of helium burning. For instance, a small increase of the helium convective core can dramatically decrease the carbon yield. Also, the carbon yield is sensitive to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction which is still somewhat uncertain. Nitrogen is produced by the CNO cycle (in the deep envelope by the CN cycle). In some massive stars, it is possible that the convective helium shell dredges up carbon with the consequent production of large amounts of nitrogen (cf. Timmes et al. 1995). In general, the newly synthesized yields of C and N increase with initial stellar mass ($m \gtrsim 8\ M_\odot$) and metallicity, which both have a favourable effect on the hydrostatic burning of He.

Oxygen is formed both during helium burning and neon burning and is the most abundant heavy element synthesized in massive stars. A substantial oxygen contribution comes from the SNII phase especially for stars born with low metallicities. The remaining oxygen is produced during the pre-SN phase (which is sensitive to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate as well).

Magnesium and aluminium are mostly products of hydrostatic carbon and neon burning (e.g. Arnett & Thielemann 1985). In general, the yields of Mg and Al decrease with increasing metallicity. This is due both to the strong reduction of the helium core mass with increasing metallicity and to the corresponding increase in mass loss with metallicity (as discussed above). The same is true for intermediate mass elements (Si–Sc) which are produced mostly during oxygen burning (i.e. both during hydrostatic oxygen shell burning and explosive oxygen burning, in proportions that vary from star to star). Intermediate mass elements are burned to iron group elements during and after shock passage. However, these elements are created by explosive oxygen burning in amounts similar to those previously burned (see below). When moving upwards in atomic mass, the nucleosynthesis becomes increasingly sensitive to abundance changes that occur during
the SN explosion (and less sensitive to the initial stellar abundances). In particular, the iron-peak elements are mainly synthesized during the explosive burnings of Si and O. Therefore, the yields of these elements are relatively insensitive to initial metallicity.

\[ \text{SNII yields: } Z\text{-dependence (Geneva/Nomoto)} \]

\[ \Delta M_{ej}, \text{tot} \quad \text{[M}_\odot\text{]} \]

\[ \text{Initial Mass [M}_\odot\text{]} \]

\[ \text{Figure 3.9 Total yields of SNII progenitors with initial masses } m \gtrsim 8 \text{ M}_\odot, \text{ and initial metallicities } Z = 0.04, 0.02, 0.008, 0.004, \text{ and 0.001 (see legend). Data are taken from the Geneva group and Nomoto et al. (1994). Total yields for AGB stars with } m \lesssim 8 \text{ M}_\odot \text{ are shown for comparison (standard model; cf. Fig. 3.3).} \]

- Pre-SN and SNII yields from Woosley and Weaver

Recently, Woosley & Weaver (1995; hereafter WW) presented a comprehensive set of Type II supernova models for various initial masses \( (11 \lesssim m/\text{M}_\odot \lesssim 40) \) and metallicities \( (Z= 0., 10^{-4}, 10^{-2}, 0.1 \text{ and } 1. \text{Z}_\odot) \). We here discuss these pre-SN and SNII yields and compare them to the yields from Nomoto et al. discussed in the previous section.
Fig. 3.10 shows the total synthesized and ejected yields of $H$, $He$, $C$, $N$, $O$, $\alpha$-elements, $Al$, $Fe$, and total $Z$, as a function of initial stellar mass and metallicity (data from Woosley & Weaver 1995; their A-models were used). Yields were linearly extrapolated for stars with $m \gtrsim 40$ $M_\odot$. For stars with $m \lessim 8$ $M_\odot$, the AGB yields discussed in Sect. 3.3.2 are shown.

Comparison of the yields from WW with those from Nomoto et al. (and the Geneva group) reveals that for stars with $m \lesssim 20$ $M_\odot$ the yields of $H$, $He$, $C$, $N$, and $O$ are similar (i.e. within a factor of 2). In contrast, for stars with $m \gtrsim 20$ $M_\odot$, mass-loss becomes important and the $H$, $He$, $C$, $N$, and $O$ yields predicted by WW are considerably smaller (in some cases by more than one order of magnitude) than those from Nomoto et al. In addition, the metallicity dependence of these element yields is relatively weak in the WW models which is due to the fact that the strong metallicity dependence of the mass lost by massive
stars was not taken into account by WW. This illustrates the importance of the inclusion of mass loss for the heavy element yields of massive stars.

Although a larger initial helium abundance generally leads to larger helium cores in the WW models, the trend expected breaks down for stars more massive than \( \sim 30 \, M_\odot \). This is due to the inclusion of the reverse shock in the WW models which results in additional accretion of matter onto the iron core (this extra amount increases towards initially more massive, larger gravity stars). As a consequence, the heavy element yields of stars with \( m \gtrsim 30 \, M_\odot \) are drastically reduced. The combined effects of the reverse shock (included in the WW models) and the metallicity dependent mass loss during the pre-SN phase (ignored in the WW models) are the main explanation for the differences between the WW and Nomoto et al. data in the production of intermediate mass elements by stars more massive than \( \sim 20 \, M_\odot \). Similarly, the yields of iron-peak elements are reduced by large factors (up to \( \sim 25 \)) in the WW models, due to the more efficient accretion of products of explosive Si and O burnings. Note the high sensitivity of the iron yields to the initial stellar metallicity in the WW models.

In contrast to the WW models, Nomoto et al. considered stars of solar composition only and used evolved helium stars to model the SNII explosion (instead of taking into account in detail the pre-SN evolution). This results e.g. in \( M_\alpha(m) \) relations distinct from that in the WW models (e.g. \( M_\alpha \) is \( \sim 9.2 \) and \( \sim 8 \, M_\odot \) for a 25 \( M_\odot \) star born with \( Z = Z_\odot \), in the WW and Nomoto et al. models, respectively). Furthermore, the calculations by Nomoto et al. were performed by depositing energy at a specific radius inside the Fe core while neglecting the effects of neutrino transport through the stellar mantle as well as ignoring the effects of a reverse shock in the more massive SN progenitors. In contrast, WW used a piston located at the outer edge of the iron core (which may differ for stars differing in mass). The actual explosion was delayed for some time during which the neutrino deposited energy built up to a critical value. This results e.g. in large differences between the two sets of models in the final iron core masses predicted.

Overall, the WW models have some important advantages over those from Nomoto et al. in combination with the Geneva group tracks. However, a disadvantage of the WW models is that they do not account for mass-loss during the pre-SN stage which depends on initial stellar metallicity and strongly affects the resulting yields as discussed above. We will consider the IMF-weighed yields for the two sets in Sect. 3.4.

### 3.3.4 Supernovae Type Ia

Type-I supernovae are subclassified in Type Ia, Ib, and Ic according to their photospheric spectra (see e.g. review by Harkness and Wheeler 1990). The lack of strong hydrogen lines in the spectra of SNI implies that the progenitors have lost most of their hydrogen-rich envelope at time of the explosion, e.g. by mass-transfer in a binary system. SNIa are characterised by spectra showing a deep absorption line produced by blueshifted SiII \( \lambda 6355 \) which is absent in SNIIb and Ic. The optical spectra of SNIa exhibit also lines of Ca, S, Mg, and O (e.g. Branch et al. 1981) and are sensitive to the production of \( ^{56}\text{Ni} \) which decays via \( ^{56}\text{Co} \) to stable \( ^{56}\text{Fe} \).

SNIa are generally accepted to originate from mass-accreting C+O white dwarfs (WD) in close binary systems, based on models of the thermonuclear explosion of an electron-degenerate C+O core (e.g. Woosley & Weaver 1986; Nomoto 1986; Yamaoka et al. 1992). In this case, the accreting WD experiences a steady supply of gravitational binding energy associated with the H-rich matter accreted from the secondary. Part of this energy is radiated away, while the remaining part heats the interior of the WD (e.g. Nomoto 1982). The rate of mass accretion onto the WD surface determines the degree to which compressional heating and radiative cooling of the WD occurs. When the WD mass increases and reaches the Chandrasekhar limit, either thermonuclear explosion or collapse occurs, depending on the accretion rate and the CO-mass at the end of the C-burning phase. Besides its mass and accretion rate, the fate of the WD depends on the composition of the accreted matter and thus on the evolution of the companion star (see further Shigeyama et al. 1992).

At present, no unique explosion model exists that can explain all SNIa observed. Apart from the amount of matter surrounding the WD at time of explosion, the companion of the WD either may be an evolved star supplying matter onto the WD (i.e. the single degenerate model; e.g. Wheeler & Iben 1973; Nomoto et al. 1991; Renzini 1993) which ultimately results in the SNIa explosion, or may be a WD as well and the SNIa explosion may occur when the two orbiting WDs merge after loosing angular momentum by gravitational wave radiation (i.e. the double degenerate model; Iben & Tutukov 1984; Paczynski 1985; cf. Renzini 1993). Observations of slowly rising SNIa light curves, appear to require explosion models in which the C+O WD is surrounded by an unburnt extended envelope of typically 0.2 to 0.4 \( M_\odot \) (Müller & Höflich 1993). Also, the WD may have a He or O+Ne+Mg composition although models suggest that the fate of such WDs is inconsistent with the observations (e.g. Müller 1990; Myaji & Nomoto 1987).

In order to retain the energy required to power the thermonuclear explosion of SNIa, the propagation of
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the mixing front through a significant part of the star may be supersonically (detonation), subsonically (deflagration), or subsonically in the outer regions and supersonically in the inner regions (delayed detonation). We here adopt the nucleosynthesis yields for SNIa based on the carbon deflagration wave model in a C+O WD as presented by Nomoto et al. (1984) and Thielemann et al. (1986). The nuclear energy released in these models is large enough to disrupt the white dwarf completely. However, nova-like explosions based on the ignition of degenerate hydrogen may prevent the WD from reaching an explosive configuration (He+C double detonation models; Nomoto 1993) so that in this case the SNIa explosion would not occur at all (or very late).

![Figure 3.11](image-url) Newly synthesized and ejected element yields of a 5 $M_\odot$ SNIa progenitor (top left; data from Nomoto et al. 1984) and a 12 $M_\odot$ helium star SNIb progenitor (top right; data from Woosley et al. 1995). Yields of SNII progenitors of 20 $M_\odot$ (bottom left) and 40 $M_\odot$ (bottom right; data from Woosley & Weaver 1995) are shown for comparison. Solar metallicity has been assumed for the SN progenitor stars.

The standard deflagration model W7 of Nomoto et al. (1984) and Thielemann et al. (1986) for SNIa accounts well for the observed light curve and spectra at both early and late times of SNIa. We here adopted the SNIa yields of the W7 model for an initial stellar mass of $\sim 5$ $M_\odot$ which leaves a WD remnant of 1.37 $M_\odot$. Metallicity dependent SNIa yields were used for $Z = Z_\odot$ and $Z = 0$. of the accreted material, respectively. Heavy element yields for other SNIa progenitor masses are assumed to be the same since SNIa yields are argued to depend only on the critical mass for Ni ignition (e.g. Nomoto 1991).

We note that the outcome of carbon deflagration depends on the propagation speed of the shock wave, which is uncertain (e.g., depending on the mixing length of convection; cf. Wheeler and Harkness 1990). Also, late detonation models may reduce the SNIa yields considerably for elements up to Mg, while the yields for elements more massive than Si will be somewhat increased (e.g. Yamaoka et al. 1992; Thielemann et al. 1993). Furthermore, it is possible that the accreting C+O WD collapses rather than explodes (Nomoto et al. 1991), which would have considerable effect on the resulting SNIa yields.

In Fig. 3.11 we compare the element yields of SNIa (Nomoto et al. 1984) with those of SNIb (Woosley et al. 1995) and SNII (Woosley & Weaver 1995), for stars born with solar metallicity. For SNIa, the ejecta from mass-loss phases preceding the explosion were not taken into account. In general, the SNIa yields of
iron-peak elements exceed those of SNII and SNIb by about one order of magnitude. Typical amounts of iron produced are \( \sim 0.8 \, M_\odot \) for SNIa, \( \sim 0.07 \, M_\odot \) for SNII, and \( \sim 0.1 \, M_\odot \) for SNIb/c. Observations require that a total amount of \( 0.4–1 \, M_\odot \) of \(^{56}\text{Ni}\) should be ejected in each SNII event (e.g. Thielemann et al. 1989). This is in good agreement with the sum of the Fe and Ni yields predicted by the SNII models shown. Note that the oxygen production by SNII is practically negligible compared to that by SNII and SNIb/c (e.g. Nomoto et al. 1984; Hashimoto et al. 1989; Woosley and Weaver 1991). SNII progenitors are thought to be a major source of the iron-peak elements and of intermediate mass elements like Si, S, and Ca (cf. Fig. 3.11). SNII (and SNIb/c) are the main contributors to elements such as He, O, Ne, and the \( \alpha \)-elements (cf. Fig. 3.9).

We consider the yields of SNIb/c in more detail below.

### 3.3.5 Supernovae Type Ib/c

SNIb/c explosions are probably associated with the core collapse of massive stars that lost their entire hydrogen-rich envelope (Ib) and even lost (most of) their helium-rich envelope (Ic; e.g. Yamaoka & Nomoto 1991; Woosley et al. 1993). The hydrogen-rich envelope may be lost due to mass-transfer in a close binary system or by means of a strong stellar wind around single massive stars (e.g. Woosley, Langer & Weaver 1993, hereafter WLW; Maeder & Meynet 1994). Once the hydrogen envelope is entirely lost, the remaining helium star may be identified as a WR star (e.g. Maeder & Meynet 1994). The strong winds observed around helium stars with \( m \gtrsim 4 \, M_\odot \) may be due to instabilities associated with radial pulsations (Glatzel, Kirakisdik & Frick 1993; WLW) which may imply a convergence of final masses near \( \sim 4 \, M_\odot \) (Langer et al. 1994). The optical light curves and spectra of SNIIb/c are found to be in agreement with explosion models for low-mass helium stars between 3 and 5 \( M_\odot \) (which correspond to main-sequence masses of \( \sim 12–18 \, M_\odot \); see e.g. Yamaoka & Nomoto 1991; Nomoto et al. 1994; Woosley et al. 1995). Since SNIIb/c presumably undergo iron core collapse as in SNII, their yields are expected to resemble those of low-mass SNII progenitors (e.g. Shigeyama et al. 1990; Hachisu et al. 1990).

Recently, WLW presented models for the chemical evolution of helium stars with masses in the range \( 4–20 \, M_\odot \) which are stripped off their hydrogen-rich envelopes. The final masses of these helium stars, left over after the hydrostatic nuclear burnings and mass-dependent mass loss (e.g. Langer 1989), were found by WLW to converge to a very narrow range between 2.26–3.55 \( M_\odot \) and were considered as the immediate progenitors of SNIIb/c. Explosive nucleosynthesis of these final masses was treated in a way similar to that for SNII (see WW; WLW). For helium stars more massive than \( \sim 7 \, M_\odot \), some matter was found to fall back to the collapsed core during the explosion (as for SNII).

We will adopt the SNIIb/c yields for the helium stars models presented by WLW (their A models were used) as the sum of the pre-SN and SN yields of massive stars stripped off their hydrogene-rich envelopes. Furthermore, we will ignore any variation of the SNIIb/c yields with metallicity as the models by WLW were performed for solar metallicity only.

In Fig. 3.11 we show an example of the SNIIb/c yields for a 12 \( M_\odot \) helium star as predicted by the models of WLW. The progenitor mass of this helium star is probably as massive as \( \sim 40 \, M_\odot \) (see below). Overall, the SNIIb/c yields are similar to that for a SNII progenitor of roughly the same mass (\( \sim 40 \, M_\odot \)). Exceptions include the SNIIb/c yields of He and C, and the mean O/Fe abundance ratio in their ejecta, which are substantially larger than in SNII.

In Fig. 3.12 we compare the yields of the most abundant elements of SNIIb/c with those of SNIIa and SNII (for stars formed with solar metallicity). The SNIIb/c yields shown were related to a main-sequence star mass using the initial mass vs. helium star mass relation given in Eq. (3.20) below. In principle, SNIIb/c contribute to the same elements as SNII. However, SNIIb/c are more important contributors to He, C, and Fe (over the entire range of SNII progenitor mass considered), even after weighing with the IMF and with the relative SN frequencies (see below). Also, SNIIb/c may be important for the enrichment of elements like Na, Al, and Ti (cf. WLW).

The SNIIb/c yields shown in Fig. 3.12 are associated with stars that lost their hydrogen-rich envelopes early in their evolution while the SNII yields shown have been computed for stars that did not experience any mass loss. Comparison reveals that the yields of intermediate mass elements are strongly reduced when mass loss by massive stars is included. In WLW it was argued that the chemical composition of helium stars with stripped envelopes differs from stars which evolved without efficient mass loss even though the density structure of such stars is the same. Thus, the chemical composition of a helium-burning helium star keeps memory of its initial mass. Furthermore, the immediate progenitors of SNIIb/c have their helium-burning shell relatively close to the core compared to stars without mass loss (cf. WLW), due to the small sizes of the final masses of helium stars that lost their envelopes. At the time of the SNIIb/c explosion, this causes an intense neutron irradiation when the shock wave passes through the helium shell which stimulates the
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Figure 3.12 Comparison of the yields of SNII, SNIa, and SNIb progenitors in the mass range $2.5 - 60 \, M_\odot$ formed with solar metallicity. In case of SNIa, the immediate progenitor is assumed to be a WD remnant with $m_{WD} \sim 1.2 \, M_\odot$. The SNIb/c yields shown were related to a main-sequence star mass using the initial mass vs. helium star mass relation given in Eq. (3.20). Literature sources for the yields are as given in Fig. 3.11.

formation of massive elements such as Ti, Fe, and Ni. Note that if a major part of the helium envelope is lost as well (as has been proposed for SNIc), the overall nucleosynthesis can be considerably altered (WLW).

It is clear that the yields of massive stars that experience substantial mass loss before they ultimately explode as SN, are sensitive to the detailed mass-loss history (which affects both the mass of the resulting helium star and its chemical composition; see WLW). In addition, these yields are sensitive to e.g. the initial composition of the star, the nucleosynthesis and convection that may occur in the envelope before mass loss occurs, and — in the case of massive stars in close binary systems — the amount of enriched material that is (re-)accreted from the secondary before the primary actually explodes. These effects have not been accounted for in the models of WLW. Nevertheless, as discussed above, the largest uncertainty in applying the SNIb/c yields of WLW in galactic chemical evolution models arises from the unknown initial mass vs. helium star mass relation.

For single stars, loss of the entire hydrogen envelope is restricted to stars more massive than $m \sim 30 \, M_\odot$ with solar metallicity at birth (e.g. Langer 1989; Maeder 1992). These stars will leave relatively large helium stars (e.g. $\gtrsim 10 \, M_\odot$) while their surface layers will be enriched in carbon (and possibly also oxygen) since a substantial fraction of the helium-burning lifetime can be spent before the hydrogen-rich envelope is lost. In contrast, in the case of binary stars, stars with initial masses as low as $\sim 10 \, M_\odot$ can lose their entire hydrogen envelope. These stars leave relatively small helium stars (i.e. $4 - 5 \, M_\odot$) surrounded by a substantial mantle of pure helium, provided that they loose their hydrogen-rich envelopes before the onset of core helium burning (e.g. Vanbeveren 1991). Thus, SNIb/c progenitors resulting from massive stars in close
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binary systems will be, on average, less massive and will have surface layers that are chemically less evolved at the time of explosion than SNIb/c originating from single stars. Therefore, both the pre-SN and SN yields of SNIb/c associated with binary stars are substantially less than those associated with single stars.

For stars that ultimately explode as SNIb/c, we will adopt an initial mass vs. helium star mass relation which favours the formation of low-mass helium stars (i.e. $4 \sim 5 \, M_\odot$). This seems justified because: 1) the minimum initial stellar mass that can lose its entire hydrogen-rich envelope is much lower for stars in binary systems ($\sim 10 \, M_\odot$) than for single stars ($\sim 30 \, M_\odot$), and 2) the formation probability of mass-losing stars of $\sim 10 \, M_\odot$ in close binaries is probably larger than that of $\sim 30 \, M_\odot$ single stars (assuming a Salpeter like IMF; cf. Greggio & Matteucci 1990).

For simplicity, we will assume that the helium star mass $m_{\text{He}}(m)$ distribution of massive stars that end as SNIb/c is the same as the IMF for these stars. This implies a linear relation between initial stellar mass and the resulting helium star mass (independent of metallicity). If we assume that only helium stars with masses between 4 and $20 \, M_\odot$ are formed, we have

$$m_{\text{He}}(m) \ [M_\odot] = 4 + 16 \frac{(m - m_{\text{SNIb/c}})}{(m_u - m_{\text{SNIb/c}})}$$

(3.20)

where $m_{\text{SNIb/c}}$ and $m_u$ are the lower mass limit of stars that presumably end as SNIb/c and the upper mass limit at birth assumed (cf. Sect. 3.2), respectively. If anything, this will overestimate the contribution by SNIb/c since massive stars ($\gtrsim 30 \, M_\odot$) eventually may end as low-mass helium stars as well (i.e. $\sim 4 - 5 \, M_\odot$; cf. Woosley et al. 1993: WLW). In this manner, a first order approximation can be made of the yields of massive stars that lose their hydrogen-rich envelope prior to core collapse, either by means of strong stellar winds or due to mass-transfer in close binary systems.

3.3.6 Close binary stars

Between 75 and 90% of all stars in the SNBH have been identified as a member of a binary or multiple stellar system (e.g. Abt 1983; Halbwachs 1986). This is expected to have several important consequences for the chemical evolution of the Galactic disk. The ultimate fate of a single star (or effectively a star in a wide binary) is primarily determined by its initial mass $m$ and mass-loss history. In the case of close binary stars, the fate of a star is further determined by the rate of mass exchange with the companion star. In general, transfer of envelope material from the primary (initially more massive star) to the secondary may leave less material to turn into metals. If we assume that the bulk of the mass transfer occurs from the primary to the secondary, the general tendency would be to reduce the overall nucleosynthesis yields of heavy elements. The reverse is probably true when mass-transfer occurs from the secondary to the compact object.

The formation of a WD from an intermediate mass progenitor star requires the loss of the envelope due to a strong stellar wind (Reimers 1975) which prevents the core from growing beyond $1.4 \, M_\odot$ (e.g. Iben & Renzini 1984). When massive stars in binaries experience drastic mass loss due to Roche-lobe overflow of the primary component (e.g. upon leaving the main-sequence), this may lead to the formation of a WD instead of a neutron star remnant (which would have been formed if the star had not experienced substantial mass loss). These massive stars may ultimately end as a SNIa instead of SNII with important consequences for their yields.

When material is transferred, it may be further processed by the secondary which may result in an enhancement of its stellar yields depending on the relative time scales for mass accretion and mass loss in the stellar wind. However, when the secondary later on fills its own Roche lobe, mass-transfer from the secondary to the primary can strongly alter the yields of both stars. Other effects which e.g. may play an important role include: 1) stellar envelope mass which is directly returned to the ISM instead of being transferred to and further processed by the companion star, and 2) mass accretion to the surface of a massive star which results in an enhancement of the envelope opacity thereby increasing the stellar mass loss by radiation pressure (i.e. mass-accretion induced mass loss by the primary which results in a reduction of the core helium mass of the secondary, with corresponding effects on the heavy element yields).

The integrated effect of binary systems on stellar nucleosynthesis is poorly known. The tendency of a detailed inclusion of the evolution of a binary system is expected to result in a substantial reduction of the heavy element yields as compared to the sum of both single star yields. For low-mass elements (i.e. lighter than neon) the net effect of binary stars may be an enhancement of the total ejected element mass compared to the sum of the single star yields. However, this effect strongly depends on: e.g. 1) the evolutionary phase(s) during which mass-transfer occurs, 2) the underlying physical mechanism and amount of mass loss, 3) the composition of the envelope mass transferred, and 4) the distributions of the initial mass-ratios and orbital separations of the binary components (cf. Iben 1991).
3.4 Overall comparison of the yields of intermediate and massive stars

We will assume that the IMF-weighed element yields of the single star progenitors of AGB stars, SNII, SNIa, and SNIb/c discussed above are representative for a stellar population that consists of predominantly binary stars. If anything, the enrichment by the single star population overestimates that of the binary population depending on the frequency of close binary systems in which mass-transfer leads to substantial changes in the yields of their single star components.

3.4 Overall comparison of the yields of intermediate and massive stars

We give a brief overview of the literature sources for the metallicity dependent stellar yields described in the previous section. Thereafter, we compare these yields for stellar generations differing in IMF, initial metallicity, and relative frequency of AGB stars, SNIa, SNIb/c, and SNII, by means of the cumulative IMF-weighed yield and the net yield function.

3.4.1 Overview of the stellar yields adopted

Table 3.10 summarizes the literature sources for the stellar yields adopted. Stellar yields for the wind and post-wind evolutionary stages are distinguished. For SNII progenitors, we use either the yields from the Geneva group for the pre-SN phase combined with the yields from Hashimoto & Nomoto (1992, hereafter HN), or the data from Woosley and Weaver (1995, hereafter WW) both for the pre-SN and SN phases. The stellar yields listed in Table 3.10 have been discussed extensively in the previous section and usually cover the range in initial metallicity from \( Z = 0.001 \) to \( Z = 0.04 \), except for the SNII yields from HN (see also Thielemann et al. 1993, 1996) and the SNIb/c yields from Woosley, Langer, and Weaver (1995; hereafter WLW) which were computed with \( Z = 0.02 \) only. When necessary, the yields are linearly interpolated (or extrapolated) down to \( Z = 0.0001 \) and up to 0.04.

Table 3.10

<table>
<thead>
<tr>
<th>( \Delta m [M_\odot] )</th>
<th>Phase</th>
<th>Elements</th>
<th>Metallicity range</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 – 60</td>
<td>Wind-phase</td>
<td>H,He,C,N,O,Ne</td>
<td>( Z = 0.001 - 0.04 )</td>
<td>Geneva group</td>
</tr>
<tr>
<td>0.8 – 8</td>
<td>EAGB+AGB</td>
<td>H,He,C,N,O</td>
<td>( Z = 0.0001 - 0.04 )</td>
<td>GJ93 &amp; HG96</td>
</tr>
<tr>
<td>2.5 – 8</td>
<td>SNIa</td>
<td>H–Zn</td>
<td>( Z = 0.02 )</td>
<td>NO84 &amp; TH86</td>
</tr>
<tr>
<td>8 – 60</td>
<td>SNIb/c</td>
<td>H–Zn</td>
<td>( Z = 0.001 )</td>
<td>WW95</td>
</tr>
<tr>
<td>8 – 60</td>
<td>SNIa(^*)</td>
<td>H–Zn</td>
<td>( Z = 0.02 )</td>
<td>WW95</td>
</tr>
<tr>
<td>8 – 60</td>
<td>SNIb/c(^*)</td>
<td>H,He,C,N,O</td>
<td>( Z = 0.001 )</td>
<td>M92</td>
</tr>
<tr>
<td>8 – 60(^*)</td>
<td>SNIa(^*)</td>
<td>F–Zn</td>
<td>( Z = 0.02 )</td>
<td>HN92</td>
</tr>
</tbody>
</table>

Notes:

* SNII yields were taken either from the combined M92 + HN92 data sets or from the WW95 models
\(^*\) yields of elements heavier than Si were neglected for stars with \( m \lesssim 13 M_\odot \) (cf. Sect. 3.3.3)

References: Geneva group: Schaller et al. (1992); Schaerer et al. (1993, 1995); Charbonnel et al. (1993); Meynet et al. (1994); GJ93: Groenewegen & de Jong (1993); HG96: van den Hoek & Groenewegen (1997); NO84: Nomoto et al. (1984); TH86: Thielemann et al. (1986); WLW95: Woosley, Langer, and Weaver (1995); WW95: Woosley & Weaver (1995); M92: Maeder (1992); HN92: Hashimoto & Nomoto (1992)

The data set listed in Table 3.10 is the most comprehensive and uniform set of metallicity dependent stellar yields currently available. The underlying stellar evolution models from these sources are based on up-to-date input physics and have been checked against many independent observational constraints. In addition, these models cover a wide range both in initial mass and metallicity, and extend from the main-sequence up to the final stages of stellar evolution. Although many uncertainties are still involved, we believe that the set of stellar yields adopted is an important step further towards an adequate and representative data base of nucleosynthesis yields well suited for use in galactic chemical evolution studies.

3.4.2 Cumulative IMF-weighed yields

Cumulative IMF-weighed yields are a measure of the relative contribution by stars, with masses in a given mass range, to the total mass of element \( j \) ejected by the stellar generation to which these stars belong.
Cumulative IMF-weighed yield distributions are computed by integrating the IMF-weighed stellar yields over a given initial mass range \((m_l, m_l + \Delta m)\) and dividing this number by the same integration over the entire mass interval \((m_l, m_u)\):

\[
rmCY(\Delta m) = \frac{\int_{m_l}^{m_l + \Delta m} \int_{m_u}^{m_u} p_j(m, Z) M(m) dm}{\int_{m_l}^{m_u} \int_{m_u}^{m_u} p_j(m, Z) M(m) dm}
\]  \tag{3.21}

where \(p_j(m, Z)\) is the stellar yield of element \(j\) (see Eq. 3.12) and \(m_l, m_u\) denote the lower and upper stellar mass limits at birth, respectively. In the following, we will refer to these distributions shortly as cumulative yields.

Although cumulative yields can be used e.g. to estimate the contribution by AGB stars to the total amount of helium returned by a stellar population, there are some limitations in their application. First, the ejecta of AGB stars are returned on time scales much longer than those of more massive stars (e.g. SNII progenitors) that belong to the same stellar generation. Second, due to their long lifetimes, AGB stars usually have initial abundances substantially less than those of e.g. SNII progenitors. Third, among the stars that nowadays return the main part of their ejecta to the ISM, AGB stars may have been formed with SFRs much higher in the past than those of massive stars. The latter effect would imply that the cumulative yields of AGB stars would be much more important than in the above definition of the cumulative yields.

For the cumulative yields considered in this section, we assume that the initial abundances of e.g. AGB stars and SNII progenitors are identical and that both low and high-mass stars return their ejecta to the ISM simultaneously. Furthermore, for the present purpose, we exclude the the yields of SNIa and SNII/c since their contributions heavily depend on the assumed fraction of stars that ultimately end as SNI. Consequently, some of the quantitative conclusions presented below may be altered when we take into account the detailed star formation histories, initial metallicities, and ejection time scales of these stars (cf. Chapter 4).

- Comparison of individual stellar yields: H, He, C, N, and O

Fig. 3.13 shows the wind and post-wind element yields of H, He, C, N, O, and Z as a function of initial stellar mass and initial metallicity. Data for stars with \(m > 8\ M_\odot\) have been adopted from the Geneva group and Nomoto et al. as discussed in Sect. 3.3. For stars with \(m < 8\ M_\odot\), we adopted the standard model for AGB stars (cf. Fig. 3.5). Yields for the wind-phase (i.e. pre-AGB phase) of AGB stars are usually negligible and have not been plotted separately from their post-wind (i.e. AGB phase) yields. Newly synthesized and ejected element masses are given by the sum of the wind and post-wind yields. Note that these yields do not include any unprocessed material initially present in the star (cf. Fig. 3.9). For hydrogen, the total amount of H consumed is plotted.

Before discussing these yields for each element individually, we show in Fig. 3.14 the corresponding cumulative IMF-weighed total yield distributions of stars born with \(Z = 0.02\) for various IMFs. Cumulative yields are considered for the IMF derived from star counts in the solar neighbourhood (cf. Scalo 1986) and for two power law IMFs with \(\gamma = -2.35\) (Salpeter IMF) and \(-3\), respectively. The cumulative distributions were normalised to one using initial mass limits of \(m_l = 0.1\ M_\odot\) and \(m_u = 60\ M_\odot\).

**Hydrogen** is consumed predominantly by massive stars during their wind phases. However, most hydrogen is deposited in long lived low-mass stars \((m < 1\ M_\odot)\) rather than being consumed. As can be seen from Fig. 3.13, total hydrogen yields are insensitive to the initial stellar metallicity. In contrast, both the wind and post-wind hydrogen consumptions do depend on the initial metal-abundance. For the local IMF, more than \(~85\%\) of hydrogen is consumed (or deposited) by stars less massive than \(8\ M_\odot\), independent of \(Z\) (see below). For a power-law IMF with slope \(\gamma = -2.35\) this fraction is \(~70\%\). In principle, the interstellar hydrogen abundance can be used to constrain the depletion and consumption of gas by the total galactic stellar population.

In Figs. 3.15 and 3.16, we show the variations of the cumulative yields with initial stellar metallicity for the Nomoto et al. (1994) and Woosley & Weaver (1995) models, respectively, assuming a Salpeter IMF. Both data sets show very similar cumulative hydrogen yields and are insensitive to initial metallicity. Note that despite the similarity in the cumulative hydrogen yields for the two sets, the corresponding total hydrogen masses returned may differ considerably (see below).

**Helium** is predominantly synthesized during the wind-phase of massive stars (cf. Fig. 3.13). Because of the large convective cores and reduced mass-loss rates of massive stars born with relatively low metallicities, the helium yields for the wind-phase of these stars increase with initial metallicity. For the same reason, the helium yields of massive stars for the post-wind phase decrease with increasing metallicity. AGB stars
3.4 Overall comparison of the yields of intermediate and massive stars

Figure 3.13 Yields for the wind and post-wind phases of SNII progenitors with initial masses $m \geq 8$ $M_\odot$ and initial metallicities $Z = 0.04, 0.02, 0.008, 0.004$, and $0.001$ (see legend). Data has been adopted from the Geneva group and Nomoto et al. (1994). Yields for AGB stars with $m \leq 8$ $M_\odot$ are shown for comparison (standard model; cf. Fig. 3.5)
Figure 3.14 Cumulative IMF-weighed yields for solar metallicity stars in the mass range 0.1−60 M⊙ formed according to a power law IMF with γ = −2.35 (i.e. Salpeter), γ = −3, and the IMF derived for stars in the solar neighbourhood (Scalo 1986; see legend). Data adopted from the Geneva group, Nomoto et al. (1994), and van den Hoek & Groenewegen (1997)

produce relatively small amounts of helium, typically less than 0.5−1 M⊙ while most of the helium enrichment by AGB stars originates from stars with initial masses between ∼4 and 8 M⊙.

However, after weighing by the Salpeter IMF, AGB stars are found to account for more than ∼65% of the helium enrichment of the ISM. For the local IMF, this number increases to ∼80% (cf. Fig. 3.14). Since the cumulative helium yields are dominated by AGB stars and are relatively insensitive to initial metallicity, the interstellar helium abundance provides a valuable constraint to the formation history of low and intermediate mass stars.

Carbon is mainly produced during the wind-phase of massive stars (as is helium). Large amounts of carbon (i.e. up to ∼6 M⊙) can be ejected during this phase while post-wind carbon yields are usual small (i.e. ∼0.5 M⊙; cf. Fig. 3.13). The trend with initial metallicity is similar to that for helium except at Z ∼ 0.02. This is related to the reduced convective helium core in this case. Production of carbon in AGB stars is low compared to that in massive stars. In fact, stars with masses ranging from 4–8 M⊙ may consume carbon by HBB burning depending on their initial metallicity (cf. Sect. 3.2).

The IMF-weighed carbon contribution by AGB stars formed with Z = Z⊙ is ∼30% assuming a Salpeter IMF (∼50% for the local IMF). For the WW models, these numbers are substantially larger, i.e. ∼60 and ∼80%, respectively (cf. Fig. 3.16 and 3.17) since the carbon contribution by massive stars is relatively low in the WW models (see Sect. 3.3). For the Geneva/Nomoto models, the AGB contribution to the cumulative carbon yields is relatively insensitive to initial metallicity. In this case, most interstellar carbon is produced by stars more massive than ∼8 M⊙ (dependent of Z). In contrast, the AGB star carbon contribution decreases from ∼65% at Z=0.04 to ∼35% at Z=0.001 in the WW models. In this case, the interstellar carbon abundance is expected to be dominated by massive stars at early epochs of galactic evolution, while AGB stars start to dominate the stellar carbon ejection rate at more recent evolution times.

Nitrogen is mainly formed by CNO burning in massive AGB stars as well as during the wind-phase of massive stars born with metallicities Z ∼ 0.01 (cf. Fig. 3.13). In particular, nitrogen is not formed in significant amounts during the post-wind evolution of stars more massive than ∼8 M⊙. Thus, the IMF-weighed nitrogen contribution is strongly dominated by AGB stars, i.e. AGB stars contribute at least 80% to 90% to the interstellar nitrogen abundance (cf. Figs. 3.14−3.16) depending on the assumed IMF and initial metallicity.
3.4 Overall comparison of the yields of intermediate and massive stars

Figure 3.15 Cumulative IMF-weighed yields for stars in the mass range 0.1–60 $M_\odot$ formed according to a Salpeter IMF with initial metallicities $Z = 0.04, 0.02, 0.008, 0.004,$ and 0.001 (see legend). Data from the Geneva group, Nomoto et al. (1994), and van den Hoek & Groenewegen (1997)

of the stars. This is true both for the Geneva/Nomoto and WW models. Therefore, interstellar nitrogen is a valuable tracer of the AGB star population formed in the past. This is especially true for low metallicities ($Z \lesssim 0.005$) at which CNO cycling in massive stars becomes negligible (cf. Maeder 1992).

Oxygen production is dominated by massive stars during their post-wind phases (i.e. SN explosions). Small amounts of oxygen are returned during the wind-phase of these stars (cf. Fig. 3.13). The variation of the explosive oxygen yields with initial metallicity can be explained in a way similar to that for helium and carbon (see above).

For a Salpeter IMF, the IMF-weighed oxygen contribution by AGB stars formed with $Z = Z_\odot$ amounts $\sim 20\%$ (cf. Fig. 3.14). However, the initial metallicity assumed for these stars is certainly too high because of their formation at relatively early epochs in galactic chemical evolution. Therefore, the actual oxygen contribution by AGB stars is much less than that shown in Fig. 3.14. In addition, the variation of the cumulative oxygen yields with metallicity is mainly due to the initial abundances of AGB stars assumed (cf. Figs. 3.15 and 3.16). In general, stars more massive than $\sim 10 \ M_\odot$ do contribute more than $\sim 90\%$ of the oxygen returned by a stellar generation.

Since oxygen is basically synthesized in SN progenitors with initial masses between 20 and $\sim 40 \ M_\odot$, the interstellar abundance puts severe constraints on the past population of massive stars. This constraint is independent of initial metallicity (or IMF) and thus provides a severe upper limit to the total number of SNII (and SNIb/c) that has occurred during the lifetime of the Galactic disk. The same is true for the cumulative yields of the element integrated metallicity $Z$ (cf. Figs. 3.13-3.16).
3 Basics of modelling the chemical evolution of the Galactic disk

Figure 3.16 Cumulative IMF-weighted yields for stars in the mass range 0.1–60 M_☉ formed according to a Salpeter IMF with initial metallicities Z = 0.04, 0.02, 0.008, 0.004, and 0.001 (see legend). Data from Woosley & Weaver (1995), and van den Hoek & Groenewegen (1997)

- Comparison of individual stellar yields: Mg, Al, Si, S, Ca, and Fe

Elements heavier than oxygen are synthesized in considerable amounts by SNIa and in massive stars (m \geq 12 M_☉). We here ignore the ejecta from SNIa since their contribution strongly depends on the formation history of low and intermediate mass progenitors (cf. Sect. 3.2). However, we note that SNIa may substantially contribute to elements such as Fe and Ni.

In Figs. 3.15 (WW models) and 3.16 (Geneva/Nomoto models), we show the cumulative yields of Mg, Al, Si, S, Ca, and Fe for stars formed with initial metallicities Z = 0.04, 0.02, 0.008, 0.004, and 0.001. In general, the cumulative yields of these elements strongly depend on initial mass, i.e. more massive stars in general eject larger amounts of heavy elements. However, in case of the WW models, stars more massive than \sim 30 M_☉ experience strong reverse shocks during which substantial amounts of envelope material are accreted by the iron core formed during the preceding collapse (cf. Sect. 3.3). Therefore, cumulative heavy element yields in the WW models are dominated completely by stars with initial masses between 10 and \sim 30 M_☉. Furthermore, the cumulative yields for the heavy elements shown are insensitive to initial metallicity as compared to the Geneva/Nomoto models. This is because stars in the WW models do not experience metallicity dependent mass loss.

The cumulative yields in case of the Geneva/Nomoto models do strongly depend on initial metallicity for elements such as Mg, Al, S, and Ca. This is mainly due to variations of the helium core mass and stellar mass-loss rates with initial metallicity. Iron is an exception since it is mainly produced during explosive silicon
3.4 Overall comparison of the yields of intermediate and massive stars

and oxygen burning. At relatively high metallicities \( Z \geq 0.02 \), stars with \( 10 \leq m[\text{M}_\odot] \leq 30 \) contribute \( \sim 50 \% \) of the total heavy element enrichment (this fraction is somewhat larger for the WW models). Note that the Geneva/Nomoto models predict substantially larger heavy element contributions by stars with \( m \geq 30 \text{ M}_\odot \) compared to the WW models.

In principle, the ISM abundances of intermediate mass elements like Mg, Al, and Si, provide contraints to the total number of massive stars \( (m \geq 8 \text{ M}_\odot) \) ever born in the Galactic disk. However, the use of these contraints is complicated by the fact that SN1a may contribute substantially to the ISM abundances of such elements as well (which usually originate from stars less massive than \( \sim 8 \text{ M}_\odot \)). As discussed in Sect. 3.2.3, the abundances of heavy elements in the ejecta of SNII and SN1a differ considerably. Therefore, interstellar abundance ratios like \([\text{O/Fe}]\) and \([\text{Ne/Fe}]\) may put more severe constraints on the total number of SNII and SN1a as well as on their relative importance for the chemical enrichment of the Galactic disk. However, the use of such constraints is further complicated by the contribution of SN1b/c (and/or SN1II progenitors which lost their hydrogen-rich envelopes), e.g. for elements like Mg, Al, and Si (cf. Sect. 4.3).

3.4.3 Net yield function

The net yield of a stellar generation is a useful quantity when comparing stellar input data for galactic chemical evolution models. The theoretical net yield \( Y_j \) is defined as the newly synthesized and ejected mass of element \( j \), IMF-weighed and integrated over all stars, divided by the net mass locked up by the same stars (see e.g. Tinsley 1980):

\[
Y_j(t) = \frac{\int_{m_0(t)}^{m_u(t)} mp_j(m, Z_j) M(m) \, dm}{\int_{m_1}^{m_u} m M(m) \, dm - \int_{m_0(t)}^{m_u(t)} (m - m_{\text{rem}}(m, Z_j)) M(m) \, dm}
\]  

(3.22)

where \( m_0(t) \) denotes the turnoff mass at galactic lifetime \( t \) and \( m_1, m_u \) denote the mass boundaries between which stars are formed. Net yields \( Y_j(t) \) can be used as a first approximation to the element abundance \( Z_j(t) \) for a given IMF and set of stellar yields \( p_j \) without incorporating the detailed galactic star formation and chemical enrichment history (e.g. Tinsley 1980). Note that the net yield depends on the initial metal-abundance \( Z_\text{int} \) of the stars under consideration by means of the stellar yields \( p_j(m, Z_\text{int}) \).

The net yield may be also expressed as:

\[
Y_j = \frac{1}{1 - R} \left( \frac{\int_{m_0(t)}^{m_u(t)} mp_j(m, Z_j) M(m) \, dm}{\int_{m_1}^{m_u} m M(m) \, dm} \right)
\]  

(3.23)

where the returned fraction \( R(t) \) is defined as the ratio of the stellar mass ejected at instant \( t \) and the total mass of stars formed for a given stellar generation (cf. Tinsley 1980):

\[
R(t) = \frac{\int_{m_0(t)}^{m_u(t)} (m - m_{\text{rem}}(m)) M(m) \, dm}{\int_{m_1}^{m_u} m M(m) \, dm}
\]  

(3.24)

The net yield \( Y_{j,i} \) can be calculated also for different stellar subsamples \( i \) (such as AGB stars) by replacing the integral in the nominator of Eqs. 3.21 (and 3.22) by:

\[
\int_{m_{l,i}}^{m_{u,i}} f^i(m) mp_j(m, Z_j) M(m) \, dm
\]  

(3.25)

where the specific mass boundaries \( m_{l,i}, m_{u,i} \) correspond to the subsample of evolved stars which belong to the same stellar generation, and the formation probabilities \( f^i(m) \) correct for the possible different evolutionary outcomes for stars of the same mass. For a stellar generation containing \( N \) stellar subsamples \( i \) (which have no stars in common), one has \( Y_j = \sum_{i=1}^{N} Y_{j,i} \). For convenience, we assume that the formation probabilities \( f_i \) are independent of initial mass. In this case, the net yields \( Y_{j,i} \) are directly proportional to the formation probabilities \( f_i \).

3.4.4 Comparison of net yields

We have calculated the net yields \( Y_j \) of H, He, C, N, O, Mg, Al, Si, S, Ca, Fe, and Z at initial metallicities \( Z = 0.001 \) and 0.02 either assuming a Salpeter IMF or assuming the local IMF derived by Scalo (1986) at a Galactic age of \( T_0 = 15 \text{ Gyr} \) (cf. Tables 3.11–3.12). We further assumed a present-day turnoff mass \( m_0 = \)
Table 3.11 Net yields $Y_j$ at $Z = 0.02$ and 0.001 for a Salpeter IMF

<table>
<thead>
<tr>
<th>$Z = 0.02$</th>
<th>AGB</th>
<th>SNII, SNII$^{60}$</th>
<th>SNII$^{30}$</th>
<th>SNII$^{60}$</th>
<th>SNII$^{30}$</th>
<th>SNIIa</th>
<th>SNIIb/c</th>
<th>Total$^{30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>-8.36(-3)</td>
<td>-1.71(-2)</td>
<td>-2.82(-2)</td>
<td>-1.19(-2)</td>
<td>-1.91(-2)</td>
<td>-</td>
<td>-</td>
<td>-1.01(-2)</td>
</tr>
<tr>
<td>He</td>
<td>7.07(-3)</td>
<td>1.11(-2)</td>
<td>1.64(-2)</td>
<td>6.21(-3)</td>
<td>9.26(-3)</td>
<td>1.20(-6)</td>
<td>5.80(-3)</td>
<td>2.38(-2)</td>
</tr>
<tr>
<td>C</td>
<td>7.00(-4)</td>
<td>8.67(-4)</td>
<td>3.17(-3)</td>
<td>3.22(-4)</td>
<td>4.74(-4)</td>
<td>2.03(-5)</td>
<td>1.04(-3)</td>
<td>2.62(-3)</td>
</tr>
<tr>
<td>N</td>
<td>1.25(-3)</td>
<td>1.10(-4)</td>
<td>2.30(-4)</td>
<td>1.93(-4)</td>
<td>2.66(-4)</td>
<td>-</td>
<td>7.55(-5)</td>
<td>1.43(-3)</td>
</tr>
<tr>
<td>O</td>
<td>-3.75(-5)</td>
<td>3.18(-3)</td>
<td>1.64(-2)</td>
<td>8.45(-3)</td>
<td>5.98(-3)</td>
<td>6.90(-4)</td>
<td>3.88(-3)</td>
<td>-</td>
</tr>
<tr>
<td>Ne</td>
<td>-</td>
<td>8.69(-4)</td>
<td>1.04(-3)</td>
<td>6.84(-4)</td>
<td>9.98(-4)</td>
<td>1.90(-6)</td>
<td>2.32(-4)</td>
<td>1.10(-3)</td>
</tr>
<tr>
<td>Mg</td>
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<th>SNII$^{60}$</th>
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<td>4.82(-4)</td>
<td>5.75(-5)</td>
<td>1.70(-4)</td>
<td>6.28(-4)</td>
<td>-</td>
</tr>
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<td>5.73(-4)</td>
<td>3.03(-3)</td>
<td>1.12(-2)</td>
</tr>
</tbody>
</table>

GN: refers to the stellar evolution tracks from the Geneva group and the SNII models from Hashimoto & Nomoto (1992)

WW: refers to the pre-SN and SN models from Woosley and Weaver (1995)

$^{30}$ and $^{60}$ indicates that an upper mass limit for SNII (and SNIIb/c) of $m_{\text{SNII}} = 30$ and 60 $M_\odot$, respectively, has been assumed

1. $M_\odot$ and stellar mass limits at birth of $m_l = 0.1$ $M_\odot$ and $m_u = 60$ $M_\odot$, respectively. These particular choices were used to allow for a detailed comparison of the net yields presented here and those calculated by Maeder (1992, 1993).

For the stellar subsamples considered, we adopted initial mass boundaries and formation probabilities $f_i$ (cf. Table 3.3) as follows: AGB stars ($m_l = 1$ $M_\odot$, $m_u = 8$ $M_\odot$, $f(m) = 1$), SNII ($8$, $8$, $0.01$), SNIIb/c ($8$, $m_{\text{SNIIb/c}}$, $0.33$), and SNII ($8$, $m_{\text{SNII}}$, $0.66$). For comparison, we list the net yields of SNII assuming $m_{\text{SNII}} = m_{\text{SNIIb/c}} = 30$ and 60 $M_\odot$, respectively, both for the Geneva/Nomoto and the WW data sets.

The yields computed are given in Tables 3.11 and 3.12 for the AGB, SNII, SNIIa, and SNIIb/c stages separately. Total net yields are tabulated as the sum of the yields in case of the Geneva/Nomoto data with $m_{\text{SNII}} = 30$ $M_\odot$. Although a detailed discussion of the net yields is beyond our scope, we like to note several important points:

- AGB stars dominate the net yields of nitrogen. Massive stars dominate the hydrogen consumption and contribute to about 60% of the helium enrichment of the ISM at $Z = Z_\odot$. SNII are the main contributors to all other elements listed except carbon (dominated by SNIIb/c at $Z = 0.02$), and iron ($\sim 30-40\%$ originates from SNIIa). Except for the iron peak elements, the contribution by SNIIa to the net yields of heavy elements is usually negligible. SNIIb/c contribute about 20–30% of the net yields of elements heavier than oxygen;

- net yields of SNII increase by $\sim 50\%$ when raising the upper mass limit for SNII from $m_{\text{SNII}} = 30$
### 3.4 Overall comparison of the yields of intermediate and massive stars

Table 3.12 Net yields \( Y_j \) at \( Z = 0.02 \) and 0.001 for a Scalo IMF

<table>
<thead>
<tr>
<th>( Z = 0.02 )</th>
<th>AGB</th>
<th>SNII(^{10})</th>
<th>SNII(^{60})</th>
<th>SNII(^{10})</th>
<th>SNII(^{60})</th>
<th>SNIIa</th>
<th>SNIIb/c</th>
<th>Total(^{10})</th>
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<td>-1.19(-2)</td>
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<td>-4.41(-2)</td>
<td></td>
</tr>
<tr>
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<td>1.26(-2)</td>
<td>1.62(-2)</td>
<td>6.57(-3)</td>
<td>8.87(-3)</td>
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<td>6.27(-3)</td>
<td>3.21(-2)</td>
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<td>3.10(-4)</td>
<td>4.02(-4)</td>
<td>3.44(-5)</td>
<td>1.03(-3)</td>
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</tr>
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<td>-</td>
<td>8.31(-5)</td>
<td>2.29(-3)</td>
</tr>
<tr>
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<td>9.64(-6)</td>
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<th>SNII(^{60})</th>
<th>SNII(^{10})</th>
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<th>SNIIa</th>
<th>SNIIb/c</th>
<th>Total(^{10})</th>
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</thead>
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</tr>
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<td>9.79(-3)</td>
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</tr>
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<td>4.26(-3)</td>
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<td>5.33(-6)</td>
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See Table 3.11 for the meaning of symbols used.

- the net yields of SNII for the GN and WW data sets differ considerably due to e.g. the in- and exclusion of pre-SN mass loss in these models (cf. Sect. 3.4). In particular, large deviations (i.e. more than factors 2–3) are present for elements like N, Al, Mg, and Fe, especially at low metallicities. Net iron yields strongly depend on initial metallicity in the WW models, in contrast to the Geneva/Nomoto models. Net oxygen yields for the two sets, however, agree remarkably well;
- variations in the net yields with initial metallicity are generally less than \( \sim 50\% \) when going from \( Z = 0.02 \) to 0.001;
- the net yields computed with the Scalo IMF are generally a factor 2–3 larger for AGB stars and SNIa than those computed using the Salpeter IMF. For SNII and SNIIb/c progenitors, the net yields are relatively insensitive to the adopted IMF.

In principle, the net yields included in Tables 3.11 and 3.12 can be used: 1) to estimate the magnitude of the element contributions by AGB stars, SNIa, SNIIb/c, and SNII (by means of the IMF, the adopted mass boundaries, and/or the formation probabilities \( f_i \)), 2) to compare the net yields for the GN and WW models, and 3) to predict the dependence of the net yields on initial metallicity. In addition, many galactic chemical evolution models predict simple relations between the resulting ISM abundance and the net yield of a given element (see below). For instance, in the instantaneous recycling approximation (IRA) one can show that for two elements \( k \) and \( l \) (cf. Maeder 1992):

\[
\frac{dZ_k}{dZ_l} = \frac{Y_k}{Y_l}
\]

so that before performing detailed model calculations, observational constraints such as the helium-to-metal enrichment \( dY/dZ \sim 4–5 \pm 1 \) (e.g. Pagel 1992) immediately imply that: 1) the contribution by AGB stars
Table 3.13 Comparison of total net yields $Y_j$ at $Z = 0.02$ and 0.001 for a Salpeter IMF

<table>
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<th>GN Total$^{30,*}$</th>
<th>Total$^{10}$</th>
<th>Total$^{60}$</th>
<th>Total$^{10,*}$</th>
<th>WW Total$^{10}$</th>
<th>Total$^{60}$</th>
<th>Maeder</th>
<th>A</th>
<th>C</th>
</tr>
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<tr>
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<td>1.50(-3)</td>
<td>1.59(-3)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<th>Total$^{10}$</th>
<th>Total$^{60}$</th>
<th>Total$^{10,*}$</th>
<th>WW Total$^{10}$</th>
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<tr>
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<td>3.07(-2)</td>
<td>3.66(-2)</td>
<td>2.51(-2)</td>
<td>2.70(-2)</td>
<td>3.42(-2)</td>
<td>3.56(-2)</td>
<td>2.72(-2)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2.57(-3)</td>
<td>3.12(-3)</td>
<td>4.36(-3)</td>
<td>1.88(-3)</td>
<td>2.65(-3)</td>
<td>3.68(-3)</td>
<td>2.19(-3)</td>
<td>9.94(-4)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>7.65(-4)</td>
<td>8.33(-4)</td>
<td>8.46(-4)</td>
<td>6.87(-4)</td>
<td>7.77(-4)</td>
<td>8.00(-4)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>O</td>
<td>5.35(-3)</td>
<td>4.46(-3)</td>
<td>1.92(-2)</td>
<td>5.29(-3)</td>
<td>4.41(-3)</td>
<td>6.62(-3)</td>
<td>2.13(-2)</td>
<td>8.67(-3)</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>1.28(-2)</td>
<td>1.27(-2)</td>
<td>2.18(-2)</td>
<td>1.06(-2)</td>
<td>1.12(-2)</td>
<td>1.50(-2)</td>
<td>3.24(-2)</td>
<td>7.66(-3)</td>
<td></td>
</tr>
</tbody>
</table>

See Table 3.11 for the meaning of GN and WW.
Maeder: refers to the A and C models from Maeder (1992, 1993)

$^{30,*}$: indicates that an upper mass limit for SNII (and SNIb/c) of $m_{\text{SNII}}^{\text{upper}} = 30$ M$_\odot$ has been used
* for this column the contributions from SNIa and SNIb/c have been excluded (see text)

to the He enrichment of the Galactic disk ISM is much larger than predicted in the IRA (i.e. the IRA is not valid for AGB stars), and 2) the average past SFR must have been substantially larger than at present (see below).

In Tables 3.13 and 3.14 we compare the total net yields of H, He, C, N, O, and Z, with those from Maeder (1992 & 1993). This is done both for the Salpeter and Scalo IMF, and for stars born with initial metallicities $Z = 0.001$ and 0.02.

We list the total net yields both for the GN and WW data sets in three distinct cases: 1) without contributions from SNIa and SNIb/c: ($\phi^{\text{SNIa}}, \phi^{\text{SNIb/c}}, m_{\text{SNII}}^{\text{upper}} = (0, 0, 30$ M$_\odot$), 2) with SNIa and SNIb/c contributions (0.001, 0.33, 30 M$_\odot$) included as in Tables 3.11 and 3.12, and 3) as for 2) but with $m_{\text{SNII}}^{\text{upper}} = 60$ M$_\odot$. Net yields from Maeder have been tabulated for his case A, providing an upper limit to $Y_j$ because all onion skin layers surrounding the remnant were assumed to be ejected, and for his case C for which an upper mass limit for SNIII of 25 M$_\odot$ was assumed (cf. Maeder 1992, 1993).

In his original paper, Maeder (1992) computed net yields according to the Salpeter IMF while ignoring the consumption of gas by low-mass stars with $m \lesssim 1$ M$_\odot$ (e.g. he used a returned fraction $R \sim 0.8$). However, the corrected net yields assuming a Salpeter IMF with $m_1 = 0.1$ M$_\odot$ and the correct value of $R \sim 0.29$ were never published (cf. Maeder 1993). It can be shown that this correction results in a reduction of the net yields included in Table 7 from Maeder (1992) by roughly a factor seven (cf. Eq. 3.21). The corrected net yields from Maeder (1992) are included in Table 3.13. In his erratum, Maeder (1993) revised his net yields for the Scalo IMF using a returned fraction $R \sim 0.46$ at $Z = 0.02$ (both for case A and C). At metallicities $Z = 0.001$, Maeder used somewhat lower values of $R = 0.45$ (case A) and 0.41 (case C). The net yields from Maeder (1993) for the Scalo IMF have been directly included in Table 3.14.

The main differences between the two sets of stellar yields discussed above and the one from Maeder (1992, 1993) are the detailed inclusion of the metallicity dependent yields and remnant masses of AGB stars (i.e. the models from the Geneva group do not extend beyond the EAGB for stars with $m \lesssim 8$ M$_\odot$). Furthermore, the set of SNI yields is somewhat different from that presented by Maeder (1992).

In general, differences in the net yields due to the differences in the adopted data for stellar remnant masses are small. For our models, we find $R \sim 0.4$ and 0.29 for the Scalo and Salpeter IMF, respectively, for stars born with $Z = 0.02$. These values are somewhat lower than the corresponding values of $R \sim 0.46$ and 0.35 listed by Maeder (1993; case A). Furthermore, we adopted an an upper stellar mass limit $m_u = 60$ M$_\odot$ instead of $m_u = 120$ M$_\odot$ as used by Maeder. In general, the combined differences in $R$ and $m_u$ do not attribute to variations larger than $\sim 10\%$ in the net yields.
3.4 Overall comparison of the yields of intermediate and massive stars

Table 3.14 Comparison of total net yields $Y_j$ at $Z = 0.02$ and 0.001 for a Scalo IMF

<table>
<thead>
<tr>
<th>Element</th>
<th>$Z = 0.02$</th>
<th>$Z = 0.001$</th>
<th>( \text{GN} )</th>
<th>( \text{WW} )</th>
<th>( \text{Maeder} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total(^{30,\ast})</td>
<td>Total(^{30})</td>
<td>Total(^{60})</td>
<td>Total(^{30,\ast})</td>
<td>Total(^{30})</td>
</tr>
<tr>
<td>H</td>
<td>-4.22(-2)</td>
<td>-4.41(-2)</td>
<td>-5.48(-2)</td>
<td>-3.31(-2)</td>
<td>-3.80(-2)</td>
</tr>
<tr>
<td>He</td>
<td>3.21(-2)</td>
<td>3.21(-2)</td>
<td>3.71(-2)</td>
<td>2.31(-2)</td>
<td>2.60(-2)</td>
</tr>
<tr>
<td>C</td>
<td>3.03(-3)</td>
<td>3.63(-3)</td>
<td>5.80(-3)</td>
<td>2.10(-3)</td>
<td>3.00(-3)</td>
</tr>
<tr>
<td>N</td>
<td>2.27(-3)</td>
<td>2.29(-3)</td>
<td>2.38(-3)</td>
<td>2.39(-3)</td>
<td>2.37(-3)</td>
</tr>
<tr>
<td>O</td>
<td>4.52(-3)</td>
<td>3.80(-3)</td>
<td>5.47(-3)</td>
<td>2.31(-2)</td>
<td>2.60(-2)</td>
</tr>
<tr>
<td>Z</td>
<td>1.12(-2)</td>
<td>1.21(-2)</td>
<td>1.65(-2)</td>
<td>1.07(-2)</td>
<td>1.17(-2)</td>
</tr>
</tbody>
</table>

Notes: see Table 3.13

Comparison of the total net yields for the GN data set with those of Maeder (case A) reveals that both the net helium and carbon yields at $Z = 0.02$ are considerably smaller (up to $\sim 80\%$) than the values given by Maeder. This is due to the fact that the element contributions by AGB stars were not incorporated by the Geneva group. At $Z = 0.001$ the differences are much smaller for helium while the net yields of carbon are considerably larger than those presented by Maeder. This is due to the relatively high carbon yields of AGB stars at low metallicities (cf. Sect. 3.3). Net yields of oxygen and Z are smaller than the corresponding yields of Maeder, typically by factors $\sim 2$. At $Z = 0.001$, the difference in $m_u$ becomes noticeable because Maeder has included stars with $m > \sim 60$ M\(_\odot\) with relatively high oxygen yields at low initial metallicities.

For the elements included in Tables 3.13 and 3.14, the net yields increase considerably (up to a factor 2) when $m_u^{\text{SNII}}$ is increased from 30 to 60 M\(_\odot\). However, we find that this effect is in general small compared to the difference between $m_u^{\text{SNII}} = 25$ and 120 M\(_\odot\) for the C and A models from Maeder, respectively. In general, the inclusion of SNIb/c has only limited effect on the resulting net yields except for carbon. Finally, the net yields for the WW data set are $\lesssim 30\%$ smaller than those for the GN set (both at $Z = 0.02$ and 0.001).

3.4.5 Concluding remarks

In principle, the effective net yield $Y_{j}^{\text{eff}}$ of element $j$ for the current generation of evolved stars can be theoretically related to its ISM abundance $Z_j$. For instance, the following relations can be derived if one considers (A) a closed box model assuming instantaneous recycling or, (B) a similar model but with gas infall (zero metallicity) exactly balancing gas consumption by star formation (e.g. Tinsley 1980; Marshall 1982):

$$Y_{j}^{\text{eff}} = \left\{ \begin{array}{ll} \frac{Z_j}{\ln \left( \frac{1}{\mu} \right)} & \text{• closed box model} \\ \frac{Z_j}{1 - \exp \left( 1 - \frac{1}{\mu} \right)} & \text{• gas infall balancing SFR} \end{array} \right.$$  \hspace{1cm} (3.27)

where $Z_j$ is the present-day ISM abundance by mass of element $j$ and $\mu$ the present-day gas-to-total mass-ratio in the Galactic disk. Assuming $\mu = 0.1$ (see Sect. 3.1) and a current mean oxygen abundance in the local ISM of $Z_\odot^{\text{ISM}} \sim 0.6 Z_\odot^\odot \approx 0.0055$ ($Z_\odot \sim 0.009$; Grevesse & Noels 1993; see also Chapter 5) one finds: $Y_{O}^{\text{eff}} = 0.0055 \pm 0.0025$. This value is in good agreement with the net oxygen yield given in Tables 3.13 and 3.14 (at $Z = Z_\odot$).
As discussed above, another way to compare the net stellar yields with the observations is by means of their ratios (cf. Eq. (3.25)). In Table 3.15 we list the net yield ratios $\Delta\text{He}/\Delta\text{Z}$, $\Delta\text{He}/\Delta\text{O}$, $\Delta\text{O}/\Delta\text{Fe}$, $\Delta\text{Fe}/\Delta\text{Z}$, and $\Delta\text{O}/\Delta\text{Z}$, respectively, which correspond to the net yields given in Tables 3.11 and 3.12. These theoretical yield ratios can be compared directly with the corresponding present-day abundance ratios observed in the ISM and can provide constraints on the stellar input data, initial stellar abundances, and IMF used. A more detailed discussion of the net yield ratios in Table 3.15 is postponed to Sect. 4.3, but here we like to emphasize the following points: 1) the effect of initial metallicity on the yield ratios is usually much stronger than that of varying the IMF, 2) large differences exist between the GN and WW models, 3) AGB stars are very important in maintaining the $\Delta\text{He}/\Delta\text{Z}$ abundance ratio in the ISM, 4) SNIb/c determine the to a large extent the $\Delta\text{He}/\Delta\text{O}$ ratio, 5) SNIa and SNIb/c are important in reducing the mean $\Delta\text{O}/\Delta\text{Fe}$ ratio, and 6) SNII alone predict $\Delta\text{O}/\Delta\text{Z}$ abundance ratios of $\sim 0.55 \pm 0.1$ so that values considerably less require a substantial contribution by SNIa and SNIb/c (see Sect. 4.2).

<table>
<thead>
<tr>
<th>$\Delta\text{He}/\Delta\text{Z}$</th>
<th>IMF</th>
<th>AGB</th>
<th>SNII$^{30}$</th>
<th>SNII$^{60}$</th>
<th>SNIIP$^{30}$</th>
<th>SNIIP$^{60}$</th>
<th>SNIa</th>
<th>SNIb/c</th>
<th>Total$^{30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>Salp.</td>
<td>5.8</td>
<td>1.8</td>
<td>1.5</td>
<td>1.0</td>
<td>1.1</td>
<td>–</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>6.8</td>
<td>1.7</td>
<td>1.1</td>
<td>1.6</td>
<td>1.7</td>
<td>–</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>6.7</td>
<td>2.0</td>
<td>1.3</td>
<td>1.7</td>
<td>1.8</td>
<td>–</td>
<td>2.3</td>
<td>2.9</td>
</tr>
<tr>
<td>$\Delta\text{He}/\Delta\text{O}$</td>
<td>0.02</td>
<td>Salp.</td>
<td>–</td>
<td>3.5</td>
<td>3.2</td>
<td>1.8</td>
<td>1.9</td>
<td>–</td>
<td>8.4</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>3.6</td>
<td>1.9</td>
<td>2.7</td>
<td>2.7</td>
<td>–</td>
<td>8.0</td>
<td>8.9</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>4.2</td>
<td>3.6</td>
<td>2.1</td>
<td>2.1</td>
<td>–</td>
<td>9.0</td>
<td>9.4</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>4.4</td>
<td>2.4</td>
<td>3.0</td>
<td>2.9</td>
<td>–</td>
<td>8.5</td>
<td>9.8</td>
</tr>
<tr>
<td>$\Delta\text{O}/\Delta\text{Fe}$</td>
<td>0.02</td>
<td>Salp.</td>
<td>–</td>
<td>7.9</td>
<td>8.9</td>
<td>30.1</td>
<td>35.1</td>
<td>0.2</td>
<td>1.8</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>8.7</td>
<td>16.6</td>
<td>31.5</td>
<td>440.6</td>
<td>0.2</td>
<td>2.2</td>
<td>4.0</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>8.9</td>
<td>8.5</td>
<td>26.7</td>
<td>31.2</td>
<td>0.2</td>
<td>1.7</td>
<td>2.9</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>7.9</td>
<td>14.4</td>
<td>284.2</td>
<td>387.4</td>
<td>0.2</td>
<td>2.0</td>
<td>3.1</td>
</tr>
<tr>
<td>$\Delta\text{Fe}/\Delta\text{Z}$</td>
<td>0.02</td>
<td>Salp.</td>
<td>–</td>
<td>0.07</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.54</td>
<td>0.14</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>0.05</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
<td>0.02</td>
<td>0.54</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>0.06</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>$\Delta\text{O}/\Delta\text{Z}$</td>
<td>0.02</td>
<td>Salp.</td>
<td>–</td>
<td>0.52</td>
<td>0.46</td>
<td>0.57</td>
<td>0.60</td>
<td>0.10</td>
<td>0.25</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>0.48</td>
<td>0.59</td>
<td>0.59</td>
<td>0.64</td>
<td>0.10</td>
<td>0.28</td>
<td>0.40</td>
</tr>
<tr>
<td>0.02</td>
<td>Salp.</td>
<td>–</td>
<td>0.51</td>
<td>0.47</td>
<td>0.56</td>
<td>0.59</td>
<td>0.10</td>
<td>0.24</td>
<td>0.31</td>
</tr>
<tr>
<td>0.001</td>
<td>Salp.</td>
<td>–</td>
<td>0.44</td>
<td>0.56</td>
<td>0.58</td>
<td>0.62</td>
<td>0.10</td>
<td>0.27</td>
<td>0.29</td>
</tr>
</tbody>
</table>

In the following, we will use the stellar yields of SNII based on the Geneva group and Hashimoto & Nomoto (1992) data. As discussed above, many uncertainties are still involved with these yields, in particular with the coupling of the different data sets, the details of stellar mass loss, the chemical evolution during the Wolf-Rayet stage, and the treatment of convection in the stellar interior (especially for stars more massive than $m \sim 30 \text{ M}_\odot$; see Maeder 1992; Woosley & Weaver 1995). Furthermore, the metallicity dependent yields of SNII and SNIb/c strongly depend on the core helium mass, the chemical structure of the mantle layers of the SN progenitor, and on the details prescribing how the shock wave proceeds through the stellar envelope (both inwards and outwards). These uncertainties also affect the stellar remnant masses and their dependence on initial metallicity and progenitor mass.

The yields of SNIa and SNIb/c are relatively uncertain due to unknown details of the evolution of the progenitor stars (in particular during their final evolution stages) and the explosion mechanism (see e.g. Smecker-Hane & Wyse 1992; Woosley et al. 1993). The yields of AGB stars are sensitive to the detailed description of hot bottom burning which is still a delicate problem in the chemical evolution of AGB stars. Other uncertainties involved with the yields of individual stars include the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate and may give an uncertainty of at least a factor of two in the carbon and oxygen yields of massive stars (e.g. Prantzos et al. 1994).

In the near future, improvements in the theoretical description of many of these quantities, as well as in the corrections for mass exchange in close-binary systems, will provide an even more detailed picture of the stellar enrichment of the Galactic ISM. For now, we expect that the above uncertainties are unimportant for the qualitative results obtained below and that the adopted set of stellar yields provides a reasonable input basis for the galactic chemical evolution models presented in the following chapters.
Appendices

A Approximate and exact solutions of the metallicity equation

The integro-differential equation for the evolution of the metallicity of an interstellar gas cloud can be written as:

\[
M_g \frac{dZ}{dt} = E_{\text{new}}(t) + E_{\text{old}}(t) + F_z(t) + T_z(t) - Z(t) \left[ E(t) + F(t) + T_g(t) \right]
\]

(A1)

where \(M_g\) is the gas content of the cloud, \(Z(t)\) the interstellar metal-abundance by mass, \(C\) the star formation rate, \(E_{\text{new}}\) and \(E_{\text{old}}\) the supply rates of old and newly returned metals, respectively, \(F\) and \(F_z\) the gas infall rate and infall rate of metals, respectively, and \(T_g\) and \(T_z\) the supply rates of gas and metals associated with stars moving into the galactic region under consideration.

In general, the \(E_{\text{new}}(t)\) terms in Eq. (A1) depend on the time retarded metallicity \(Z(t - \tau(m, Z_\ast))\) with \(\tau(m, Z_\ast)\) the metallicity dependent lifetime of a star of initial mass \(m\). For this reason, only a numerical solution of the metallicity equation is possible. However, if we consider the term \(E_{\text{new}}(t) - Z(t)E(t)\) in Eq. (A1) in more detail we get:

\[
E_{\text{old}}(t) - Z(t)E(t) = \int_{m_\ast(t)}^{m_\ast} \Delta m \, \left( Z(t - \tau(m)) - Z(t) \right) \, M(m) S(t - \tau(m, Z_\ast)) \, dm
\]

(A2)

where \(\Delta m\) is \((m - m_\ast(m, Z_\ast))\), and \(S(t)\) and \(M(m)\) denote the SFR and IMF, respectively, and if we assume that \(Z(t - \tau(m, Z_\ast)) \ll Z(t) \forall t\), we have \(E_{\text{old}}(t) \ll Z(t)E(t) \forall t\) with the following extremes:

If \(Z(t - \tau(m, Z_\ast)) \approx Z(t) \rightarrow E_{\text{old}}(t) - Z(t)E(t) \approx 0\)

If \(Z(t - \tau(m, Z_\ast)) \ll Z(t) \rightarrow E_{\text{old}}(t) - Z(t)E(t) \approx -E(t)Z(t)\)

In general, one can define: \(E_{\text{old}}(t) - Z(t)E(t) \equiv -G(t)Z(t)\) so that Eq. (A1) can be written as:

\[
\frac{dZ(t)}{dt} = \frac{1}{M_g(t)} \left[ -Z(t) \cdot (G(t) + F(t) + T_g(t)) + E_{\text{new}}(t) + F_z(t) + T_z(t) \right]
\]

(A3)

\[
= -Z(t)P(t) + Q(t)
\]

(A4)

where

\[
P(t) = \frac{1}{M_g(t)} \cdot \left[ G(t) + F(t) + T_g(t) \right]
\]

(A5)

\[
Q(t) = \frac{1}{M_g(t)} \cdot \left[ E_{\text{new}}(t) + F_z(t) + T_z(t) \right]
\]

(A6)

Hence, the general solution for the gas metallicity at instant \(t\) can be written as:

\[
Z(t) = e^{-\int P(\tau)d\tau} \cdot Z(t = 0) + \int_0^t \frac{Q(\tau)}{e^{\int P(\tau)d\tau}} d\tau
\]

(A7)

for which minimum and maximum solutions can be found using \(G(t) = E(t)\) and \(G(t) = 0\), respectively. Note that \(P(t)\) and \(Q(t)\) usually are functions of the metallicity \(Z(t - \tau(m, Z_\ast))\) in the past, by means of the metallicity dependent stellar lifetimes, remnant masses, and nucleosynthesis yields involved.
B Normalisation of the SFR

A useful relation is derived between the normalisation constants $C_0^A$ and $C_0^B$ of the SFR for two models A and B which differ with respect to their present-day gas-to-total mass-ratio $\mu_1$, galactic age $T$, initial galaxy mass $M_{\text{tot}}(0)$, infall fraction $\alpha_{\text{inf}}$, and/or stellar mass function $M(m,t)$ at birth. Such a relation allows beforehand scaling of the SFR normalisation constant for a particular SFR model without the need to solve the galactic chemical evolution equations. Such scaling is e.g. necessary when models appropriate for the Galactic disk are applied to other galaxies, such as the Magellanic Clouds, that differ from the Galactic disk in terms of their total mass, IMF, SFR, etc.

If we neglect any transport of stars and assume that infall of gas is proportional to the SFR, i.e. $F(t) = \alpha_{\text{inf}} C(t)$, the net amount of gas converted into stars at galactic evolution time $t = T$ can be written as:

$$M^*_{\text{net}} = (1 - \mu(T)) M_{\text{tot}}(T) = (1 - \mu(T)) [M_{\text{tot}}(0) + \alpha_{\text{inf}} T < C > T]$$

in which the average SFR is given by:

$$< C > T = \frac{1}{T} \int_0^T \int_{m_1}^{m_u} m M(m,t) S(t) \, dm \, dt$$

where $m_1$, $m_u$ denote the stellar mass limits at birth and $S(t)$ the SFR by number at evolution time $t$.

We define the average stellar returned fraction $< R > T$ in terms of the mean SFR as follows:

$$< R > T = \frac{1}{T < C > T} \int_0^T \int_{m_1}^{m_u} \Delta m M(m,t) S(t - \tau(m)) \, dm \, dt$$

where $m_u(t)$ is the turnoff mass at galactic age $t$ and $\Delta m$ is the total mass returned to the ISM by a star of initial mass $m$. Therefore, the net amount of gas converted into stars at age $t = T$ also can be written as:

$$M^*_{\text{net}} = T < C > T [1 - < R > T]$$

Equating Eq. (B1) and (B4) for two SFR models A and B results in the following relation between the corresponding SFR normalisation constants:

$$\frac{C_0^A}{C_0^B} = \frac{\bar{C}_B T_B \left(1 - \mu_A(T_A)\right) M_{\text{tot}}^A(0)}{\bar{C}_A T_A \left(1 - \mu_B(T_B)\right) M_{\text{tot}}^B(0)} \begin{cases} 1 - < R_B > T_B - \alpha_{\text{inf}}^B (1 - \mu_B(T_B)) \overline{< R_A > T_A - \alpha_{\text{inf}}^A (1 - \mu_A(T_A))} \end{cases}$$

where $\bar{C} \equiv < C > T / C_0$ is defined as the normalised SFR averaged over galactic evolution time $t = T$. Note that $< R > T$ is independent of the SFR normalisation $C_0$.

As an example, when we consider two SFR models with similar IMFs and star formation histories and further neglect infall, Eq. (B4) reduces to:

$$\frac{C_0^A}{C_0^B} = \frac{M_{\text{tot}}^A(0) T_B (1 - \mu_A(T_A))}{M_{\text{tot}}^B(0) T_A (1 - \mu_B(T_B))}$$
Modelling the chemical evolution of the Galactic disk: basics considerations, selected models, comparison with observations

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Abstract

We model a large set of observational data related to the chemical evolution of the Galactic disk and halo using a comprehensive and up-to-date galactic evolution model that incorporates metallicity dependent stellar yields, lifetimes, and remnant masses. An iterative solution procedure is applied to solve the galactic chemical evolution equations in a self-consistent manner with the freedom to study complex relations between e.g. the IMF and the SFR. We make a distinction between the enrichment contributions by AGB stars, SNIa, SNII/p, and SNII while using state-of-the-art evolution models for the chemical evolution of these final stages of stellar evolution.

First, we consider some basic concepts of Galactic chemical evolution. We address the abundance inhomogeneities observed among similarly aged stars and open clusters in the Galactic disk. We analyse in detail the possibility that stellar orbital diffusion in combination with radial abundance gradients in the disk ISM are the main explanation for these abundance inhomogeneities. We show that in case of large errors in the derived ages and orbital parameters of the stars in the Edvardsson et al. (1993) sample, orbital diffusion as described by Wielen et al. (1996) can provide an adequate explanation for the majority of the observed stellar abundance variations. At the same time, we argue that this requires several specific assumptions which may be unjustified. In order to interpret part of the observations, we discuss several basic issues concerning the dynamical and chemical evolution of the Galaxy. We do not attempt to model individual components of the Galaxy (e.g. bulge, halo, disk at different galactocentric radii) but restrict ourselves to the star formation history and chemical evolution of the Galaxy as a whole.

Second, we investigate the sensitivity of the age-metallicity relation (AMR) to specific model assumptions and we select a set of models that can explain the mean [Fe/H] vs. age relation observed in the local Galactic disk. We study the sensitivity of the AMR to the main parameters and assumptions involved in our models. We demonstrate that a wide range of enrichment scenarios is consistent with the observed AMR, i.e. no unique model exists which is in best agreement with the observed AMR. Conversely, the observed AMR alone is insufficient to constrain tightly Galactic chemical evolution models and additional constraints are needed.

Third, we confront the models selected on their ability to fit the observed AMR with observational constraints related to the ISM abundances and stellar content of the disk:

- the present-day stellar mass function (PDMF) and IMF;
- the total number and formation rates of (post) main-sequence stars;
- the gas depletion, infall, and star formation rates in the disk ISM;
- the enrichment history of the Galactic disk as recorded by the abundance-abundance variations (i.e. the variation of the abundance of a given element as a function of the abundance of another element) and the present-day abundances observed. We investigate the impact of: 1) the adopted stellar yields, 2) the star formation history, 3) the IMF, 4) the delay time of SNIa, and 5) the upper mass limit for SNII, on the resulting abundance-abundance variations of the most abundant elements in the disk ISM including C, N, O, Mg, Al, Si, and Fe.
- the abundances in planetary nebulae (PNe);
• the luminosity function of white dwarf (WD) remnants;
• the mass distribution of WD remnants;
• the age and metallicity distributions of long-living stars in the local disk (i.e. the classical G-dwarf problem).

By means of this comparison, we attempt to converge to a set of models for the chemical evolution of the Galaxy consistent with the above constraints and we trace back eventual discrepancies between our results and the observations.

In particular, we aim to deduce the star formation history of the Galaxy both from the abundance-abundance variations observed and other independent observational constraints to the chemical evolution of the Galaxy. As a shortlist of interesting results we like to emphasize the following ones: 1) we find that evolution scenarios in which the SFR gradually increases up to a given maximum in the disk and thereafter decreases exponentially are clearly favoured by the observations. We argue that models which incorporate infall of gas regulating this kind of behaviour of the SFR in the Galactic disk with age are preferred over models which do not incorporate gas infall; 2) we demonstrate that the ejecta of SNIa, associated with stars formed early in the evolution of the Galaxy and with initial masses in the range $\sim 2.5 - 8 \, M_\odot$, need to be delayed over at least $3-5$ Gyr after the formation of their WD progenitors in order to fit the observations. Instead of such a time delay, SNIa may be associated with considerably less massive stars than previously thought, i.e. with masses between $\sim 1.5$ and $2 \, M_\odot$; and 3) we show that models in which the upper mass limit of SNIa increases as a function of galactic age during early epochs of star formation in the Galaxy are consistent with the observations for variations of $m_u$ between $\sim 20$ and $\sim 30-40 \, M_\odot$ if these variations did occur delayed with respect to the variation of the SFR. Such a behaviour of the upper mass limit of SNIa may be supported by the formation of massive stars both in the Galactic disk and in external galaxies (see Chap. 2).

Fourth, we briefly compare our main results with those presented in several other recent investigations dealing with Galactic chemical evolution. We summarize the type of chemical evolution models that are in best overall agreement with the observations and we discuss what this may imply for the chemical evolution of the Galaxy as a whole.

Combined with the detailed description in Chap. 3 of the galactic chemical evolution model assumptions and ingredients involved, the extensive results for a wide range of observations presented here make that our model is one of the best documented Galactic chemical evolution models currently available.

### Introduction

The ultimate goal of modelling various observational characteristics of the chemical evolution of the local disk ISM is to obtain information about the principal processes that have directed the star formation history and chemical enrichment of the Galactic disk. In the past decade, many of such individual constraints have been modelled by numerous authors independently, using distinct galactic chemical evolution models. The present study of the chemical evolution of the Galactic disk has several important improvements over previous investigations which include: 1) simultaneous modelling of various independent observational constraints (e.g. the luminosity functions of asymptotic giant branch (AGB) stars and white dwarfs (WD), the age and metallicity distributions of main-sequence dwarfs, the abundances in planetary nebulae, the remnant mass distribution, abundance-abundance variations, and the present-day element abundances in the disk) using one and the same galactic chemical evolution model, 2) the application of a uniform, up-to-date, and comprehensive metallicity dependent set of stellar evolution data, and 3) the use of a wide range of observational constraints which were not available until recently. While exploiting these improvements, both in theoretical and observational fields of Galactic evolution, we aim to reconstruct the star formation history of the Galactic disk and to derive essential quantities such as the current gas infall and star formation rate, the typical enrichment time scale, and the upper mass limit for SNIa. Apart from studies that concern the evolution of our Galaxy, knowledge of these quantities are of particular interest for research on the evolution of nearby systems such as the Magellanic Clouds and M31. In turn, these Local Group galaxies provide an important reference frame of observations to which the evolution of more distant galaxies in the universe can be compared.

In this chapter, we concentrate on the star formation history and chemical evolution of the local Galactic disk. In Sect. 4.1, we consider several basic issues concerning the dynamical and chemical evolution of the Galaxy as a whole. In Sect. 4.2, we investigate the sensitivity of the age-metallicity relation (AMR) to specific model assumptions and we select a set of models which can explain adequately the mean [Fe/H] vs. age relation observed in the local Galactic disk. In Sect. 4.3, the selected models are confronted with
observational constraints related to the ISM abundances and stellar content of the disk (e.g. the age and metallicity distributions of F and G dwarfs, and the abundances in planetary nebulae). In Sect. 4.4, we summarize the type of chemical evolution models that are in best overall agreement with the observations and we discuss what our results may imply for the chemical evolution of the Galaxy as a whole.

4.1 Basic considerations

In this section, we describe the selection criteria for a sample of stars suited to study the chemical evolution of the Galactic disk and we briefly discuss some observational samples that are currently available for this purpose. In particular, we address possible signatures of the effects of inhomogeneous chemical evolution of the Galactic disk and stellar orbital diffusion in combination with radial abundance gradients in the disk ISM. In addition, we discuss the evolution of the vertical structure of the stellar disk in the SNBH and the scale height corrections that we will adopt. First, we briefly recall the conventional abundance notation and calibration used here.

4.1.1 Notation and calibration of abundances

The abundance of a given element relative to that of hydrogen is usually expressed in terms of the corresponding abundance ratio in the Sun:

$$[\text{El/H}] = 10 \log (\text{El/H})_{\text{obj}} - 10 \log (\text{El/H})_\odot$$

where $(\text{El/H})_{\text{obj}}$ and $(\text{El/H})_\odot$ are the abundance ratios in the object and the Sun, respectively. Unless stated otherwise, we will refer to the abundances by mass so that $(\text{El/H})_{\text{obj}}$ denotes the total mass of element El relative to the total mass in hydrogen for the same object. A similar expression is used for the abundance ratio of any two elements P and Q.

For the solar element abundances, we will use the data presented by Anders & Grevesse (1989, 1991; hereafter AG). For instance, solar iron and oxygen abundance ratios by mass are adopted as $^{10} \log ( \text{Fe/H} )_\odot = -2.65 \pm 0.05 \text{ dex}$ and $^{10} \log ( \text{O/H} )_\odot = -1.87 \pm 0.05 \text{ dex}$, respectively (see AG$^1$). Due to the inclusion of non-LTE effects and the use of more up-to-date transition probabilities, the above value for the iron abundance in the Sun differs considerably from that given by e.g. Cameron (1982), i.e. $^{10} \log ( \text{Fe/H} )_\odot \sim -2.72$, which has been used in many previous investigations. When different sets of abundance data are compared, it should be verified that the solar abundance of a given element used is the same for all data sets. As an example, we note that the iron abundance data of Twarog (1980; described below) is calibrated using the standard $\delta m_3 (H\beta) - [\text{Fe/H}]$ relation from Crawford and Perry (1976) which implicitly is based on a solar iron abundance similar to that given by Cameron. Consequently, direct comparison with the Edvardsson et al. (1993) data, which is calibrated to the AG solar abundances, is allowed only after setting a common reference point for $[\text{Fe/H}]$. Both the observational and theoretical data discussed below will be calibrated using the accurate data presented by AG (when possible).

We recall several additional uncertainties involved with the comparison between observational and theoretical abundances:

- the observed abundance ratio $[\text{El/H}]$ is sensitive to the total (i.e. atomic+molecular+ionized) hydrogen content of the object of interest. This may introduce abundance errors if a substantial part of the hydrogen is "missed". In general, we expect that the total hydrogen abundances predicted by models for the Galactic disk are consistent with the observations within a few percent, at least during the last $\sim 5 \text{ Gyr}$. We note that the effect of a large discrepancy of $\sim 10\%$ between the predicted and observed total hydrogen abundances in the local disk ISM would be limited to a shift of $\sim 0.05 \text{ dex}$ in $[\text{El/H}]$. Substantial errors can be introduced if part of the mass contained in element El is undetected, e.g. because a considerable fraction of this element may be highly ionized, molecular, and/or contained in solid dust grains. We expect that such errors are important when physical properties —such as temperature, density, radiation field— vary strongly among the objects studied (e.g. ISM regions, stars of different spectral type, etc);

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$^1$The hydrogen density in the Sun (by number) is usually fixed to $^{10} \log N_H = 12$ to provide a convenient and uniform abundance scale for other elements. Element abundances by number relative to hydrogen are usually given as $A_{el} = ^{10} \log ( N_{el} / N_H ) + 12$. (solar abundance scale). Conversion of the astronomical/meteoritic abundance scale ($N_{el} = 10^6$) to the solar abundance scale ($\log N_H = 12$) involves a correction factor $R \sim 35.8$ with which the meteoritic abundances of all elements should be multiplied.

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• in many previous investigations, the theoretical $[\text{Fe}/\text{H}]$ abundance ratio was computed using a conversion between $[\text{Fe}/\text{H}]$ and $[Z/Z_{\odot}]$, such as $[\text{Fe}/\text{H}] \approx [Z/Z_{\odot}] + 0.07$ dex (e.g. Twarog 1980; Tosi 1988; Rocca-Volmerange & Schaeffer 1990). We stress that such abundance conversion relations (because of a lack of appropriate observational data) may introduce considerable systematic and stochastic errors. Careful analysis is needed to disentangle the artefacts caused by such errors and the real trends present in the data. As discussed in Chap. 3, we compute the abundances of elements $M$ and $N$ individually when determining $[M/N]$;

• if iron is produced in substantial amounts by the thermonuclear explosion of intermediate mass stars (as is probably the case; see Chap. 3), the overall chemical evolution of the Galactic disk is not well represented by $[\text{Fe}/\text{H}]$. Instead, it is expected that $[\text{O}/\text{H}]$ is a much more reliable indicator of metallicity since the bulk synthesis of oxygen comes from massive stars ($m > \sim 10 M_{\odot}$; cf. Wheeler, Sneden & Truran 1989).

• in the models discussed below, we assume that the ISM is enriched homogeneously. However, if a substantial fraction of the ISM would be unavailable for star formation and stellar enrichment, the remaining part of the ISM would be enriched much more efficiently than predicted by our models. Similarly, selective enrichment of preferred regions in the Galactic ISM over substantial fractions of the lifetime of the disk (e.g. galactic winds, chimney material) would slow down the enrichment of the global disk ISM as compared to our results.

4.1.2 Sample selection criteria

An ideal sample of stars suited to study the chemical evolution of the local disk has to meet at least two conditions: 1) the sample must be unbiased and representative for the stars born within the region of interest over the lifetime of the Galactic disk, 2) the sample needs to be complete within a certain volume (e.g. the solar cylinder with a radius of 50 pc). The first condition implies that the sample stars are representative with respect to e.g. age and metallicity. Furthermore, this condition implies that the sample must be homogeneous, i.e. not contaminated by stars formed outside the region, and is unbiased towards stars that are atypical for the stellar generations formed within the volume of interest. The second condition requires e.g. that corrections should be made for stars which: 1) have a relatively low detection probability within the volume considered (e.g. low-luminosity stars), 2) did move out of the volume during their lifetime, and/or 3) did evolve to other evolutionary phases (e.g. white dwarf remnants) and, therefore, are missed according to the usual selection criteria for stars of a given spectral type.

It has been noted previously that stars nowadays observed in the solar neighbourhood (SNBH) are not exclusively related to the chemical evolution of the local disk ISM but rather are associated with a much more extended region of the Galactic disk from which these stars evolved and recently moved to the SNBH due to their galactocentric orbits (e.g. Grenon 1989; Wielens, Fuchs & Dettbarn 1996, hereafter WIEL). As a consequence, nearby stars do trace the evolution of the Galactic ISM over a much wider range in galactocentric distance than they are observed. Other aspects of the orbits of stars is that stars born long ago in the SNBH may have travelled to regions elsewhere in the Galaxy and that even the Sun itself may have formed at a galactocentric distance much different from its present-day position in the disk. Therefore, detailed information is required about the present-day and past orbits of the sample stars (e.g. positions and space velocities), to fulfil the selection criteria of a stellar sample suited for Galactic evolution studies. In addition to the effects of orbital diffusion of stars born in the Galactic disk, the stellar populations observed in the SNBH may be contaminated by e.g. stars which formed long ago in the Galactic halo or which formed in galactic systems that merged with the Galaxy in the past. If this is the case, a useful analysis of the stars observed in the SNBH can be made only when: 1) it is possible to trace back the star formation history and chemical evolution of each of the Galactic components which are involved separately, and 2) to correct for the dynamical evolution of the stellar populations associated with each component. However, if the contamination is small and/or the enrichment history of the contributing subsystems is similar to that of the stellar populations in the disk, a sample of stars observed in the SNBH may be suited to constrain the evolution of the local Galactic disk. In the following, we will ignore any possible contamination by other galactic stellar components (unless stated otherwise). In particular, we will assume that the star formation history and chemical evolution as deduced from a sample of stars in the SNBH applies to the evolution of the Galactic disk.
4.1 Basic considerations

4.1.3 Inhomogeneous chemical evolution of the Galactic disk

Element abundances of long-lived stars (e.g. F and G dwarfs) provide a record of the nucleosynthesis history of the Galaxy since the onset of star formation therein (e.g. Twarog 1980; Carlberg et al. 1985; Meusinger, Reimann & Stockhum 1991; Sommer-Larsen 1991; Pagel 1992; Edvardsson et al. 1993). Accordingly, abundance differences among such stars born at the same galactic age may trace spatial abundance variations throughout the Galaxy revealing differential rates of enrichment in distinct ISM regions while stars born at the same time and same place provide information about local ISM inhomogeneities and the efficiency of mixing (and/or infall) of interstellar gas.

Studies related to the heavy element enrichment of the local Galactic disk have long shown that stars similar in age exhibit large abundance variations (e.g. Mayor 1976; Twarog 1980; Carlberg et al. 1985; Meusinger et al. 1991; see Chap. 5). Recently, Edvardsson et al. (1993) presented accurate abundance data for nearly 200 F and G main-sequence dwarfs in the SNBH. Their spectroscopic data, analysed with up-to-date input physics, confirms abundance variations as large as ~0.6 dex in $\Delta[\text{El}/\text{H}]$ (where El = Fe, O, Mg, Al, Si) among similarly aged stars (see Fig. 4.1). Such variations are much in excess of experimental uncertainties and demonstrate that the abundance spread among stars observed in the SNBH is similar in magnitude to the overall increase in metallicity during the lifetime of the disk.

![Figure 4.1 Observational data on the chemical evolution of the Galactic disk: $[\text{Fe}/\text{H}]$ vs. age. Left panel: F and G main-sequence dwarfs observed in the SNBH (data from Edvardsson et al. 1993). Bars indicate the maximum range in $[\text{Fe}/\text{H}]$ observed in age bins of ~1.5 Gyr width, asterisks indicate the mean $[\text{Fe}/\text{H}]$ ratio in each age bin. Right panel: open disk clusters (triangles: uncorrected for radial metallicity gradient, data from Boesgaard 1989; Friel & Boesgaard 1990; full circles: data corrected for radial metallicity gradient of $-0.1$ dex kpc$^{-1}$ in $[\text{Fe}/\text{H}]$ from Carraro & Chiosi 1994).](image)

Additional support for the existence of large abundance inhomogeneities in the Galactic disk has been provided by studies of stars in open clusters (e.g. Nissen 1988; Boesgaard 1989; Lambert 1989; García-López et al. 1993; Friel & Janes 1993: Carraro & Chiosi 1994) and B stars in star forming regions in the SNBH (e.g. Gies & Lambert 1992; Cunha & Lambert 1992). In Fig. 4.1 we compare the abundance data for two samples of open clusters in the Galactic disk: data mainly based on Friel & Janes (1993) and data from Carraro & Chiosi (1994). The main difference between the two data sets is that the latter data set has been corrected for a radial gradient in $[\text{Fe}/\text{H}]$ of $-0.1$ dex kpc$^{-1}$ when moving outwards in the Galactic disk, according to the galactocentric distances at birth estimated for the sample clusters (see Carraro & Chiosi 1994). For both samples, the abundance variations among similarly aged clusters is large, i.e. $\geq 0.5$ dex in $[\text{Fe}/\text{H}]$. It is evident that the scatter in $[\text{Fe}/\text{H}]$ at a given age is much larger than any possible trend of $[\text{Fe}/\text{H}]$ with galactic age for open clusters (Nissen, 1988).

Corrections for stellar orbital diffusion do not seem to affect the large abundance inhomogeneities observed. However, direct comparison of the two data sets, as well as comparison of the open cluster data with the abundances of F and G dwarfs in the SNBH, is hindered by the lack of an accurate and adequate method to determine the galactocentric distances of stars at birth from their present-day orbits (see below). Therefore, the abundance data corrected for orbital diffusion from Carraro & Chiosi still may suffer from considerable systematic errors.
In any case, these studies show that the concept of a well-defined tight age-metallicity relation for the Galactic disk ISM is unfounded (Edmunds 1993) and that the chemical enrichment of the disk has been inhomogeneous on time scales as short as $\lesssim 10^8 - 10^9$ yr. Similar conclusions were obtained by Merchant-Boesgaard (1989) from high-quality spectroscopic data on selected F dwarfs in six young disk clusters.

In Chap. 5, we will deal with the origin of the abundance variations observed among similarly aged objects in the Galactic disk. For the moment, we like to emphasize that the AMR of stars observed in the SNBH is determined by various processes associated with: 1) the star formation history of the Galactic disk (e.g. radial, longitudinal, and/or vertical variations of the SFR), 2) the occurrence of small-scale abundance variations in the ISM, 3) the individual evolution of specific stars, and 4) the birthplaces (e.g. halo, bulge, outer disk) and epicyclic orbits of stars in the Galaxy. Each of these processes may lead to considerable variations in the abundances of stars nowadays observed in the SNBH. Therefore, to interpret the observed abundance variations in a correct manner, detailed knowledge about the dynamical evolution (and mixing history) of the Galaxy and its constituent components is required.

### 4.1.4 Stellar orbital diffusion

**Evolution of stellar orbits in the Galactic gravitational potential**

Kinematic properties (e.g. total orbital energy and angular momentum) of stars nowadays observed in the SNBH are determined by the corresponding properties of the gas clouds from which these stars formed as well as by the subsequent dynamical evolution of the stellar orbits in the gravitational potential of the Galaxy (e.g. Gilmore et al. 1989). Therefore, for a given sample of stars in the SNBH, accurate stellar ages as well as abundances, positions, and kinematical properties at time of birth, are needed to deduce the star formation history and chemical evolution of the regions in the Galaxy where these stars formed.

Direct evidence for the diffusion of stellar orbits is provided by observations which show that the velocity dispersion of stars in the SNBH increases with stellar age (e.g. Wielen 1977; Wielen et al. 1992). The observed mean radial velocity dispersion of stars in the SNBH is $\sim 55$ km s$^{-1}$. This corresponds to a spatial dispersion of $\sim 3-4$ kpc in 10 Gyr for stars formed at a galactocentric distance of about 8.5 kpc. An immediate consequence is that, the mixture of stars at a given galactocentric distance becomes more and more contaminated by stars from distant regions in the Galactic disk with age.

In first approximation, the mean galactocentric distance $R_m$ of a stellar orbit may be unchanged by the diffusion process since the exponential radial density profile of the Galactic disk seems hardly affected by orbital diffusion (Fuchs et al. 1994). However, this approximation neglects local variations in the Galactic potential which may have systematic effects on the variation of $R_m$ with galactic age as we will discuss below.

With the use of a kinematical model for the formation and evolution of the Galaxy (e.g. disk, halo, and bulge), one can distinguish between stars which formed in the local disk and stars that probably did not (e.g. Grenon 1989; Sommer-Larsen 1991). This can be done on the basis of the present-day positions and orbits of stars in the Galactic disk which are then integrated back in time according to the kinematical evolution model. However, this method is sensitive to uncertain assumptions about the detailed evolution of the gravitational potential in the Galaxy, both as a function of location and time. Apart from this, kinematical decompositions with respect to stellar birthplaces often result in subsets of stars that are too small for use in statistical studies of the chemical evolution of the local disk (e.g. Edvardsson et al. 1993; hereafter EDV).

By using a chemo-dynamical model for the Galactic disk, it is possible to derive approximate positions of stars at birth on the basis of their ages and abundances. In this case, radial abundance gradients observed in the Galactic disk are used to estimate the stellar galactocentric distances at birth. This is done by transforming the difference between the metallicity of the star and the mean metallicity predicted by the chemo-dynamical model at the galactocentric distance this star is observed into a distance scale (WIEL). Apart from the uncertainties involved in chemo-dynamical evolution models, use of this method may be further complicated by abundance inhomogeneities which may occur on small scales in the local disk ISM (see above; Chap. 5). Furthermore, this method is relatively sensitive to errors in the observed ages and metallicities of the sample stars and heavily depends on the radial abundance gradients in the disk assumed. In the following, we will compare the results obtained from the chemo-dynamical and kinematical methods in more detail and discuss the main assumptions involved. Such a comparison is important to understand what the abundances of stars in the SNBH can tell us about the chemical evolution of the Galactic disk.

**Comparison of EDV and WIEL galactocentric distances at birth**

In Fig. 4.2 we compare the galactocentric distances at birth presented by EDV and WIEL for a subsample of F and G dwarfs for which accurate iron and oxygen abundances are available.
4.1 Basic considerations

The **EDV galactocentric distances at birth** were obtained using the stellar orbits reconstructed from their present-day galactocentric distances, proper motions, and radial velocities, according to both theoretical and empirical models for the Galactic potential (see EDV). In brief, these kinematical models predict the evolution of stellar orbits in the gravitational potential of the Galactic disk which in turn is self-consistently coupled to the distribution of stars and gas in the disk. Back integration of the orbits of stars with predicted kinematical properties that are on average the same as those observed for a given sample star, provides an estimate of the galactocentric distance at birth of this star.

Most stars in the EDV sample were found to have peri- and apo-centric distances within ~2.5 kpc from the actual galactocentric distance of the Sun ($R_\odot = 8.4$ kpc). Mean galactocentric distances at birth were estimated from $R_m = (R_{apo} + R_{peri})/2$ where $R_{apo}$ and $R_{peri}$ are the apo- and perigalactocentric distances, respectively. However, there are two main caveats involved with this procedure. First, this estimate ignores the different times spent by stars in different parts of their orbits so that $R_m$ underestimates the mean stellar galactocentric distance at birth. This suggests that stars nowadays observed in the SNBH (such as R by Grenon (1989). Since changes in the disk gravitational potential probably have occurred from the onset of star formation in the disk, conservation of $R_m$ with galactic age may be unrealistic, at least for individual stars, and substantial errors in $R_m$ may be present especially for old stars. According to these assumptions, nearly 85% of the sample stars have estimated birthplaces $R_m$ within 1 kpc from the Sun (i.e. with $R_m = R_\odot \pm 1$ kpc).

The **WIEL galactocentric distances at birth** were derived by directly translating the abundance variations observed among similarly aged stars in the SNBH into a spatial dispersion of the stellar orbits due to the diffusion process (Wielen et al. 1996). In this case, the mean dispersion $\Delta[Z/H]$ in metallicity at a given galactocentric distance $R_0$ (relative to the mean stellar metallicities at this distance) can be related to the mean spatial dispersion by (Fuchs et al. 1994):

$$\Delta[Z/H] \equiv \left( \langle [Z/H] \rangle - \langle [Z/H] \rangle_{R_0} \right)^2 = \frac{\partial}{\partial R} \left( \langle [Z/H] \rangle_{R_0} \cdot \langle (R - R_0)^2 \rangle_{R_0} \right)^{\frac{1}{2}}$$

(4.2)

where $[Z/H]$ and $R$ correspond to the metallicity and galactocentric distance of a star at birth, respectively, and $\langle [Z/H] \rangle_{R_0}$ is the mean radial metallicity gradient at a galactocentric distance $R_0$.

In principle, this relation may be used to estimate the mean effect of orbital diffusion on the abundance variations of a given element among similarly aged stars which are observed at a galactocentric radius $R_0$ (but originate from a wide range in galactocentric distance). However, application of Eq. (4.2) to individual stars may be erroneous because: 1) abundance inhomogeneities among stars formed at roughly the same time and position may be present (e.g. due to local gas infall or sequential stellar enrichment; see Chap. 5), i.e. the initial dispersion in $[Z/H]$ among stars formed at roughly the same time $t$ and $R$ can be large, 2) stars formed at a given galactocentric distance may show abundance variations due to tangential variations in metallicity (e.g. in the regions associated with the spiral arms), 3) individual stars may have experienced peculiar gravitational perturbations of their orbits that are uncommon to the majority of the stars observed at a given galactocentric radius $R$, and 4) both radial metallicity gradients and stellar orbits may have varied substantially over the time interval needed for a given star to travel from its birthsite to the galactocentric distance at which it is nowadays observed. Another aspect which is not clearly visible from Eq. 4.2, is that both the radial dispersion velocity (or spatial dispersion) and the radial metallicity gradients itself may be functions of galactocentric distance.

The complete Edvardsson et al. data suggest that: 1) the mean metallicity $\langle [Z/H] \rangle$ of stars of a given age $\tau$ decreases with mean galactocentric distance $(R_m)$ at birth (see EDV), and 2) the mean metallicity $\langle [Z/H] \rangle$ of stars born at a given galactocentric distance $R_{ini}$ decreases with mean stellar age ($\tau$) (see e.g. Nissen 1995; Joench-Sørensen 1995; however, we note that accurate quantitative relations for these variations are still lacking). If indeed true, a mean relation between stellar metallicity, age, and galactocentric distance at birth can be derived from the complete Edvardsson et al. data provided that the radial metallicity gradient in the Galactic disk has been constant both in time and space (Wielen et al. 1996):

$$\langle [Fe/H] \rangle = +0.05 - 0.048 \left( \frac{\tau}{\text{Gyr}} \right) - 0.09 \left( \frac{(R_{ini}) - R_\odot}{\text{kpc}} \right)$$

(4.3)

where $\langle [Fe/H] \rangle = +0.05$ dex is the mean iron abundance of stars formed at the present-day galactocentric distance of the Sun. A mean gradient $\alpha_{[Fe/H]} = -0.09$ dex kpc$^{-1}$ in [Fe/H] at $R = R_\odot$ was adopted (e.g. Friel 1995; see below). Conversely, the mean galactocentric distance at birth $\langle R_{ini} \rangle$ of stars of a given age $\tau$
and metallicity $[\text{Fe}/\text{H}]_*$ can be derived from Eq. (4.3) by:

$$\langle R_i \rangle - R_\odot = -11 \ [\text{Fe}/\text{H}]_* - 0.53 \left( \frac{\tau}{\text{Gyr}} \right) + 0.6 \ [\text{kpc}]$$

(4.4)

assuming that the initial variation in metallicity among similarly aged stars born at $\langle R_i \rangle$ is relatively small$^2$.

$$\Delta[\text{El}/\text{H}]_* < \alpha^*_{R[\text{El}/\text{H}]} \cdot \langle R_i \rangle - R_\odot \tag{4.5}$$

In general, each element for which Eq. (4.5) is valid can be used to constrain the galactocentric distances at birth of stars observed in the SNBH. For instance, similar relations can be derived when oxygen abundances are considered:

$$\langle [\text{O}/\text{H}] \rangle = +0.035 - 0.038 \left( \frac{\tau}{\text{Gyr}} \right) - 0.07 \left( \frac{\langle R_i \rangle - R_\odot}{\text{kpc}} \right)$$

(4.6)

where $\langle [\text{O}/\text{H}] \rangle = +0.035$ dex is the mean present-day iron abundance of stars at $R \sim R_\odot$ and the mean gradient in $[\text{O}/\text{H}]$ at $R \sim R_\odot$ is assumed to be $\Delta R[\text{O}/\text{H}] = -0.07$ dex kpc$^{-1}$ (e.g. Matteucci et al. 1992; see below).

$$\langle R_i \rangle - R_\odot = -14.3 \ [\text{O}/\text{H}]_* - 0.54 \left( \frac{\tau}{\text{Gyr}} \right) + 0.5 \ [\text{kpc}]$$

(4.7)

We emphasize again that Eqs. (4.2-4.7) implicitly rely on the assumptions that: 1) the rate of enrichment of the disk ISM with age is independent of galactocentric distance, and 2) the mean value of $[\text{El}/\text{H}]$ in the disk ISM increases linearly with galactic age.

Fig. 4.2 compares the stellar galactocentric distances at birth derived using the kinematical (EDV) and chemo-dynamical models (WIEL) both from the stellar $[\text{Fe}/\text{H}]$ and $[\text{O}/\text{H}]$ abundances observed. Values of $R_{\text{ini}}$(EDV) derived from the kinematical data range between 6 and 10 kpc for the majority of the sample stars. In contrast, the range in $R_{\text{ini}}$(WIEL) between 3 and 13 kpc as predicted by the chemo-dynamical model is much larger. It is evident that the values of $R_{\text{ini}}$ predicted by the two methods disagree (except for a few stars which may be relatively young and have unperturbed orbits).

$^2$We will argue in Chap. 5 that this condition probably is not fulfilled for a substantial fraction of the stars in the Edvardsson et al. sample as well as for other objects in the SNBH. In principle, inhomogeneous chemical enrichment of the disk ISM may be important in determining the abundance variations observed among similar aged stars in the SNBH, in addition to the effects of orbital diffusion.
4.1 Basic considerations

Figure 4.3 Stellar galactocentric distances at birth $R_{\text{ini}}$ [kpc] vs. $\Delta$[O/H] for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993). **Left panel:** $R_{\text{ini}}$ (EDV) according to a model for the gravitational potential of the Galactic disk (see Edvardsson et al.), **Right panel:** $R_{\text{ini}}$(WIEL, [O/H]) vs. $\Delta$[O/H] relation (similar to that proposed by WIEL for [Fe/H]). A present-day galactocentric distance of the Sun of $R_\odot = 8$ kpc was assumed.

Fig. 4.3 amplifies the differences in galactocentric distance predicted by the two models in another way: the kinematical model predicts large scatter in the $R_{\text{ini}}$ vs. $\Delta$[O/H] with no clear correlation, while the chemo-dynamical model implicitly assumes no scatter and a linear correlation between $R_{\text{ini}}$ and $\Delta$[O/H]. Which of the two figures is appropriate to the chemical evolution of the Galactic disk is still matter of debate but we will argue below that independent observations probably favour the relation from the chemo-dynamical model proposed by WIEL.

Fig. 4.4 shows the galactocentric distances at birth derived from the chemo-dynamical model using the stellar [Fe/H] and [O/H] abundance ratios, respectively. A correlation is found although considerable scatter is present, i.e. $\Delta R_{\text{ini}} \sim 4$ kpc at a given value of $R_{\text{ini}}$. Part of this scatter may be reduced when one accounts for: 1) errors in the stellar ages derived by EDV, 2) uncertainties in $R_{\text{ini}}$ due to the unknown phase of the star in its epicyclic orbit around the Galactic center, and 3) experimental errors in the iron and oxygen abundances measured as well as in the radial metallicity gradients assumed. Derived values of $R_{\text{ini}}$ are very sensitive to the radial abundance gradients $\alpha_R$ adopted (see Eqs. 4.3 and 4.6). Thus, errors in $\alpha_R$ may cause a substantial part of the observed scatter. However, apart from the fact that these values have been derived empirically, a reduction of $\alpha_R$ would increase the range in galactocentric distance at birth predicted for stars nowadays observed in the SNBH which would increase the scatter correspondingly.

Conversely, an enhancement of $\alpha_R$ would reduce the scatter but this possibility is probably excluded by the observations (see below). Corrections for errors in the measured abundances of individual stars do not necessarily have to result in a reduction of the scatter apparent from Fig. 4.4. Unless systematic errors are present, the same is probably true when one would account for possible errors in the derived stellar ages. Note that systematic deviations from the line $y = x$ appear to be present in the abundance dispersion data. These deviations are due to the different radial gradients in [Fe/H] and [O/H] in the Galactic disk derived from the observations (see below).

The corresponding correlation between the abundance dispersions (relative to the mean abundance of stars formed at a given galactic age) reveals that the dispersion in [O/H] is correlated with that in [Fe/H] apart from the scatter discussed before. However, we stress that this correlation is initially present in the EDV abundance data (see Chap. 5) and may not provide additional support for orbital diffusion as the main explanation for the observed abundance variations because stellar abundance variations caused by metal-deficient gas infall and/or sequential stellar enrichment can result in similar correlations between the abundance dispersions (see Chap. 5).

We have shown that the stellar galactocentric distances at birth obtained from kinematical and/or abundance data for individual stars usually give very different results. Apart from the large number of assumptions and uncertainties involved, we argue that both the kinematical and the chemo-dynamical methods are likely to give erroneous results if used to determine the galactocentric distances of individual stars because these models are essentially a statistical description of the orbital diffusion process. In other words, although both the kinematical and chemo-dynamical methods may provide good descriptions of what is happening
Modelling the chemical evolution of the Galactic disk

Figure 4.4 Stellar galactocentric distances at birth $R_{ini}$ [kpc] and abundance-dispersions $\Delta[M/H]$ for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993). **Left panel**: $R_{ini}(\text{WIEL}, [O/H])$ vs. $R_{ini}(\text{WIEL}, [Fe/H])$ obtained according to the relations proposed by Wielen et al. (1996). **Right panel**: $\Delta[O/H]$ vs. $\Delta[Fe/H]$ for the data shown in the left panel. For comparison, the dotted line indicates $y$ equals $x$. A present-day galactocentric distance of the Sun of $R_\odot = 8$ kpc was assumed.

with the orbit of a star in the gravitational field of the Galactic disk in a statistical manner, these methods usually cannot be used to determine the orbital characteristics of individual stars.

From the data currently available, we conclude that stellar orbital diffusion is probably a common and important process in the Galactic disk which, at least in a statistical manner, can provide an adequate explanation for the large abundance variations observed among similarly aged stars in the SNBH. At the same time, however, we are forced to conclude that stellar orbital diffusion alone is probably insufficient to explain these abundance inhomogeneities for all stars in the EDV sample (see Chap. 5). The relative importance of orbital diffusion and other processes (such as metal-deficient gas infall) in determining these abundance variations is unclear from the present data. Clarification of this situation is extremely important for our interpretation of the abundance data of stars observed in the SNBH and will greatly improve our understanding of the chemical evolution of the Galactic disk as a whole. In addition, solving this problem can facilitate the selection of samples of stars suited to study specific aspects of Galactic chemical evolution.

Stellar orbital diffusion: observations vs. theory

As part of the basic considerations presented in this section, we discuss some of the differences in the orbital diffusion predictions by EDV and WIEL. In particular, we will argue that part of the discrepancies between the orbital parameters and galactocentric distances at birth as derived by EDV and WIEL can be explained if the kinematical data and stellar ages presented by Edvardsson et al. are subject to large errors. We highlight several interesting aspects of stellar orbital diffusion (i.e. the birthplace of the Sun in the Galactic disk and the dispersion in metallicity as a function of Galactic age as pointed out by Wielen et al. 1996) with particular emphasis on the abundance variations observed among similarly aged stars in the SNBH.

- Predicted stellar orbital parameters: additional constraints

If orbital diffusion is the dominant mechanism responsible for the abundance variations observed among similarly aged stars in the SNBH, it is expected that old stars, which have travelled relatively large distances through the Galactic disk before finally reaching the SNBH, show a larger spread in abundance than do young stars.

Fig. 4.5 displays the variation of the dispersion of metallicity with stellar age, both for all stars in the EDV sample with accurate metallicities $^{10}\log(Z/Z_\odot)$ and for a subsample of these stars with $R_{ini}(\text{WIEL}, [Fe/H]) = 6-10$ kpc. From these data, Wielen et al. conclude that the abundance spread observed among similarly aged stars in the SNBH decreases with stellar age. We believe that this conclusion is rather uncertain on the basis of the EDV data (e.g. due to the selection criteria that were used by Edvardsson et al). For the subsample of stars with a restricted range in galactocentric distance at birth as derived by Wielen et al. the spread in metallicity at a given age is considerably reduced and a mean AMR for such stars
4.1 Basic considerations

Figure 4.5 Stellar age vs. heavy element integrated metallicity $^{10}\log(Z/Z_\odot)$ both for all F and G stars in the SNBH with accurate metallicities (left panel) and for a subsample of these F and G dwarfs with $R_{\text{ini}}$(WIEL, [Fe/H]) between 6 and 10 kpc (right panel). Data from Edvardsson et al. 1993.

remains. This concept forms the basis of the idea of stellar orbital diffusion as the primary cause for the abundance variations observed among similarly aged stars in the SNBH (Wielen et al. 1996).

The method presented by Wielen et al. is based on the translation of a stellar abundance deviation (from the mean abundance observed among stars of a given age) in terms of a shift in galactocentric distance. We like to emphasize that this method in principle can be applied to any set of stellar abundance data no matter the origin of the abundance inhomogeneities present in the data set. Therefore, the basic results presented by Wielen et al. probably remain unchanged even when the abundance spread among similarly aged stars would be entirely due to processes other than orbital diffusion. This is true provided that such processes: 1) do not result in mean stellar abundances considerably different from the mean of the EDV data at a given age, and 2) predict the abundance inhomogeneities for different elements to be correlated in a progressive manner.

Figure 4.6 $\Delta$[Fe/H] (left panel) and $R_{\text{ini}}$(WIEL, [Fe/H]) (right panel) vs. stellar age for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993).

Fig. 4.6 shows that there is no clear trend from the EDV data for an increase in the abundance dispersion $\Delta$[Fe/H] with stellar age. The same is true for the variation in the range of $R_{\text{ini}}$ with stellar age. This may be due to errors in the stellar ages derived by EDV and/or due to selection effects which concern the galactocentric distances of the sample stars. Alternatively, processes other than stellar orbital diffusion may
be involved which might erase the initial trend.

Fig. 4.7 confirms that there are also no signs of a correlation between $\Delta[O/H]$ and stellar age or between $[O/H]$ and the stellar $W$ velocity perpendicular to the Galactic plane. If stellar orbital diffusion is the dominant mechanism causing the observed abundance variations among similarly aged stars in the SNBH, one again would expect that $\Delta[O/H]$ increases with stellar age. Similarly, one would expect that the dispersion in $W$ increases with decreasing values of $\Delta[O/H]$ (such a relation is observed for F, G, and K dwarfs in the Galactic disk between W-velocity and $[Fe/H]$; see Bahcall et al. 1992). However, these correlations appear not to be present in the current data. This either suggests that orbital diffusion is not the dominant mechanism causing the observed abundance variations, or that the EDV data: 1) are biased towards young stars (old, metal-rich stars are probably under-represented), and/or 2) are considerable in error for what concerns the ages and $W$ velocities of the sample stars.

![Figure 4.7](image)

*Figure 4.7* Stellar ages (left panel) and $W$ velocities (perpendicular to the Galactic plane; right panel) vs. $\Delta[O/H]$ for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993).

Similarly, Fig. 4.8 illustrates that the EDV data do not provide evidence in support of a larger maximum distance $Z_{\text{max}}$ above the Galactic plane for older stars or for larger dispersions $|\Delta[Fe/H]|$. Thus, the EDV data do not comply with the generally accepted idea that $Z_{\text{max}}$ increases with age for stars in the Galactic disk due to the effects of orbital diffusion (e.g. Wielen 1977; Sommer-Larsen 1991). This suggests that the stars in the EDV sample suffer from severe selection effects (e.g. birthplace, age) and/or that the derived stellar ages as well as values of $Z_{\text{max}}$ contain considerable systematic errors.

In Fig. 4.9 we show the variations of $\Delta[Fe/H]$ and $R_{\text{ini}}([Fe/H])$ vs. the heavy element integrated metallicity $Z$. Both $\Delta[Fe/H]$ and $R_{\text{ini}}([Fe/H])$ were computed according to Eqs. 4.3 and 4.4 while $Z$ was taken directly from the data presented by EDV. Although the scatter is considerable, Fig. 4.9 suggests a correlation between the magnitude of $|\Delta[Fe/H]|$ and $10\log(Z/Z_\odot)$ (i.e. the deviation of the stellar metallicity $Z$ from solar). Such a relation is expected if orbital diffusion is responsible for the majority of the abundance variations observed among stars in the SNBH. Apart from the large scatter, a radial metallicity gradient of $10\log(Z/Z_\odot) = -0.11$ dex kpc$^{-1}$ is found. This suggests that, on average, stars with $\Delta[Fe/H] > 0$ were formed more inwards in the Galactic disk, i.e. at galactocentric distances $R_{\text{ini}} \lesssim R_\odot$. The scatter in these relations may be due to errors in the stellar ages derived by EDV (which lead to incorrect values of $\Delta[Fe/H]$ according to Eq. 4.3) and/or to processes other than orbital diffusion which cause considerable spread in $Z$ at a given $\Delta[Fe/H]$.

We conclude that, for the majority of the stars in the EDV sample, the results presented by Wielen et al. may be correct, at least to first order, provided that: 1) large errors are present in both the stellar ages and kinematical data presented by Edvardsson et al. and 2) the EDV data suffer from substantial selection effects both with respect to stellar age, metallicity, and galactocentric distance at birth. To what extent these effects are indeed present in the EDV data is unclear although the possibility of large errors in the absolute stellar ages has been emphasized by Edvardsson et al. Furthermore, if the stellar ages derived by EDV are indeed incorrect by large factors, how must one interpret Eqs. 4.3–4.7 which were derived by Wielen et al. and are based on these ages?
4.1 Basic considerations

Figure 4.8 Maximum present-day distances from the Galactic plane \( Z_{\text{max}} \) vs. age (left panel) and vs. \( \Delta[\text{Fe}/\text{H}] \) (right panel) for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993).

Figure 4.9 \( \Delta[\text{Fe}/\text{H}] \) (left panel) and \( R_{\text{ini}} \) (WIEL, \( [\text{Fe}/\text{H}] \) (right panel) vs. heavy element integrated metallicity \( 10^{\log(Z/Z_\odot)} \) for F and G dwarfs in the SNBH (data from Edvardsson et al. 1993).

• The birthplace of the Sun in the Galactic disk

Stellar abundances are usually given in terms of solar abundances. Therefore, it is important to know to what extent the abundances in the Sun are representative for the abundances of stars born \( \sim 4.5 \) Gyr ago at the present-day galactocentric radius of the Sun.

According to the overabundance of the Sun of \( \Delta[\text{Fe}/\text{H}] = +0.17 \pm 0.04 \) dex, compared to the mean iron abundance of stars formed about 4.5 Gyr ago which are observed at the present-day galactocentric distance of the Sun, it is argued by Wielen et al. that the galactocentric radius of the Sun at birth is \( R_{\text{ini},\odot} = 6.6 \pm 0.9 \) kpc. This estimate is based on the assumption of a radial gradient in iron of \( \alpha_{R}[\text{Fe}/\text{H}] = -0.09 \pm 0.02 \) dex kpc\(^{-1} \) and a present-day galactocentric radius of the Sun of \( R_\odot = 8.5 \) kpc. A similar calculation of \( R_{\text{ini},\odot} \) from the oxygen overabundance in the Sun leads to \( R_{\text{ini},\odot} = 6.4 \pm 1.0 \) kpc which is in good agreement with the previous estimate. This suggests that the Sun formed \( \sim 1.9 \) kpc more inwards in the Galactic disk than its present-day galactocentric distance (Wielen et al. 1996).

As an alternative to this interpretation, we emphasize that: 1) the Sun may well have been enriched sequentially in oxygen and iron by roughly similar amounts (relative to the mean element abundances of stars formed \( \sim 4.5 \) Gyr ago at the galactocentric distance of the Sun) since independent observations suggest...
that the material out of which the Sun formed has been enriched sequentially by massive stars (see Sect. 5.5; Chap. 2). If indeed true, the agreement between $R_{\text{ini},\odot}(\text{[O/H]})$ and $R_{\text{ini},\odot}(\text{[Fe/H]})$ in the orbital diffusion scenario would be just an odd coincidence and would provide no constraint to the orbital diffusion effect on the galactocentric distance of the Sun; and 2) for a substantial fraction of the stars in the EDV sample $R_{\text{ini},\odot}$, as derived from their [Fe/H] abundances is probably inconsistent with that derived from their [O/H] abundances. Thus, orbital diffusion may be insufficient as well to explain the abundance variations of stars (such as the Sun) with consistent values of $R_{\text{ini},\odot}$. We argued above that the effects of orbital diffusion on the abundance variations among similarly aged stars in the SNBH may provide an adequate statistical description of inhomogeneous chemical evolution of the Galactic disk. In contrast, we believe that application of the orbital diffusion theory to individual stars can give erroneous results.

To first order, the regular orbit of a disk star in the mean gravitational field of the Galaxy can be described by an epicyclic orbit (see e.g. Scheffler & Elsässer 1982). If the guiding center of the epicycle is assumed to move around the Galactic center in a circular orbit with radius $R_m$, one can derive the present-day semi-axis $A_R$ (of the (elliptical) epicycle in the $R$ direction) of a disk star using its observed space velocities. In this manner, Wielen (1982) derived $A_R = 0.7$ kpc and $R_{\text{m,\odot}} - R_\odot = +0.6$ kpc. Combined with $R_{\text{ini},\odot} \sim 6.5$ kpc, this suggests a change in $R_{\text{m,\odot}}$ of $\sim 0.6 + 1.9 = 2.5 \pm 0.7$ kpc over the past $\sim 4.5$ Gyr if the orbital prediction is correct (Wielen et al. 1996). From these results, it seems improbable that the diffusion in galactocentric distance of the Sun can be ascribed only to variations in the epicycle movement of the Sun. Nevertheless, variations in galactocentric distance due to the epicycle movement of stars around the guiding centers of their orbits may explain part of the scatter observed in the $R_{\text{ini}}$(WIEL, [Fe/H]) vs. $R_{\text{ini}}$(WIEL, [O/H]) and $\Delta$[Fe/H] vs. $\Delta$[O/H] diagrams discussed above, in particular for stellar orbits with $A_R \gtrsim 1$ kpc.

• Dispersion in metallicity as a function of stellar age

In general, models for the orbital diffusion of stars in the Galactic disk predict that the radial diffusion is proportional to the diffusion in velocity (e.g. Fuchs et al. 1994). For such models, it is expected that the abundance dispersion among similarly aged stars observed at a given galactocentric distance is related to the velocity dispersion of these stars by (WIEL):

$$\sigma_{[\text{El/H}]}(\tau) = | \alpha_{R}^{[\text{El/H}]} | \cdot f_U \sigma_U(\tau) = | \alpha_{R}^{[\text{El/H}]} | \cdot f_V \sigma_V(\tau)$$

(4.8)

where $\sigma_U$ and $\sigma_V$ are the velocity dispersions in $U$ (towards the galactic center) and in $V$ (total peculiar velocity), respectively. Assuming an isotropic diffusion coefficient and an expression for $\sigma_U$ (or $\sigma_V$) following from the diffusion model, the empirical values of the diffusion coefficients $f_U = 3.8$ kpc / 100 km s$^{-1}$ and $f_U / f_V = 1.3$ can be well explained (Wielen 1977). If one further assumes that the radial abundance gradients $\alpha_{R}^{[\text{El/H}]}$ in the Galactic disk are independent both of galactic age and location, the dispersion in the abundance of a given element (relative to the mean abundance among stars observed at a given galactocentric distance) may be related to stellar age by (WIEL):

$$\sigma_{[\text{El/H}]}(\tau) = | \alpha_{R}^{[\text{El/H}]} | \cdot \langle (\Delta R(\tau))^2 \rangle^{\frac{1}{2}}$$

(4.9)

where

$$\langle (\Delta R(\tau))^2 \rangle^{\frac{1}{2}} = [\langle (\Delta R(\tau))^2 \rangle + \frac{1}{2}\langle (\Delta A_R(\tau))^2 \rangle^{\frac{1}{2}}]^{\frac{1}{2}} = \sim 0.92 \left( \frac{\tau}{\text{Gyr}} \right)^{\frac{1}{2}}$$

[ kpc]

(4.10)

is the variation of the diffusion in galactocentric distance with stellar age as derived from the observations (Wielen 1977). For 10 and 4.5 Gyr old stars, Eq. (4.10) gives $\Delta R = 3$ and $\sim 2$ kpc, respectively. Note that this equation takes into account the diffusion of stellar orbits, both due to the stochastic increase in $R_m$ and to the increase in the semi-axis of the epicycle $A_R$ with stellar age (WIEL).

According to Eqs. 4.9 and 4.10, variations in $\sigma_{[\text{Fe/H}]}$ and $\sigma_{[\text{O/H}]}$ with age for stars observed at a given galactocentric distance result in:

$$\sigma_{[\text{Fe/H}]}(\tau) = 0.083 \left( \frac{\tau}{\text{Gyr}} \right)^{\frac{1}{2}} \quad \text{and} \quad \sigma_{[\text{O/H}]}(\tau) = 0.064 \left( \frac{\tau}{\text{Gyr}} \right)^{\frac{1}{2}}$$

(4.11)

which leads to:

$$\sigma_{[\text{O/H}]} = 0.78 \sigma_{[\text{Fe/H}]}$$

(4.12)

This equation predicts that the scatter in the stellar [O/H] abundances is smaller by a factor of 0.78 than the scatter in [Fe/H] independent of stellar age. The factor 0.78 is simply determined by the ratio of the radial
gradients in [O/H] and [Fe/H]. This simple result is new and not addressed by Wielen et al. (1996). It shows that the model for orbital diffusion of stars in space proposed by Wielen et al. predicts that the ratio of the abundance variations of two elements P and Q among similarly aged stars observed at a given galactocentric distance $R_{\text{obs}}$, is simply given by to the ratio of the radial abundance gradients of these elements at $R_{\text{obs}}$. For the variations in oxygen and iron this result is in good agreement with the observations despite the uncertainties involved with the radial abundance gradients (see at the end of this section). This is true even though Eqs. 4.11 and 4.12 are clearly inconsistent with the Edvardsson et al. data (see Fig. 4.6). For elements other than iron and oxygen, the prediction is more difficult to check since radial abundance gradients for such elements are relatively uncertain. It is interesting to note that the relative abundance scatter observed among similarly aged stars in the SNBH for two elements P and Q may provide direct information about their relative abundance gradients in the local disk.

As far as the discrepancy between the predictions of Eqs. 4.11 and 4.12 and the EDV data is concerned, we can think of three effects which may play an important role: 1) the stellar ages determined and presented by EDV are systematically in error for stars with abundances that deviate considerably from the mean abundance of similarly aged stars; 2) the EDV data are incomplete with respect to stellar age; this effect may arise when the EDV sample stars were selected in such a way that they were more or less evenly distributed over galactic age. Since the abundance dispersion is predicted to increase with stellar age, such a pronounced effect probably shows up only in samples of stars that contain relatively large numbers of old stars compared to young ones, provided that the stellar abundance variations (relative to the mean abundance of similarly aged stars) are distributed in a Gaussian manner; and 3) the EDV sample is biased towards stars which formed at galactocentric distances relatively close to the present-day galactocentric distance of the Sun, i.e. stars with $R_{\text{ini}} > R_{\odot}$ or $R_{\text{ini}} < R_{\odot}$ in the EDV sample are under-represented (see Figs. 4.2–4.6).

At present, it is unclear to what extent the stars in the sample of Edvardsson et al. (1993) is statistically representative with respect to age, galactocentric distance at birth, and metallicity. We expect that the EDV data are subject to each of the three effects discussed above. If indeed true, this would favor the possibility that stellar orbital diffusion is the dominant mechanism causing the abundance variations observed among similarly aged stars in the SNBH. At the same time, this would weaken the possibility that infall of metal-deficient gas and sequential stellar enrichment (as discussed in Chap. 5) provide an adequate explanation for the dominant part of the abundance variations observed (although these processes are likely to be important on small scales in the disk ISM). On the other hand, it is unclear whether the results of Wielen et al. would be altered when both the effects of abundance inhomogeneities in the disk ISM (due to metal-poor gas infall and sequential stellar enrichment) and stellar orbital diffusion would be present in the Edvardsson et al. data. To definitely solve the problem of the abundance variations observed among similarly aged stars in the SNBH, a large, statistically complete sample of stars with accurate abundances, ages, and orbital parameters is needed. Here, we have argued that stellar orbital diffusion may be a much more important process than previously thought and certainly much more important than suggested by Edvardsson et al. (1993) and Grenon (1990). In particular, the galactocentric distance $R_{\text{ini}}$ of the guiding center of an epicyclic orbit of a star moving in the gravitational potential of the Galactic disk is strongly affected by orbital diffusion as argued in the work of Wielen et al. (1996).

In the near future, the chemical and orbital properties of a sample of stars nowadays observed in the SNBH may be corrected for diffusion in galactocentric radius on the basis of accurate kinematical data as well as on the basis of more reliable models for the kinematical evolution of stars in the disk gravitational potential. At present, such corrections for individual stars are probably premature and, therefore, we will simply assume that the total number of stars within a given volume around the Sun remains unaffected by radial orbital diffusion. This is probably untrue in case of a stellar surface density that decreases exponentially when going outwards in the Galactic disk, but it is an inevitable and convenient first order approximation. For what concerns the corrections for the evolution of the vertical structure of the stellar disk in the SNBH, the diffusion effect may be more substantial and we will apply correction factors as discussed below.

### 4.1.5 Evolution of the vertical structure of the Galactic stellar disk

Stars adopt the velocity dispersion of the protostellar gas at their birth and are accelerated by gravitational perturbations of their orbits later in their evolution. This leads to an increase of the vertical scale height of the Galactic stellar disk with age as shown by Wielen’s (1977) empirical stellar velocity dispersion-age relation $\sigma \propto \tau^\lambda$ where $\lambda \sim 0.5$. Possible acceleration mechanisms may include collisions with massive gas clouds (e.g. Spitzer & Schwarzschild 1951; Lacey 1984), large scale fluctuations in the galactic gravitational potential such as spiral arms (e.g. Sellwood & Carlb erg 1984; Jenkins & Binney 1990), collisions with massive black holes (e.g. Lacey & Ostriker 1985; Fuchs et al. 1994), and merging with satellite galaxies which deposit
their orbital energy as kinetic energy of disk stars (Tóth & Ostriker 1992; Quinn et al. 1993; Athanassoula 1993). Stellar acceleration laws with λ between 0.2 and 0.5 can be generally reconciled with the observations (e.g. Wielen 1977; Jahrein & Wielen 1983; Strömgren 1987; Lacey 1991; Rana 1991).

The increase of the scale height of the stellar disk in the SNBH with age is an important effect which should be taken into account when statistical properties are considered of stellar samples that refer to a small volume around the Sun. In particular, such scale height corrections are necessary when converting the local volume densities of stars observed in the SNBH to z-integrated volume (or surface) densities for the entire solar cylinder (in general model results refer to the latter volumes). To correct local volume limited samples to represent the entire solar cylinder, a kinematical model for the variation of the scale height of disk stars with age is needed. In this manner, mean correction factors f would be determined as a function of stellar age (or as a function of metallicity when the mean AMR is used).

A simple and convenient parametrization of the evolution of scale height with age is (e.g. Rana 1991):

\[ h_z = h_0 \left(1 + \frac{T - t}{\tau_0}\right)^\eta \]  

(4.13)

where \( h_0 \) is the scale height of the stellar disk at the time a star is born, (T - t) the stellar age, \( \tau_0 \) a characteristic time scale, and \( \eta = 1/2 \) or 1/3 depending on the adopted mechanism of scattering stars out of the Galactic plane. The correction factor for a subpopulation of the Galactic disk born at a given galactic age \( t \) is then given by \( f(t) \propto \rho_{SNBH}/\Sigma_z \) where \( \rho_{SNBH} \) and \( \Sigma_z \) are the volume density of stars in the SNBH and the z-integrated surface density of the local solar cylinder of the stellar subpopulation of interest, respectively.

We here consider the scale height correction factors presented by Sommer-Larsen (1991) which essentially are based on two distinct kinematical models for the evolution of the vertical structure of the Galactic disk. The first model is that from Norris & Ryan (1989; hereafter NR), the second that of Kuijken & Gilmore (1989; hereafter KG). Resulting correction factors \( f/(T - t) \) usually range between \( \sim 0.2 \) and 1 for values of \( [\text{Fe/H}] \approx -1 \) dex and \( [\text{Fe/H}] \gtrsim 0.05 \), respectively (Sommer-Larsen 1991). Due to the vertical expansion of the stellar disk, the oldest stars are distributed over z-distances \( \sim 5 \) times larger than stars recently formed in the disk. At values of \( [\text{Fe/H}] \) between \( -0.4 \) and 0, the NR kinematical model predicts \( 1/f \) factors that are substantially less than the KG model (by more than a factor of two). Assuming \( \tau_0 = 0.5 \) Gyr (Rana 1991) both model predictions comply with the range of \( 1/f \)-values resulting from Eq. (4.13) by applying \( \eta = 1/3 \) and 1/2. In general, the older and more metal-poor stars that have more perturbed orbits and, on average, are kinematically more excited are subject of relatively large scale height corrections (e.g. Grenon 1989; Sommer-Larsen 1991). We emphasize that such corrections, in principle, should take into account the fact that the present-day position of the Sun is offset from the Galactic plane. Whenever possible we will use the scale height corrections for the KG model as presented by Sommer-Larsen (1990, 1991). Furthermore, we note that the scale heights of main-sequence stars and their remnants are expected to have the same dependence on stellar age since the velocity dispersion is essentially an effect of the kinematical evolution of the stellar disk (e.g. Fuchs & Wielen 1987).

### 4.1.6 Distinction between halo, thick and thin disk stars

We briefly discuss the usual distinctions made between halo, thick and thin disk stars. In principle, such a distinction is important when the properties of stars nowadays observed in the SNBH are considered. The data of Edvardsson et al. suggest the existence of at least two discrete populations of stars observed in the SNBH: 1) a thin disk component with age smaller than 10 Gyr, \( -0.4 \lesssim [\text{Fe/H}] \lesssim +0.3 \), \( \langle V \rangle \sim -10 \) km/s, velocity dispersions that increase with age, and a radial metallicity gradient that probably depends on age as well. For this component, the data of Beers & Sommer-Larsen (1994) indicate a mean rotational velocity of 210 km/s in the SNBH, mean velocity dispersions in U, V, W of 40, 30, 20 km/s, respectively, an exponential scale height of 300 pc, and a radial scale length of \( \sim 4.5 \) kpc. The thin disk contains most (i.e. 90–95 %) of the luminous mass in the local Galaxy; and 2) a thick disk component with age older than 10 Gyr, \( -0.8 \lesssim [\text{Fe/H}] \lesssim -0.4 \), \( \langle V \rangle \sim -50 \) km/s, a mean rotational velocity in the SNBH of 190 km/s, velocity dispersions in U, V, W = 60, 40, 40 km/s, respectively, an exponential scale height of \( \sim 1000 \) pc, and a radial scale length of \( \sim 4.5 \) kpc (Beers & Sommer-Larsen 1994). The onset of the thick disk becomes apparent in the rapid change in the vertical velocity distribution at metallicities \( [\text{Fe/H}] \sim -0.4 \). Most of the thick disk stars have abundances in the range \( -1 \lesssim [\text{Fe/H}] \lesssim -0.5 \) but a tail beyond \( [\text{Fe/H}] \lesssim -1 \) is present (e.g. Bahcall et al. 1992). The thick disk provides about 5–10 % of the galactic luminous mass.

There is considerable overlap of the thick and thin disk components for stars with \( [\text{Fe/H}] \) between \(-0.75\) and \(-0.3\) as the old thin disk probably extends down to at least \( [\text{Fe/H}] \sim -0.75 \) (e.g. Wyse & Gilmore 1995; hereafter WG). Star count models indicate a local ratio of thick disk to thin disk stars of about 2–10 percent (e.g. Gilmore et al. 1989; Majewski 1993; Ojha 1994). In contrast, studies of the abundance
distribution of long-living stars in the SNBH (e.g. Pagel 1989; Sommer-Larsen 1991) suggest that thick disk stars with $[\text{Fe/H}] \lesssim -0.4$ contain as much as $\sim 25\%$ of the local sample stars. However, it was pointed out recently by WG that the majority of the stars near the Sun with metallicities around the peak of the thick disk metallicity distribution (i.e. $[\text{Fe/H}] \sim -0.6$) have kinematics appropriate to that of the thin disk and not of the thick disk. Thus, the thick disk contribution to stars with $[\text{Fe/H}] \lesssim -0.4$ has been overestimated by considerable factors in samples of long-living stars in the SNBH. This has important consequences for the kinematical corrections applied to e.g. the age and metallicity distributions of such samples (see WG; Sect. 4.3.8).

The rapidly rotating (thin+thick) disk components observed in the SNBH are distinct from the non-rotating metal-poor halo population with a mean rotational velocity of $\sim 0$, and velocity dispersions of $U$, $V$, $W = 140$, 100, 100 km/s, respectively. The stellar halo starts to dominate at metallicities $[\text{Fe/H}] \lesssim -1$ (e.g. Marquez & Schuster 1994).

The combination of kinematics, spatial distribution, and chemical abundances for samples fo stars selected according to different selection criteria is required to understand how different components (halo, thick, and thin disk) contribute to a local sample of stars observed. However, to what extent the chemical properties of the stellar populations observed in the SNBH are affected by differences in dynamical evolution of the stars associated with e.g. the thin and thick disk components is not well known. Such corrections may be particularly important when these properties are compared to predictions of chemical evolution models for the entire solar cylinder (see e.g. Wyse & Gilmore 1995). Nevertheless, we will ignore such corrections in the following and assume that the stellar disk as a whole has experienced a common star formation and chemical evolution history.

### 4.1.7 Radial abundance gradients

The existence of radial abundance gradients play an important role in the chemical evolution of the Galactic disk. In our models, however, we will not deal with radial abundance gradients and simply consider the chemical evolution of the Galactic disk as a whole. Nevertheless, comparison of our results with the abundances of stars observed in the SNBH may require an interpretation in terms of different rates of enrichment at different galactocentric radii. For this reason, we like to discuss briefly the existence of radial abundance gradients in the Galactic disk and to address several interesting aspects involved.

**Abundance gradients from H\textsc{i} regions.** Observations of both gas (e.g. Shaver 1983) and stars (e.g. Friel & Janes 1993) indicate that substantial radial abundance gradients are present in the Galactic disk. Gradients derived from optical and radio spectroscopy of H\textsc{i} regions (Shaver et al. 1983; Fich & Silkey 1991) are $-0.07 \pm 0.02$ dex kpc$^{-1}$ in $[\text{O/H}]$ and $-0.09 \pm 0.02$ dex kpc$^{-1}$ in $[\text{N/H}]$ for H\textsc{i} regions with galactocentric distances between 4 and 14 kpc. Steeper gradients are found for H\textsc{i} regions in the innermost parts of the Galaxy which are relatively metal-rich. Abundance gradients of $[\text{Fe/H}]$ among H\textsc{i} regions in the local disk are of the order of $-0.05 \pm 0.02$ dex kpc$^{-1}$ (Clarke 1989; Panagia & Tosi 1981). We note that abundance gradients derived from H\textsc{i} regions may be systematically affected by: 1) errors in the specific conditions (e.g. high temperatures and densities, intense radiation fields, highly ionized gas, large extinction factors) assumed in the models for these regions (e.g. Osterbrock et al. 1992), and 2) depletion of heavy elements by dust grains which is expected to be more severe in high-metallicity regions (see Chap. 7).

**Abundance gradients from disk stars.** For comparison, Friel & Janes (1993) observed a mean gradient in $[\text{Fe/H}]$ of $-0.09 \pm 0.02$ dex kpc$^{-1}$ from the metallicities of 24 open clusters ($R \sim 8$–16 kpc, ages 1–8 Gyr). In contrast, gradients determined from B-main-sequence stars in young clusters and H\textsc{i} regions indicate much smaller gradients of $0.00 \pm 0.01$ and $-0.03 \pm 0.01$ dex kpc$^{-1}$ in $[\text{O/H}]$ and $[\text{N/H}]$, respectively (e.g. Rolleston et al. 1994; Kilian et al. 1994; Kaufer et al. 1994). At a galactocentric distance of $\sim 5$–6 kpc these gradients appear to increase inwards. Gradients of O, N, Mg, Si, and Fe, in the outer part of the Galactic disk covered by B-star observations are flat or almost negligible (Kaufer et al. 1994). In particular, Kaufer et al. conclude that the B-type stars have oxygen abundance gradients in $[\text{O/H}]$ of $0.00 \pm 0.01$ dex kpc$^{-1}$ between 7 and 16 kpc and that the steep abundance gradient in $[\text{O/H}]$ of $-0.09 \pm 0.02$ dex kpc$^{-1}$ reported by Shaver et al. (1983) is due to linear fitting of the H\textsc{i} region abundances over the full range in Galactocentric distance.

In principle, abundance gradients are best traced by the present-day disk ISM and by young stars which have not moved far from their birthsites. However, several selection effects may present, such as a strong bias to regions containing young, massive stars. Since star forming regions may be contaminated by earlier generations of massive stars, gradients derived from such regions may be systematically larger than those obtained from old disk stars. Abundance gradients from old stars in the disk can be derived if the orbital parameters of these stars can be determined. However, the derivation of abundance gradients from
old stars is complicated by the fact that reliable corrections for the radial diffusion of their orbits must be applied. Since stellar orbital diffusion causes \( R_m \) to increase with stellar age, it can be shown that the abundance gradients derived from old stars omitting corrections for the diffusion of \( R_m \) may underestimate the actual gradients by roughly a factor of \( \sim 2 \) (Wielen et al. 1996).

Due to the effects of orbital diffusion (Wielen et al. 1996) and because the surface density of stars increases inwards in the Galactic disk (e.g. Bahcall & Soneira 1980), the mean metallicity of stars nowadays observed at \( R = R_\odot \) is substantially larger than the metallicity of similarly aged stars born at \( R_{ini} = R_\odot \). Indeed, this is suggested e.g. by the presence of old, metal-rich stars in the SNBH with \( +0.3 \lesssim [\text{Fe}/\text{H}] \lesssim +0.6 \), ages of \( \sim 10 \) Gyr, and kinematical properties which indicate that they were formed in the inner part of the Galaxy (e.g. Grenon 1990).

The magnitude of radial abundance gradients in the Galactic disk probably varies with galactocentric distance (e.g. Kaufer et al. 1994; Minniti et al. 1995). Radial gradients may depend on galactic age as well (e.g. Vila-Costas & Edmunds 1992) since the stellar surface density decreases outwards and stellar yields depend on initial metallicity. In this manner, stellar orbital diffusion may lead to systematic changes in the variation of abundance gradients with galactic age. Differential movement of stars between their time of formation and the time of ejection of their heavy elements (e.g. for SNIa and SNII progenitors) may affect the variation of abundance gradients in the Galactic disk as well. Furthermore, radial transport of gas associated with infall of material may affect abundance gradients in the disk ISM (e.g. Mayor & Martinet 1977; Yoshii & Sommer-Larsen 1987).

It is generally accepted that the radial abundance gradients observed in the Galactic disk originate from the inside-out formation and evolution of the Galaxy in which the bulge formed first and star formation proceeded outwards in the disk thereafter (e.g. Rocca-Volmerange & Schaeffer 1990; Burkert & Hensler 1994). This idea is consistent with models for which the SFR is very sensitive to the gas density (e.g. Schmidt 1959; Dopita 1985; see also Edmunds & Pagel 1984; Phillipps & Edmunds 1991) and by observations which suggest that the origin of large abundance gradients may be linked to the existence of (un-barred) spiral structure in gas-rich disk galaxies (Edmunds & Roy 1993).

### 4.1.8 Concluding remarks

In this section, we considered several interesting aspects of stellar orbital diffusion with particular emphasis on the abundance variations observed among similarly aged stars in the SNBH and the interpretation of these abundance variations by Edvardsson et al. (1993; EDV) and Wielen et al. (1996; WIEL).

We summarize the main results obtained in this section as follows:

- the stellar galactocentric distances at birth derived from kinematic (EDV) and abundance data (WIEL) for individual stars are inconsistent. Apart from the large number of assumptions and uncertainties involved, both the kinematical and the chemo-dynamical models are likely to give erroneous results when used to determine the galactocentric distances at birth for **individual stars** because these models are essentially a **statistical** description of the orbital diffusion process;

- the basic results presented by WIEL probably remain unchanged even when the abundance spread among similarly aged stars would be entirely due to processes other than orbital diffusion (e.g. due to metal-poor gas infall and sequential stellar enrichment). This is true provided that such processes: 1) do not result in mean stellar abundances considerably different from the mean of the EDV data at a given age, and 2) predict the abundance inhomogeneities for different elements to be correlated in a progressive manner;

- for the majority of the stars in the EDV sample, the results presented by WIEL may be correct, at least to first order, provided that: 1) large errors are present in both the stellar ages and kinematical data presented by EDV, and 2) the EDV data suffer from substantial selection effects both with respect to stellar age, metallicity, and galactocentric distance at birth. However, if the stellar ages derived by EDV are indeed incorrect by large factors, how must one interpret the abundance dispersion relations derived by WIEL which are based on these ages?;

- the results presented by WIEL imply that the scatter in the stellar \([\text{O}/\text{H}]\) abundances among similarly aged stars in the SNBH is smaller by a factor of 0.78 than the scatter in \([\text{Fe}/\text{H}]\) **independent of stellar age**. The factor 0.78 is simply determined by the ratio of the local radial gradients in \([\text{O}/\text{H}]\) and \([\text{Fe}/\text{H}]\). This simple result is new and has not been addressed by WIEL. It shows that the model for orbital diffusion of stars in space proposed by WIEL predicts that the **ratio of the abundance variations of two elements** \(P\) and \(Q\) among similarly aged stars observed at a given galactocentric distance \(R_{obs}\), is simply given by to the ratio of the radial abundance gradients of these elements at \(R_{obs}\). Conversely, the relative abundance scatter observed among similarly aged
stars in the SNBH for two elements P and Q may provide direct information about their relative abundance gradients in the local Galactic disk;

- it is unclear to what extent the chemical properties of the stellar populations observed in the SNBH are affected by the differences observed in the dynamical evolution of stars associated with the thin and thick disk components. Such corrections may be particularly important when these properties are compared to predictions of chemical evolution models for the entire solar cylinder;

- from the data currently available, we conclude that stellar orbital diffusion is probably a common and important process in the Galactic disk which, at least in a statistical manner, can provide an adequate explanation for the large abundance variations observed among similarly aged stars in the SNBH. At the same time, however, we are forced to conclude that the theory of stellar orbital diffusion alone as presented by WIEL is probably insufficient to explain these abundance inhomogeneities for all stars in the EDV sample (see Chap. 5).

In this section, we have paid attention to some important observational aspects of Galactic chemical evolution. These considerations may help to interpret some of the observations and results presented below. In particular, it is important to realize that the chemical properties of stars observed in the solar neighbourhood (or elsewhere in the Galaxy) are determined by many independent processes which operate simultaneously in the Galactic disk such as stellar orbital diffusion and inside-out enrichment of the disk ISM. As a consequence, huge samples of stars \((N \gg 1000)\) are needed to study specific aspects of Galactic chemical evolution in detail. Until such samples become available in the near future, we have to cope with relatively small samples of stars which may or may not reflect the chemical evolution of the Galactic disk in all its aspects. Severe selection effects may bias our interpretation of specific observational characteristics of the sample stars. For the moment, it seems justified to restrict ourselves to the chemical evolution of the Galactic disk as a whole when dealing with the relatively small samples of stars that are currently available — especially when such samples are confined to the solar neighbourhood — since the sample stars probably do originate from regions widespread throughout the Galaxy.
4.2 Modelling the Age–Metallicity Relation

In this section, we model the mean interstellar [Fe/H] vs. age relation (hereafter AMR) observed for F and G dwarfs present in the Galactic disk. First, we investigate the sensitivity of the predicted AMR to the boundary conditions and main input parameters assumed. Many of the qualitative results obtained for the dependence of the AMR on these specific assumptions apply to other heavy elements as well. Thereafter, we concentrate on the star formation and gas accretion history of the Galactic disk and we select a set of models that meet the observational constraints associated with the AMR.

In the next section, we will examine in detail how the models selected behave with respect to various additional constraints to the present-day stellar content (e.g. main-sequence stars, AGB stars, different types of supernovae, white dwarfs) and interstellar abundances of elements up to Zn observed in the Galactic disk. In the final section, models in best overall agreement with the observations will be discussed. These models will be used as a reference set to which the chemical evolution of nearby galaxies, such as the Magellanic Clouds, can be compared.

4.2.1 Model assumptions

We briefly recall the entire set of initial conditions and assumptions used for each galactic chemical evolution model run:

- boundary conditions, e.g. the galactic age \( t_{ev} \) and total initial mass: \( M_g(t = 0) \) (cf. Sect. 3.1). In particular, we adopt a present-day total galaxy mass \( M_g(t_{ev}) = 1.8 \times 10^{11} \, M_\odot \) (see Table 3.1).
- the adopted set of stellar evolution tracks and stellar yields. These have been described in detail in Chapter 3. Unless stated otherwise, we will use the standard AGB model and the Geneva/Nomoto stellar evolution data (see Sect. 3.3).
- parameters which describe the relative frequency of evolved stars such as AGB stars and SNIa. These include e.g. the initial mass range from which SNIa, SN Ib/c, and SNII originate, and the fraction \( F_{SNII} \) of WD progenitors ending their lives as SNIa (cf. Table 3.3; Sect. 3.1).
- assumptions related to the IMF, SFR, and infall of gas. These include e.g. the IMF slope, the upper and lower stellar mass limits at time of star formation, and the normalisation constant \( S_0 \) of the SFR. We will discuss these assumptions below.

4.2.2 Basic set of models

The base of models from which we start our computations is given in Table 4.1. Parameters related to the IMF and SFR are listed assuming a present-day gas-to-total mass-fraction \( \mu_1 = 0.1 \) and a galactic age \( t_{ev} = 14 \) Gyr (indices 1 and 0 will be used when referring to quantities related to the current and initial epoch of Galactic disk evolution, respectively). The majority of the parameter values used has been chosen arbitrarily within ranges appropriate for the Galactic disk since our main purpose here is to illustrate the sensitivity of the AMR to a number of relevant quantities. In the next section, we will consider specific parameter choices that are in best agreement with the observations.

For the IMF, we either assume a power law \( M(m) \propto m^{-\gamma} \) (label PL in Table 4.1), or we derive the IMF from the present-day mass function observed in the SNBH according to the star formation history adopted (see Sect. 4.3). This is accomplished using either the present-day mass function (PDMF) presented by Scalo (1986; label SCA) or that presented by Rana (1991; RA). Unless stated otherwise, the upper stellar mass limit at birth is taken as \( m_u = 60 \, M_\odot \) (see Sect. 3.1). In columns 2–5, we list the adopted IMF, lower mass limit \( m_l \), IMF-slope \( \gamma \), and the mean stellar mass \( <m> \) at birth (averaged over the Galactic lifetime \( t_{ev} \)).

For the SFR, we consider two principal cases: A) a density dependent SFR assuming no gas infall, i.e. \( C(t) \propto \rho^{n}_{gas} \) with index \( n = 1 \) (see Schmidt 1959\(^3\)) and B) as A) but with gas infall decaying exponentially at a rate \( F(t) \propto \exp(-t/t_{inf}) \) assuming a gas infall time scale \( t_{inf} = 3 \) Gyr. Unless stated otherwise, the normalisation constant of the SFR has been chosen such that \( \mu_1 = 0.1 \) is achieved at \( t = t_{ev} \) (i.e. present epoch).

\(^3\)Schmidt’s star formation function (Schmidt 1959, 1963) \( C(t) \propto \rho^{n}_{gas} \) is given in terms of the volume density of the gas. In terms of gas surface density, the relation is identical both for \( n = 1 \) (no matter the scale of the gas) and for any value of \( n \) provided that the scale height of gas is independent of galactocentric radius over the galactic region considered (cf. Lacey & Fall 1985). We here consider the Galactic disk up to a galactocentric distance of \( \sim 15 \) kpc as a whole and use Schmidt’s law in terms of the gas-to-total mass-ratio \( \mu \).
4.2 Modelling the Age-Metallicity Relation

For each SFR case, we tabulate in columns 6-11 the average past and present SFR (respectively denoted by $<C>$ and $C_1$) and the returned fraction $<R>$ which is a measure for the recycling efficiency of interstellar gas by evolved stars. Quantities related to case B (i.e. with gas infall) are given in columns 9–11. In this case, the initial disk mass has been assumed to be one tenth of the present-day total system mass, i.e. $M_g(0) \equiv \delta_0 M_{tot}(t_{ev})$ where $\delta_0 = 0.1$ (unless stated otherwise).

Table 4.1 Basic set of models: parameters related to the IMF and SFR

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<th>IMF</th>
<th>$m_l$</th>
<th>$\gamma$</th>
<th>$&lt;m&gt;_t$</th>
<th>$&lt;C&gt;^A$</th>
<th>$C_1^A$</th>
<th>$&lt;R&gt;^A$</th>
<th>$&lt;C&gt;^B$</th>
<th>$C_1^B$</th>
<th>$&lt;R&gt;^B$</th>
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<td>7.4</td>
<td>0.49</td>
<td>$\delta_0 = 0.2$</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>PL var</td>
<td>2.35</td>
<td>0.37</td>
<td>16.7</td>
<td>3.2</td>
<td>0.33</td>
<td>15.3</td>
<td>3.8</td>
<td>0.29</td>
<td>$\delta_0 = 0.2$</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>PL 0.1 var</td>
<td>0.46</td>
<td>17.5</td>
<td>4.7</td>
<td>0.36</td>
<td>16.8</td>
<td>5.0</td>
<td>0.32</td>
<td>$\gamma = 2$ to 2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>PL var var</td>
<td>0.18</td>
<td>15.9</td>
<td>3.4</td>
<td>0.34</td>
<td>13.5</td>
<td>5.3+</td>
<td>0.32+</td>
<td>$\delta_0 = 0.2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* IMF computed from the PDMF in the solar neighbourhood.
+ derived for $\mu_1 = 0.2$.

The models listed in Table 4.1 are used to illustrate particular effects of the adopted IMF and SFR on the AMR. Models 1–4 (IMF-slope) are computed for a power-law IMF with various slopes $\gamma$ between 2.35 to 2.9 assuming a fixed lower mass limit $m_l = 0.1 \, M_\odot$. Models 5–8 (lower mass limit) are computed for various values of $m_l$ between 0.01 $M_\odot$ and 0.5 $M_\odot$ using a fixed IMF-slope $\gamma = 2.35$. The IMF used in models 9–15 is computed iteratively from the present-day mass function (PDMF) observed in the SNBH using appropriate correction factors related to the past SFR and metallicity dependent stellar lifetimes (see Sect. 4.3). These models either incorporate an IMF computed according to the PDMF presented by Scalo (1986; models 9–13) or according to the PDMF from Rana (1991; models 14+15) and cover the above range in $m_l$. Models 16–20 deal with the effects of an age-dependent IMF on the AMR. In total, a set of 40 basic models is contained in Table 4.1. These models serve to illustrate the effects of the main model parameters on the resulting AMR.

4.2.3 Model results

In this section, we investigate the sensitivity of the AMR to the following quantities: 1) the IMF, 2) the stellar mass limits at birth, 3) the age of the Galactic disk, 4) the present-day gas-to-total mass ratio, 5) the upper mass limit of SNII, 6) the contribution by SNIa, and 7) the star formation history. We note that the enrichment by SNIa has been excluded (unless stated otherwise) in order to show the effect of the individual model parameters in a more explicit way.
Standard model and observed AMR

In Fig. 4.10 we show the [Fe/H] vs. age relation for the standard model (i.e. model 7A) with a density dependent SFR ($n = 1$), power law IMF ($\gamma = 2.35$), and lower mass limit $m_l = 0.2 M_\odot$. We adopt this model as the standard model since it fits the observed AMR for the set of basic parameters listed in Table 3.1. Galactic evolution time runs from the instant of disk formation, assumed to be 14 Gyr ago, to the current epoch. Observational data provided by the large sample of 329 F and G dwarfs in the SNBH, based on narrowband $ubvy$-$H\beta$ photometry presented by Twarog (1980a, b), has been plotted for comparison.

Twarog’s data (averaged over 1 Gyr age bins) are found to be in good agreement with the more recent samples of Meusinger, Reimann, and Stecklum (1991; hereafter MRS), who reanalysed the AMR using 536 stars both from the Twarog (1980) and Carlberg (1985) data samples and made several improvements in the reduction details, and of Edvardsson et al. (1993) who presented new and accurate data for $\sim$200 disk F and G dwarfs. However, the data of EDV should not be used to determine the mean age-metallicity relation in the SNBH as old, metal-rich stars have been excluded. Except for dwarfs older than 12 Gyr, the data of these three samples are consistent within the observational errors of $\sim$0.1 dex in [Fe/H] at Galactic ages less than 12 Gyr. In contrast, the error-box for the oldest dwarfs may extend up to ages of $\sim$16 Gyr, depending on the theoretical stellar evolution tracks used. It is clear that more accurate age determinations for such stars are needed to constrain the chemical evolution models at these early epochs of disk evolution (cf. Edvardsson et al. 1993).

We included data for open clusters in the Galactic disk presented by Boesgaard (1989) and by Friel & Boesgaard (1990). Ages and metallicities for older clusters, based on medium resolution spectroscopic data, were taken from Friel & Jones (1990) except for the old, metal-rich cluster NGC 6791 (with [Fe/H]$=+0.01$, age $\approx 10$ Gyr) for which the tabulated data is probably in error. Alternatively, NGC 6791 is not a member of the Galactic open cluster system, has been enriched sequentially, or originates from a region much closer to the Galactic center than the SNBH (see below).

Globular cluster data from Kraft (1979) and Armandroff (1989) have been included which cover metallicities [Fe/H] $\lesssim -1.4$ and ages $\gtrsim 12$ Gyr, i.e. the era during which cluster formation in the Galactic halo presumably took place. Evidence in support of globular cluster ages in the range 14–17 Gyr has been reviewed e.g. by Hesser (1993; cf. Chap. 1). Nevertheless, globular cluster ages continue to be a controversial topic, primarily due to uncertainties in stellar evolution theory and scenarios of globular cluster formation. We have assigned an age of 14 Gyr to all globulars included in Fig. 4.10.
Both the cluster and F+G dwarf data included in Fig. 4.10 exhibit substantial scatter in [Fe/H] of ±0.3 dex, clearly in excess of experimental errors (e.g. Edvardsson et al. 1993; see Chap. 5). No corrections were made for orbital diffusion or for other sample inhomogeneities. However, it has been shown that considerable scatter in the AMR remains even when such corrections are made (cf. Sect. 4.1). In particular, the recent analysis of Carraro & Chiosi (1995) reveals that variations in [Fe/H] of ±0.4 dex exist among similarly aged open clusters in the SNBH, even after correcting for radial (and vertical) abundance gradients in the Galactic disk.

We concentrate on the AMR of iron as constrained by the best determined ages and metallicities of F and G dwarfs in the local Galactic disk currently available. Within the error bars of the mean data, a gradually increasing AMR is found with [Fe/H] ≈ −1 at the early epoch of star formation in the disk about 12 Gyr ago, rising to solar metallicity ∼4.5 Gyr ago, and climbing another ∼0.15 − 0.2 dex during the last few Gyr. As discussed in Sect. 4.1, it is probably incorrect to use the observed AMR as representative for the enrichment history of gas and stars in the SNBH. Instead, we will use this AMR as typical for the Galactic disk as a whole, while assuming that the scatter in metallicity among stars observed in the SNBH is representative for the Galactic disk (R < 15 kpc), and we compare it to the basic set of galactic chemical evolution models presented below. The shape of AMR predicted by the standard model is characteristic of exponentially decreasing SFRs (or, more general, for SFRs that were considerably higher in the past than present) and is in good agreement with the observations. In contrast, constant SFR models are inconsistent with the observations. As we will discuss below, the shape of the resulting AMR is predominantly determined by the SFR, IMF, and iron yields of massive stars.

We note that the [Fe/H] ratio for the standard model shown in Fig. 4.10 has been computed using the stellar nucleosynthetic yields of both Fe and H. Therefore, in contrast to many previous investigations, an ad hoc conversion between e.g. the heavy element integrated metallicity Z and the iron abundance was not required.

Initial Mass Function

The impact of the IMF-shape on the [Fe/H] vs. age relation is basically twofold. First, a steeper IMF means an increase of the formation probability of low-mass stars compared to that of high-mass stars, which results e.g. in a direct reduction of the (iron) enrichment by SNII. Second, a steeper IMF results in a decrease of the returned gas fraction (cf. Table 4.1) since low-mass stars deplete gas more efficiently than high-mass stars (m > ∼1 M⊙). Therefore, for a fixed amount of gas converted into stars, a steeper IMF generally yields a reduction of the total number of post main-sequence stars with m > m⊙(tev). Both effects strongly decrease the rate of enrichment by massive stars in case of relatively steep IMF models. In Fig. 4.11 we illustrate these effects for a power-law IMF: an increase in the IMF slope from γ =2.35 to 2.55 (i.e. models 1A and 2A, respectively) leads to a reduction of ∼0.5 dex in [Fe/H] at all galactic ages.

**Figure 4.11** Effects of IMF slope on the AMR. Resulting AMRs are shown for power-law IMFs with γ =2.35 (dash-dotted curve) and 2.55 (dotted), and for IMFs computed iteratively using the PDMF observed in the SNBH, using data from Scalo (1986; solid line) and Rana (1991; dashed). In all cases, the SFR has been normalised such that μ₁ = 0.1.
In case of an exponentially decreasing SFR, the overall level of the AMR is determined mainly by the iron production by SNII at early Galactic phases ($t \lesssim 2$ Gyr). If no gas would be returned to the ISM by evolved stars, the AMR would continue to rise rapidly with age (even for decreasing SFRs). The fact that the AMR flattens is caused by the mixing of low-metallicity gas from relatively long-living, low-mass stars evolving off the main-sequence. Such flattening is consistent with the shape of the observed AMR (cf. Fig. 4.11). At values of $\mu < \sim 0.1$, the [Fe/H] ratio in the ISM approaches the limit set by the yield function of iron for the actual generation of evolving stars (see Sect. 3.4). Note that low-mass stars (i.e. $m \lesssim 1$ M$_\odot$) that do not evolve off the main-sequence within the galactic lifetime $t_{ev}$, determine the rate of gas consumption (and thus the gas fraction $\mu$) at all evolution times.

For IMFs flattening towards low-mass stars, the enrichment by massive stars becomes increasingly important due to the effects of gas depletion discussed above. For instance, IMFs computed using the observed PDMF show a huge reduction (i.e. up to two orders of magnitude) of the formation probability of low-mass stars compared to that for the Salpeter IMF ($\gamma = 2.35$). Conversion of the PDMF to a time-averaged IMF requires knowledge of the star formation history over the lifetime of the galactic region of interest, in order to correct for stars that evolved off the main-sequence in the past and are therefore "missed" in present-day star counts. Under the assumption of a separable SFR, both Scalo (1986) and Rana (1991) computed the IMF using the observed PDMF for stars in the SNBH. Using their PDMF data, we followed a similar procedure according to the specific SFR assumed and galactic age adopted (cf. Sect. 4.3).

In Fig. 4.11 we show the resulting AMRs for IMFs computed from the Scalo and Rana PDMF data using an exponentially decreasing SFR (models 11A and 15A, respectively; detailed IMFs are discussed in Sect. 4.3). Clearly, the overall level of the AMR is increased drastically and illustrates that the formation probability of low-mass stars can strongly affect the resulting AMR. Both models appear inconsistent with the observations for the set of parameter values used, especially in case of the Rana PDMF, since the inclusion of SNIa would heighten the resulting AMRs by at least 0.15–0.2 dex (see below). However, since the detailed SFR history assumed plays an important role as well, further analysis is needed before any conclusions can be drawn from this discrepancy (see Sect. 4.3).

- **Age-dependent IMF**

As the mass spectrum of stars at birth is likely to vary with the environment (e.g. molecular cloud density, ambient radiation field), the assumption of an IMF that is constant in time and space is rather unrealistic (cf. Chap. 2). However, the reason that this assumption is frequently used in spite of its poor approximation of reality, is that the galactic chemical evolution equations can be solved conveniently for a time-invariant IMF.

![Figure 4.12](image.png)

**Figure 4.12** Effects of an age-dependent IMF on the AMR. Resulting AMRs are shown for models with a density dependent SFR ($n = 1$), power law IMF, and age-dependent IMF slope: $\gamma(t) = \gamma_0 + \Delta \gamma [1 - (C(t)/C_0)\beta]$ where $C(t)$ denotes the SFR and $(\gamma_0, \Delta \gamma, \beta) = (2.2, 0.2, 1; \text{ solid line}), (2.2, 0.4, 1; \text{ dash-dotted}), (2, 1, 2; \text{ dotted})$, and $(2.2, 0.4, 1; \text{ but for an exponentially decreasing SFR with } \tau_{decr} = 3 \text{ Gyr; dashed})$. In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$. 

• **Age-dependent IMF**

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4.2 Modelling the Age-Metallicity Relation

We here investigate the resulting AMR for models which incorporate an IMF varying with galactic age. As an example, we consider a power law IMF that favours the formation of high-mass stars at high levels of star formation (see e.g. Larson 1989). In case of a density dependent SFR \((n = 1)\), this means that low-mass star formation preferentially occurs at relatively low gas densities. We simulate this effect by considering an IMF slope that depends on the SFR as:

\[
\gamma(t) = \gamma_0 + \Delta\gamma [1 - (C(t)/C_0)^{\beta}].
\]

Fig. 4.12 shows resulting AMRs for various combinations of \(\gamma_0\), \(\Delta\gamma\), and \(\beta\). Models consistent with the observed AMR appear restricted to values of \(\beta \sim 1\), \(\gamma_0 \sim 2 \pm 0.2\) and \(\Delta\gamma \lesssim 0.5\). Note that the flattening of the AMR is directed by the rate at which the IMF steepens with galactic age. The AMR flattens as soon as the ejection of low-abundant matter by low-mass stars dominates the enriched material returned by high-mass stars (cf. Fig. 4.12). These models illustrate the high sensitivity of the AMR to variations in the IMF with galactic age.

Stellar mass limits at birth

In Fig. 4.13 we show theoretical AMRs for different choices of the lower stellar mass limit \(m_l = 0.05\), 0.1, and 0.2 \(M_\odot\) (models 1A, 6A, and 7A, respectively) in case of the Salpeter IMF (with fixed slope \(\gamma = 2.35\)). The overall level of the AMR is found to increase by \(\sim 0.4\) dex when \(m_l\) is raised from 0.05 to 0.2 \(M_\odot\). In general, the formation probability of massive stars increases with \(m_l\). As discussed before, this is due to the increase in both the average stellar mass formed and the returned gas fraction with \(m_l\) (cf. Table 4.1). More precisely, the magnitude of the shift in the AMR due to variations in \(m_l\) is determined by the detailed slope of the IMF at the low mass end. However, this effect is negligible for an IMF computed from the PDMF (Scalo 1986; models 10A, 11A, and 12A, respectively; see Fig. 4.13), that roughly remains constant at low masses \((m < \sim 0.5 \ M_\odot)\). In general, the shape of the AMR is insensitive to the adopted value of \(m_l\) for values of \(m_l \lesssim 1 \ M_\odot\).

![Figure 4.13 Effects of the lower stellar mass limit on the AMR. Results are shown for AMRs computed with \(m_l = 0.05\), 0.1, and 0.2 \(M_\odot\) (solid lines; bottom to top, respectively) using the Salpeter IMF (\(\gamma = 2.35\)). For comparison, the same models but for the IMF computed using the PDMF (Scalo 1986) are shown (dashed curves). In all cases, the SFR was normalised to satisfy \(\mu_1 = 0.1\).](image)

In general, a maximum value \(m_{l\ max}\) exists at which the overall level of the AMR exceeds the observed level (for a given IMF, SFR, and set of input parameters). According to the model inputs described above while excluding the iron contribution by SNIa, we find that \(m_{l\ max} \sim 0.2 \ M_\odot\) for a power law IMF with \(\gamma = 2.35\), and similarly \(m_{l\ max} \sim 0.38 \ M_\odot\) (\(\gamma = 2.55\)), 0.57 \(M_\odot\) (2.7), and 0.80 \(M_\odot\) (2.9). Thus, the observed AMR can be explained equally well using any combination of \(\gamma\) and \(m_l\) along the relation shown in Fig. 4.14 while assuming a density dependent SFR \((n = 1)\) normalised such that \(\mu_1 = 0.1\) is achieved, no contribution by SNIa, and parameter values as in Table 3.3. The corresponding relation for models which include the enrichment by SNIa is shown for comparison (i.e. \(m_{l\ SNIA} = 2.5 \ M_\odot\) and \(F_{SNIA} = 0.015\); cf. Table 3.3). In this case, the value of \(m_{l\ max}\) at a given value of \(\gamma\) is considerably smaller. The main point illustrated in Fig. 4.14 is that observational constraints other than the AMR are needed to determine the IMF-shape and lower mass limit of stars formed in a galactic region such as the Galactic disk. Furthermore, a small and limited range in \(m_l\) is consistent with the observations for a given IMF (and \textit{vice versa}).
Modelling the chemical evolution of the Galactic disk

Figure 4.14 Combinations of $\gamma$ and $m_l$ for which the resulting AMR is consistent with the observations. A density dependent SFR ($n = 1$) and model parameters as in Table 3.3 were assumed: curves are drawn both for models with SNIa enrichment (open circles; $m_{\text{SNIA}} = 2.5$ $M_\odot$ and $F_{\text{SNIA}} = 0.015$) and without SNIa enrichment (filled circles). Preferred values of $\gamma$ and $m_l$ for stars formed in the Galactic disk are indicated (hatched region). Values of $\gamma \lesssim 2.2$ and $m_l \lesssim 0.05$ $M_\odot$ are not supported by the observations (dashed lines; see Chap. 2).

- Age-dependent lower stellar mass limit

We investigate the effect of an age-dependent lower mass limit $m_l(t)$ on the AMR in a manner similar to that in case of an age-dependent IMF slope $\gamma(t)$. We concentrate on models for which $m_l$ is large at early phases of star formation in the Galactic disk and decreases with age to $m_l \sim 0.05 - 0.1$ $M_\odot$ (as nowadays observed in the SNBH; cf. Scalo 1986). In this case, the early rise of the AMR proceeds at a substantially higher rate than models for which $m_l$ is assumed constant in time. In addition, the amount of metal-poor gas returned by low-mass stars is substantially larger and the flattening of the AMR is more pronounced than for the former models.

Figure 4.15 Effect of an age-dependent lower stellar mass limit on the AMR. Resulting AMRs are shown for models with $m_l(t) = m_l(0) - \Delta m_l[1 - (C(t)/C_0)]$ for $(m_l(0), \Delta m_l, \gamma) = (0.5$ $M_\odot$, $0.4$ $M_\odot$, $2.35$; top solid curve) and $(1$ $M_\odot$, $0.5$ $M_\odot$, $2.9$; top dashed curve). For comparison, corresponding AMRs are shown in case of a constant $m_l(t) = m_l(0) = 0.5$ and $0.1$ $M_\odot$ (bottom solid and dashed lines, respectively). In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$.

In Fig. 4.15 we show the AMRs for a Salpeter IMF with $m_l(t) = m_l(0) - \Delta m_l[1 - (C(t)/C_0)^\beta]$ with $m_l(0) = 0.5$ $M_\odot$, $\Delta m_l = 0.4$ $M_\odot$, and $\beta = 1$. In this case, the AMR remains roughly constant at recent times due to the increase in gas consumption by low-mass stars while the formation probability of massive stars decreases. At early phases, massive star formation is enhanced and adds to the overall level of the AMR.
We illustrate these effects also for a model with a power-law IMF with $\gamma = 2.9$, $m_1(0) = 0.5 \, M_\odot$, $\Delta m_1 = 0.5 \, M_\odot$, and $\beta = 1$. The possible ranges over which $m_1$ and $\gamma$ may have varied in the past are limited by the shape of the observed AMR. For instance, rapid variations of $m_1$ with age would lead to large fluctuations in the mean AMR which are inconsistent with the observations. However, such fluctuations in the AMR would be suppressed in case of simultaneous variations in the IMF slope. We conclude that the shape of the AMR is very sensitive to the variation of the lower mass limit with galactic age. Furthermore, the lower stellar mass limit at birth is important in determining the overall level of the AMR for IMFs which continue to increase towards low-mass stars. Therefore, we emphasize that $m_1$ is an important ingredient in galactic chemical evolution models.

As far as the upper stellar mass limit at birth is concerned, we discussed in Sect. 3.3 arguments in favour of $m_u = 60 \, M_\odot$ (see also Chap. 2). Variations in $m_u$ between $\sim 60$ and $\sim 120 \, M_\odot$ are expected to have a negligible effect on the present-day gas fraction. This effect is even more pronounced for the Scalo IMF $\sim 10$ and $17 \, Gyr$, respectively. In the latter case, the assumed lifetime $t_{\text{ev}}$ increases with the adopted IMF or present-day gas fraction. The time of onset of star formation in the Galactic disk is a crucial quantity in chemical evolution models. The age of the stellar disk in the Galaxy may have been different from those in the past (e.g. Grenon 1989; Sect. 4.1). We here assume a Galactic disk age of $t_{\text{disk}} = 11 \pm 1 \, Gyr$. In our models, the value of $t_{\text{disk}}$ is not used explicitly but is determined by: 1) the assumed lifetime $t_{\text{rmev}}$ of the Galaxy as a whole, and 2) the instant of onset of main star formation in the disk as set by the detailed variation of the SFR with galactic age assumed (see below).

For the Galaxy as a whole, we will adopt an age of $t_{\text{ev}} = 14 \pm 3 \, Gyr$ as suggested by recent age determinations of globular clusters (e.g. Walker 1992; Hesser 1993; Sandage 1993; Shi 1995; Chaboyer et al. 1996). The latter authors presented arguments in support of a considerable age spread of $\gtrsim 5 \, Gyr$ among the bulk of the globular clusters observed in the outer halo of the Galactic disk. This suggests that, independent of the uncertainties involved with the absolute age calibration of globular clusters, the onset of star formation in the outer Galaxy occurred at least $14 \, Gyr$ ago.

Fig. 4.16 illustrates the influence of galactic age on the AMR. It can be seen that the shape of the resulting AMR is rather sensitive to the instant of onset of star formation in the disk. This is primarily due to the sensitivity of the metal-abundance to the actual gas fraction (cf. Eq. 3.12). For models ending at a given gas fraction (e.g. $\mu_1 = 0.1$), larger values of $t_{\text{ev}}$ yield higher present-day levels of the AMR. This is related to the fact that the stellar turnover mass $m_0(t_{\text{ev}})$ decreases with increasing galactic age, e.g. $m_0 = 0.91, 0.82,$ and $0.76 \, M_\odot$ for $t_{\text{ev}} = 10, 14,$ and $17 \, Gyr$, respectively (cf. Fig. 4.6). For larger galactic lifetimes, the total gas reservoir is less rapidly exhausted since the total amount of matter returned by low-mass stars increases with $t_{\text{ev}}$. Part of this matter is converted into massive stars within the galactic lifetime $t_{\text{ev}}$ and the overall level of the AMR is increased accordingly. This effect is even more pronounced for the Scalo IMF models (see Fig. 4.15), for which the returned gas fraction increases from $< R > = 0.42$ to 0.53 for $t_{\text{ev}} = 10$ and $17 \, Gyr$, respectively. In the latter case, $\sim 10\%$ of the total system mass is extra available for star formation which results in a corresponding increase in the AMR. For the Salpeter IMF models, the effect is negligible because this IMF continues to rise towards low-mass stars so that the returned gas fraction $(< R > \sim 0.30)$ does not vary considerably over the range in galactic lifetime considered.

We conclude that the $[\text{Fe}/\text{H}]$ ratio at a given value of $\mu(t_{\text{ev}})$ basically is determined by the stellar recycling efficiency of gas averaged over the galactic lifetime $t_{\text{ev}}$. Thus, both the shape and overall level of the resulting AMR heavily depends on the adopted age of the Galactic disk since the onset of star formation. Current estimates for the age of the Galaxy tend to converge to $t_{\text{ev}} = 14 \pm 3 \, Gyr$ and uncertainties in the shape of the AMR due to the adopted value of $t_{\text{ev}} = 14 \, Gyr$ are similar in magnitude as those due to e.g. the adopted IMF or present-day gas fraction.
Figure 4.16 Effect of galactic age on the AMR. Results are shown for \( t_{\text{ev}} = 17, 14, \) and \( 10 \) Gyr assuming a density dependent SFR model \((n = 1)\) with either a Scalo IMF (solid lines) or Salpeter IMF (dashed). For each model, the SFR has been normalised such that the condition \( \mu_1 = 0.1 \) is satisfied.

Present-day gas-to-total mass-ratio

The present-day gas fraction \( \mu_1 \) in the Galactic disk is determined by the initial disk mass, the star formation history of the disk, and the amount of gas accreted from the Galactic halo since the onset of star formation in the disk. We investigate the sensitivity of the theoretical AMR to the adopted value of \( \mu_1 \) in the disk for which observational values are generally in the range \( \mu_1 = 0.05 - 0.2 \) (see Sect. 4.1).

Figure 4.17 Effect of the present-day gas fraction on the AMR. Results are shown for two normalisations of the SFR such that \( \mu_1 = 0.05 \) (solid curve) and \( \mu_1 = 0.2 \) (dashed), respectively. A density dependent SFR model \((n = 1)\) with the Salpeter IMF with \( m_1 = 0.1 \, M_\odot \) has been assumed.

In Fig. 4.17 we plot the resulting AMRs for two normalisations of the SFR corresponding to \( \mu_1 = 0.05 \) and 0.2, respectively. For SFRs which vary directly proportional to the gas density \((n = 1)\), the precise value of the normalisation constant \( C_0 \) of the SFR (or initial system mass) has a cumulative effect both on \( \mu(t) \) and on the AMR. In general, a reduction of \( C_0 \) (or increase in \( M_L(0) \)) results in larger values of \( \mu \) and, consequently, leads to smaller [Fe/H] ratios at all evolution times.

If the SFR normalisation constant is reduced from \( C_0 \sim 60 \, M_\odot \, \text{yr}^{-1} \) \((\mu_1 = 0.05)\) to \( 30 \, M_\odot \, \text{yr}^{-1} \) \((\mu_1 = 0.2)\), the shift in the overall level of the AMR is about 0.3 dex which is primarily caused by the reduction of the total number of massive stars formed. However, the effect is weakened by the amount of unprocessed gas returned by low-mass stars (for the \( \mu_1 = 0.05 \) model this amount is about twice as large as for \( \mu_1 = 0.2 \)).
When IMFs are considered that flatten towards low-mass stars (e.g. the Scalo and Rana IMFs), the shift in the AMR is \( \sim 0.3 \) dex as well. For such IMFs, however, the amount of unprocessed gas returned by low- and intermediate mass stars is considerably larger. Therefore, the overall level of the AMR is considerably higher than for the Salpeter IMF model ending at the same \( \mu_1 \) (cf. Fig. 4.12). As a consequence, the magnitude of the shift in the AMR due to changes in \( \mu_1 \) as determined by the SFR normalisation constant (and initial system mass) is relatively insensitive to the IMF adopted (for reasonable IMFs).

Another way to reduce \( \mu_1 \) is to extend the period of time during which star formation in the Galactic disk did occur (e.g. 14 instead of 11 Gyr). In this case, the increase in the AMR is found to be less than \( \sim 0.1 \) dex at values \( \mu_1 \lesssim 0.2 \).

We conclude that uncertainties in \( \mu_1 \) and/or the age of the Galactic disk do account for an uncertainty of \( \pm 0.15 \) dex in the overall level of the AMR for exponentially decreasing SFR models ending at \( \mu_1 = 0.05-0.2 \). We emphasize that this is true for the AMRs of any element heavier than helium. Note that scaling the SFR and the total system mass simultaneously, in such a way that the present-day value of \( \mu \) remains unchanged, does not affect the solutions of the galactic chemical evolution equations (provided that the variation of the SFR with \( \mu \) remains roughly the same; cf. Appendices to Chap. 3). In the following we will adopt \( \mu = 0.1 \) as the most probable value for the present-day gas-to-total mass-ratio in the Galactic disk (e.g. Twarog 1980; Güsten & Mezger 1983; Scalo 1986; Kulkarni & Heiles 1987).

**SNII contribution to the AMR**

Observational constraints to the current SNII rate and to the iron peak element contribution of SNIa relative to SNII provide valuable information about the importance of massive stars (\( m > \sim 20-25 M_\odot \)) in maintaining the abundances of specific heavy elements in the ISM (cf. Sect. 3.4). Conversely, the AMR can be used to constrain model assumptions related to the average past and current formation rates of e.g. SNII and SNIa. However, such constraints do also concern the IMF and star formation history adopted and careful analysis is needed to draw reliable conclusions from the AMR alone.

We here consider the impact on the AMR of the upper mass limit \( m_{\text{SNII}} \) of SNII, i.e. the initial stellar mass above which stars presumably will become a black hole (instead of ending their lives as a neutron star; cf. Sect. 3.2). For stars more massive than \( m_{\text{SNII}} \), most heavy elements will be locked up in the rapidly contracting core provided that the collapse does not lead to an explosive shock wave (see Sect. 3.4). Fig. 4.18 shows that the overall level of the AMR is reduced by \( \sim 0.25 \) dex when \( m_{\text{SNII}} \) is decreased from 45 to 25 \( M_\odot \) (model 1A). A somewhat smaller shift of \( \sim 0.2 \) dex is found when for the Scalo IMF model (i.e. model 11A). In principle, the shift in the AMR of a given element for various values of \( m_{\text{SNII}} \) can be derived directly from the cumulative IMF weighed yields in Fig. 3.16 (i.e. without the need to solve the galactic chemical evolution equations).

![Figure 4.18](image-url) Effect of the upper mass limit of SNII on the AMR. Results are shown for a density dependent SFR model \( (n = 1) \), Salpeter IMF with \( \mu = 0.1 M_\odot \), and \( m_{\text{SNII}} = 25 \) and 45 \( M_\odot \) (bottom and top dashed lines), respectively. For comparison, corresponding AMRs are shown in case of the Scalo IMF (solid curves). In all cases, the SFR was normalised to satisfy \( \mu_1 = 0.1 \).
In the following, the minimum mass above which stars presumably become SNII will be fixed at $m_{\text{SNII}}^{\text{min}} = 8 \, M_{\sun}$. This value is mainly based on theoretical models for the core collapse of massive stars and does depend on the initial metallicity of the star (cf. Sect. 3.3). For solar metallicity stars, SNII progenitors more massive than $\sim 12 \, M_{\sun}$ are predicted to contribute substantially to the interstellar iron abundance. We note that the IMF-weighted enrichment of stars with $m \lesssim 15 \, M_{\sun}$ is very important (if not dominant). This implies that the AMR is rather sensitive to the precise value of $m_{\text{SNII}}^{\text{min}}$ as well as to its variation with metallicity. Nevertheless, we will concentrate in the following on the detailed value of the upper mass limit for SNII.

It is evident that, depending on the adopted IMF and SFR, the overall levels of the AMR observed for different elements can provide independent constraints to the enrichment contribution by SNII+Ib/c and SNIa (Sect. 4.3). Here we find that the detailed choice of $m_{\text{SNII}}^{\text{min}}$ (between $\sim 20$ and $60 \, M_{\sun}$) can result in a shift of at most 0.3 dex in the AMR of iron. The shape of the AMR remains unchanged for different values of $m_{\text{SNII}}^{\text{min}}$. However, we will show that when SNIa are taken into account, the shape of the AMR is altered at galactic ages at which SNIa start to dominate the iron enrichment.

### SNIa contribution to the AMR

We investigate the sensitivity of the AMR to the heavy element contribution by SNIa in terms of: 1) the progenitor mass range ($m_{\text{SNIa}}^{\text{min}}$, $m_{\text{SNIa}}^{\text{max}}$) of stars that leave an accreting WD remnant massive enough to explode as SNIa (for simplicity, we will assume $m_{\text{SNIa}}^{\text{min}} = 8 \, M_{\sun}$), and 2) the fraction $F_{\text{SNIa}}$ of such WDs which ultimately end as SNIa. Furthermore, we will assume that $F_{\text{SNIa}}$ is constant in time (i.e. independent of the WD mass or initial metallicity; cf. Sect. 3.3). Constraints related to the frequencies of SNIa will be considered in Sect. 4.3.

- **SNIa delay time**
  
  The heavy element enrichment by a SNIa associated with a star formed at galactic age $t$ will take place at age $t + \Delta t_{\text{SNIa}}$ where $\Delta t_{\text{SNIa}}$ is determined by: 1) the lifetime of the progenitor star that leaves the WD, and 2) the characteristic delay time between the formation of the WD remnant and the actual SNIa occurrence. In turn, the delay time is either determined by the time required for the accreting WD to become massive enough to collapse (in the single WD scenario) or by the time needed to allow for coalescence of the WDs after the orbital separation of the binary system has become sufficiently small (in the double WD scenario; cf. Sect. 3.3).

  In general, $\Delta t_{\text{SNIa}}$ is a complex function of progenitor mass and further depends on the detailed evolution scenario of the WD until the SNIa. For the most massive SNIa progenitor, i.e. $m_{\text{SNIa}}^{\text{min}} \sim 8 \, M_{\sun}$, the typical delay time is estimated $\Delta t_{\text{SNIa}} = 2.5 \, \text{Gyr}$ after formation of the progenitor, dominated by the time for the WD to end as SNIa (see Sect. 4.3). For the least massive SNIa progenitor, i.e. $m_{\text{SNIa}}^{\text{min}} \sim 2.5 \, M_{\sun}$, a substantial part of the delay time comes from the main-sequence lifetime of the SNIa progenitor (e.g. $\tau_{\text{MS}}(2.5 \, M_{\sun}) \sim 8 \times 10^8 \, \text{yr}$ for $Z = Z_{\odot}$; see Sect. 3.1) and for such stars presumably $\Delta t_{\text{SNIa}} \gtrsim 3.5 \, \text{Gyr}$. We will return to the value of $\Delta t_{\text{SNIa}}$ in more detail in Sect. 4.4. For now, we will assume that the SNIa delay time is determined only by the lifetime of the progenitor star leaving a WD, i.e. $\Delta t_{\text{SNIa}} = \tau(m)$, so that the SNIa is assumed to occur instantaneously when the WD is formed. In this case, we can simply simulate the effects of an additional delay time due to the detailed evolution of the WD by reducing the lower mass limit for SNIa.

- **Lower mass limit of SNIa**

  In Fig. 4.19 we show the resulting AMRs for various values of $m_{\text{SNIa}}^{\text{min}} = 3.5$, 2.5, and 1.5 $\, M_{\sun}$ assuming a density dependent SFR ($n = 1$) and Salpeter IMF (model 1A). Furthermore, we assumed $F_{\text{SNIa}} = 0.015$ and $m_{\text{SNIa}}^{\text{SNII}} = 30 \, M_{\sun}$ (cf. Table 3.3). It can be seen that the contribution by SNIa to the AMR can be substantial and may increase the present-day [Fe/H] ratio by as much as 0.45 dex as compared to the case in which SNIa are ignored. In particular, reducing $m_{\text{SNIa}}^{\text{min}}$ from 3.5 to 1.5 $\, M_{\sun}$ gives rise to an increase in the AMR of $\sim 0.3$ dex.

  For the same models, Fig. 4.20 shows the detailed variation of the total ejection rate of iron (i.e. the sum of newly synthesized iron and iron initially present in the material out of which the progenitor star formed) with galactic age, both for SNII+Ib/c, SNIa, and AGB stars. In case $m_{\text{SNIa}}^{\text{SNII}} \gtrsim 2.5 \, M_{\sun}$, the ejecta of SNIa are returned at relatively short time scales and closely follow the exponential decrease of the SFR (in the same manner as do the ejecta of SNII+SNIb/c). In this case, about 40% of the total ejection rate of newly synthesized iron originates from SNIa while SNII dominate at all galactic ages. Predicted present-day iron ejection rates of SNII and SNIa are $E_{\text{Fe}} = 3.8 \times 10^{-3}$ and $1.6 \times 10^{-3} \, M_{\sun} \, \text{yr}^{-1}$, respectively. Interestingly, the iron ejection rate of AGB stars, i.e. $E_{\text{Fe}} = 2.6 \times 10^{-3} \, M_{\sun} \, \text{yr}^{-1}$, constitutes $\sim 25\%$ of the total stellar iron
4.2 Modelling the Age-Metallicity Relation

Figure 4.19 Effect of the lower mass limit of SNIa on the AMR. Results are shown for $m_{\text{SNIa}} = 3.5 \, M_\odot$ (solid curve), $2.5 \, M_\odot$ (dashed), and $1.5 \, M_\odot$ (dotted). A density dependent SFR model ($n = 1$), Salpeter IMF with $m_i = 0.1 \, M_\odot$, and $F_{\text{SNIa}} = 0.015$ were assumed. For comparison, the corresponding AMR is shown when the enrichment by SNIa is omitted (dash-dotted curve).

Ejection rate and is even larger than that of SNIa. This is true even though AGB stars only return metals that were initially present in the material from which they formed. The reason for their high contribution is that, for an exponentially decreasing SFR model, the total number of AGB stars with low-mass progenitors (i.e. $\lesssim 1.5 \, M_\odot$) increases rapidly with galactic age. As a consequence, the ejection rate of iron by AGB stars becomes approximately constant a few Gyr after the onset of star formation in the disk. In contrast, the iron ejection rates of SNII+Ib/c and SNIa closely follows the SFR and decreases exponentially with galactic age.

Fig. 20c shows the variations in the mean iron abundances $Y_{Fe}$ within the stellar ejecta corresponding to Fig. 4.20a. For AGB stars, SNIa, and SNII+SNIIb/c, mean values of $Y_{Fe}$ were derived as the ratio of the total IMF-weighed stellar iron ejection rate and the total rate of matter returned by the same stars. In case of SNIa, we divided the iron ejection rate of all SNIa by the total rate of matter returned by all AGB stars (i.e. with $m_i(t) \lesssim m \lesssim 8 \, M_\odot$). This is done to allow for a comparison with the iron initially present and returned by AGB stars. To derive SNIa abundances for individual stars in the single WD scenario, the abundances $Y_{Fe}$ should be multiplied by a factor $(1/F_{\text{SNIa}})$. Iron abundances predicted in the ejecta of SNII+Ib/c are similar to the solar iron abundance by mass: $Z_{Fe,\odot} \sim 4 \times 10^{-3}$ (e.g. Anders and Grevesse 1989). For comparison, the present-day mean iron abundance predicted in the stellar ejecta is $Y_{Fe} \sim 3.5 \times 10^{-3}$.

Mean iron abundances in SNII+Ib/c gradually increase with age primarily because of the increase of their initial abundances with galactic age. The corresponding decrease of $Y_{Fe}$ for the ejecta of SNIa just reflects the increase in the total mass ejection rate of their low-mass progenitors which dilute the SNIa enrichment strongly (again this is a result of the normalisation of the SNIa ejecta to the matter returned by AGB stars). It can be seen that the value of $Y_{Fe}$ averaged over all stellar ejecta remains roughly constant with galactic age. This is mainly due to the increase in the ejection rate of unprocessed material returned by AGB stars. As can be seen, the interstellar iron abundance becomes equal to the mean iron abundance within the enriched stellar ejecta for low values of the gas-to-total mass-ratio $\mu$.

When $m_{\text{SNIa}} \lesssim 1.5 \, M_\odot$, the shape of the AMR changes considerably (cf. Fig. 4.19) since in this case most SNIa return their ejecta to the ISM with a typical delay time $\tau_{\text{MS}}(1.5 \, M_\odot) \sim 2.2$ Gyr. Consequently, the SNIa ejection rate of iron peaks $\sim 2.2$ Gyr after the onset of main star formation in the exponentially decreasing SFR model (cf. Fig. 4.20b). A reduction of $m_{\text{SNIa}}$ from 2.5 to 1.5 $M_\odot$ increases the mean iron ejection rate of SNIa by a factor $\sim 2.5$. In this case, SNIa dominate the iron ejection rate at all evolution times. Similarly, the iron abundance averaged over all stellar ejecta is considerably larger than in case $m_{\text{SNIa}} = 2.5 \, M_\odot$. Furthermore, a slight increase in $E_{Fe}$ as well as in $Y_{Fe}$ is found both for SNII+Ib/c and AGB stars. This is due to the increase in the initial iron content of these stars as a result of the increase of the overall level of the AMR (cf. Figs. 4.20c and d).
Figure 4.20 Variation of the stellar ejection rate of iron (top panels) and mean iron yield (i.e. approximately equal to the mean iron abundance) within the stellar ejecta (bottom) vs. galactic age for the upper two AMRs plotted in Fig. 4.19 with \( m_{\text{SNIA}}^{\text{SNIa}} = 2.5 \, M_\odot \) (left panels) and \( m_{\text{SNIA}}^{\text{SNIa}} = 1.5 \, M_\odot \) (right), respectively. Results are shown for the ejecta of SNII+Ib/c \( (m > 8 \, M_\odot; \text{dashed curves}) \), SNIa \( (m_{\text{SNIA}}^{\text{SNIa}} < m < 8 \, M_\odot; \text{dash-dotted}) \), and AGB stars \( (m_\odot(t) < m < 8 \, M_\odot; \text{dotted}) \). Total iron ejection rates are indicated by solid curves. For comparison, the AMRs from Fig. 4.19, which give the variation of the iron abundance in the ISM with galactic age, are repeated in the bottom panels (thick solid lines). Mean iron abundances in the ejecta of SNIa were normalised to the total amount of matter returned by AGB stars (see text).

- Fraction of WDs that ultimately end as SNIa

In the previous section, we discussed the effect of \( m_{\text{SNIA}}^{\text{SNIa}} \) on the iron enrichment by SNIa. In our models, the rate of SNIa is determined also by \( F_{\text{SNIA}} \), i.e. the fraction of all WDs left by stars in the mass range \( [m_{\text{SNIA}}^{\text{SNIa}}, m_{\text{SNIA}}^{\text{SNIa}}] \) which ultimately end as a SNIa. Fig. 4.21 shows resulting AMRs for values of \( F_{\text{SNIA}} = 0.005, 0.015 \) and 0.025, respectively, assuming \( m_{\text{SNIA}}^{\text{SNIa}} = 2.5 \, M_\odot \) (further model 1A). When \( F_{\text{SNIA}} \) is increased from 0.005 to 0.025, the AMR is shifted upwards by \( \sim 0.2 \) dex.

A closer examination of the AMR in case \( F_{\text{SNIA}} = 0.025 \) reveals that SNIa dominate the iron ejection rate at early evolution times (i.e. during the first Gyr after the onset of star formation in the disk; cf. Fig. 4.20). Such models are in marked contrast with the observational idea that SNII are the dominant (or only) contributors to the iron enrichment at early epochs in the evolution of the Galactic disk (see Sect. 4.5). This would imply that values of \( F_{\text{SNIA}} \geq 0.02 \) are excluded by the observations. For a value of \( F_{\text{SNIA}} = 0.005 \), the mean present-day ejection rates of iron by SNII and SNIa are \( 4 \times 10^{-3} \) and \( 5 \times 10^{-4} \, M_\odot \, \text{yr}^{-1} \), respectively. In this case, the mean value of \( Y_{Fe} \) in the ejecta of all stars and of SNIa, respectively, is reduced by 0.3 and 0.7 dex as compared to the \( F_{\text{SNIA}} = 0.025 \) case. Note that for values of \( F_{\text{SNIA}} \sim 0.005 \), the resulting AMR is still consistent with the observations (at least during the last 10 Gyr).

We conclude that the particular choices of \( m_{\text{SNIA}}^{\text{SNIa}} \) and \( F_{\text{SNIA}} \) have considerable impact on the AMR. The iron contribution by SNIa may shift the AMR by \( \leq 0.4 \) dex for reasonable values of \( m_{\text{SNIA}}^{\text{SNIa}} = 2.5 \pm 1 \, M_\odot \)
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Figure 4.21 Effect on the AMR of the fraction of WDs which ultimately end as SNIa. Results are shown for values of $F_{\text{SNIa}} = 0.005$ (dotted curve), 0.015 (dashed) and 0.025 (solid). A density dependent SFR model ($n = 1$), Salpeter IMF with $m_1 = 0.1 \, M_\odot$, and $m_{\text{SN}} = 2.5 \, M_\odot$ were assumed. For comparison, the corresponding AMR is shown when the enrichment by SNIa is omitted (dash-dotted curve).

Figure 4.22 Stellar ejection rate of iron (top panels) and mean iron abundance within the stellar ejecta (bottom panels) vs. galactic age for the models shown in Fig. 4.22 with $F_{\text{SNIa}} = 0.005$ (left side) and 0.025 (right). A lower mass limit for SNIa of $m_{\text{SN}} = 2.5 \, M_\odot$ has been assumed. Curves have the same meaning as in Fig. 4.20.
and $F_{\text{SNII}} = 0.015 \pm 0.010$. The effect of SNIa becomes smaller when the overall level of the AMR is higher. We note that the contribution by SNIa depends on the IMF as well. In case of the Salpeter IMF, the iron yields of SNIa progenitors generally do not exceed those of SNII. However, when the formation probability of stars in the mass range $1-8 \, M_\odot$ is enhanced (e.g. for a Scalo IMF), the contribution by SNIa dominates over that by SNII (cf. Table 3.12).

The relative contribution of the two main sources of interstellar iron, i.e. SNIa and SNII+Ib/c, plays a key role in galactic chemical evolution. In principle, the AMR can be explained by models in which either SNIa or SNII+Ib/c act as the dominant supplier of interstellar iron. As we will discuss in Sect. 4.4, these two types of models give rather different results e.g. for the abundance-abundance variations in the disk ISM.

Star formation history

We investigate the impact of the galactic star formation history (SFH) on the AMR. A distinction is made between SFR models without and with gas infall. In view of the limited information about the early phase of disk formation, we will consider infall primarily as a mechanism which may regulate the shape of the AMR (especially at early evolution times). Empirical SFHs will be considered at the end of this section. Unless stated otherwise, we will use the Salpeter mass function with $m_1 = 0.1 \, M_\odot$ and model parameters as given in Table 3.3.

- Models without gas infall

Fig. 4.23 shows resulting AMRs for density dependent SFR models $C(t) \propto \mu^n$ without gas infall. Such models may be appropriate to the Galaxy if most of the Galactic disk ISM settled before the onset of main star formation therein (see Chap. 2). For the SFR model with $n = 1$, the SFR averaged over the lifetime of the disk, i.e. $t_{\text{ev}} = 14 \, \text{Gyr}$, is $<C> \sim 15 \, M_\odot \, \text{yr}^{-1}$. This value will be used as reference value to the absolute SFR scale when comparing the models described in this section. The rate at which the theoretical AMR increases with age, both at early and late epochs in the evolution of the disk, suggests that constant SFR models ($n \sim 0$) are not supported by the observations. In contrast, density dependent SFR models with $n = 1 - 2$ appear consistent with the shape of the observed AMR (although this depends as well on e.g. the adopted IMF).

For a given IMF and stellar mass boundaries, a higher sensitivity of the SFR to the gas density (i.e. $n \approx 2$) results in a more rapid increase of the [Fe/H] ratio at early epochs in the evolution of the disk (cf. Fig. 4.23). Thereafter, the total amount of gas returned by low-mass stars flattens the AMR more severely as compared to models with $n \sim 1$. We conclude that SFR models with $n \gtrsim 1.5$ for the Galactic disk probably can be excluded on the basis of the observed AMR.

![Figure 4.23](image-url)

**Figure 4.23** Impact of the star formation history on the AMR: models without infall. **Left panels:** normalised SFR vs. galactic age. The mean SFR for the "standard" model, i.e. $<C> \sim 15 \, M_\odot \, \text{yr}^{-1}$, is indicated as reference to the absolute SFR scale whenever possible (full dot). The following SFR models are shown: constant SFR ($n = 0$; dotted line), density dependent SFR with $n = 1$ (dashed), $n = 1.5$ (solid), and $n = 2$ (thick solid), and a bimodal SFR (dash-dotted). In all cases, the SFR has been normalised such that $\mu_1 = 0.1$ is achieved. **Right panels:** corresponding AMRs assuming the Salpeter IMF with $m_1 = 0.1 \, M_\odot$. 


This conclusion is in agreement with the finding that SFR models with $n \gtrsim 1.3$ are inconsistent with observations of star formation in nearby galaxies (e.g. Dopita 1985, 1990; Donas et al. 1987; Rana & Williams 1988; Dopita & Ryder 1995).

For comparison, we consider a bimodal SFR model (see Sect. 2) for which the formation of low-mass stars (i.e. $m \lesssim 1 \, M_\odot$) proceeds at a constant rate, independent of galactic age, and the formation rate of high-mass stars varies proportional to the gas density ($n = 1$; cf. Fig. 4.23). In this case, the mean stellar mass formed decreases rapidly with age. The shape of the resulting AMR is similar to that in case of a constant SFR model. This is due to the fact that the depletion of gas is determined mainly by the accumulation of long-living, low-mass stars formed ($m \lesssim 1 \, M_\odot$). Note that the present-day [Fe/H] ratio is roughly equal (within 0.1 dex) for all SFR models.

We conclude that the shape of the AMR is sensitive to the assumed formation history of both low and high-mass stars. For exponentially decreasing SFRs in particular, the metal-poor material returned by low-mass stars formed during the early epoch of Galactic evolution is very important in regulating the enrichment efficiency of the disk ISM by more massive stars. In principle, this sensitivity allows one to derive the variation of the global SFR with galactic age using accurate measurements of the AMR for different elements, provided that other model parameters can be constrained tightly by independent observations.

- Models with gas infall

Fig. 4.24 shows resulting AMRs for SFR models including gas infall. We assumed a disk initial-to-final mass ratio $\delta_0 = 0.1$ and normalised the infall-rate such that the present-day system mass, $M(t_{\text{eq}})$, is equal to the initial system mass $M_g(0) = 1.8 \times 10^{11} \, M_\odot$ assumed in the no infall case (cf. Table 3.1). Furthermore, we assumed metal-poor gas infall, i.e. $X = 0.76$, $Y = 0.24$ and $Z_{\text{el}} = 0$ for elements heavier than helium (unless stated otherwise).

**Figure 4.24** Impact of the star formation history on the AMR: models with infall. Resulting AMRs are shown for 1) a constant SFR with model assumptions similar to those made by Twarog (1980; solid line), and 2) a density dependent SFR ($n = 1$) with infall proportional to the SFR (dotted curve) or with exponentially decaying infall on a time scale $\tau_{\text{inf}} = 3 \, \text{Gyr}$ with either $m_1 = 0.1 \, M_\odot$ (dash-dotted curve) or $m_1 = 0.2 \, M_\odot$ (dashed curve). A disk initial-to-final mass ratio $\delta_0 = 0.1$ was assumed (except for the constant SFR model for which $\delta_0 = 0.5$). In all cases, the SFR was normalised such that $\mu_1 = 0.1$.

Constant SFR models with constant infall of gas, i.e. $F = \alpha \, C_0$ with $\alpha = 0.5$, result in gradual flattening of the AMR caused by mixing of relatively unprocessed gas from the halo to the disk. In this case, the observed AMR can be explained adequately as already has been shown by Twarog (1980) who assumed $\mu_1 \sim 0.05$, $\delta_0 = 0.5$, $F = 0.43 \, C_0$, and $Z_{\text{inf}} \approx 0.1 \, Z_\odot$. We here relaxed the instantaneous recycling approximation, neglected the contribution by SNIa, and used a set of stellar evolution data distinct from that used by Twarog. Nevertheless, we find good agreement with the observed AMR for similar values of $\mu_1 = 0.1$ and $Z_{\text{Fe}} = 2 \times 10^{-4}$ of the infalling gas.

For a density dependent SFR model ($n = 1$) with infall, we consider first the case in which the SFR is directly determined by the gas infall rate, i.e. $C(t) \propto F(t)$. In this case, the gas infall rate regulates the SFR: a high gas infall rate implies a high SFR (and vice versa). As a consequence, most of the disk is built from infalling gas within $\sim 10^8 \, \text{yr}$ after the onset of star formation in the disk. After this initial
burst of self-induced star formation, gas exhaustion strongly reduces the gas infall rate and the SFR ceases correspondingly. This results in a rapid rise of the AMR at early evolution epochs. Thereafter, the shape of the AMR is found to flatten strongly due to: 1) the low enrichment rate of the ISM (i.e. low SFR), and, 2) the dilution of the ISM both by the metal-poor material returned by low-mass stars and by the residual infall of unprocessed gas from the halo (cf. Fig. 4.24). Such a strong dependence of the SFR on the gas infall rate probably is not supported by the observations. Instead, the disk ISM may have been built up more gradually (see below).

Exponentially decaying infall of primordial (or near primordial) matter is motivated by the assumption that the disk component of the Galaxy built up to many times its initial value during the early phase of evolution while infall rapidly ceases beyond the time scale for disk formation (e.g. Clayton 1982). For the same SFR model, we also consider gas infall exponentially decaying with age: $F(t) \propto \exp(-t/\tau_{\text{inf}})$ with an infall time scale $\tau_{\text{inf}} = 3$ Gyr (cf. model 1B; see e.g. Bravo, Isern, & Canal 1993). In this case, about 50% of the present-day disk mass is settled within $\sim 2$ Gyr after the onset of star formation and a more or less linear $[\text{Fe/H}]$ vs. age relation steadily rising with age is found. When the lower mass limit is increased from $m_\text{l} = 0.1$ to 0.2 $M_\odot$, good agreement with the observed AMR is obtained. However, we note that this agreement can be achieved also by e.g. including the contribution of SN Ia, reducing $\tau_{\text{inf}}$, and/or incorporating infall of metal-rich gas.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.25}
\caption{Infall models: dependence of the AMR on the gas infall time scale. Resulting AMRs are shown for $\tau_{\text{inf}}$ [Gyr] = 1 (solid curve), 3 (dashed), 5 (dash-dotted), and $\infty$ (i.e. no gas infall; dotted curve). A density dependent SFR ($n = 1$), Salpeter IMF with $m_\text{l} = 0.1$ $M_\odot$, and disk initial-to-final mass-ratio $\delta_0 = 0.1$ were assumed. In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$.}
\end{figure}

Fig. 4.26 illustrates the effect of the gas infall time scale on the AMR for various values of $\tau_{\text{inf}}$ [Gyr] = 1, 3, 5, and $\infty$. As before, a density dependent SFR ($n = 1$) was used with $\delta_0 = 0.1$ and $m_\text{l} = 0.1$ $M_\odot$. Again, it can be seen that a reduction of the gas infall time scale results in a steeper rise of the AMR at early phases and a more severe flattening thereafter. Clearly, both the initial rise and overall level of the AMR are affected by the infall time scale. A similar but less pronounced effect is found when the disk initial-to-final mass-ratio $\delta_0$ is varied. Fig. 4.26 shows resulting AMRs for values of $\delta_0 = 0.1$, 0.3, 0.5, and 1 with $\tau_{\text{inf}} = 3$ Gyr. Smaller values of $\delta_0$ generally lead to a more rapid increase of the AMR at early phases. In contrast, the final level of the AMR is hardly affected when going from $\delta_0 = 0.1$ to 1. Thus, in case of a density dependent SFR and exponentially decaying gas infall rate, the amount of gas infall after the onset of star formation in the disk predominantly affects the shape of the AMR during the first few Gyr.

We conclude that the shape of the AMR can be strongly affected by the infall (or accretion) of gas. The effect depends on the interstellar iron abundance in the disk ISM, the abundances within the infalling gas, the degree to which infall regulates the star formation in the disk, and on the magnitude and variation of the infall rate with galactic age. For high infall rates of relatively unenriched gas, the AMR rapidly "stabilizes" at early evolution phases of the disk for density dependent SFR models. Stabilization occurs at later ages for lower infall rates and/or infall of metal-rich gas. At early epochs in the evolution of the Galactic disk, theoretical AMRs for models incorporating gas infall differ substantially from those excluding gas infall (cf. Fig. 4.24). However, when gas infall has ceased at later epochs, the AMR for such models becomes very similar.
4.2 Modelling the Age-Metallicity Relation

Initial-to-final mass-ratio

Figure 4.26 Infall models: dependence of the AMR on the disk initial-to-final mass-ratio $\delta_0$. Resulting AMRs are shown for $\delta_0 = 0.1$ (solid curve), 0.3 (dashed), 0.5 (dash-dotted) and 1 (i.e. no infall; dotted curve). A density dependent SFR ($n=1$), Salpeter IMF with $m_1 = 0.1 \, M_\odot$, and $\tau_{\text{inf}} = 3 \, \text{Gyr}$ were assumed. In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$

- SFR dependence on both total system mass and gas density

An interesting dependence of the SFR on the total amount of matter that has fallen onto the disk (since the onset of star formation therein) has been proposed by Dopita (1985, 1989), who argued that the SFR varies with total system mass and gas density as follows (see Chap. 2):

$$C(t) \propto M_8^{4/3} M_{\text{tot}}^{1/3}$$ \hspace{1cm} (4.14)

In this case, the disk gradually builds up by exponentially decaying gas infall from the halo (cf. Russell & Dopita 1992; hereafter RD) but in a manner different from the cases discussed before. Note that the values in the exponents are sensitive to specific model assumptions (see Dopita 1989; Dopita & Ryder 1995).

Figure 4.27 Resulting AMRs for models in which the SFR depends on both the total system mass and gas density (e.g. Dopita 1985). Exponentially decaying gas infall has been assumed with $\tau_{\text{inf}} \, [\text{Gyr}] = 1$ (solid curve), 3 (dashed), and 5 (dotted). A Salpeter IMF with $m_1 = 0.1 \, M_\odot$ and $\delta_0 = 0.1$ was assumed. In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$

In Fig. 4.27 we illustrate resulting SFRs and AMRs according to the Dopita SFR model with gas infall time scales $\tau_{\text{inf}} = 1, 3$ and 5 Gyr, respectively. A disk initial-to-final mass-ratio $\delta_0 = 0.1$ was assumed. We relaxed the bimodality of the IMF adopted by RD and simply used the Salpeter IMF. In the Dopita SFR case, an early, extended burst of star formation is initiated by the rapid increase of the gas density in the disk due to gas infall. Thereafter, the SFR decreases exponentially while the disk ISM is replenished by the gas returned by low-mass stars formed during the preceding burst. The shape of the observed AMR can be
explained well by the Dopita SFR model, especially for short infall time scales $\tau_{\text{inf}} \sim 0.5-1$ Gyr. When the infall time scale is increased, the overall level of the AMR decreases substantially and the inclusion of e.g. SNIa would be required to provide agreement with the observations. We confirm that Dopita's SFR model is consistent with the AMR observed in the Galactic disk and may provide a natural and physical expression for the variation of SFR with age, both in our own and in external galaxies (see Chap. 2).

- Double exponential SFR

Under the assumption of sustained infall of gas onto the disk during the early evolution of the Galaxy, Clayton (1988) derived an analytical relation for an SFR that is directly proportional to $\mu(t)$.

$$C(t) \propto \left[ \exp(-t/\tau_{\text{cons}}) - \exp(-t/\tau_{\text{inf}}) \right]^\beta$$

where $\tau_{\text{cons}}$ is the gas consumption time scale and $\beta \sim 1$. Basically, this model is similar to that proposed by Dopita (1985) since the instant of onset of star formation in the disk can be adjusted both in the double exponential and Dopita SFR models. However, in the double exponential SFR model of Clayton, the time scale for gas consumption is a free parameter whereas in Dopita's model it is fixed by the dependence of the SFR on the gas-density and the total system mass.

Fig. 4.28 Resulting AMRs for the double exponential SFR model including gas infall (see Clayton 1988). Results are shown for combinations ($\tau_{\text{cons}}$, $\tau_{\text{inf}}$ in Gyr) = (4, 0.5; solid curve), (4, 3.5; dash-dotted), (14, 0.5; dashed curve) and (14, 3.5; dotted). A Salpeter IMF with $m_1 = 0.1$ $M_{\odot}$ and $\delta_0 = 0.1$ was assumed. In all cases, the SFR was normalised to satisfy $\mu_1 = 0.1$

Fig. 4.28 shows resulting AMRs for the double exponential SFR model for combinations of $\tau_{\text{cons}} = 14$ and 4 Gyr and $\tau_{\text{inf}} = 0.5$ and 3.5 Gyr. We restricted ourselves to the plausible cases $\tau_{\text{inf}} \lesssim \tau_{\text{cons}}$. Reasonable agreement is obtained with the observations, similar to the Dopita SFR model, for short infall time scales of $\tau_{\text{inf}} \sim 0.5$ Gyr and gas consumption time scales $\tau_{\text{cons}} \lesssim 4$ Gyr. The initial rise of the AMR relation can be sustained by extending the star formation burst over a longer period of time. Models incorporating a more recent and/or more extended burst of star formation appear inconsistent with the observed shape of the AMR. However, we note that values of $\tau_{\text{inf}}$ and $\tau_{\text{cons}}$ in good agreement with the observations are sensitive to e.g. the contribution by SNIa (neglected here).

- Empirical SFR

Fig. 4.29 displays AMRs for the empirical SFR derived using: 1) the age distributions of F and G main-sequence dwarfs (Twarog 1980), 2) the chromospheric Ca\ii emission-line ages for $\sim 100$ dwarf stars (Barry 1988), and 3) the present-day stellar mass function (Rana 1991). Empirical SFRs presented by Twarog and Rana are similar but differ substantially from that inferred by Barry. Since we here deal with empirical SFRs, we like to make a useful comparison with the observed AMR. This is done by including the contribution of SNIa assuming $F_{\text{SNla}} = 0.015$ and $m_{\text{SNla}} = 2.5$ $M_{\odot}$ in the models shown in Fig. 4.29. Furthermore, we used the Salpeter IMF with $m_1 = 0.1$ $M_{\odot}$. We note that Rana used an IMF rather distinct from the Salpeter IMF.
4.2 Modelling the Age-Metallicity Relation

We find that the resulting AMRs for the Rana and Twarog SFRs are in reasonable agreement with the observed AMR. In these cases, the resulting AMR is similar to that for a density dependent SFR model ($n = 1$) without gas infall. In contrast, the variations found in the AMR using Barry’s empirical SFR are not supported by the observations. Both selection effects and errors in the calibration of the chromospheric ages of the sample stars may have affected the data (cf. Barry 1988; Kennicutt 1992).

We conclude that the detailed variation of the SFR with galactic age as well as the gas infall history determine to a large extent the AMR. It is evident that no unique model for the chemical evolution of the Galactic disk exists on the basis of the AMR alone, although only a limited set of star formation and infall histories provides results consistent with the observed AMR. These results are consistent with those derived by earlier investigations (e.g. Twarog 1980; Tinsley 1980; Tosi 1988; Matteucci & Francois 1992; Ferrini et al. 1994; Pardi & Ferrini 1994). Here we attempted to show in a more quantitative way the assumptions and uncertainties involved with the current generation of galactic chemical evolution models (with particular attention the AMR of iron). Other observational constraints, preferably independent of the AMR, are required to narrow down the possible range of models appropriate to the star formation history and chemical evolution of the Galactic disk. In the next section, we will investigate whether we can distinguish between various models that are consistent with the observed AMR on the basis of additional observational constraints. Before, we briefly discuss the set of SFR models selected for this purpose.

4.2.4 Selected models for the Galactic disk

In the previous section we have shown that the AMR observed in the Galactic disk can be explained adequately by a wide range of models with different star formation histories. Apart from the effect of the SFR, we have illustrated that the influence of the main model parameters on the resulting AMR can be large and can affect both the shape and the overall level of the AMR. Since simultaneous variations in individual model parameters usually have a cumulative effect on the AMR, each star formation model comprises a set of models that are able to adequately reproduce the observed AMR. Therefore, it is impossible to reconstruct the Galactic star formation history from the observed AMR alone. For the same reason, it is hard to constrain other model assumptions, such as the IMF and stellar mass limits for SNIa and SNII, by means of modelling the AMR in its own.

We here choose to follow a trial and error method: we select a number of SFR models consistent with the AMR and confront these models with various independent observational constraints to the star formation history and chemical evolution of the Galactic disk. Such constraints include the properties (e.g. total number, formation rate, luminosity distribution) of the present-day stellar populations in the Galactic disk (e.g. main-sequence stars, AGB stars, different types of supernovae, white dwarfs) as well as the interstellar abundances of elements up to Zn observed in the disk ISM. After examining how the selected models behave with respect to these constraints, we aim to converge to a more or less uniform set of models that apply to the chemical evolution of the Galactic disk.
A set of six SFR models that fit the observed AMR but are distinct in the adopted star formation history, IMF, and gas infall history, has been selected:

- density dependent SFR model ($n = 1$) without gas infall (model A, cf. Fig. 4.23)
- density dependent SFR model ($n = 1$) with exponentially decaying gas infall (model B, cf. Fig. 4.24)
- double exponential SFR model with infall (model C, cf. Fig. 4.28)
- bimodal SFR model without infall. The formation rate of low-mass stars ($m \lesssim 1 \, \text{M}_\odot$) was assumed to be constant in time and the formation rate of high-mass stars was assumed to vary proportional to the gas density in the disk (model D, cf. Fig. 4.23)
- density dependent SFR model with $C(t) \propto M_k^{1/3} \dot{M}_\text{tot}^{1/3}$ with infall (cf. Dopita 1989; model E, Fig. 4.27)
- density dependent SFR model ($n = 1$) with an IMF slope depending on the SFR: $\gamma(t) = \gamma_0 + \Delta \gamma [1 - (C(t)/C_0)]$ without gas infall (model F, cf. Fig. 4.12)

In Table 4.2 we list the basic characteristics of the models selected (columns 1–3), the mean stellar mass formed $\langle m \rangle$, the average past and present-day star formation rates ($\langle C \rangle$ and $C_1$) and infall rates ($\langle F \rangle$ and $F_1$), and the mean returned gas fraction $\langle R \rangle$. Parameters related to the SFR and infall rate are given in the last column of Table 4.2. The star formation rate for each model was normalised to satisfy the condition $\mu_1 = 0.1$. Infall was assumed to decay exponentially on the time scale $\tau_{\text{inf}}$ as indicated. Unless stated otherwise, a Salpeter IMF ($\gamma = 2.35$) with $m_1 = 0.1 \, \text{M}_\odot$ was assumed. For each model, values of $\dot{F}_{\text{SNIa}}$ and $\dot{F}_{\text{SNIb}}$ were adjusted to fit the observed AMR (canonical values are $\dot{F}_{\text{SNIa}} = 0.015$ and $\dot{F}_{\text{SNIb}} = 0.33$). For the remaining parameters, values are as listed in Tables 3.2 and 3.3.

Table 4.2 Selected models for the Galactic disk

<table>
<thead>
<tr>
<th>Model</th>
<th># SFR</th>
<th>Infall</th>
<th>$\langle m \rangle$ [M$_\odot$]</th>
<th>$\langle C \rangle$ [M$_\odot$ yr$^{-1}$]</th>
<th>$\langle F \rangle$ [M$_\odot$ yr$^{-1}$]</th>
<th>$\langle R \rangle$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>n = 1</td>
<td>no</td>
<td>0.35</td>
<td>15.9</td>
<td>4.8</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>n = 1</td>
<td>yes</td>
<td>0.35</td>
<td>15.7</td>
<td>5.6</td>
<td>0.51</td>
<td>0.29</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>yes</td>
<td>0.35</td>
<td>16.2</td>
<td>2.3</td>
<td>11.6</td>
<td>0.00</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>no</td>
<td>0.39</td>
<td>17.7</td>
<td>11.3</td>
<td>--</td>
<td>0.36</td>
</tr>
<tr>
<td>E</td>
<td>total mass</td>
<td>yes</td>
<td>0.35</td>
<td>16.2</td>
<td>5.5</td>
<td>12.7</td>
<td>0.57</td>
</tr>
<tr>
<td>F</td>
<td>n = 1</td>
<td>no</td>
<td>0.32</td>
<td>15.3</td>
<td>3.9</td>
<td>--</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Notes:
1. $C(t) \propto [\exp(-t/\tau_{\text{cons}}) - \exp(-t/\tau_{\text{inf}})]$ with $\tau_{\text{cons}} = 4$ Gyr
2. stars with $m \lesssim m_{\text{cutoff}}$ form at a constant rate of 0.35 $S_0$ yr$^{-1}$ independent of Galactic age,
   stars with $m > m_{\text{cutoff}}$ form at a rate proportional to the gas density: $S_0 \cdot \mu(t)$ yr$^{-1}$
3. $C(t) \propto M_k^{1/3} \dot{M}_\text{tot}^{1/3}$ (e.g. Dopita 1985).
4. SFR dependent IMF slope was assumed: $\gamma(t) = \gamma_0 + \Delta \gamma [1 - (C(t)/C_0)]$

Figs. 4.30 and 4.31 show the resulting star formation and infall histories as well as [Fe/H] vs. age relations for the models selected. Resulting AMRs are shown both for the Geneva/Nomoto and Woosley/Weaver element yields (the star formation histories shown in the left panels are very similar for both sets of yields). For comparison, we also considered the corresponding AMRs if one would assume that the stellar lifetimes, remnant masses, and element yields would be independent of initial metallicity. In these cases, the solar metallicity values of these quantities were adopted. As the iron yields in the Geneva/Nomoto set are insensitive to initial metallicity, the resulting AMRs are very similar to those resulting when the variation of the stellar yields with metallicity are taken into account in detail. This is also due to the fact that the abundances in the disk ISM over the past 5–10 Gyr are roughly solar (within factors ~2–3). Stellar iron production in the Woosley/Weaver set of yields heavily depends on initial metallicity and the resulting AMRs shift accordingly when stellar quantities for solar metallicity are assumed (cf. Figs. 4.30 and 4.31).

It is evident that there exists no unique galactic chemical evolution model that best fits the observed AMR. However, as we will argue below, many of these models can be excluded on other observational grounds. In this manner, we will try to reconstruct the star formation history of the Galaxy on the basis of a wide range of independent observational constraints which comply with our current knowledge of the chemical evolution of the Galactic disk.
4.2 Modelling the Age-Metallicity Relation

Figure 4.30 Selected models for the Galactic disk: • Model A: density dependent SFR model \( (n = 1) \) without gas infall, • Model B: density dependent SFR model \( (n = 1) \) with exponentially decaying gas infall, and • Model C: double exponential SFR model (see Table 4.2). **Left panels:** Variation with galactic age of the model SFR (solid curve), the ejection rate of gas returned by evolved stars (dotted), the gas infall rate normalised to maximum infall rate (dash-dotted), and the gas-to-total mass-fraction \( \mu \) (dashed, for models A and B dashed curve coincides with solid curve). In all cases, the SFR and stellar ejection rate were normalised to SFR \( \text{max} \) \( [M_\odot \text{yr}^{-1}] \) (indicated in the top right corner of left panels) determined by the condition \( \mu_1 = 0.1 \). Gas infall rates were normalised to the maximum infall rate (not indicated). **Full circles** refer to the galactic age at which the absolute SFR (or infall rate) is equal to \( 15 \ M_\odot \text{yr}^{-1} \). **Right panels:** Resulting \( [\text{Fe/H}] \) vs. age relation (solid curve) according to the SFR model shown in the left panel and the Geneva/Nomoto yields. For comparison, corresponding AMRs are shown in case of the Woosley/Weaver yields both including (dotted curve) and excluding the dependence of the stellar quantities on initial metallicity (dashed curve; quantities at solar metallicity were assumed). Observations are as shown in Fig. 4.10 (symbols).
Figure 4.31 Selected models for the Galactic disk: • Model D: bimodal SFR \((n = 1)\) with the formation rate of low-mass stars \((m < \sim 1 \, \text{M}_\odot)\) independent of age and a density dependent SFR for high-mass stars, • Model E: SFR model with \(C(t) \propto M_k^{3/4} M_{\text{tot}}^{1/3}\) (Dopita 1989), and • Model F: density dependent SFR model \((n = 1)\) with an IMF-slope \(\gamma(t)\) depending on the SFR (cf. Table 4.2). In all cases, the SFR was normalised to \(\text{SFR}_{\text{max}}\) to satisfy \(\mu_1 = 0.1\). Curves and symbols have the same meaning as in Fig. 4.30.
4.3 Results

We confront the set of models selected in the previous section with the following observational constraints to the star formation history and chemical evolution of the Galaxy:

- the present-day stellar mass function (PDMF) and IMF;
- the total number and formation rates of (post) main-sequence stars;
- the gas depletion, infall, and star formation rates in the disk ISM;
- the enrichment history of the Galactic disk as recorded by the abundance-abundance variations and present-day abundances observed;
- the abundances in planetary nebulae (PNe);
- the luminosity function of white dwarf (WD) remnants;
- the mass distribution of WD remnants;
- the age and metallicity distributions of long-living stars in the local disk (i.e. the classical G-dwarf problem).

By means of this comparison, we attempt to converge to a set of models for the chemical evolution of the Galaxy which are consistent with the above observational constraints and we try to trace back eventual discrepancies between the models and the observations. For each of the constraints listed above, we present results in a separate subsection below. Each subsection consists of: 1) a brief introduction, 2) a description of the observational constraints used, 3) an outline of the main model assumptions, and 4) a results+discussion part. We summarize the main results obtained at the end of each subsection. In the next section (Sect. 4.4), we will briefly discuss the combined results presented below.

4.3.1 The present-day and initial mass function

Introduction

The observed present-day mass function (PDMF) of field stars in the SNBH provides an important constraint to models for the chemical evolution of the Galactic disk. First, the PDMF can be converted to the stellar mass spectrum at birth averaged over the galactic lifetime, i.e. the stellar initial mass function (IMF). The stellar IMF is a fundamental ingredient in theoretical models for the evolution of stellar populations. Second, the observed PDMF in principle can be used as an independent check on the adopted star formation history and stellar evolution tracks for models incorporating a theoretical IMF.

We here briefly summarize observations related to the derivation of the PDMF and its conversion to the stellar IMF. Thereafter, we describe the basic method to compute the PDMF theoretically while taking into account metallicity dependent stellar main-sequence lifetimes, and compare model results with the observed PDMF.

Observations

In the following we will denote the PDMF by \( P(m) \) and the IMF by \( M(m) \). The observed PDMF (cf. Miller & Scalo 1979; Scalo 1986 & 1987; Rana 1991) is usually defined as the total number \( N \) of main-sequence stars per logarithmic mass interval (per unit area of the local Galactic disk). Here, we will define the PDMF per unit mass. The PDMF can be derived from the observed stellar luminosity function \( \phi_{\text{LF}}(M_V) \) (corrected to the solar cylinder) and the mass-visual luminosity relation for main-sequence stars in the SNBH (e.g. Scalo 1986):

\[
P(m) \equiv \frac{dN}{dm} = \phi_{\text{LF}}(M_V) \frac{1}{m} \frac{dM_V}{dm} 2\bar{h}_z(\tau(m))
\]

where \( \bar{h}_z \) is the scale height of stars of mass \( m \) averaged over their main-sequence lifetimes \( \tau(m) \) (if \( \tau \) exceeds the lifetime \( t_{ev} \) of the disk then \( \bar{h}_z \) is averaged over \( t_{ev} \)). We note that in order to obtain the luminosity function for main-sequence stars corrections are usually made for the presence of evolved (i.e. post-MS) stars selected according to photometric criteria (cf. Scalo 1986).
We show in Fig. 4.32 the PDMF as constructed from the stellar luminosity function for stars observed within about \(\sim 40\) pc \((m \lesssim 2\,\text{M}_\odot)\) to 5 kpc \((m \gtrsim 10\,\text{M}_\odot)\) from the Sun by Scalo (1986) and, more recently, by Rana (1991). The PDMF in fact samples a much larger volume of the disk than usually is referred to as the SNBH (see Sect. 4.1). This is because stars due to their orbital motions, traversed large fractions of the Galactic disk during their lifetimes. This effect is strongest for low-mass, long-living stars with \(m \lesssim 2\,\text{M}_\odot\).

Differences between the Scalo and Rana PDMFs are caused by different kinematical corrections for the inflation of the disk with stellar age and are further due to uncertainties in the calibration of the \(M_V\) vs. mass relation for main-sequence stars, the adopted stellar lifetimes, and corrections for the possible multiplicity of the sample stars (e.g. unresolved binaries; see further Scalo 1986, Rana 1991; see Chap. 2). Marked differences between Scalo’s and Rana’s PDMF occur at low mass stars \((m < \sim 0.4\,\text{M}_\odot)\) for which Rana’s PDMF is higher up to a factor 3.

![Figure 4.32](image)

**Figure 4.32** Log-log plot of the solar neighbourhood PDMF and IMF \([\#\,\text{pc}^{-2}\,\text{M}_\odot^{-1}]\) vs. initial stellar mass. Top axis indicates the initial stellar mass \([\text{M}_\odot]\). *Dotted lines* represent the PDMFs adopted from Scalo (1986; *full squares*) and Rana (1991; *open circles*). *Solid lines* show the corresponding IMFs. For comparison, the slope of the Salpeter IMF with \(\gamma = 2.35\) (cf. Salpeter 1955) is indicated.

To determine the PDMF using Eq. (4.16) involves additional difficulties related to the metallicity dependence of both the luminosity and main-sequence lifetime of a star of initial mass \(m\). Due to the metallicity dependence of stellar lifetimes, the stellar scale height \(h_z\) needs to be weighed by metallicity dependent factors related to the variation of the SFR over the time during which the main-sequence stars of interest were formed. In particular, this is important for low-mass stars with \(m \lesssim 1\,\text{M}_\odot\). To determine these scale height corrections accurately, the absolute scale-height distribution of stars with luminosities in given luminosity bins are required (up to large scale heights \(z\) above the Galactic plane). Additional correction factors are related to the net effect of orbital diffusion which causes the mean galactocentric distances of the orbits of stars to increase with age (i.e. after their birth stars move on average outwards in the Galactic disk; Wielen et al. 1996).

We note that the stellar luminosity function is particularly uncertain between \(M_V = 5\) and 10 (corresponding to a mass range of \(\sim 1\) to 0.3 \(\text{M}_\odot\)) and between \(M_V = 10\) and 18 (i.e. from 0.3 to \(\lesssim 0.1\,\text{M}_\odot\)). For instance, large uncertainties may be present for stars with \(m \lesssim 0.3\,\text{M}_\odot\) due to the relatively unknown mass-luminosity relation for stars approaching the minimum hydrogen burning mass (see further Scalo 1986; Zinnecker 1988). These uncertainties allow the PDMF to continue to rise down to the minimum stellar mass observed. The sensitivity of the PDMF to these effects is not well established and has been neglected in the constructions of the PDMF by Scalo and Rana. However, it is evident that large uncertainties are present in the PDMF of stars observed in the local Galactic disk.

To derive the time-averaged IMF from the PDMF one needs to know the stellar birth function \(B(m, t)\) which is defined as the number of stars of mass \(m\) formed at galactic age \(t\) in units \([\text{yr}^{-1}\,\text{M}_\odot^{-1}]\). However, \(B(m, t)\) is expected to depend on the local properties of the star forming region such as the density,
temperature, chemical composition, velocity structure, ambient radiation field and star formation efficiency. Therefore, the stellar birth function is usually assumed to be composed of two more tractable quantities, i.e. the SFR \(S(t)\) describing the time variation of the number of stars formed and the IMF \(M(m)\) describing the relative formation probability of stars of initial mass \(m\). This assumption is well known as a “separable” SFR:

\[
B(m, t) = \begin{cases} 
S(t)M(m) & \text{separable SFR} \\
S(t, m)M(m) & \text{bimodal SFR} \\
S(t)M(m, t) & \text{variable IMF}
\end{cases}
\]

(4.17)

However, other ways to decompose the stellar creation function are known e.g. as: 1) bi- or multimodal SFRs where \(S(t)\) is assumed to vary for different mass ranges, and 2) variable IMF models for which the relative formation probability for stars of different mass varies as a function of time. These examples are probably more realistic approximations to the stellar creation function in nature but involve many uncertainties. Note that the term ”bimodal” with respect to the IMF refers to different slopes of the IMF for different mass ranges and has nothing to do with a bimodal SFR. We will return to these assumptions below.

The stellar IMF is the essential link between small-scale processes which initiate star formation and the overall properties of entire stellar populations, e.g. present in the Galaxy. In general, it is assumed that the field star IMF is independent of time and location in the Galactic disk. Derivation of the IMF from the local PDMF requires explicit knowledge of: 1) the history of the star formation in the local Galactic disk, 2) the age of the disk since the onset of star formation, and 3) the lifetimes \(\tau(m, Z_*)\) of stars with initial mass \(m\) and metallicities \(Z_*\). Using these quantities, the PDMF can be corrected for stars which have evolved off the main-sequence in the past and, therefore, are “missed” in present-day star counts. Assuming a separable SFR, the IMF can be expressed in terms of the PDMF as follows (cf. Scalo 1986):

\[
M(m) = \begin{cases} 
P(m) & \text{for } \tau(m, Z_*) \gtrsim t_{ev} \\
\frac{1}{t_{ev}} \int_{t_{ev} - \tau(m, Z_*)}^{t_{ev}} \mathcal{Y}(t) \, dt & \text{for } \tau(m, Z_*) < t_{ev}
\end{cases}
\]

(4.18)

where \(\mathcal{Y}(t) \equiv <S>/S(t)\) is the ratio of the SFR averaged over the lifetime of the Galactic disk and the SFR at time \(t\). Observational constraints to the stellar birthrate history suggest \(\gamma(t)\) to vary between 1 and \(\sim 8 - 10\) (cf. Miller & Scalo 1979; Twarog 1980; Dopita 1990; see Sect. 3.1). We note that the dependence of the stellar main-sequence lifetimes on the metallicity \(Z_*\) is usually ignored and has not been taken into account by Scalo (1986) and Rana (1991).

Under the assumption of a separable SFR both Rana and Scalo reconstructed the IMF from the PDMF. The resulting IMFs are shown in Fig. 4.32. The local IMF peaks both at 0.3 \(M_\odot\) and at about 3 \(M_\odot\), and exhibits a bimodal behaviour in the mass-slope (cf. Scalo 1986 and 1987) with \(\gamma\) varying from 1.5 for low mass stars \((m \lesssim 1 \, M_\odot)\) to 2.7 for stars more massive than 10 \(M_\odot\). The IMF behaviour at very low masses \((\sim 0.1 \, M_\odot)\) is uncertain due to the unknown mass-luminosity relation at the low end of the stellar mass range, and due to selection effects.

It can be seen that the IMF is identical with the PDMF for low-mass stars which have main-sequence lifetimes exceeding the age of the disk (i.e. \(m \lesssim 0.8 \, M_\odot\)). At these low masses, the resulting IMFs are rather flat compared to the Salpeter mass function (i.e. \(M(m) \propto m^{-2.35}\); cf. Fig. 4.32). Depending on the adopted variation of the SFR over the lifetime of the disk, the IMF for massive stars \((m \sim 60 \, M_\odot)\) can be a factor of \(10^4\) higher than the PDMF for the same stars. Furthermore, the IMFs reconstituted by Rana and Scalo differ considerably due to differences e.g. in the adopted past SFR and scale-height corrections. These different IMFs have direct consequences for chemical evolution models, e.g. for the rate of gas consumption by low-mass stars or enrichment of the ISM by massive stars (see Sect. 4.2). For the models discussed in 4.2 which incorporate the Scalo or Rana PDMF, we converted the PDMF to the IMF using the specific model SFR and disk age \(t_{ev}\) assumed (cf. Eq. 4.18).

Assuming stellar mass limits at birth of \(m_l = 0.1 \, M_\odot\) and \(m_u = 60 \, M_\odot\), the mean stellar mass \(<m>\) for the Scalo and Rana IMFs is 0.81 and 0.75 \(M_\odot\), respectively. For comparison, the Salpeter IMF results in a considerably lower mean stellar mass of \(<m> = 0.35 \, M_\odot\). Observed minimum and maximum observed stellar masses are (cf. Scalo 1986): \(m_l \sim 0.08 \, M_\odot\) and \(m_u \sim 120 \, M_\odot\), respectively. However, uncertainties exist both at the low-mass end (due to the relatively unknown \(m_l\) vs. \(M_\odot\) relation) and at the high-mass end (due to possible binarity or multiplicity). Although the precise value of the upper mass limit is in general not important for galactic evolution models (differences are negligible assuming \(m_u = 60 \, M_\odot\) instead of 120 \(M_\odot\)), galactic chemical evolution model results are rather sensitive to the minimum stellar mass at birth adopted. Unless stated otherwise, we will assume \(m_l = 0.1 \, M_\odot\) and \(m_u = 60 \, M_\odot\) independent of galactic
age. We note that the common and convenient assumption of constant stellar mass limits (i.e. independent of age and location in the Galactic disk) is probably unrealistic since the stellar mass range at birth is expected to vary with the physical properties (e.g. gas density, temperature, velocity dispersion) of the star forming molecular cloud (see Chap. 2).

In the following, we describe the derivation of the PDMF from a given theoretical IMF in the more general case of a semi-separable stellar creation function (such as bimodal star formation or a age-dependent IMF) while taking into account the metallicity dependence of the stellar main-sequence lifetimes. Thereafter, we present resulting PDMFs and IMFs for the models selected in Sect. 4.2.

Model assumptions

The mass spectrum of stars at birth averaged over the lifetime $t_{\text{ev}}$ of the Galaxy, can be in general written as:

$$M(m) = \frac{\int_{0}^{t_{\text{ev}}} B(m, t) \, dt}{\int_{0}^{t_{\text{ev}}} \int_{m_l(t)}^{m_u(t)} B(m, t) \, dm \, dt} \, [M_{\odot}^{-1}]$$

(4.19)

where $B(m, t)$ is the stellar birth function and $m_l(t)$, $m_u(t)$ the stellar mass limits at birth which are allowed to depend on galactic age. It can be verified that this general equation applies also to the case of a separable SFR. Also, note that the IMF $M(t_{\text{ev}})$ depends on the lifetime $t_{\text{ev}}$ of the disk since the onset of star formation.

If we allow for a time-dependent mass function $M(m, t)$, the normalisation of the stellar mass function at any galactic age $t$ is given by:

$$\int_{m_l(t)}^{m_u(t)} M(m, t) \, dm = 1$$

(4.20)

We define $t = t_1$ as the galactic age at which a star of initial mass $m$ evolves off the main-sequence at the present epoch $t = t_{\text{ev}}$. This is equivalent to $t_1 \equiv t_{\text{ev}} - \tau(m, Z_*)$ where $\tau(m, Z_*)$ is the main-sequence lifetime of the star born with metallicity $Z_*$ at time $t$. Note that to determine $t_1$, the resulting AMR $\dot{Z}(t)$ for a given SFR and IMF model is required.

The main-sequence turnoff mass corresponding to $t_1$ will be denoted by $m_{\text{to}}(t_1)$. According to these definitions, the PDMF at evolution time $t_{\text{ev}}$ can be written as:

$$P(m) = \frac{\int_{0}^{t_{\text{ev}}} B(m, t) \, dt}{\int_{0}^{t_{\text{ev}}} \int_{m_l(t)}^{m_u(t)} B(m, t) \, dm \, dt} \, [M_{\odot}^{-1}]$$

(4.21)

where the metallicity dependence of the stellar main-sequence lifetimes has been accounted for. For stars with main-sequence lifetimes $\tau(m, Z_*) \gtrsim t_{\text{ev}}$ one has $t_1 = 0$. Furthermore, if $m_{\text{to}}(t_1) \gtrsim m_*(t)$ at a given galactic age $t$, one has $m_{\text{to}}(t_1) = m_*(t)$. It can be verified that in case of a separable SFR and constant stellar mass boundaries at birth, Eq. (4.21) reduces to Eq. (4.18). In case of a time-dependent IMF, bimodal SFR, or otherwise non-separable SFR, no simple relation between the PDMF and IMF exists since both the star formation history and stellar mass function affect the PDMF independently.

To allow for a detailed comparison with the observations, we normalise the IMF to the total number of stars ever formed. Furthermore, we normalise the PDMF to the resulting present-day total number of stars on the main-sequence (cf. Eqs. (4.18) and (4.21)). Observations and theoretical computations were normalised in exactly the same manner. Note that, since most stars formed during the lifetime of the Galactic disk are still on the main-sequence, normalisations of the IMF and PDMF are usually similar in magnitude.

Results

We investigate the sensitivity of the PDMF to the star formation history, stellar lower mass limit at birth, IMF slope, the underlying AMR, and the Galactic lifetime assumed. Figs. 4.33 and 4.34 show the resulting PDMFs for the SFR models selected in Sect. 4.2. Unless stated otherwise, these models were computed using a power law IMF with $\gamma = 2.35$ and stellar mass boundaries at birth $m_1 = 0.1$ and $m_u = 60 \, M_{\odot}$ (independent of Galactic age).
4.3.1 The present-day and initial mass function

Figure 4.33 Left panel: resulting PDMFs for selected SFR models A (solid curve), B (dotted), and C (dashed). Error bars for the Scalo PDMF data indicate observational uncertainties. Right panel: the Salpeter IMF used as input is shown for comparison. Observational data from Scalo (1986; full squares with error bars) and Rana (1991; open circles) are plotted for comparison (see Fig. 4.32). Note that the vertical scales of the left and right panels are different.

- Dependence on star formation history

First, it can be seen from Fig. 4.33 that the overall shape of the observed PDMF is reasonably well reproduced for stars with $m \gtrsim 0.5 \, M_\odot$. This indicates that the metallicity dependent lifetimes assumed for stars with $m \gtrsim 0.5 \, M_\odot$ as well as the Salpeter IMF adopted for such stars are essentially correct. At the high-mass end, the PDMF is primarily sensitive to the level of star formation in the Galactic disk during the last few Gyr and to the slope of the IMF during this period. The shift between the observed and predicted PDMFs is mainly due to the normalisation of the PDMF relative to the present-day total number of main-sequence stars. In fact, the observed and predicted total number of main-sequence stars differ considerably because: 1) too many low-mass stars ($m \lesssim 0.5 \, M_\odot$) are predicted by Salpeter IMF models compared to the observations, and 2) $m_1 = 0.1 \, M_\odot$ as assumed in the models differs from $m_1 = 0.08 \, M_\odot$ for the observations. Thus, in principle, the assumption of a Salpeter IMF provides good agreement with the observations for stars with $m \gtrsim 0.5 \, M_\odot$ while the total number of stars with $m < 0.5 \, M_\odot$ is considerably too large for such an IMF. Usually, to get around the differences in normalisation, both the predicted and observed PDMF are shifted vertically to one and the same well-defined reference point (e.g. the observed PDMF at $m \sim 1.5 \, M_\odot$). Here we choose not follow this method of comparison since: 1) the interpretation of the results depends strongly on the reference point chosen, and 2) observational errors can bias such comparisons systematically.

Second, the predicted PDMF at masses below the turnoff mass $m_\odot(t_{\text{ev}}) \sim 0.82 \, M_\odot$ is insensitive to the Galactic star formation history assumed (see also Fig. 4.34). Such stars have not evolved off the main-sequence within the lifetime of the Galaxy so that the PDMF is identical with the IMF. In principle, the critical stellar mass range to derive the past SFR is between $\sim 0.8 \, M_\odot$ and $2.5 \, M_\odot$ (see Scalo 1986). For the models selected, the PDMF is most sensitive to the SFR for high-mass stars ($m \sim 60 \, M_\odot$). At such masses, the resulting PDMF differs by more than an order of magnitude for the SFR models selected (cf. Fig. 4.33). However, due to the uncertainties associated with the normalisation of the PDMF, it is difficult to constrain the adopted SFR by means of the observed PDMF. Instead, the PDMF is more suited to constrain the assumed IMF.
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.34 Left panel: resulting PDMFs for selected SFR models D (solid curve), E (dotted), and F (dashed). Right panel: corresponding IMFs averaged over the lifetime of the Galactic disk. Observational data from Scalo (1986; full squares with error bars) and Rana (1991; open circles) are plotted for comparison (see Fig. 4.32).

- Dependence on IMF

In Fig. 4.34 two selected SFR models with an IMF different from Salpeter are shown: 1) the bimodal SFR model which involves the Salpeter IMF but with different normalisations for the low and high-mass parts at \( m = 1 \) M\(_\odot\) (model D). These kind of SFR models basically can be considered as models with bimodal IMFs different from the Salpeter IMF; and 2) an IMF with a slope \( \gamma(t) \) decreasing with the SFR (model F). We conclude that the kind of bimodal SFR models considered here are inconsistent with the observations as such models predict too many massive stars (\( m > \sim 1 \) M\(_\odot\)). We will briefly consider other types of bimodal IMFs below. In contrast, the PDMF and IMF resulting from a model with a SFR-dependent IMF slope between \( \gamma = 2.1 \) (at \( t = 0 \)) and 2.6 (at \( t \sim t_{\text{ev}} \)) are found to be very similar to that of a constant IMF slope model with roughly the same SFR. Therefore, it is not possible to exclude age-dependent (e.g. SFR dependent) IMF models (with a reasonable range in IMF slope) from the observed PDMF.

Fig. 4.35 shows the influence of the adopted IMF on the PDMF for the standard SFR (model A). The Kroupa empirical IMF, which takes into account corrections for binary stars and the age and metallicity dependences of the stellar luminosity, is essentially a three-slope power law IMF with \( \gamma_1 \sim 2.7 \) for stars more massive than \( \sim 1 \) M\(_\odot\), \( \gamma_2 = 2.2 \) for 0.5 \( \lesssim m/\) M\(_\odot\) \( \lesssim 1 \), and \( \gamma_3 \) between 0.7 and 1.85 for 0.08 \( \lesssim m/\) M\(_\odot\) \( \lesssim 0.5 \) (Kroupa et al. 1993). The latter slope depends on details of the analysis and may be sensitive to the Galactic region considered. We here assume \( \gamma_3 = 1.5 \) which is the value derived by Kroupa et al. for reasonable assumptions. Thus, the empirical IMF at low masses (\( m \lesssim 0.5 \) M\(_\odot\)) is significantly flatter than at high masses. The flattening of the observed PDMF towards low-mass stars appears to be an essential feature and should be explained by any model for the stellar IMF at birth (see Chap. 2). For the standard SFR model, the resulting PDMF using the Kroupa empirical IMF is in good agreement with the data from Scalo (1986) and Rana (1991), except for stars with \( m \sim 0.2 \) M\(_\odot\).

For comparison, the PDMF resulting from a power-law IMF with slope \( \gamma = 2.7 \) is clearly inconsistent with the observations as it predicts too many stars at the low-mass end and/or too little at the high-mass end. As another test, we computed the IMF iteratively from the observed PDMF from Scalo (1986). In this case, we derived the stellar IMF at birth from the empirical PDMF according to the model SFR assumed (see Eq. 4.18). This is an iterative procedure since a density-dependent SFR itself is sensitive to the rate of gas consumption by stars formed according to the unknown IMF. In general, this procedure rapidly converges to the searched IMF which indeed perfectly matches the empirical PDMF. For the standard SFR model, the resulting IMF is in reasonable agreement with the one derived by Scalo (1986) even though he used a somewhat different SFR and set of stellar lifetimes. We will discuss in Sect. 4.3.4 how the Kroupa IMF and Scalo IMF may affect the resulting abundance-abundance variations for stars in the Galactic disk and halo.
4.3.1 The present-day and initial mass function

• Effect of bimodal IMFs and SFRs

In general, the effect of a bi or multi-modal SFR is differentiation of the formation history of stars with masses in distinct ranges. This may give rise to discontinuities in the mean IMF if the stars are formed according to one and the same mass function. In principle, the distinction between bimodal SFRs and age-dependent IMFs is artificial as such models give very similar results. We consider bimodal SFRs of the following mathematical forms:

\[
S(t) = \begin{cases} 
L \rho(t) & \text{for all } m \\
H e^{-t/\tau} & \text{for } m \geq m_{bi}
\end{cases} \quad \text{(type A)}
\]

\[
S(t) = \begin{cases} 
H e^{-t/\tau} & \text{for } m \geq m_{bi} \text{ and } t < \tau \\
L \rho(t) & \text{for all } m \text{ and } t \geq \tau
\end{cases} \quad \text{(type B)}
\]

\[
S(t) = \begin{cases} 
L(t) \rho(t)^{\alpha} & \text{for } m < m_{bi} \\
H(t) \rho(t)^{\beta} & \text{for } m \geq m_{bi}
\end{cases} \quad \text{(type C)}
\]

where \(m_{bi}\) is the transition mass between the low- and high-mass SFR modes, and \(L, H\) are additional multiplication factors associated with the low and high-mass SFR modes, respectively. For the underlying physical mechanism of star formation (see also Chap. 2), we refer e.g. to Larson (1986; type A), Vangioni-Flam & Audouze (1988; type B) and Dopita (1989; type C).

In Fig. 4.36, we investigate the effect of bimodal star formation on the PDMF in more detail for the bimodal SFR types defined above. We used \(m_{bi} = 0.5 \, M_\odot\), \(\tau = 1 \, \text{Gyr}\), \(L/H = 1/3\) (types A and B) and \(L(t) = 1, H(t) = \sigma T^{1/3}\) where \(\sigma T\) denotes the total surface mass density in the disk (type C). In addition, we adopted (type C only) \(\alpha = 1, \beta = 4/3\), an infall time scale \(\tau_{inf} = 1 \, \text{Gyr}\), and a disk initial-to-final mass ratio \(\delta_0 = 0.1\). Results are also shown for a bimodal SFR (different from the types discussed above but similar to model D) with a constant mode of low-mass star formation (\(m_{bi} < 1 \, M_\odot; \, L = 1\)) and a density-dependent high-mass SFR mode with \(H = 1/3\). In all cases, the SFR was normalised to satisfy \(\mu_1 = 0.1\) and the Salpeter mass function at birth was assumed.

Comparison of Figs. 4.34 and 4.36 reveals that bimodal SFR models (e.g. type B) can improve the agreement with the observed PDMF substantially. To achieve this, however, the high-mass mode of star formation needs: 1) to extend to masses down to \(\lesssim 0.5 \, M_\odot\), and 2) to have an amplitude which is relatively high compared to that of the low-mass mode. In principle, the shifts between the low and high-mass mode
Figure 4.36 Effect of a bimodal SFR on the PDMF. **Left panel**: resulting PDMFs for bimodal SFRs of type A (dotted curve), type B (solid) and type C (thick solid). For comparison, the resulting PDMF for a bimodal SFR with a constant mode of low-mass star formation and a density dependent mode of high-mass star formation (dashed; see text). Observational data as in Fig. 4.32. **Right panel**: Corresponding IMFs for the bimodal SFR models shown in the left panel.

ranges of the IMF are determined mainly by the SFR amplitudes $L$, $H$ (see above) while the underlying star formation mechanism is irrelevant in this. However, the variation of the SFR with time determines the enrichment contribution of the high-mass mode with galactic age and is, therefore, constrained by e.g. the abundance-abundance variations and AMRs observed among Galactic disk and halo stars. In this manner, type C bimodal SFR models result in values $[\text{Fe/H}]$ which are $\sim 0.4$ dex too high compared to the observations (for model parameters as listed in Table 3.3). Consequently, such models are probably inadequate to explain the observed PDMF and abundance-abundance variations both at the same time.

For these reasons, we will restrict ourselves to the bimodal SFR model selected in Sect. 4.2 when confronting such SFR models with the observed abundance-abundance variations. We note that, bimodal SFR models are equivalent to bimodal IMF models in case of identical variations of the low and high-mass SFR modes with galactic age apart from different multiplication amplitudes $L$ and $H$ (both constant in time). Therefore, variations in the lower and upper stellar mass limits at birth with galactic age, which simply change the formation probability of low and high-mass stars with age, have similar effects on the PDMF as bimodal SFR models (see below). Thus, the resulting PDMF can be very similar for a wide range of IMF and SFR models and it is generally difficult to constrain the latter quantities using the observed PDMF. The abundance-abundance variations observed among long-living stars in the Galaxy are better suited for this purpose because different SFR and IMF models usually predict divergent chemical enrichment histories (see Sect. 4.3.4).

- **Dependence on the lower stellar mass limit**

Fig. 4.37 shows the resulting PDMF for different lower stellar mass limits at birth in case of the standard SFR (model A). When $m_l$ is decreased from 0.2 to 0.05 $M_\odot$, the formation probability of stars with $m \gtrsim m_l$ is decreased considerably. The main effect on the resulting PDMF is an downward shift due to the normalisation of the PDMF. The model with $m_l = 0.2$ $M_\odot$ is in good agreement with the observed PDMF over the mass range considered but ignores the fact that stars with $m_l = 0.2$ $M_\odot$ are present in large numbers in the Galactic disk and halo (e.g. Scalo 1986; Kroupa et al. 1993; see Chap. 2).

The resulting PDMF in case of an SFR-dependent lower mass limit decreasing from $m_l = 0.5$ to 0.05 $M_\odot$ strongly affects the shape of the PDMF at the low-mass end. Although the agreement with the empirical PDMF is improved, the present-day number of stars with $m \gtrsim 1$ $M_\odot$ is underestimated by about an order of magnitude. This is due to the predominance of low-mass stars formed at the current epoch in models for which $m_l$ decreases strongly with galactic age. This suggests that: 1) the observed PDMF does not support large variations of $m_l$ over the lifetime of the Galaxy, and 2) values of $m_l \lesssim 0.1$ $M_\odot$ result in large
4.3.1 The present-day and initial mass function

discrepancies between the observed and predicted PDMF at the high-mass end. We note that, in contrast to the previous case, variations of the upper stellar mass limit of stars with galactic age are not well traced by the PDMF (unless they did occur at recent epochs in the evolution of the disk or were very large). Values of \( m_u \) between \( \sim 2 \) and 150 M\(_\odot\) at early epochs in the evolution of the Galaxy cannot be traced back from the observed PDMF.

![Figure 4.37 Resulting PDMFs for different lower stellar mass limits at birth. Results are shown for the standard SFR (model A) with the Salpeter IMF. Left panel: resulting PDMFs for \( m_l = 0.05 \) M\(_\odot\) (dotted curve), 0.1 M\(_\odot\) (dashed) and 0.2 M\(_\odot\) (solid). For comparison, the resulting PDMF for an SFR-dependent \( m_l(t) \) decreasing with galactic age from 0.5 to 0.05 M\(_\odot\) (thick solid line). Observational data as in Fig. 4.32. Right panel: Corresponding IMFs.](image)

- **Dependence on gas fraction, AMR, and lifetime assumed**

For density dependent SFR models, the resulting PDMF shifts upwards at masses \( m > \sim 1 \) M\(_\odot\) for larger values of the gas-to-total mass ratio. For instance, the shift in the PDMF is about 0.3 dex in case of the standard SFR (model A) with normalisations such that \( \mu_1 = 0.2 \) and \( \mu_1 = 0.05 \), respectively.

Assuming a disk lifetime of 10 Gyr instead of 14 Gyr (as above), leads to a small shift upwards (\( \sim 0.2 \) dex) of the PDMF for stars with \( m > \sim 1 \) M\(_\odot\). For models ending at the same gas fraction, the SFR at a given galactic age is somewhat larger in the 10 Gyr compared to the 14 Gyr case.

In principle, the resulting PDMF is insensitive to the detailed chemical enrichment of the Galactic ISM because stars more massive than \( \sim 1 \) M\(_\odot\) represent only a tiny fraction (\( \leq 1\% \)) of the total number of main-sequence stars currently present in the disk. Since the majority of the stars ever formed in the Galaxy has not yet evolved off the main-sequence, the effect of the AMR on the PDMF is negligible (except perhaps for the most massive stars).

In general, the sensitivity of the PDMF to the present-day gas fraction, AMR, and Galactic lifetime assumed is small compared to that to the adopted star formation history and IMF.

**Conclusion**

We summarize the main results obtained in this section as follows:

- the observed PDMF can be reasonably well reproduced by the models selected in Sect. 4.2 for stars with \( m > \sim 0.5 \) M\(_\odot\). This indicates that the metallicity dependent lifetimes assumed for stars with \( m > \sim 0.5 \) M\(_\odot\) as well as the Salpeter IMF adopted for such stars are essentially correct;

- the flattening of the observed PDMF towards low-mass stars appears to be an essential feature for stars observed in different regions of the Galaxy (Kroupa et al. 1993). Although large uncertainties are still involved in the PDMF for stars with \( m \sim 0.5 \) M\(_\odot\), the flattening of the IMF over this mass
range should be explicitly taken into account in Galactic chemical evolution studies since it can lead to significantly different results as compared to Salpeter IMF models;

- the predicted PDMF at masses below the turnoff mass \( m_{\text{TO}}(t_{\text{ev}}) \sim 0.82 \, M_\odot \) is insensitive to the Galactic star formation history assumed: the PDMF is identical with the IMF over this mass range.

- the critical stellar mass range to derive the past SFR using the observed PDMF is between \( \sim 0.8 \, M_\odot \) and \( \sim 2.5 \, M_\odot \) (see also Scalo 1986). However, the resulting PDMF can be very similar for a wide range of IMF and SFR models and it is generally difficult to constrain the latter quantities using the observed PDMF. This is mainly due to the uncertainties involved with the normalisations of the predicted and observed PDMF.

- the resulting PDMF in case of a SFR-dependent IMF slope between \( \gamma = 2.1 \) (at \( t = 0 \)) and \( 2.6 \) (at \( t = t_{\text{c}} \)) is found to be similar to that in case of a constant IMF.

- the kind of bimodal SFR models selected in Sect. 4.2 is inconsistent with the observations as such models predict too many massive stars \( (m \lesssim 1 \, M_\odot) \). For bimodal SFR models, the shifts between the low and high-mass mode ranges of the IMF are determined mainly by the ratio of the amplitudes of the SFR for these modes while the underlying star formation mechanisms assumed for the low and high-mass modes are relatively unimportant;

- the shape of the resulting PDMF at the low-mass end \( (m \lesssim 0.5 \, M_\odot) \) is very sensitive to the variation of the lower stellar mass limit at birth with galactic age. In case of a SFR-dependent stellar lower mass limit (i.e. \( m_1 \propto C(t) \)), the PDMF for stars observed in the SNBH can be explained adequately even when these stars are assumed to form according to the Salpeter IMF over the entire stellar mass range independent of galactic age.

### 4.3.2 Post main-sequence stars: total number and formation rates

The total number and present-day formation rates of evolved stars such as AGB stars, SNIa, and SNI are sensitive probes to the formation history of their progenitors. In this section, we investigate how selected models for the Galactic disk behave with respect to these quantities.

**Observations**

We concentrate on the present-day and average past formation rates of main-sequence, AGB stars, SNIa, SNII/c, and SNI in the Galactic disk.

The current formation rate of main-sequence stars in the Galactic disk is estimated from the observed present-day SFR by mass, i.e. \( C_1 \sim 3.6 \pm 1 \, M_\odot \, \text{yr}^{-1} \) (see Sect. 3.1). The mean stellar mass formed according to a Salpeter IMF with \( m_1 = 0.1 \) and \( m_u = 60 \, M_\odot \), is \( (\langle m \rangle = 0.35 \, M_\odot) \). This gives a present-day SFR by number of \( S_1 \sim 10 \pm 5 \, \text{yr}^{-1} \). Corresponding values for the Scalo IMF are \( (\langle m \rangle = 0.81 \, M_\odot \) and \( S_1 = 4 \pm 2 \, \text{yr}^{-1} \). Uncertainties in the present-day mass function of low-mass stars provide the main uncertainty in \( S_1 \). Observations suggest that the average past SFR by mass is \( (C) \sim 16.5 \, M_\odot \, \text{yr}^{-1} \), i.e. larger by a factor 3–5 than the present-day SFR (Sect. 3.1). Thus, provided that the stellar IMF has been constant over the lifetime of the Galactic disk, the average past SFR by number has been larger than the present-day SFR observed by about the same factor.

Estimates of the formation rate of AGB stars in the Galactic disk are based on (e.g. Pottasch 1993): 1) the total number of planetary nebulae (PNe) observed in the Galactic disk (\( \sim 25000 \)), the mean lifetime of PNe before they become invisible (\( \sim 2 \times 10^4 \, \text{yr} \)), and on the fraction of all AGB stars which ultimately end as a PN (this fraction is assumed arbitrarily to be one third and is highly uncertain). This leads to a present-day formation rate of \( R_{1}^{\text{AGB}} \sim 1.5 \pm 0.5 \, \text{yr}^{-1} \). The total number of AGB stars in the Galaxy is estimated to be \( \gtrsim 5 \times 10^5 \) (Jura & Kleinmann 1992a,b). With a theoretical IMF-weighed mean lifetime \( (t_{\text{AGB}}) = 2.8 \times 10^5 \, \text{yr} \) of AGB stars in the Galactic disk that do experience third dredge-up (e.g. Groenewegen & de Jong 1993; see Sect. 4.3.6), we derive a present-day formation rate of \( R_{1}^{\text{AGB}} \gtrsim 1.8 \, \text{yr}^{-1} \) which is consistent with the previous estimate.

Present-day rates of SNIa, SNII/c, and SNI in the Galactic disk are estimated from (e.g. van den Bergh & Tammann 1991; Tammann et al. 1994): 1) the corresponding rates in external galaxies and their variation with Hubble type as well as with galaxy luminosity, and 2) a few historically observed SNe in the Galaxy. These observations are affected by variations both in the mean age of the stellar populations since the onset of main star formation (among different galaxies of a given Hubble type), the IMF, and the amount of internal extinction, as well as by uncertainties in the distance, and in the detection probabilities of SNe in galaxies differing in luminosity and Hubble type. Assuming that the Milky Way is a Sbc galaxy
with $L_B = 2 \times 10^{10} \, L_{\odot}$, Tammann et al. (1994) derive a total SN rate $0.026 \pm 0.010 \, yr^{-1}$, and present-day SNIa, SNIIb/c, and SNII rates of $3.1 \pm 3 \times 10^{-5} \, yr^{-1}$, $3.3 \pm 1.5 \times 10^{-3} \, yr^{-1}$, and $1.8 \pm 1.0 \times 10^{-2} \, yr^{-1}$, respectively.

The SNII-rate derived for the MW has been argued difficult to reconcile with the observed luminosity function of OB stars in the Galaxy as there seem not enough precursors present to produce such a high number of core-collapse SNe (e.g. van den Bergh & Tammann 1991). However, this argument relies on the mean lifetime of O and B stars in the Galactic disk, assumed to be $8 \times 10^6 \, yr$, which may well be too long (e.g. Garmany, Conti & Chiosi 1982). In addition, the total number of O and B stars in the Galactic disk is probably underestimated since these stars are often members of compact associations. Another possibility is that the lower mass limit for the progenitors of core-collapse SNe adopted, i.e. $m_{\text{SNII}} = 8 \, M_{\odot}$, must be shifted downwards (e.g. $\sim 5 \, M_{\odot}$) if mass transfer in close binary systems adds substantially to the total number of objects massive enough to explode as core-collapse SNe.

In all external galaxies in which SNIb/c occur, the ratio of SNIb/c to SNII is about 1/5 (Tammann et al. 1994). This ratio is expected to be constant provided that: 1) these SN types differ in progenitor mass range only, and 2) the shape of the mass spectrum of massive stars is similar for different galaxies. Under these assumptions, the present-day SNIb/c in the Galactic disk is $\sim 15\%$ of the total SN rate.

Concerning SNIa, Tammann et al. assumed that the SNIa rate per unit B luminosity of the parent galaxy is roughly independent of Hubble type. This assumption implies that $\sim 15\%$ of all SNe in Sbc galaxies are SNIa. Thus, the ratio of the present-day rate of SNIa in the Galaxy and that of SNII+Ib/c is $\epsilon_1 = 0.18$. Although this ratio is independent of galaxy luminosity, it is strongly affected by selection effects which operate predominantly against faint SNe (both towards galaxies with high surface brightnesses and/or high amounts of internal extinction). Since SNII and SNIb/c are associated with the core collapse of massive stars (see Sect. 3.3), type II+Ib,c supernovae are found to be confined to sites of massive star formation, i.e. in the spiral arms in Sbc galaxies. Since both surface brightness and dust extinction are relatively large in the spiral arm regions in Sbc galaxies (as compared to the interarm regions, e.g. Walterbos 1991; see Chap. 9), the rates of SNIa and SNIb/c are underestimated considerably.

As discussed in Sect. 3.3, we consider SNIa as accreting white dwarfs (WD) in binary systems that explode after thermonuclear burning of their electron-degenerate core. We assume SNIa to originate from main-sequence stars with masses in the range $\sim 2.5 - 8 \, M_{\odot}$ (e.g. Nomoto et al. 1984). Since there is a considerable time delay (of a few Gyr or more) between the WD formation and the ultimate explosion of the WD as SNIa (see below; Sect. 4.2), SNIa are usually not associated with the regions of star formation and are found to be spread throughout the galaxy and less confined to the galactic plane (e.g. van den Bergh & Tammann 1991; Tutukov, Yungelson & Iben 1992). Thus, although SNIa are generally fainter at maximum by $2 - 3$ mag than are SNII, the selection effects discussed above are expected to be more severe for SNIa(+Ib,c).

Therefore, the value of $\epsilon_1 = 0.18$ suggested by the observations is probably a severe upper limit to the actual value in the Galactic disk. We will adopt $\epsilon_1 = 0.1 \pm 0.05$ as a reasonable value for the Galactic disk. Similarly, the predicted rates of SNe in the Galactic disk are probably all lower limits (e.g. van den Bergh & Tammann 1991). We note that the star formation history, IMF, and e.g. radial distribution of the stellar populations in the Galactic disk, may differ considerably from that typical in Sbc galaxies. For this reason, and because of the uncertainties discussed above, the present-day SN rates as predicted for the Galactic disk may be off by perhaps a factor of two.

Model assumptions

The current formation-rate of stars with initial masses in a given mass interval that enter a given evolutionary phase (such as the horizontal branch or AGB) at galactic age $t$ can be expressed theoretically as:

$$R_{PH}(t) = \int_{\Delta m_{PH}} f_{PH}(m) S(t - \tau_{PH}(m, Z)) M(m) \, dm$$

(4.23)

where the subscript PH refers to the phase under consideration, $f_{PH}(m)$ corrects for the different evolutionary outcomes of stars with the same initial mass $m$, $\tau_{PH}(m, Z)$ denotes the age of a star of initial mass $m$ born with metallicity $Z$, entering the phase of interest, and $\Delta m_{PH}$ is the initial mass range for stars that pass through this phase later in their evolution. In principle, the correction factors $f_{PH}(m)$ may be a function of $(t - \tau_{PH})$ as well. However, for convenience we will assume that $f_{PH}$ is constant in time as well as constant over the mass interval $\Delta m_{PH}$ considered. The formation rate of main-sequence stars can be derived from Eq. (4.23) assuming $\Delta m_{MS} = [m_o(t), m_M]$ where $m_o(t)$ is the main-sequence turnoff-mass at galactic age $t$ and $m_M$ the upper stellar mass limit at birth. In a similar way, the total number of stars in evolutionary
phase PH at galactic age \( t \) can be written as:

\[
N_{\rm PH}(t) = \int_{\Delta m_{\rm PH}}^{\tau_{\rm PH, u}(m, Z_*)} f_{\rm PH}(m) S(t - \tau) M(m) \, d\tau \, dm \tag{4.24}
\]

where \( [\tau_{\rm PH, l}(m, Z_*), \tau_{\rm PH, u}(m, Z_*)] \) defines the age interval during which a star spends its life in the phase of interest. Note that this number differs from the total number of stars that ever entered phase PH during the galactic lifetime \( t \):

\[
T N_{\rm PH}(t) = \int_{\tau = 0}^{t} R(\tau) \, d\tau \tag{4.25}
\]

Defining further \( \Delta \tau_{\rm PH}(m, Z_*) \equiv [\tau_{\rm PH, l}(m, Z_*), \tau_{\rm PH, u}(m, Z_*)] \), the SFR and IMF-weighed average time spent in evolutionary phase PH for all stars that are in that phase at galactic age \( t \) is given by:

\[
< \tau_{\rm PH}(t) > = \frac{1}{N_{\rm PH}(t)} \int_{\Delta m_{\rm PH}}^{\tau_{\rm PH, u}(m, Z_*)} f_{\rm PH}(m) S(t - \tau) M(m) \Delta \tau_{\rm PH}(m, Z_*) \, d\tau \, dm \tag{4.26}
\]

In principle, stars that pass through a specific evolutionary phase do not necessarily have to originate from a single range in progenitor mass. Therefore, \( \Delta m_{\rm PH} \) may include several mass intervals (depending on e.g. the initial stellar abundances, evolutionary phases, etc).

We note that the impact of e.g. mass-exchange in binary systems on the total number and formation rates of single stars in a given evolutionary phase can be accounted for through the phase correction factors \( f_{\rm PH}(m) \). For SNIa, SNIb/c, and SNII, we applied such corrections by assuming \( f_{\rm PH} = \phi_{\rm SN} \) with values of \( \phi_{\rm SN} \) as listed in Table 4.3. For other evolutionary phases, binary corrections were neglected (unless stated otherwise).

### Results

Table 4.3 lists the predicted present-day and average past formation-rates of main-sequence (MS) stars, AGB stars, SNIa, SNIb/c, and SNII, for the star formation models selected in Sect. 4.2. Present-day and average past formation rates are denoted by \( R_1 \) and \( \langle R \rangle \), respectively. The present-day ratio of the rate of SNIa and that of SNII+SNIb/c is listed as \( \epsilon_1 \) in Table 4.3. The Salpeter IMF (\( \gamma = 2.35 \)) and standard model parameters as discussed in Sect. 3.1 were adopted (unless stated otherwise in the last column of Table 4.3).

For model A, we considered the effect of the IMF on the formation rate of (post) main-sequence stars in more detail by means of (cf. Table 4.3): 1) an IMF computed iteratively from the PDMF given by Scalo (1986; **model A1**), 2) the IMF presented by Kroupa et al. (1993; see Sect. 4.3; **model A2**), and 3) a power law IMF with slope \( \gamma = 2.7 \) (**model A3**).

### Table 4.3 Predicted and observed formation rates of (post) main-sequence stars\(^{1,2} \)

<table>
<thead>
<tr>
<th>Model</th>
<th>MS ( \langle R \rangle ) [yr(^{-1})]</th>
<th>AGB ( \langle R \rangle ) [yr(^{-1})]</th>
<th>SNIa ( \langle R \rangle ) ( \times 10^{-3} ) yr(^{-1})</th>
<th>SNIb/c ( \langle R \rangle ) ( \times 10^{-2} ) yr(^{-1})</th>
<th>SNII ( \langle R \rangle ) ( \times 10^{-2} ) yr(^{-1})</th>
<th>( \epsilon_1 )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13.8 46.2</td>
<td>2.3</td>
<td>0.74 2.4</td>
<td>0.47 1.6</td>
<td>2.7 8.9</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>8.4 26.3</td>
<td>5.9</td>
<td>2.44 7.4</td>
<td>0.84 2.7</td>
<td>4.8 15.0</td>
<td>3.8</td>
<td>Scalo IMF</td>
</tr>
<tr>
<td>A2</td>
<td>9.9 23.2</td>
<td>3.8</td>
<td>0.86 2.7</td>
<td>0.36 1.2</td>
<td>2.0 6.7</td>
<td>3.6</td>
<td>Kroupa IMF</td>
</tr>
<tr>
<td>A3</td>
<td>16.3 53.9</td>
<td>5.2</td>
<td>0.31 1.0</td>
<td>0.13 0.4</td>
<td>0.7 2.3</td>
<td>3.6</td>
<td>( \gamma = -2.7 )</td>
</tr>
<tr>
<td>B</td>
<td>16.2 45.7</td>
<td>2.4</td>
<td>0.87 2.4</td>
<td>0.55 1.5</td>
<td>3.1 8.8</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>6.8 47.1</td>
<td>2.5</td>
<td>0.38 2.4</td>
<td>0.23 1.6</td>
<td>1.3 9.0</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>42.9 45.1</td>
<td>2.2</td>
<td>0.84 3.3</td>
<td>0.49 2.2</td>
<td>2.8 12.0</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>16.1 46.9</td>
<td>2.3</td>
<td>0.87 2.4</td>
<td>0.73 2.1</td>
<td>2.9 8.4</td>
<td>2.4</td>
<td>( \phi_{\rm SN}^{\text{SNIb/c}} = 0.2 )</td>
</tr>
<tr>
<td>F</td>
<td>14.5 47.9</td>
<td>2.2</td>
<td>0.49 2.5</td>
<td>0.39 1.3</td>
<td>0.9 7.5</td>
<td>3.8</td>
<td>( \phi_{\rm SN}^{\text{SNIb/c}} = 0.3 )</td>
</tr>
</tbody>
</table>

\(^{1}\) A total mass of the Galactic disk of \( M_{\text{tot}} = 1.8 \times 10^{11} \) M\(\odot\) was assumed

\(^{2}\) SFRs were normalised to satisfy the condition \( \mu_1 = 0.1 \)

\(^{3}\) Present-day rates are listed

\(^{4}\) Salpeter IMF was assumed
Comparison of the predicted and observed present-day formation rates of MS and post-MS stars in the Galactic disk reveals that:

- the present-day formation rates of MS stars predicted by the models selected in Sect. 4.2 are in reasonable agreement with the observations except for the bimodal SFR model (model D; for which the variation of the IMF with galactic age results in $\langle m \rangle_1 = 0.26 \, M_\odot$);
- the formation rates of AGB stars are larger than observed by $\gtrsim 30\% \text{--} 50\%$ for all models (except for the steep IMF model A3). We believe that this result is significant and may indicate that part of the stars in the mass range $0.8 \text{--} 8 \, M_\odot$ do not reach the AGB;
- a large discrepancy of a factor of $\sim 4\text{--}5$ is found between the predicted and observed formation rate of SNIa. This is true for all models, except models C (double exponential SFR) and F (age-dependent IMF), which are off by even larger factors, and model A1 (Scalo IMF) for which the discrepancy is a factor $\sim 2$;
- the predicted formation rates of SNIb/c are in reasonable agreement with the observations (for models E, A1, and A3, within a factor of about two);
- the SNII rates predicted are consistent with the observations (except for model A1);
- SFR models ending at $\mu_1 = 0.1$ with stars formed according to the Scalo (1986) or Kroupa et al. (1993) IMFs are probably excluded by the observations on the basis of the high formation rates of AGB stars and SNII predicted by such models.

**Discussion**

Present-day formation rates of MS stars, SNIb/c and SNII as predicted by the selected models for the Galactic disk are in reasonable agreement with the observations. Therefore, it is difficult to distinguish between different star formation models for the Galactic disk on the basis of these quantities (i.e. for SFR models with input parameters in Table 3.1).

An exception may be the bimodal SFR model for which the formation rate of low-mass stars (i.e. $m \lesssim 1 \, M_\odot$) is assumed constant as a function of galactic age, and the formation rate of more massive stars is assumed to vary directly proportional to the gas density, may be inconsistent with the observations since it predicts too many low-mass main-sequence stars formed at present (cf. Table 4.3; model D). A reduction of the SFR of low-mass stars by a factor 3 would lead to overproduction of heavy elements by massive stars and/or a present-day gas-to-total mass-ratio inconsistent with the observations. Instead, decreasing the mass cut of $\sim 1 \, M_\odot$ would improve the agreement with the observations and mimic conventional unimodal SFR models. Clearly, there are no indications for a bimodal SFR from the present-day formation rates of evolved stars.

- **AGB stars**

The present-day formation rate of AGB stars predicted by the selected models A--F, i.e. $\sim 2.3 \, yr^{-1}$, is inconsistent with the observed value of $R_{1\,AGB} \sim 1.5 \pm 0.5 \, yr^{-1}$. This result is insensitive to the star formation history assumed. If one excludes the possibility that the total number of (post-) AGB stars observed in the Galactic disk is considerably underestimated, this suggests that the total number of main-sequence stars (with initial masses between $\sim 0.82 \, M_\odot$, i.e. the turnoff mass $m_{\odot}(t_{ev})$ at galactic age of 14 Gyr, and 8 $M_\odot$) experiencing third dredge-up is overestimated and/or that the IMF assumed for stars in this mass range is in error.

Low-mass AGB stars ($m \lesssim 1 \, M_\odot$) are the dominant contributors to the present-day formation rate of AGB stars in the Galactic disk because: 1) the formation probability of low-mass stars is strongly favoured for any reasonable IMF, and 2) the pre-AGB lifetimes of low-mass AGB stars are much longer than for high-mass ones, and 3) the SFR in the past has been substantially higher than at present.
Comparison of the predicted formation rate of AGB stars for various IMFs in case of model A (see Table 4.3) indicates that steep IMFs \(\gamma \geq 2.5\) and/or lower values of the minimum stellar mass limit at birth \(m_i \lesssim 0.1\ M_\odot\) may provide better agreement with the observations ⁴.

A value of \(m_l \lesssim 0.1\ M_\odot\) would imply rapid exhaustion of the disk ISM by very low mass stars and would strongly reduce the formation rates of both AGB stars, SNe, and other post main-sequence stars. Although this possibility cannot be excluded entirely, current observations do not support the formation of stars with masses much lower than \(~0.1\ M_\odot\) to be a common phenomenon in the Galactic disk (e.g. Scalo 1986). Therefore, we propose that not all stars with initial masses between 0.82 and 8 \(M_\odot\) enter the AGB at the end of their lives. Hence, the minimum initial mass of AGB stars is considerably larger than \(~0.82\ M_\odot\) and/or a substantial fraction of low-mass stars with initial masses between \(~0.82\) and \(\sim 1.2\ M_\odot\) does not reach the AGB (or leave a PN). We will return to these possibilities when dealing with the luminosity function of AGB stars in the Galactic disk (Sect. 4.3.6).

**SNIa**

In our models, the present-day SNIa rate is at least a factor 4–5 too low compared to the observations. This finding is independent of the assumed SFR and IMF (provided that a flat IMF at low masses can be excluded by the current formation rate of AGB stars, see above). Simply increasing the fraction \(\phi_{\text{SNIa}}\) of WD progenitors which ultimately end as SNIa is not an option, since this will lead to overproduction of iron and will result in an AMR inconsistent with the observations. This is true also for any other option that would increase \(\phi_{\text{SNIa}}\) over the entire galactic lifetime. In addition, we will argue below that the discrepancy above remains no matter the set of stellar yields used. Instead, we will show that the discrepancy can be explained adequately when the delay time of SNIa after formation of the WD progenitor is taken into account.

For models with the Geneva/Nomoto data, the discrepancy poses a serious problem since the remaining model parameters were chosen such that the iron contribution by SNIb/c and SNII cannot be further reduced considerably (see Table 3.3) while the present-day contribution of SNIa to the total stellar iron ejection rate is \(~20\%) in these models. For the same models but with the Woosley/Weaver data, we have shown that the resulting overall level of the AMR is typically 0.3 dex below that observed. In such models, the present-day contribution of SNIa to the iron enrichment of the disk ISM is \(~30\%). Thus, to provide agreement with the observed SNIa rate, the SNIa iron contribution would have to be increased by a factor of 4–5 for models using the WW data. In this case, the SNIa contribution to interstellar iron would be \(~70\%) and the overall level of the AMR would be increased by \(~0.3\) dex (i.e. a factor of 2). Although this would still be consistent with the observed AMR, we will discuss in Sect. 4.3.4 arguments against such a large iron contribution by SNIa over the entire galactic lifetime. Therefore, we argue that the discrepancy found between the predicted and observed rate of SNIa cannot be solved by simply assuming a higher rate of SNIa at all galactic ages. In other words, an age-dependent increase of the SNIa rate is required to provide agreement with both the overall level of the AMR and the present-day SNIa rate observed in the Galactic disk.

In principle, the rate of SNIa at a given galactic age \(t = T\) is determined by: 1) the total number of stars (with initial masses presumably between 2.5 and 8 \(M_\odot\)) that leave a WD at galactic ages \(t \lesssim T\), and 2) the detailed evolution scenario of the WD that ultimately leads to the occurrence of a SNIa (e.g. Nomoto 1991; Yamaoka 1993). In case of a binary (single or double WD) origin, the delay time between the formation of the WD progenitor and the actual SNIa explosion is expected to be a substantial fraction of the lifetime of the Galactic disk and to vary in a complex manner with the detailed evolution of the binary members, the initial mass-ratio of the binary components, and the decay of the orbital separation (e.g. Smecker-Hane & Wyse 1992; Renzini 1994).

We will show in Sect. 4.3.4 that such a delay has interesting implications for the iron enrichment by SNIa of the disk ISM. Here, we briefly discuss the first order effects of such a delay on the present-day formation rate of SNIa. Other possibilities to explain the observed rate of SNIa, e.g. by means of variations in e.g. \(m_i^{\text{SNIa}}\) and/or \(\phi_{\text{SNIa}}\) with galactic age are not supported by the observations.

---

⁴For a fixed amount of gas converted into stars according to a given IMF and star formation history, a steeper IMF towards low-mass stars results in a smaller SFR at present. This is due to the fact that gas is depleted more efficiently by low-mass, long-living stars. Consequently, the total number and current formation rates of (post-)main-sequence stars (e.g. AGB stars, SNIa, etc.) for a given SFR model is strongly reduced for steeper IMFs. When the lower mass limit of stars at birth is decreased, a similar effect occurs (specific combinations of \(m_l\) and \(\gamma\) may give very similar results in terms of total number and formation rates of post-main-sequence stars; see Sect. 4.2). Conversely, the large returned gas fraction of the stellar populations formed according to the Scalo (1986) or Kroupa (et al. 1993) IMFs, will result in relatively high formation rates and total numbers of (post-)MS stars. However, such a steep IMF at low and intermediate masses probably can be excluded by the observations (see Sect. 4.3). Note that in this manner, the total number and present-day formation rate of e.g. AGB stars observed in the Galactic disk can provide valuable constraints to the past SFR and IMF of low- and intermediate mass stars.
Variations with galactic age of the lower stellar mass limit at birth will be considered in Sect. 4.3.4 and are probably inconsistent with the observations.

In general, when the time delay for the occurrence of a SNIa after the formation of its WD progenitor is increased, the present-day SNIa rate for decreasing SFR models can be increased considerably. For model A, Table 4.4 illustrates the sensitivity of the present-day and average past SNIa rate to: 1) the minimum SNIa delay time \( \Delta t^{\text{min}} \) after formation of a WD progenitor formed at a given galactic age \( T \), and 2) the maximum delay time \( \Delta t^{\text{max}} \) after formation of the same WD progenitor. In this manner, SNIa which are associated with one and the same generation of WDs formed at galactic age \( t = T \), are allowed to go off with a constant probability anywhere between galactic age \( T + \Delta t^{\text{min}} \) and \( T + \Delta t^{\text{max}} \) (see Sect. 4.3.4 for the detailed probability profile).

### Table 4.4 Dependence of SNIa rate on the WD progenitor lifetime (model A)

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Delta t^{\text{min}} ) [Gyr]</th>
<th>( \Delta t^{\text{max}} ) [Gyr]</th>
<th>( R_1^{\text{SNIa}} ) [10(^{-3}) yr(^{-1})]</th>
<th>( \langle R^{\text{SNIa}} \rangle ) [10(^{-3}) yr(^{-1})]</th>
<th>( \epsilon_1 ) [10(^{-2})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>0.0</td>
<td>0.74</td>
<td>2.40</td>
<td>2.3</td>
</tr>
<tr>
<td>A-a</td>
<td>1.0</td>
<td>1.0</td>
<td>1.29</td>
<td>2.07</td>
<td>4.1</td>
</tr>
<tr>
<td>A-b</td>
<td>2.5</td>
<td>1.0</td>
<td>0.73</td>
<td>1.90</td>
<td>5.5</td>
</tr>
<tr>
<td>A-c</td>
<td>2.5</td>
<td>3.5</td>
<td>2.11</td>
<td>2.17</td>
<td>6.7</td>
</tr>
<tr>
<td>A-d</td>
<td>4.0</td>
<td>2.5</td>
<td>1.12</td>
<td>2.14</td>
<td>3.5</td>
</tr>
<tr>
<td>A-e</td>
<td>4.0</td>
<td>5.0</td>
<td>1.31</td>
<td>2.11</td>
<td>4.1</td>
</tr>
<tr>
<td>A-f</td>
<td>5.0</td>
<td>5.0</td>
<td>2.65</td>
<td>1.39</td>
<td>8.2</td>
</tr>
<tr>
<td>A-g</td>
<td>5.0</td>
<td>14.0</td>
<td>3.02</td>
<td>1.22</td>
<td>9.5</td>
</tr>
</tbody>
</table>

| Observed | 3±1 | 10±5 |

In case of an exponentially decreasing SFR (model A), the present-day formation rate of SNIa can be increased from \( R_1^{\text{SNIa}} = 0.7 \times 10^{-3} \) to \( 2.1 \times 10^{-3} \) yr\(^{-1}\) for values of \( \Delta t^{\text{min}} = 0 \) to 4 Gyr, respectively, for \( \Delta t^{\text{max}} = 5 \) Gyr. Similarly, a value of \( \Delta t^{\text{min}} = 2.5 \) Gyr (e.g. Smecker-Hane & Wyse 1992) and \( \Delta t^{\text{max}} = 14 \) Gyr equal to the lifetime of the Galaxy results in \( R_1^{\text{SNIa}} = 2.7 \times 10^{-3} \), which is a factor of \( \sim 4 \) larger compared to the case in which the SNIa delay time is omitted. In this manner, the present-day rate of SNIa can be increased by large factors (up to \( 10 \)–15) depending on the star formation history and SNIa delay time profile assumed.

From this exercise we find that the present-day SNIa rate can be increased easily by a factor \( \sim 4 \) for exponentially decreasing SFRs by assuming longer SNIa delay times after WD formation (model A; the corresponding decrease in the average past SNIa rate is \( \sim 50\% \)). The ratio \( \epsilon_1 = R_1^{\text{SNIa}} / R_1^{\text{SNIa+Ib/c}} \) increases by the same factor so that the delay of SNIa can provide good agreement with the observed ratio \( \epsilon_1 \sim 0.10 \pm 0.05 \). Conversely, the observed ratio \( \epsilon_1 \) would imply a typical SNIa delay time of \( \sim 5.5 \) Gyr for exponentially decreasing SFRs (model A and parameters as in Table 3.3). We note that \( \epsilon_1 \) increases as well when IMFs steeper than Salpeter are considered. When instead \( m_1 \) is increased from 0.01 to 0.1 M\(_{\odot}\) for a given IMF, \( \epsilon_1 \) remains unchanged since it is primarily sensitive to the shape of the IMF.

If the IMF has been constant over the lifetime of the Galactic disk and there are no age-dependent quantities involved that substantially affect the relative frequency of SNIa and SNII-Ib/c, \( \epsilon_1 \) is an indirect measure of the average past to present SFR provided that SNIa are delayed over considerable fractions of the lifetime of the disk. In contrast, if the IMF has changed with galactic age (e.g. by means of an age-dependent IMF slope, or lower stellar mass limit at birth) and/or other quantities which determine the relative frequency of SNIa and SNII-Ib/c did vary with galactic age (e.g. minimum initial stellar mass for SNIa, SNIa delay times, fraction of WDs that ultimately end as SNIa), the observed value of \( \epsilon_1 \) or the present-day rate of SNIa cannot be related simply to the past SFR and additional observational constraints are needed. In particular, theoretical estimates for SNIa quantities such as \( m_1^{\text{SNIa}} \), \( \phi^{\text{SNIa}} \), and \( \Delta t^{\text{min}} \) are closely related to each other and are extremely sensitive to possible variations in these quantities (or in the IMF) with galactic age. We will discuss additional constraints on such quantities in Sect. 4.3.4.

**Conclusion**

We summarize the main results obtained in this section as follows:

- we find that the present-day formation rates of MS stars, SNIb/c and SNIII as predicted by our models (which are selected on the basis of their ability to fit the observed AMR of iron in the Galactic disk) are in reasonable agreement with the observations except for bimodal SFR models;
the present-day formation rate of AGB stars is considerably overestimated in our models. This suggests that more than \( \sim 30\% - 50\% \) of the stars that are nowadays in their final stages of evolution and have initial masses between \( \sim 0.82 \) and \( 8 \) \( \odot \) do not reach the AGB at the end of their lives. We will consider this discrepancy in more detail in Sect. 4.3.7;

- an increase of the SNIa rate with galactic age is probably required to provide agreement with both the overall level of the AMR and the present-day SNIa rate observed in the Galactic disk. This conclusion is independent of the Galactic star formation history assumed;

- when the time delay for the occurrence of a SNIa after the formation of its WD progenitor is increased, the present-day SNIa rate can be increased by a factor up to \( \sim 4-5 \) for models in which the SFR decreases exponentially (over the main part of the lifetime of the disk). For such SFR models, mean SNIa time delays of \( 4-6 \) Gyr result in present-day SNIa rates which are consistent with the observations (depending on the assumed probability function for the occurrence of SNIa with WD age; see Sect. 4.3.4).

### 4.3.3 Star formation, gas depletion, and infall rates

We compare observational estimates of the star formation, gas depletion, and infall rates in the Galactic disk with corresponding predictions for the models selected in Sect. 4.2.

#### Observations

The present-day SFR of \( \sim 3.6 \pm 1.0 \ M_\odot \ yr^{-1} \) observed in the Galactic disk is derived mainly from tracers of the formation of massive O and B stars in nearby molecular clouds (e.g. IR radiation, Ly \( \alpha \) continuum photons, CO emission, etc.). These numbers are corrected for: 1) the formation of low-mass stars according to the local IMF (see Chap. 2), and 2) the variation of the SFR as a function of location in the Galactic disk. Apart from the uncertainties involved in the conversion of the observed SFR tracers to formation rates of massive stars, substantial uncertainties may arise from the fact that part of the stars formed nowadays in the Galactic disk is missed because of extinction effects and/or because low-mass stars are too faint to detect in associations of massive OB stars. Extrapolation of the SFR observed in the SNBH to the entire Galactic disk (including spiral arms, bulge, radial variations of the gas density in the disk, etc) forms an additional source of uncertainty.

If we assume a total amount of molecular hydrogen in the Galactic disk within \( R \sim 15 \) kpc of \( 3 \times 10^9 \ M_\odot \) (Scoville & Sanders 1987) and further assume a mean lifetime of a star forming molecular cloud of \( \sim 3 \) \( 10^6 \) yr (e.g. Garmany et al. 1982; see Sect. 5.5.2), a present-day SFR of \( 3.6 \ M_\odot \ yr^{-1} \) would imply that only \( \sim 0.5\% \) of the total disk molecular content is currently forming stars. Conversely, many of the uncertainties mentioned above are contained in observational estimates of the latter percentage. Therefore, poorly known quantities such as the star formation efficiency and the star formation frequency of the molecular material in the Galactic disk cannot be used to tightly constrain the present-day SFR in the disk.

We consider \( 3.6 \ M_\odot \ yr^{-1} \) as a reasonable estimate for the present-day SFR in the Galactic disk within a factor of \( \sim 2 \). A minimum estimate for the present-day rate \( E_1 \) of gas returned by evolved stars can be made by assuming that the returned fraction for stars formed according to a given IMF and constant SFR is a first order approximation to the mean returned fraction for such stars formed according to a variable SFR, averaged over the lifetime of the Galactic disk. In case of exponentially decreasing SFRs, the present-day returned fraction is always larger than in the constant SFR approximation. For reasonable IMFs and SFRs, the difference is usually less than 50%. In case of the Scalo IMF with \( m_l = 0.1 \) and \( m_u = 60 \ M_\odot \), \( \langle R \rangle \sim 0.30 \) this implies a present-day ejection rate by evolved stars of \( E_1 \gtrsim 1.1 \ M_\odot \ yr^{-1} \). Corresponding values for the Scoville & Sanders IMF are \( \langle R \rangle \sim 0.4 \) and \( E_1 \gtrsim 1.5 \ M_\odot \ yr^{-1} \). Therefore, we estimate that the actual ejection rate of gas by evolved stars is \( E_1 = 1.5 \ M_\odot \ yr^{-1} \) within a factor of \( \sim 2 \) (this value scales with the present-day SFR). In this manner, we derive for the present-day gas consumption rate in the Galactic disk: \( G_1 \equiv C_1 - E_1 \sim 2 \ M_\odot \ yr^{-1} \) (also within a factor of two).

Estimates for the present-day infall rate of gas onto the Galactic disk range from \( 0.2-0.5 \ M_\odot \ yr^{-1} \) based on high-velocity clouds (e.g. Mirabel 1989; Lépine & Duvert 1994) to \( \sim 1.5 \ M_\odot \ yr^{-1} \) based on observations of atomic hydrogen (Oort 1970). However, these numbers are rather uncertain since part of the material currently falling onto the disk may have been previously injected into the Galactic halo. This may occur when multiple supernova explosions cause gas to expand out of the disk (e.g. the chimneys recently observed in the Perseus arm, see Normandeau et al. 1996). After cooling down, the gas may fall back in the disk gravitational potential and return to the disk ISM (see Sect. 5.5.3). Apart from infall, the disk may be replenished by gas accreted at large galactocentric distances. We consider \( F_1 \sim 0.5 \ M_\odot \ yr^{-1} \) as a reasonable lower limit to the present-day infall rate onto the Galactic disk. We note that, although we
choose not to reject models that do not incorporate gas infall after the onset of star formation in the disk, it seems highly unlikely that the Galactic disk settled entirely before star formation started.

**Model assumptions**

Present-day star formation, gas depletion, and infall rates were computed according to the equations given in Sect. 3.1. Apart from the IMF and star formation history assumed, these rates are sensitive to the adopted set of theoretical metallicity dependent stellar lifetimes, yields, and remnant masses. In particular, the adopted set of stellar evolution data predicts a turnoff mass of \( m_o(t_{ev}) \sim 0.82 \, M_\odot \) at a galactic age of \( t_{ev} = 14 \, \text{Gyr} \) (see Sect. 3.2). This value differs considerably from \( m_o \sim 0.98 \, M_\odot \) for stars born with solar metallicity that has been used in many previous investigations. As we will discuss below, this difference has important implications for the gas depletion rate in the disk ISM.

**Results**

Table 4.5 lists the present-day star formation, gas ejection, depletion, and infall rates predicted by the models selected in Sect. 4.2. In addition, the current returned fraction \( R_1 = E_1 / C_1 \), depletion ratio \( P_1 = E_1 / G_1 \), and gas depletion time scale \( \tau_{depl} = M_{g,1} / G_1 \) are included. For all models, the present-day gas mass is \( M_{g,1} = 1.8 \times 10^{10} \, M_\odot \) according to the condition \( \mu_1 = 0.1 \). For model A, we considered the effect of the IMF in more detail by means of (cf. Table 4.5): 1) an IMF computed iteratively from the PDMF given by Scalo (1986; model A1), 2) the IMF presented by Kroupa et al. (1994; see Sect. 4.3.1; model A2), and 3) a power law IMF with slope \( \gamma = 2.7 \) (model A3).

**Table 4.5 Theoretical present-day star formation, gas depletion, and infall rates**

<table>
<thead>
<tr>
<th>Model</th>
<th>( \langle m \rangle )</th>
<th>( C_1 )</th>
<th>( E_1 )</th>
<th>( G_1 )</th>
<th>( F_1 )</th>
<th>( R_1 )</th>
<th>( P_1 )</th>
<th>( \tau_{depl}^{\text{obs}} )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.35</td>
<td>4.8</td>
<td>2.2</td>
<td>2.6</td>
<td>–</td>
<td>0.46</td>
<td>0.84</td>
<td>6.9 ( \gamma = 2.35 )</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>0.85</td>
<td>7.1</td>
<td>5.4</td>
<td>1.7</td>
<td>–</td>
<td>0.76</td>
<td>3.18</td>
<td>10.6 Scalo IMF</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>0.50</td>
<td>4.9</td>
<td>2.7</td>
<td>2.2</td>
<td>–</td>
<td>0.55</td>
<td>1.23</td>
<td>8.2 Kroupa IMF</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>0.24</td>
<td>3.9</td>
<td>0.9</td>
<td>3.0</td>
<td>–</td>
<td>0.23</td>
<td>0.30</td>
<td>6.0 ( \gamma = 2.7 )</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.35</td>
<td>5.6</td>
<td>2.4</td>
<td>3.2</td>
<td>0.51</td>
<td>0.43</td>
<td>0.78</td>
<td>5.8</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.35</td>
<td>2.3</td>
<td>1.6</td>
<td>0.7</td>
<td>0.00</td>
<td>0.74</td>
<td>2.43</td>
<td>25.7 Double exp. SFR</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.26</td>
<td>11.3</td>
<td>2.4</td>
<td>8.9</td>
<td>–</td>
<td>0.20</td>
<td>0.26</td>
<td>2.0 Bimodal SFR</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.35</td>
<td>5.5</td>
<td>2.5</td>
<td>3.0</td>
<td>0.57</td>
<td>0.45</td>
<td>0.81</td>
<td>5.8 Dopita SFR</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.27</td>
<td>3.9</td>
<td>1.5</td>
<td>2.4</td>
<td>–</td>
<td>0.38</td>
<td>0.63</td>
<td>7.5 ( \gamma(t) )</td>
<td></td>
</tr>
</tbody>
</table>

Resulting present-day SFRs range from \(~2\) to \(7 \, M_\odot \, \text{yr}^{-1} \). This is more or less consistent with the observed value. An exception is the bimodal SFR model D which predicts \( C_1 = 11.3 \, M_\odot \, \text{yr}^{-1} \). This is clearly inconsistent with the observations and shows that bimodal SFR models with a constant SFR mode for low-mass stars, as well as constant SFR models, can be excluded on the basis of the observed SFR.

Present-day ejection-rates predicted range from \(~1\) to \(5.4 \, M_\odot \, \text{yr}^{-1} \). The latter value for the Scalo IMF model A1 is probably too large compared to the observations. In general, IMFs with an averse stellar mass of \( \langle m \rangle \gtrsim 0.5 \, M_\odot \) are excluded by the observations since such models imply both high SFRs and relatively large returned gas fractions \( (R_1 \gtrsim 0.5) \) according to the theoretical stellar remnant masses adopted (see Sect. 3.2). This usually leads to ejection rates that exceed the observationally estimated value considerably.

Fig. 4.38 illustrates the present-day normalised and cumulative mass-loss contributions by evolved stars for models A and D, respectively. It can be seen that the current rate of gas returned by evolved stars is dominated by low-mass stars, i.e. with \( m \lesssim 1.5 \, M_\odot \), both for exponentially decreasing and bimodal (or constant) SFR models. Due to the combined effect of the IMF and SFR, the highest total integrated mass-loss rates originate from stars with masses roughly equal to the turnoff mass \( (m_o(t) \sim 0.82 \, M_\odot) \). However, assuming a constant instead of an exponentially decreasing SFR for low-mass stars results in a shift of stars with the largest integrated mass-loss rate from \( m \sim 0.82 \) to \(~1 \, M_\odot \).

In Fig. 4.39 we illustrate the effect of the IMF on the normalised and cumulative mass-loss rates in case of SFR model A. For the Scalo IMF, the contribution by stars with masses between \(~1.5\) and \(4 \, M_\odot \) is enhanced compared to Salpeter IMF models. The effect is limited, however, and does not alter the dominance of stars less massive than \(~1.5 \, M_\odot \) to the present-day total mass ejection rate of evolved stars. For comparison, the Kroupa IMF mass-loss rates fall off somewhat steeper towards more massive stars than in case of the Salpeter IMF but are otherwise similar.
These results show that low-mass stars are very important in locking-up and recycling the disk ISM. Therefore, the minimum turnoff mass of stars as determined by the lifetime $t_{\text{ev}}$ of the Galactic disk since the onset of star formation, sensitively affects the chemical evolution of the disk ISM. Stars with masses less than $m_o(t_{\text{ev}})$ have not evolved off the main-sequence within $t_{\text{ev}}$ and do not contribute to the replenishment of the disk. For $t_{\text{ev}} = 14$ Gyr and the set of stellar isochrones discussed in Sect. 3.2, we find that $\sim 80\%$ of the matter currently returned by evolved stars is not enriched in elements heavier than nitrogen. Thus, the abundances of such elements will be strongly diluted by the majority of the mass-losing stars in the Galactic disk.

Assuming a lifetime of the Galactic disk of $t_{\text{ev}} = 10$ Gyr instead of 14 Gyr, would enhance $m_o$ from $\sim 0.82$ to $\sim 1.1$ $M_\odot$ for solar metallicity stars. This would reduce the present-day total stellar ejection rate by about 30% (see Fig. 4.38) with substantial effects on both the enrichment and dilution of the disk ISM. A similar shift in $m_o$ may arise when distinct sets of stellar evolution data are used or when the metallicity at early epochs in the evolution of the Galactic disk differs strongly between different models (see Sect. 3.2). We emphasize that the dependence of $m_o(t)$ on initial stellar metallicity is a critical ingredient in galactic chemical evolution studies, a point which has not been realized in many previous investigations.

For all models, the current gas consumption rates predicted are considerably less than the corresponding SFRs. This is due to the fact that large amounts of material are returned by evolved stars which replenish the ISM at high rates, especially for exponentially decaying SFRs. Depending on the SFR and IMF, gas consumption rates can be either larger or smaller than the present-day stellar ejection rates. In terms of gas consumption time scales, the disk ISM is consumed entirely within $\sim 2$ Gyr in case of bimodal and constant SFR models without infall, to time scales comparable to the Hubble time (models A2 and C). Both large returned gas fractions and SFRs strongly peaked at evolution times $t_{\text{ev}} < 14$ Gyr (e.g. model C) can greatly enhance the lifetime of the disk ISM against gas depletion by long-living stars.
4.3.3 Star formation, gas depletion, and infall rates

Models B, C, and E predict present-day infall rates between 0 and $\sim 0.6$ M$_\odot$ yr$^{-1}$. In these models, the SFR has been related directly to the gas infall rate by means of the gas density in the disk ISM while most of the total mass of the disk was assumed to fall in after the onset of star formation in the disk. In principle, for a given star formation history, models with or without infall do not differ significantly in terms of gas consumption and chemical evolution as long as the SFR varies proportional to the gas infall rate. Such models do not predict present-day gas infall rates larger than $F_1 = 0.5$–1 M$_\odot$ yr$^{-1}$. In constrast, much higher values of $F_1$ can be achieved when the coupling between the SFR and gas infall rate is relaxed. In that case, however, both the SFR and infall rate should vary with galactic age in such a way that the observed present-day gas-to-total mass-ratio $\mu_1 = 0.1$ is explained. The latter condition would lead to star formation and gas infall histories that are not motivated by theory or observations when star formation would not be regulated directly by means of gas infall. Thus, if the present-day infall rate of $F_1 \sim 0.5$ M$_\odot$ yr$^{-1}$ is considered as a reasonable lower limit (as is suggested by the observations), this would imply that: 1) the star formation and gas infall rate vary in a similar manner with galactic age, and 2) the present-day gas infall rate in the Galactic disk cannot be much larger than $F_1 \sim 0.5$–1 M$_\odot$ yr$^{-1}$.

Conclusion

We summarize the main results obtained in this section as follows:

- bimodal or constant SFR models probably can be excluded by the present-day SFR observed in the Galactic disk. However, we find that it is in general difficult to constrain evolution models for the Galactic disk by means of the present-day star formation, gas consumption, and infall rates observed. This is mainly due to the observational uncertainties in these quantities;
- low-mass stars are very important in recycling the disk ISM. For a Galactic lifetime since the onset of star formation of $t_{ev} = 14$ Gyr, we find that $\sim 80\%$ of the matter currently returned by evolved stars
comes from stars with $m \lesssim 8 \, M_\odot$. Therefore, the abundances of elements heavier than nitrogen will be strongly diluted by the majority of the mass-losing stars in the Galactic disk;

- if gas infall is indeed important in the Galactic disk at a present-day rate of $\gtrsim 0.5 \, M_\odot \, \text{yr}^{-1}$, as suggested by the observations, the variations of the SFR and gas infall/accretion rate with galactic age probably have been similar;

- models for which both the SFR and gas infall (or accretion) rate on average decay exponentially with galactic age are favoured by the observations.
4.3.4 Element abundances in the Galactic disk and halo

In this section, we present results for the abundance-abundance variations in the Galactic disk and halo as predicted by the models selected in Sect. 4.2. Interstellar abundances of elements up to the iron peak elements Ni and Zn are computed using the model described in Chap. 3 and are compared with the corresponding abundances observed in stars in the local Galactic disk and halo.

Introduction

The abundances of stars observed in the local Galactic disk and halo probably provide the most stringent test for galactic chemical evolution models. In this section, we present the results of such a test for a preselected set of models. For the same set of models, we consider a number of interesting options which may improve the agreement with the observations.

First, we interpret our results in terms of the uncertainties involved with the stellar yields resulting from two state-of-the-art models for nucleosynthesis in exploding massive stars, i.e. from the group of Nomoto, Hashimoto, Thielemann et al. and from the group of Woosley and Weaver et al. Second, we investigate the effect of the adopted star formation history and IMF on the resulting abundance-abundance variations in the Galactic disk. In particular, the effects of an age-dependent IMF by means of e.g. variations in the lower and upper mass limit of stars at birth are analysed. Both models with and without gas infall regulating the SFR are considered while the effect of the abundances in the infalling gas on the abundance-abundance variations is discussed. Third, we consider the effect of the maximum progenitor mass for SNII and its possible variation with age in combination with the fraction of massive stars that ultimately explodes as SNII/c. Fourth, the influence of the fraction of all WDs that ultimately explode as SNIa is investigated while the delay time between the formation and the ultimate thermonuclear explosion of WDs as SNIa is taken into account.

In the following, we briefly describe the observational data used to constrain our models and we recall the main model assumptions. Thereafter, we confront the resulting abundance-abundance variations with the observations for the models selected in Sect. 4.2. We investigate what kind of models are able to explain adequately the chemical evolution of the Galaxy as traced by the abundance-abundance variations observed among long-living stars and we discuss our results with respect to the validity of the various assumptions made. In Sect. 4.4, we briefly compare the main results obtained in this section with some important clues from other recent Galactic chemical evolution studies.

Observations

The data base of accurate abundances of stars observed in the Galactic disk and halo has been growing rapidly over the past few years. In Table 4.6 we list a subset of this database which has been used to compare our model results with. In columns 1 to 5, we give subsequently the literature source, number of sample stars, main elements up to the iron peak for which abundances were determined, an indication of disk (D) or halo (H) stars, and additional notes to the sample selection criteria. The data set in Table 4.6 is incomplete but is probably well suited to study the average trends in the abundance-abundance variations observed among disk and halo stars in the Galaxy. We have chosen neither to make a distinction between the method that was used to determine the element abundances in each investigation, nor to make a distinction according to accuracy, possible selection effects, stars in common, etc. This may be justified since this set of abundance data is primarily used to constrain models for the chemical evolution of the Galaxy as a whole and we draw attention to general trends rather than the abundances of individual stars observed. Furthermore, we expect that, for most of the stars included in Table 4.6, stellar evolution and mass-loss have not affected the initial abundances significantly.

For stars in the Galactic disk, we predominantly rely on the accurate abundances of nearly 200 F+G main-sequence dwarfs in the SNBH as presented by Edvardsson et al. (1993) and by Andersson & Edvardsson (1994). For halo stars, we rely on many different sources of which it is worth to mention the recent studies from Kipper et al. (1996) of cool metal-poor carbon stars and from Cayrel (1996) who presented the mean abundances of halo dwarfs at four distinct metallicities [Fe/H] = −4, −3.5, −3, and −2.4. We emphasize that the inhomogeneous set of data for halo stars used may give rise to substantial scatter in the abundance variations even for stars in common between different investigations. Furthermore, it is important to note that while the abundances of disk stars are predominantly confined to long-living main-sequence dwarfs, the abundances of halo stars are based on a wide variety of sources, including main-sequence stars, subgiants, and giants. Apart from differences in age, both the masses and spectral types of the sample halo stars may be much more inhomogeneous compared to that of the disk samples.
4.3 Modelling the chemical evolution of the Galactic disk: results

Table 4.6 Element abundances of stars in the Galactic disk and halo: literature sources

<table>
<thead>
<tr>
<th>Source</th>
<th># Elements</th>
<th>D/H Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andersson &amp; Edvardsson (1994)</td>
<td>85 C</td>
<td>D F + early G dwarfs</td>
</tr>
<tr>
<td>Barbuy (1988)</td>
<td>20 C, N, O, Fe</td>
<td>H halo giants</td>
</tr>
<tr>
<td>Barbuy &amp; Erdelyi-Mendes (1989)</td>
<td>24 O, Fe</td>
<td>D+H old + thick disk stars</td>
</tr>
<tr>
<td>Bessell et al. (1991)</td>
<td>8 O, Fe</td>
<td>H metal-poor G dwarfs</td>
</tr>
<tr>
<td>Cayrel (1996)</td>
<td>4 CNO, Na, α, Fe-peak</td>
<td>H averages of population III stars</td>
</tr>
<tr>
<td>Clegg et al. (1981)</td>
<td>20 C, N, O, Fe</td>
<td>D field F+G main-sequence stars</td>
</tr>
<tr>
<td>Edvardsson et al. (1993)</td>
<td>189 O, Na, α, Fe</td>
<td>D field F+G dwarfs</td>
</tr>
<tr>
<td>Fernley &amp; Barnes (1996)</td>
<td>9 O, Ca, Fe</td>
<td>D+H bright field RR Lyrae stars</td>
</tr>
<tr>
<td>Gratton &amp; Sneden (1991)</td>
<td>19 Fe, Ni</td>
<td>D+H field stars</td>
</tr>
<tr>
<td>Kipper et al. (1996)</td>
<td>5 C, N, Na, α, Fe peak</td>
<td>H metal-poor carbon stars</td>
</tr>
<tr>
<td>Laird (1985)</td>
<td>120 C, N, Fe</td>
<td>D+H field dwarfs</td>
</tr>
<tr>
<td>Magain (1987a)</td>
<td>18 Mg, Ca, Fe</td>
<td>H</td>
</tr>
<tr>
<td>Magain (1987b)</td>
<td>7 Si, Fe</td>
<td>H</td>
</tr>
<tr>
<td>Laird (1989)</td>
<td>20 Mg, Ca, Ti, Fe</td>
<td>H</td>
</tr>
<tr>
<td>Nissen et al. (1994)</td>
<td>9 O, Mg, Ca, Ti, Cr, Fe</td>
<td>H metal-poor halo stars</td>
</tr>
<tr>
<td>Sneden et al. (1991)</td>
<td>19 Fe, Cu, Zn</td>
<td>D+H field stars</td>
</tr>
<tr>
<td>Sneden et al. (1996)</td>
<td>37 Fe, Zn</td>
<td>D+H field stars</td>
</tr>
<tr>
<td>Tautvaišienė &amp; Straižys (1989)</td>
<td>10 Mg, Ca, Ti, Fe</td>
<td>H stars with [Fe/H] (\lesssim -0.8) used</td>
</tr>
<tr>
<td>Tomkin et al. (1992)</td>
<td>34 C, O, Fe</td>
<td>H dwarfs [Fe/H] = -3 to -0.8</td>
</tr>
</tbody>
</table>

*Elements up to the iron peak are listed only

Table 4.7 Observed abundances in the Galactic disk

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>0.01</td>
<td>-0.01</td>
<td>-</td>
<td>-0.38</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-2.4</td>
<td>-1.5</td>
<td>0.10</td>
<td>0.16 -2.63</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>-3.0</td>
<td>-0.37</td>
<td>-0.01</td>
<td>0.41 -3.14</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>-1.9</td>
<td>-0.24</td>
<td>-0.18</td>
<td>0.38 -1.98</td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>-0.09</td>
<td>0.06</td>
<td>-</td>
<td>-2.67</td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td>-2.2</td>
<td>-0.03</td>
<td>-</td>
<td>0.47 -3.03</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>-2.7</td>
<td>0.20</td>
<td>-</td>
<td>-8.86</td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>-2.1</td>
<td>0.20</td>
<td>-</td>
<td>0.33 -2.92</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>-0.09</td>
<td>0.01</td>
<td>-</td>
<td>-3.41</td>
<td></td>
</tr>
<tr>
<td>Ca</td>
<td>-2.1</td>
<td>0.12</td>
<td>-</td>
<td>0.31 -4.28</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>-2.1</td>
<td>0.20</td>
<td>-</td>
<td>-5.46</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>-2.4</td>
<td>0.05</td>
<td>-</td>
<td>0.41 -2.72</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>-2.4</td>
<td>-0.02</td>
<td>-</td>
<td>0.33 -4.14</td>
<td></td>
</tr>
</tbody>
</table>


Another way of comparison between observations and model results is to consider the mean abundances of stars at a given Galactic age for various elements simultaneously. Although this method is relatively new and heavily depends on the extent to which the abundances of the objects considered reflect the mean abundances in the Galactic ISM at the time these objects were formed, we find it useful to examine the resulting abundances at three specific ages in the Galaxy. For this purpose, we list in Table 4.7 the abundances by mass relative to solar found in: 1) halo dwarfs at the time the mean iron abundance in the Galaxy was [Fe/H] \(\sim -2.4\) (Cayrel 1996), 2) the young, well-studied supergiant Canopus (e.g. Reynolds, Hearshaw, and Cottrell 1988; Hill et al. 1996; see also references in Russell & Dopita 1992), 3) H II regions in the local Galactic disk (Cunha & Lambert 1992, 1994), and 4) young disk stars for which we considered the maximum abundances observed (e.g. Edvardsson et al. 1993; see Table 4.6). Solar abundances by mass are given in the last column of Table 4.7.
4.3.4 Element abundances in the Galactic disk and halo

We will restrict ourselves to a comparison of the model results with the data from halo dwarfs at the galactic age that \([\text{Fe/H}] \sim -2.4\), at the time the Sun was formed \(\sim 4.5\) Gyr ago, and at present in the Galactic disk. When comparing the resulting abundances with the solar abundances one should keep in mind that the abundances in the Sun are larger by \(\sim 0.2\) dex compared to the mean abundances of elements, such as O, Mg, and Fe, observed in disk stars with ages of \(\sim 4.5\) Gyr (see Chap. 5). Whether this is true also for elements such as C and N, as suggested by the abundances of Canopus but which is not supported by the abundances observed in both \(\text{Hii}\) regions and B-stars in Orion (e.g. Gies & Lambert 1992; Cunha & Lambert 1994), remains an open question. Table 4.7 shows that the highest abundances observed in disk stars are considerably larger than the abundances observed in \(\text{Hii}\) regions in the Galactic disk. This may be due to: 1) the effect of orbital diffusion of the disk stars which are at least \(2-3\) Gyr old (Sect. 4.1), 2) sequential stellar enrichment of the most metal-rich disk stars, 3) dust depletion of heavy elements in \(\text{Hii}\) regions (or possible errors in the \(\text{Hii}\) region abundances, see above), and/or 4) the possibility that the \(\text{Hii}\) region abundances do not reflect the mean present-day abundances in the disk ISM. Although we do not compare our model data with the extreme abundances observed in metal-rich disk stars, it is important to keep in mind these abundances when interpreting the results presented below.

Results

We subsequently investigate the impact of: 1) the stellar yields adopted, 2) the star formation history, 3) the IMF, 4) the delay time of SNIIa, and 5) the upper mass limit for SNII, on the resulting abundance-abundance variations of stars in the Galactic disk and halo.

![Figure 4.40 Resulting abundance-abundance variations (O, Fe, Mg, and C): dependence on stellar yields. Results are shown for the standard model (model A) with the Geneva/Nomoto (dashed curve) and the Woosley/Weaver stellar yields (solid). For comparison, observational data from the various sources listed in Table 4.6 are plotted (filled circles) while the data of disk F+G dwarfs from Edvardsson et al. (1993) and Andersson & Edvardsson (1994) are represented by (open circles).](image-url)
• Initial results and dependence on stellar yields

In Fig. 4.40 we show resulting abundance-abundance variations of the standard model (model A) in case of the Geneva/Nomoto (GN) and Woosley/Weaver (WW) yields with model parameters as listed in Table 3.1 (except for the WW case for which we adopted $F_{\text{SNIa}} = 0.02$ in order to fit the observed AMR of iron; see Fig. 4.31). Although the predicted ranges in the abundances of Fe, O, Mg, and C are in reasonable agreement with the observations for both sets of yields, the standard model is clearly inconsistent with the mean trends present in the observational data. The same is in general true for the abundance-abundance variations shown in Fig. 4.41 which concentrate on variations of C and N with O and Fe. We note that the lowest abundance point drawn for the models is determined by the age resolution of the models (in this case $\Delta t_{\text{cal}} = 2 \times 10^7$ yr) and the enrichment rate predicted at early epochs in the evolution of the Galaxy.

![Figure 4.40](image)

Figure 4.40 Resulting abundance-abundance variations (C, N, O, and Fe): dependence on stellar yields. Same as Fig. 4.40 but with emphasis on abundance variations of C and N with O and Fe.

At a given value of $[\text{Fe}/\text{H}]$, the standard model with the GN yields is found to predict $[\text{O}/\text{H}]$ ratios that are $\sim 0.6$ dex below the observations for stars with $[\text{Fe}/\text{H}] \lesssim -0.6$ dex (cf. Fig. 4.40). This is characteristic of all models selected which incorporate the GN yields. Increasing the upper mass limit for SNII from $m_{\text{SNII}} = 30$ to e.g. $60$ M$_\odot$ would not improve the situation since the increase in $[\text{O}/\text{H}]$ would be accompanied by a corresponding increase in $[\text{Fe}/\text{H}]$. In this case, the resulting $[\text{O}/\text{Fe}]$ ratio would increase slightly while the resulting AMR for iron would increase far above that observed, since the standard model with $m_{\text{SNII}} = 30$ M$_\odot$ fits the AMR perfectly (see Fig. 4.30). To overcome this problem, the nucleosynthesis of iron should be reduced considerably (when $m_{\text{SNII}}$ is increased) to explain both the $[\text{O}/\text{H}]$ and $[\text{O}/\text{Fe}]$ abundance ratios observed. Inspection of the iron contribution by SNIa for the standard model with the GN yields reveals that SNIa do not contribute more than $\sim 20\%$ to the present-day stellar ejection rate of iron (see below). Therefore, a sufficient reduction of the synthesis of iron is not possible when $m_{\text{SNII}}$ is increased at the same time. This implies that our models with the GN yields are unable to explain adequately the $[\text{O}/\text{Fe}]$ ratios observed in Galactic halo stars. However, the abundance ratios of other elements such as C and N can be explained reasonably well (see Fig 4.6).
We consider this argument also from another point of view. The mean \([O/Fe]\) ratio observed in Galactic halo stars is \(\sim0.6\) dex at \([Fe/H] = -2\) (see Fig. 4.40). This means that the oxygen-to-iron ratio by mass in such halo stars is \(\sim25\) (with \(^{16}\log (O/Fe)\) by mass is 0.8; cf. Table 4.7). Thus the IMF weighed oxygen-to-iron ratio required to explain the observations is \(\sim25\). This is much larger than the values of \(\Delta O/\Delta Fe\sim16\) given in Table 3.15 for models which incorporate the GN yields (assuming \(m_{SNH}^{Fe} = 30\) M\(_\odot\)). This remains true no matter the value of \(m_u\gtrsim30\) M\(_\odot\) assumed and is further independent of the (reasonable) IMF or initial stellar metallicity assumed (cf. Table 3.15). Therefore, we conclude that our models incorporating the GN yields are unable to explain the \([O/Fe]\) ratios observed in Galactic halo stars. This implies that the oxygen production by stars with metallicities \(Z \sim 0.001\) is underestimated by a considerable amount in the yield models of Maeder (1992) and/or Nomoto et al. (1995). The reason for this may be related to uncertainties in the initial mass vs. helium-core mass relation for massive stars (cf. Sect. 3.3.3). For models with the GN yields, the discrepancy of \(\sim0.6\) dex between the predicted and observed \([C/O]\) for stars with \([O/H]\) \(\lesssim -0.7\) dex is due to this underestimate of oxygen as well.

We investigate additional constraints implied by the element abundances observed at three distinct ages in the evolution of the Galaxy. For \(\sim20\) elements up to the iron peak, Fig. 4.42 shows the difference between predicted and observed abundance ratios \([El/H]\) at the following epochs: 1) at the age that \([Fe/H] = -2.4\) is reached in the models (i.e. \(\gtrsim10\) Gyr ago), 2) \(\sim4.5\) Gyr ago corresponding with the age of the Sun, and 3) at present in the Galactic disk. The element abundances observed at these respective epochs are listed in Table 4.7 and have been briefly discussed above.

We concentrate on the differences between the GN and WW models for the standard model. At low metallicities \([Fe/H] = -2.4\), the WW abundances are substantially larger than the GN abundances for all elements shown except Al and Fe. This is primarily due to the large differences between the Fe yields for massive stars as predicted by the GN and WW data (see Sect. 3.4). The IMF-weighed iron yields of SNII at \(Z \lesssim 0.001\) in the WW case are smaller by a factor of \(\sim35\) (1.5 dex) than in the GN case (assuming a Salpeter IMF and \(m_{SNH}^{Fe} = 30\) M\(_\odot\); see Table 3.11). Overall, in case of the standard model, the GN yields are somewhat better in agreement with the mean abundances of halo dwarfs although large discrepancies (\(\gtrsim0.5\) dex) are present for elements such as O, Cu, and Ti.

At 4.5 Gyr ago, the standard model both with the GN and WW yields is in satisfactory agreement with the solar abundances for most elements plotted in the bottom panels of Fig. 4.42, except for elements such as Co, Ni, and Cu. These elements are overproduced in the WW model. In contrast, Cu is severely underproduced in the GN model (as is K). Thus, while the abundances at \([Fe/H] = -2.4\) predicted for the GN and WW set of yields differ by huge factors, the corresponding abundances at \([Fe/H] \sim0\) are rather similar. This reflects the different metallicity dependence of the GN and WW yields as discussed above and in Sect. 3.4. Present-day abundances predicted by the standard model with the GN and WW yields differ considerably e.g. for C, O, Co, Ni, and Cu (see Tables 4.8 and 4.9). However, the overall trend is very similar and elements for which the abundances disagree considerably with the present-day abundances observed in Canopus and H\(_\upalpha\) regions in the SNBH are common for the two sets.

We conclude that the GN and WW yields behave very similarly at roughly solar metallicities and above, while marked differences are present at metallicities below solar (see Sect. 3.4). Since the element abundances predicted at given galactic age do strongly depend on the SFR and IMF adopted, a firm conclusion cannot be drawn about whether the GN or WW yield set is favoured on the basis of the differences in abundances predicted at \([Fe/H] = -2.4\) alone. In contrast, from the abundance-abundance variations discussed above, models with the GN yields cannot be reconciled with e.g. the \([O/Fe]\) and \([C/O]\) ratios observed in Galactic halo stars. This conclusion is independent of the SFR and IMF model used and is insensitive to the parameter values assumed (for reasonable ranges). Therefore, in the following we will restrict ourselves predominantly to models which incorporate the WW yields.

- **Differences between the abundance-variations observed among halo and disk stars**

We examine in more detail the observational data in Figs. 4.40 and 4.41. It can be seen that marked differences in the abundance-abundance variations are present between disk and halo stars which roughly can be separated at values of \([Fe/H] \sim -1\), \([O/H] \sim -0.6\), and \([C/H] \sim -1\). It is commonly accepted that these transition metallicities mark the epoch in the evolution of the Galaxy during which the major part of the disk material accumulated. This era probably coincides with the onset of main star formation in the disk ISM. The reason why the detection probability of stars with metallicities roughly equal to the transition metallicity is relatively low, is probably related to the rapid contraction and settling of the disk ISM. Either star formation at such metallicities has been suppressed or the kinematical properties of stars which formed during the transition phase are strikingly different from that of the halo and disk stars included in the samples listed in Table 4.6.
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.42 Comparison of predicted and observed abundances by mass at three distinct epochs of Galactic evolution. **Left panels**: results for the standard model with the Geneva/Nomoto (GN) yields. **Right panel**: same as left panel but for the Woosley/Weaver (WW) yields. From top to bottom, resulting abundances are compared to the mean abundances of halo dwarfs with [Fe/H] = −2.4 (Cayrel 1996; roughly 10 Gyr ago), the abundances in the Sun (Anders & Grevesse 1989; AG; ~4.5 Gyr ago), and the abundances observed in Canopus (e.g. Hill et al. 1995) and Hii regions in the SNBH (e.g. Cunha & Lambert 1994; represented by triangles; present-day abundances). Error bars indicate the uncertainty in the measured abundances (varying between ±0.05 and ±0.2 dex). For comparison, solar abundances by number (10 log (El/H) +12; data from AG) are plotted in the bottom panel for elements included in the computations.

The possibility that the enrichment in the Galaxy proceeded at a relatively rapid rate at the transition phase would imply that the formation of massive stars during this phase was not accompanied by a corresponding enhancement in the formation of low and intermediate mass stars (i.e. not many of such stars are nowadays observed). This would point to a difference in the IMF of stars formed during the transition phase as compared to the bulk of halo and disk stars that are nowadays observed. Selective enrichment of the material out of which the main part of the disk formed may be an alternative explanation for the possible enhanced rate of enrichment during the transition phase. Observational selection effects against stars formed during the transition phase and/or systematic errors in the analysis also may provide an explanation for the fact that stars with transition metallicities are underrepresented in the observational data currently available. We will return to the origin of the transition in metallicities below.
Apart from the different trends in the abundance-abundance variations among disk and halo stars, it can be seen from Figs. 4.40 and 4.41 that the scatter in the abundances of halo stars is generally much larger than for disk stars. The large spread observed among halo stars at a given value of [Fe/H] is probably in excess of experimental errors since a substantial part of the scatter usually remains within each of the samples of halo stars listed in Table 4.6. The origin of these abundance variations is unknown but the scatter appears to be dominated by variations in [Fe/H]. The spread in e.g. [O/H] and [C/H] seems to be substantially less than that in [Fe/H] (cf. Fig. 4.41). Abundance inhomogeneities in the halo ISM and/or the effects of stellar orbital diffusion may play an important role. Alternatively, the large abundance variations among halo stars in the Galaxy may be associated with stellar populations accreted from nearby galaxies that merged with the Galaxy at early epochs in its evolution. However, in either case it appears difficult to explain why the abundance variations among halo stars appear larger in [Fe/H] than in [O/H] (or [C/H]). Therefore, it seems more plausible that these abundance variations are due to small-scale enhancements in the iron enrichment of the halo ISM. This suggests that the large spread in iron abundances observed among halo stars in the Galaxy is related to the nucleosynthesis by intermediate mass stars ($m = 2 \simto 8 M_\odot$) which do not produce both iron and oxygen in substantial amounts at the same time. We propose that these abundance variations are primarily due to the local enrichment of the halo ISM by SNIa.

Since the progenitor stars of SNIa in general are more massive than those of SNIa, SNIa progenitors may travel considerable distances from their birthplace in the Galactic disk before they ultimately explode. This implies SNIa ejecta to be returned in the vicinity of the progenitor birthplace while this probably is not true for the ejecta of SNIa (primarily Fe). Furthermore, this implies that SNIa generally go off in relatively high-density environments as compared to SNIa. This probably has important consequences for the mixing time scales and mixing efficiencies of the SNI and SNIa ejecta. The distance which SNIa progenitors travel before they ultimately explode heavily depends on the progenitor mass as well as on the appropriate evolution scenario of the WD (e.g. single or double-WD degenerates; cf. Nomoto 1991; Renzini 1994). Part of the scatter in the abundance-abundance variations observed among Galactic halo stars may be due to sporadic contamination by SNIa ejecta and/or due to variations in e.g. the IMF, the upper mass limit for SNIa, etc.

**Table 4.8** Present-day abundances for selected models (GN yields)

<table>
<thead>
<tr>
<th>Model</th>
<th>H</th>
<th>He</th>
<th>Z</th>
<th>[C/H]</th>
<th>[N/H]</th>
<th>[O/H]</th>
<th>[Mg/H]</th>
<th>[Si/H]</th>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.67</td>
<td>0.30</td>
<td>0.032</td>
<td>0.48</td>
<td>0.41</td>
<td>0.09</td>
<td>0.33</td>
<td>0.29</td>
<td>0.12</td>
</tr>
<tr>
<td>A1</td>
<td>0.61</td>
<td>0.35</td>
<td>0.044</td>
<td>0.69</td>
<td>0.82</td>
<td>0.19</td>
<td>0.43</td>
<td>0.49</td>
<td>0.40</td>
</tr>
<tr>
<td>A2</td>
<td>0.69</td>
<td>0.29</td>
<td>0.022</td>
<td>0.31</td>
<td>0.35</td>
<td>−0.13</td>
<td>0.15</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>A3</td>
<td>0.74</td>
<td>0.25</td>
<td>0.008</td>
<td>−0.14</td>
<td>−0.25</td>
<td>−0.58</td>
<td>−0.30</td>
<td>−0.29</td>
<td>−0.44</td>
</tr>
<tr>
<td>B</td>
<td>0.68</td>
<td>0.30</td>
<td>0.027</td>
<td>0.41</td>
<td>0.35</td>
<td>0.01</td>
<td>0.25</td>
<td>0.22</td>
<td>0.03</td>
</tr>
<tr>
<td>C</td>
<td>0.66</td>
<td>0.31</td>
<td>0.031</td>
<td>0.48</td>
<td>0.42</td>
<td>0.08</td>
<td>0.33</td>
<td>0.29</td>
<td>0.12</td>
</tr>
<tr>
<td>D</td>
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<td>0.31</td>
<td>0.033</td>
<td>0.50</td>
<td>0.45</td>
<td>0.12</td>
<td>0.36</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
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<td>0.30</td>
<td>0.028</td>
<td>0.44</td>
<td>0.37</td>
<td>0.02</td>
<td>0.26</td>
<td>0.23</td>
<td>0.06</td>
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<tr>
<td>F</td>
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<td>0.021</td>
<td>0.30</td>
<td>0.24</td>
<td>−0.12</td>
<td>0.14</td>
<td>0.15</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 4.9** Present-day abundances for selected models (WW yields)

<table>
<thead>
<tr>
<th>Model</th>
<th>H</th>
<th>He</th>
<th>Z</th>
<th>[C/H]</th>
<th>[N/H]</th>
<th>[O/H]</th>
<th>[Mg/H]</th>
<th>[Si/H]</th>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.66</td>
<td>0.30</td>
<td>0.035</td>
<td>0.19</td>
<td>0.44</td>
<td>0.29</td>
<td>0.19</td>
<td>0.46</td>
<td>0.13</td>
</tr>
<tr>
<td>A1</td>
<td>0.61</td>
<td>0.34</td>
<td>0.051</td>
<td>0.44</td>
<td>0.82</td>
<td>0.41</td>
<td>0.34</td>
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<td>0.57</td>
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<td>−0.07</td>
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<td>0.11</td>
</tr>
<tr>
<td>A3</td>
<td>0.74</td>
<td>0.25</td>
<td>0.009</td>
<td>−0.23</td>
<td>−0.25</td>
<td>−0.44</td>
<td>−0.57</td>
<td>−0.23</td>
<td>−0.35</td>
</tr>
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<td>0.030</td>
<td>0.09</td>
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<td>0.20</td>
<td>0.11</td>
<td>0.39</td>
<td>0.06</td>
</tr>
<tr>
<td>C</td>
<td>0.65</td>
<td>0.31</td>
<td>0.035</td>
<td>0.21</td>
<td>0.46</td>
<td>0.29</td>
<td>0.19</td>
<td>0.47</td>
<td>0.15</td>
</tr>
<tr>
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<td>0.037</td>
<td>0.21</td>
<td>0.48</td>
<td>0.32</td>
<td>0.21</td>
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<tr>
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<td>0.031</td>
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<td>0.21</td>
<td>0.13</td>
<td>0.40</td>
<td>0.09</td>
</tr>
<tr>
<td>F</td>
<td>0.69</td>
<td>0.28</td>
<td>0.023</td>
<td>0.13</td>
<td>0.27</td>
<td>0.04</td>
<td>−0.04</td>
<td>0.27</td>
<td>0.09</td>
</tr>
</tbody>
</table>

- **Dependence on star formation history**

In this section, we investigate the dependence of the abundance-abundance variations for various elements on the Galactic star formation history. Figs. 4.43–4.45 show results for the abundance-abundance variations of C, N, O, Fe, Mg, Si, and Cu for the SFR models A–F selected in Sect. 4.2 (WW yields and model parameters as listed in Table 3.3 except for $F_{\text{SNIa}} = 0.02$).
First, it can be seen from Figs. 4.43–4.45 that the influence of the SFR on the resulting abundance-abundance variations is large, in particular for elements which are predominantly synthesized in massive stars ($m \gtrsim 8$ M$\odot$). For abundance-variations of elements such as C and N, which partly originate from intermediate mass stars, the effects of the SFR are relatively small (see Fig. 4.44). Thus, in principle, the abundance-abundance variations recorded by long-living stars in the Galaxy for elements such as O, Ne, Mg, Al, and Si (which are mainly produced by massive stars; see Sect. 3.4) provide stringent constraints to the Galactic star formation history.

Second, none of the SFR models plotted provides an adequate explanation of the detailed abundance-abundance variations observed among Galactic halo and disk stars. In particular, the different trends in these variations when going from halo to disk stars (as discussed above) suggest that an important ingredient is missing in the models which takes into account additional variations of the element productions with galactic age.

Third, although large scatter is present in the abundance data for halo stars, we claim from the SFR models shown in Figs. 4.43–4.45 that Galactic star formation histories which incorporate gas infall, and show a maximum in the SFR several Gyr later than the onset of star formation in the Galaxy, are clearly favoured by these data. This is most clearly visible from the observed variations in [C/Fe], [Si/Fe] and [Ca/Fe] with [Fe/H]. For monotonically decreasing SFRs (after a very rapid initial onset of star formation), variations in [El/Fe] (e.g. El = C, O, Mg, Si, and Ca) with [Fe/H] (or [O/H]) continue to increase with decreasing metallicity. Thus, such SFR models usually result in [El/Fe] ratios which exceed the observations.

Figure 4.43 Resulting abundance-abundance variations (O, Fe, Mg, and C): dependence on star formation history. Results are shown for models A−F selected in Sect. 4.2 with the WW yields. Model parameters are as listed in Table 3.3 except for $F_{\text{SN1a}} = 0.02$. Results are shown for the following SFRs: 1) density dependent SFR with $n=1$ without infall (thick dashed curve; model A) and with infall (solid; model B), 2) double exponential SFR with infall (thick dotted; model C), 3) bimodal SFR without infall (dashed, model D), 4) Dopita SFR with infall (thick solid, model E), and 5) density dependent SFR ($n=1$) with SFR-dependent IMF slope (dash-dotted, model F). For comparison, observational data from the various sources listed in Table 4.6 are plotted (filled circles) while the data of disk F+G dwarfs from Edvardsson et al. (1993) and Andersson & Edvardsson (1994) are represented by (open circles).
4.3.4 Element abundances in the Galactic disk and halo

Figure 4.44 Resulting abundance-abundance variations (C, N, O, and Fe): dependence on star formation history. Curves and symbols have the same meaning as in Fig. 4.43.

Figure 4.45 Resulting abundance-abundance variations (Si and Cu): dependence on star formation history. Curves and symbols have the same meaning as in Fig. 4.43.
by substantial factors. In contrast, models for which the SFR gradually increases to a given maximum and thereafter decreases exponentially predict \([\text{El/Fe}]\) to increase with \([\text{Fe/H}]\) up to a given value and to decrease thereafter. A natural manner to explain such a behaviour of the SFR with galactic age, is to allow the disk ISM to build up gradually over several Gyr by means of gas infall and/or accretion of material combined with a density dependent SFR law with \(n \geq 1\).

In principle, the predicted variations of \([\text{El/Fe}]\) with \([\text{Fe/H}]\) for elements such as O, Mg, Si, and Ca, mimic the behaviour of the underlying SFR with galactic age. From the observed variations of e.g. \([\text{Mg/Fe}]\) and \([\text{Ca/Fe}]\) with \([\text{Fe/H}]\), we argue that models for which the Galactic SFR increases over several Gyr up to a given maximum and thereafter decreases exponentially are favoured by the observations. The agreement of such models with the observed abundance-abundance variations depends on the contraction time of the disk ISM before the maximum SFR in the disk ISM is reached. In case of an instantaneous onset of star formation and thereafter monotonically decreasing SFR, the agreement with the observations could be improved considerably by assuming an upper mass limit of stars at birth at early epochs in the evolution of the Galaxy that is considerably smaller than at present. We will discuss these possibilities below.

Tables 4.8 and 4.9 demonstrate the effect of the adopted SFR on the resulting present-day abundances for the models selected in Sect. 4.2. In columns 2–4, present-day mass fractions of H, He, and Z (i.e. the metal-abundance integrated over all elements heavier than helium) are given. Columns 5–10 list the element abundance ratios \([\text{El/H}]\) by mass (relative to solar) for C, N, O, Mg, Si, and Fe. We give the abundances predicted for the SFR models A–F selected in Sect. 4.2 as well as for the standard model with different IMFs (models A1–A3; see below).

**Figure 4.46** SFR models: effect of star formation history. **Top panels:** SFR histories assumed and resulting AMRs. **Bottom panels:** corresponding \([\text{Mg/Fe}]\) and \([\text{O/Fe}]\) vs. \([\text{Fe/H}]\) relations. Curves have the following meaning: burst SFR model with exponentially decreasing infall (solid curve; \(\tau_{\text{inf}} = 0.5\) Gyr; see text), double exponential SFR with infall (dotted; model C; \(\tau_{\text{inf}} = 0.5\) Gyr; see Sect. 4.2), and Dopita SFR model with infall (dash-dot; model E; \(\tau_{\text{inf}} = 3\) Gyr; see Sect. 4.2). Observational data as in Fig. 4.40 (symbols).
4.3.4 Element abundances in the Galactic disk and halo

Figure 4.47 SFR models: effect of Galactic age $t_{burst}$ at which initial burst occurs. Top panels: SFR histories assumed and resulting AMRs. Bottom panels: corresponding [Mg/Fe] and [O/Fe] vs. [Fe/H] relations. Curves have the following meaning: $t_{burst} = 1$ Gyr (solid curve), 3 Gyr (dashed), and 5 Gyr (dotted). For all models, exponentially decreasing gas infall was assumed ($t_{inf} = 0.5$ Gyr and $\delta_0 = 0.1$). Observational data as in Fig. 4.40 (symbols).

- Dependence on burst profile and burst SFR threshold

To investigate the effect of the initial burst of the SFR on the abundance-abundance variations we consider a different class of models in which the SFR is forced to follow a given burst profile. The burst profile assumed consists of two parts (see Fig. 4.46): 1) a Gaussian half for the rising part of the burst (with width $\sigma_{burst}$), and 2) an exponential decaying half for the decreasing part of the burst (with decay time scale $\tau_{inf}$). This type of SFR profile is chosen for convenient parameterization of the burst profile in terms of $\sigma_{burst}$ and $\tau_{inf}$. We note that the results presented below do not depend on the detailed description of the burst. In addition, we assume gas infall to decay exponentially on a time scale $\tau_{inf}$ as well. Since infall is assumed to take place from Galactic age $t = 0$ on, the initial burst of the SFR is delayed with respect to the onset of infall. The physical idea behind such a delay is that first a critical gas density must be reached in the disk ISM before star formation can be initiated at very high rates. When star formation triggers more star formation, a rapid burst occurs which is determined exclusively by the threshold gas density at which the chain of star formation is initiated. Thereafter, the SFR decays exponentially on a similar time scale as the gas infall/accretion rate. Note that this concept is different from the one in which star formation is determined by the gas density in the disk ISM. Although the idea of a star formation threshold has been known for a long time (see Chap. 2), we believe that the concept of a delayed SFR by means of a burst threshold is new and has not been studied in detail before.

Fig. 4.46 shows the threshold burst SFR model and the resulting AMR as well as the resulting [Mg/Fe] and [O/Fe] variations with [Fe/H]. Comparison with the double exponential and the Dopita SFR models (selected in Sect. 4.2) suggests that, basically, threshold burst SFR models may provide an equally adequate explanation for the abundance-abundance variations observed among Galactic halo and disk stars, in particular for [Mg/Fe]. The discrepancies seen for the variation of [O/Fe] with [Fe/H] do occur for all SFR models selected in Sect. 4.2 (computed with the WW yields) and must be associated with processes other than the SFR. Note that the present-day abundances ratios predicted by these models are all the same and only depend on the current gas fraction $\mu_1 = 0.1$ assumed.
Motivated by the above results, we investigate both the effects of: 1) the Galactic age $t_{\text{burst}}$ at which the main SFR burst occurs, and 2) the burst duration $\sigma_{\text{burst}}$ (during which the halo presumably collapses and the major part of disk is formed).

Fig. 4.47 illustrates the effect of $t_{\text{burst}}$ on the AMR and the variations of $[\text{Mg/Fe}]$ and $[\text{O/Fe}]$ with $[\text{Fe/H}]$. Burst ages $t_{\text{burst}} \gtrsim 1.5$ Gyr are probably inconsistent with the observations. However, different parts of the Galaxy may have collapsed at later times. Although large scatter is present in the abundance data of Galactic halo stars, the trend predicted in the variation of $[\text{Mg/Fe}]$ with $[\text{Fe/H}]$ suggest that the main episode of star formation occurred within $1 \sim 2$ Gyr after the onset of star formation in the Galaxy. Fig. 4.48 shows the effect of burst duration (or collapse time) on the same quantities as discussed before. When the burst duration is increased from $\sigma_{\text{burst}} = 0.25$ to $\gtrsim 0.5$ Gyr, the period of time during which enrichment of the ISM is balanced by infall of metal-deficient matter is increased compared to that for rapidly increasing SFR models. This leads to a reduction of the effect of the burst on the abundances of stars with $[\text{Fe/H}] \lesssim -1$. This suggests that the abundance-abundance variations for these stars do favour relatively short collapse times $\lesssim 1$ Gyr during which the SFR rapidly increases. However, other explanations of these abundance data may apply equally well and careful interpretation is necessary.

Figure 4.48 SFR models: effect of burst duration $\sigma_{\text{burst}}$. Top panels: SFR histories assumed and resulting AMRs. Bottom panels: corresponding $[\text{Mg/Fe}]$ and $[\text{O/Fe}]$ vs. $[\text{Fe/H}]$ relations. Curves have the following meaning: $\sigma_{\text{burst}} = 0.25$ Gyr (solid curve; $t_{\text{burst}} = 1$ Gyr), 1 Gyr (dashed; $t_{\text{burst}} = 3$ Gyr), and 1.75 Gyr (dotted; $t_{\text{burst}} = 5$ Gyr). For all models, exponentially decreasing gas infall was assumed ($t_{\text{inf}} = 0.5$ Gyr and $\delta_0 = 0.1$). Observational data as in Fig. 4.40 (symbols).

We conclude that models in which the SFR increases up to a given maximum during several Gyr and thereafter decreases exponentially are favoured by the abundance-abundance variations observed among halo and disk stars in the Galaxy. Such a behaviour of the SFR with galactic age is naturally explained by a rapid (or gradual) accumulation of the disk ISM by gas infall (and/or gas accretion), in combination with star formation regulated predominantly by the gas density in the Galactic ISM. As a consequence, gas infall onto the Galactic disk seems to be required to explain the observed abundance-abundance variations. In the following, we investigate whether this conclusion is altered by the effects on the resulting abundance-abundance variations of: 1) the stellar IMF, and 2) the enrichment contributions by SNII and SNII+SNIIb/c.
4.3.4 Element abundances in the Galactic disk and halo

- Dependence on the IMF

Fig. 4.49 shows the effect of the IMF on the stellar abundance-abundance variations for the Dopita SFR (model E). Since the formation of low-mass stars \((m < 1 \, M_\odot)\) in the Scalo and Kroupa IMF models is substantially less than for the Salpeter IMF, the enrichment of predominantly intermediate mass stars is considerably enhanced in the former models. This leads to a reduction in the \([\text{O/Fe}]\) and \([\text{Mg/Fe}]\) ratios compared to the Salpeter IMF model. In the same manner, the production of C and N is enhanced in the Scalo and IMF models compared to Salpeter. At low metallicities, these effects are less severe in case of the Kroupa IMF for which the IMF slope at low masses is \(\gamma \sim 1.5\) which is relatively steep compared to the Scalo IMF (see Sect. 4.3.1). The agreement with the observations for O and the \(\alpha\)-elements is improved when IMFs are considered which flatten towards low-mass stars as compared to the Salpeter IMF. In contrast, elements such as C and N are overproduced by intermediate mass stars in case of such flat IMFs. The latter discrepancy is severe and argues against such strong flattening of the IMF towards low-mass stars. Alternatively, the carbon and nitrogen yields of stars with metallicities \(Z \lesssim 0.001\) may be substantially in error (see below). In general, different IMFs result in a shift of the predicted abundance-abundance variations while the shape of these variations is predominantly determined by the underlying star formation (and infall) history.

![Figure 4.49 Effect of IMF on stellar abundance-abundance variations. Results are shown for the Dopita SFR (model E) with: 1) the Salpeter IMF (solid curve), 2) the Kroupa IMF (dotted), and 3) the IMF computed iteratively from the Scalo PDMF (dashed). Observational data as in Fig. 4.40 (symbols).](image)

For the models shown in Fig. 4.49, we have introduced a SNIa delay time between the formation and the ultimate explosion of the WD (see Sect. 4.2) for reasons discussed below. This by no means affects the conclusions concerning the effect of the IMF on the resulting abundance-abundance variations. We do not consider this delay of SNIa when discussing the effects of the lower (LML) and upper (UML) stellar mass limits at birth on the abundance-abundance variations as shown in Figs. 4.50 and 4.51.
When the LML is assumed to increase with the SFR, the formation of intermediate and massive stars is favoured at high rates of star formation. This leads to a relatively large production of elements such as O, Mg, and Si which are synthesized predominantly in massive stars during the burst. Consequently, the resulting present-day abundances of most elements for such models are much in excess of the observations (cf. Fig. 4.50; note that when the SFR ceases, the resulting abundance-ratios approach the values for constant LML models). This confirms our earlier finding that there is no observational support for large variations of the LML over the lifetime of the Galaxy (see Sect. 4.3.1). The formation probability of massive stars at early epochs in the evolution of the Galaxy may have been higher than at present as suggested by observations in external disk galaxies (Chap. 2). If the formation of low-mass stars has been suppressed in the Galaxy as well, such episodes of enhanced massive star formation must have been short compared to the lifetime of the Galaxy since this otherwise will lead to abundance ratios of e.g. \([\text{O/H}]\) and \([\text{Fe/H}]\) > 1 (cf. Fig. 4.50).

An alternative manner to explain enhanced massive star formation is by means of an increase in the UML at high SFRs. In this case, the formation rate of low-mass stars continues while the formation rate of massive stars is increased during the burst. Fig. 4.51 shows the resulting abundance-abundance variations for a model for which the UML increases with the SFR. In this case, the effect on the stellar abundance-abundance variations is determined by the abundance-ratios in the material returned by the most massive stars formed (as compared to the same ratios in the ejecta of intermediate mass stars). For the WW yields, this gives rise to a decrease of the \([\text{O/Fe}]\) and \([\text{C/Fe}]\) ratios at low metallicities during the burst (see also Fig. 3.10). The impact of an increase of the UML with the SFR on the abundance-abundance variations...
is different from that of raising the LML as distinct mass ranges of stars become relatively important in enriching the ISM. An increase in the UML with the SFR is not supported by the abundance-abundance variations observed in the Galaxy. Other variations of the stellar mass boundaries at birth with galactic age are beyond the scope of this investigation. However, such variations may alter the results obtained here.

We conclude that IMFs which flatten towards low-mass stars (as compared to the Salpeter IMF) predict C (and N; see Tables 4.8 and 4.9) abundances which are much too high compared to the observations. Thus, such IMFs are difficult to reconcile with the abundances observed among Galactic disk and halo stars unless the formation rate of intermediate mass AGB stars is suppressed at the same time. Small changes in the IMFslope (e.g. between $\gamma = 2.1$ and 2.35) with Galactic age may have occurred but a present-day slope steeper than $\gamma = 2.35$ is inconsistent with the abundance ratios observed among Galactic disk stars. Weak support is found for enhanced massive star formation by means of a modest increase of the LML when the SFR was high during early epochs in the evolution of the Galaxy. Models for which the UML increases with the SFR are not supported by the observations (unless simultaneous variations in e.g. the LML did occur). We note that part of the spread in the abundances observed in halo stars may be due to rapid, small-scale variations of the IMF,

- **Dependence on the enrichment by SNIa**

We investigate the influence of the contribution of SNIa to the enrichment of the ISM in the Galaxy. Since the main element synthesized in SNIa explosions is iron (see Sect. 3.3), SNIa predominantly affect the iron enrichment in the Galaxy. In principle, the element contribution by SNIa at galactic age $t$ depends on: 1) the past formation rate of SNIa progenitors, and 2) the characteristic delay time $\Delta t_{\text{SNIa}}$ between the formation of a SNIa progenitor and its ultimate explosion as SNIa. Apart from the adopted SFR and IMF, the past formation rate of SNIa progenitors is determined by the initial mass range $[m_{\text{SNIa,l}} \sim 2.25 \, M_\odot, m_{\text{SNIa,u}} \sim 8 \, M_\odot]$ of stars which leave a WD massive enough to explode as SNIa, and by the fraction $F_{\text{SNIa}} \sim 0.005$ of these WDs that ultimately ends as SNIa (see Sect. 4.2; Table 3.3). The dependence of the iron contribution by SNIa on the assumed values of $m_{\text{SNIa,l}}$ and $F_{\text{SNIa}}$ has been discussed in Sect. 4.2. The SNIa delay time $\Delta t_{\text{SNIa}}$ is determined by: 1) the lifetime $\tau_m(Z_\star)$ of the progenitor star leaving a WD, and 2) the time $\Delta t_{\text{WD}}$ between the formation of this WD and its actual explosion as SNIa:

$$\Delta t_{\text{SNIa}} = \tau_m(Z_\star) + \Delta t_{\text{WD}}$$

(4.27)

For SNIa progenitors with initial masses $\gtrsim 2.5 \, M_\odot$, the SNIa delay time is dominated by the delay $\Delta t_{\text{WD}}$ between the WD formation and the actual SNIa explosion (see Sects. 3.3 and 4.2) so that for such stars $\Delta t_{\text{SNIa}} \sim \Delta t_{\text{WD}}$. We here consider the impact of the SNIa delay time $\Delta t_{\text{SNIa}}$ on the stellar abundance-abundance variations.

![Figure 4.52](image-url) Schematic outline of the WD delay time distribution function for a given WD born at galactic age $t$. Several examples of distribution functions are shown: 1) exponentially increasing and thereafter constant (dashed curve), and 2) exponentially increasing, constant, and thereafter exponentially decreasing (solid and dotted curves). For clarity, the maximum WD delay time probability has been assigned to one in this figure. In our models, the probability function is normalised to the integral of the distribution function over WD age.

The WD delay time $\Delta t_{\text{WD}}$ depends on many details, such as the assumed evolution scenario of the WD ultimately exploding as SNIa (see Sect. 3.3) which, in fact, are unknown. In principle, $\Delta t_{\text{WD}}$ may vary strongly for WDs similar in mass or which belong to the same stellar generation. Observational estimates for the WD delay time range from zero to a Hubble time. Theoretical estimates for $\Delta t_{\text{SNIa}}$ are in the range 0.06–8 Gyr (e.g. Smecker-Hane & Wyse 1992; Renzini 1994). For a SNIa progenitor of 2.5 $M_\odot$ ($\tau \sim 0.8$)
the upper value would imply \((\Delta t_{\text{WD}} \gtrsim 7 \, \text{Gyr})\). Before discussing the implementation of the WD delay time in our models, we briefly address several related observations concerning the occurrence of SNIa.

In principle, the characteristic SNIa delay time may be determined by the turning point in the \([\text{O}/\text{Fe}]\) vs. \([\text{Fe}/\text{H}]\) relation at roughly \([\text{Fe}/\text{H}] \sim -1\) (e.g. Bravo et al. 1993; King 1995; Ishimaru & Arimoto 1995). However, such estimates heavily depend on: 1) the assumed lifetime of the Galaxy at the onset of star formation, and 2) the variation and magnitude of the iron enrichment rate up to ages at which \([\text{Fe}/\text{H}] \sim 1\) is reached in the Galactic ISM. In turn, the iron enrichment rate is determined by many uncertain quantities such as the history and magnitude of the SFR, IMF, stellar mass limits at birth, gas infall rate, and the metallicity dependent iron yields and contributions by SNII and SN Ib/c (as well as SNIa) during the early evolution of the Galaxy. Therefore, such estimates involve many assumptions and may give rather uncertain results. We here choose not to rely on the SNIa delay times derived in this manner.

The relatively high frequency of SNIa in late type spiral galaxies (van den Bergh 1991) suggests that either not all SNIa have old progenitors or that these rates are related to the high past SFRs in these systems. Della Valle & Livio (1994) find that the SNIa rate (per unit K luminosity of the parent galaxy) is \(~5\sim10\) times larger in late spirals than in ellipticals. Bartunov & Tsvetkov (1996) claim that the SNIa rates in spiral galaxies are higher in the neighbourhood of the spiral arms. These observations suggest that part of the SNIa is associated with a relatively young stellar population for which the WD delay time is probably less or comparable to the lifetime of the SNIa progenitor.

The expansion velocities of SNIa are found to correlate strongly with Hubble type of the parent galaxy in the sense that low expansion velocities of SNIa are observed only in early type galaxies (van den Bergh 1993; hereafter vdB93). This suggests that the SNIa expansion velocity is related to the age of the SNIa. It has been found that the rates of SNIa with large expansion velocities appear enhanced in post-starburst galaxies (vdB93). This leads to the hypothesis that SNIa with low expansion velocities belong to relatively old stellar populations while those with high expansion velocities may be associated with young stellar populations (Branch & van den Bergh 1993). In the latter case, the WD delay times are probably comparable to or smaller than the main-sequence lifetimes of relatively massive SNIa progenitors. In the former case, either the WD delay times are very small (for SNIa progenitor masses \(m \lesssim 1 \, \text{M}_\odot\)) or the WD delay times are substantial with \(\Delta t_{\text{WD}} \sim 1 \sim 10 \, \text{Gyr}\) (for SNIa progenitor masses \(m \gtrsim 1 \, \text{M}_\odot\)). SNIa progenitors with masses \(m \lesssim 1 \, \text{M}_\odot\) presumably leave WDs which are not massive enough to end as SNIa and, consequently, would require substantial amounts of matter to be accreted after the formation of the WD (e.g. from the companion star). Although we cannot exclude this possibility, it seems unlikely that such WDs do explode as SNIa within a Hubble time. Therefore, we propose that WD delay times up to \(~10 \, \text{Gyr}\) are required to explain at least part of the SNIa observed in early type galaxies.

On average, SNIa are brighter at maximum in late-type (spiral and irregular) galaxies than in early-type (lenticulars and elliptical galaxies). In combination with the slower decay of brighter SNIa, this suggests a broader dispersion in the ages of SNIa progenitors in late-type compared to early-type galaxies (Ruiz-Lapente et al. 1996) and may imply that, on average, younger (and perhaps more massive) SNIa progenitors are present in late-type galaxies. This is consistent with the observations of SNIa discussed above. Thus, no single class of progenitors of SNIa emerges when a variety of observational characteristics are considered (Livio et al. 1995)

It has been argued that the double WD (DD) scenario for SNIa may be relatively important in early-type galaxies, i.e. for old stellar populations (Ruiz-Lapente et al. 1996). In this case, the Chandrasekhar mass (\(\sim 1.38 \, \text{M}_\odot\)) is reached at the time that the WDs coalesce and explode as SNIa. The DD SNIa time scale (or WD coalescence time) is primarily determined by the orbital shrinking due to the loss of angular momentum by gravitational wave radiation. In contrast, the single WD (SD) scenario for SNIa may be the dominant SNIa mechanism in late-type galaxies. In this case, the WD mass can be sub-Chandrasekhar and a wider range in WD masses exploding as SNIa is expected. This would be consistent both with the brighter SNIa and larger expansion velocities of SNIa in late-type galaxies discussed above. The SD SNIa time scale is determined by the lifetime of the secondary (for single degenerate binary systems with a primary mass \(\lesssim 8 \, \text{M}_\odot\)) in late-type galaxies, the spread in SNIa time scales and the continuous process of star formation probably causes a wide range both in the ages and masses of the exploding WDs (Ruiz-Lapente et al. 1996).

Motivated by the above observational and theoretical indications of WD delay times as large as \(\Delta t_{\text{WD}} \gtrsim 10 \, \text{Gyr}\), we investigate the effect of such delays on the enrichment by SNIa. For this purpose, we assume an ad hoc probability distribution of the delay time for a given WD (hereafter briefly SNIa profile) as shown in Fig. 4.52. For simplicity, the SNIa profile is assumed to be constant, i.e. independent of the WD mass, initial metallicity of the WD progenitor, or galactic age. Furthermore, we assume that the SNIa profile can be characterized by a minimum and maximum WD delay time denoted by \(t_{\text{WD}, \text{min}}\) and \(t_{\text{WD}, \text{max}}\), respectively. In the following, we will restrict ourselves to a SNIa profile as illustrated in Fig. 4.52 (dotted curve). For such
4.3.4 Element abundances in the Galactic disk and halo

Figure 4.53 Effect of SNIa delay time on $\text{[O/Fe]}$ and $\text{[Mg/Fe]}$ vs. $\text{[Fe/H]}$ abundance ratios. Top panels: Results are shown for the Dopita SFR (model E) with minimum and maximum WD delay times as follows: $(t_{\text{WD}}^{\text{min}}, t_{\text{WD}}^{\text{max}}) = (0, 0; \text{dashed curve}), (1, 1.1; \text{dotted}),$ and $(3, 3.1; \text{solid})$. Bottom panels: as for top panels but with $(t_{\text{WD}}^{\text{min}}, t_{\text{WD}}^{\text{max}}) = (3, 3.1; \text{solid curve})$ and $(3, 14; \text{dotted})$. A SNIa fraction $F^{\text{SNIa}} = 0.01$ was assumed. Observational data as in Fig. 4.40 (symbols).

SNII profiles, WD delay times between $t_{\text{WD}}^{\text{min}}$ and $t_{\text{WD}}^{\text{max}}$ occur with equal probability while the probability of WD delay times outside this range is effectively zero. In Sect. 4.3.2, we considered already the influence of the choices of $t_{\text{WD}}^{\text{min}}, t_{\text{WD}}^{\text{max}}$ on the present-day SNIa rate. It was found that for values of $t_{\text{WD}}^{\text{min}} \gtrsim 3 - 5$ Gyr, the present-day SNIa rate can be increased by factors up to $\sim 4 - 5$ for SFRs exponentially decreasing with age.

Fig. 4.53 shows the impact of the WD delay time on the contribution by SNIa to the ISM enrichment in case of the Dopita SFR (model E) with $F^{\text{SNIa}} = 0.01$ (WW yields and remaining model parameters as in Table 3.3). Clearly, the predicted abundance-abundance variations are inconsistent with the observations when no WD delay times are considered (cf. Fig. 4.53). This is true for all SFR models selected in Sect. 4.2. It can be seen that the shapes of the $\text{[O/Fe]}$ and $\text{[Mg/Fe]}$ ratios for stars with $\text{[Fe/H]} \lesssim -1$ become more consistent with the observations for larger WD delay times. We find that optimal agreement with the observed shapes is reached for values of $t_{\text{WD}}^{\text{min}} = 3 - 5$ Gyr. It is important to realize that such WD delay times, instead of implying real delays between the SNIa occurrence and the formation of its WD progenitor, may point to the possibility that the majority of SNIa are not progenitors with initial masses in the range between $\sim 2.5$ and $8$ $M_\odot$ but instead originate from stars with much lower masses, e.g. between $\sim 1.5$ and $2.5$ $M_\odot$ (see Sect. 4.2).

We note that the resulting abundance-abundance variations are sensitive as well to the magnitudes and variations of the SFR and IMF of intermediate mass stars $(m \sim 1 - 8 M_\odot)$ at early epochs in the evolution of the Galaxy. Apart from the value of $t_{\text{WD}}^{\text{min}}$, the detailed SNIa profile assumed is relatively unimportant for the shape of the $\text{[O/Fe]}$ and $\text{[Mg/Fe]}$ ratio variations with $\text{[Fe/H]}$. In particular, the precise value of $t_{\text{WD}}^{\text{max}}$ assumed marginally affects the resulting abundance-abundance variations (cf. Fig. 4.53).
Fig. 4.54 shows that the impact of the WD delay time also depends on the fraction $F_{\text{SNIa}}$ of WDs which ultimately end as SNIa (cf. Sect. 3.3). For values of $F_{\text{SNIa}} = 0.05 - 0.025$, both the present-day [Fe/H] ratio and the shape of the abundance-abundance variations are affected. For the Dopita SFR model we find that the [O/Fe] and [Mg/Fe] vs. [Fe/H] ratios observed among Galactic disk and halo stars are best fitted for values of $F_{\text{SNIa}} \sim 0.02$ and $t_{\text{WD}}^\text{min} \sim 3 - 5$ Gyr. However, both O and Mg abundances are too large compared to the observations, in particular at values of [Fe/H] $\lesssim -1$. Since these elements are predominantly synthesized in massive stars (see Sect. 3.4), this suggests that: 1) the upper mass limit for SNII assumed, i.e. $m_{\text{SNII}}^u = 30 \, M_\odot$, is somewhat too large, and/or 2) the Salpeter IMF overestimates the formation probability of stars with $m \sim 20 - 30 \, M_\odot$. For instance, when the Dopita SFR model is computed with the Scalo IMF instead of the Salpeter IMF (with $t_{\text{WD}} = 5$ Gyr, $F_{\text{SNIa}} = 0.01$, $m_{\text{SNII}}^u = 25 \, M_\odot$), the agreement with the observations can be considerably improved (see Fig. 4.49). However, we have argued above that models with IMFs flattening more strongly towards low-mass stars than the Salpeter IMF are probably inconsistent with the observations because such models overproduce C and N by large amounts. Therefore, we consider the effect of the upper mass limit of SNIa on the abundance-abundance variations in more detail below.

![SNIa contribution](image)

**Figure 4.54** Effect of SNIa contribution on [O/Fe] and [Mg/Fe] vs. [Fe/H] abundance ratios. Results are shown for the Dopita SFR (model E) with SNIa fractions $F_{\text{SNIa}} = 0.005$ (dashed curve), 0.015 (dotted), and 0.02 (solid). Minimum and maximum WD delay times ($t_{\text{WD}}^\text{min}, t_{\text{WD}}^\text{max}$) = (3, 3.1) were assumed. Observational data as in Fig. 4.40 (symbols).

We note that the effect of the WD delay time on the iron enrichment by SNIa leads to an underestimate of the iron abundance at early epochs in the evolution of the Galaxy. This discrepancy can be solved by assuming a larger age of the Galaxy since the onset of star formation. Also, the iron contribution by massive stars may have been much larger at such epochs than previously assumed. This may be due to variations in the IMF, stellar mass limits at birth, the fraction of massive stars ending as SNIb/c (which have larger mean [Fe/O] ratios in their ejecta than SNII, cf. Sect. 3.4), and/or the upper mass limit of SNII at early evolution epochs. Alternatively, the iron yields of massive stars from WW may be too small at low metallicities or the ages of stars e.g. in the Edvardsson et al. sample may be in error (see below).

- **Dependence on the upper mass limit of SNII**

  We investigate the influence of the upper mass limit of SNII on the predicted abundance-abundance variations. Figure 4.55 shows that the specific value of $m_{\text{SNII}}^u = 20 - 30 \, M_\odot$ assumed strongly affects the resulting abundance-abundance variations. Stars with masses in this range are important in determining e.g. the [O/Fe] and [Mg/Fe] ratios at early epochs in Galactic evolution. Since the SNII yields of e.g. O, Mg, Al, Si, and Fe strongly vary with initial mass in case of the WW data (see Fig. 3.10), the upper mass limit for SNII assumed has great impact on the [El/Fe] and [El/O] abundance ratios. For the Dopita SFR model, a value of $m_{\text{SNII}}^u \sim 30 \, M_\odot$ appears in best agreement with the observations. We emphasize, however, that this value is sensitive to e.g. the IMF, and the SNIa contribution to the iron enrichment. The agreement with the observations can be further improved by assuming a somewhat larger SNIa contribution of $F_{\text{SNIa}} \sim 0.02$.

  From Fig. 4.55, it appears that both O and Mg may be overproduced at metallicities [Fe/H] $\lesssim -1$. Therefore, another way to improve the agreement with the observations is to allow for an increase of the upper mass limit of SNII with the SFR. We consider an ad hoc variation of $m_{\text{SNII}}^u$ with the SFR as follows: a constant value of $m_{\text{SNII}}^u = 20 \, M_\odot$ at galactic ages for which the SFR has not yet reached its maximum
4.3.4 Element abundances in the Galactic disk and halo

Figure 4.55 Effect of upper mass limit of SNII on the stellar abundance-abundance variations. Results are shown for the Dopita SFR (model E) with $m_{\text{SNII}}^{\text{upper mass}} = 20 \ M_\odot$ (dashed curve), 25 $M_\odot$ (dot), and 30 $M_\odot$ (solid). A WD delay time $\Delta t_{\text{WD}} = 3.5 \ \text{Gyr}$ and SNIa fraction $F_{\text{SNIa}} = 0.01$ were assumed. Observational data as in Fig. 4.40 (symbols).

Figure 4.56 Effect of SFR-dependent upper mass limit of SNII on the stellar abundance-abundance variations. Results are shown for the Dopita SFR (model E) with: 1) $m_{\text{SNII}}^{\text{upper mass}} = 20 \ M_\odot$ (dashed curve), 2) $m_{\text{SNII}}^{\text{upper mass}}$ increasing with the SFR between 20 and 30 $M_\odot$ (solid), and 3) $m_{\text{SNII}}^{\text{upper mass}}$ increasing with the SFR between 20 and 40 $M_\odot$ (dotted). A WD delay time $\Delta t_{\text{WD}} = 3.5 \ \text{Gyr}$ and SNIa fraction $F_{\text{SNIa}} = 0.01$ were assumed. Observational data as in Fig. 4.40 (symbols).

Fig. 4.56 illustrates the effect of an SFR-dependent upper mass limit for SNII on the stellar abundance-abundance variations. It can be seen that the agreement with the observations is considerably improved compared to models incorporating constant values of e.g. $m_{\text{SNII}}^{\text{upper mass}} = 20 \ M_\odot$. Variations of $m_{\text{SNII}}^{\text{upper mass}}$ between 20 and 30 $M_\odot$ are in reasonable agreement with the observations.

In the following, we will view in retrospect the above results and examine the kind of Galactic chemical evolution models that we find to be in best agreement with the observations. In addition, we will draw attention to several discrepancies that remain.

Discussion

For a given set of stellar metallicity dependent yields, there is a very limited range of models for the star formation history and chemical evolution of the Galaxy which are able to explain the abundance-abundance variations and present-day abundances observed among Galactic disk and halo stars. There appears no unique galactic chemical evolution model in best overall agreement with the abundance and
age data currently available for stars in the Galaxy. Therefore, instead of a discussion of the model most consistent with these observations, we prefer to point out the main "pros and cons" of the kind of models that can explain adequately several basic characteristics of the observations and to draw attention to those trends that are hard to explain by the models presented here.

- Towards an appropriate model for the chemical evolution of the Galaxy

We restrict ourselves in this discussion to two models which are based on the Dopita SFR (model E) but differ slightly in some aspects related to the enrichment contributions by SN Ia and SN II. We will denote the models by E− and E+ as follows:

**Table 4.10 Parameters for models E− and E+ (Dopita SFR)**

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_u^{SNIa}$ [M$_\odot$]</th>
<th>$F_{SNII}^{SNII}$</th>
<th>$t_{WD}^{min}$ Gyr</th>
<th>$t_{WD}^{max}$ Gyr</th>
<th>$F_{SNII/c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E−</td>
<td>20</td>
<td>0.01</td>
<td>3.5</td>
<td>3.6</td>
<td>0.2</td>
</tr>
<tr>
<td>E+</td>
<td>25</td>
<td>0.02</td>
<td>3.0</td>
<td>3.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The star formation history and IMF for these models are identical with that for the Dopita SFR model selected in Sect. 4.2 and discussed above. Remaining model parameters are as listed in Table 3.3. Although the parameter values listed in Table 4.10 are not strikingly different, we will show that the results for models E− and E+ differ substantially.

Figure 4.57 displays resulting abundance-abundance variations of O, Fe, Mg, and C for models E− and E+. Both models include a substantial delay of the SN Ia occurrence after WD formation (see above) and are in good agreement with the trends in the [O/Fe] and [Mg/Fe] variations with [Fe/H] observed for disk stars with [Fe/H] $\leq -1$. Since the Woosley/Weaver yields vary strongly for stars with masses between 20 and 30 M$_\odot$, the enrichment of elements such as O, Mg, and Si, differs substantially when $m_u^{SNII}$ is increased from 20 to 25 M$_\odot$.

To improve the agreement with the observed [Mg/Fe] values for model E−, the SN II contribution can be increased to $F_{SNII/c}^{SNII} \sim 0.3$. This will have small corresponding effects on both the O and Fe abundances since the contribution by SN II to these elements is relatively low ($\sim 20\%$; see Sect. 3.4). Alternatively, the iron contribution by SN Ia can be decreased but this reduces the agreement between the models and the observations for abundance ratios such as [O/Fe] and [C/Fe]. To improve the agreement with the observed [O/Fe] values for model E+, the oxygen enrichment should be reduced considerably. For instance, this can be achieved by decreasing the upper mass limit for SN Ia assumed (e.g. $m_u^{SNII} = 20$ M$_\odot$ as in model E−) but this gives rise to an underestimate of the abundances of elements such as Mg and Si. Alternatively, to explain the observed [O/Fe] ratios with model E+, the iron contribution by SN Ia could be enhanced but this would result in considerable overproduction of iron. Altogether, to explain simultaneously the abundance-abundance variations of e.g. [O/Fe] and [Mg/Fe] would require very precise finetuning of the parameters affecting the heavy element production by massive stars ($m \gtrsim 8$ M$_\odot$).

In spite of the fact that models E− and E+ can explain adequately the basic trends observed in the abundance-abundance variations of elements such as O, Fe, and Mg in Galactic halo and disk stars, we have shown before that this can be achieved only when the enrichment by SN Ia is delayed for several Gyrs after formation of their WD progenitors. We have discussed both theoretical and observational arguments in support of such SN Ia delays. It is difficult to extract information about the detailed SN Ia delay time profile (cf. Fig. 4.52) from the observed abundance-abundance variations. However, a substantial delay of a large number of SN Ia over at least several Gyr after the major period of star formation is needed to strongly affect the slope of the [O/Fe] ratio vs. [Fe/H] at values of [Fe/H] $\gtrsim -1$. We like to recall that such a delay results in less efficient enrichment of iron during the early epochs of star formation in the Galaxy. As a consequence, SN Ia delay models are inconsistent with the observed AMR of iron at early epochs of star formation in the Galaxy during the time interval that the SN Ia enrichment is delayed (e.g. $\sim 3.5$ Gyr for model E−). Thus, such models are difficult to reconcile with the age and iron abundance data currently available for F and G dwarfs in the Galaxy, unless the ages derived for such stars are systematically too large (i.e. by at least 3-4 Gyr). Indeed, large systematic errors in the ages of the sample stars from Edvardsson et al. may be present but are not yet confirmed (see Sect. 4.1).

Apart from the impact of the enrichment of SN Ia with galactic age, we have argued that the detailed variation of the SFR with galactic age at early epochs in the evolution of the Galaxy strongly affects the resulting [O/Fe] and [Mg/Fe] ratio variations with [Fe/H]. From our models, it appears difficult to explain
Figure 4.57 Resulting abundance-abundance variations (O, Fe, Mg, C, and N) for models E− (solid curve; Dopita SFR, WW yields, $m_{\text{SNII}}^\text{SN} = 20 M_\odot$, $f_{\text{SNII}} = 0.01$, $t_{\text{WD min}}^{\text{WD}} = 3.5$ Gyr, other parameters as in Table 3.3) and E+ (dotted; Dopita SFR, WW yields, $m_{\text{SNII}}^\text{SN} = 25 M_\odot$, $f_{\text{SNII}} = 0.02$, $t_{\text{WD min}}^{\text{WD}} = 3$ Gyr). Observational data as in Fig. 4.40.
the trends observed for these ratios by means of one and the same star formation and chemical evolution model both for Galactic halo and disk stars. Models incorporating an early, a short ($\lesssim 1$ Gyr) period of intense star formation appear consistent with the abundance-abundance variations observed among halo stars in the Galaxy. Such models, however, are inconsistent with the abundance-abundance variations observed in disk stars. On the basis of the results presented above, we find it hard to explain the abundance-abundance variations in Galactic disk and halo stars unless: 1) variations did occur in e.g. the IMF, stellar mass limits at birth, enrichment contributions by SNII, SNIb/c, and SNII, with galactic age, and/or 2) the formation histories of stars in the Galactic halo and disk were decoupled, i.e. different processes have initiated and regulated the star formation history in the Galactic halo and disk ISM. Since both the kinematics, abundance-ratios, and abundances differ substantially between halo and disk stars, it seems plausible that such decoupling may have taken place (see Chap. 1). Thus, if it turns out that the delay of SNII is not the primary cause for the change in slope of the variation of $[\text{O}/\text{Fe}]$ with $[\text{Fe}/\text{H}]$, the process of star formation in the Galactic halo probably has been different from that in the disk. This may imply corresponding differences in e.g. the IMF, lower stellar mass limit, and/or upper mass limit for SNII.

Besides the distinction between the halo and disk ISM, the disk itself probably cannot be treated as a single component since substantial radial gradients appear to be present (sect. 4.1) and the gas and stellar surface densities, ISM abundances, total-to-gas ratio, and star-to-gas ratio decrease substantially when moving from the Galactic center outwards in the Galactic disk (Chap. 1). We modelled the Galaxy as a whole in order to keep the number of parameters involved as small as possible and to investigate the problems such models encounter when confronted with abundance data of both halo and stars in the Galaxy. Concerning these assumptions, the uncertainties still involved with the theoretical stellar yields in particular, and the large scatter present in the abundance data (especially for halo stars), it may be even surprising that our models are able to explain the main trends observed in the abundance-abundance variations at all.

We have argued that SFR models that exhibit an early maximum in their variation with Galactic age and are regulated by gas infall/accretion are probably favoured by the observations. However, this argumentation is based on weak observational support and further depends strongly on the adopted stellar yields as well as on the homogeneity and accuracy of the constraining stellar abundance data. We here have concentrated on the resulting abundance-abundance variations for the Dopita SFR model but e.g. double exponential SFR models may explain the observations equally well, depending on the gas infall and gas consumption time scales assumed. The origin of the scatter observed in the abundances of Galactic halo stars is important for a further distinction between models appropriate for the star formation history of the Galaxy on the basis of the abundance-abundance variations observed.

- Model discrepancies

Two clear shortcomings are present in the theoretical stellar yields discussed in Sect. 3.3. First, inspection of Fig. 4.57 (as well as earlier figures) reveals that the $[\text{C}/\text{O}]$ and $[\text{C}/\text{Fe}]$ ratios in stars with $[\text{Fe}/\text{H}] \lesssim -1$ predicted by our models are too large by $\sim 0.7$ dex (e.g. for halo stars values of $[\text{C}/\text{O}] = -0.6 \pm 0.1$ are observed). We argue that this is probably due to the overproduction of carbon by SNII at low metallicities both in the Geneva/Nomoto (GN) and Weaver/Woosley (WW) yield sets. From Tables 3.12 and 3.13 we find that the minimum $[\text{C}/\text{O}]$ ratio in the ejecta of SNII predicted by the GN and WW yield sets are $-0.32$ and $-0.45$ dex, respectively, in case of a Salpeter IMF and for $Z = 0.001$. In case of a Scalo IMF these ratios are $-0.16$ and $-0.45$ dex, respectively. Either increasing or reducing the upper mass limit of SNII does not improve the situation as can be seen from Figs. 3.10 and 3.11 and Tables 3.12 and 3.13. When the carbon contributions by SNIb/c and AGB stars are included these ratios become even larger and more inconsistent with the observations.

Comparison of the GN and WW SNII yields with the (wind+explosion) SNII yields from Maeder (1992; Table 3.13) gives $[\text{C}/\text{O}]$ ratios $-0.98$ and $-0.94$ at $Z = 0.001$ for the Salpeter and Scalo IMFs, respectively. This would give good agreement with the observations as has been shown recently by Prantzos et al. (1994, 1996), although it was needed to reduce the carbon yields from Maeder by 30% in order to explain the ratios $[\text{C}/\text{O}] \lesssim -0.5$ observed in Galactic halo stars. These authors argue that large uncertainties may be introduced in the yields e.g. by means of the $^{12}\text{C}(\alpha, \gamma)$ reaction rates adopted in the stellar evolution models. Although a detailed investigation of this problem is beyond the scope of this study, we like to emphasize that the Maeder (1992) carbon yields would be in perfect agreement with the observed $[\text{C}/\text{O}]$ and $[\text{C}/\text{Fe}]$ ratios observed in the halo stars considered here, as well as with the observed trends of these ratios with e.g. $[\text{O}/\text{H}]$ and $[\text{Fe}/\text{H}]$ (cf. Fig. 4.57). This suggests that the amount of carbon produced during the SNII explosion of massive stars as predicted both by the GN and WW yields is considerably too large. This may be e.g. related to the $^{12}\text{C}(\alpha, \gamma)$ rates adopted. As an alternative to the possible errors in the theoretical yields, the carbon abundances observed in low-metallicity stars may be considerably in error. However, this
seems unlikely since the carbon abundances determined by independent groups (which used different analysis techniques) are very similar for low-metallicity stars (see also Prantzos et al. 1996).

A second problem of our models is the overproduction of nitrogen by \( \sim 0.3 \)–\( 0.4 \) dex at apparently all metallicities. Nitrogen is predominantly synthesized in AGB stars. At solar metallicities, low-mass (\( m \lesssim 4 \, M_\odot \)) AGB stars are the main source of nitrogen, at metallicities \( Z \lesssim 0.001 \) nitrogen is mainly synthesized in AGB stars with \( m \gtrsim 4 \, M_\odot \) during hot bottom burning (see Sect. 3.3). This suggests that: 1) too many stars reach the AGB in our models, and/or 2) the effect of HBB is too large in our models. The first possibility would imply that the minimum initial mass for stars that reach the AGB is considerably larger than the turnoff mass \( m_{\text{TO}} \sim 0.82 \, M_\odot \). This is also suggested on the basis of the predicted rates of AGB stars and PNe in the Galaxy (see Sect. 4.3.2). The second possibility would imply that the effect of HBB is overestimated in massive AGB stars. This could be interpreted in terms of: a) the value of \( m_{\text{HBB}} = 0.8 \, M_\odot \) assumed is too low (see Sect. 3.3), and/or b) the fraction of stars more massive than \( \sim 4 \, M_\odot \) reaching the AGB is too large (either the assumed upper mass limit of AGB stars \( m_{\text{AGB}} = 8 \, M_\odot \) is too high, or not all stars in the range \( m \sim 4 \, \cdots \, 8 \, M_\odot \) reach the AGB/experience HBB). Clearly, this result needs further investigation.

Note that the above artefacts reduce (but do not take away completely) the main objections against IMFs that flatten towards low-mass stars from the point of view of the observed abundance-abundance variations.

- **Predicted present-day abundance contributions by AGB stars, SNIa, SNIb/c, and SNII**

We are left with a discussion of the contributions by AGB stars, SNIa, SNIb/c, and SNII to the present-day stellar ejection rates and abundances of the most abundant elements in the disk ISM. We concentrate on models E− and E+ discussed above.

Fig. 4.58 shows the abundances predicted at three distinct epochs of Galactic evolution for models E− and E+. At solar metallicities, the resulting abundances for model E+ are in better agreement with the observations than that for model E−. At early and late epochs, the discrepancies between the predicted and observed abundances are large, in particular for C, N, and O. At early epochs, the model abundances are determined by the detailed variation of the SFR with Galactic age and by the ejecta of predominantly massive stars (i.e. SNII and SNIb/c). We have discussed above the artefacts in the stellar yields of C and N, especially at low metallicities. Apart from C and N, the agreement between the observed and predicted abundances by model E+ at these epochs is satisfactory, taking into account the observational uncertainties in the abundances of halo stars with mean ratios \([\text{Fe/H}] \sim -2.4 \) (see Cayrel 1996).

### Table 4.11 Present-day IMF-weighed net element masses (Model E+, WW)*

<table>
<thead>
<tr>
<th>El</th>
<th>SNIa</th>
<th>SNIb/c</th>
<th>SNII</th>
<th>AGB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
</tr>
<tr>
<td>H</td>
<td>0.00</td>
<td>-5.60</td>
<td>-2.77</td>
<td>-0.036</td>
</tr>
<tr>
<td>He</td>
<td>0.03</td>
<td>2.10</td>
<td>1.70</td>
<td>0.026</td>
</tr>
<tr>
<td>C</td>
<td>0.05</td>
<td>0.30</td>
<td>0.10</td>
<td>0.002</td>
</tr>
<tr>
<td>N</td>
<td>0.00</td>
<td>0.03</td>
<td>0.05</td>
<td>0.005</td>
</tr>
<tr>
<td>O</td>
<td>0.14</td>
<td>0.25</td>
<td>0.53</td>
<td>-0.000</td>
</tr>
<tr>
<td>Mg</td>
<td>8.6 (3)</td>
<td>3.3 (2)</td>
<td>3.1 (2)</td>
<td>-</td>
</tr>
<tr>
<td>Al</td>
<td>9.9 (4)</td>
<td>2.8 (3)</td>
<td>3.5 (3)</td>
<td>-</td>
</tr>
<tr>
<td>Si</td>
<td>0.15</td>
<td>0.05</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td>Ca</td>
<td>1.2 (2)</td>
<td>2.2 (3)</td>
<td>4.2 (3)</td>
<td>-</td>
</tr>
<tr>
<td>Ti</td>
<td>2.5 (4)</td>
<td>1.5 (4)</td>
<td>6.9 (5)</td>
<td>-</td>
</tr>
<tr>
<td>Fe</td>
<td>0.74</td>
<td>0.13</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>Ni</td>
<td>0.14</td>
<td>0.02</td>
<td>0.14</td>
<td>-</td>
</tr>
<tr>
<td>Z</td>
<td>1.37</td>
<td>0.90</td>
<td>1.01</td>
<td>5.3 (3)</td>
</tr>
</tbody>
</table>

*Negative values indicate consumption

### Table 4.12 Present-day element contributions (Model E+, WW)*

<table>
<thead>
<tr>
<th>El</th>
<th>SNIa</th>
<th>SNIb/c</th>
<th>SNII</th>
<th>AGB</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
<td>([M_\odot])</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.00</td>
<td>-0.14</td>
<td>-0.44</td>
<td>-0.41</td>
<td>-0.170</td>
</tr>
<tr>
<td>He</td>
<td>0.03</td>
<td>0.12</td>
<td>0.39</td>
<td>0.49</td>
<td>0.119</td>
</tr>
<tr>
<td>C</td>
<td>0.05</td>
<td>0.23</td>
<td>0.31</td>
<td>0.42</td>
<td>0.56 (-3)</td>
</tr>
<tr>
<td>N</td>
<td>0.00</td>
<td>0.02</td>
<td>0.10</td>
<td>0.88</td>
<td>1.12 (-2)</td>
</tr>
<tr>
<td>O</td>
<td>0.04</td>
<td>0.10</td>
<td>0.86</td>
<td>-0.02</td>
<td>1.66 (-2)</td>
</tr>
<tr>
<td>Mg</td>
<td>0.04</td>
<td>0.20</td>
<td>0.76</td>
<td>-</td>
<td>1.11 (-3)</td>
</tr>
<tr>
<td>Al</td>
<td>0.04</td>
<td>0.16</td>
<td>0.80</td>
<td>-</td>
<td>1.20 (-4)</td>
</tr>
<tr>
<td>Si</td>
<td>0.24</td>
<td>0.11</td>
<td>0.66</td>
<td>-</td>
<td>3.39 (-3)</td>
</tr>
<tr>
<td>Ca</td>
<td>0.33</td>
<td>0.08</td>
<td>0.59</td>
<td>-</td>
<td>1.94 (-4)</td>
</tr>
<tr>
<td>Ti</td>
<td>0.31</td>
<td>0.24</td>
<td>0.45</td>
<td>-</td>
<td>4.20 (-6)</td>
</tr>
<tr>
<td>Fe</td>
<td>0.75</td>
<td>0.16</td>
<td>0.09</td>
<td>-</td>
<td>5.22 (-3)</td>
</tr>
<tr>
<td>Ni</td>
<td>0.16</td>
<td>0.03</td>
<td>0.82</td>
<td>-</td>
<td>4.68 (-3)</td>
</tr>
<tr>
<td>Z</td>
<td>0.14</td>
<td>0.12</td>
<td>0.52</td>
<td>0.23</td>
<td>5.29 (-2)</td>
</tr>
</tbody>
</table>

*Contributions are normalised to the total ejection rate of newly synthesized element mass as given in the last column. Negative values indicate consumption.

At present, the predicted abundances show large deviations from the mean abundances observed in HII regions in the SNBH and in Canopus. This suggests that the abundances of young objects in the solar vicinity are not well suited for comparison with the mean present-day abundances predicted in Galactic disk stars. Most elements for which a comparison between the mean abundances observed in young objects in
the SNBH and the maximum abundances observed in Galactic disk stars is possible, are more abundant in the latter objects by about 0.3–0.5 dex. The latter sources are probably contaminated by: 1) metal-rich stars formed much more inwards to the Galactic center than the present-day galactocentric distance of the Sun (see Sect. 4.1), and 2) old, high-metallicity stars. If we assume that stars in the Galactic disk have roughly mean present-day abundances between $[\text{El}/\text{H}] = 0$ and $+0.2$ dex, the predicted abundances of most elements would be in reasonable agreement with the observations.

![Figure 4.58](image.png)

**Figure 4.58** Comparison of predicted and observed abundances by mass at three distinct epochs of Galactic evolution. **Left panels:** results for the $E^-$ model (Dopita SFR, WW yields, $m_u^{\text{SNH}} = 20$ M$_\odot$, $F_{\text{SNIa}} = 0.01$, $t_{\text{WD}}^{\min} = 3.5$ Gyr, other parameters as in Table 3.3). **Right panel:** same as left panel but for the $E^+$ model (Dopita SFR, WW yields, $m_u^{\text{SNH}} = 25$ M$_\odot$, $F_{\text{SNIa}} = 0.02$, $t_{\text{WD}}^{\min} = 3$ Gyr, other parameters as in Table 3.3). From top to bottom, resulting abundances are compared to the mean abundances of halo dwarfs with $[\text{Fe}/\text{H}] = -2.4$ (Cayrel 1996; roughly 10 Gyr ago), the abundances in the Sun (Anders & Grevesse 1989; AG; ~4.5 Gyr ago), and the abundances observed in Canopus (e.g. Hill et al. 1995) and Hii regions in the SNBH (e.g. Cunha & Lambert 1994; represented by triangles; present-day abundances). Error bars indicate the uncertainty in the measured abundances (varying between $\pm 0.05$ and $\pm 0.2$ dex). For comparison, solar abundances by number ($10^{\log (\text{El}/\text{H})} + 12$; data from AG) are plotted in the bottom panel for elements included in the computations.

For model $E^+$, we show in Tables 4.11 and 4.12 the net IMF weighed masses and present-day rates of the most abundant elements nowadays ejected by SNIa, SNIb/c, SNII, and AGB stars. For other SFR models (computed with the Salpeter IMF, Woosley/Weaver yields and input parameters as given in Tables 4.10 and 3.3), these quantities are usually similar to the values listed in Tables 4.11 and 4.12. Total present-day
stellar ejection rates for the most abundant elements are given in the last column of Table 4.12. For instance, AGB stars contribute \( \sim 50\% \) to the present-day total stellar ejection rate of newly synthesized helium (see Table 4.12) while AGB stars account for \( \sim 40\% \) of the present-day stellar consumption rate of hydrogen. Furthermore, AGB stars contribute about 90\% to the present-day stellar ejection rate of nitrogen. SNIa contribute \( \sim 75\% \) to the current ejection rate of newly synthesized Fe and are predicted to be significant contributors to elements such as Si, Ca, Ti, and Ni. SNIb contribute \( \sim 80\% \) to the current stellar ejection rates of newly synthesized O, Mg, Al, and Ni.

Figs. 4.59 and 4.60 show the present-day ejection rate contributions and mean element abundances (relative to solar) in the ejecta of SNIa, SNII, SNIb/c, and AGB stars in more detail for models E− and E+. We note that these figures were constructed while accounting in detail for the star formation history and IMF of the stellar populations considered, the metallicity dependent stellar lifetimes, remnant masses, and stellar yields, and the delay of SNIa after the formation of the WD progenitor. For each element, a distinction is made between the ejection of: 1) newly synthesized material (New), 2) material initially present (Old), and 3) the sum of newly synthesized and old materials (Total). We restrict ourselves to Fig. 4.60 (i.e. model E+). The values displayed in the top left panel of Fig. 4.60 are listed in Table 4.12 (for elements in common).

While SNII contribute \( \sim 80\% \) to the present-day stellar ejection rate of newly synthesized O, the contribution of SNII to the total stellar ejection rate of O, which includes both newly synthesized oxygen and the oxygen already present in the material at time the stars were formed, is drastically reduced to \( \sim 50\% \) (Fig. 4.60, top right panel). The large oxygen contribution by AGB stars is due to the ejection of considerable amounts of oxygen initially present in the material from which low-mass AGB stars formed and the intensifying effects of both the stellar IMF and the variation of the SFR with Galactic age (see Sect. 4.2). This effect is illustrated in Fig. 4.60 (top center panel). For all elements heavier than He, AGB stars contribute \( \sim 75\% \) to the total rate of metals that were initially present in stars and are nowadays returned to the ISM. For SNII, SNIb/c, and SNIa these contributions are \( \sim 20\% \), 5\%, and 0.5\%, respectively. Similarly, AGB stars contribute \( \sim 20\% \) and \( \sim 25\% \), respectively, to the present-day rates of iron and Z (i.e. all elements heavier than helium) returned by stars (Fig. 4.60, top right panel). This is true even though AGB stars are unimportant for the heavy elements newly synthesized in stars. For model E−, the element contributions by SNIa, SNII, SNIb/c, and AGB stars are similar to that of model E+ apart from small differences related to the somewhat lower value of \( m_{\text{SNII}}^{\text{SNII}} = 20 \, M_\odot \) compared to model E+ (see Table 4.10).

Corresponding mean element abundances in the IMF-weighted and SFR-integrated ejecta of SNIa, SNII, SNIb/c, and AGB stars are shown for models E− and E+ in the bottom panels of Figs. 4.59 and 4.60, respectively. For instance, the abundance of newly synthesized carbon in the present-day ejecta of SNIb/c as predicted by model E+ is \( [C/H]_{\text{SNII}} \sim +0.9 \) dex (relative to solar; cf. Fig. 4.60). Similarly, the mean present-day abundance of newly synthesized oxygen in the ejecta of SNII is \( +0.45 \) dex (Fig. 4.60; bottom left panel). When the amount of oxygen initially present in the material from which massive stars formed is included, the mean present-day oxygen abundances in SNII ejecta are about \( +0.65 \) dex (Fig. 4.60, bottom right panel). This value is \( +0.30 \) dex for model E− (Fig. 4.59, bottom right panel) due to the value of \( m_{\text{SNII}}^{\text{SNII}} = 20 \, M_\odot \) assumed.

The abundances in the stellar ejecta of elements heavier than N in the material out of which stars formed are in most cases substantially less than solar (Figs. 4.59 and 4.60, bottom center panels). The initial abundances in stars nowadays on the AGB are much lower than the abundances in SNII and SNIb/c since, on average, stars nowadays on the AGB formed long ago. It is interesting to note that the initial abundances in SNIa progenitors are lower than those in AGB stars. This reflects the time delay of the SNIa occurrence after the formation of the WD so that SNIa are associated with progenitors substantially older than those associated with AGB stars.

Since the mean present-day abundances of elements heavier than He in SNIa, SNII, SNIb/c, and SNII are much larger than solar, the Galactic disk ISM is currently being enriched by these populations. In contrast, the mean abundances in the material nowadays returned by AGB stars are substantially below solar except for C and N (see Figs. 4.59 and 4.60, bottom right panels). As a consequence, AGB stars delay the heavy element enrichment of the Galactic disk ISM considerably and are responsible for the characteristic flattening of the AMR as discussed in Sect. 4.2.
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.59 Present-day enrichment contributions by AGB stars, SNIa, SNIIb/c, and SNII for the E− model discussed in the text. Results are shown for the elements C, N, O, Mg, and Fe. **Top panels:** IMF and SFR integrated present-day ejection rates by mass of element X (normalised to total) for: SNIa (*wide hatched histograms*), SNIIb/c (*open*), SNII (*solid*), and AGB stars (*dense hatched*). **Bottom panels:** As top panels but for the logarithm of the mean abundances (relative to solar) in the ejecta of SNIa, SNIIb/c, SNII, and AGB stars. Abundances [El/H] greater than 1 and less than −1 have been assigned to 1 and −1, respectively. A distinction is made between newly synthesized material ejected (*New*), material initially present ejected (*Old*), and the sum of the two (*Total*).

Figure 4.60 Present-day enrichment contributions by AGB stars, SNIa, SNIIb/c, and SNII for the E+ model discussed in the text (see Fig. 4.59 for details).
Conclusion

We summarize the main results obtained in this section as follows:

- overall, reasonable agreement is found between the predicted and observed abundance-abundance variations for stars in the Galactic disk and halo. In detail, however, none of the SFR models selected on their ability to fit the observed AMR of iron can provide an adequate explanation of the abundance-abundance variations observed, unless additional variations of the element productions by (massive) stars with galactic age are taken into account;

- the standard SFR model (density dependent SFR $n = 1$, no infall) selected in Sect. 4.2 is found to be inconsistent with the mean trends present in the abundance-abundance variations observed among Galactic disk and halo stars. This is true even though this model explains adequately the mean AMR of iron, as well as age and metallicity distributions of long-living stars observed in the SNBH;

- our results support: 1) a gradual increase of the SFR up to a maximum several Gyr after the onset of star formation in the Galaxy, 2) an exponentially decrease of the SFR past its maximum, and 3) an SFR in the disk ISM regulated by gas infall/accretion of matter. Gas infall onto the Galactic disk seems to be required to explain the stellar abundance-abundance variations observed. Infall time scales between 0.5 and 3 Gyr appear in best agreement with the observations when exponential decaying gas infall is assumed. The agreement of the SFR models above with the observed abundance-abundance variations is very sensitive to the contraction time of the disk ISM before the maximum SFR in the disk ISM is reached;

- the influence of the SFR on the resulting abundance-abundance variations is generally large, in particular for elements which are predominantly synthesized in massive stars ($m > \sim 8 \, M_\odot$). In principle, the abundance-abundance variations recorded by long-living stars in the Galaxy for elements such as O, Ne, Mg, Al, and Si provide stringent constraints to the Galactic star formation history;

- our models suggest that the ejecta of SNIa associated with intermediate mass stars formed at early epochs in the evolution of the Galaxy have been delayed over at least 3−5 Gyr after the formation of their WD progenitors. It is difficult to extract information about the detailed SNIa delay time profile from the observed abundance-abundance variations. However, a substantial delay of a large number of SNIa over at least several Gyr after the major period of star formation is needed to strongly affect the slope of the [O/Fe] vs. [Fe/H] variation at values of [Fe/H] $\sim -1$. Instead of such a time delay, the majority of SNIa may be associated with considerably less massive stars than previously thought, i.e. with masses between $\sim 1.5$ and $2 \, M_\odot$. The WD delay time effect on the iron enrichment by SNIa leads to an underestimate of the iron abundance at early epochs in the evolution of the Galaxy. Although there are several ways out to compensate for this effect, we favour the possibility that the ages of stars in the Edvardsson et al. (1993) sample are systematically too large by at least 3−4 Gyr;

- if the time delay of SNIa is not the primary cause for the change in slope of the variation of [O/Fe] with [Fe/H], it appears difficult to explain the observed abundance-abundance trends for these elements unless different processes have initiated and regulated the star formation history in the Galactic halo and disk ISM. This may involve corresponding differences in e.g. the IMF, lower stellar mass limit, and/or upper mass limit for SNII;

- our models combined with the Geneva/Nomoto yields are unable to explain adequately the [O/Fe] and [C/O] ratios observed in Galactic halo stars. This conclusion is independent of the SFR and IMF model used and is insensitive to the parameter values assumed;

- we have argued that the amount of carbon produced during the SNII explosion of massive stars as predicted both by the Geneva/Nomoto and Woosley/Weaver yield sets is considerably too large. This may be e.g. related to the $^{12}\text{C}(\alpha, \gamma)$ rates adopted;

- we find that nitrogen is overproduced in our models by $\sim 0.3−0.4$ dex. This suggests that: 1) too many stars reach the AGB, and/or 2) the effect of hot bottom burning is too large in our models. This result needs further investigation;

- in general, an IMF distinct from the Salpeter IMF results in a shift of the abundance-abundance variations predicted, while the shape of these variations is predominantly determined by the underlying star formation (and infall) history;

- the agreement with the observations for oxygen and the $\alpha$–elements is improved when IMFs are considered that flatten towards low-mass stars as compared to the Salpeter IMF. However, elements such as C and N formed in intermediate mass AGB stars, probably are overproduced in case of such flat IMFs. Therefore, such flat IMFs are excluded by the observed abundance-abundance variations unless
the formation rate of intermediate mass AGB stars is suppressed at the same time. Alternatively, the
carbon and nitrogen yields of stars with metallicities \( Z \lesssim 0.001 \) are substantially in error;
• no observational support is found for large variations of the stellar lower mass limit at birth over the
lifetime of the Galaxy. If such variations did occur, episodes of relatively massive star formation must
have been very short with respect to the lifetime of the Galaxy and/or simultaneous variations in the
enrichment contributions by massive stars must have occurred to prevent overproduction of heavy
elements in the disk ISM;
• models for which the stellar upper mass limit at birth increases substantially with the SFR are not
supported by the observations (unless e.g. simultaneous variations in the lower stellar mass limit at
birth did occur);
• models for which the upper mass limit of SNII increases as a function of galactic age during early
epochs of star formation in the Galaxy are consistent with the observations for variations of \( m_u \) with
between \( \sim 20 \) and \( \sim 30–40 \ M_\odot \), if these variations occur delayed with respect to the variation in the
SFR with age. We emphasize, however, that the precise value and variation of \( m_u^{\text{SNII}} \) favoured by the
observed abundance-abundance variations is rather sensitive to e.g. the IMF, and the contribution by
SNIa to the iron enrichment;
• the Dopita SFR and Salpeter IMF models are found in best agreement with the observed stellar
abundance-abundance variations in the Galaxy for values of \( m_u^{\text{SNII}} \) between 20 and 25 \( M_\odot \) at the early
epoch of star formation in the Galaxy, a SNIIb/c fraction \( F^{\text{SNIIb/c}} \sim 0.2 \) for stars with masses between
\( \sim 8 \ M_\odot \) and \( m_u^{\text{SNII}} \), a SNIa fraction \( F^{\text{SNIa}} \) between 0.01 and 0.02 for stars with masses between \( \sim 2.5 \) and
\( \sim 8 \ M_\odot \), and a SNIa delay time after formation of the WD progenitor of \( \sim 3–5 \) Gyr;
• for these models, we find that AGB stars roughly account for \( \sim 40\% \) of the present-day stellar con-
sumption rate of hydrogen, and contribute \( \sim 50\% \) and \( \sim 90\% \) to the present-day ejection rates of newly
synthesized helium and nitrogen, respectively. SNII are found to contribute \( \sim 80\% \) to the current stel-
lar ejection rate of \textit{newly} synthesized oxygen. When oxygen initially present in stars at time of their
formation is included in the total stellar ejection rate of oxygen, we find that the contribution by SNIi
is reduced to \( \sim 50\% \) and that AGB stars contribute \( \sim 35\% \) to this rate;
• models in best agreement with the observations and computed with the Woosley/Weaver stellar yields,
the Salpeter IMF, and parameters as listed hereabove imply typical contributions by AGB stars, SNIa,
SNIb/c, and SNII, to the total present-day stellar ejection rates of C, O, and Fe as follows (normalised
to one):

\[
\begin{array}{cccc}
\text{El} & \text{AGB} & \text{SNIa} & \text{SNIb/c} & \text{SNII} \\
C & 0.45 & – & 0.30 & 0.25 \\
O & 0.35 & – & 0.15 & 0.50 \\
Fe & 0.25 & 0.50 & 0.10 & 0.15 \\
\end{array}
\]

• the present-day abundances observed in the Galactic disk ISM are not suited to distinguish between
different SFR models;
• the present-day abundances predicted by our models deviate strongly from the mean abundances
observed in HI regions in the SNBH and in Canopus. This suggests that the abundances of young
objects in the solar vicinity are not representative of the mean present-day abundances in Galactic
disk stars;
• the possibility that the enrichment in the Galaxy proceeded at a relatively rapid rate during the
transition phase in the \([\text{O/Fe}]\) vs. \([\text{Fe/H}]\) relation may imply that the formation of massive stars
during this phase has not been accompanied by a corresponding enhancement in the formation of low
and intermediate mass stars (i.e. not many of such stars are nowadays observed). This may point to
a difference in the IMF of stars formed before and after the transition phase as compared to the bulk
of disk stars nowadays observed;
• we suggest that the large spread in abundances observed among Galactic halo stars is related to small-
scale variations in the nucleosynthesis of intermediate mass stars \( (m = 2–8 \ M_\odot) \) which do not produce
both at the same time iron and e.g. oxygen in substantial amounts. These abundance variations may
be primarily due to the local enrichment of the halo ISM by SNIa.
4.3.5 Planetary nebulae abundances

Introduction

We compare the mean abundances in the envelopes of the late stages of Asymptotic Giant Branch (AGB) stars as predicted by the models selected in Sect. 4.2 with the abundances observed in planetary nebulae (PNe) in the Galactic disk. Apart from a brief discussion of the main uncertainties involved, we consider in some detail the effects of second dredge-up, Hot Bottom Burning (HBB), and the initial abundances of AGB stars on the predicted PN abundances. The analysis presented here closely follows that by Groenewegen et al. (1995; hereafter GHJ) and van den Hoek & Groenewegen (1997; hereafter HG) to whom we refer the reader for a more detailed discussion (see also Sect. 3.3).

Observations

The abundances of PNe in the Galactic disk are taken from various sources, i.e. mainly from Aller & Cryzack (1983), Zuckerman & Aller (1986), Aller & Keyes (1987), and Kaler et al. (1990). The few halo PNe contained in these samples are excluded since the present comparison concentrates on AGB stars in the Galactic disk. Errors in the observed abundances are typically 0.015 in He/H and about 0.2−0.25 dex in all other number ratios considered below.

Model assumptions

In the model, the abundances in PNe are estimated by averaging the abundances in the ejecta of AGB stars over the final $\tau_{PN} = 25000$ yr (e.g. Pottasch 1995). We neglect any changes in the ejected shell abundances during the post-AGB phase, e.g. due to a late thermal pulse (Schönberner 1983), which is expected to be a rare event, or due to selective element depletion by dust formation. The latter process may affect the composition both in the wind of an AGB star and during the post-AGB phase (e.g. Bond 1992; van Winckel et al. 1992) but is neglected here for simplicity.

We assume an upper mass limit of 8 $M_\odot$ for stars that ultimately end as PN (with final core mass less than $\sim 1.2 M_\odot$) and ignore the possibility that not all our model AGB stars will become PNe. In reality, some of the low-mass AGB stars may evolve so slowly during the post-AGB phase that the material previously collected in the wind is dispersed before the central star has become hot enough to ionize this material. Also, the upper mass limit for AGB stars is matter of debate and may range between 6 and $\sim 9 M_\odot$, depending on the critical mass for carbon ignition in an electron degenerate core and on details of the stellar mass-loss scenario (cf. GJ; Vassiliadis & Wood 1993; Hashimoto et al. 1993). Furthermore, we assume a constant value of $\tau_{PN} = 25000$ yr. In reality, the time during which the mass accumulated in a PN has been swept up on the AGB may depend on the mass and initial composition of the progenitor. Nevertheless, we do not expect that these simplifications will alter our qualitative conclusions given below.

Planetary nebulae nowadays observed in the Galactic disk originate from AGB stars covering a wide range in initial mass, i.e. with masses between $\sim 0.85$ and 8 $M_\odot$. According to the Geneva tracks (see e.g. Schaller et al. 1992), the progenitors of these PNe were born at galactic evolution times ranging from $\sim 10−15$ Gyr to 50 Myr ago. Since the enrichment of the Galactic disk ISM over this time interval has been substantial (e.g. Twarog 1980; Edvardsson et al. 1993), the initial abundances of the PN progenitors differ considerably. We account for this important effect when we compare the abundances predicted in the envelopes of AGB stars with those observed in PNe. For this purpose, a self-consistent model for the chemical evolution of the Galactic disk was used (see Chap. 3). In particular, it was necessary to apply an iterative solution method to predict the abundances in actual population of PNe in the local Galactic disk (see Sect. 3.1).

Results

Resulting abundance-ratios (by number) in PNe are shown in Fig. 4.61 in case of the standard model assuming pre-AGB evolution according to the Geneva tracks (see HG). We verified that the resulting abundances are insensitive to the adopted PN lifetime up to $\tau_{PN} = 50000$ yr. In general, good agreement is found between the observed and predicted PN abundances despite the uncertainties involved. In particular, we find that the overall trend of the observations is reproduced well by the standard model. However, some discrepancies are present especially at values of He/H $\gtrsim 0.15$ and $\log (N/H) \lesssim -4.5$ which we will address below.

For comparison, we show in Fig. 4.61 the PN abundances predicted by the standard model with pre-AGB evolution according to the recipes outlined in Groenewegen & de Jong (1993) and HG. In this case, the enhanced effect of second dredge-up can account for massive AGB stars with He/H up to $\sim 0.18$
in their envelopes. This suggests that second dredge-up has been relatively important at least for some PNe with He/H ≤ 0.15 in our sample. Alternatively, a substantial fraction of the hydrogen contained in the outer envelope may have turned into helium. Since PNe may evolve from a H and/or He-shell burning AGB star, the relative importance of H and He shell burning during the latest AGB stages will determine the distribution of the He/H abundance ratios observed for a PN progenitor of a given mass.

The effect of HBB on the predicted abundances can be seen in Fig. 4.61 by comparison of the standard model with $m_{\text{HBB}} = 0.8$ and $1.3 \, M_\odot$ (i.e. no HBB), respectively (see also Fig. 3.6). Our results indicate that the standard model overestimates the effect of HBB on the resulting N/O abundance ratios in PNe with progenitor masses $\geq 5-6 \, M_\odot$. We note that the standard model takes into account the maximum effect of HBB as described by RV so that values of the mixing length parameter $\alpha < 2$ in case of RV are probably more appropriate for massive AGB stars. On the other hand, models without HBB are inconsistent with the observed N/O abundances as well as with other independent observations discussed in Sect. 3.3. Therefore, the range of N/O abundances observed in the envelopes of post-AGB stars allows for variations in the importance of HBB roughly covering the range from $m_{\text{HBB}} = 0.8$ to $0.9 \, M_\odot$.

![Figure 4.61](image_url)

**Figure 4.61** Planetary nebulae abundances (by number) predicted by the standard SFR (model A) with pre-AGB evolution according to the Geneva tracks (solid curves) and according to the recipes outlined in GJ (dotted curves). The latter model without HBB is shown for comparison (dashed curves). A Salpeter IMF was assumed. Abundances observed in PNe in the Galactic disk are shown by open circles (data mainly from Aller & Cryzack (1983), Zuckerman & Aller (1986), Aller & Keyes (1987), and Kaler et al. (1990)). Typical errors in the observations are indicated at the bottom right corner of each panel.

The procedure to approximate the effect of HBB in a semi-analytical way has been described in the Appendix of GJ. In fact, the temperature structure of the envelope is expected to change when the number of thermal pulses decreases with increasing values of $\eta_{\text{AGB}}$. This may reduce the amount of HBB occurring in the convective envelope and affect the resulting abundances as observed for PNe with $\log (N/O) \lesssim -0.5$ and He/H ≥ 0.15 (cf. Fig. 4.61; see Sect. 3.3).
Apart from the importance of processes that occur before or during the AGB, we investigated the effect of the adopted star formation history and IMF on the resulting PNe abundances for a comprehensive set of models discussed in Sects. 4.2 and 4.3.4. In brief, we find that the resulting PNe abundances are insensitive to the star formation history assumed for models that fit the observed AMR (see Sect. 4.2). Furthermore, we find that the stellar yields adopted for massive stars (i.e. the Geneva/Nomoto or Woosley/Weaver yields; see Sect. 3.3) affect the PNe abundances more strongly, in particular for C and O. This is basically due to the high sensitivity of the yields of AGB stars to the initial stellar abundances (such as He, C, N, and O).

Fig. 4.62 demonstrates this effect in another way by means of the resulting PNe abundances for various IMFs in case of standard SFR model selected in Sect. 4.2 (model A; Woosley/Weaver yields). In these models, differences in the PNe abundances predicted are due to differences in the initial abundances of the model AGB stars only. Because the enrichment of the ISM proceeds at different rates for models incorporating different IMFs, the initial abundances of stars that currently evolve off the AGB strongly affect the mean abundances in the envelope material ejected as PNe. Thus, in principle, the observed abundances in nearby PNe can be used to constrain the detailed enrichment history of the Galactic ISM over the lifetime their progenitor stars were formed.
From Fig. 4.62 we argue that models with the Scalo or Kroupa IMFs (which flatten towards low-mass stars compared to the Salpeter IMF models) strongly overproduce C and N (see Sect. 4.3.4). The discrepancy between the observed and predicted PNe abundances for these IMF models suggest that the C and N yields of AGB stars may be too large, in particular at low metallicities $[\text{Fe/H}] \lesssim -1$. In contrast, models with IMFs steeper than (or as steep as) the Salpeter IMF are consistent with the C and N abundances in PNe, since for such IMFs the formation probability of AGB stars is considerably reduced.

We emphasize that the abundances of PNe currently formed in the Galaxy are very sensitive to the initial abundances of their progenitors. Thus, in principle, the abundances of PNe nowadays observed in the Galactic disk provide tight constraints to the past chemical evolution of the Galactic ISM. For the set of AGB and massive star yields discussed in Sect. 3.3, Salpeter IMF models result in PNe abundances that appear more consistent with the observations. However, the latter conclusion strongly depends on the stellar yields adopted.

A considerable part of the observed scatter in Fig. 4.61 is expected to be due to substantial variations in the initial abundances of the PN progenitors as set by the inhomogeneous chemical evolution of the Galactic disk ISM (see Chap. 5). Furthermore, the PN progenitors probably formed over a large range in galactocentric distance (e.g. Wielen et al. 1996; see Sect. 4.1), thus covering a wide range in initial abundances according to the radial gradients in the disk ISM (e.g. Shaver et al. 1983). We expect that the agreement between the predicted and observed PN abundance-ratios can be improved further, when a range in initial composition is considered for a given progenitor mass.

**Conclusion**

We summarize the results obtained in this section as follows:

- the abundance-ratios predicted by the standard model are consistent with the observed abundances in virtually all the PNe in our sample when we allow for plausible variations in strength of second dredge-up and HBB;
- the resulting PNe abundances are relatively insensitive to the adopted star formation history for models that fit the observed AMR;
- the abundances in the PNe nowadays left by AGB stars in the Galactic disk depend strongly on the initial element abundances of their progenitors. Thus, in principle, the observed PNe abundances may trace in detail the chemical enrichment of the Galactic ISM over the time interval that the progenitors of these PNe were formed;
- according to the adopted set of stellar yields, Salpeter IMF models are favoured by the PNe abundances observed;
- overall, the good agreement between the predicted and observed abundances in PNe nowadays observed in the Galaxy suggests that the main assumptions made both in the AGB star and Galactic chemical evolution models are essentially correct.
4.3.6 The white dwarf luminosity function

Introduction

White dwarfs (WDs) are the cooling electron-degenerate remnants of the vast majority of all main-sequence stars formed. They probably cool at such low rates that even the remnants of the earliest stellar generations have had no time to fade to invisibility during the lifetime of the Galaxy (Wood 1992).

The observed disk WD luminosity function (WDLF) is defined as the derived space density of WDs in the SNBH per bolometric (or absolute visual) magnitude interval. Since the cooling process of WDs, from the planetary nucleus stage through the crystallization stage is at least qualitatively well understood (e.g. Iben & Tutukov 1984; d’Antona & Mazzitelli 1990; Wood 1992; García-Berro et al. 1996), the local WDLF may provide a valuable constraint to the star formation history in the SNBH.

We briefly discuss recent observations related to the local WDLF. Extensive reviews on this subject have been given by Wood (1992) and Bergeron, Safer & Liebert (1992; hereafter BSL). We describe the theoretical model to compute the WDLF and confront the observed WDLF with that predicted by the models selected in Sect. 4.2. We discuss the dependence of the theoretical WDLF on other model assumptions such as the age of the Galactic disk.

Observations

Data on the local WDLF has been presented by Liebert (1979), and Liebert, Dahn & Monet (1988; hereafter LDM). The WDLF from LDM is based on a sample of 353 hot DA (i.e. hydrogen-rich) dwarfs ($12000 \lesssim T_{\text{eff}} [K] \lesssim 80000$) from the Palomar-Green survey stars listed by Fleming, Liebert & Green (1986; hereafter FLG) and on complementary data for 43 proper-motion selected and spectroscopically confirmed cool WDs in the LHS Catalog (stars with proper motions larger than $0.5 \text{"} \text{yr}^{-1}$; Luyten 1979) with $M_V \gtrsim 13$. The WDLF in the SNBH corresponding to the data of LDM (see their tables 4 and 6) is shown in Fig. 4.63.

The WDLF exhibits a dropoff of about one order of magnitude in the distribution of cool degenerate dwarfs near $\log (L/L_\odot) = -4.4 \pm 0.2$ (depending on the bolometric corrections for cool WDs with $M_V \sim +16$). White dwarfs fainter than $M_V \sim +17$ have been observed but, unfortunately, in numbers too low to allow for any completeness corrections. The rise of the WDLF is mainly caused by the increase of the WD cooling time with decreasing luminosity (see below). This effect is magnified because the average past SFR has been considerably larger than at present (Sect. 3.1). The youngest WDs observed in the visible as the nuclei of planetary nebulae with effective temperatures $T_{\text{eff}} \gtrsim 10^5$ K have been excluded from the sample. This is justified since very hot WDs with $T_{\text{eff}} > 7 \times 10^4$ K have been argued to make a negligible contribution to the space density of degenerate stars (cf. LDM).

![Figure 4.63](image-url)

**Figure 4.63** The observed WDLF in the SNBH. Data has been taken from Fleming, Liebert & Green (1986: full dots) and from Liebert, Dahn & Monet (1988: open circles). The WDLF has been normalised to the value at $\log (L/L_\odot) \sim -2.6$, corresponding to $M_V \sim +12.5$, at which errors in the observations are relatively small (cf. LDM and Wood 1992).

Using the $1/V_{\text{max}}$ method, LDM corrected their WD data for kinematical bias associated with proper-motion samples (cf. Schmidt 1975). This was done by weighing each object contribution to the space density (within its absolute magnitude bin) by the inverse of the maximum volume $V_{\text{max}}$ within which the star hypothetically could be found given the presumed completeness of the sample in both proper motion and apparent magnitude. Proper-motion bias may produce a WDLF which is too large at the faint end while overcorrection for kinematical bias would result in a WDLF that is too low at the faint end (cf. LDM). Consequently, while the existence of the cutoff is not in question, the exact luminosity at which it occurs as well as its shape are not yet well determined (see further LDM; Wood 1992).
4.3 Modelling the chemical evolution of the Galactic disk: results

The observed WDLF at the faint end is particularly uncertain because the WD number density in this region is extremely low. However, the plotted upper limits of the WDLF at these low luminosity bins are considered to be reliable as discussed in detail by LDM. Uncertainties in the observed WDLF may also be due to the kinematical corrections and to the choice of the luminosity intervals into which the WDLF has been binned (Iben & Laughlin 1989). In the following, we will assume that the observed WDLF shown in Fig. 4.63 is representative for WDs with distances to the Sun less than about 50 pc (at which the spectroscopically confirmed WDs with $M_V > 14$ have parallaxes that are large enough to be measured).

The downturn in the observed WDLF at luminosities $L_{	ext{dt}} \sim 10^{-4} \, L_\odot$ may be explained by: 1) a sudden decrease in the WD cooling time at these luminosities, 2) incompleteness of the WDLF at the faint end, 3) low-luminosity WDs which have moved out of the SNBH region during their lifetime, and/or 4) WDs with ages larger than the critical time needed to cool down to $\sim L_{	ext{dt}}$ simply may not have formed. We will discuss these possibilities in turn below.

- The WD cooling time scales have been argued to increase approximately exponentially with age (Wood 1992). Therefore, low-luminosity objects are expected to dominate the WDLF in the SNBH. A sudden increase in the WD cooling rate at a critical luminosity $L_{	ext{dt}}$ is not supported by theory. In addition, this would require the majority of the WDs to have about the same age $t_{	ext{dt}}$ when cooling down to a luminosity $L_{	ext{dt}}$ which is improbable. Alternatively, low-luminosity, old WDs may have evolved to other (unknown) evolutionary stages after which they become undetectable in the visible. For instance, part of such WDs may have exploded as SNIa. Nevertheless, the fraction of WDs ending as SNIa is probably much too small to be significant for the large downturn in the observed WDLF.

- The possible incompleteness of the observed WDLF at the faint end has been discussed extensively by LDM. They argue that such incompleteness is highly unlikely and even when present is probably insufficient to explain the severe dropoff in the observed WDLF. Depending on the bolometric corrections for cool WDs, which depend on the model atmospheres adopted for such WDs, the exact luminosity of the turndnouf may be located between log ($L/L_\odot$) = −4.3 and −4.7 (cf. LDM).

- The increase of the scale height of stars in the Galactic disk with stellar age results in a depression of the contribution by the old, low-luminosity WDs to the observed WDLF. Due to the vertical expansion of the stellar disk, the oldest dwarfs are distributed over $z$–distances $\sim$ 5 times larger than young WDs (cf. Wood 1992; Sommer-Larsen 1991ab; Sect. 4.1). Therefore, the observed local WDLF needs to be corrected for WDs which moved to large heights above the Galactic plane but originate from the same volume as the young WDs. If the drop in the observational WDLF would be due to the inflation of the stellar disk with age, then there must be an explanation for the large difference in scale heights reached by WDs with luminosities below and above $L_{	ext{dt}}$. Again this would require most WDs to have about the same age $t_{	ext{dt}}$ at which they have cooled down to a luminosity $L_{	ext{dt}}$. Since the velocity dispersion is essentially an effect of the kinematical evolution of the stellar disk (e.g. Fuchs & Wielen 1987), the scale heights of main-sequence stars and their remnants should have the same dependence on stellar age. A sudden increase of the disk scale height by about one order of magnitude is not found in statistical studies of main-sequence stars within the Galactic disk and is not supported by theoretical models for the kinematical evolution of the stellar disk (e.g. Sommer-Larsen 1991a,b). Consequently, the vertical expansion of the stellar disk as a possible mechanism to explain the drop in the observed WDLF is rather unlikely.

- Given the shape of the derived luminosity function it is difficult to attribute the downturn at the faint end to any physical effect other than the near-equality of the ages of the lowest luminosity observed WDs to the lifetime of the Galactic disk since the onset of main star formation (cf. Wood 1992). Consequently, the critical luminosity around which the cutoff in the empirical WDLF occurs probably measures the age of the Galactic disk since the epoch at which the progenitors of low-luminosity WDs started to form in the Galactic disk.

We note that the downturn observed in the local WDLF might be related to the steep increase of the stellar main-sequence lifetimes with decreasing stellar mass at $m \lesssim 1 \, M_\odot$ (see Sect. 3.2). In this case, the major fraction of faint WDs would need to be formed by low-mass stars. The main-sequence lifetimes of such stars could be too long to allow for many faint WDs at the present epoch (either these WDs have not formed yet or have not had the opportunity to become very faint). Alternatively, the progenitors of such WDs may not have formed in significant numbers during the early epoch in the evolution of the Galaxy (e.g. due to variation of the stellar lower mass limit at birth with age, IMF effects, or low SFRs at these ages).
We describe the assumptions made to compute the WDLF while accounting for the metallicity dependence of the stellar lifetimes and WD remnant masses. The theoretical WDLF involves both the mass and age distribution of all WDs which have been formed during the lifetime of the Galactic disk. Since the ensemble of WD remnants formed at galactic age $t$ originates from a range of progenitor stars (with various initial masses and initial metal-abundances) which were born at distinct epochs in the past, the WDLF is a complex function depending on the formation history of stars which evolve up to the end of the AGB and on the cooling history of the WD remnants left behind by these stars.

We assume WDs to originate from AGB stars and ignore the impact on the evolution of the WD being a member of a close binary system (common envelope evolution). We use the $Z$-dependent initial-final mass relations and evolutionary tracks for single AGB stars from Groenewegen et al. (1992; see Sect. 3.2) as well as the pre-AGB lifetimes (e.g. including the RGB and HB) from Schaller et al. (1992).

We adopt the mass and composition dependent WD evolutionary sequences computed by Wood (1990, 1992) for WD masses in the range 0.4–1.2 $M_{\odot}$ with spacing in mass of 0.1 $M_{\odot}$. The two main quantities which determine the evolutionary time scales are the WD core composition and the mass of the He-layer. The pure carbon core model sequences result in ages roughly 2 Gyr larger than the corresponding pure oxygen sequences. We here use the sequences for C/O-core compositions of the WD with a helium mass layer of $m_{\text{He}} = 10^{-4} M_{\odot}$. These sequences comprise DB models (helium-rich atmospheres) since surface convection is likely to be efficient enough to mix helium to the thin surface hydrogen layers of most DA (hydrogen-rich atmosphere) dwarfs (cf. Wood 1992).

Fig. 4.64 shows the WD effective temperature $T_{\text{eff}}$, corresponding bolometric correction (BC), bolometric luminosity log ($L/L_{\odot}$) and absolute visual magnitude $M_V$ both as a function of WD-age and WD-mass. The WD cooling sequences were linearly interpolated in log (Age), log ($L/L_{\odot}$), $T_{\text{eff}}$ and log ($m_{\text{He}}$). It can be seen that the WDs cool more rapidly at higher $T_{\text{eff}}$ although the cooling rate also depends on the WD mass. Furthermore, massive WDs are considerably hotter than low-mass WDs at similar WD ages between $\sim 10^7$ and $3 \times 10^6$ yr. This trend is reversed for WD ages in excess of $\sim 3 \times 10^9$ yr: at these ages more massive WDs start to cool rapidly.

To convert the WD bolometric luminosities from the theoretical sequences to absolute $V$-magnitudes $M_V$, we adopted the BC vs. $T_{\text{eff}}$ relation for WDs from LDM between 3500 and 90000 K. As discussed by LDM, the BCs for WD temperatures below $T_{\text{eff}} \sim 4000$ K are uncertain and hard to determine accurately. Below $T_{\text{eff}} \sim 5500$ K, the hydrogen Balmer lines are too weak to detect, and it is uncertain whether the dominant atmosphere constituent is H or He (affecting the theoretical $T_{\text{eff}}$ vs. BC relation). Helium-rich WDs appear to be the major group at $T_{\text{eff}} < 6000$ K but the assumption of hydrogen-rich atmospheres for these WDs probably has no significant effect on the resulting WDLF (cf. LDM). For WD ages in excess of $\sim 1.5$ Gyr, the BCs are less than a few tenths of a magnitude. When the WD effective temperature drops below $\sim 6000$ K the BC slowly increases again.

The WD bolometric luminosity distribution reveals that WDs fade from about 10 $L_{\odot}$ to $\sim 10^{-5} L_{\odot}$ (independent their mass) within the lifetime of the Galactic disk since the onset of main star formation, e.g. assumed to be $t_{\text{ev}} = 14$ Gyr. From the corresponding absolute visual magnitude distribution it can be seen that WDs fade from $M_V \sim 7$ to about 20 during this lifetime. White dwarfs with $m_{\text{He}} \sim 1.2 M_{\odot}$ reach $M_V < 18$ at ages of $\sim 3$ Gyr. In contrast, low-mass WDs ($m_{\text{He}} \sim 0.5 M_{\odot}$) fade down to these luminosities after cooling down over a period of more than 10 Gyr. We note that all WDs in the mass-range 0.4–1.2 $M_{\odot}$ roughly pass through a similar luminosity range from $\geq 1 L_{\odot}$ down to $\leq 10^{-7} L_{\odot}$. However, each mass spends different fractions of its cooling time at distinct Galactic epochs within a specific luminosity bin. Consequently, WDs observed within a particular magnitude range $\Delta M_V$ cover the entire WD mass range from 0.4 to 1.2 $M_{\odot}$ and were left at different epochs in the past.

### Details of the WD cooling scenario

We briefly describe some relevant theoretical aspects of the WD cooling scenario. Shell hydrogen and helium burning (early phases $\lesssim 10^5$ yr), neutrino losses (intermediate phases), and the effects of liquification and crystallization (of carbon and oxygen) are incorporated when the central star of a planetary nebula cools to the stage of complete internal crystallization (Iben & Tutukov 1984). Heat transfer at the center is determined mainly by electron heat conduction while in the outer layers it is determined by the photon opacity (cf. Blinikov & Dunina-Barkovskaya 1994).

Once the loss of neutrinos from the core becomes unimportant (at about $\log(L/L_{\odot}) \sim -1.5$) latent heat associated with the phase transition from liquid to solid is released and all WD masses have about the same slope in the log (Age) vs. log($L$) diagram (Iben & Tutukov 1984). At high WD luminosities, the gas
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.64 Theoretical WD cooling tracks from Wood (1992). Data is shown for carbon/oxygen WDs with a helium mass layer of $m_{\text{He}} = 10^{-4}$ $M_\odot$. WD effective temperature (top left panel), bolometric correction (top right), bolometric luminosity (bottom left), and absolute visual magnitude (bottom right) are shown as a function of the logarithmic WD-age and WD-mass. Effective temperature ranges from several thousands K (light) to $\sim 70,000$ K (dark). Bolometric correction ranges from about $-6$ to $-0.25$ mag. Note that the bolometric luminosity has been plotted on a logarithmic scale with $\log \left( \frac{L}{L_\odot} \right)$ between $-7$ and $+1$. Corresponding absolute visual magnitudes range from $M_V \sim 7$ to 20.

of ions makes the main contribution to the heat capacity and the WD cooling rate is determined by heat diffusion through the nondegenerate envelope. At low luminosities, opacity in the outer atmosphere is by far the most crucial parameter in determining the cooling times of WDs (d’Antona & Mazzitelli 1989).

The degenerate interior of a WD is a mixture of carbon, oxygen and impurities coming from the initial metal contents of the progenitor star. Thus, the cooling time depends on the degree of mixing of oxygen and carbon before the crystallization starts (e.g. Wood 1992). The presence of impurities in the interiors of WDs may delay the cooling of dwarfs by several Gyrs preventing the WD from crystallization due to the gravitational energy release in particular by the settling of $^{22}$Ne (small charge-mass ratio) in the WD interior instead of being distributed uniformly all over the star (Isern et al. 1993).

Matter starts to crystallize when the rate of nuclear energy production (once again) falls below the cooling rate and the WD relies primarily on its interior thermal energy reserves. After crystallization the onset and development of Debye cooling may become important. Once the WD interior has completely
crystallized and the central temperature has dropped considerably below the Debye temperature, the time scale for cooling is \( \sim 1 \) yr times the optical depth of the surface layer (Iben & Tutukov 1984). Note that Debye cooling is not able to give rise to the sharp drop in the observed WDLF due to the progressive onset in the star (d’Antona & Mazzitelli 1990).

The WD cooling process may be affected by the occurrence of accretion of material from the surrounding ISM as is indicated by the observed abundances of metals in the photospheres in cool WDs (see e.g., Dupuis et al. 1993). This suggests that WDs experience several encounters with denser parts of the ISM during which they may accrete material at rates sufficiently large to leave (temporarily) detectable traces of metal-rich material. Nevertheless, the majority of cool WDs do not show this metal-line phenomenon while a substantial fraction of all WDs do not show evidence for hydrogen at their surfaces (e.g., Liebert 1980) which would be expected if accretion of material from interstellar clouds would be a frequently operating process. We note that accretion of matter would substantially increase the WD cooling times, in particular at small luminosities \( L_{\text{WD}} \sim 10^{-4} \, L_\odot \).

The evolutionary sequences terminate shortly after the crystallization boundary reaches the core-envelope transition and sequences have been extrapolated in the same manner as described by Wood (1992). The typical WD cooling time required to reach \( (L/L_\odot) = -4.5 \) is about 10 Gyr. The models for more massive WDs have higher central temperatures and start crystallization at higher luminosities while the remaining thermal energy from the core is radiated away. White dwarfs cool at a rate dependent mostly on their temperature, and hence luminosity, with weaker dependences on mass and composition (cf. Wood 1992). Furthermore, the WD cooling tracks strongly depend on the opacity, the equation of state (especially in the outer layers of the WD), and the magnitude of convection and diffusion.

- Computation of the theoretical WDLF

To compute the theoretical WDLF, one needs to know: 1) the formation rate of WDs (with masses \( m_{\text{rem}} \)) as a function of Galactic age, and 2) the initial temperature and cooling of the WD from the time it has been left behind by an evolved star. The visual luminosity of a WD remnant with mass \( m_{\text{WD}} \) and age \( t_{\text{WD}} \) has been computed using the WD cooling tracks discussed above and is denoted by: \( L_{\text{WD}} = F_{\text{cool}}(m_{\text{WD}}, t_{\text{WD}}) \).

The numerical method used here to compute the WDLF is similar to the one used by Yuan (1989) and Iben & Laughlin (1990) except for including metallicity dependent stellar lifetimes and WD remnant masses. The total number of WDs within a specific WD mass range ever born within the lifetime \( t_{\text{ev}} \) of the Galaxy can be written as:

\[
\frac{dN_{\text{WD}}}{dm_{\text{WD}}} = \int_{t_{\text{b}}}^{t_{\text{ev}}} \int_{\Delta D} S(t^b)M(m) \frac{dm}{dm_{\text{WD}}} \bigg|_{t^b} \, dm \, dt^b
\]  

(4.28)

where \( t^b \) is time of birth of a star of initial mass \( m \) and \( \Delta D \) is the mass range of stars that leave a WD remnant with mass between \( r_1 \) and \( r_2 \) within evolution time \( t_{\text{ev}} \).

The metallicity dependent initial-final mass relation \( m = m(m_{\text{WD}}, Z(t^b)) \) is required to compute Eq. (4.27). Defining \( t_{\text{c}}(m_{\text{WD}}, L) \) as the cooling time of a WD remnant with mass \( m_{\text{WD}} \) to cool down to a luminosity \( L \), we can write the total number of WDs within a specific mass range and with luminosities between \( L \) and \( L + \Delta L \) (see also Yuan 1989) as:

\[
\frac{dN_{\text{WD}}(L, L + \Delta L)}{dm_{\text{WD}}} = \int_{\Delta T} \int_{\Delta D} S(t^b)M(m) \frac{dm}{dm_{\text{WD}}} \bigg|_{t^b} \, dm \, dt^b
\]  

(4.29)

where the birthtime interval \( \Delta T \) is defined as the period of time during which progenitor stars with lifetimes \( \tau_{\text{AGB}}(m, Z(t^b)) \) up to the end of the AGB, leave a WD remnant that has cooled down to luminosities between \( L \) and \( L + \Delta L \) at time \( T \), i.e. \( \Delta T = \left[ T(m, m_{\text{WD}}, t^b, L), T(m, m_{\text{WD}}, t^b, L + \Delta L) \right] \) where \( T(m, m_{\text{WD}}, t^b, L) = t_{\text{ev}} - \tau_{\text{AGB}}(m, Z(t^b)) - t_c(m_{\text{WD}}, L) \). The theoretical WDLF can be derived by integrating Eq. (4.28) over all WD remnant masses:

\[
N_{\text{WD}}(L, L + \Delta L, t_{\text{ev}}) = \int_{W_{\text{min}}}^{W_{\text{max}}} \int_{W_{\text{min}}}^{W_{\text{max}}} \frac{dN_{\text{WD}}(L)}{dm_{\text{WD}}} \, dm_{\text{WD}}
\]  

(4.30)

where \( W_{\text{min}} \) and \( W_{\text{max}} \) denote the minimum and maximum WD remnant masses formed, respectively. We here will assume \( W_{\text{min}} = 0.5 \) and \( W_{\text{max}} = 1.2 \, M_\odot \). The cooling time \( t_c \) of a WD is simply related to the galactic evolution time \( t^d \) (at which the progenitor star evolved off the AGB) by \( t_c = t_{\text{ev}} - t^d \). Therefore, we may express Eq. (4.29) in terms of the stellar die time \( t^d \) instead of birthtime \( t^b \). As can be seen from Fig. 4.64, a range in cooling time \( \Delta t_c(m_{\text{WD}}) \) exists for each WD mass in such a way that the WD will have a luminosity \( L_{\text{WD}} \) at evolution time \( t_{\text{ev}} \) within a specific luminosity interval \( (L, L + \Delta L) \).
With the above conversion of cooling time \( t_c \) to \( t^d \), we can compute the range \( \Delta t^d \) for a progenitor star leaving a WD that will have a luminosity between \( L \) and \( L + \Delta L \) at time \( t^d = t_{ev} \). Since this range in \( t^d \) solely depends on the WD cooling tracks, \( \Delta t^d \) can be calculated for each WD mass \( m_{WD} \) and for each luminosity bin \( \Delta L \) in advance of the actual derivation of the theoretical WDLF.

According to the above transformations, we can rewrite Eq. (4.29) and express the theoretical WDLF at \( t_{ev} \) as:

\[
N_{WD}(L, L + \Delta L, t_{ev}) \sim \int_{m_{AGB}(t_{ev})}^{m_{AGB}} S(t^d - \tau_{AGB}(m, Z_u))M(m) \, dt^d \, dm 
\]

(4.31)

where \( m_{AGB}(t_{ev}) \) is the initial stellar mass evolving off the AGB at time \( t_{ev} \) and \( m_{u}^{AGB} \) the upper stellar mass producing a WD remnant. To allow for scale height corrections, the integrand in Eq. (4.30) must be divided by \( 2h_u(T(m, m_{WD}, t^b, L)) \). These corrections can be substantial for the older, fainter WDs (see Sect. 4.1). We note that analogous expressions can be derived for WD distributions over \( M_V \), \( M_{bol} \), \( T_{eff} \), etc.

The remnant mass \( m_{WD} \) of a star of initial mass \( m \) born at time \( t^b \) is computed using the initial-final mass relation \( m = m(m_{WD}, Z(t^b)) \) and is subsequently used to derive the corresponding range in \( \Delta t^d \). For an assumed Galactic lifetime of \( t_{ev} = 14 \) Gyr, we have a present-day turnoff mass of \( m_{AGB}(t_{ev}) = 0.83 \ M_\odot \) (see Sect. 3.2). This is the lowest mass of stars that leave a WD during the lifetime of the Galaxy. We assume that the most massive star which leaves a WD is determined by the upper mass limit for AGB stars, i.e. \( m_{AGB} = 8 \ M_\odot \). Furthermore, we ignore WDs eventually formed from stars that do not enter the AGB, such as perhaps low-mass RGB stars.

We normalise the observed and predicted WDLF to the total number of WDs within the entire magnitude range considered. Thereafter, we scale the theoretical values to the value of the empirical WDLF at log (\( L/L_\odot \)) = −2.6 at which observational errors are probably small (cf. Noh & Scalo 1990; Wood 1992). We verified that the sum of all WDs binned in luminosity is approximately equal to the total number of WDs derived from integration of Eq. (4.27) over WD remnant mass.

We compute the theoretical WDLF according to Eq. (4.30) while we take into account the metallicity dependence of the stellar lifetimes (up to the end of the AGB) and WD remnant masses. Numerical computation of the theoretical WDLF according to Eq. (4.30) is found to be relatively fast (i.e. as compared to Eq. (4.29)) and accurate (see also Noh & Scalo 1989; Wood 1992). The WDLF is derived for absolute visual luminosities between \( M_V = 8 \) and 20 (in 0.5 mag bins).

**Results**

We have computed the WDLF for the star formation models selected in Sect. 4.2 which incorporate metallicity dependent stellar lifetimes and remnant masses and the cooling tracks from Wood (1992) for C/O WDs with a helium-rich surface layer. Overall, the results discussed below are very similar to the results presented in a comprehensive study of the WDLF by Wood (1992). However, we investigated a different set of star formation histories and IMFs and we included the detailed metallicity dependence of the evolution of the WD progenitors. The latter inclusion leads to new and important results concerning the WDLF.

Fig. 4.65 shows the resulting WDLF for the standard SFR (model A). The predicted WDLF is in reasonable agreement with the observations down to values of \( M_V \sim 15.5 \) mag. At these values, the predicted WDLF start to deviate from the observed WDLF with discrepancies of more than an order of magnitude at values \( M_V \sim 16.5 \) mag. Inspection of Fig. 4.64 reveals that WD remnants with masses between 0.4 and 1.2 \( M_\odot \) and cooling times \( \lesssim 4 \) Gyr have absolute visual magnitudes \( M_V \sim 15.5 \) mag. Since the stellar IMF favours low-mass stars and such stars predominantly leave low-mass WD remnants (see Fig. 3.3), the discrepancy between the predicted and observed WDLF can be interpreted as due to the overproduction of low-mass WDs with masses \( m_{rem} \sim 0.55 - 0.6 \ M_\odot \), i.e. the lowest WD mass included in the AGB models (see Sect. 3.2). In our models, such WDs are produced by stars with masses between \( m_{AGB}(t_{ev}) \sim 0.82 \ M_\odot \) and \( \sim 2 \ M_\odot \), depending on the initial metallicities of these stars (see Fig. 3.3) and assuming \( t_{ev} = 14 \) Gyr. Thus, the observed WDLF suggests that the total number of stars with \( 0.82 \lesssim m/ \ M_\odot \lesssim 2 \) which have left a WD remnant during the lifetime of the Galaxy is greatly overestimated in our models.

- **Effect of scale height corrections**

We have corrected the resulting WDLF for the vertical diffusion of the Galactic stellar disk with age by applying scale height corrections from Sommer-Larsen (1991; see Sect. 4.1). Such corrections result in a reduction of the total number of long-living, low-mass stars predicted to be present in the SNBH, and therefore reduce the total number of faint WDs associated with low-mass remnants. The corresponding effect is illustrated in Fig. 4.65 which demonstrates that reasonable scale height corrections are unable to solve the discrepancies between the predicted and observed WDLFs at values \( M_V \gtrsim 15.5 \) mag.
4.3.6 The white dwarf luminosity function

Figure 4.65 Present-day luminosity function of WDs in the Galaxy. Results are shown for the standard SFR (model A) after applying scale height corrections (Sommer-Larsen 1991; see Sect. 4.1) for the vertical dispersion of the Galactic stellar disk with age (dotted curve) and without such corrections (solid). A Salpeter IMF and model parameters as in Table 3.3 were assumed. The WDLF observed in the SNBH is shown for comparison (data from Fleming, Liebert & Green (1986; full dots) and from Liebert, Dahn & Monet (1988; open circles). Both the empirical and theoretical WDLFs were normalised to the value at log (L / L⊙) ≈ −2.6.

- Effect of the SFR

Fig. 4.66 shows the effect of the assumed star formation history on the present-day WDLF. The WDLF is insensitive to the SFR at values MV ≲ 15.5 mag, while at fainter magnitudes variations of at most a factor of two in the predicted WDLF are due to variations in the SFR at early epochs in the evolution of the Galaxy. The WDLF is relative insensitive to the star formation history assumed because the WDLF essentially is a measure of the total number of WDs formed weighed by the respective WD cooling times. Thus, the discrepancies between the observed and predicted WDLFs cannot be explained in terms of the SFR for reasonable star formation histories.

Figure 4.66 Present-day luminosity function of WDs in the Galaxy: effect of the SFR. Results are shown for SFR models selected in Sect. 4.2: 1) density dependent SFR without infall (model A; solid curve), 2) double exponential SFR with infall (model C; dash-dot), and 3) Dopita SFR (model E; dotted). Scale height corrections from Sommer-Larsen (1991) were applied to the model data. Observational data as in Fig. 4.65. Both empirical and theoretical WDLFs were normalised to the value at log (L / L⊙) ≈ −2.6.
• Effect of the IMF

Fig. 4.67 illustrates the impact of the adopted IMF on the WDLF. At faint magnitudes ($M_V > 15.5$), the predicted WDLF is more sensitive to the IMF than to the SFR (cf. Fig. 4.66). This is partly due to the fact that the mass distribution of WDs is sensitive to the adopted IMF (see Sect. 4.3.7). The WDLF at these faint magnitudes can be reduced considerably when the formation probability of stars with masses between $\sim 0.82$ and $\sim 2 M_\odot$ is reduced. This can be achieved by assuming an IMF much steeper than Salpeter (e.g. a power-law IMF with $\gamma = 2.7$; cf. Fig. 4.66). Alternatively, the formation probability of such stars can be reduced strongly by variations in e.g. the lower stellar mass limit at birth or the IMF slope with Galactic age. Nevertheless, it seems unlikely that reasonable reductions of the formation probabilities of stars with $0.82 < m/ M_\odot < 2$ can provide results consistent with both the observed WDLF and other constraints to the evolution of the Galaxy.

![Figure 4.67](image-url) Present-day luminosity function of WDs in the Galaxy: effect of the IMF. Results are shown for the standard SFR (model A) with the following IMFs: 1) the IMF computed iteratively from the PDMF presented by Scalo (1986; solid curve), 2) the empirical IMF from Kroupa et al. (1993; dash-dot), and 3) a power-law IMF with slope $\gamma = 2.7$ (dotted). Scale height corrections from Sommer-Larsen (1991) were applied to the model data. Observational data as in Fig. 4.65. Both empirical and theoretical WDLFs were normalised to the value at $\log (L / L_\odot) \sim -2.6$.

• Effect of the age of the Galactic disk

As discussed above, the location of the dropoff in the WDLF is given approximately by the age-luminosity relation for the peak of the WD mass distribution ($m_{WD} \sim 0.6 M_\odot$; see Sect. 4.3.7). Contributions beyond this point (i.e. to the faint end of the WDLF) are from: 1) less massive WDs which formed at relatively high rates in the past which evolve slowly, and 2) more massive WDs which were formed more recently at much lower rates and which evolve more quickly. The contribution of the more massive WDs to the faint end of the WDLF increases with decreasing disk age while at the same time the steepness of the falloff decreases (Wood 1992).

If the assumed age of the Galactic disk is too small, no WDs are predicted at luminosities where the observed WD number density is still increasing (Iben & Laughlin 1989). Conversely, if the assumed disk age is too large, the total number of WDs at the faint end of the WDLF would be overproduced as appears to be the case in our models. In the low-luminosity domain, the more massive WDs begin to cool more rapidly than the lighter ones (cf. Fig. 4.64). This leads to a so called "levelling off" at the faint end of the WDLF past its maximum (Iben & Laughlin 1989).

Fig. 4.68 shows the effect of the assumed age of the Galactic disk on the WDLF. For $t_{ev} = 9$ Gyr, the turnoff mass is $m_o \sim 0.95 M_\odot$ at $Z = 0.001$ (see Fig. 3.2). In this case, stars with masses $m \lesssim 0.95 M_\odot$ have not evolved off the main-sequence during the lifetime of the Galactic disk so that the contribution of low-mass stars to the faint end of the WDLF is reduced drastically. However, the reduction in galactic
The white dwarf luminosity function

Figure 4.68 Present-day luminosity function of WDs in the Galaxy: effect of the Galactic age. Results are shown for the standard SFR (model A) in case of lifetimes of the Galaxy of $t_{\text{ev}} = 14$ Gyr (dotted curve) and 9 Gyr (solid). See further Fig. 4.65.

age assumed is insufficient to explain the observed WDLF, in contrast to what has been argued in earlier investigations (e.g. Liebert et al. 1988; Iben & Lauglin 1989; Wood 1992). The main difference is that in previous investigations main-sequence lifetimes for solar metallicity stars were assumed to be independent of the initial metallicities of these stars. This leads to: 1) much larger turnoff masses for a given lifetime of the Galactic disk (see Fig. 3.2), and 2) an overestimate of the main-sequence lifetimes of low-mass stars ($m \lesssim 2 M_\odot$) since the lifetimes for such stars decrease with metallicity (see Fig. 3.1). Both effects result in a considerable reduction of the contribution by low-mass stars at the faint end of the WDLF as compared to models (discussed here) which incorporate the metallicity dependence of the evolution tracks for low-mass stars in detail. For the latter models, a lifetime of the Galactic disk as short as 6 Gyr would be required to improve the agreement at the faint end of the observed WDLF. Apart from the fact that such lifetimes of the Galactic disk are inconsistent with many other observational constraints, discrepancies would remain since substantial numbers of cooling WDs associated with relatively massive AGB stars would populate the faint end of the WDLF (cf. Fig. 4.64).

Discussion

From the results presented above, we arrive at the conclusion that our models are unable to explain the observed WDLF at values $M_V > \sim 15.5$ mag. Possible errors in: 1) the initial-final mass relations for AGB stars (Sect. 3.2), and/or 2) the assumed lower and upper mass limits for stars that presumably leave a WD are (by far) insufficient to overcome the above discrepancies (see also Wood 1992). Therefore, we suggest one or more of the following possibilities:

- the observed WDLF is seriously underestimated at the faintest luminosities. Indirect arguments against this possibility have been discussed extensively by Liebert et al. (1988). However, this possibility cannot be excluded and may be supported by the fact that no dwarfs fainter than $M_V \sim 17$ were found by Liebert et al. even though such dwarfs are predicted to exist even for lifetimes of the Galactic disk as short as 6 Gyr;
- uncertainties in the bolometric corrections of WDs result in too bright absolute visual magnitudes for low-mass WDs (cf. Wood 1992; LDM);
- the cooling times of WDs are considerably in error, in particular for low-mass WDs. If the cooling rates would be much higher, low-mass WDs would rapidly reach luminosities much fainter than $M_V \sim 16.5$ mag and their contribution to the faint end of the WDLF would be drastically reduced (see also Garcia-Berro et al. 1996). Here we used the cooling tracks for C/O WDs from Wood (1992). The same tracks but for O-rich WDs reveals that such WDs cool more rapidly and evolve faster by about 2 Gyr at the luminosity peak of the observed WDLF than do C-rich WDs (Wood 1992). The composition of the thin outer layer around a WD heavily affects the opacity and thus the cooling rate of the WD. In relation to the input physics used, theoretical cooling ages of WDs urgently need more reliable empirical calibrations;
the WD remnants associated with low-mass stars \((m \lesssim 1 \, M_\odot)\) are not as massive as previously thought and, therefore, have luminosities which are outside the range in absolute visual magnitude of the observed WDLF. It is unclear what implications this would have on the chemical evolution of the Galaxy but at first sight there seem no clear objections against this possibility. Alternatively, the WD remnants associated with low-mass stars may not have formed yet because of errors in the theoretical main-sequence lifetimes of such stars. In this case, the actual libraries of theoretical stellar evolution tracks (e.g. from the Geneva group or Bertelli et al. 1994) would need to underestimate the lifetimes of stars with \(m \lesssim 1 \, M_\odot\) by considerable amounts (\(\sim\) factors of 2). Other options are: 1) low-mass WDs evolve differently from the conventional cooling scenario in such a way that they become more rapidly faint with age of the WD than in the cooling scenario. Clearly, the possibility that such low-mass WDs remain bright during long times is excluded by the observed WDLF, or 2) the progenitors of low-mass WDs may have formed in much smaller numbers than predicted by conventional chemical evolution models. In this case, variations in e.g. the stellar mass limits at birth, IMF, etc. with galactic age may have suppressed the formation of low-mass stars at early epochs in the evolution of the Galaxy.

Conclusion

We summarize the results obtained in this section as follows:

- our models are unable to explain the observed WDLF at values \(M_V \gtrsim 15.5\) mag because the total number of stars with \(0.82 \lesssim m/ M_\odot \lesssim 2\) which have left a WD remnant during the lifetime of the Galaxy is greatly overestimated. We have argued that this result does not depend on the assumed scale height corrections, star formation history, IMF, stellar mass limits at birth, and assumed age of the Galactic disk. In contrast, our models are in good agreement with the observed WDLF at values \(M_V \lesssim 15.5\) mag;

- although observational errors and selection effects as well as errors in the theoretical cooling rates of WDs cannot be excluded, we favour the possibility that the WD remnants associated with low-mass stars \((m \lesssim 1 \, M_\odot)\) are not as massive as suggested by the observations (to which our AGB star models are calibrated; see Sect. 3.2), and, therefore, have present-day luminosities that are outside the absolute visual magnitude range covered by the observed WDLF (see also Sect. 4.3.7);

- the results presented by Wood (1992) and confirmed here emphasize that the dropoff in the observed WDLF in fact is determined by the length of time that low-mass WDs have been formed in the local disk. Therefore, the WDLF is more sensitive to the mean SFR at early Galactic evolution epochs than to the detailed variation of the SFR with Galactic age. Except for the faint luminosity tail of the WDLF, which is affected by the past birthrates of low-mass WDs as well as by their mass distribution, the WDLF is quite insensitive to the IMF, stellar mass limits at birth, etc. (see also Noh & Scalo 1990). This result is important since it is notoriously difficult to disentangle the SFR from the IMF using other indirect methods (Wood 1992);

- previous estimates for the lifetime of the Galactic disk from modelling the dropoff in the empirical WDLF are considerably too large and highly unreliable (e.g. \(t_{\text{disk}} 7.5-11\) Gyr (Wood 1992) or \(t_{\text{disk}} \sim 9 \pm 1\) Gyr (Iben & Laughlin 1989)). This is due to the fact that previous investigations incorporated main-sequence lifetimes for solar metallicity stars independent of the initial metallicities of the progenitors of the WDs. This leads to: 1) much larger turnoff masses for a given lifetime of the Galactic disk, and 2) an overestimate of the main-sequence lifetimes of low-mass stars \((m \lesssim 2 \, M_\odot)\) since the lifetimes for such stars strongly decrease with metallicity. Both effects result in a considerable reduction of the contribution by low-mass stars at the faint end of the WDLF as compared to the models discussed here which incorporate in detail the metallicity dependence of the evolution tracks for low-mass stars. The latter inclusion leads to the new and important result that the reduction at the faint end of the WDLF, when smaller and smaller disk ages are considered, is difficult to reconcile with the observed WDLF, in contrast to what has been argued in earlier investigations (e.g. Wood 1992; LDM 1988).
4.3.7 Remnant mass-distribution of low and intermediate mass stars

Introduction

The present-day mass-distribution of white dwarf (WD) remnants provides valuable information concerning the past star formation rate of their low and intermediate mass progenitors. We describe a theoretical model developed to compute the mass distribution of WDs using metallicity dependent stellar evolution data including stellar lifetimes and remnant masses. The method is similar to that applied for the WDLF as presented in the previous section. We briefly examine the WD mass distributions observed in the Galaxy and describe our main model assumptions. Thereafter, we compare results with the observations for the SFR models selected in Sect. 4.2 and discuss possible implications for the Galactic star formation history.

Observations

In Fig. 4.69 we compare the mass distribution of 129 DA (atmospheric composition dominated by hydrogen) WDs in the Galactic disk from Bergeron, Saffer & Liebert (1992; hereafter BSL) with the normalised distributions of the core masses of 303 PNe from Zhang & Kwok (1993; hereafter ZK) and of 95 PNe in the Galactic bulge from Tylenda et al. (1991; hereafter TSAS).

- White dwarf mass distribution

The DA WDs were selected by BSL from the spectroscopically identified WD catalogue by McCook & Sion (1987) which in turn is predominantly based on proper motion catalogs. White dwarfs with \( T_{\text{eff}} \lesssim 15000 \) K were excluded since their atmospheres are thought to be convective and model results for such WDs would strongly depend on e.g. the adopted mixing length (BSL). Using a spectroscopic technique for fitting detailed hydrogen line profiles, the surface gravity of the selected hot WDs with radiative hydrogen-rich atmospheres can be derived. By comparing the derived WD radius with the theoretical models from Wood (1992) for WDs with carbon core composition, the WD mass distribution has been obtained by BSL. In the ideal case, the WD mass distribution can provide exactly the same constraint to Galactic chemical evolution models as the WDLF. However, determination of the WD mass is relatively indirect and requires additional observational (spectroscopic) data as well as more stringent selection criteria. In general, the selection criteria required for a sample of WDs suited to study the WD mass distribution differ considerably from those required to study the WD luminosity function. As a consequence, the WDLF and WD mass distribution do provide more or less independent observational constraints to the models. We here consider the WD mass distribution for comparison reasons only since we will compare our results with the core mass distribution of PNe discussed below.

The empirical WD mass distribution over 0.05 \( M_{\odot} \) mass bins is shown in Fig. 4.69 and has a mean WD mass of \( \langle m_{\text{WD}} \rangle = 0.56 \ M_{\odot} \) with a standard deviation of 0.14 \( M_{\odot} \). More than 30\% of the WDs is found to have masses in the range 0.5−0.55 \( M_{\odot} \) which is significantly lower than the canonical value of \( \sim 0.6 \ M_{\odot} \) used in most studies. As was emphasized by BSL, the apparent sequence of low-mass DA dwarfs (~10 stars with \( m_{\text{WD}} \) less than 0.4 \( M_{\odot} \)) cannot be explained by single star evolution theory within the lifetime of our Galaxy due to the critical mass for core helium ignition. Consequently, the low-mass end of the WD mass distribution indicates that these stars must have undergone phases of common envelope evolution in close binary systems (BSL).

When results for several WD samples are compared, it is found that the mean and mass distribution of WDs depends on the range of \( T_{\text{eff}} \) used (e.g. Oke et al. 1983; Weidemann & Koester 1984; McMahan 1989; Kaler, Shaw & Kwitter 1990). Since the above WDs were selected towards higher \( T_{\text{eff}} \), meaning younger and thus more recently formed WDs left by predominantly low-mass progenitor stars (see below), the BSL sample is probably biased towards less massive WDs. Furthermore, the absolute values of the WD surface gravity may suffer from a zero-point offset which may result in somewhat larger WD masses over the entire mass range (see further BSL). Another effect which may be important for the empirical WD mass distribution is the higher detection probability of WDs formed in close binary systems, in particular in spectroscopically selected samples. The actual magnitude-limited WD sample is estimated to be complete down to \( V \sim 15.5 \) and corrections should be made to obtain a volume-complete sample (see also BSL).

- Planetary nebulae nuclei mass distribution

The mass distribution of the nuclei of visible PNe (PNN; cf. Fig. 4.70) peaks at significantly larger remnant masses than the WD mass distribution (which are selected according to different criteria). These PNN masses are comparatively better determined due to the high sensitivity of the central star luminosity on core mass (e.g. Weidemann & Koester 1984). We have plotted the PNN mass distribution presented by ZK based
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.69 Observed mass distribution of WDs (Bergeron et al. 1992) compared to the central mass distribution of PNe observed in the solar neighbourhood (data from Zhang & Kwok 1993 and Tylenda et al. 1991.

Figure 4.70 Observed distribution of the central masses of planetary nebulae in the solar neighbourhood.

on radio and IR measurements in Fig. 4.70. These PNe were selected by their detection in at least three of the four IRAS bands for which radio continuum flux densities at 5 GHz, electron density measurements, and the nebular angular diameters are available. The authors used the interacting wind model from Kwok (1982), in which the formation of a PN results from the sweeping up of the remnant of the circumstellar envelope formed during the AGB phase by a fast wind emanating from the central star of a PN, to derive the radio continuum temperature \( T_b \) of the PN and of the central star \( T_* \). Using further Schönberner’s (1981, 1983; Blöcker & Schönberner 1990) post-AGB evolution tracks, ZK derived the PNN mass distribution by comparing model calculations and observations in the \( T_b \) vs. \( T_* \) diagram. This diagram provides an unique opportunity to derive the core mass for individual PNe directly from their \( (T_b, T_*) \) position without the need to know their distances (see further ZK; Zhang 1993).

The observed mass distribution of 303 PNN derived by ZK is shown in Fig. 4.69 (rebinned to 0.05 \( M_\odot \) bins) as well as in Fig. 4.70 at a resolution of 0.01 \( M_\odot \) bins. The resulting distribution peaks at \( \sim 0.6 \ M_\odot \). This is somewhat different from that found by Schönberner (1981) for which the PNN mass distribution peaks at \( m_{WD} = 0.55 \ M_\odot \), and from that for PNN in the Galactic bulge as presented by TSAS which shows a maximum at \( \sim 0.58 \ M_\odot \) (see Fig. 4.70). Since PNN are associated with visible PNe they must have been formed during the most recent \( \tau_{PN} = 2 \times 10^4 \) yr which is approximately the characteristic lifetime of a PN in the SNBH before becoming invisible due to dispersion into the ISM (e.g. Pottasch 1992). However, PNN with masses below \( m_{WD} = 0.55 \ M_\odot \) cannot be observed since for smaller core masses the evolution away from the AGB probably is slowed down so much that the nebulae are dispersed before the PNN is hot.
enough to ionize them (Schönberner 1983). As a consequence, WDs with masses between $0.45 - 0.55 \, M_\odot$ usually do not go through a stage with a visible PN and are therefore excluded in the PNN samples. The fraction of WDs born without a visible PNe is estimated to be 30–45% (cf. Weidemann & Koester 1984).

Results for individual PNN in common with samples from Mendez et al. (1992) and by Tylenda et al. (1991) compare remarkably well and individual errors in the derived PNN masses are quoted to be less than ~0.02 $M_\odot$. Unfortunately, no attempt has been made to account for inhomogeneity and incompleteness of the sample. Since detectability both in the radio-continuum and in the far-IR is the major selection criterion for the PNN included, the sample cannot be complete at the end of the fainter nebulae due to the sensitivity limits of the instruments used. Therefore, we expect that the true PNN mass distribution: 1) peaks less severe than in the actual PNN sample, and 2) has a maximum at less massive remnants than in the actual sample.

The sample of PNN present in the Galactic bulge studied by TSAS is predominantly based on spectroscopic surveys from Acker et al. (1990) and Stasinska et al. (1990). Since the evolution of PNN is extremely sensitive to their mass, comparison of observations with post-AGB evolutionary tracks may be used to estimate the PNN mass. The post-AGB tracks from Schönberner (1983) used by TSAS were corrected for the presence of the nebula surrounding the PNN as were the observations. Assuming a PN expansion velocity of 20 km s$^{-1}$ and a distance of 7.8 kpc to the Galactic center (Feast 1987), TSAS derived a PNN mass distribution for which about 50% of the PNN have masses below 0.585 $M_\odot$ (cf. Fig. 4.69). The bulge PNN mass distribution has an average of $<m_{\text{WD}}>$ = 0.593 $M_\odot$ with standard deviation of 0.025 $M_\odot$. The distribution apparently peaks at lower PNN masses around ~0.58 $M_\odot$ compared to the IR + Radio selected PNN sample from ZK as can be seen from Fig. 4.69. A rather restricted PNN mass range between 0.55 $M_\odot$ and 0.67 $M_\odot$ is found which is much narrower than obtained for the ZK sample. However, all the low mass end strong selection effects operate while at the high mass end the total luminosity of a PNN drops rapidly and the nebulae fall below the detection limit of the present survey (TSAS). In addition, high-mass PNN are difficult to observe because they are hot ($\log (T_\ast[K]) > 5$) so that most of the time they have a faint visual continuum compared to the nebula (TSAS).

Since the stellar population of the Galactic bulge and disk are different one may expect their PNN mass distribution to be different too (cf. TSAS). Unfortunately, both samples suffer from severe observational selection effects which are quantitatively not well known and prevent a detailed comparison between the two samples. When rebinned to 0.05 $M_\odot$ bins both distributions seem to be consistent in the sense that the majority (i.e. > 75%) of the observed PNN fall in the mass range ~0.55 $M_\odot$ to 0.65 $M_\odot$ while both distributions peak around 0.55–0.6 $M_\odot$. Furthermore, both PNN distributions tend to be on average more massive than the DA WD mass distribution which is consistent with discussion above.

Although the remnant mass distributions compared above are among the most accurate and extended samples currently available, we emphasize that these samples probably are still far from being statistically complete and severe selection effects may be present. We estimate the total number of WDs present in the Galactic disk to be of the order of $10^{10}$ (see Sect. 4.3.6) while for a current PN birth rate of ~1 yr$^{-1}$ and $\tau_{\text{PN}} \sim 2 \times 10^4$ yr (e.g. Pottasch 1993; Peimbert 1993; Pottasch 1996), the total number of PNe in the disk is nearly two orders of magnitude larger than the most complete sample currently available. For the Galactic bulge, the total number of PNN with ages $\tau_{\text{PN}} < 2 \times 10^4$ yr is estimated to be 600–700 (Stasinska et al. 1991) after a careful correction for possible selection effects. This number suggests that at present about 40% of all the Galactic bulge PNe have been identified optically and about 15% has been included in the sample by TSAS.

Since PNe are very diverse in their appearance, while both detector sensitivities and interstellar extinction probably play an important role in causing severe selection effects, it is a difficult task to get a subsample that is representative for the present-day population of PNN in the Galactic disk at least with respect to their mass distribution. We consider the bulge sample of PNN to be the most representative subsample currently available with respect to its mass distribution (although this sample is perhaps not the most accurate and complete one currently available). Keeping these limitations in mind, we investigate the dependence of the theoretical WD and PNN mass distributions on the assumed star formation history and IMF, and compare the predicted with the observed remnant mass distributions.

Model assumptions

Low and intermediate mass single stars ($m \lesssim 8 \, M_\odot$) lose the largest part of their mass on the AGB (see Sect. 3.3). Since mass loss on the AGB is found to be sensitive to the stellar metal-abundance on the main-sequence (e.g. Knapp 1985; see also Groenewegen & de Jong 1992; see Sect. 3.3), the remnant mass of a star of initial mass $m$ depends on the metallicity of the progenitor star as well. For single AGB stars, the WD remnant mass ranges between the critical mass for core helium ignition, i.e. ~0.45–0.5 $M_\odot$ and the
4.3 Modelling the chemical evolution of the Galactic disk: results

Chandrasekhar limit for the formation of a neutron star, i.e. $\sim 1.4 \, M_\odot$.

As a consequence, stars of initial mass $m$ which evolve off the AGB during the lifetime of the Galactic disk produce a range of remnant masses. Fig. 4.71 shows the WD mass left by a star of initial mass $m$ vs. the age $t^b$ at which the progenitor star was born in the Galaxy (with metallicity $Z_* = Z(t^b)$). To obtain Fig. 4.71 we used the theoretical AMR from model A selected in Sect. 4.2 together with the remnant masses from the AGB evolution model from Groenewegen & de Jong (1993; see also Sect. 3.3). We note that Fig. 4.71 is identical for all models which are consistent with the AMR observed for stars in the Galactic disk. In particular, quantities related to the SFR and IMF are not incorporated in Fig. 4.71.

![Figure 4.71: Theoretical WD remnant mass (greyscale) for stars of initial mass $m$ born at Galactic age $t^b$ with corresponding metallicity $Z_* = Z(t^b)$. Greyscale varies between $m_{WD} = 0.545 \, M_\odot$ (white) and $m_{WD} \sim 1.2 \, M_\odot$ (black; cf. legend bottom right). **Vertical axis**: $\log (t/\text{Gyr}) = 8$, 8.5, 9, 9.6 and 10.15 corresponding to initial metallicities $(Z/Z_\odot) = 0.01, 0.03, 0.1, 0.3$ and 1.5, respectively. **Horizontal axis**: progenitor mass between the current turnoff mass $m_o(14 \, \text{Gyr}) \sim 0.82 \, M_\odot$ and $m_{AGB}^u$ (plotted on a non-linear scale). **Solid curve** illustrates the contour of constant remnant mass $m_{WD} = 0.6 \, M_\odot$. **Dotted line** indicates the variation of the turnoff mass $m_o$ with Galactic age $t$.](image)

The WD remnant mass increases with progenitor mass at all galactic evolution times (i.e. initial metallicities). This can be seen also from the bottom histogram in Fig. 4.71 which displays the remnant mass vs. initial mass averaged over all age bins (sensitive predominantly to the AMR). The predicted range in WD mass is largest for progenitor stars with $m \sim 3.5 \, M_\odot$ which leave remnants with masses between 0.65 $M_\odot$ and $\sim 0.95 \, M_\odot$.

In contrast, $m_{WD}$ decreases with increasing initial metallicity. For instance, a progenitor mass of $m = 2.5 \, M_\odot$ leaves a WD remnant with $m_{remn} \sim 0.8$ and $\sim 0.6 \, M_\odot$ when born with $Z_* \sim 10^{-2} Z_\odot$ and $Z_* \approx Z_\odot$, respectively (see also Fig. 4.71; top right histogram which displays the remnant mass vs. Galactic age averaged over all mass bins). The solid curve in Fig. 4.71 shows the contour of remnants with masses $m_{WD} = 0.6 \, M_\odot$. Progenitor stars in the mass range $m = 1.25 \, M_\odot$ to $\sim 2.8 \, M_\odot$ will leave a $\sim 0.6 \, M_\odot$
4.3.7 Remnant mass-distribution of low and intermediate mass stars

WD remnant depending on their initial metallicity. The variation of the turnoff mass with Galactic age determines the instant at which stars of a given mass \( m \) start to evolve off the AGB and contribute to the Galactic present-day WD mass distribution. For instance, stars with \( m \leq 1 \, M_\odot \), born with low metallicities \( Z_\star \sim 10^{-2} Z_\odot \) do not contribute to the WD mass distribution until a Galactic age of \( \sim 7.5 \) Gyr has been reached after the onset of star formation in the Galaxy.

For single WDs, the progenitor mass ranges between the present-day turn-off mass from the AGB \( m_{\text{b}, \text{AGB}} \) and the assumed upper stellar mass that leaves a WD remnant, i.e. \( m_{\text{u}, \text{WD}} \). If we assume all AGB stars to leave a WD remnant at the end of their evolution, the WD mass-distribution \( N_{\text{WD}}(r_1, r_u, T) \), defined as the total number of remnants with masses between \( r_1 \) and \( r_u \) at galactic age \( t = T \), can be written as:

\[
N_{\text{WD}}(r_1, r_u, T) = \int_0^T \int G(r_1, t) S(t - \tau_{\text{AGB}}(m, t^b)) M(m) \, dm \, dt
\]

(4.32)

where \( \tau_{\text{AGB}}(m, t^b) \) is the stellar lifetime up to the end of the AGB of a star of initial mass \( m \) born at a metallicity \( Z_\star = Z(t^b) \). The stellar birth time \( t^b \) is related to the galactic age by \( t^b = t - \tau_{\text{AGB}}(m, t^b) \), and \( G(m_{\text{WD}}, t^b) \) is the progenitor mass \( m \) of a remnant with mass \( m_{\text{WD}} \) formed at galactic age \( t \).

We neglect the stellar evolution time spent in the post-AGB phase, i.e. during which the star leaves the AGB and ultimately forms a WD (usually accompanied by the formation of a PN). Since the stellar birth age \( t^b \) in Eq. (4.31) needs to be computed iteratively as stellar lifetimes depend on the initial metallicity, it is more convenient to integrate over \( t^b \) instead of galactic age \( t = t^b + \tau_{\text{AGB}}(m, t^b) \) and to rewrite Eq. (4.31) in the form:

\[
N_{\text{WD}}(r_1, r_u, T) = \int_{t^b=0}^T \int_{D} S(t^b) M(m) \, dm \, dt^b
\]

(4.33)

where the inner integral integrates over the mass range \( D \) defined as the intersection of the respective ranges \( (m_{\text{b}, \text{AGB}}(T - t^b), m_{\text{u}, \text{WD}}) \) and \( (G(r_1, t^b), G(r_1, t^b)) \). The former mass range comprises stars born at \( t = t^b \) evolving off the AGB at galactic age \( t = T \). The latter mass range includes all progenitor stars born at \( t = t^b \) which ultimately end as a WD with mass between \( m_{\text{WD}} = r_1 \) and \( r_u \). We ignored the possibility that some stars may leave multiple times a PN (e.g. Pottasch 1996).

To compare the theoretical WD mass distribution to the observed distribution we have normalised \( N_{\text{WD}}(r_1, r_u, T) \) in Eqs. (4.31) and (4.32) to the total number of WDs present in the Galactic disk as predicted by the adopted SFR model. To derive the theoretical mass distribution of the central stars of visible planetary nebulae we used an equation similar to Eq. (4.32) except for integrating from \( t_{\text{ev}} - \tau_{\text{PN}} \) to \( t_{\text{ev}} \), i.e. including only WD remnants formed during the most recent \( \tau_{\text{PN}} \sim 2 \times 10^4 \) yr (e.g. Pottasch 1992) and assuming \( t_{\text{ev}} = 14 \) Gyr. We refer the reader to Sect. 4.3.5 for the uncertainties related to the assumption of a constant value of \( \tau_{\text{PN}} \) (e.g. independent of galactic age, PN and WD masses). We note that relations similar to Eq. (4.32) can be derived for the metallicity, age, and luminosity distributions of PNe and WDs.

Results

We have computed the age-integrated WD mass distribution and present-day mass distribution of the nuclei of PNe for several SFR and IMF models discussed in Sect. 4.2. This has been done while neglecting corrections for the increase in the vertical diffusion of the Galactic stellar disk or for stellar orbital diffusion in general (see Sect. 4.1). Since we will compare the results with the mass distribution of PNN in the Galactic bulge (Tylenda et al. 1991), such corrections are uncertain and probably are of minor importance compared to those involved with the mass determination of PNN in the Galaxy (e.g. distances, post-AGB evolution tracks).

The mass distribution of bulge PNN may be biased towards progenitors with large initial metallicities. Inspection of Fig. 3.3 reveals that WD remnant masses of single stars decrease with increasing metallicity. Therefore, the mass distribution of bulge PNe is expected to have a mean remnant mass substantially less massive (by \( \sim 0.1 \, M_\odot \)) than that for PNN associated with stellar populations formed in the SNBH. However, the metallicity effect on the mass distribution of PNe is much smaller than suggested above because: 1) the formation probability of low-mass stars \( (m \sim 1 \, M_\odot) \) is relatively large due to IMF effects, 2) the low-mass progenitors of PNe were formed \( \geq 7.5 \) Gyr ago when the metallicity in the Galactic ISM was substantially less than at present, and 3) the metallicity dependence of the initial-final mass relation for low-mass stars is weak (see Fig. 3.3).

We will compare the mass distribution of bulge PNN both with the time-integrated WD mass distribution and the PNN mass distribution. The present-day PNN mass distribution peaks at smaller remnant masses than the WD mass distribution due to: 1) the finite period of time during which a PN remains visible
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.72 a) Remnant mass distribution of low and intermediate mass stars: effect of the SFR. **Left panels:** Resulting mass distribution of the nuclei (PNN; *solid lines*) of the planetary nebulae formed in the Galaxy during the last \( \tau_\text{PN} = 2 \times 10^4 \) yr for the SFR models (label A–C) selected in Sect. 4.2. A Salpeter IMF and model parameters as in Table 3.3 were assumed. **Right panels:** Corresponding WD mass distribution integrated over the lifetime of the Galaxy. Mean PNN and WD masses predicted are indicated in the top right corner of each panel, respectively. For comparison, observational data are shown for PNN in the Galactic bulge (Tylenda et al. 1991; *dotted*). Note the different bin sizes for the left and right panels.

- Predicted remnant mass distributions: dependence on SFR

Fig. 4.72 shows the resulting PNN and WD mass distributions for the SFR models selected in Sect. 4.2. For all SFR models, the remnant mass at maximum in the PNN and WD mass distributions are in good agreement with the observations, i.e. \( m_{\text{rem}}^{\text{max}} \sim 0.58 \ M_\odot \). In all cases, the PNN mass distribution predicted contains substantially less high-mass remnants with \( m_{\text{rem}} \gtrsim 0.58 \ M_\odot \) than observed. This discrepancy is probably related to selection effects which favour the more massive, more luminous PNe in the Galactic bulge in the sample from Tylenda et al. (1991). Therefore, we expect that resulting PNN mass distributions which produce substantial more low-mass remnants \( (m_{\text{rem}} \lesssim 0.58 \ M_\odot) \) are in better agreement with the actual PNN mass distribution in the Galaxy than models which perfectly match the Tylenda et al. data in this mass-range. This is also justified by the fact that the application of corrections for stellar orbital
diffusion mainly will enhance the PNN mass distribution at the low-mass end since the long-living, low-mass stars produce relatively low-mass remnants. Thus, in spite of the good agreement between the observed and predicted PNN distribution of the bimodal SFR (model D), this model can be excluded for these reasons. Since the magnitude of the selection effects towards more massive PNe is not well known, a clear distinction between the different SFR models considered on the basis of the PNN mass distribution is difficult. However, SFR models A, B, and F may be reasonable predictions of the actual mass distribution of PNN in the Galactic disk.

For the WD mass distribution, we use a bin width in remnant mass equal to that used by BSL. In general, good agreement between the predicted WD mass distribution and the observed PNN mass distribution is found for the models considered here. However, we emphasize that comparison with the ZK sample of PNN (or the WD mass distribution from BSL) reveals that marked differences exist between these observations and the model predictions. The PNN masses derived by ZK may be even more strongly biased towards massive PNe than for the TSAS sample and the disagreement of the predicted mass distributions is not worrying. In contrast, the mass distribution derived by BSL extends down to \( m_{\text{rem}} \sim 0.3 \, M_\odot \) while about half of the sample WDs have masses less than \( \sim 0.55 \, M_\odot \). Our models do not predict WD masses much less than 0.54 \( M_\odot \). This suggests that: 1) the initial-final mass relation predicted for low-mass AGB stars is considerably in error, 2) another mechanism is required to explain WD masses \( \lesssim 0.55 \, M_\odot \), and/or 3) the surface gravities (and WD masses) determined by BSL are considerably underestimated.
Although the latter possibility cannot be excluded, we favour the idea that such low-mass WDs are produced by low-mass stars which do not reach the AGB but loose substantial amounts of their envelope earlier in their evolution\(^5\).

This process may be uncommon unless a large population of very low-mass WDs \((m \lesssim 0.55 \, M_\odot)\) is present in the Galaxy. The existence of large numbers of these WDs cannot be excluded since these faint WDs are not associated with PNe (perhaps they may be observed as dwarf novae). The observed WD mass distribution from BSL predicts a substantial number of such WDs and this raises the question why the BSL sample would be strongly biased towards these low-mass WDs. Perhaps the answer is in part that such WDs are much more common than previously thought. Also, these very low-mass WD remnants may be produced predominantly in binary systems.

We recall from Fig. 4.72 that remnants with \(m_{\text{rem}} \sim 0.55 \, M_\odot\) are left by \(\sim 1.4 \, M_\odot\) stars formed recently (i.e. \(\sim 2\) Gyr ago) with metallicities \(Z* \approx Z_\odot\) or by \(\sim 0.85 \, M_\odot\) stars formed about 12.5 Gyr ago with low metallicities \(Z* \approx 0.1 Z_\odot\). Therefore, the present-day WD mass distribution \(N_{\text{WD}}(r_1, r_u, T)\) cannot provide detailed information about the Galactic star formation history since even strong bursts of star formation are readily smeared out in the remnant mass distribution (cf. Eqs. (4.31) and (4.32)).

### Predicted remnant mass distributions: dependence on IMF

Fig. 4.74 illustrates the effect of the IMF on the PNN and WD mass distributions in case of the standard SFR (model A). We find that the resulting mass distributions for the Scalo and Kroupa IMF models are consistent with the observations. In contrast, an IMF with slope \(\gamma = 2.7\) predicts too many low-mass PNN in a manner probably inconsistent with the expected impact of the selection effects on the observations discussed above. Even though the adopted IMF strongly influences the theoretical PNN mass distribution (e.g. Yuan 1989), it appears very unlikely that the high-mass end of the PNN distribution of the Galactic disk or bulge is as important as indicated by the TSAS distribution (unless the IMF and/or stellar mass limits at birth are significantly different from that usually found in the Galactic disk; see Chap. 2). In this case, e.g. the stellar lower mass limit would need to be rather large (i.e. \(m_l \gtrsim 0.5 \, M_\odot\)) in combination with a flatter IMF, or the SFR would need to increase with Galactic age which is not supported by the observations (see Sect. 4.3.4).

### Total number and present-day formation rates of WDs

Table 4.13 lists the total number, present-day birthrates, and average masses of PNN and WDs for the SFR and IMF models discussed above. An average lifetime of PNe in the Galactic disk of \(\tau_{\text{PN}} = 2 \times 10^4\) yr was assumed. The mean PNN and WD masses are \(\sim 0.58 \pm 0.01\) \(M_\odot\) and \(\sim 0.62 \pm 0.05\) \(M_\odot\), respectively, for all models except the bimodal SFR (model D). This may be compared to \(\langle m_{\text{PNN}} \rangle = 0.593 \pm 0.025 \, M_\odot\) (TSAS) and \(\langle m_{\text{WD}} \rangle = 0.56 \pm 0.14 \, M_\odot\) (BLS) as obtained from the observations. Considering the selection effects and uncertainties present in the observations, the agreement is satisfactory. For the mean PNN mass, the agreement is surprising because: 1) observational selection effects operate against low-mass PNN, and 2) PNN with masses below \(\sim 0.55\) \(M_\odot\) are difficult to detect as the planetary nebula may be dispersed before the central star is hot enough to ionize the nebula.

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<th>(N_{\text{PNN}}) ([10^4])</th>
<th>(\langle m_{\text{PNN}} \rangle) ([M_\odot])</th>
<th>(N_{\text{WD}}) ([10^{10}])</th>
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</tbody>
</table>

\(^5\)For WDs with masses \(\lesssim 0.55 \, M_\odot\), the progenitors may not have passed through the luminous AGB or Mira phases as discussed in Sect. 3.3 (see also BSL) but may have experienced sufficiently higher rates of mass loss that truncate evolution on the "early" AGB or even at the horizontal branch. White dwarfs with \(m_{\text{WD}} \sim 0.5 \, M_\odot\) may originate in subdwarf B and perhaps subdwarf O stars if such stars are core helium burning objects on the "extended" horizontal branch. It is known that these hot subdwarfs exist in large numbers in the Galaxy (see Saffer, Liebert & Green 1992).
4.3.7 Remnant mass-distribution of low and intermediate mass stars

Figure 4.74 Predicted remnant mass distribution of low and intermediate mass stars: effect of the IMF. **Left panels:** Resulting mass distribution of the nuclei (PNN; solid lines) of the planetary nebulae formed in the Galaxy during the last $\tau_{PN} = 2 \times 10^4$ yr for the standard SFR (model A) with: 1) the Salpeter IMF (labeled A), 2) the IMF computed iteratively from the PDMF presented by Scalo (1986; A1), 3) the empirical IMF from Kroupa et al. (1993; A2), and 4) a power law IMF with slope $\gamma = 2.7$ (A3). **Right panels:** Resulting WD mass distribution integrated over the lifetime of the Galaxy. Mean WD and PNN masses predicted are indicated in the top right corners of each panel, respectively. For comparison, observational data are shown for PNN in the Galactic bulge (Tylenda et al. 1991; dotted).
Predicted total numbers of PNN and WDs are given for comparison. The birthrate of the immediate progenitors of WDs such as PNe, post-AGB objects and post-HB stars, is estimated to be $1.6 \times 10^{-3}$ pc$^{-3}$ Gyr$^{-1}$ (cf. Drilling & Schönberner 1985). This corresponds to a present-day WD birthrate of about 0.4 to 1 yr$^{-1}$ and a WD total number of $N_{\text{WD}} \sim (5-14) \times 10^9$ in the Galactic disk (assuming a disk radius of 20 kpc, a disk stellar scale height at birth of 100 pc, $t_\text{ev} \sim 14$ Gyr, and a mean WD formation rate about 1/3 of the present-day formation rate of WDs independent of location in the disk).

We note that the present-day birthrate of WDs is much larger than the birthrate of WDs in the past. This is due to the fact that the mean progenitor mass of the oldest dwarfs in the Galaxy must be considerably larger than the mass of the oldest stars nowadays on the main-sequence (e.g. Iben & Tutukov 1984). Both the IMF and Galactic star formation history intensify this effect.

The present-day formation rate of PNN predicted by standard SFR model is $R_{\text{PNN}} = 2.3$ yr$^{-1}$ and appears somewhat too large compared to the observed rate of $R_{\text{PNN}} \sim 0.5-2$ yr$^{-1}$ (Pottasch 1992; this value is similar to the present-day birthrate of WDs previously estimated). However, in our models we assumed all stars with masses between $m_o(t_\text{ev}) \sim 0.82$ and 8 $M_\odot$ to reach the AGB and to leave a PN while, in fact, a substantial fraction ($\sim 30 - 40\%$) of these stars may end as WDs without associated (or visible) PNe.

**Conclusion**

We summarize the main results obtained in this section as follows:

- The agreement between the observed and predicted mass distributions of WDs and PNN as well as their formation rates in the Galaxy is satisfactory. However, severe selection bias in the observations prevents a clear distinction between different models on the basis of these constraints;
- Our models do not predict WD remnants less massive than $\sim 0.54$ $M_\odot$. This appears inconsistent with the observed WD mass distribution derived by Bergeron, Saffer, & Liebert (1992). These authors propose that such WDs originate from low-mass stars which experience substantial mass-loss during the helium flash at the end of the horizontal branch. Alternatively, it has been suggested that such low-mass WDs may be formed in binary systems. If indeed true, our models would predict too massive WD remnants, in particular for low-mass stars ($m < \sim 1$ $M_\odot$). This result would be consistent with that derived for the WDLF (see Sect. 4.3.6). However, we emphasize that uncertainties in the observed remnant mass distribution at low masses are relatively large and prevent a conclusion;
- Mean PNN and WD masses in the Galaxy are predicted as $m_{\text{PNN}} \sim 0.58 \pm 0.01$ $M_\odot$ and $m_{\text{WD}} \sim 0.62 \pm 0.05$ $M_\odot$, respectively, for an assumed average lifetime of PNe in the Galactic disk of $\tau_{\text{PN}} = 2 \times 10^4$ yr;
- The present-day remnant mass distribution of intermediate mass stars cannot provide detailed tests of the Galactic star formation history since, depending on the initial metallicity of the progenitors, the effects of star formation bursts are spread over a wide range in remnant mass.
4.3.8 Metallicity and age distributions of long-living stars

The age and metallicity distributions of long-living stars observed in the solar neighbourhood provide tight constraints on the star formation history and chemical evolution of the local Galactic disk. We here give a brief summary of recent observational abundance data on F, G, and K dwarfs and describe the model adopted to compute the age and metallicity distributions for stars covering a given range in spectral type. To allow for a meaningful comparison with the observations, we study the effect on the resulting metallicity and age distributions of: 1) the inflation of the scale height of the Galactic stellar disk with age, 2) the large abundance inhomogeneities observed among similarly aged F and G dwarfs in the SNBH (Edvardsson et al. 1993; see Chap. 5), and 3) the metallicity dependence of the range in initial mass of stars with present-day effective temperatures (spectral types) within a given range. We compare the observed [Fe/H], [O/H] and age-distributions of F, G, and K main-sequence dwarfs in the SNBH with the corresponding distributions predicted by the SFR models selected in Sect. 4.2 while taking into account the metallicity dependence of the stellar yields and lifetimes. We discuss our results in the context of the many solutions that have been proposed in the past to explain the classical G-dwarf problem.

Introduction

Simple models for the chemical evolution of the Galaxy have long been found inconsistent with the observed abundance distributions of disk G dwarfs (e.g. Pagel & Patchett 1975; Twarog 1980; Pagel 1987; Guzik & Struck-Marcell 1988; Casuso & Beckman 1989; Francois et al. 1990; Sommer-Larsen & Yoshii 1990; Taylor 1990; Sommer-Larsen 1991a,b; Meusinger 1994) in the sense that such models usually predict too many metal-deficient stars, a problem which is known as the G-dwarf problem (see for a recent review Pagel 1992). During the last decades, many possible explanations have been proposed:

- prompt initial enrichment (Talbot & Arnett 1972; Pagel & Patchett 1975) from the halo and the bulge: first halo star formation with subsequent infall to the Galactic disk (Ostriker and Thuan 1975; Köppen & Arimoto 1991);
- metal-enhanced star formation (Talbot 1974);
- movement of the oldest stars either towards the Galactic centre (Grenon 1989) or away from the Galactic disk reaching such large scale heights that they are missed in local samples even after weighing by velocity perpendicular to the Galactic plane (Pagel 1987);
- inhomogeneous chemical evolution of the Galactic disk with intermitting periods of remixing (Malinie et al. 1993);
- larger yields at lower metallicities either by effects on stellar evolution (Maeder 1991) or to the IMF (Schmidt 1963);
- gradual formation of the Galactic disk: inflow of (nearly) unprocessed material (Twarog 1980; Pagel 1989, 1992; Sommer-Larsen 1991);
- bimodal star formation, i.e. different star formation histories for low and high-mass stars either by different SFRs (Larson 1986; Wyse & Silk 1987; Francois, Vangioni-Flam, and Audouze 1991) or by different IMFs (e.g. by an SFR enhanced lower mass limit as earlier suggested to explain observations of starburst galaxies (Rieke et al. 1980, 1985; Silk 1987; Larson 1988) and for massive OB-stars observed in giant molecular clouds in the Galactic disk (Myers et al. 1986; Blitz & Stark 1986).

Until recently, the observed G-dwarf metallicity distribution has been based predominantly on UBV photometry data originally collected by Pagel & Patchett (1975) which has been revised by Sommer-Larsen (1991) and Pagel (1989, 1992). Recently, new data on the local G-dwarf abundance distribution have been presented by Gilmore & Wyse (1995) and Rocha-Pinto & Maciel (1996). These data are based on Strömgren photometry data adopted from the Third Gliese Catalog and allow for a more reliable abundance determination than in the case of UBV photometry. Also, accurate abundance data have become available recently for K dwarfs in the SNBH (e.g. Flynn & Morell 1996).

In this section, we present results for the age and abundance distributions of long-living stars in the Galaxy using the galactic chemical evolution model discussed in Chap. 3. We compare these results with new observational data on the abundance and age distributions of F, G, and K dwarfs in the SNBH. In particular, we show that no G-dwarf problem is encountered by our models. We consider this important
result in detail and discuss to what extent the observed abundance and age distributions can be used to constrain models for the chemical evolution of the Galactic disk.

**Observations**

We concentrate on the best samples available to date of F, G, and K main-sequence dwarfs observed in the SNBH. Table 4.14 lists the selection criteria and characteristics for a number of samples adopted from the literature. Sample reference, number of sample stars, ranges in effective temperature $T_{\text{eff}}$, spectral type, and metallicity, selection criterion, data homogeneity, and literature source are given subsequently. In principle, the age and metallicity distributions of the stars in these samples provide independent constraints on the chemical evolution of the Galactic disk. We note that for stars with spectral types much later than G, the sampling volume in the Galaxy decreases rapidly (e.g., samples of M dwarfs are not well suited for Galactic disk studies according to the actual generation of detectors).

Table 4.14 Selected samples of F and G main-sequence dwarfs

<table>
<thead>
<tr>
<th>Sample</th>
<th>#</th>
<th>$\Delta T_{\text{eff}}$[K]</th>
<th>Sp. Type</th>
<th>$\Delta$[Fe/H]</th>
<th>Selection</th>
<th>I/H</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAG</td>
<td>132</td>
<td>5400–5850$^*$</td>
<td>G2V–G8V</td>
<td>$-1.2/\pm0.2$</td>
<td>within $\sim$25 pc from Sun</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>MRS</td>
<td>536</td>
<td>5500–6760</td>
<td>F ($\sim$10% G)</td>
<td>$-1.8/\pm0.4$</td>
<td>magnitude $m_V \lesssim 6.5$</td>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>EDV</td>
<td>446</td>
<td>5600–6800</td>
<td>F ($\sim$25% G)</td>
<td>$-1.2/\pm0.45$</td>
<td>within $\sim$40 pc from Sun</td>
<td>H</td>
<td>3</td>
</tr>
<tr>
<td>SCH</td>
<td>1214</td>
<td>5620–6980$^*$</td>
<td>$\sim$F–M</td>
<td>$-2.0/\pm0.5$</td>
<td>high-velocity stars</td>
<td>H</td>
<td>4</td>
</tr>
<tr>
<td>RPM</td>
<td>287</td>
<td>5190–5920$^*$</td>
<td>G0V–G9V</td>
<td>$-1.2/\pm0.4$</td>
<td>within $\sim$25 pc from Sun</td>
<td>H</td>
<td>5</td>
</tr>
<tr>
<td>FLM</td>
<td>87</td>
<td>4650–5190$^*$</td>
<td>K0V–K3V</td>
<td>$-1.4/\pm0.6$</td>
<td>within $\sim$25 pc from Sun</td>
<td>I</td>
<td>6</td>
</tr>
<tr>
<td>RAB</td>
<td>60</td>
<td>3900–5190$^*$</td>
<td>K0V–K9V</td>
<td>$-1.1/\pm0.4$</td>
<td>various criteria</td>
<td>I</td>
<td>7</td>
</tr>
</tbody>
</table>


In Fig. 4.75 we compare the observed [Fe/H] and age distributions of main-sequence F+G dwarfs in the SNBH from Meusinger, Reimann & Stecklum (1991; hereafter MRS) and Edvardsson et al. (1993; hereafter EDV). Both samples contain internal errors less than $\sim$0.1 dex in [Fe/H] and $\sim$2 Gyr in age.

The sample of MRS is essentially based on two earlier samples described by Twarog (1980) and Carlberg (1985). However, several improvements in the reduction and analysis of the combined data were made by MRS. The magnitude limited sample of MRS ($m_V \lesssim 6.5$) comprises 536 stars with $-2 \lesssim [\text{Fe/H}] \lesssim +0.4$ and $T_{\text{eff}} = 5500$ to 6760 K. According to the spectral type classification by Harmanec (1988), we find that about 40 G-dwarfs are included in the sample. Thus, the MRS sample is strongly dominated by F-dwarfs. Since the absolute magnitude of a star depends on its initial metal-abundance, each star contribution was weighed by the inverse of the volume of the sphere with radius equal to the maximum distance at which the star can be placed and still falls within the apparent magnitude limit of the sample (see further MRS). The normalised [Fe/H]-distribution (bin width 0.1 dex) peaks at [Fe/H] $\approx -0.05$ (cf. Fig. 4.75). The corresponding age distribution is shown in the right panel of Fig. 4.75. The age distribution peaks towards small ages since the fraction of F+G dwarfs (i.e. with $m \sim 0.96$ to 1.2 $M_\odot$) nowadays on the main-sequence is larger for more recently formed stars. This is true in spite of observations which strongly suggest that the average past SFR has been considerably higher than at present (e.g. Mayor & Martinet 1977; Dopita 1990; see Sect. 3.1).

In the recent paper by EDV, the authors discuss a sample of 446 stars with distances less than 40 pc to the Sun for which accurate [Fe/H] ratios have been determined. The EDV sample appears to be volume complete and contains approximately 20–25% G-stars. Both the [Fe/H] and age-distributions of the EDV sample are roughly consistent with that of MRS considering the different fractions of G-dwarfs. The [Fe/H] distribution of the EDV sample is somewhat flatter than that of MRS and shows a maximum around [Fe/H] $= -0.15$. The corresponding age distribution extends to older ages. This is mainly due to the different age calibration used by EDV and because of the larger fraction of G-dwarfs in the EDV sample. In order to derive the age distribution of the EDV volume complete sample of stars, we corrected the age distribution of the well studied EDV subsample of 189 stars for incompleteness according to the local volume correction factors given by EDV. These corrections were applied also in a consistent manner when obtaining the [O/H] distribution of these stars. We emphasize that both the MRS and EDV distributions are not corrected to the solar cylinder but refer to a small volume around the Sun (see below).
Figure 4.75 Observed [Fe/H] (left panel) and age (right panel) distributions of field F+G main-sequence dwarfs from Edvardsson et al. (1993; dotted line) and Meusinger, Reimann & Stecklum (1991; solid).

Figure 4.76 Left panel: Observed [Fe/H] distributions of G main-sequence dwarfs in the solar vicinity from Pagel (1989; full dots) and for all main-sequence stars independent of spectral type from Schuster et al. (1993; solid line). Note that the bin width in [Fe/H] is 0.15 and 0.1 dex for the SCH and PAG samples, respectively. Right panel: Corresponding [O/H] distribution for the G-dwarfs shown in the left panel adopted from Pagel (1989; full dots) and corrected to the solar cylinder by Sommer-Larsen (1991ab; solid line).

The [Fe/H]-distributions of combined samples of F and G dwarfs discussed above can be compared to the corresponding distributions for G-dwarfs (132 stars; Pagel & Patchett 1975; Pagel 1989; PAG) and main-sequence stars of all spectral types (1214 stars; Schuster et al. 1993; SCH) as shown in Fig. 4.76. The empirical [Fe/H] distribution of G-dwarfs by Pagel & Patchett and revised by Pagel (1989) has not been corrected to the solar cylinder (i.e. not weighed by the modulus of the stellar velocity perpendicular to the Galactic plane; see also Rana 1991). G-dwarfs with metallicities below [Fe/H] ≈ −1.2 are not present in the PAG sample. The SCH sample consists of high-velocity disk and halo stars (the halo stars in this sample appear at values of [Fe/H] ≲ −1.2; stars with [Fe/H] ≲ −1.5 were excluded in Fig. 4.76). These kinematically excited stars are expected to be relatively old (compared to all main-sequence disk stars) and thus predominantly late in spectral type. This is also suggested by the fact that the Schuster et al. sample appears similar to the PAG and EDV samples (which include both F and G dwarfs). Consequently, the bias in the SCH sample is mainly a kinematical one (and not related to e.g. metallicity) since stars with orbital velocities similar to that of the local standard of rest are excluded by the selection criteria. Thus, the Schuster distribution is expected to refer to a considerably larger volume of the local Galactic disk compared to the distributions discussed before. Furthermore, the SCH sample may contain post main-sequence stars. The SCH distribution peaks at [Fe/H] values of −0.1 dex, while stars with [Fe/H]-ratios between −1.2 and −1.5 are argued to be either halo or disk stars. We emphasize that the selection bias of the kinematically selected (proper motion limited) samples is rather different from that of spectroscopically selected (magnitude limited) ones. In particular, preferential selection of the younger, more luminous stars may give rise to significant incompleteness corrections at low metallicities which would alleviate the G-dwarf problem.
In Fig. 4.77 we compare the normalised [Fe/H] distributions of G and K dwarfs in the solar vicinity (not corrected to the solar cylinder). The data for G dwarfs have been adopted from Rocha-Pinto and Maciel (1996; RPM) and differs considerably from the sample of Pagel (1989), in particular at metallicities [Fe/H] = −0.3 to +0.2. The RPM stars have been selected from the Third Catalogue of Nearby Stars (Gliese & Jahreiss 1991) and Strömgren photometry have been obtained from the catalogues of Olsen (1990) and Hauck & Mermilliod (1990). To exclude subgiants and white dwarfs, a visual absolute magnitude criterion was applied for stars with uncertain classifications (similar to that used by Pagel & Patchett 1975). To obtain the stellar metallicity using Strömgren photometry data of main-sequence stars, the metallicity calibrations from Schuster & Nissen (1989) were used. These calibrations allow for more accurate stellar metallicity distributions as compared to the metallicity distributions obtained by Pagel & Patchett (1975) and Pagel (1989) which were based on UBV photometry (see further RPM). Stars with [Fe/H] ≤ −1.2 were excluded as halo stars by RPM (only few stars with [Fe/H] < −0.8 and [Fe/H] ≥ −1.2 were found depending on the metallicity calibration used). While the RPM data peak at values of [Fe/H] ∼ −0.2 ± 0.1, the PAG data show a minimum in the G-dwarf metallicity distribution at these metallicities. Clearly, marked differences are present between the two distributions. We note that the metallicity distribution of the sample of local disk G dwarfs presented by Wyse & Gilmore (1995) is in good agreement with that of the RPM sample (both samples were selected in a similar manner).

For comparison, the [Fe/H] distributions of K-dwarfs presented by Flynn & Morell (1996; FLM) and Rana & Basu (1990) are shown. These data sets are subject to severe selection effects as discussed by FLM but appear in overall agreement. In the following, we will use the Rocha-Pinto and Maciel (1996) data for G dwarfs and the Flynn & Morell (1996) for K-dwarfs when a comparison is made with the model results.

In addition to the [Fe/H] and age distributions discussed above, it is interesting to consider the [O/H] distribution of long-lived stars in the SNBH. In principle, the [O/H] distribution can be considered as an independent observational constraint for models that do not incorporate the instantaneous recycling approximation and predict the [Fe/H] distribution independently. Unfortunately, the number of long-lived stars for which accurate oxygen abundances are available is yet low. For instance, the sample presented by Edvardsson et al. contains only ∼80 stars for which [O/H]-ratios have been determined accurately. To overcome this problem, [Fe/H]-abundances of stars are usually converted to [O/H]-ratios using an empirical relation for the variation of [O/H] with [Fe/H] in the local disk ISM (e.g. Sommer-Larsen 1991a,b; Pagel 1992):

\[
[O/H] \approx \begin{cases} 
[Fe/H] + 0.6 & \text{for } [Fe/H] \leq -1.2 \\
0.5[Fe/H] & \text{otherwise}
\end{cases}
\]

In spite of the fact that such conversions may lead to erroneous results (see Sect. 4.1), we here choose to apply such a conversion to the [Fe/H] abundance distribution of G dwarfs from Rocha-Pinto and Maciel (1996). The [O/H] distribution for nearby G dwarfs derived in this manner will be used below as an independent observational constraint.

Our alternative is to compare the predicted [O/H] distribution of G dwarfs in the SNBH to the early data from Pagel & Patchett (1975) and Pagel (1989) which was analysed with relatively inaccurate (and somewhat out-of-date) metallicity calibrations (see above). For comparison reasons, we show this distribution...
in Fig. 4.76 (right panel). The empirical [O/H]-distribution of G-dwarfs is considerably more narrow than that of the corresponding [Fe/H]-distribution. This is mainly due to the fact that the oxygen enrichment during the early evolution of the Galaxy proceeded much more rapidly than that of iron because: 1) oxygen is about one order of magnitude more abundant in SNII ejecta than is iron (see Sect. 3.3), and 2) SNIa progenitors usually have lifetimes much longer than SNII progenitors (see Sect. 4.3.4). After star formation ceased in the Galaxy, the disk interstellar [Fe/H]-ratio increased at much higher rate than [O/H].

The distributions discussed above all refer to a local volume around the Sun. The G-dwarf data corrected to the solar cylinder is clearly different from the original data. However, we emphasize that the magnitude of these corrections strongly depend on the adopted kinematical model for the Galactic stellar disk. As an example, we show in Fig. 4.76 the resulting [O/H]-distribution of G dwarfs from Pagel (1989) after applying corrections for the evolution of the disk vertical structure from Sommer-Larsen (1991a,b). Corrections for stellar orbital diffusion in galactocentric distance may also affect the data significantly (see Sect. 4.1). We will discuss the influence of such corrections below.

Model assumptions

The abundance distribution of stars of a given spectral type (e.g. G-stars) at time $t$ within a specific Galactic region can be derived theoretically if one knows: 1) the star formation history and the chemical history (e.g. gas infall, radial flows, mixing processes) of the region of interest, 2) the orbital evolution of the stars born in this region, and 3) a conversion relation between the spectral type of the main-sequence stars considered (essentially measured by the effective temperature $T_{\text{eff}}$) and the initial stellar mass $m$. A simple expression for the total number of stars born at Galactic ages between $t = t_1$ and $t_u$, which are still on the main-sequence at $t = T$ is:

$$N(t_1, t_u, T) = \int_{t_1}^{t_u} \int_{m_l}^{m_u(T-t, Z_*)} S(t)M(m) \, dm \, dt$$

where $m_l$ is the stellar lower mass limit and $m_u(T-t, Z_*)$ the turnoff-mass of a stellar population, born at metallicity $Z_*(t)$, with age $(T-t)$. Since Galactic evolution times can be simply transformed into stellar ages and abundances, Eq. (4.33) can be used to derive theoretical distributions over metallicity and stellar age. Abundance ranges (e.g. in terms of [O/H] and [Fe/H]) can be converted to stellar birth times according to the corresponding age-metallicity relations (AMR). These conversions are straightforward both for AMRs increasing monotonically and AMRs varying in non-monotonic ways. By intersecting the inner integral with a specific range in stellar mass, $\text{Eq. (4.33)}$ can be applied to stars confined to a particular spectral type, e.g. G-stars.

In general, samples of normal main-sequence stars with different spectral types trace the evolution of the Galaxy during different epochs. Taking into account the metallicity dependence of the stellar lifetimes, F-stars can provide information about the Galactic star formation history during the last $\sim$6-7 Gyr. This maximum look-back time increases towards later spectral types. For e.g. G-stars, the look-back time is $\sim$12 Gyr, i.e. about twice as long as for F-stars (cf. Schaller et al. 1992). Distributions of main-sequence stars that cover a wide range in spectral type (or initial mass) are usually not suited to determine in detail the variation of the SFR with galactic age. To allow for a comparison with the observations, model results are normalised to the predicted total number of stars (of the spectral type of interest) that are still on the main-sequence at the present epoch. We emphasize that the normalised distributions usually depend on variations of the SFR during the period of Galactic evolution over which the stars of interest are formed, i.e. for F stars during the past 6-7 Gyr.

- Spectral type vs. initial mass conversion

For the $m$ vs. $T_{\text{eff}}$ relation of main-sequence stars, we adopt the accurate data presented by Harmanec (1988) based on stars observed in eclipsing binaries. We assumed F-stars to have effective temperatures in the range $T_{\text{eff}} = 6980$ to 5920 K and G-stars between $T_{\text{eff}} = 5920$ and 5190 K. According to these data, main-sequence F and G-dwarfs have initial masses $[M_\odot]$ in the ranges (1.19 to 1.50) and (0.96 to 1.19), respectively. These values differ somewhat from the values given e.g. by Bowers and Deeming (1984; based on Allen 1973) which are (1.10 to 1.69) and (0.77 to 1.10) for F and G-dwarfs, respectively. To compare results with the K-dwarf data from Flynn & Morell (1996), we will consider dwarfs with spectral-type K0V–K3V to have masses in the approximate range (0.69 to 0.96).

In principle, the conversion of $T_{\text{eff}}$ (spectral type) to initial stellar mass requires knowledge of how $T_{\text{eff}}$ varies with initial metallicity. The relation between $T_{\text{eff}}$ and spectral type itself is sensitive to log $g$ (i.e. stellar mass), age, and metallicity so that it is in general not justified to assume that stars of a given spectral type have a fixed range in initial mass. A detailed discussion of how the initial mass range of stars of a given
4.3 Modelling the chemical evolution of the Galactic disk: results

Figure 4.78 Theoretical dependence of initial mass range of stars with present-day effective temperatures within a specified interval (spectral type) on stellar age and initial metallicity. **Top panels:** Initial mass range for stars with present-day $T_{\text{eff}} = 5920-6980$ K (approximately spectral type F) both for main-sequence stars (MS; left panel) and stars in any evolutionary stage including post main-sequence phases (All; right). The initial mass range is delimited by lines of the same texture for stars born with $Z = 0.02$ (solid curves) and $Z = 0.001$ (dotted). **Center panels:** As top panels but for stars with present-day $T_{\text{eff}} = 5190-5920$ K (approximately spectral type G). **Bottom panels:** As top panels but for stars with present-day $T_{\text{eff}} = 4650-5190$ K (approximately spectral type K0V–K3V). Hatched areas indicate the initial stellar mass range (for stars with effective temperatures as given in the top left corner of each panel) according to the empirical relation of Harmanec (1988).
spectral type varies with e.g. stellar age and metallicity is beyond the scope of this paper. However, we like to emphasize that large uncertainties are involved with this conversion which can significantly alter the resulting abundance and age distributions of long-living stars. To illustrate this effect, we used the Geneva tracks to derive the initial mass range $\Delta m$ of stars with present-day effective temperatures in a given range $\Delta T_{\text{eff}}$, both as a function of stellar age and initial metallicity. This non-trivial conversion was made both for main-sequence stars and stars in any evolutionary stage and is shown in Fig. 4.78 for fixed ranges in effective temperature for F, G, and K dwarfs according to the empirical relation from Harmanec (1988) discussed above. Fig. 4.78 shows that: 1) $\Delta m(\Delta T_{\text{eff}})$ is strongly dependent on of both stellar age and metallicity, 2) $\Delta m(\Delta T_{\text{eff}})$ deviates considerably from the fixed mass range given by Harmanec (1988) for stars of a given spectral type (i.e. with $T_{\text{eff}}$ in a given range). Although at solar metallicity reasonable agreement is found between the derived values of $\Delta m(\Delta T_{\text{eff}})$ and that predicted by Harmanec (1988), large discrepancies are present at low metallicities.

We conclude from Fig. 4.78 that it is erroneous to assume that stars of a given spectral type have a fixed range in initial mass since this mass range strongly depends on the age and initial metallicity of the stars involved. This implies that very careful selection of observational samples of stars in terms of their spectral type, age, and metallicity is required to allow for a meaningful investigation of their age and abundance distributions. In particular, to prevent contamination by old, low-metallicity stars and post main-sequence stars, the effective temperature criterion is insufficient and additional selection criteria are needed. We will briefly consider below to what extent these discrepancies affect the resulting abundance and age distributions of e.g. F, G, and K dwarfs. Nevertheless, unless stated otherwise, we will ignore any dependence of the $m$ vs. $T_{\text{eff}}$ relation on other quantities and assume for convenience the empirical conversion relation from Harmanec (1988).

We recall that in order for the metallicity distribution of long-living stars to provide an unbiased observational constraint to the chemical evolution of a Galactic region, the sample stars must be representative of the majority of stars ever formed in that region with the same properties, e.g. with respect to mass, age, and metallicity. Thus, a suited sample must not contain a significant number of stars formed outside the Galactic region of interest, or in an evolutionary phase not representative of the majority of stars formed in that region. For instance, a sample of long-living stars that is used to study the chemical evolution of the Galactic disk must not include a considerable contribution by e.g. halo stars or post main-sequence stars. Clearly, selection of stars with spectral types in a restricted range does not achieve this (e.g. Wyse & Gilmore 1995) and additional (e.g. chemical, kinematical, age) criteria are required to allow for a convenient and useful comparison with theoretical age and metallicity distributions of long-living stars.

- Dispersion and orbital diffusion corrections

The samples of F, G, and K dwarfs discussed above refer to a small volume around the Sun. No corrections were made for the effect of inflation of the vertical scale height in the SNBH. Such corrections are necessary when converting observed local volume densities of stars in the SNBH to z-integrated volume (or surface) densities for stars in the entire solar cylinder to which model calculations generally refer. To correct volume limited stellar samples to the entire solar cylinder, a kinematical model for the variation of the disk stellar scale height with age is needed. Correction factors $f$ for the solar cylinder can then be determined as a function of metallicity using the observed [Fe/H] vs. age relation (see Sect. 4.1).

In principle, high-velocity stars which spend only a small fraction of their orbit in the vicinity of the Sun will be under-represented in local samples of long-living stars (e.g. Wyse & Gilmore 1995, hereafter WG). To correct for this, a kinematical weighing factor has to be applied which is proportional to the product of (WG): 1) the vertical speed of the star when passing through the plane of Galactic disk, and 2) the vertical oscillation period of the orbit of the same star.

In an harmonic Galactic gravitational potential model, the density distribution of the gravitating material is constant with height above the Galactic plane so that the vertical orbital period of a star is constant as well. This may be a reasonable approximation for low-velocity stars that do not reach much beyond one disk scale height. In this case, weighing by the the mean vertical velocity of stars in a given metallicity bin may be an adequate correction. For high-velocity stars, however, which reach large distances above the Galactic plane and for which the weighing is most important, the assumption of an harmonic potential is a rather poor assumption (see WG). In an nonharmonic potential model, the vertical potential gradient causes the vertical oscillation period of stars, in particular those with high velocities at the disk plane, to be considerably longer than in case of an harmonic potential model (up to a factor $\sim 3$, cf. WG; Kuijken & Gilmore 1989). In a more detailed comparison, one accounts also for the variation of the disk vertical potential with galactic age.
It is clear that the assumption of an harmonic potential may considerably underestimate the contribution by high-velocity (i.e., relatively old, metal-poor) stars in the observed age and metallicity distributions discussed above. Nevertheless, we will not correct for this effect since such corrections depend strongly on the spread in the vertical oscillation period among the sample stars. Furthermore, the metallicity distribution of the local sample of G dwarfs presented by Wyse & Gilmore (1995; who have corrected for the vertical period effect) is similar to that presented by Rocha-Pinto & Maciel (1996; uncorrected). Thus, we will only deal with (scale height) corrections related to the vertical speed of a star at the disk plane.

Since observational samples suffer both from intrinsic and artificial dispersions in metallicity, the observed distributions need to be deconvolved before scale height corrections for the contents of different metallicity bins can be applied (cf. Rana 1991). In principle, the best way to correct an observed local metallicity distribution for the disk vertical inflation would be to weigh each star according to the increase in scale height of the stellar disk (at the Galactocentric radius the star was born) during the lifetime of the star. However, the resulting scale height corrections would be erroneous since the absolute ages of the stars included in the above samples are inaccurate with common errors of a few Gyrs (with larger uncertainties for the older stars; see Sect. 4.1). The result after deconvolving the observations may not be unique and in fact may depend both on the accuracy of the observational data and on the deconvolution method used.

We decide to use the following method to compare the model results with the observations. Since the model stars have metallicities according to a single valued AMR, we first convolve the model predicted metallicity contributions with a Gaussian dispersion in metallicity (or equivalently stellar age in our models) to account both for observational and intrinsic spread in the abundances of stars born at the same age. We use constant Gaussian standard deviations of \( \sigma \) for each stellar metallicity (or age). The spread in the abundances and ages of stars in e.g. the sample from Edvardsson et al. (1993; see Sects. 4.1−4.2) may be larger but the above values are suited to illustrate the effect.

Subsequently, we apply inverse scale height corrections to the convolved model data in order to convert the theoretical distribution to a local volume around the Sun (instead of referring to the entire solar cylinder). We here will apply the scale height correction factors presented by Sommer-Larsen (1991a,b) which are based on two distinct kinematical models for the evolution of the vertical structure of the Galactic disk (cf. Sect. 4.1). In principle, the dispersion convolved and scale-height corrected theoretical distributions can be compared directly with the observed age and metallicity distributions of stars born within a local volume around the Sun.

The samples discussed above are not subdivided according to e.g. stellar galactocentric birthplace so that these samples probably contain stars that are nowadays observed but were not necessarily born in the SNBH. We expect that corrections for radial orbital diffusion of the sample stars will not alter significantly the qualitative results presented below. However, depending on the sample selection criteria and on the mean effect of stellar orbital diffusion, such corrections tend to reduce the number of the relatively old, high-metallicity stars that moved outwards from the more central regions of the Galactic disk to the SNBH during their lifetimes (see Sect. 4.1). Consequently, mean corrections for radial orbital diffusion are likely to shift the age and metallicity distributions of long-living stars to somewhat smaller ages and lower metallicities.

### Results

We investigate the sensitivity of the resulting present-day age and metallicity distributions of F, G, and K main-sequence dwarfs in the Galactic disk to: 1) the \( \Delta m \) vs. \( \Delta T_{\text{eff}} \) relation, 2) the scale height and dispersion corrections, and 3) the underlying star formation history and IMF assumed. For the star formation history, we will adopt the SFR models selected in Sect. 4.2 together with the resulting AMRs, e.g. for oxygen and iron. We neglect radial flows and large scale mixing of interstellar gas (i.e. the ISM is homogeneous at all evolution times). Furthermore, we relax the instantaneous recycling approximation and compute the ISM abundances for each element explicitly. In this manner, the oxygen and iron abundance distributions of stars observed in the SNBH provide independent constraints to our models (see e.g. Tayler 1990).

- **Effect of \( \Delta m \) vs. \( \Delta T_{\text{eff}} \) relation assumed**

Fig. 4.79 illustrates the impact of the detailed metallicity and age dependent \( \Delta m \) vs. \( \Delta T_{\text{eff}} \) conversions derived from the Geneva tracks on the resulting age and metallicity distributions of F, G, and K dwarfs. In general, these conversion relations result in a substantial contribution by old, metal-poor stars in contrast to the case of a constant \( \Delta m \) vs. \( \Delta T_{\text{eff}} \) relation. These old, metal-poor stars have present-day effective temperatures in the range derived by Harmaneke (1988) for spectral types F and G. For K dwarfs, the impact on the resulting age and abundance distributions is relatively small.
We conclude from Fig. 4.79 that the present-day age and metallicity distributions of F and G main-sequence dwarfs in the SNBH are substantially altered when one accounts for the detailed metallicity and age dependence of the $\Delta m$ vs. $\Delta T_{\text{eff}}$ relation. Thus, the initial mass range of stars observed and classified as e.g. G dwarfs depends in a complex manner on the metallicities and ages of the sample stars. This has two important consequences: 1) the usual assumption of a constant $\Delta m$ vs. $\Delta T_{\text{eff}}$ relation in the context of age and abundance distribution studies introduces errors, e.g. by excluding a significant contribution by old, low-metallicity stars, and 2) apart from effective temperature, additional observational selection criteria are required to select a sample of stars suited to study in detail the age and metallicity distributions of long-living stars in the SNBH. In particular, a star’s spectral type cannot be simply translated in terms of initial stellar mass in the absence of both age, metallicity, and log $g$ data of the star.

![Figure 4.79](image_url)

**Figure 4.79** Age and metallicity distributions of F, G, and K main-sequence dwarfs: effect of $\Delta m$ vs. $\Delta T_{\text{eff}}$ relation assumed. Results are shown for the standard SFR (model A) with: 1) constant $\Delta m$ vs. $\Delta T_{\text{eff}}$ (spectral type) conversions as given by Harmanec (1988; solid curve), and 2) with the stellar metallicity and age dependent $\Delta m$ vs. $\Delta T_{\text{eff}}$ conversions derived from the Geneva tracks (dotted curve) as discussed in the text.

Although these consequences can be important, we will assume in the following that stars of a given spectral type have masses within the constant initial mass range derived by Harmanec (1988) for each sample listed in Table 4.14. The reason is that, in general, it is unclear to what extent these samples are contaminated by stars with masses outside the mass range derived by Harmanec (1988) for the spectral type of interest (see Fig. 4.78). Therefore, it is not possible to correct for such effects without additional information about the sample stars (e.g. age, mass, and metallicity). Nevertheless, comparison with the observations suggests that F stars with $[\text{Fe}/\text{H}] \gtrsim -0.8$ and G stars with $[\text{O}/\text{H}] \gtrsim -0.5$ are not observed (cf. Figs. 4.75 and 4.76). This indicates that the observational selection criteria used have been sufficiently tight to exclude stars with masses outside the mass ranges of F and G main-sequence dwarfs derived by Harmanec (1988). This may partly justify the usual assumption of a constant $\Delta T_{\text{eff}}$ vs. $\Delta m$ relation.

We emphasize that the detailed variation of the $\Delta T_{\text{eff}}$ vs. $\Delta m$ relation with e.g. stellar age and metallicity has not been considered before. However, in the absence of an adequate conversion of the observational selection criteria (e.g. $\Delta T_{\text{eff}}$ or spectral type) to theoretical criteria (e.g. $\Delta m$), the effect of such variations can be important and should be kept in mind when a comparison is made between the theoretical and observed age and metallicity distributions of in particular F and G stars.
• Effect of scale height and dispersion corrections

Fig. 4.80 shows the resulting present-day [Fe/H] distributions of main-sequence stars of F, G, K, and all spectral types in case of the standard SFR (model A) selected in Sect. 4.2. In addition, the age and [O/H] distributions of, respectively, F and G stars are shown. When dispersion corrections ($\sigma_{\text{age}} = 2$ Gyr, $\sigma_{[\text{El/H}]} = 0.2$ dex) and scale height corrections are applied to the model data, the resulting distributions are considerably reduced at low values of [Fe/H] and [O/H], in particular for stars with G, K, and all spectral types. Dispersion corrections predominantly smooth the data over a wider range in metallicity (or age) than the original distribution, while scale height corrections decrease the predicted number of stars at low metallicities and large ages. The effects of these corrections on the age and [O/H] distributions of F and G stars are usually small. For stars of spectral type F (and earlier), scale height corrections are usually negligible since these stars have main-sequence lifetimes which are too short for the stars to move to large scale heights ($\gtrsim 100$ pc). Scale height corrections for stars with spectral types later than G are similar in magnitude.

![Figure 4.80](image.png)

**Figure 4.80** Age and metallicity distributions of F, G, and K main-sequence dwarfs: effect of scale-height + dispersion corrections. Results are shown for the standard SFR (model A) before (solid curve) and after (dotted) applying corrections to the model data. Corrections were made as follows (in order): 1) age and abundance dispersion corrections (with Gaussian dispersions $\sigma_{\text{age}} = 2$ Gyr and $\sigma_{[\text{El/H}]} = 0.2$ dex for the contents of each age and [El/H] bin, respectively), and 2) scale height corrections for the vertical dispersion of the Galactic stellar disk with age (Sommer-Larsen 1991; see Sect. 4.1).

Instead of applying these corrections to each of the resulting distributions presented below, we prefer to compare the uncorrected model data with the observations. This may be partly justified as it is unclear to what extent the observational data are subject to different kinds of selection effects and how corrections for these effects would alter the data (see above). We will keep in mind the magnitude and behaviour of the effects of dispersion and scale height corrections on the model data discussed here. In the following, we investigate the effect of the underlying SFR and IMF on the resulting age and metallicity distributions.

• Dependence on SFR

In principle, the SFR history weighed by the AMR (or present-day stellar age) determines the metallicity (or age) distributions of long-living stars nowadays observed in the Galaxy. Fig. 4.81 demonstrates the effect of the star formation history on the metallicity and age distributions of F, G, and K dwarfs for different SFR
4.3.8 Metallicity and age distributions of long-living stars

Figure 4.81 Age and metallicity distributions of F, G, and K main-sequence dwarfs: effect of SFR. Results are shown for SFR models selected in Sect. 4.2 as follows: 1) density dependent SFR without infall (model A; solid curve), 2) as 1) but with infall (model B; dotted), and 3) bimodal SFR model (model D; dashed). A Salpeter IMF and model parameters as in Table 3.3 were assumed. No corrections were made for dispersions in abundance/age or inflation of the stellar disk.

models selected on their ability to explain the observed AMR of iron (cf. Sect. 4.2). We here consider the models computed with the Geneva/Nomoto yields only, since these appear in somewhat better agreement with the observed distributions than the same models computed with the Woosley/Weaver yields. This particular choice of stellar yields does not affect the qualitative conclusions given below.

It can be seen from Fig. 4.81 that the resulting distributions for the different SFR models shown are similar. However, depending on the metallicity in the ISM at which the SFR exhibits its maximum, the metallicity distributions differ considerably, in particular for stars with spectral types later than G. For instance, in case of the bimodal SFR model for which the formation rate of low-mass stars ($m < \sim 1 M_\odot$) is assumed to be constant with galactic age, relatively large number of high-metallicity stars of late spectral types are predicted. For all other models, metallicities and ages at maximum are comparable. In case of the standard SFR model with infall (model B), the maxima in the metallicity distributions occur at somewhat lower metallicities than in the same model without infall (model A). For SFR models that fit the observed AMR of iron, the resulting metallicity and age distributions of F, G, and K main-sequence dwarfs are found to be similar (especially after correcting for the intrinsic dispersion in metallicity observed).

For long-living, low-mass stars, the predicted metallicity (or age) distributions peak at values $[\text{Fe/H}]$ (or stellar ages) at which the SFR was relatively high and the metallicity dependent lifetimes of the stars of the spectral type of interest are such that the majority of these stars nowadays are still on the main-sequence. Both metallicity and age distributions are sensitive to the underlying AMR by means of the metallicity dependent stellar lifetimes. For models with AMRs and SFRs nearly constant over the past 10 Gyr or so, the resulting age distributions of stars with spectral types earlier than $\sim$F merely reflect the metallicity dependence of the stellar lifetimes.

In Fig. 4.82 we compare the observed present-day age and metallity distributions of F, G, and K main-sequence dwarfs with those predicted by the standard and Dopita SFR models selected in Sect. 4.2. A Salpeter IMF was assumed. Considering the observational uncertainties and selection effects, the model results are found to be in remarkably good agreement with the observations. For all distributions, abundance+age dispersion corrections will further improve the agreement with the observations as demonstrated in Fig. 4.80, for reasonable values of $\sigma_{[\text{Fe/H}]} \sim 0.2 - 0.3$ dex, $\sigma_{[\text{O/H}]} \sim 0.1$ dex, and $\sigma_{\text{Age}} \sim 2 - 4$ Gyr. Scale height corrections somewhat reduce the agreement with the observations in case of the $[\text{Fe/H}]$ distributions.
Figure 4.82 Age and metallicity distributions of F, G, and K main-sequence dwarfs. Results are shown for the standard SFR (model A; solid curves shown in the upper six panels and for the Dopita SFR (model E; solid curves in the bottom six panels. For comparison, observational data (dash-dotted) has been included from the following sources: 1) Meusinger et al. (1991; F dwarfs, both age and [Fe/H]), 2) Flynn & Morell (1996; K dwarfs, [Fe/H]), 3) Rocho-Pinto & Maciel (1996; G dwarfs, both [Fe/H] and [O/H]), and 4) Schuster et al. (1994; all spectral types, [Fe/H]).
4.3.8 Metallicity and age distributions of long-living stars

of stars with K and all spectral types. For the observational data from Schuster et al. (1993), which is a proper motion selected sample of stars of all spectral types, we expect the older, more metal-deficient stars to be over-represented compared to our predictions. This can explain why scale height corrections applied to the model data would not improve the agreement with the Schuster et al. data substantially. For the K-dwarf data, metal-weak stars are argued to be over-represented as well (see Flynn & Morell 1996), so that scale height corrections would further improve the agreement when such selection bias is taken into account. Thus, we find that our models are in very good overall agreement with the present-day age and metallicity distributions of F, G, and K dwarfs observed in the SNBH.

Interestingly, no "classical G-dwarf problem" is encountered in any of the metallicity distributions resulting from the SFR models selected in Sect. 4.2. Since the models cover a wide range in star formation and gas infall histories, this suggests that the inclusion of the metallicity dependent stellar lifetimes and yields in our model calculations are the main reason for the absence of the otherwise so evidently present G-dwarf problem. This is true also for the metallicity distributions of both F and K dwarfs.

The impact of metallicity dependent stellar lifetimes on the G-dwarf problem has been earlier studied by Bazan & Mathews (1990). They concluded that inclusion of the metal-dependent lifetimes could explain (at least) part of the G-dwarf problem. However, they incorporated the instantaneous recycling approximation and further used the different set of metallicity dependent stellar lifetimes from van den Berg & Lasherides (1987) and from Mengel et al. (1979). Also, they did not use metallicity dependent stellar yields. Furthermore, they assumed upper and lower mass limits for G-dwarfs of 1.09 and 0.79 M⊙, respectively, given by Bowers & Deeming (1984). These values differ somewhat from the 1.19 and 0.96 M⊙ limits used here (see above). In addition, Bazan & Mathews used the Miller-Scalo IMF (1979) which is markedly different from the Salpeter IMF applied here. Alltogether, these differences can explain why we arrive at the distinct conclusion that the metallicity dependent set of stellar yields and lifetimes do not lead to the G-dwarf problem encountered in previous investigations.

Comparison of the results for the standard and Dopita SFRs reveals that these SFR models predict similar age and metallicity distributions. Overall, for the Dopita SFR model the agreement with the observations is somewhat better but a firm conclusion is prevented by the observational uncertainties. We conclude that our SFR models, which were selected on the basis of their ability to fit the observed AMR of iron and were computed while accounting in detail for the metallicity dependence of the stellar lifetimes and yields, are in extremely good agreement with the observed age and metallicity distributions of F, G, and K dwarfs in the SNBH. In particular, no "classical G-dwarf problem" is found for stars of these spectral types.

• Dependence on IMF

Finally, we investigate whether it is possible to constrain the IMF by the observed stellar age and metallicity distributions. Fig. 4.83 illustrates the dependence on the IMF of the present-day age and metallicity distributions of long-living stars in the Galaxy for the standard SFR model. Comparison with Fig. 4.82 reveals that the effect of the IMF on these distributions is much larger than that of the underlying star formation history. However, the IMF models shown were not tuned to fit the observed AMR of iron and, in fact, Fig. 4.83 shows the sensitivity of the resulting distributions to the AMR. Since the models shown were computed with input parameters as listed in Table 3.3, there seems to be no obvious way to fit the observed age and metallicity distributions for models with either very steep IMFs (e.g. γ = 2.7) or for the IMF computed iteratively from the PDMF presented by Scalo (1986). In contrast, the Kroupa IMF model appears in reasonable agreement with the observations (apart from the effects of dispersion and scale height corrections). We conclude that the observed age and metallicity distributions are very sensitive to the underlying AMR and, in principle, can be used to distinguish between the IMF of the population of long-living stars formed in the Galaxy.
Figure 4.83 Age and metallicity distributions of F, G, and K main-sequence dwarfs: effect of IMF. Results are shown for the standard SFR (model A) in case of: 1) the IMF computed from the PDMF presented by Scalo (1986; dotted line), 2) the empirical IMF from Kroupa et al. (1993; thick solid), and 3) a power-law IMF with $\gamma = 2.7$ (thin solid). No corrections were made for dispersion in abundance/age or inflation of the stellar disk.

Conclusion

We summarize the main results obtained in this section as follows:

- our models are in very good agreement with the present-day age and metallicity distributions of F, G, and K dwarfs observed in the SNBH. This is true for values of $\sigma_{[\text{Fe/H}]} \sim 0.2-0.3$ dex, $\sigma_{[\text{O/H}]} \sim 0.1-0.15$ dex, and $\sigma_{\text{Age}} \sim 2-4$ Gyr, and scale height corrections as derived by Sommer-Larsen (1991). We emphasize that this result was achieved without correcting for: 1) the dependence of the spectral type vs. initial mass range relation on initial metallicity and age of the sample stars (we simply assumed the constant $\Delta T_{\text{eff}}$ vs. $\Delta m$ relation from Harmanec 1988), and 2) the spread in the vertical oscillation period of the Galactocentric orbit among the sample stars (this may overestimate the contribution by high-velocity – relatively old, metal-poor – stars to the theoretical distributions). The magnitude of these corrections is uncertain due to observational uncertainties;

- no "classical G-dwarf problem" is encountered for any of the SFR models studied here. We conclude that the inclusion of the metallicity dependent stellar lifetimes and yields in our model calculations are the main reason for the absence of the G-dwarf problem. This is true also for the metallicity distributions of both F and K dwarfs. This conclusion is in contrast with that from Bazan & Mathews (1990) who demonstrated that the inclusion of metallicity dependent stellar lifetimes can explain only a limited part of the G-dwarf problem. However, their model assumptions and stellar input data used differs strongly from the stellar evolution data applied here;

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We note that Wyse & Gilmore (1995) have recently argued that a large fraction (~45%) of the metal-poor stars with $[\text{Fe/H}] \sim -0.5\pm0.1$ dex in the SNBH is associated with the thin disk (i.e. the kinematics of these stars confine them close to the plane of the Galactic disk). This implies that previous attempts to derive the column-integrated abundance distribution of long-living stars near the Sun (by kinematic-weighing of the local sample data) will over-estimate the true number of metal-poor stars in the Galactic disk. In these cases, for stars with metallicities $[\text{Fe/H}] \lesssim -0.4$ dex thick-disk kinematics were used in the weighing applied to correct the abundance distributions of these stars to the solar cylinder. Rectification of this will reduce the total number of metal-poor stars present in the solar cylinder and will aggravate the G-dwarf problem e.g. encountered by the simple model.
• for long-living, low-mass stars, the predicted metallicity (or age) distributions peak at values \([\text{[El/H]}]\) (or stellar ages) at which the SFR was relatively high and the metallicity dependent lifetimes of the stars of the spectral type of interest are such that the majority of these stars nowadays are still on the main-sequence. Both metallicity and age distributions are sensitive to the underlying AMR by means of the metallicity dependent stellar lifetimes. For models with AMRs and SFRs nearly constant over the past 10 Gyr or so, the resulting age distributions of stars with spectral types earlier than \(\sim F\) merely reflect the metallicity dependence of the stellar lifetimes. The age and metallicity distributions of long-living stars nowadays observed in the Galaxy are determined by the the SFR history weighed by the AMR (or stellar ages);

• for SFR models that fit the observed AMR of iron, the resulting metallicity and age distributions of F, G, and K main-sequence dwarfs are found to be similar (especially after correcting for the intrinsic dispersions in age and metallicity observed e.g. in the Edvardsson et al. (1993) data). From the observational data currently available it is not possible to draw strong conclusions about the underlying SFR model provided that such models fit the observed AMR. For instance, we find that models with and without infall can equally well explain the observed G-dwarf abundance distribution (in contrast to the finding of Rocha-Pinto and Maciel (1996) which favours infall models). Since the age and metallicity distributions of long-living stars are very sensitive to the underlying AMR, the observed distributions, in principle, can be used to distinguish between IMFs appropriate for the population of long-living stars formed in the Galaxy. For the adopted set of metallicity dependent stellar yields, we find that the Salpeter (1955) and Kroupa (1993) IMFs are consistent (and the Scalo IMF and \(\gamma = -2.7\) power law IMF are inconsistent) with the observed abundance and age distributions of long-living stars in the SNBH.
4.4 Discussion and Conclusion

In the previous sections, we have modelled a large set of observational data related to the chemical evolution of the Galactic disk and halo using a comprehensive and up-to-date galactic evolution model which incorporates metallicity dependent stellar yields, lifetimes and remnant masses. In this section, we will restrict ourselves to the main results obtained in Sect. 4.3.4 where we modelled the element abundances and abundance-abundance variations observed among stars in the Galaxy. We briefly discuss how these results compare to the results obtained in several recent studies concerning the chemical evolution of the Galactic disk. This comparison is far from being complete and is meant to address the present state of Galactic chemical evolution models and to highlight some of their main results in common.

We emphasize that the model used here is one of the state-of-the-art models currently available to investigate the chemical evolution of the Galactic disk. In Chap. 3, particular attention was paid to describe in detail the major assumptions and model ingredients involved. In combination with the extensive results derived in the previous section for a wide range of observations, this model is currently one of the best documented Galactic chemical evolution models available. We note that interpretation of a direct comparison of our results with those presented in previous Galactic evolution studies is often hampered by differences in: 1) the stellar input data (e.g. yields, evolution tracks), 2) the choices of parameter values (e.g. upper mass limit of SNII, Galactic age) and assumptions (e.g. relaxation of the instantaneous recycling approximation, the number of distinct Galactic components considered), 3) the implementation and parametrization of model ingredients (e.g. inflow, SNII contribution, IMF, star formation history), and 4) the method of solving the galactic chemical evolution equations (e.g. iteratively, adaptive stepsize inclusion, accuracy).

Timmes et al. (1995) considered a dynamical evolution model with inflow on a 4 Gyr e-folding time scale onto an exponential disk and 1/1/r² bulge, a Salpeter IMF, and quadratic Schmidt (1955) law for the SFR. In order to prevent overproduction of oxygen, an upper limit of \( m_{\text{SNII}}^{\text{SNII}} \sim 30 \text{ M}_\odot \) was derived of stars that eject all material external to the iron core. Such a cutoff in the mass stars that ultimately end as SNII is also suggested by the observed helium-to-metal enrichment ratio \( \Delta Y / \Delta Z \sim 4 \) (see e.g. Maeder 1992; Schaller et al. 1992; Sect. 3.2.2). However, Giovagnoli & Tosi (1995) pointed out that the latter observation does not provide a direct constraint because of the usual but erroneous assumption of a linear relation between the stellar oxygen abundance and Z. Apart from this, different slopes are used by different authors and careful interpretation of this constraint is needed.

Our results confirm earlier results (e.g. Maeder 1992; Timmes et al. 1995; Giovagnoli & Tosi 1995) that in order to avoid over-production of heavy elements, an upper limit for massive stars that explode as SNII of \( m_{\text{u}}^{\text{SNII}} \sim 25 \text{--} 30 \text{ M}_\odot \) is required. This value may imply a lower mass limit for black hole formation larger than \( \sim 25 \text{ M}_\odot \) in order to prevent over-production of oxygen. Alternatively, this may indicate that stars more massive than \( \sim 25 \text{ M}_\odot \) lose large parts of their envelopes before they ultimately collapse. In either case, we emphasize that the precise value of the mass cutoff is sensitive to the detailed variation of the heavy element yields of stars with \( m \gtrsim 8 \text{ M}_\odot \) with initial metallicity and stellar mass (as determined by e.g. mass-loss history, fraction of binary stars, ratio of SNII/c and SNII), as well as on the adopted stellar IMF at birth and its possible variation with Galactic age. It must be noted that Timmes et al. did not take into account the contribution by SNII/c (which have considerably smaller [O/Fe] ratios in their ejecta than SNII), so that the precise cutoff in the mass of stars that become SNII in fact may be larger than obtained. In addition, it was found that the reduction of the yields of SNII would improve the agreement with the observations for most elements.

Pagel & Tautvaisienè (1995; hereafter PT) modelled analytically the abundance-abundance variations observed among both Galactic disk and halo stars using the gas inflow formalism of Clayton (1985; see Sect. 4.2.3) and the delayed production approximation introduced by Pagel (1989). In this method, the IMF-weighted effective yields of massive stars are derived from fitting the observations while assuming that these yields do not depend explicitly on initial composition. PT found no evidence for a change in the yields of stars formed in the SNBH and those formed in the inner galaxy. As an order of magnitude estimate, PT determined the IMF-weighted effective yields of massive stars (e.g. lifetime of the Galaxy, inflow time scale, lifetime of SNII progenitors, power index of the SFR) we
refer the reader to Yoshii et al. (1996). From the break in the observed [O/Fe] vs. [Fe/H] relation and the behaviour of the metal-poor tail in the abundance distribution of long-living stars, these authors derive best agreement for $\tau_{inf} = 5$ Gyr and $\tau_{SNIa} = 0.5$–3 Gyr (typically $\tau_{SNIa} = 1.5$ Gyr) assuming a Galactic lifetime of 15 Gyr. In particular, Yoshii et al. argue that infall does not need to be completed before SNIa contributed significantly to the iron enrichment of the Galactic disk ISM. These results are similar to the preferred values of $\tau_{inf} = 3$ Gyr and $\tau_{SNIa} = 1$–3 Gyr obtained here. We recall, however, that the time scales derived are very sensitive to: 1) the adopted age of the Galaxy since the onset of star formation, 2) the heavy element contribution by massive stars during early evolution epochs ($m \gtrsim 8$ M$_\odot$), as determined by e.g. the assumed IMF, stellar mass loss rates, and binary fraction, 3) the detailed star formation and enrichment history of the Galaxy before the onset of star formation in the disk, and 4) the gas infall and accretion history of the disk (see also Sect. 5.5.1).

Mihara & Takahara (1996) argued that the chemical evolution of the SNBH is best modelled using the Salpeter IMF, exponentially decaying gas infall ($\tau_{inf} = 5$ Gyr) with primordial abundances in the infalling matter, and a fraction of close binary systems that ultimately ends as SNIa of about 0.01–0.05 with $\epsilon_1 \sim 0.1$. These results are consistent with the results obtained in Sect. 4.3.4. In contrast, these authors find that the Salpeter IMF models result in amounts of helium insufficient to reproduce the solar helium abundance. This is not supported by our results (see e.g. Tables 4.8 and 4.9).

Carigi (1994) considered a model with an SFR proportional to the surface mass gas density, decreasing infall rate with e-folding time 3.5 Gyr, Salpeter IMF, and inclusion of the dependence of the stellar yields on initial metallicity (with two classes of SNIb progenitors, WR stars, and binary evolution). She found that the models with the Salpeter IMF cannot reproduce the observed abundance patterns in the SNBH and that best agreement with the observations is obtained with the Salpino IMF (although e.g. C and [C/Fe] disagree with the observations).

Giovagnoli & Tosi (1995) have argued that a significant amount of gas must have been accreted by the Galactic disk over its lifetime. They argue that models without infall must be rejected (see also Tosi 1988) because they do not allow to solve the G-dwarf problem or to fit the O and C abundances. Although these motivations for gas infall are not supported by our results (see also Sect. 4.3.8), our results do favour infall of gas onto the Galactic disk over its lifetime. The need for strong infall at early Galactic evolution times is also supported by the models of Meusinger (1994). He argues that a decreasing SFR, which is nearly constant over the second half of the disk evolution, combined with gas infall with a similar temporal run as the SFR is in best agreement with the observations (including the observed WDLF) assuming a disk age of 12 Gyr.

In the model of Prantzos & Aubert (1995), the disk is subdivided in independent concentric rings which are built up gradually by infall of primordial gas (neglecting complicated effects of radial inflows or the interaction of gas and stars; see also Matteucci 1990). An infall time scale $\tau_{inf} = 3$ Gyr, Schmidt (1963) like SFR with an extra dependence on galactocentric distance based on spiral wave theory (e.g. Onishiishi 1975; Wyse & Silk 1989), and Kroupa et al. (1993) IMF is assumed. The model is able to explain several observational constraints including the evolution of C and O isotope abundances. For a comprehensive discussion of the uncertainties involved in the stellar nucleosynthesis of C and O we refer the reader to Prantzos (1996).

In general, it is found that the inclusion of the metallicity dependence of the stellar nucleosynthesis yields and stellar mass loss strongly affect galactic chemical evolution results (e.g. Maeder 1992; Carigi 1994; Giovagnoli & Tosi 1995; Timmes et al. 1995; Portinari 1996; Chiappini et al. 1996; Prantzos et al. 1996). Consequently, the IMF and SFR derived from fitting e.g. the abundances and abundance-abundance variations observed among Galactic disk and halo stars depend strongly on the adopted set of stellar yields and associated model parameters. It is clear that the ability of a Galactic chemical evolution model to fit these observational constraints does not guarantee the adequateness of the model. In other words, the relative success of a given Galactic evolution model does not prove the validity of the underlying assumptions, i.e. essentially the adopted IMF and SFR (Prantzos et al. 1996).

Apart from the adopted set of stellar yields, galactic evolution models can be distinguished according to the different Galactic regions to which they are applied as well as the underlying formation and evolution scenarios adopted for these regions. Eggen, Lynden-Bell, and Sandage (1962; hereafter ELS) proposed that the Galaxy formed in a single large rotating gas cloud in which the halo collapsed dissipatively to form the disk. Toomre (1977) and Searle & Zinn (1978) argued that the Galaxy formed by a merger of several chemically distinct and unique fragments and that the Galactic halo consists of dwarf systems that have been captured over an extended period of time. These scenarios probably lead to rather different star formation histories and chemical evolution paths of the Galactic halo, thick disk and thin disk components. Recent models for the Galaxy attempt to account in detail for such differences in evolution (see below).
An important aspect for such models is the distinction between stars that belong to different Galactic regions. It has been argued recently that the distinction between e.g. thick disk and halo stars is possible only on the basis of both kinematical and chemical properties (e.g. Schuster et al. 1993; Wyse & Gilmore 1995; PT). Also, the spheroidal (bulge and halo) and disk (thick and thin disks) components of the Galaxy have substantial different angular momentum distributions (Wyse & Gilmore 1992; Ibata & Gilmore 1995). Apart from these properties, stellar age may be an important quantity to distinguish between different Galactic regions at birth. This is true provided that stars associated with different regions were formed at different epochs in Galactic evolution. Observations reveal that the thick disk population extends to low metallicities so that stars which were previously classified as halo stars on the basis of their low metallicity actually belong to the disk (e.g. Wyse & Gilmore 1995; PT; see Sect. 4.1.6).

In principle, multi-phase models for the chemical evolution of the Galaxy should be constrained by the observational characteristics of the stars associated with different regions in the Galaxy, i.e. by considering samples of stars that were formed (but not necessarily are nowadays observed) in each region. Such samples are still rare and often limited to a detailed comparison of model predictions with the observations for stars belonging e.g. to the halo, thick, and thin disk (as well as for stars born at different ranges in galactocentric distance in the Galactic disk). Therefore, the current generation of Galactic chemical evolution models is restricted to a very limited set of observations dealing predominantly with stars nowadays observed in the local Galactic disk and halo. This apparently prevents one to draw strong conclusions about the chemical evolution of the halo, thick and thin-disk regions as well as about their mutual interactions during Galactic evolution. Nevertheless, we like to discuss briefly several interesting and promising models for the chemical evolution of the Galactic disk. These models tend to converge to a model in which both the kinematical and chemical properties of stars as a function of Galactic age and location are modelled simultaneously.

The chemical evolution of a self-gravitating, star forming viscous disk has been studied by Tsujimoto et al. (1995). Such a disk is able to reproduce the exponential distribution of the surface density of stars in the Galactic disk as well as a flat rotation curve in the disk. The main chemical properties of the Galactic disk, e.g. the radial abundance variations of O and Fe and the dispersion in the [O/Fe] vs. [Fe/H] diagram, can be explained adequately in the framework of the star-forming viscous disk model (see also Yoshii & Sommer-Larsen 1989). In this model, the metal enrichment of the bulge is enhanced by viscosity driven radial inflow of metal-rich gas over an extended period of a few Gyr.

Pardi & Ferrini (1994) considered the effect of different halo collapse times and star formation efficiencies in the disk. From modelling a wide range of observational constraints including the PDMF, these authors argued that the SFR in the Galaxy did rise slowly during the first \( \sim 2 \) Gyr and thereafter decreased. Further, they argued that the disk formed from the halo on a time scale \( \gtrsim 5 \) Gyr so that the slow collapse naturally allows for a thick-disk phase with abundances intermediate to that of the thick disk population and old population II stars. For a density dependent SFR, the initial rise of the SFR and its subsequent decay is determined by the rate of gas accumulation from the halo in the disk. Self-regulation of the SFR leads to a constant low SFR over the last few Gyr. The Dopita SFR and burst models discussed in Sect. 4.3.4 give very similar results. As discussed by Pardi & Ferrini (1994), an important difference between constant SFR and burstlike SFR models is that the latter models imply much shorter time scales for the enrichment of the disk ISM. For this reason, it is important to determine accurately the ages and kinematical properties at birth of the sample stars in order to derive this time scale observationally and to tightly constrain the models.

In a subsequent study, Pardi et al. (1995) considered a multiphase three zone description of the Galaxy: halo, thick and thin disk, as coupled but separate components. They consider the formation of the Galaxy as the continuous collapse process during which the formation and evolution of the different galaxy components are connected. From modelling the metallicity distributions of long-living stars associated with each component, Pardi et al. argue that star formation ceased in the halo within \( \lesssim 2 \) Gyr, in the thick disk within \( \gtrsim 8 \) Gyr, while in the thin disk star formation is still in progress. In this manner, the different Galaxy components experienced different star formation histories. While in the halo and thick disk the star formation is very intense at the beginning and then declines rapidly, star formation in the thin disk rises more slowly and declines more smoothly. The possible delays between the main star formation episodes in each phase are a natural consequence of the formation of density enhancements through the accumulation of diffuse gas. Such delays in main episodes of star formation lead to different enrichment paths (e.g. [O/Fe]) for each dynamically distinct Galaxy region. In this manner, there is considerable overlap between the stars belonging to the different components, at least concerning the chemical properties of the stars as is indicated by the observations (e.g. Wyse & Gilmore 1995). Similarly, it is argued that the presence of these distinct stellar populations can provide a natural explanation for part of the abundance spread among stars observed in the SNBH.
Chiappini et al. (1996) consider two main episodes of infall for the formation of the thick disk and thin disk. In their model, the formation of the thin disk occurs much later than that of the thick disk. These authors propose that most of the thin disk was formed by accretion of extra-galactic material. They argue that a revision is needed of the common picture in which the gas shed from the halo was the main contributor to the material out of which the thin disk formed. As has been argued by Wyse & Gilmore (1992), the angular momentum distribution of halo stars makes it unlikely that halo gas ended up anywhere near the local disk. This may imply that the chemical evolution of the SNBH started from scratch without any initial contribution from halo gas (see Pagel & Tautvaisienė 1995). The assumed decoupling of the rate of gas lost from the halo/thick-disk and that of gas falling onto the thin disk allows for much longer formation time scale of the thin disk (\(\sim 8\) Gyr) as compared to that of the halo/thick disk (\(\lesssim 1\) Gyr). In addition, a threshold in the star formation process was assumed. This threshold naturally produces a time gap in the SFR at the end of the thick disk phase as may be indicated by the observations (e.g. Gratton et al. 1996). The threshold limits the star formation in the halo and allows for the delay in the onset of star formation in the thin disk. This delay model is different from that of Pardi et al. (1995) in which simultaneous evolution of and star formation in all Galactic components occurs but at different evolutionary rates. Furthermore, the assumption of a threshold in the gas density prevents the growth of abundances in the outer regions of the exponential disk and results in: 1) the inner abundance gradients to steepen with Galactic age, and 2) the abundance gradients in the inner disk to be steeper than in the outer disk.

Raiteri et al. (1996) presented N-body hydrodynamical simulations to investigate the chemical evolution of the Galaxy. They assumed that the Galaxy formed by the collapse of a rotating cloud of gas and dark matter. Dissipative effects lead to the formation of a gaseous disk wherein stars are allowed to form in locally unstable regions. The gas radiates away efficiently the energy gained during the collapse and settles into a rotationally supported disk. The local enrichment of gas clouds can be followed in detail and the concept of a star formation threshold can be applied in a relatively simple manner. This kind of model enables the simultaneous study of the chemical enrichment and dynamical evolution of the Galaxy and can provide an adequate explanation for the scatter in the [O/Fe] vs. [Fe/H] relation discussed in Sect. 4.1 (see also Chap. 5). Such models seem very promising for detailed Galactic evolution studies in the near future.

Apart from the recent developments in modelling the chemical evolution of the Galaxy, there are several aspects which have not yet been (fully) accounted for in the models but which may play an important role:

- the impact of the accretion of satellite galaxies (e.g. Quinn et al. 1993; Ibata et al. 1994)
- radial mixing and gas flows
- variations in the IMF of stars at birth with Galactic age (see e.g. Sect. 4.3.4)
- a delay of several Gyr in the formation of the thin disk due to the energy deposited by supernovae (e.g. Burkert et al. 1992).
- the dynamical interaction between gas and stars
- metal-rich outflow of gas from the disk into the halo
- effects of sequential stellar enrichment and infall induced star formation (see Chap. 5)
- the effects of spiral arms on Galactic chemical evolution
- effects of stellar orbital diffusion (e.g. Wielen et al. 1996)
- simultaneous modelling of the widest range of observational constraints possible
- variations in the upper mass limit of stars ending as SNII (see Sect. 4.3.4)
- the inclusion of binary star evolution
- the inclusion of the dependence of stellar yields, lifetimes, and remnant masses on the abundances of individual elements such as He, O, and Fe (instead of the heavy-element integrated metallicity Z)
- the influence of the evolution of the bulge and central parts of the Galaxy

In general, we find that it is a rather complex task to obtain hard results concerning the chemical evolution of the Galactic disk due to the many observational and theoretical uncertainties still involved. In this chapter, we have shown the kind of problems encountered when modelling different aspects of Galactic chemical evolution. In particular, we have illustrated, for a wide range of observational constraints, the sensitivity of our results to the main parameters and assumptions inherent in such models. We hope that the results obtained here can be used in conjunction with other Galactic chemical evolution studies and may help to converge to a consistent and adequate model for the chemical evolution of the Galaxy as a whole.
Inhomogeneous chemical evolution of the Galactic disk: evidence for sequential stellar enrichment?

van den Hoek, L.B., and de Jong, T. 1

Abstract

We investigate the origin of the abundance variations observed among similarly aged F and G dwarfs in the local Galactic disk. We argue that orbital diffusion of stars in combination with radial abundance gradients is probably insufficient to explain these variations.

We show that episodic and local infall of metal-deficient gas can provide an adequate explanation for iron and oxygen variations as large as $\Delta[M/H] \sim 0.6$ dex among stars formed at a given age in the solar neighbourhood (SNBH). However, such models appear inconsistent with the observations because they: 1) result in current disk ISM abundances that are too high compared to the observations, 2) predict stellar abundance variations to increase with the lifetime of the disk, and 3) do not show substantial scatter in the $[\text{Fe}/\text{H}]$ vs. $[\text{O}/\text{H}]$ relation. Notwithstanding, our results do suggest that metal-deficient gas infall plays an important role in regulating the chemical evolution of the Galactic disk.

We demonstrate that sequential enrichment by successive stellar generations within individual gas clouds can account for substantial abundance variations as well. However, such models are inconsistent with the observations because they: 1) are unable to account for the full magnitude of the observed variations, in particular for $[\text{Fe}/\text{H}]$, 2) predict stellar abundance variations to decrease with the lifetime of the disk, and 3) result in current abundances far below the typical abundances observed in the local disk ISM.

We present arguments in support of combined infall of metal-deficient gas and sequential enrichment by successive stellar generations in the local Galactic disk ISM. We show that galactic chemical evolution models which take into account these processes simultaneously are consistent with both the observed abundance variations among similarly aged F and G dwarfs in the SNBH and the abundances observed in the local disk ISM. For reasonable choices of parameters, these models can reproduce $\Delta[M/H]$ for individual elements $M = \text{C, O, Fe, Mg, Al, Si}$ as well as the scatter observed in abundance-abundance relations like $[\text{O}/\text{Fe}]$. For the same models, the contribution of sequential stellar enrichment to the magnitude of the observed abundance variations can be as large as $\sim 50\%$.

We discuss the impact of sequential stellar enrichment and episodic infall of metal-deficient gas on the inhomogeneous chemical evolution of the Galactic disk.

5.1 Introduction

The chemical enrichment of the interstellar medium (ISM) by successive generations of stars is a key issue in understanding the chemical evolution of galaxies in general, and the formation history and abundance distributions of the stellar populations in our Galaxy in particular.

Observational studies related to the heavy element enrichment of the local Galactic disk have long shown that stars of similar age exhibit large abundance variations (e.g. Mayor 1976; Twarog 1980a; Twarog & Wheeler 1982; Carlberg et al. 1985; Gilmore 1989; Klochkova et al. 1989; Schuster & Nissen 1989; Meusinger et al. 1991). Recently, Edvardsson et al. (1993a) presented accurate abundance data for nearly 200 F and G main-sequence dwarfs in the solar neighbourhood (SNBH). Their spectroscopic data, analysed with up-to-date input physics, confirms abundance variations as large as $\sim 0.6$ dex in $\Delta[M/H]$ (where $M=\text{Fe,O,Mg,Al,Si}$) among similarly aged stars.

1Sects. 5.2 and 5.3 as well as Appendix B were added to the published version of this paper.
In contrast to previous understanding, these variations are much in excess of experimental uncertainties and demonstrate that the abundance spread for stars born at roughly the same galactocentric distance is similar in magnitude to the overall increase in metallicity during the lifetime of the disk.

Additional support for the existence of large abundance inhomogeneities in the Galactic disk has been provided by studies of stars in open clusters (e.g. Nissen 1988; Boesgaard 1989; Lambert 1989; García-Lopez et al. 1993; Friel & Janes 1993; Carraro & Chiosi 1994) and B stars in star forming regions in the SNBH (e.g. Gies & Lambert 1992; Cunha & Lambert 1992). These studies show that the concept of a well-defined tight age-metallicity relation (AMR) for the Galactic disk ISM is unfounded (Edmunds 1993) and that the chemical enrichment of the disk has been inhomogeneous on time scales as short as $\sim 10^8$–$10^9$ yr. Similar studies of objects in the Magellanic clouds (e.g. Cohen 1982; Da Costa 1991; Olsewski et al. 1991) and dwarf galaxies (Pilyugin 1992; Kunth et al. 1994; Thuan et al. 1995) suggest that inhomogeneous chemical evolution is a common phenomenon in nearby galaxies as well.

The origin of the abundance variations observed in the local Galactic disk is investigated in this paper. Clearly, large abundance variations in the ISM on time scales at least an order of magnitude shorter than the lifetime of the disk cannot be reproduced by simple galactic evolution models incorporating monotonously increasing age-metallicity relations (AMR). In the past few years, various ideas have been put forward as possible explanations for the intrinsic abundance variations among similarly aged stars:

- stellar orbital diffusion in combination with radial abundance gradients in the Galactic disk (e.g. Francois & Matteucci 1993; Wielen, Fuchs, & Dettbarn 1996);
- sequential enrichment by successive stellar generations (e.g. Edmunds 1975; Olive & Schramm 1982; Gilmore 1989; Gilmore & Wyse 1991; Cunha & Lambert 1992; Roy and Kunth 1995);
- local infall of metal-poor gas (e.g. Edvardsson et al. 1993a; Roy & Kunth 1995; Pilyugin & Edmunds 1995a);
- cloud motions in the ISM (Bateman & Larson 1993);
- inefficient mixing in the disk ISM: isolated chemical evolution of individual parcels of interstellar gas during considerable fractions of the lifetime of the disk (Lennon et al. 1990; Wilmes & Köppen 1995);
- major galaxy merger events resulting in multiple stellar populations in the Galactic disk (Strobel 1991; Pilyugin & Edmunds 1995b);
- chemical fractionization processes such as grain formation (e.g. Henning & Gürtler 1986) and/or element diffusion (e.g. Bahcall & Pinsonneault 1995) so that measured abundances do not reflect initial stellar abundances.

As discussed by Edvardsson et al. (1993a) stellar orbital diffusion is probably inadequate as main explanation for the observed abundance variations. However, recently Wielen et al. (1996) claimed that stars can be born at galactocentric distances very different from those derived using their present-day orbits. In this case, a major fraction of the observed abundance scatter could be due to stellar orbital diffusion. Since it appears unlikely that diffusion can explain the observed abundance variations for all stars in the Edvardsson et al. sample (see below) as well as those observed among young stars (e.g. present in star forming regions) other processes are probably important as well.

Based on the assumption of short mixing time scales of $\sim 10^7$ yr in the local disk ISM, we argue that the underlying physical mechanisms causing the observed abundance inhomogeneities and those initiating star formation in the disk ISM are the same. No observational support exists for chemical fractionization in low mass F and G main-sequence stars. Therefore, the scatter in stellar metallicities probably reflects the original inhomogeneities in the interstellar gas (e.g. Gilmore 1989). We here restrict ourselves mainly to the processes of sequential stellar enrichment and episodic infall of gas onto the Galactic disk as possible explanation for the observed stellar abundance variations. Since these processes are observed to operate simultaneously in the SNBH (see below), it is important to investigate their combined effect on the chemical evolution of the Galactic disk.

The process of star formation initiated by stars formed during a preceding star formation event nearby, is known as sequential star formation. Support for sequential star formation in the SNBH is provided by observations of spatially separated subgroups of OB stars that appear aligned in a sequence of ages in many OB associations (e.g. Blaauw 1991) and by observations of stars forming at the interfaces of HII regions and their surrounding molecular clouds (e.g. Genzel & Stutzki 1989; Pismis 1990; Goldsmith 1995). Sequential star formation may be induced by the blast waves of nearby supernova explosions compressing the ambient
5.2 Observations

ISM (e.g. Ogelman & Maran 1976) and/or by propagating ionization and shock fronts from an OB association causing the gravitational collapse of a nearby molecular cloud (e.g. Elmegreen & Lada 1977). In either case, efficient self-enrichment through mixing of enriched material by successive generations of massive stars is expected.

On the other hand, stellar abundance variations can be attributed to infall of relatively unprocessed gas and star formation within the accreted material before efficient mixing wipes out any local chemical inhomogeneities (e.g. Edvardsson et al. 1993a; Pilyugin & Edmunds 1995a). Observational support for star formation in the SNBH initiated by infall of high velocity clouds, has recently been presented by Lépine & Duvert (1994). These authors claim that episodic gas infall is a dominant process in the local disk ISM and is associated with all prominent star forming molecular clouds seen near the Sun (see Sect. 5.5.2). Furthermore, ongoing gas infall has been emphasized by models of dissipative protogalactic collapse (Larson 1969, 1976) and on the basis of time scale arguments of gas consumption in the local disk (Larson et al. 1980; Kennicutt 1983).

In this paper, we present a chemical evolution model for a star forming gas cloud which incorporates stellar enrichment and mixing processes (including infall) and which allows for temporal and/or spatial inhomogeneities in the ISM. This study differs from previous work (e.g. Pilyugin & Edmunds 1995a) in that we investigate in detail the combined effect of metal deficient gas infall and sequential stellar enrichment by successive stellar generations on the chemical evolution of multiple gas clouds in the Galactic disk. In particular, each gas cloud is allowed to follow its individual star formation, mixing, and infall history, as is suggested by the observations. With this model, we fit the stellar abundance variations and current local ISM abundances of C, O, Fe, Mg, Al, and Si observed in the SNBH, the present gas-to-total mass-ratio, and actual star formation and supernova rates. We note that previous investigations were restricted to abundance variations in oxygen and iron only.

We will show that models taking into account the above processes simultaneously are in good agreement with the observations and provide an adequate explanation for the stellar abundance variations with respect to the mean abundances observed in the local disk. Furthermore, we will argue that the contribution of sequential stellar enrichment to the magnitude of the observed stellar abundance variations can be as large as ∼50%, i.e. much larger than suggested by previous investigations (e.g. Pilyugin & Edmunds 1995a; Wilmes and Köppen 1995). Corresponding theoretical age and abundance-distributions related to the G-dwarf problem will be discussed in a separate paper.

The paper is organized as follows. In Sect. 5.2, we briefly review observations related to the inhomogeneous heavy element enrichment of the local Galactic disk ISM. In Sect. 5.3, we describe characteristics of the inhomogeneous chemical evolution model proposed for the Galactic disk (model equations and details are given in the Appendix to the electronic version of this paper). In Sect. 5.4, we present model results for episodic infall of metal-deficient gas and sequential stellar enrichment, and examine which of these mechanisms can account satisfactorily for the observations. In Sect. 5.5, we discuss these results in the more general context of the chemical evolution of the Galactic disk and adduce both observational arguments in support of sequential star formation and metal deficient gas infall in the local disk ISM.

5.2 Inhomogeneous chemical evolution of the local Galactic disk: observations

5.2.1 Main-sequence F and G dwarfs

We concentrate on the abundance data of nearly 200 main-sequence field F and G dwarfs with actual distances ≲70 pc from the Sun as recently presented by Edvardsson et al. (1993a; hereafter EDV). This sample provides the largest sample of stars available to date for studies related to the chemical evolution of the local disk. Fig. 5.1 displays all F and G dwarfs for which both [O/H] and [Fe/H] abundance-ratios have been determined by EDV.

Large abundance variations of ∼0.9 dex in [Fe/H] and ∼0.7 dex in [O/H] among stars of a given age are seen to be present (abundance variations in e.g. Mg, Al, and Si resemble those in [Fe/H]). At intermediate stellar ages, these variations are no doubt significant since typical observational errors are ∼0.1 dex both in [M/H] and log(Age) (see EDV). At ages in excess of ∼15 Gyr and less than ∼2 Gyr, the data probably are undersampled (see EDV). Note that the sample is biased against old, high-metallicity stars through the minimum $T_{\text{eff}}$ limit assumed by EDV.
The observed spread in [Fe/H] is tightly correlated with that in [O/H]. This suggests that different nucleo-synthesis sites, which contribute different elements to the initial abundances in stars, mix their products together well. Furthermore, this suggests that stellar abundance variations for different elements are due to the same process. Current observations support the idea that the magnitude of the stellar abundance variations has remained constant over the lifetime of the disk (see also Mayor 1976; Twarog 1980a; Meusinger et al. 1991; Carraro & Chiosi 1994). In the following, we will assume that these abundance variations are randomly distributed within the metallicity range observed at a given stellar age. This is particularly important when considering possible explanations for the observed stellar abundance variations in detail (see Sect. 5.4; cf. Wielen et al. 1996).

Figure 5.1 Observed iron, oxygen, and magnesium abundance ratios for main-sequence F and G dwarfs in the solar neighbourhood (data from Edvardsson et al. 1993a). Open circles represent stars with mean stellar galactocentric distances at birth within 0.5 kpc from the Sun ($R_{⊙} = 8.4$ kpc). Full dots indicate stars with average distances within $\sim 2$ kpc from the Sun. Typical errors are indicated at the bottom right of each panel. Note that the abundances of the most metal-poor disk stars included in this sample resemble those of metal-rich halo dwarfs and giants (e.g. Bessell et al. 1991; Gratton & Sneden 1991; Nissen et al. 1994). We assumed solar abundance ratios by number of $10 \log (O/\text{H})_{⊙} = -3.13$, $10 \log (\text{Fe}/\text{H})_{⊙} = -4.51$, and $10 \log (\text{Mg}/\text{H})_{⊙} = -4.42$ and a hydrogen mass fraction in the Sun of 0.68 (see Anders & Grevesse 1989; Grevesse & Noels 1993). Top panels: Distributions of [Fe/H] (left) and [O/H] abundance ratios vs. galactic age. Bottom panels: [Fe/H] vs. [O/H] (left) and [Mg/H] vs. [O/H].

Ages, abundances, and kinematical properties of the dwarfs belonging to the EDV sample are consistent with earlier investigations (e.g. Twarog 1980a; Carlberg et al. 1985; Meusinger et al. 1991). An extensive discussion of the possible sources of errors in the abundance and age analysis as well as several consistency checks can be found in Edvardsson et al. (1993a,b). Errors due to data reduction uncertainties are estimated
to lead to errors of at most 0.05 to 0.1 dex in abundance ratios [M/Fe] as well as in [Fe/H] (see EDV). These errors are not expected to vary in a systematic manner with the derived stellar abundances and corresponding corrections will probably not reduce the observed variations. Edvardsson et al. estimated errors in relative ages of ~25% for stars with similar abundances (absolute errors may be considerably larger). Thus, ages of stars as old as the Sun are estimated to be accurate within ~1–2 Gyr. We conclude that errors in the abundances and ages of the sample stars are unlikely to account for the observed abundance variations, at least for stars with intermediate ages of ~5 Gyr.

Knowledge of the formation sites of the sample stars is important to decide whether or not orbital diffusion in combination with radial abundance gradients in the Galactic disk can provide an adequate explanation for the observed abundance variations. Galactocentric distances of the sample stars at birth were obtained using stellar orbits reconstructed from their present-day galactocentric distances, proper motions, and radial velocities, and using both theoretical and empirical models for the Galactic potential as discussed by EDV. Accordingly, nearly 85% of the sample stars were found to have mean galactocentric distances $R_m$ at birth within 1 kpc from the Sun (assuming $R_\odot \sim 8.4$ kpc at present). However, predictions of the diffusion of stellar orbits in space, based on the observed relation between velocity dispersion and age for nearby stars, suggest that many stars may have been formed at galactocentric distances as large as ~4 kpc from where they are nowadays observed in the SNBH (Wielen et al. 1996). In either case, these nearby stars trace the evolution of the Galactic disk ISM over a much wider range in galactocentric distance than they are observed.

As an independent test to examine whether stellar orbital diffusion can be the main cause for the observed abundance variations, we translated stellar abundance deviations $\Delta [M/H]$ from the mean abundances of similar aged stars born at $\sim R_\odot$ into galactocentric distance differences $R_m - R_\odot$. This was done independently for [Fe/H] and [O/H] abundance ratios assuming present-day local radial abundance gradients of $-0.07 \pm 0.015$ dex kpc$^{-1}$ in [O/H] (e.g. Shaver et al. 1983; Grenon 1987; see also Wilson & Matteucci 1992) and $-0.1$ dex kpc$^{-1}$ in [Fe/H] (see e.g. EDV). Clearly, distances $R_m$ based on oxygen and iron abundances are expected to be similar (e.g. $\Delta R_{m,Fe}^O \lesssim 1$ kpc) when orbital diffusion is important for the observed stellar abundance variations.

In this manner, we find that $\Delta R_{m,Fe}^O \gtrsim 1$ kpc for 47 stars in the EDV sample (i.e. ~56%). Similarly, we find $\Delta R_{m,Fe}^O \gtrsim 2$, 3, and 4 kpc, for ~31, 14, and 8% of the sample stars, respectively. We note that the derived values of $\Delta R_{m,Fe}^O$ are insensitive to the stellar age but depend on the assumed radial abundance gradients as well as on the mean [M/H] vs. age relations adopted for stars born at $R_\odot$. Although a detailed investigation of the uncertainties involved is beyond the scope of this paper (e.g. the variation of abundance gradients with disk age; cf. Grenon 1987), we estimate that $\Delta R_{m,Fe}^O \lesssim 0.8$ (1.5) kpc for typical errors of 0.05 (0.1) dex in both [Fe/H] and [O/H] for most of the sample stars (assuming a gaussian error distribution). This suggests that a substantial part of the observed abundance variations is difficult to explain by stellar orbital diffusion only. Also, Edvardsson et al. argued that the magnitude of the observed variations will reduce to $\Delta [M/H] \sim 0.3$ dex if one accounts properly for systematic errors and possible effects of stellar orbital diffusion. However, a reduction of the abundance spread among field dwarfs to $\Delta [Fe/H] \sim 0.3$ dex seems contradicted by the observed variations of $\Delta [Fe/H] \sim 0.5 \pm 0.1$ dex among similarly aged open clusters after correcting for radial abundance gradients across the Galactic plane (Carraro & Chiosi 1994). Apart from this, such a reduction appears inconsistent with the observed abundance spread of [O/H] $\gtrsim 0.4$ dex among B stars at a given galactocentric radius between 7 and 16 kpc in the disk (Gehren et al. 1985; Kaufer et al. 1994) and with the large abundance variations of ~0.7 dex observed among young open clusters over a distance scale of only ~1 kpc at a galactocentric radius of ~13 kpc (Rolleston et al. 1994).

What fraction of the current disk stellar population actually formed in the Galactic halo depends on the detailed dynamical evolution of the disk which is not well known (e.g. Pagel & Tautvaisiene 1995). However, most stars in the EDV sample have derived maximum distances from the Galactic plane at birth of $h_{\text{max}} < 0.5$ kpc. This largely excludes halo stars from the sample and further implies that abundance gradients perpendicular to the Galactic plane are inadequate as explanation for the observed abundance variations (e.g. Carney et al. 1990).

From these arguments, we conclude that orbital diffusion of stars from elsewhere in the Galactic disk is probably insufficient as explanation for the observed variations in [Fe/H] and [O/H] among F and G dwarfs in the SNBH. This conclusion is consistent with the finding that abundance variations for subsamples of stars restricted to be born within 1 and 0.5 kpc from the Sun, respectively, are similar to those for the complete sample (see EDV; cf. Fig. 5.1). Therefore, we believe that differential chemical evolution and mixing of interstellar gas must be an important cause for the large stellar abundance variations observed in the SNBH as well. The abundances of the Sun and of open clusters in the Galactic disk fit well into this picture, as is argued below.
5.2.2 Chemical evolution of the solar neighbourhood

Detailed comparison of abundances within local Hii regions and the Sun have shown that oxygen (among other heavy elements) is underabundant in the Hii regions by about 0.15–0.3 dex (e.g. Shaver et al. 1983; Peimbert 1987; Baldwin et al. 1991; Osterbrock, Tran & Veilleux 1992). Also, CNO-abundances of Hii-regions and B main-sequence stars in the Orion nebula were found smaller than corresponding abundances in the Sun (Cunha & Lambert 1992; Gies & Lambert 1992). The remarkable result that the Sun is metal-rich by \( \sim 0.15–0.2 \) dex in \([O/H]\) compared to its surroundings is also supported by observations of B stars in nearby associations and young clusters (Fitzsimmons et al. 1990), diffuse interstellar clouds (e.g. York et al. 1983), and disk planetary nebulae (de Freitas Pacheco 1993; Peimbert et al. 1993). Although abundance determinations in the SNBH may be biased towards regions associated with infall of metal-poor gas or suffer from heavy element depletion by dust, the existence of many metal-poor regions in the SNBH would be difficult to reconcile with efficient mixing in the local disk ISM (e.g. Roy & Kunth 1995).

The above observations are consistent with the Edvardsson et al. data which suggest that the Sun is metal-rich by \( 0.2–0.25 \) dex in \([O/H]\) and by \( 0.25–0.3 \) dex in \([Fe/H]\) compared to the mean abundances of stars which formed in the SNBH \( \sim 4.5 \) Gyr ago (cf. Fig. 5.1). These observations support the idea that the Sun is metal-rich for its age (see also Steigman 1993) and that abundance inhomogeneities in the local disk ISM did exist. The fact that the Sun is metal-rich by a factor of \( \sim 1.5–2 \) compared to nearby regions currently experiencing star formation may be explained by self-enrichment of the gas cloud out of which the Sun was born (e.g. Gies & Lambert 1992; Peimbert et al. 1993). Alternatively, orbital diffusion of the Sun may play an important role (Wielen et al. 1996). We will discuss arguments in support of the former possibility in Sect. 5.2.

5.2.3 Open clusters

Variations in \([Fe/H]\) among disk open clusters of a given age are known to be larger than any possible trend of \([Fe/H]\) with age (e.g. Nissen 1988; Boesgaard 1989; Garcia-Lopez et al. 1993; Friel and Janes 1993; Dufton et al. 1994). Recently, abundance variations of \( \sim 0.5 \pm 0.1 \) dex in \([Fe/H]\) among clusters of a given age after correcting for the radial abundance gradient across the Galactic plane have been reported by Carraro & Chiosi (1994). The observed abundance variations among open clusters appear somewhat smaller (i.e. by \( \sim 0.2–0.3 \) dex in \([Fe/H]\)) than those among field F and G dwarfs in the EDV sample. However, the magnitude of the observed variations suggests that the processes responsible for the abundance inhomogeneities among field stars in the SNBH and among open clusters widespread throughout the Galactic disk may well be the same.

The lack of a tight age-metallicity relationship for open clusters in the Galactic disk suggests that the chemical enrichment of the disk ISM has been inhomogeneous on time scales less than \( \sim 10^8–10^9 \) yr, consistent with the abundance variations observed for intermediate age F and G dwarfs discussed above.

5.3 Model characteristics and assumptions

In the previous section, we have argued that differential chemical evolution and mixing of interstellar gas probably provides the main explanation for the large abundance variations observed among similarly aged stars in the SNBH. In this case, abundance inhomogeneities in the global disk ISM may result from local mixing of metal-deficient material (e.g. infall) and/or local mixing of metal-enhanced material (e.g. stellar enrichment). When star formation is initiated within the mixed material before any abundance fluctuations are wiped out, these inhomogeneities can be recorded by long-living stars.

Efficient mixing by stellar winds and supernova explosions is generally accepted to occur within \( \sim 10^7–10^8 \) yr (e.g. Edmunds 1975; Ciotti et al. 1991; Roy & Kunth 1995). This suggests that the processes responsible for the onset of star formation and those causing substantial abundance inhomogeneities in the disk ISM are the same. We consider this as a strong argument in favour of sequential star formation and/or infall induced star formation as the main processes responsible for the observed abundance variations (ample observational support for the occurrence of these processes in the local disk ISM are briefly discussed in Sect. 5.5.2). Obviously, the quantitative effect of these processes on stellar abundance variations, relative to the mean abundances in the local ISM, depends on the detailed chemical evolution of the disk ISM.

5.3.1 Model description

We present a model for the inhomogeneous chemical evolution of a star forming gas cloud. The basis for this model forms the individual star formation history and chemical evolution of multiple subclouds that
mutually exchange interstellar material. We here restrict ourselves to a brief outline of the basic assumptions and model characteristics. A more detailed description of the equations and input physics used is given in the (Appendix to the) electronic version of this paper.

We start from a homogeneous gas cloud with total mass $M_{cl}$. At any time the cloud is subdivided into $N_{scl}$ star forming, active subclouds (with corresponding masses $M_{i,scl}$) and a quiescent, inactive cloud part (with mass $M_{qcl}$) not experiencing star formation. Each subcloud is allowed to follow its individual star formation, infall, and mixing history. Infall of matter is considered by allowing episodic mixing of metal-deficient material to each subcloud separately.

![Figure 5.2](image)

**Figure 5.2** Schematic model for the inhomogeneous chemical evolution of a star forming cloud: evolutionary sequence of star formation, enrichment and mixing processes. Shown is a star forming cloud region. Each of the processes indicated in this region may occur in other regions of the cloud as well. Symbols have the following meaning: a subclouds indicated by hatched areas, b star formation indicated by asterisks, c stellar enrichment shown as shaded areas enclosing white asterisks, d subcloud core dispersal indicated as blanked out area surrounding stars, e break up of entire subcloud and initiation of star formation in a nearby subcloud, f arrow indicates stars entering a subcloud from elsewhere. Each of the processes indicated may occur frequently during the cloud evolution time $t_{ev}$.

The adopted set of processes that modify the distribution of gas and stars within a star forming region of a molecular cloud are illustrated in Fig. 5.2. Different subfigures refer to the following processes:

(a) subcloud formation from the inactive cloud ISM (and/or from infalling material);
(b) conversion of gas into stars (star formation at distinct subcloud cores);
(c) ejection of material by stars to their immediate surroundings;
(d) mixing of dispersed core material with subcloud after star formation event;
(e) break up of entire subcloud, mixing with inactive cloud ISM, and induced star formation;
(f) enrichment of subcloud by stars not formed within the subcloud.

In our model, the inhomogeneous chemical evolution of a star forming gas cloud, consisting of many subclouds, is determined by the combined effect of the above processes. During a time-interval $\Delta t$, these processes may occur simultaneously within each subcloud. In this manner, the initial abundances of a newly formed stellar generation are determined by: 1) the enrichment of the subcloud by preceding stellar generations, and 2) the mixing history of the subcloud with the ambient ISM.
In brief, the adopted evolution scenario is as follows. Subcloud formation (Fig. 5.2a) is assumed to occur either from the inactive cloud ISM and/or from infalling material (details related to the infall model will be given in Sect. 5.4.3). During the lifetime \( t_{ev} \) of the entire system, a total number of \( N_{sf} \) star formation events is assumed to occur. Each star formation event is assumed to take place in an active subcloud (Fig. 5.2b). Each subcloud is allowed to experience numerous star formation events and/or to remain inactive during a substantial part of its lifetime. Consequently, each subcloud can be enriched by one or multiple star formation events dictating its chemical evolution (Fig. 5.2c). When the active subcloud core is dispersed by stellar winds and/or supernova shocks, part of the enriched matter is assumed to mix homogeneously with the surrounding subcloud material (Fig. 5.2d). The remaining part is assumed to mix homogeneously either to a nearby subcloud hosting the next star formation event or to the ambient inactive cloud part (Fig. 5.2e). No mass-exchange is assumed between the subcloud and the ambient inactive cloud ISM during the time interval in which two or more star formation events occur within the same subcloud. In addition, subcloud material may be enriched by stars formed outside the subcloud. In this case, stars from elsewhere in the inactive cloud occasionally enter the subcloud region and enrich the subcloud by means of their ejecta (Fig. 5.2f). Stellar enrichment by old stellar generations is assumed to proceed continuously with time but is considered in detail only at specific evolution times corresponding to the occurrence of any of the discontinuous processes referred to in Fig. 5.2.

We define the subcloud core dispersal time \( \Delta t_{disp} \) as the time between onset of star formation within a subcloud core region and the complete dispersal of this region. This time interval constraints the mass of the most massive star that is able to enrich the subcloud core material before the core ultimately breaks up. Before dispersal of an entire subcloud, the subcloud will be enriched by the stellar populations it is hosting. After subcloud dispersal (i.e. after a typical mixing time scale \( \Delta t_{mix} \)), stars and gas belonging to the subcloud are assumed to mix instantaneously and homogeneously with the inactive cloud ISM. Subsequently, different cloud fragments may combine to form new subclouds wherein star formation occurs as soon as the critical conditions for star formation are met. The mixing history of each subcloud determines the inhomogeneous chemical evolution of the inactive cloud part as well as that of nearby subclouds. For simplicity, we do not consider partial mixing of subcloud material to the inactive cloud.

### 5.3.2 Outline of model computations

We perform Monte-Carlo simulations of the inhomogeneous chemical evolution of a star forming gas cloud. The continuous process of formation and break up of subclouds and of the formation and dispersion of subcloud core regions associated with star formation, are followed as outlined in the previous section. During the evolution of the cloud, we keep track of the total mass contained in gas and stars as well as the stellar and interstellar abundances of H, He, C, O, Fe, Mg, Al, and Si, both within each subcloud and the inactive cloud part. No instantaneous recycling is assumed, i.e. metallicity dependent stellar lifetimes are taken into account.

### 5.3.3 Model input parameters

Model input parameters for the reference model are listed in Table 5.1. We distinguish parameters related to: 1) the entire cloud and inactive cloud part, 2) active subcloud regions, and 3) individual star formation events:

- **Cloud and inactive cloud part**: The initial cloud mass \( M_{cl} \) is treated as a mass scaling parameter (i.e. resulting abundances are not altered for different values of \( M_{cl} \)). We here adopted \( M_{cl} = 5 \times 10^{10} \ M_{\odot} \) similar to that of the Galactic disk (e.g. Binney & Tremaine 1987). We assume a cloud evolution time \( t_{ev} = 14 \) Gyr. This is comparable to the age of the Galaxy as derived from the age of the oldest globular clusters, i.e. 14±3 Gyr (e.g. Buonanno et al. 1989). In our model, the impact of processes causing stellar abundance variations does not depend on the specific age of the Galactic disk assumed.

  We consider a total number of star formation events during the cloud evolution time of typically \( N_{sf} = 100 \). In practice, \( N_{sf} \) is limited only by the preferred model run time, i.e. 1-2 hours on a HP Apollo 715 machine. The total number of subclouds \( N_{scl} \) is determined by the number of star formation events within each subcloud. For the reference model, we assume a maximum number of star formation events within one subcloud \( N_{sf}^{max} = 1 \) so that \( N_{scl} = N_{sf} \). Cloud initial abundances \( X_{scl} \) are as given in Table 5.1. Initially, the cloud is considered homogeneous, metal-free, and void of stars.

- **Active subclouds**: In case of the reference model, we force subclouds to form at regular intervals of \( t_{ev} / N_{scl} = 1.4 \times 10^8 \) yr. We assume the subcloud mass \( M_{scl} \) directly proportional to the entire cloud gas-to-total mass-ratio \( \mu \) at time of subcloud formation \( t_{scl} \). This implies more massive subclouds to form at relatively...
## 5.3 Model characteristics and assumptions

### Table 5.1 List of input parameters (values listed for reference model)

<table>
<thead>
<tr>
<th>Cloud and inactive cloud part</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{cl}}$</td>
<td>$5 \times 10^9 , M_\odot$</td>
</tr>
<tr>
<td>$t_{\text{ev}}$</td>
<td>14 Gyr</td>
</tr>
<tr>
<td>$N_{\text{sf}}$</td>
<td>100</td>
</tr>
<tr>
<td>$N_{\text{scl}}$</td>
<td>100</td>
</tr>
<tr>
<td>$X_{\text{scl}(0)}$</td>
<td>H=0.76</td>
</tr>
</tbody>
</table>

**For each subcloud $i$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{scl}}$</td>
<td>$1.4 \times 10^5 , \text{yr}$</td>
<td>time at which subcloud is formed, i.e. each $t_{\text{ev}} \div N_{\text{scl}} = 1.4 \times 10^5 , \text{yr}$</td>
</tr>
<tr>
<td>$M_{\text{scl}}$</td>
<td>$\exp \left( t_{\text{scl}} \right)$</td>
<td>exponentially decaying subcloud mass at time of formation $(M_{\text{scl}}(t = 0) = 6 \times 10^9 , M_\odot)$</td>
</tr>
<tr>
<td>$N_{\text{sf}}^{\text{max}}$</td>
<td>1</td>
<td>maximum number of SF events within one subcloud</td>
</tr>
<tr>
<td>$\Delta t_{\text{mix}}$</td>
<td>$\Delta t_{\text{disp}}$</td>
<td>mixing time of entire subcloud</td>
</tr>
</tbody>
</table>

**For each star formation event $j$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{sf}}$</td>
<td>$t_{\text{scl}}$</td>
<td>evolution time at which SF-event $j$ occurs</td>
</tr>
<tr>
<td>$\Delta t_{\text{disp}}$</td>
<td>$10^7 , \text{yr}$</td>
<td>subcloud core dispersal time</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.50</td>
<td>subcloud star formation efficiency $(1)$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0</td>
<td>efficiency of sequential enrichment $(2)$</td>
</tr>
</tbody>
</table>

**Notes:** (1) $\epsilon^{\text{sf}}$ is the mass fraction of the subcloud converted into stars during dispersal time $\Delta t_{\text{disp}}$; (2) $\lambda^{\text{disp}}$ refers to the amount of stellar material returned to the subcloud core hosting the next star formation event.

high gas fractions $\mu(t)$. Assuming a constant star formation efficiency, this results in an exponential decrease of subcloud mass with disk age $t$ (e.g. Clayton 1985):

$$M_{\text{scl}} = M_{\text{scl}}(0) \exp(-t/t_{\text{dec}}) \quad (5.1)$$

where $M_{\text{scl}}(0)$ is the mass of a subcloud formed at $t=0$ (which may vary between different models) and $t_{\text{dec}}$ a characteristic time scale at which the mass of subsequent subclouds formed is assumed to decay (identical for all models). The assumption of a star formation rate (SFR) directly proportional to the subcloud formation rate is not essential for the results discussed here.

The decay time scale $t_{\text{dec}}$ is constrained observationally by the ratio of the average past to present SFR in the Galactic disk ($\sim 3$--7; e.g. Mayor & Martinet 1977; Dopita 1990). We here assume an exponentially decaying SFR with $t_{\text{dec}} = 6 \, \text{Gyr}$. As will be shown in Sect. 5.4.1, this SFR can account simultaneously for the actual gas-to-total mass-ratio in the disk of $\mu_1 \sim 0.05$--0.2 (Kulkarni & Heiles 1987; Binney & Tremaine 1987; see also Basu & Rana 1992), the smooth increase in the global AMR for elements such as O and Fe, and the magnitude of the current SFR in the Galactic disk (i.e. $\sim 3.5 \, M_\odot \, \text{yr}^{-1}$; e.g. Dopita 1987). In contrast, constant SFR models are inconsistent with these observations (Twarog 1980a; see also Clayton 1985).

The time between the formation and complete mixing of a subcloud to the inactive cloud part is defined as $\Delta t_{\text{mix}}$. This time scale has been considered to allow for the individual chemical evolution of a subcloud isolated from the inactive ISM (see below).

- **Individual star formation events:** We assume the onset of star formation within each subcloud to coincide with the formation of the subcloud itself in case of the reference model. This results in a grid of regularly spaced star formation times $t_{\text{sf}} = t_{\text{scl}}$.

We define the core dispersal time $\Delta t_{\text{disp}}$ as the time between onset of star formation $t_{\text{sf}}$ within a subcloud core and the moment star formation ends due to the actual break up of this core. Observational estimates of this time scale are generally $\lesssim 10^7 \, \text{yr}$ (e.g. Garmany et al. 1982; Leisawitz 1985; Genzel & Stutzki 1989; Rizzo & Bajaja 1994; Haikala 1995). For the reference model, we assume the entire subcloud to break up at time of dispersal of the star forming subcloud core, i.e. $\Delta t_{\text{mix}} = \Delta t_{\text{disp}}$.

The star formation efficiency $\epsilon^{\text{sf}}$ is defined as the amount of subcloud matter $\Delta M_{\text{scl}}$ turned into stars during star formation event $j$. In fact, the star formation efficiency determines the amount of material to which the stellar ejecta of a previous stellar generation are mixed within a given star forming cloud. Observational estimates for $\epsilon$ in molecular clouds in the Galactic disk span a wide range: between a few tenths of a percent to $\sim 50\%$ (e.g. Wilking & Lada 1983). We will discuss the values assumed for $\epsilon^{\text{sf}}$ in
Sect. 5.4. Sequential stellar enrichment is taken into account by assuming that a fraction $\lambda^j$ of enriched material associated with star formation event $j$ is mixed homogeneously to the subcloud hosting the next star formation event. Subcloud material not converted into stars is mixed to the inactive cloud part after complete dispersal of the subcloud. The relative importance of these model parameters on the resulting stellar abundance variations will be discussed in Sect 5.4.2.

Table 5.2 IMF related parameters and stellar enrichment

| $\gamma$ | -2.35 | slope of power-law IMF $m^\gamma$ |
| $m_{min}, m_{\alpha}$ | 0.1, 60 $M_\odot$ | stellar mass limits at birth |
| $m_{SNII}^\alpha, m_{SNII}^\beta$ | 8, 30 $M_\odot$ | progenitor mass range for SNII and SNIb/c |
| $m_{SNIa}^\alpha, m_{SNIa}^\beta$ | 2.5, 8 $M_\odot$ | progenitor mass range for SNIa |
| $\phi_{SNIa}$ | 0.005 | fraction of progenitors ending as SNIa |
| $\phi_{SNIb/c}$ | 0.33 | fraction of SNII progenitors ending as SNIb/c |

5.3.4 Stellar evolution data

We follow the stellar enrichment of the star forming cloud in terms of the characteristic element contributions of Asymptotic Giant Branch (AGB) stars, SNII, SNIa, and SNIb/c. This treatment is based on the specific abundance patterns observed within the ejecta of each of these stellar groups (e.g. Trimble 1991; Groenewegen & de Jong 1992; van den Hoek et al. 1996). We take into account metallicity dependent stellar element yields, remnant masses, and ages, while assuming the stellar ejecta to be returned at the end of the lifetime of the star (see e.g. Maeder 1992; Schaller et al. 1992). The respective time delays in enrichment by SNIa and SNII are accounted for in detail. A more detailed description of the combined set is given elsewhere (e.g. van den Hoek et al. 1996; see also electronic version of this paper).

For AGB stars (initial mass $m \lesssim 8 M_\odot$), we adopt the metallicity dependent yields presented by Groenewegen & de Jong (1992). These yields are based on a synthetic evolution model for AGB stars and are successful in explaining the observed abundances in carbon stars and planetary nebulae in the Galactic disk (Groenewegen et al. 1995; van den Hoek & Groenewegen 1996). For Type-II SNe, we use the explosive nucleo-synthesis yields (independent of initial metallicity) described in detail by Hashimoto et al. (1993) and Thielemann et al. (1993) for stars with $8 \lesssim m[M_\odot] \lesssim 60$. The 20 $M_\odot$ model of this set accounts well for the observed abundances in SN1987A (Nomoto et al. 1991). Explosive nucleo-synthesis yields for Type-Ia SNe are adopted from Nomoto et al. (1984; model W7 for SNIa at $Z = Z_\odot$ and $Z = 0.0$ of the accreted material; see also Yamaoka 1993) and for SNIb/c from Woosley et al. (1995). According to these yields, typical amounts of iron produced are $\sim 0.08 M_\odot$ for SNIa, $\sim 0.8 M_\odot$ for SNII, and $\sim 0.1 M_\odot$ for SNIb/c.

The adopted yields for SNIa, b/c are relatively uncertain due to unknown details of the progenitor history and the explosion mechanism (either binary or single star evolution; see e.g. Smecker-Hane & Wyse 1992; Woosley et al. 1993). However, we do not believe that these uncertainties are relevant for the qualitative results obtained in this paper (cf. Sect. 5.5.2).

Metallicity dependent stellar yields for stars during their wind (i.e. pre-SN) phase have been adopted from Maeder (1992, 1993), to whom we refer the reader also for a definition of the stellar element yields as used in the Appendix. For stars with $m \gtrsim 20 M_\odot$ we used the higher mass loss rates in case $Z = 0.02$ (cf. Maeder 1992; Schaller et al. 1992). The mass $m_u$ of the helium core left at the end of the He-burning phase (or C-burning phase for massive stars) has been used as input for the SNII and SNIb/c nucleosynthesis models referred to above. Yields were linearly interpolated both in $m$ and $m_u$. Errors due to the coupling of these sets of stellar evolution data are probably small and are neglected here (see Chap. 3). Remnant masses and stellar lifetimes were adopted from the Geneva group as well (e.g. Schaller et al. 1992).

For the reference model, the adopted IMF-slope, stellar mass limits at birth, and progenitor mass ranges for stars ending their lives as SNIa and SNII(+SNIb/c) are listed in Table 5.2. Stars with $m > 60 M_\odot$ have been excluded because their theoretical yields are rather uncertain (e.g. Maeder 1992). We expect that the IMF-weighted contribution by such stars to the enrichment of the ISM is relatively low.

Stars more massive than $m_{\alpha}^{SNII}$ are assumed not to explode as supernova but to end as black hole (cf. Maeder 1992; Nomoto et al. 1994; Prantzos 1994; Tsujimoto et al. 1995). Consequently, stars with $m \gtrsim m_{\alpha}^{SNII}$ contribute to the ISM enrichment during their stellar wind phase only. When no upper mass limit $m_{\alpha}^{SNII} = 25 - 30 M_\odot$ is introduced, models using up-to-date SNII yields predict abundances that are too high compared to those observed in the ISM, in particular for helium and oxygen (e.g. Twarog & Wheeler
5.4 Results

We present results for the inhomogeneous chemical evolution model described in the previous section. First, we consider the reference model which does not incorporate stellar abundance variations at a given age. Thereafter, we discuss models that do incorporate stellar abundance variations due to: 1) sequential stellar enrichment, 2) infall of metal-deficient matter, and 3) combined infall of metal-deficient matter and sequential enrichment.

5.4.1 Reference model

We consider a homogeneous gas cloud with initial conditions as listed in Tables 5.2 and 5.3. Within this cloud, active subclouds are formed at regular time intervals of \(1.4 \times 10^8\) yr so that in total \(N_{\text{cl}} = 100\) subclouds form during cloud evolution time \(t_{\text{ev}} = 14\) Gyr. We assume no time-delay between the formation of the subcloud and the actual onset of star formation within that subcloud, i.e. \(t_{\text{sf}} = t_{\text{sc}},\) and further assume each subcloud to experience a single star formation event. During this event, lasting \(\Delta t_{\text{disp}} = 10^7\) yr, half of the subcloud mass is converted into stars, i.e. \(\epsilon = 0.50\). After each event, both gas and stars contained within the subcloud are mixed homogeneously to the inactive cloud part. We note that the assumption of \(\epsilon = 0.50\) has no physical meaning here other than defining the gas consumption rate as a function of cloud age. Model related quantities are given in Table 5.3 (see Sect. 5.5.2).

![Figure 5.3](image.png)

Figure 5.3 Reference model: **a** Stellar-to-total (dashed curve) and gas-to-total (solid line) mass-ratios vs. age. The gas-to-total mass-ratio for the inactive cloud part coincides with that for the entire cloud, **b** Subcloud mass \(\Delta M_{\text{sc}} = \epsilon M_{\text{sc}}\) converted into stars (dashed curve) and amount of gas returned to the subcloud by newly formed stars during each star formation event. During this event, lasting \(\Delta t_{\text{disp}} = 10^7\) yr, half of the subcloud mass is converted into stars, i.e. \(\epsilon = 0.50\). After each event, both gas and stars contained within the subcloud are mixed homogeneously to the inactive cloud part. We note that the assumption of \(\epsilon = 0.50\) has no physical meaning here other than defining the gas consumption rate as a function of cloud age. Model related quantities are given in Table 5.3 (see Sect. 5.5.2).

Figure 5.3a shows resulting stellar and gas-to-total mass-ratios vs. age for the reference model. According to the assumed variation of subcloud mass with cloud evolution time (cf. Sect. 5.3.2), the gas-to-total mass-ratio decreases exponentially from \(\mu_{\text{st}} = 1\) to 0.1 (corresponding decrease in subcloud mass converted into stars is shown in Fig. 5.3b). Figure 5.3b illustrates that the amount of gas returned by massive stars during each star formation event is less than \(\sim 5\%\) of the total amount of gas converted into stars during the same event. This ratio is determined primarily by \(t_{\text{sf}}\) and \(\epsilon\) (see below). The amount of gas returned during \(\Delta t_{\text{disp}}\) by the *entire* stellar population within the inactive cloud part vs. cloud age is plotted for comparison.
Inhomogeneous chemical evolution of the Galactic disk

**Figure 5.4** Reference model: Stellar [Fe/H] and [O/H] abundance ratios vs. age. Stellar and ISM abundances at a given age are exactly the same. 

- **a** [Fe/H]: stellar abundances at birth (full circles) coinciding with ISM abundances (solid curve). Each full circle represents a stellar generation with total mass of approximately $\Delta M_{\text{cl}}$. Asterisks with error bars indicate the maximum stellar abundance variations observed in the Edvardsson et al. (1993a) data for main-sequence F and G dwarfs (averaged over 1.5 Gyr bins). Data for stars older than 15 Gyr have been omitted because of incompleteness (see Sect. 5.2).
- **b** [O/H]: curves and data similar to those for [Fe/H].
- **c** Theoretical [Fe/H] vs. [O/H]-relations including SNIa (full circles) and excluding SNIa (dashed curve). Mean [Fe/H] vs. [O/H] relation for the Edvardsson et al. data is shown as a straight line (horizontal marks indicate the observational range).

Corresponding stellar and interstellar [Fe/H] and [O/H] abundance ratios are shown in Figs. 5.4a and 5.4b, respectively. At a given age, stellar and ISM abundances are exactly the same so that abundance inhomogeneities do not occur. Note that the resulting AMRs do not depend on the adopted value for $M_{\text{cl}}$ as long as the normalisation of the SFR remains such that the condition of a current gas-to-total mass-ratio $\mu_1$ of 0.1 is met. The reference model predicts [Fe/H] and [O/H] abundance ratios that are consistent with the mean EDV data for stars younger than $\sim 10$ Gyr. For stars older than 10 Gyr, agreement with the observations may be improved e.g. by considering cloud ages in excess of $t_{\text{ev}} = 14$ Gyr or by detailed modeling of the halo-disk enrichment at early epochs of Galaxy evolution. We here concentrate on the stellar abundances observed during the last 10 Gyr of Galactic disk evolution.

Our adopted values of $\phi_{\text{SNIa}} = 0.005$ and $m_{\text{SNII}} = 30$ M$_{\odot}$ provide optimal consistency with the mean observed [Fe/H] vs. [O/H] relation (cf. Fig. 5.4c). Clearly, the slope of the resulting [Fe/H] vs. [O/H] relation in case of enrichment by SNIa only ($m_{\text{SNII}} = 60$ M$_{\odot}$) is inconsistent with the observations. Thus, the data provided by Edvardsson et al. imply that SNIa and SNIb/c nucleo-synthesis sites mixed their products together well. In addition, dilution of the supernova ejecta by more metal-deficient material is needed to comply with the range in [Fe/H] and [O/H] observed for F and G dwarfs in the SNBH. This is simply because theoretically predicted (lifetime-integrated) mean [Fe/H] ratios within the ejecta of supernova progenitors are in general much larger than those observed for long-living stars in the SNBH (see Sect. 5.3.4). Thus, whatever process is responsible for the observed stellar abundance variations, both mixing of ejecta from different SN-types and dilution with metal-deficient material are involved.

The resulting well-defined tight AMRs for the reference model are similar to those predicted by conventional single-zone chemical evolution models (e.g. Twarog 1980a; Tinsley 1980). Such models account for the global chemical enrichment of the Galactic disk ISM during the last 10 Gyr, at least for elements like O and Fe, but they obviously provide no explanation for the observed variations in stellar abundances at a given age.

### 5.4.2 Sequential stellar enrichment

In case of sequential star formation, efficient self-enrichment of a star forming gas cloud by successive stellar generations may result in abundance enhancements relative to the abundances in the ambient ISM. When the local mixing time scale is larger than the time between two successive star formation events in such a cloud, these abundance enhancements can be deposited and recorded by newly formed stars.

In our model, the impact of sequential enrichment on abundance inhomogeneities in the ISM is determined by: a) the dispersal time of the star forming region, b) the total number of stellar generations formed within one and the same cloud, c) the efficiency of sequential enrichment, i.e. the mass-ratio of the enriched stellar material and the cloud to which this material is mixed, d) details of stellar enrichment: e.g. the relative number of SNII and SNIa, and e) the IMF and stellar mass limits at birth. We distinguish the effect of single and multiple sequential stellar enrichment on the stellar abundance variations. We will refer to single sequential enrichment as the case in which a star formation event induces subsequent star formation in a nearby cloud (when mixing enriched material to this cloud).


5.4 Results

Table 5.3 Summary of model input parameters and resulting quantities related to the SFR \((M_\odot = 2 \times 10^{11} \, M_\odot)\)

<table>
<thead>
<tr>
<th>Model</th>
<th>Fig.</th>
<th>(M_{\text{sc}}(0)) ([M_\odot])</th>
<th>(N_\text{sf})</th>
<th>(\epsilon_{\text{max}})</th>
<th>(m_{\text{SNII}}) ([M_\odot])</th>
<th>(\mu_1)</th>
<th>SFR (<em>1) ([M</em>\odot \text{yr}^{-1}])</th>
<th>INF (_1) ([\text{yr}^{-1}])</th>
<th>(R_{\text{SNII}}) ([\text{yr}^{-1}])</th>
<th>(R_{\text{SNIIa}}) ([\text{yr}^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>5.3+4</td>
<td>1.3 (10)</td>
<td>100</td>
<td>0.50</td>
<td>30</td>
<td>0.09</td>
<td>2.9</td>
<td>-</td>
<td>1.3 (-2)</td>
<td>7.1 (4)</td>
</tr>
<tr>
<td>Seq.(single)</td>
<td>5.5-1/2</td>
<td>2.1 (10)</td>
<td>100</td>
<td>0.95</td>
<td>25</td>
<td>0.11</td>
<td>3.7</td>
<td>-</td>
<td>1.5 (-2)</td>
<td>8.2 (4)</td>
</tr>
<tr>
<td>Seq.(multiple)</td>
<td>5.5-3</td>
<td>1.5 (10)</td>
<td>200</td>
<td>0.50</td>
<td>25</td>
<td>0.21</td>
<td>2.5</td>
<td>-</td>
<td>1.0 (-2)</td>
<td>6.4 (4)</td>
</tr>
<tr>
<td>Infall</td>
<td>5.6</td>
<td>6.3 (9)</td>
<td>100</td>
<td>0.95</td>
<td>40</td>
<td>0.16</td>
<td>5.4</td>
<td>3.1</td>
<td>2.5 (-2)</td>
<td>1.6 (-3)</td>
</tr>
<tr>
<td>Infall+Seq.</td>
<td>5.7-1/2</td>
<td>6.4 (9)</td>
<td>136</td>
<td>0.90</td>
<td>25</td>
<td>0.19</td>
<td>5.2</td>
<td>1.8</td>
<td>2.1 (-2)</td>
<td>1.3 (-3)</td>
</tr>
<tr>
<td>Infall+Seq.</td>
<td>5.7-3</td>
<td>6.4 (9)</td>
<td>166</td>
<td>0.90</td>
<td>25</td>
<td>0.32</td>
<td>4.4</td>
<td>3.1</td>
<td>1.8 (-2)</td>
<td>1.1 (-3)</td>
</tr>
<tr>
<td>Observations*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.05–0.2</td>
<td>3.6±1.</td>
<td>2–4 (-2)</td>
</tr>
</tbody>
</table>

*References:


\(\mu_1\): Kulkarni & Heiles (1987); Binney & Tremaine (1987); see also Basu & Rana (1992).

SFR: Dopita (1987); Walterbos (1988; based on IR observations); Mezger 1988


SNI & SNII: van den Bergh & Tammann (1991); Tutukov et al. (1992); Cappellaro et al. (1993); Strom (1993).

Single sequential stellar enrichment

We present results for models incorporating single sequential stellar enrichment \((N_{\text{max}} = 1)\). For illustration purposes, we consider only the alternating half of the subclouds to experience sequential enrichment. We define initial masses of subclouds that are sequentially enriched as \(M_{\text{sc}} = \vartheta M_\odot\), where \(M_\odot\) is the total mass of gas converted into stars during the previous enriching star formation event. Initial masses of subclouds not involved with sequential enrichment are assumed to decrease exponentially (as for the reference model).

To maximize the effect of sequential enrichment on the stellar abundance variations, we assume \(\vartheta = 0.2\), \(\epsilon = 0.95\), \(m_{\text{SNII}} = 25 \, M_\odot\), \(\phi_{\text{SNIIa}} = 0.005\), and an enrichment efficiency \(\lambda = 0.95\) (i.e. nearly all enriched stellar material ejected is mixed to the material wherein the next star formation event is induced). We consider cloud dispersal times \(\Delta t_{\text{disp}} \sim 10^7\) yr. Such dispersal times are among the largest ones deduced from observations of nearby star forming molecular clouds (see Sect. 5.3.3). Using a theoretical age vs. turnoff-mass relation (e.g. Schaller et al. 1992), \(\Delta t_{\text{disp}}\) can be related to the least massive star \(m_{\text{env}}\) able to enrich material before cloud dispersal. For instance, values of \(\Delta t_{\text{disp}} \sim 5 \times 10^5\), \(10^7\), and \(2 \times 10^7\) yr, correspond to \(m_{\text{env}} \sim 40, 15,\) and \(12 \, M_\odot\), respectively.

Figures 5.5-1 and 5.5-2 (top and center panels in Fig. 5.5, respectively) illustrate the effect of sequential enrichment on the stellar abundance variations for cloud dispersal times of \(\Delta t_{\text{disp}} \sim 10^7\) and \(2 \times 10^7\) yr, respectively. The extent to which sequential enrichment contributes to the stellar abundance variations is determined by the chemical evolution of the ambient ISM. In general, large cloud dispersal times give rise to efficient locking up of metals in long living stars and enhanced stellar abundance variations relative to the abundances in the ISM. Stellar abundance variations due to single sequential stellar enrichment are found to be maximal for \(\Delta t_{\text{disp}} \sim 2 \times 10^7\) yr. Larger values of \(\Delta t_{\text{disp}}\) allow stars less massive than \(m \sim 12 \, M_\odot\) to dilute the metal-rich ejecta of more massive stars (e.g. Hashimoto et al. 1993).

In case \(\Delta t_{\text{disp}} = 2 \times 10^7\) yr, resulting stellar abundance variations due to sequential enrichment are sufficiently large to explain the observed variations in \([O/H]\). In contrast, corresponding variations in \([Fe/H]\) are much smaller than observed. This is true even though the models presented here do account for sequential enrichment by SNIb/c which usually show theoretical \([O/Fe]\) ratios much lower than SNII (e.g. Woosley et al. 1995). Results disagree with the observational fact that stellar abundance variations in \([Fe/H]\) are considerably larger than in \([O/H]\) (see also Gilmore & Wyse 1991; Edvardsson et al. 1993a). An enhanced contribution of SNIb/c (i.e. assuming \(\phi_{\text{SNIIb/c}} > 0.33\)) or participation of SNIa to the process of sequential enrichment seems to be excluded by the observations (van den Bergh & Tammann 1991; Tutukov et al. 1992).

We conclude that single sequential stellar enrichment models are inconsistent with the observations because: 1) result in \([O/H]\) variations that are larger than those in \([Fe/H]\), 2) predict current ISM abundances far below those observed, and 3) are difficult to reconcile with the apparent age independency of the stellar abundance variations (see Sect. 5.2). This conclusion is independent of the assumed cloud dispersion time scale \(\Delta t_{\text{disp}}\), sequential enrichment efficiency \(\lambda\), value of \(\vartheta\), background level of iron-group elements set by SNIa (i.e. \(\phi_{\text{SNIIa}}\)), star formation efficiency \(\epsilon\), and adopted IMF. Also, omitting the enrichment during the stellar wind phase of supernova progenitors does not alter this conclusion.
Figure 5.5 Model results for single and multiple sequential stellar enrichment. Top panels: Single sequential enrichment assuming a cloud dispersal time $t_{\text{disp}} = 10^7$ yr. Center panels: Single sequential enrichment assuming $t_{\text{disp}} = 2\times10^7$ yr. Bottom panels: Multiple sequential enrichment (see text). Model results are shown for variations of stellar and interstellar [Fe/H] and [O/H] abundance ratios: a [Fe/H] vs. age, b [O/H] vs. age, c [Fe/H] vs. [O/H]. Stellar abundances are indicated by filled circles. Mean interstellar abundances (averaged over both active and inactive clouds) are indicated by solid curves. Average interstellar abundances within the inactive cloud ISM only (indicated by short dashed curve) do approximately coincide with the overall mean abundances. Remaining symbols and curves have the same meaning as in Fig. 5.4

**Multiple sequential stellar enrichment**

The effect of sequential stellar enrichment on the abundance variations among successive stellar generations can be very large, especially for high sequential enrichment efficiencies and/or small amounts of cloud material to which the stellar ejecta are mixed before star formation is initiated. These conditions are naturally fulfilled when isolated gas clouds experience multiple star formation events (i.e. $N_{\text{sf}}^{\text{max}} > 1$) before mixing with the surrounding ISM. Since the gas content of an isolated cloud is reduced by each star formation event, cloud abundances rapidly increase when enriched by successive generations of massive stars.

We consider two possible scenarios of multiple sequential enrichment. In the first scenario, earlier generations of stars actually separate from the remaining subcloud material after dispersal of the subcloud core and do not further participate in the enrichment of the subcloud. Such models result in substantial stellar abundance variations only under conditions and assumptions similar to those for the single sequential enrichment case. In the second scenario, all stellar generations formed in the subcloud continue to contribute to the enrichment until the entire subcloud has been dispersed. In this case, large stellar abundance variations...
arise due to efficient recycling of the stellar ejecta from successive generations formed within the same cloud.

We apply the second scenario and consider all subclouds to experience multiple sequential enrichment. As maximum number of star formation events within one and the same subcloud we assume \( N_{\text{max}} = 4 \) (e.g. suggested by observations of OB subgroups in Orion; see Blaauw 1991). We adopt a sequential enrichment efficiency \( \lambda = 1 \) (by definition within the same subcloud), a star formation efficiency \( \epsilon \) between 0.3 and 0.6 for each star formation event, and \( \Delta t_{\text{disp}} = 10^7 \) yr. Other values of these parameters may provide similar results. Remaining model parameters are taken as for the single sequential enrichment model.

In Fig. 5.5-3, we plot resulting stellar abundance variations in case of multiple sequential stellar enrichment. Although variations caused by the first sequential enrichment event are relatively small (similar to the single sequential enrichment case assuming \( \Delta t_{\text{disp}} = 10^7 \) yr; see Fig. 5.5-1), abundance variations caused by subsequent events can be as large as \( \sim 0.2 \)–\( 0.3 \) dex (depending on the subcloud abundances). As mentioned before, such large abundance variations are mainly due to ongoing sequential enrichment of the remaining cloud material by stellar generations formed earlier in the cloud. We find that models incorporating multiple sequential stellar enrichment encounter the same problems as single sequential enrichment models. However, the former models appear observationally far more justified. This is true in particular for the sequential enrichment and star formation efficiencies, as well as the cloud core dispersal times, required to obtain a given stellar abundance variation.

We emphasize that the local conditions at the cores of star forming molecular clouds are likely to determine the IMF, the relative formation rate of different supernova progenitors, the fraction of binaries, etc. Therefore, in a more detailed treatment of sequential stellar enrichment, it seems natural to account for variations from one star formation event to another in e.g. \( t_{\text{disp}}, m_u^{\text{SNH}}, \) the fraction \( \phi^{\text{SNB/c}} \) of SNII progenitors which ultimately end as SNB/c, etc. We have verified that for reasonable variations in: 1) \( m_u^{\text{SNH}}, \) 2) the contribution by SNB/c, and 3) the contribution by SNIa (e.g. SNIa exploding in the vicinity of a subcloud; see Fig. 5.2f) may add at most \( \sim 0.1 \) dex to the observed stellar abundance variations at solar metallicity. Similarly, we find that stellar abundance inhomogeneities caused by variations in the IMF (and/or stellar mass limits at birth) among different star forming regions are highly sensitive to the IMF slope\(^2\) and may result in variations in \([\text{O}/\text{H}]\) of more than 0.15 dex at solar abundance.

We conclude that the observed stellar abundance variations are difficult to explain by sequential stellar enrichment only. This conclusion is not altered when allowing for variations in sequential enrichment between distinct star formation events (e.g. by considering variations in the IMF, relative formation rates of SNII and SNB/c, etc.). Possible exceptions may be selective mixing of SNII nucleo-synthesis products to the material wherein star formation is induced and/or cloud conditions that determine both the composition of the ejecta returned by a stellar generation (e.g. by means of the IMF, \( m_u^{\text{SNII}}, \Delta t_{\text{disp}}, \) contribution by SNB/c, binaries) and the sequential enrichment efficiency. In addition, conditions that regulate the amount of cloud material to which the stellar ejecta are mixed before star formation is initiated (i.e. the star formation efficiency) may be important for the effects of sequential stellar enrichment. However, the impact of such conditions on the stellar abundance variations, which would imply that the IMF weighed SN yields used here would be considerably modified, is beyond the scope of this paper.

5.4.3 Episodic infall of metal-deficient matter

Infall of metal-poor material can account for abundances of newly formed stars which lie substantially below the abundances in the global disk ISM. In principle, abundance variations due to metal-deficient gas infall are determined by the abundances within the infalling gas, the gas infall rate, and the amount of disk ISM to which the infalling material is mixed before star formation is initiated. Stellar abundance variations due to infall of metal-rich material associated with SN ejecta from massive stars in the Galactic disk can be considered as a special case of sequential stellar enrichment and is not discussed here.

Element abundances within the infalling gas are constrained by the lowest abundances observed for disk stars with \([\text{Fe}/\text{H}] \gtrsim -1\). The Edvardsson et al. data imply infall abundances of \([\text{M}/\text{H}]_{\text{inf}} \leq -0.8\) to \(-1.2\) for e.g. \( \text{M}=\text{C}, \text{Mg}, \text{Al}, \) and \( \text{Si} \). These abundances are consistent with observations of interstellar clouds in the halo (see Sect. 5.5.2) and suggest that infall induced star formation is associated with the lowest abundances observed among disk F and G dwarfs in the SNBH.

\(^2\)It is evident that the stellar abundance variations observed among similarly aged stars in the SNBH certainly do not exclude variations in e.g. the IMF among distinct star formation events.
We assume infall abundances similar to the abundances observed among the oldest metal-poor disk stars, i.e. $[\text{Fe}/\text{H}] = -1$, $[\text{O}/\text{H}] = -0.65$, and a hydrogen mass fraction of $X \sim 0.72$ (e.g. Bessell et al. 1991). For simplicity, we do not account for abundance inhomogeneities within the infalling gas and assume infall abundances to be constant in time. Furthermore, we consider infall to occur as soon as the infall abundances are reached in the global disk ISM (presumably corresponding with the onset of star formation in the disk).

The detailed manner in which gas infall varies with time is not essential for the results presented here as long as infalling gas plays an important role in determining the stellar abundances when it induces star formation. Here, we deal with the concept of infall induced star formation, i.e. the infalling gas initiates star formation when falling onto the disk. This concept is based on observations of infalling high-velocity clouds that strongly interact with disk ISM and initiate star formation therein as soon the critical density for star formation is reached (see Sect. 5.5.2). Since star formation occurs by definition within active subclouds according to our model, infall is associated with subclouds experiencing star formation shortly after their formation. For simplicity, we assume each subcloud to contain an amount of infalling gas accumulated at the time star formation is induced. This amount is taken as a random fraction of the initial subcloud mass (i.e. between 0 and 1). In this manner, we allow for local and episodic gas infall onto the Galactic disk ISM. On average, the gas infall rate is assumed to decay exponentially on a time scale $t_{\text{dec}} = 6 \, \text{Gyr}$ while its overall amplitude is constrained by $\mu_1 = 0.05 - 0.2$ (cf. Eq. (1); see Table 5.3).

We assume an initial disk mass $M_\text{cl} = 3 \times 10^{10} \, M_\odot$ before the onset of gas infall. This results in a disk initial-to-final mass-ratio $\zeta = 0.55$ according to an exponential decrease of subcloud mass with cloud evolution time ($M_\text{scl}(0) = 2 \times 10^9 \, M_\odot$; cf. Table 5.3). Clearly, stellar abundance variations due to metal-deficient gas infall are small at low levels of enrichment of the global disk ISM (i.e. large initial disk mass). Furthermore, continuous and large scale metal-deficient gas infall not associated with star formation results in relatively small stellar abundance variations. Thus, the magnitude of the stellar abundance variations is affected by the ratio of initial disk mass and total amount of infalling matter, as well as the amount of infalling gas that is involved with induced star formation in the disk ISM.

Figure 5.6 displays resulting AMRs for iron and oxygen in case of episodic infall of metal-deficient gas. We assumed $\phi^{\text{SN}}_{\text{SNIa}} = 0.015$, $\phi^{\text{SN}}_{\text{SNIb/c}} = 0.33$, and $m^{\text{SN}}_{\text{u}} = 40 \, M_\odot$. The value of $m^{\text{SN}}_{\text{u}}$ is taken larger than for the reference model (cf. Table 5.2) to obtain somewhat better agreement with the observations. Resulting abundances in the disk ISM follow the upper end of the abundances observed in F and G dwarfs younger than $\sim 10 \, \text{Gyr}$. Although the effect of global infall of metal deficient gas is generally to dilute the enrichment of the ISM, the inflow model results in larger ISM abundances than the reference model. This is mainly due to: 1) the assumption of a low initial disk mass which allows for a rapid early enrichment of the disk, and 2) the assumption of local infall of metal-deficient gas and subsequent star formation therein so that infalling material has relatively small effect on the dilution of the global disk ISM.

By varying the ratio of infalling matter and disk ISM within each subcloud, stellar abundance variations of $\Delta [\text{Fe}/\text{H}] \sim 0.8$ dex and $\Delta [\text{O}/\text{H}] \sim 0.65$ dex naturally can be accounted for. In addition, the scatter in $[\text{Fe}/\text{H}]$ remains larger than in $[\text{O}/\text{H}]$ since the iron abundances within the infalling gas relative to solar (i.e. $[\text{Fe}/\text{H}]_{\text{inf}} = -1$) are much smaller than that of oxygen ($[\text{O}/\text{H}]_{\text{inf}} = -0.65$). We note that the current gas infall rate of $\sim 3.1 \, M_\odot \, \text{yr}^{-1}$ predicted by the model shown in Fig. 5.6 is larger than suggested by the observations (see Sect. 5.5.1). However, other choices of model parameters, e.g. $\zeta$, predict much lower infall rates while providing similar abundance results.
Our models incorporating metal-deficient gas infall are in good agreement with the observed magnitude of stellar abundance variations and the slope of the [Fe/H] vs. [O/H] relation observed. However, these models predict current interstellar [Fe/H] and [O/H] abundance ratios of \( \sim 0.2 \) dex above solar. This is in marked contrast with [O/H] abundance ratios of \( \sim 0.15 \) dex below solar observed both in interstellar gas and recently formed stars in the SNBH (see Sect. 5.2). In addition, these models appear to disagree with the observations on two other grounds. First, no significant scatter in the [Fe/H] vs. [O/H] relation is predicted, contrary to what is observed for intermediate age disk stars (cf. Fig. 5.1). Part of the observed scatter may be due to experimental errors but variations of at least \( \pm 0.1 \) dex in the [Fe/H] vs. [O/H] relation are probably real and have to be explained by any satisfactory model. A way to account for such scatter would be to allow for considerable abundance variations among different parcels of infalling gas. Secondly, these models predict stellar abundance variations to increase with time. This is inconsistent with the apparent constancy of the abundance scatter observed. Possible ways out may be uncertainties in the ages of stars older than \( \sim 10 \) Gyr or disk evolution times in excess of \( t_{ev} \sim 14 \) Gyr.

We conclude that models dealing with metal-deficient gas infall can probably be excluded as the complete answer to the stellar abundance variations observed, even though such models are in good agreement with both the observed abundance variations and [Fe/H] vs. [O/H] relation. This conclusion is primarily based on the fact that such models predict mean current ISM abundances \( \sim 0.4 \) dex larger than those observed in the SNBH.

### 5.4.4 Metal-poor gas infall combined with sequential enrichment

Motivated by the results previously discussed, we study the combined effect of sequential stellar enrichment and episodic infall of metal-deficient gas on the inhomogeneous chemical evolution of the Galactic disk. Such investigation is important also because these processes are observed to operate simultaneously in the SNBH (see Sect. 5.5.2). Attractive features of combined infall of metal-poor gas and sequential stellar enrichment are that a self-consistent explanation can be obtained for: 1) the presence of high metallicity stars at early epochs of star formation in the Galactic disk (due to sequential enrichment), 2) the presence of metal-poor stars at recent epochs of Galactic evolution (as a result of metal-deficient gas infall), 3) the nearly constant magnitude of the stellar abundance variations during the lifetime of the disk, and 4) abundances in the local disk ISM that are currently below solar (as observed for oxygen).

We show in Fig. 5.7 results for combined sequential stellar enrichment and metal-deficient gas infall. Model assumptions concerning each of these processes are similar to those described in the previous sections (e.g. \( \epsilon_{\text{max}} = 0.90, \lambda = 0.95, m_{\text{SN}} = 25 M_\odot, \phi^{\text{SNIa}} = 0.005, \) and \( \phi^{\text{SNB}/c} = 0.33; \) cf. Tables 5.2 and 5.3). The three models shown in Fig. 5.7 differ only in the amounts of disk ISM involved with sequential stellar enrichment and infalling gas accreted during the lifetime of the disk. For each of these models, resulting stellar abundance variations and [Fe/H] vs. [O/H] relation are consistent with the observations. Clearly, models with combined sequential enrichment and metal-poor gas infall do not encounter the specific problems involved when each of these processes is considered separately.

We study the relative impact of metal-poor gas infall and sequential stellar enrichment on the resulting stellar abundance variations as well as the global enrichment of the ISM. First, we investigate the effect of varying the fraction of subclouds (i.e. the amount of star forming disk ISM) experiencing multiple sequential stellar enrichment (\( N_{\text{sf}}^{\text{max}} = 4, \Delta t_{\text{disp}} = 10^7 \) yr, \( \epsilon_{\text{max}} = 0.9; \) see Sect. 5.4.2). This fraction increases from \( \sim 10\% \) to \( \sim 25\% \) when going from top to center models shown in Fig. 5.7. The remaining part of the subclouds is assumed to experience one single star formation event. Furthermore, we assume half of the subclouds to form stars partly from infall of metal-deficient gas (i.e. Figs. 5.7-1 and 5.7-2), regardless of the number of star formation events in each subcloud. Note that subclouds involved with metal-poor gas infall form predominantly stars with abundances below those in the global disk ISM. It can be seen that mean interstellar abundance and stellar abundance variations are not significantly altered when the fraction of ISM associated with sequential stellar enrichment is increased from 10 to 25\%. However, when this fraction is further increased, more and more metals will be locked up in long living stars due to sequential enrichment and marked deviations from the observed [Fe/H] vs. [O/H] relation will occur (see Sect. 5.4.2).

Secondly, we investigate the effect when the fraction of subclouds forming stars from metal-deficient gas infall is increased from 50 to 100\%. In this case, stellar generations are all formed according to infall induced sequential star formation and the total mass of infalling gas is increased by a factor two. This results in a reduction of the interstellar [Fe/H] and [O/H] abundance ratios by \( \sim 0.1 \) dex (see Fig. 5.7-3). Interestingly, this marginally affects the magnitude of the resulting stellar abundance variations (assuming \( \epsilon_{\text{max}} = 0.9, \lambda = 0.95 \)) but strongly alters the number of stars with abundances below those present in the global disk ISM. We note that direct comparison of the abundance results with previous models is not justified because
Inhomogeneous chemical evolution of the Galactic disk

Figure 5.7 Model results for combined sequential stellar enrichment and episodic infall of metal-poor gas. Top panels: ~10% of the clouds is assumed to experience multiple sequential enrichment while half of the subclouds is involved with metal-deficient gas infall. Nearly 5% of the clouds undergo both infall of metal-deficient material and sequential stellar enrichment. Center panels: 25% of the subclouds experience multiple sequential enrichment while half of the subclouds is involved with metal-poor gas infall. Bottom panels: as center panels but all subclouds experience metal-deficient gas infall. Model results are shown for variations of stellar and interstellar [Fe/H] and [O/H] abundance ratios: a [Fe/H] vs. age, b [O/H] vs. age, c [Fe/H] vs. [O/H]. Symbols and curves have the same meaning as in Fig. 5.5.

The enhanced gas infall model results in a current gas-to-total mass-ratio $\mu_1 \sim 0.3$, i.e. considerably higher than the $\mu_1 = 0.1 - 0.2$ indicated by the observations (cf. Table 5.3). To arrive at $\mu_1 = 0.2$, a reduction in initial disk mass from $3 \times 10^{10}$ to $2 \times 10^{10} \ M_\odot$ would be required. In turn, this would result in ISM abundances and stellar abundance variations similar to that for models with more modest infall rates.

5.4.5 Additional abundance constraints

Keeping these results in mind, we study how models with combined sequential stellar enrichment and metal-poor gas infall behave when confronted with additional observational constraints provided by the stellar abundance variations and current ISM abundances of C, Mg, Al, and Si.

Carbon abundance data for 85 F and G dwarfs in the SNBH have been presented by Andersson & Edvardsson (1994). These data show that there is a weak correlation between [C/H] and [O/H] (see Fig. 5.8). The shape of this correlation differs from that between e.g. [Fe/H] and [O/H]. In addition, the variation in [C/H] (i.e. $\geq 0.6$ dex) at a given value of [O/H] is about three times larger than that in [Fe/H].
5.4 Results

If the observed stellar abundance variations are caused by infall of metal-deficient gas only, one would expect that stellar abundances for all elements heavier than helium would be mutually correlated, e.g. similar to the correlation between oxygen and iron. In case of sequential stellar enrichment only, a similar behaviour would be expected only for elements that are produced predominantly by SNII and SNIb/c. This implies that abundance-abundance variations between elements which are not synthesized predominantly within SNII and SNIb/c (such as C and N), on the one hand, and elements that are produced predominantly within supernovae (e.g. O, Si), on the other hand, may be conclusive about the importance of metal-deficient gas infall.

We show in Fig. 5.8 results for: 1) infall of metal-deficient gas only (cf. Fig. 5.6), 2) multiple sequential stellar enrichment only (cf. Fig. 5.5-1), and 3) combined metal-poor gas infall and sequential enrichment (cf. Fig. 5.7-1). The infall model predicts no substantial scatter in the [C/H] vs. [O/H] relation but follows the trend in the observations well. Although the scatter in the [C/H] vs. [O/H] relation suggests that infall is not exclusively responsible for the observed variations, the shape of this relation indicates that infall is important. Conversely, the sequential enrichment model shows large scatter in the [C/H] vs. [O/H] relation but appears to deviate from the observed trend. The correlation predicted by the combined sequential enrichment and infall model appears in reasonable agreement with the observations. However, the observed carbon abundances at [C/H]\approx -0.4 exhibit considerable more scatter than predicted by the model shown in Fig. 5.7-1 and seem to require a somewhat steeper increase of carbon relative to oxygen. This may be due to variations in sequential stellar enrichment between different star formation events and/or variations in abundances within the infalling material.

Figure 5.8 Comparison between observed and model-predicted [C/H] vs. [O/H] relation. Observations: data for F and G dwarfs in the SNBH from Andersson & Edvardsson (1995). Open circles represent stars with mean stellar galactocentric distances at birth within 0.5 kpc from the Sun (R_⊙ = 8.4 kpc). Full dots indicate stars with average distances within ∼2 kpc from the Sun. Typical errors are indicated at the bottom right (top left panel). Model results: Predicted abundances of stars (full dots) and gas (solid line) for models incorporating metal-poor gas infall and/or sequential stellar enrichment.

In our models, the overall shape of the stellar [C/H] vs. [O/H] relation is due to infall of metal-poor material with carbon abundances [C/O]_{inf} \approx -0.4. The reason why these relatively low carbon infall abundances are necessary to explain the observed trend in the [C/H] vs. [O/H] relation, is unclear. A possible
Figure 5.9 Comparison between observed (left) and model-predicted (right) [M/H] vs. [Fe/H] relations: a) [Si/Fe], b) [Mg/Fe], and c) [Al/Fe]. Observations: data for F and G dwarfs in the SNBH from Edvardsson et al. (1993a). Symbols have the same meaning as in Fig. 5.8. Typical errors are indicated at the lower right of each panel. Model results: predicted abundances of stars (full dots) and gas (solid line) for model 7-1.

Explanation may be a delayed carbon enrichment of the disk ISM, e.g. by low-mass SNIa progenitors that experience incomplete carbon burning or by low-mass AGB stars with small carbon yields.

Sequential stellar enrichment seems inevitable to explain the observed scatter in the [C/H] vs. [O/H] relation. Although large abundance inhomogeneities within the infalling gas may reproduce the observed variations as well, such inhomogeneities in [C/O] appear inconsistent with the small scatter observed in e.g. [Fe/O]. Also, uncertainties in the derived carbon abundances may be considerably larger than those in O and Fe but are not likely to exceed ~0.2 dex (see Andersson & Edvardsson 1994; see also EDV). Therefore, it seems improbable that the scatter observed in the [C/H] vs [O/H] relation is due to observational errors. Finally, it is difficult to see how chemical differentiation processes (e.g. dust depletion, element mixing to the surface, O/N-cycle, metallicity dependent nucleo-synthesis, etc.) can cause such large abundance-abundance variations among stars similar in mass and age.

From the arguments above, we conclude that models incorporating both infall of metal-poor gas and sequential stellar enrichment provide an adequate explanation for the observed [C/H] vs. [O/H] relation. For the model shown in Fig. 5.7-1, we compare the predicted stellar abundance ratios Si, Mg, and Al vs. Fe with
the observations in Fig. 5.9. We find that slight offsets in [M/H] are present between the model predicted and observed relations. These offsets, most pronounced for Al, are due to details in the adopted stellar yields and related parameters (see Table 5.3), and are not essential for the following discussion. Interestingly, a number of observed features are naturally reproduced by these models.

First, the observations suggest that variations in the stellar [M/H] abundance ratios decrease with increasing metallicity. This is theoretically predicted by the individual effects of both sequential stellar enrichment and infall of metal-deficient material (as discussed above). For the model shown in Fig. 5.7-1, the resulting scatter in [M/H] at a given value of e.g. [Fe/H] is mainly due to sequential enrichment (except at abundances [M/H] \( \leq -0.7 \)) and is strongly related to the iron contribution by SNIa in regions that do not experience sequential enrichment. It can be seen that the scatter in [M/H] strongly decreases at solar metallicities and above. This may indicate that much of the observed element-to-element variations at high metallicities is due to observational errors of \( \sim 0.1 \) dex in [M/H]. Alternatively, the observed scatter at high metallicities may imply that sequential enrichment by massive stars varies from one star formation event to another, e.g. by means of variations in the upper mass limit for SNII.

Secondly, the predicted variation in [Mg/H] at a given [Fe/H] is substantially larger than that in [Si/H], consistent with the observations. In our model, this is due to: 1) the fact that part of the Si comes from SNIa which are important contributors also to Fe (Mg is produced less efficiently in SNIa than is Si by about one order of magnitude; e.g. Nomoto et al. 1984), and 2) the predicted ISM abundance of [Mg/H] is less by about 0.1 dex than that of [Si/H] (see Fig. 5.9).

Thirdly, the observed variation in [Al/H] is similar (or even larger) than that in [Mg/H] (see EDV). This behaviour is also found for the model shown in Fig. 5.7-1. The predicted ISM abundance of Al is probably too low by \( \sim 0.1 \) dex so that the resulting abundance scatter would be slightly reduced when correcting for this. The impact of sequential stellar enrichment is more pronounced for Al than for Mg and Si, due to the somewhat lower ISM abundance of Al (even after correction). Overall, we conclude that the magnitudes of the resulting stellar variations in Si, Mg, and Al vs. Fe appear in reasonable agreement with the observations.

Comparison of variations in Mg, Si, and Al vs. O with the observations reveals a somewhat different picture from that vs. Fe. The observed variation of [Mg/H] with [O/H] has been shown in Fig. 5.1: Al and Si display a similar behaviour. No trend is observed for variations in the scatter in the \( \Delta [M/H] \) vs. [O/H] relation, in contrast to that in the \( \Delta [M/H] \) vs. [Fe/H] relation. Furthermore, the observations indicate mean variations in [M/Fe] of the same magnitude as those in [M/O]. In contrast, the model shown in Fig. 5.7-1 predicts variations in Mg, Al, and Si vs. Fe that are considerably larger than those vs. O. This is simply due to the fact that a substantial fraction of Fe originates from SNIa. Therefore, models predict hardly any scatter in e.g. [Mg/O] since both elements are synthesized predominantly within SNII (and SNIb/c). This implies that either the scatter in the observed [Mg/H] vs. [O/H] relation is due to observational errors or that an additional process is needed in the models to explain this scatter.

In the former case, there would be no reason to believe the variations observed in the [M/H] vs. [Fe/H] relations either. However, we have argued above that part of this scatter is real. In the latter case, an additional mechanism causing the scatter in [M/O] could be variations from one star formation site to another in the enrichment by SNII (and SNIb/c). Alternatively, such variations could include variations in e.g. the IMF, upper mass limit for SNII, and/or the mass distribution of binaries. We have verified that such variations generally result in abundance-abundance scatter sufficiently large to account for the observed variations of 0.1 dex in [M/O] and sufficiently small to have a negligible effect on the scatter observed in [M/Fe]. Clearly, element-to-element variations in enrichment from one star formation site to another would be a natural refinement of the sequential stellar enrichment models discussed before.

We conclude that models incorporating both sequential stellar enrichment and episodic infall of metal-poor gas provide a natural explanation for the observed stellar abundance variations and are consistent with the ISM abundances of C, O, Fe, Mg, Si, and Al. We find that the mean ISM abundances and abundance-abundance relations can provide only limited constraints on the relative importance of sequential enrichment and infall induced star formation in the Galactic disk. Therefore, improvements in observational and theoretical constraints are required to disentangle the effects of these processes on the inhomogeneous chemical evolution of the Galactic disk in a more quantitative way.
5.5 Discussion

We briefly examine how the combined sequential stellar enrichment and metal-deficient gas infall models discussed in the previous section behave when confronted with independent constraints provided by the current star formation rate in the Galactic disk and the chemical evolution of the Galactic halo. Thereafter, we discuss observational evidence in support of sequential enrichment and gas infall in the local disk ISM and consider possible implications of these processes for the chemical evolution of the Galaxy as a whole.

5.5.1 Additional constraints

SFR related constraints

The combined sequential enrichment and metal-deficient gas infall (Fig. 5.7-1) predicts a present SFR of \( \sim 5.2 \, M_\odot \, yr^{-1} \), and current rates of SNIa (excluding SNIIb/c) and SNIa of \( R_{\text{SNIa}} = 2.1 \times 10^{-2} \, yr^{-1} \) and \( R_{\text{SNIa}} = 1.3 \times 10^{-3} \, yr^{-1} \), respectively. These values are roughly consistent with the observations (i.e. within a factor of two; see Table 5.3). Adopted values of \( m_{\text{u,SNII}} = 25 \, M_\odot \) and \( \psi_{\text{SNII}} = 0.005 \) may be somewhat too low since the SN-rates scale with the predicted SFR. For the same model, the current gas infall rate is determined by the assumed disk initial-to-final mass-ratio \( \zeta = 0.5 \) and by the time scale \( \tau_{\text{inf}} = 6 \, Gyr \) on which infall decays exponentially. This results in a current gas infall rate of \( 1.8 \, M_\odot \, yr^{-1} \). Observations indicate a current gas infall rate of \( \sim 0.5 \, M_\odot \, yr^{-1} \) (e.g. Mirabel & Morras 1984). However, selection effects may account for an underestimate of a factor of 2–3 (see Sect. 5.5.2). We note that higher values of \( \zeta \) and/or lower values of \( \tau_{\text{inf}} \) may apply equally well.

The predicted rates above all scale with the amplitude of the SFR. In turn, this amplitude is determined by the total cloud mass \( M_{\text{cl}} = 2 \times 10^{11} \, M_\odot \) and SFR decay time scale \( \tau_{\text{dec}} \sim 6 \, Gyr \) assumed (see Sect. 5.3.2). Distinct values of \( M_{\text{cl}} \) and/or \( \tau_{\text{dec}} \) will not affect the predicted stellar and interstellar abundances substantially, provided that a current gas-to-total mass-ratio \( \mu_1 = 0.1 \) is maintained. We conclude that the adopted parameters for the model shown in Fig. 5.7-1 are consistent with observational constraints on the current SFR, gas infall rate, and supernova rates. Obviously, these observations do not yet provide tight constraints on e.g. \( \tau_{\text{dec}}, \mu_1, \) and \( \zeta \), thus preventing a clear distinction between chemical evolution models based on these quantities (cf. Table 5.3).

Constraints related to the enrichment of the Galactic halo

The mean plateau value of \([\text{O/Fe}]\sim-0.5 \pm 0.15 \) observed for halo stars with \([\text{Fe/H}]\leq -1 \) (e.g. Bessell et al. 1991) is presumably determined by the average \([\text{O/Fe}]\) ratio within the ejecta of SNIa (and SNIIb/c) as well as the initial abundances within the halo ISM. For our models, the plateau value implies a maximum upper mass limit of SNIa progenitors of \( m_{\text{u,SNII}} \approx 40 \, M_\odot \), assuming initial metallicities \([\text{Fe/H}] \leq -1 \), a Salpeter IMF, and stellar yields as described in Sect. 5.3.4. An even larger value for \( m_{\text{u,SNII}} \) is implied when SNIIb/c contributed substantially to the halo enrichment.

We assumed \( m_{\text{u,SNII}} = 25 \, M_\odot \) for the combined infall + sequential enrichment model (Fig. 5.7-1) discussed before. This results in \([\text{O/Fe}] \sim 0.2 \) at \([\text{Fe/H}] \leq -1 \) while omitting the contribution from SNIIb/c would have resulted in \([\text{O/Fe}] \sim 0.25 \) at \([\text{Fe/H}] \leq -1 \). This is inconsistent with the observations. Possible solutions to this discrepancy are: 1) \( m_{\text{u,SNII}} \) and/or the IMF have changed between the time stars formed in the halo and the time of onset of star formation in the disk, 2) \( m_{\text{u,SNII}} \) is actually \( \sim 40 \, M_\odot \) for disk stars so that the predicted current disk ISM abundances of e.g. O and Fe increase and the effect of sequential enrichment is reduced, and/or 3) the adopted yields for SNIa (and or SNIIb/c) are in error at metallicities below \([\text{Fe/H}] \sim -1 \).

Although the first two possibilities cannot be excluded, we favour the latter option since values of \( m_{\text{u,SNII}} \sim 30 \, M_\odot \) are suggested by recent models accounting for metallicity dependent yields of SNIa in full detail (e.g. Timmes et al. 1995). We emphasize that the detailed yields for SNIa at metallicities \([\text{Fe/H}] \leq -1 \) are not important for the sequential enrichment and infall model results for disk stars presented in this paper but we just want to note here that it is difficult to explain the mean \([\text{O/Fe}]\) ratio in halo stars using the same models.

The observed break in the \([\text{O/Fe}]\) vs. \([\text{Fe/H}]\) relation at \([\text{Fe/H}] \sim -1.0 \pm 0.2 \) (e.g. King 1994) is generally associated with the time SNIa start to contaminate the global disk ISM (e.g. Gilmore & Wyse 1991; Bravo et al. 1993; Ishimaru & Arimoto 1995). In our models, the breakpoint in the \([\text{O/Fe}]\) vs. \([\text{Fe/H}]\) relation is mainly determined by: 1) the assumed fraction \( \phi_{\text{SNIIa}} = 0.005 \) of main-sequence stars with initial masses between 2.5 and 8 \( M_\odot \) (e.g. Nomoto et al. 1984), 2) the delay time \( \tau_{\text{SNIIa}} = 2.5 \, Gyr \) after which SNIa start to contribute to the enrichment of the ISM (e.g. Smecker-Hane & Wyse 1992; Ishimaru & Arimoto 1995), and 3) the frequency distribution of SNIa as a function of age for a given stellar generation (assumed
to be constant from $\tau_{\text{SNIa}}$ to $\tau_{\text{SNIa}} + 0.5$ Gyr, and zero otherwise). These assumptions, in particular for $\tau_{\text{SNIa}}$, strongly affect the increase of [Fe/H] with disk age and thus determine the scatter in and the slope of the [O/H] vs. [Fe/H] relation predicted. In fact, SNIa provide a background signal of iron-group elements on top of which stellar abundance variations due to sequential enrichment by SNI+SNII occur.

We assumed $\tau_{\text{SNIa}} = 2.5$ Gyr for the models presented in this paper, as recently suggested by Ishimaru & Arimoto (1995). However, this assumption implies different breakpoints in the [O/Fe] vs. [Fe/H] relation for models with distinct star formation and infall histories. The model shown in Fig. 5.7-1 predicts [Fe/H] = −1 after ∼1.3 Gyr (and results [Fe/H] = −0.75 after 2.5 Gyr). This may indicate that the assumed value of $\tau_{\text{SNIa}} = 2.5$ Gyr is in error. Estimates for $\tau_{\text{SNIa}}$ based on stellar evolution calculations suffer from large uncertainties in the detailed evolution scenario for SNIa progenitors (see e.g. Smecker-Hane & Wyse 1992; King 1994) while theoretical estimates for $\tau_{\text{SNIa}}$ based on the observed breakpoint may suffer from large uncertainties in the assumed chemical evolution of the halo (e.g. Ishimaru & Arimoto 1995).

Alternatively, model assumptions related to: 1) the enrichment rate of the disk ISM by SNI (e.g. IMF and SFR, $m_i^{\text{SNII}}$ and SNII yields), or 2) the initial abundances of the material to which the SNII ejecta are mixed (e.g. disk mass at the onset of star formation, amount of gas infall, and infall abundances) may be in error. For instance, adopted iron yields for SNI at metallicities $[\text{Fe/H}] \leq -1$ may be too high by about a factor of two. This would be consistent with the discrepancy in the [O/Fe] ratio for Galactic halo stars discussed above. Furthermore, large amounts of metal-poor gas infall would improve the consistency with the assumption of $\tau_{\text{SNIa}} = 2.5$ Gyr. Other possibilities, such as higher values of the disk total-to-final mass-ratio or larger SFR decay times $t_{\text{decr}}$, seem to be excluded by the observations (e.g. van den Hoek et al. 1996).

We conclude that combined sequential enrichment and metal-poor gas infall models are consistent with the observed plateau value and breakpoint in the [O/Fe] vs. [Fe/H] relation provided that e.g. the adopted SNI yields at low metallicities [Fe/H] ≤ −1 are too high by about a factor of two. At the same time, we have illustrated how sensitive our model results are to specific assumptions related to the enrichment by e.g. SNII and SNIa. These assumptions may affect quantitative conclusions concerning the relative importance of sequential stellar enrichment and metal-poor gas infall. However, our qualitative conclusion regarding the simultaneous presence of these processes in the local Galactic disk is not altered.

### 5.5.2 Observational support

We briefly discuss observational evidence in support of sequential stellar enrichment and metal-deficient gas infall in the local disk ISM.

#### Metal-deficient gas infall

Observations of high-velocity clouds (HVCs) show that many separate interstellar clouds are present in the Galactic halo with abundances usually ~0.1 solar for elements like C, O, S, and Si (e.g. de Boer & Savage 1984; Schwarz et al. 1995). Many faint HVCs up to distances of at least ∼10 kpc above the Galactic plane are found to be a member of large scale cloud complexes (Wakker 1991). The velocity distribution of these complexes is asymmetric showing a net inflow of matter onto the Galactic disk (e.g. Mirabel & Morras 1984; Hulsbosch & Wakker 1988; Wakker 1990). These observations support the idea of high-velocity inflow of neutral hydrogen gas towards the Galactic disk.

Estimates of the current gas infall rate onto the Galactic disk range from 0.2–0.5 $M_\odot$ yr$^{-1}$ based on HVCs ($v \gtrsim 250$ km s$^{-1}$; e.g. Mirabel & Morras 1984; Mirabel 1989; Lépine & Duvert 1994), to ~0.7 $M_\odot$ yr$^{-1}$ derived from the soft X-ray background (Cox & Smith 1976), and ~1.5 $M_\odot$ yr$^{-1}$ based on observations of atomic hydrogen (Oort 1970). However, gas infall rates derived from the inflow of HVCs are likely to be underestimates both because of the preferential detection of nearby HVCs and the large uncertainties involved with e.g. distances (a factor of two uncertainty in the distance results in a factor 4 in the estimated influx of mass) and the detection probabilities of HVCs. In particular, low-velocity gas may add substantially to the total influx of matter onto the Galactic disk. This gas is hard to detect due to its less pronounced (and more diffuse) interaction with the disk ISM (e.g. Mirabel & Morras 1984).

A crude estimate of the fraction of stars recently formed from metal-poor gas fallen onto the disk can be made as follows. The current star formation rate in the Galactic disk is ∼3.5 ± 1. $M_\odot$ yr$^{-1}$ (e.g. Dopita 1987; see also Table 5). The minimum gas infall rate obtained from the observations is ∼0.5 $M_\odot$ yr$^{-1}$ but an underestimate by a factor of 3–4 appears likely. This would imply that a considerable fraction of 30–60 % of stars currently forming in the disk is associated with infalling gas provided that all infalling matter initiates star formation. On the theoretical side, models in reasonable agreement with observational constraints on the chemical evolution of the Galactic disk predict current gas infall rates in the range of...
Observational support for infall induced star formation in the SNBH has recently been presented for the Orion cloud complex (Lépine & Duvert 1994; Meyer et al. 1994), the Monoceros cloud complex (Gómez de Castro 1999), the Gould Belt (e.g. Cómeron & Torra 1994), and the ζ Sculptoris open cluster (Edvardsson et al. 1995). These observations suggest that the most prominent star forming regions in the SNBH have been partly formed from infalling clouds from the Galactic halo. Circumstantial evidence for HVC impacts on the Galactic disk is based on: 1) the existence of subgroups of young stars in star forming molecular clouds at high Galactic latitudes (like the Orion molecular cloud complex), 2) the displacement of OB star clusters with respect to the centers of their parent molecular clouds, 3) the alignment of the OB clusters in directions that are substantially inclined to the Galactic plane, 4) the age sequence of the aligned OB associations with an age of ~10^7 yr for the oldest subgroups, and 5) the large elongated or filamentary structures observed in e.g. the Orion, Taurus, Monoceros molecular clouds connecting the clouds to the Galactic plane (see also Tenorio-Tagle et al. 1986; Franco et al. 1988; Gómez de Castro 1992). Many of these phenomena can be naturally explained by the interaction of a high velocity cloud with disk ISM and are difficult to reproduce by the process of sequential star formation (Lépine & Duvert 1994). Infall induced star formation appears to be a process frequently operating in the Galactic disk.

It has been proposed that many of the HVCs are associated with the ejection of material up to large scale heights by supernovae in the Galactic disk (Mathewson & Ford 1984). Apart from the fact that HVCs cannot be fully primordial because of the presence of various metals in HVCs, considerable mixing of material in the Galactic halo with metal-rich material associated with star formation in the disk may have occurred. The height above the Galactic plane and the amount of accreted material, participating in the circulation between the halo and the disk, may determine the average abundances within the infalling material. Alternatively, infalling clouds may be related to intergalactic gas or associated with stripping of material from nearby metal-poor galaxies like the Small Magellanic Cloud (see below).

It is interesting to note that Orion’s oxygen abundance is at the low end of the oxygen abundances observed in HII regions at roughly the galactocentric distance of Orion (Cunha & Lambert 1992; see also Meyer et al. 1994). If due to metal-poor gas infall, this would be consistent with the scenario of infall induced star formation as deduced from OB associations in Orion. To preserve the chemical inhomogeneities caused by infall of metal-poor gas, infall must induce star formation on time scales short compared to the local mixing time scale. According to the interaction time scale of 10^7 yr, this condition is likely fullfilled during the impact of a HVC with the disk ISM.

This would be consistent with the generally accepted idea that Galactic HVCs usually have abundances below those present in the local disk ISM (Savage & de Boer 1981; de Boer & Savage 1983, 1984). However, recent observations indicate that both high (i.e. about solar; Spitzer & Fitzpatrick 1995) and low (i.e. about 1/10 solar; Danly et al. 1993; Savage et al. 1993; Kunth et al. 1994; Schwarz et al. 1995) metal abundances in HVCs are present. The latter authors derived abundances of C, O, S, and Si which are at most 0.1 times solar; the lowest abundances found are similar to those present in the Magellanic Stream. Since depletion of heavy elements by dust grains may play an important role and metallicities may vary considerably across a HVC complex (Schwarz et al. 1995) further analysis is needed to confirm the overall metal-deficiency of high-velocity clouds falling onto the disk ISM.

### Sequential stellar enrichment

The concept of sequential star formation is based on observations of spatially separated subgroups of OB stars that appear aligned in a sequence of ages in many OB associations (e.g. see the review by Blaauw 1991; Megeath et al. 1995; Testi et al. 1995). Sequential star formation has been argued to occur in nearby molecular cloud complexes including the well known Orion, Taurus-Auriga-Perseus, Cepheus, Carina, and Chameleon cloud complexes. Additional support is provided by observations of newly formed stellar generations at the interfaces of HII regions and their surrounding molecular clouds (e.g. Genzel & Stutzki 1989; Junkes et al. 1992; Goldsmith 1995; Megeath et al. 1996). Efficient self-enrichment is expected to occur in these regions.

Age differences between the OB subgroups (distances of 10-50 pc) in Orion are typically of the order of 2–7 × 10^6 yr (e.g. Genzel & Stutzki 1989; Blaauw 1991; Cunha & Lambert 1992; Elmegreen (1992); Risco & Bajaja 1994; Brown et al. 1994; Haikal 1995). These ages appear similar to the estimated dispersal times of 3-5 × 10^6 of molecular cloud cores associated with young star clusters after the onset of star formation (e.g. Garmaz et al. 1982; Leisawitz 1985; Leisawitz et al. 1989). This can be compared to the typical free-fall time of ~2 Myr for a giant molecular cloud which implies star formation to start shortly after cloud formation (Blitz 1990). When star formation proceeds on such time scales, massive stars belonging
to different OB subgroups may enrich the ambient molecular cloud material before a next round of star formation is initiated. This process is actually going on in various sites in the Orion A and B clouds (e.g. Brown et al. 1994).

OB associations in Orion probably display a substantial variation in their heavy element abundances among stars that form within the first OB association and those that form just before the association disperses. For instance, the oldest of four subgroups observed in the Orion OB1 association appears to have oxygen abundances which are lower by about 40% compared to the younger subgroups while C and N abundances are identical within the observational errors (see Gies & Lambert 1992; Cunha & Lambert 1992). The finding that C and N abundances are similar for all subgroups studied in OB1 (Cunha & Lambert 1994) is consistent with the idea that these elements are returned predominantly by stars less massive than those responsible for the oxygen enrichment (e.g. Timmes et al. 1995).

The winds and radiation from OB stars in the Trapezium cluster interact with the dense gas of the cloud and may be inducing another round of star formation in the cloud. As the total mass of the cloud appears less than an order of magnitude larger than the total mass in stars, star formation appears to be highly efficient (Cunha & Lambert 1992). These authors suggest that additional oxygen observed in the Trapezium stars is the product of explosive nucleosynthesis immediate prior to its formation. Since the Orion OB association is at the edge of the molecular cloud complex the ejecta of supernovae are expected to reach larger distances in the low density gas and such enrichment may be expected.

Apart from observational support for sequential enrichment in nearby star forming regions, there are strong indications that the molecular cloud out of which the Sun formed has been enriched sequentially as well. Studies related to extinct radioactive nuclides such as $^{53}$Mn both in the Sun and in meteorites suggest that the protosolar molecular cloud has been enriched by high mass stars from a preceding OB association, about 10–25 Myr prior to the actual formation of the Sun (e.g. Cameron 1993; Swindle 1993). These studies imply that the Sun formed from material metal-rich compared to the ambient ISM at that time. The estimated time scale of 10–25 Myr between cloud enrichment and actual formation of the Sun is similar to that for sequential star formation inferred from other observations. This suggests that the formation of the Sun has been initiated by an evolved massive star and that sequential stellar enrichment of the protosolar cloud occurred. In turn, this would imply that the typical abundances of 0.15 dex in [M/H] below solar, observed for the vast majority of the gas and young stars present in the SNBH, are not biased by infall of metal-poor gas but, more likely, are the result of the self-enrichment of the protosolar nebula.

### 5.5.3 Implications for Galactic chemical evolution

We have argued that infall of metaldeficient, material onto the disk is required to explain the observed abundance variations among similarly aged stars throughout the Galactic disk. Although there is a wealth of observations supporting the presence of high-velocity gas clouds at high Galactic latitudes (see above), the origin of this infalling material is not well known. As possible nature of the gaseous material observed in the Galactic halo have been proposed: 1) the residual of a slow halo collapse in which the chemical evolution of the halo is expected to be halted at the time the collapse becomes pressure-supported and the remaining gas settles to a disk, 2) accretion of intergalactic gas (Oort 1970; Hulsbosch & Oort 1973) in combination with the peculiar motion of the Galaxy in intergalactic space, 3) condensation of gas in a Galactic fountain flow (Shapiro & Field 1976; Bregman 1980), and 4) gas stripping from nearby galaxies by tidal interaction with the Galaxy, e.g. such as the Magellanic Stream (Gardiner et al. 1994; Wollf et al. 1995).

Observations support the idea that some HVCs are of extragalactic origin (e.g. McGee et al. 1983; Mirable & Morris 1984; West et al. 1985; Mirabel 1989; Songaila et al. 1989), at least at distances above the Galactic plane of $\gtrsim 10–15$ kpc. The presence of large amounts of neutral hydrogen in the Galactic halo up to such distances and beyond seems difficult to explain by ejection of material into the halo by massive stars formed in the disk. Probably, the region of the Galactic halo which is contaminated with enriched gas by supernova-driven winds out of the disk ISM (e.g. Norman & Ikeuchi 1989) is distinct from the gas which extends up to the distance of the Magellanic Clouds and beyond.

In the Galactic fountain model, HVCs of neutral hydrogen can condense from a hot, dynamic corona above the plane of the Galaxy perhaps up to distances of 30 kpc above the plane (Shapiro & Field 1976; Bregman 1980; Wakker & Bregman 1994). In the model favoured by these authors, hot gas from superbubbles (arising from multiple supernovae in OB associations; e.g. Garmany 1994) escapes from the Galactic disk, cools radiatively when it moves upwards, eventually recombines, and spirals down to the disk about 3-60 $10^7$ yr after its ejection out of the Galactic plane. The model predicts a total mass flux of HVCs onto the disk of $\sim 2.5 \ M_\odot$ yr$^{-1}$ with typical HVC masses of $5 \times 10^7$ to $5 \times 10^9 \ M_\odot$ (see also Schulman et al. 1994).

The fact that most HVCs show signatures of Galactic rotation implies that they have distances greater than $\sim 3$ kpc (e.g. Dickey & Lokajmin 1990). In this part of the halo, gas exchange with the disk ISM
probably makes a dominant contribution to the enrichment of the halo gas. If this model is correct, a substantial fraction of the HVCs may be metal-rich and induce abundance inhomogeneities (similar to those caused by sequential stellar enrichment) when they interact with the disk ISM. Interestingly, the mass of H\textsubscript{I} contained in HVCs in external galaxies appears to be correlated with the amount of star formation as traced by the IRAS far-IR flux (e.g. Schulman et al. 1994). Whether this is due to star formation initiated by the impacts of HVCs and/or is due to a relation between the total mass contained in HVCs and the SFR is yet unclear.

In contrast, the outer part of the Galactic halo at distances of \(
\sim 30\ \text{kpc}\) and above may have abundances similar to that found in the Magellanic Clouds and in fact may extend to distances up to \(
\sim 200\ \text{kpc}\). Massive haloes of spiral galaxies with luminosities similar to that of the Galaxy extend out to radii of 200 kpc (e.g. Zaritsky et al. 1989, 1993). This gas is usually not seen as neutral hydrogen but becomes ionized by the intergalactic radiation field when the opacity in the outer disk regions becomes very low. In the Milky Way this might occur at distances of at least 10-15 kpc above the Galactic plane (e.g. Savage & de Boer 1988; Kutyrev & Reynolds 1989). Below these distances, H\textalpha emission presumably is due to recombination after a shock wave created by the HVC/hot gas interaction (Kutyrev & Reynolds 1989; Ferrara & Field 1994).

The chemical composition of the outer halo gas may be traced by the interaction of the Magellanic Stream (MS) with hot halo gas. Abundances in the MS are similar to lowest abundances found in HVCs, i.e. less than \(
\sim 0.1\ \text{Solar}\) (Schwarz et al. 1995). The density concentrations observed in the Magellanic Stream (Mathewson et al. 1977) have been suggested to be due to the interaction of the gas inbetween the Clouds and that in the Galactic halo (Mathewson et al. 1987; Wayte 1991). The presence of gas in the outer halo of the Galaxy may be important for the motion of the MS by means of hydrodynamic effects (Irwin 1991). An interesting explanation has been proposed by Liu (1992) who showed that a cloud of concentrations of cold hydrogen gas at the approximate positions of the observed concentrations along the MS could be explained by a model in which gas is stripped from the region between the LMC and SMC and cools in the wake of hot halo gas behind the receding Magellanic Clouds. The MS itself contributes as well to the infall of hydrogen gas onto the Galactic disk: the tip of the stream near the South Galactic Pole has a high infall velocity of about 380 km s\(^{-1}\) (Wannier & Wrixon 1972; Murai & Fujimoto 1980). Therefore, gas stripping from nearby galaxies also may provide an explanation for gas clouds falling onto the disk.

Clearly, the temporal behaviour of the rate of gas infall onto the disk and the origin of the infalling matter are not well known. The majority of the observations suggest an external origin for most of the material contained in HVCs. This is consistent with our results indicating that a substantial fraction of matter infalling from the halo must have abundances typical to that of halo stars with \([\text{M/H}]\sim -1\), in order to explain the observed stellar abundance variations. On the other hand, a local origin of the HVCs related to the ejecta of massive stars in the disk would imply these clouds to be more metal-rich. In this case, the effect of metal-rich infall on the chemical evolution of the disk ISM would be very similar to that of sequential stellar enrichment except that the halo residence time for such clouds would be considerably longer than that for star formation initiated directly in the disk ISM.

There are several indications that the most prominent star forming regions in the SNBH, e.g. the Orion molecular cloud and the Gould belt, have been partly formed from (and undergo star formation induced by) infalling clouds from the Galactic halo (Franco et al. 1988; Comerón & Torra 1992; Edvardsson et al. 1995). Molecular clouds may form at high Galactic latitudes due to the rapid cooling and high densities achieved in the shocked layer resulting from the impact of a high-velocity cloud (HVC) with the Galactic disk or halo ISM. It is interesting to note that high-altitude molecular clouds have been observed recently in \(^{12}\text{CO}(1-0)\) (Malhotra 1995).

Hydrodynamical models for infall induced star formation events suggest that, although the accretion of infalling gas onto the shocked disk ISM layer is a continuous process during the impact, the formation of an OB cluster could inhibit star formation for a few times \(10^6\) yr until the threshold density for star formation at these high Galactic latitudes is reached again (see Lépine & Duvert 1994; Comerón & Torra 1994). After the formation of the OB cluster, the remaining part of the infalling cloud may continue to interact with the disk ISM, initiating another round of star formation, until the velocity dispersion of the cloud matter has become similar to that of the ambient ISM after \(~10^7\) yr (Lépine & Duvert 1994). It appears that such models naturally can explain the observed age sequence and spatial alignment of the OB associations in OMC1. The time scale estimated for the impact of the HVC with the disk ISM is \(~10^7\) yr, depending on the cloud velocity, direction, and density contrast with the disk ISM. The fact that the OB associations observed in Orion are displaced from the center of the molecular cloud may be a strong indication for the impact by a HVC (see above).

In the infall induced star formation model, successive stellar generations are born on time scales determined by the velocity of the HVC, the resulting density contrasts within the disk ISM, and the amounts of gas consumed by previous star formation events during the impact. In this case, there is no direct causal
effect of one group of stars upon the other although massive stars belonging to the previously formed generation may catalyze the next star formation event. While in the traditional view of sequential star formation massive stars are required to induce a next round of star formation, this is not the case for the impact of an HVC with the disk ISM (Lépine & Duvert 1994). In order to preserve the chemical inhomogeneities caused by infall of metal-poor gas for the next generation of stars, infall must induce star formation on time scales short compared to the local mixing time scale. This condition likely is fulfilled during the impact of a HVC with the disk ISM.

Apart from the possibility of sequential star formation caused by infalling clouds, sequential enrichment also may occur when expanding (and/or shock driven) stellar ejecta break out of the surface of their parent molecular cloud core and induce star formation at the interference zones with the ambient disk ISM, i.e. at the cloud edges. Sequential star formation is expected to occur preferentially in regions where density inhomogeneities are generated by the interaction of large amounts of swept up matter so that the critical conditions for star formation can be reached. Such conditions, i.e. for the collapse of high density gas under gravitational instabilities, are likely to be met in the spiral arms and nuclei of galaxies.

As discussed above, sequential enrichment may be triggered by OB-associations or by single massive stars. Efficient sequential enrichment will take place when newly synthesized heavy elements returned by a generation of massive stars are mixed to relatively small amounts of material in which star formation is induced. This is especially true in case: 1) the enriched stellar material is returned to the ambient ISM in collimated outflows and/or irregularly expanding shells, and 2) the time scale for star formation initiated by these outflows is shorter than the local mixing time scale so that stellar enrichment is restricted mainly to the region in which star formation is induced. These conditions are met in particular when star formation within the same cloud core continues over an extended period of time so that the first massive stars formed are able to enrich the material accumulating at the star forming core. In general, the effect of sequential stellar enrichment depends on the detailed mass-loss history of the actual generation of massive stars enriching the surrounding cloud material before the cloud core disperses.

Self-enrichment may also be important within the expanding shells associated with giant HII regions around young star clusters (Kunth & Sargent 1986; Pilyugin 1992). In this case, heavy elements originating from stellar winds and supernovae mix exclusively with the ionized gas within the expanding HII region. Depending on the dispersion of the heavy elements, efficient self-enrichment may occur when the expanding shell enters a new episode of star formation. Initiation of star formation within the expanding shell is determined by the detailed structure of the ambient ISM together with the underlying cluster of massive stars driving the expansion. This process has been suggested also to operate in globular clusters when a second generation of stars forms in the expanding shell around an earlier generation of stars (Brown 1991), and in case of supershell induced star formation around evolving OB associations (McCray & Kafatos 1987; Elmegreen 1992). The abundance inhomogeneities observed among similarly aged open clusters (e.g. Carraro & Chiosi 1994), as well as among stars within a given open cluster (e.g. Kilian-Montenbruck et al. 1994), may be due to a similar process of self-propagating star formation.

5.5.4 Concluding remarks

In this paper, we have presented both theoretical and observational arguments in support of combined metal-deficient gas infall and sequential stellar enrichment in the local disk ISM. We have shown that these processes can provide an adequate explanation for the abundance inhomogeneities observed among similarly aged stars in the SNBH and may play an important role for the inhomogeneous chemical evolution of the Galactic disk. In addition, these processes probably affected the star formation history and chemical evolution of the Galactic halo as suggested by the significant abundance variations observed among metal-poor halo stars (e.g. Bessell et al. 1991; Nissen et al. 1994; Sect. 4.3). Also, abundance inhomogeneities of 20.8 dex among similarly aged globular clusters in the Galactic halo have been discussed recently by Chaboyer et al. (1996). Although part of these abundance inhomogeneities may be associated with the accretion of globular clusters from nearby galaxies by the Galaxy (such as the Sgr dwarf galaxy), a substantial fraction of these clusters probably formed in the outer halo during the early collapse of the proto-Galactic cloud as indicated by the mean age-metallicity relation observed among such clusters. In this context, it seems likely that sequential stellar enrichment and metal-poor gas infall are at least in part responsible for the large spatial abundance fluctuations observed in massive disk galaxies (e.g. Roy & Kunth 1994).

Notwithstanding the results presented in this paper, combined metal-poor gas infall and sequential stellar enrichment may be a too schematic picture of the complex set of processes directing the chemical evolution of the Galactic disk. In particular, merger events with small companion galaxies may be important as well (e.g. Quinn et al. 1993). In such cases, both gas and stars in the companion galaxy may add substantially to the observed abundance inhomogeneities in the Galactic disk (Pilyugin & Edmunds 1996c).
However, the suggestion that stellar populations from merging companion galaxies contribute substantially to the observed stellar abundance variations in the Galactic disk seems difficult to reconcile with: 1) the apparent homogeneous distribution of these variations within the metallicity range observed at a given age of the disk, and 2) the relatively small scatter observed in the element-to-element variations for stars in the SNBH. Instead, we argue that most of the stars present in the Galactic disk formed from gaseous material accumulated in the disk and that the observed stellar abundance variations are due to inhomogeneities in the ISM rather than merging of stellar populations with independent star formation and chemical evolution histories. In either case, merging may be important for the chemical evolution of the Galaxy both by adding large amounts of predominantly metal-poor material and by initiating star formation in the disk.

Additional processes which probably contribute to the inhomogeneous chemical evolution of the Galactic disk include: 1) ejection of enriched material into the halo generating local abundance variations in the disk ISM, and 2) stellar orbital diffusion. We argue, however, that the time scales for these processes required to cause substantial abundance variations in the disk ISM are often much larger than those indicated by the observations. Therefore, we believe that such processes are unlikely to be the main cause for the observed abundance variations.

Inhomogeneous chemical evolution due to sequential stellar enrichment and/or metal-poor gas infall is probably important also in nearby galaxies such as the Magellanic Clouds and M31. In the Large Magellanic Cloud, large abundance variations of $\sim 0.4-0.8$ dex in [Fe/H] among similarly aged open clusters are observed (e.g. Cohen et al. 1982; Da Costa 1991; Olsewski et al. 1991). Part of the variations may be accounted for by a radial gradient of $\sim 0.15$ dex kpc$^{-1}$ in [Fe/H] (Kontizas et al. 1993). However, the main part of these variations is likely due to triggered star formation in supershells as indicated by the close association of HII complexes with large HI holes observed in the LMC (Dopita 1985; Lortet & Testor 1988; Meaburn et al. 1991). In addition, gas infall may have affected the chemical evolution of the tidally interacting Magellanic Clouds.

Observational evidence in support of shock-induced star formation by SNII in the spiral arms of M31 has been presented by Magnier et al. (1992). At these sites, young OB stars are observed to initiate recent star formation so that large abundance inhomogeneities due to sequential stellar enrichment are expected, similar to those observed among OB associations in the Orion star forming cloud complex in our own Galaxy.

Stellar and nebular abundance indicators reveal that substantial abundance fluctuations exist in the ISM of gas-rich galaxies (e.g. Roy & Kunth 1995). For instance, abundance inhomogeneities in metal-poor galaxies such as IZw 18 may be among the largest observed in external galaxies (e.g. Kunth et al. 1995) although this is still highly uncertain (Pettini & Lipman 1995). Whether the abundance fluctuations observed in dwarf galaxies are due to variations in self-enrichment of the HII-regions in these systems (e.g. Pilyugin 1992) and/or are related to selective loss of metals through galactic winds driven by massive stars (Roy & Kunth 1995; Martin 1996) is unclear.

We expect that sequential stellar enrichment is generally inefficient in dwarf galaxies because of their low gas densities, and that the effect of metal-poor gas infall on the stellar abundance variations is weak due to their low ISM abundances. Instead, star formation and abundance inhomogeneities induced by metal-rich gas infall associated with previous star formation may be relatively important in these systems.

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Appendices

A  A model for the inhomogeneous chemical evolution of a star forming gas cloud

We describe the adopted model for the inhomogeneous chemical evolution of a star forming gas cloud. The model can be applied to various mass scales, e.g. to the entire system of molecular cloud complexes in the Galactic disk or to the star forming core regions within a single molecular cloud. We start from a homogeneous, metal free gas cloud with a total mass $M_{\text{cl}}$. At any evolution time $t$ in its evolution, this cloud is subdivided into $N_{\text{scl}}$ active subclouds (with corresponding masses $M_{\text{scl}}$) involved with star formation and an inactive cloud part (with mass $M_{\text{ret}}$) not involved with star formation. We assume matter to be freely exchanged within the inactive cloud part. Each subcloud $i$ is formed at corresponding evolution times $t_{\text{sf}}^i$ and is allowed to follow its individual star formation, mixing, and infall history.

A.1 Model description, definitions and assumptions

During the lifetime $t_{\text{ev}}$ of the star forming gas cloud a total number $N_{\text{sf}}$ star formation events is assumed occur. Each star formation event $j$ presumably occurs within an active subcloud $i$. We define $N_{\text{sf}}^i$ as the total number of star formation events within subcloud $i$. For the reference model $N_{\text{sf}}^i = 1$ and each star formation event $j$ occurs in corresponding subcloud $i = j$. Subclouds are allowed to experience numerous star formation events, i.e. $N_{\text{sf}} > 1$. During each star formation event $j$ at time $t = t_{\text{sf}}^j$ within subcloud $i$, a total mass of gas $\delta M_{\text{scl}}^i = \epsilon^j M_{\text{scl}}^i (t_{\text{sf}}^j)$ is transformed into stars.

We define $\Delta t_{\text{disp}}^i$ as the time between the onset of star formation within a subcloud core and the complete dispersal of this core region by supernova explosions and/or stellar winds. During $\Delta t_{\text{disp}}^i$ the subcloud core is assumed to form stars. The profile of the star formation rate (SFR) during $\Delta t_{\text{disp}}^i$ is assumed constant and identical for all star formation events. However, quantities such as the minimum stellar mass formed and IMF-slope are allowed to vary from one star formation event to another (cf. Sect. 5.4.2). The subcloud core dispersal time determines the mass of the most massive star that is able to enrich subcloud cloud material before the core ultimately breaks up. At time of core dispersal, the newly formed generation of stars has returned an amount of material $\delta M_{\text{ret}}^i$. Accordingly, the net amount of material converted into stars during star formation event $j$ at time $t = t_{\text{sf}}^j$ within subcloud $i$, is given by: $\delta M_{\text{scl}}^i = \epsilon^j M_{\text{scl}}^i (t_{\text{sf}}^j) - \delta M_{\text{ret}}^i$.

Subclouds $M_{\text{scl}}$ are formed from the inactive cloud ISM at cloud evolution times $t = t_{\text{scl}}$. When a subcloud forms it adopts the abundances of the inactive cloud ISM at $t = t_{\text{scl}}$. For each subcloud, we define a mixing time scale $\Delta t_{\text{mix}}$ as the time between formation of the subcloud and the actual break up of the entire subcloud. The instant of break up of the subcloud may be either after one or more star formation events, or before star formation actually takes place. In this manner, material can be deposited within a subcloud region for a considerable period of time before being mixed to the surrounding ISM. The mixing history of each subcloud directs both the inhomogeneous chemical evolution of the inactive cloud and that of the neighboring subclouds.

Before an entire subcloud breaks up its constituent material will be enriched by the stellar populations it is hosting. We assume the stellar enrichment of the subcloud to proceed homogeneously. In order to allow for sequential enrichment, we consider a fraction $\lambda^j$ of enriched material ejected during star formation event $j$ to mix homogeneously with subcloud core material hosting the next star formation event. Simultaneously with the ejection of enriched material returned by newly formed stars, a substantial fraction of the ambient subcloud matter $\kappa^j M_{\text{scl}}$ may be swept up during dispersal of its star forming core. This subcloud material may mix to the subcloud hosting the next star formation event as well. The subcloud hosting the next star formation event may be either the subcloud hosting the current star formation event or a subcloud nearby. No matter exchange is assumed between the subcloud and the surrounding ISM during the time between two star formation events occurring within one and the same subcloud. In case of the reference model, we do not consider mass transfer between subclouds, i.e. $\lambda^j = \kappa^j = 0$.

After an entire subcloud breaks up its mass is assumed to mix homogeneously to the inactive cloud part. At the same time, stars associated with the dispersing subcloud become part of the stellar populations in the inactive cloud. After break up, different cloud fragments present in the ambient ISM may form new subclouds wherein star formation occurs as soon as the critical conditions for star formation are met.
In addition to the individual chemical evolution of subclouds, which is directed by their star formation history and exchange history with the surrounding ISM, we allow for local enrichment of a given subcloud by stars that were not formed within that subcloud. This may be particularly important for low mass SNIa-progenitors which travelled considerable distances from their birth sites and enrich their immediate surroundings at the time they explode as SNIa (cf. Fig. 5.2f; see below).

A.2 Basic equations

We keep track of the total mass of and abundances in stars and gas as a function of evolution time, both within each subcloud and the inactive cloud ISM. For each star formation event we use conventional chemical evolution model equations (e.g. Tinsley 1980, see below) except for including metallicity dependent stellar lifetimes, remnant masses and element yields (cf. van den Hoek et al. 1996).

Mass-exchange between subclouds and the inactive cloud ISM

We denote $\Delta Q$ as the variation of a quantity $Q$ between two cloud evolution times $t - \Delta t$ and $t$. With $M_{\text{cl}}(t = 0)$ the initial mass of the cloud and no stars initially present, i.e. $M_*(0) = 0$, we can express the variations of mass of gas and stars within the cloud as:

$$
\Delta M_{\text{cl}} = \Sigma_{i=1}^{N_{\text{cl}}} \Delta M_{\text{cl}}^i + \Delta M_{\text{qcl}} \tag{A1}
$$

$$
\Delta M_* = \Sigma_{i=1}^{N_{\text{cl}}} \left( \Delta C_*^i - \Delta E_*^i \right) - \Delta E_* \tag{A2}
$$

where $N_{\text{cl}}(t)$ is the current number of individual subclouds, $N_{\text{sf}}(t)$ the current number of star formation events within the cloud, $\Delta C^j$ the total mass of stars formed during star formation event $j$, and $\Delta E^j$ the total mass of matter returned within $\Delta t$ by stars formed during star formation event $j$. We recall conventional expressions for $\Delta C^j$ and $\Delta E^j$ (cf. Tinsley 1980):

$$
\Delta C^j = \int_{t_{\text{sf}}}^{t_{\text{sf}} + t_{\text{disp}}} \int_{m_{\text{u}}}^{m_{\text{a}}} m S_j(t) M_j(m) \, dmdt \tag{A3}
$$

$$
\Delta E^j = \int_{t - \Delta t}^{t} \int_{m_{\text{disp}}(t-t_{\text{disp}})}^{m_{\text{rem}}(t)} (m - m_{\text{rem}}(m)) S_j(t - \tau(m)) M_j(m) \, dmdt \tag{A4}
$$

where $S_j$ and $M_j$ denote the SFR by number $[\text{yr}^{-1}]$ and IMF $[\text{M}_\odot^{-1}]$ for star formation event $j$. For convenience, we ignored the index $j$ for $t_{\text{sf}}$, $t_{\text{disp}}$ as well as for the stellar mass boundaries at birth $m_u$, $m_a$. We emphasize that both the stellar remnant masses $m_{\text{rem}}(m)$, lifetimes $\tau(m)$, and turnoff-masses $m_o(t)$ are a function of the initial metallicity $Z_*$ (containing all elements heavier than He) of the stellar generation under consideration. Variations in the total gas masses within the inactive cloud and subcloud $i$, i.e. $M_{\text{qcl}}$ and $M_{\text{cl}}^i$ respectively, can be expressed as:

$$
\Delta M_{\text{qcl}} = \Delta E_* - \Sigma_{\text{form}}^k M_{\text{cl}}^k + \Sigma_{\text{disp}}^l M_{\text{cl}}^l \tag{A5}
$$

$$
\Delta M_{\text{cl}}^i = \left[ \Delta E_* - \Delta C_*^i \right] + \Delta M_{\text{sf,prev}} - \Delta M_{\text{sf,next}} \tag{A6}
$$

$$
\Delta M_{\text{cl}} = \Sigma_{i=1}^{N_{\text{cl}}} \Delta M_{\text{cl}}^i + \Sigma_{\text{form}}^k M_{\text{cl}}^k - \Sigma_{\text{disp}}^l M_{\text{cl}}^l \tag{A7}
$$

where $\Delta E_* \text{qcl}$ refers to the amount of material returned by stars present in the inactive cloud within time $\Delta t$. We followed both the stellar ejecta from recently formed stars within active subclouds and the ejecta from older stellar populations present in the inactive cloud ISM. The total amount of gas depleted by subclouds which are formed within time $\Delta t$ is denoted by $\Sigma_{\text{form}}^k M_{\text{cl}}^k$. Similarly, the amount of gas returned by subclouds which become dispersed within time $\Delta t$ is denoted by $\Sigma_{\text{disp}}^l M_{\text{cl}}^l$. We remark that the term between square brackets in Eq. (A6) refers to star formation events which occur within subcloud $i$.

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The term $\Delta E_*^i$ in Eq. (A6) is related both to stellar generations which formed within subcloud $i$ and to stars that entered the subcloud from elsewhere in the cloud. We will consider the case of SNIa progenitors stars only. Consequently, the term $\Delta E_*^i$ can be expressed as two terms, i.e. $\Delta E_*^i = \Delta E_* \text{prev}^i + \Delta E_{\text{SNIa}}$. The former term is related to star formation events which occurred within subcloud $i$ while the latter term is associated with subcloud enrichment by SNIa-progenitors formed elsewhere in the cloud. We define the total
amount of matter returned by SNIa within subcloud \( i \) during time \( \Delta t \) as: \( \Delta E_{\text{SNIa}} \equiv \alpha_{\text{SNIa}} \Delta t R_{\text{SNIa}} \Delta m_{\text{rem}}(m) \) where \( R_{\text{SNIa}} \) is the total average SNIa-rate in the entire cloud and \( \alpha_{\text{SNIa}} \) the corresponding fraction of SNIa that is assumed to go off within subcloud \( i \). In case of the reference model \( \Delta E_{\text{SNIa}} = 0 \).

**Mass-exchange between individual subclouds**

As matter may be transferred from one subcloud to another (or within one subcloud from one subcloud core to another) we include terms \( \Delta M_{\text{sf}, \text{prev}} \) and \( \Delta M_{\text{sf}, \text{next}} \) in Eq. (A6). The term \( \Delta M_{\text{sf}, \text{prev}} \) corresponds to the amount of material added from the preceding star formation event to the core of the subcloud currently experiencing star formation. The term \( \Delta M_{\text{sf}, \text{next}} \) refers to the amount of matter mixed from the subcloud core actually experiencing star formation to the subcloud hosting the next star formation event. For each star formation event \( j \) which happens to occur in subcloud \( i \) within the time interval \( \Delta t \) we may write:

\[
\begin{align*}
\Delta M_{\text{sf}, \text{prev}} &= \lambda_i^{-1} \delta M_{\text{ret}}^i + \kappa_{i}^{-1} M_{\text{scl}, \text{prev}} \\
\Delta M_{\text{sf}, \text{next}} &= \lambda_i^{j} \delta M_{\text{ret}}^i + \kappa_{i}^{j} M_{\text{scl}}^i
\end{align*}
\]

where \( M_{\text{scl}, \text{prev}} \) is the mass of the subcloud hosting the preceding star formation event. In this paper, we presented only results for \( \kappa_i^j = 0 \). In general, \( \kappa_i^j \prec 0 \) has a similar effect as when reducing the sequential enrichment efficiency \( \lambda_i \).

**Chemical evolution of subclouds and inactive cloud ISM**

Expressions for the average abundance changes of element \( X \) within the entire cloud, inactive cloud part, and subclouds can be written as:

\[
\begin{align*}
\Delta(X_{\text{cl, Mcl}}) &= \sum_{i=1}^{N_{\text{scl}}} \Delta (X_{\text{scl, Mcl}}^i) + \Delta (X_{\text{qcl, Mqcl}}) \\
\Delta(X_{\text{qcl, Mqcl}}) &= \Delta E_{X_{\text{qcl}}} - \Sigma_{N_{\text{scl}}} \Delta \Sigma_{Q_{\text{scl, Mcl}}} \\
\Delta(X_{\text{scl, Mcl}}) &= \sum_{i=1}^{N_{\text{scl}}} \Delta (X_{\text{scl, Mcl}}^i) + \Delta \Sigma_{Q_{\text{scl, Mcl}}} \\
\Delta(X_{\text{scl, Mcl}}^i) &= \Delta E_X - X_{\text{scl, Mcl}} \Delta C^i_k \Delta M_{\text{scl}, \text{prev}} - \Delta M_{\text{scl}, \text{next}}
\end{align*}
\]

where the meaning of each term can be found from its counterpart in Eqs. A5-A7. Similarly, expressions for \( \Delta M_{X, \text{prev}} \) and \( \Delta M_{X, \text{next}} \) can be written as:

\[
\begin{align*}
\Delta M_{X, \text{prev}} &= \lambda_i^{j} \delta M_X^i - (X_{\text{scl, Mcl}}^i) \text{prev} \\
\Delta M_{X, \text{next}} &= \lambda_i^{j} \delta M_X^i + \kappa_{i}^{j} M_{\text{scl}}^i
\end{align*}
\]

where \( \Delta E_X^i \) is the total mass of enriched material of element \( X \) returned within time \( \Delta t \) by a stellar generation formed during star formation event \( j \):

\[
\Delta E_X^i = \int_{t - \Delta t}^{t} \int_{m_{\text{disp}}}^{m_{\text{ret}}} \Delta M_X(m) S_{j}(t - \tau(m)) M_{j}(m) \text{ dmdt}
\]

\[
\Delta M_{X}(m) = m p_{X}(m) + (m - m_{\text{rem}}(m)) X_{\star}^j
\]

and \( \Delta M_{X}(m) \) is the total mass of element \( X \) ejected by a star of initial mass \( m \) born with metallicity \( X_{\star}^j \) during star formation event \( j \). The term \( \Delta M_{X}(m) \) includes both newly synthesized stellar material and matter initially present at the time stars were formed. Initial stellar abundances \( X_{\star}^j \) are determined by the abundances of the subcloud \( i \) (hosting star formation event \( j \)) at time \( t_{\text{sj}}^j \), i.e. \( X_{\star}^j = X_{\text{scl}}(t = t_{\text{sj}}^j) \). Literature sources for the adopted theoretical metallicity dependent stellar yields \( p_{X}(m) \), stellar lifetimes \( \tau(m) \), and remnant masses \( m_{\text{rem}}(m) \) are given in Sect. 5.3.4.
B Basic effects of metal-deficient gas infall and sequential stellar enrichment

Our goal is to estimate the amount of material to be mixed homogeneously to a star forming gas cloud in order to explain abundance variations by mass of $10\log (M/H) > 0.6$ dex relative to the initial abundances of the cloud. Such abundance variations can be achieved either by mixing metal-poor or metal-rich material to the cloud. We define $\delta_{M_i}$ as the ratio of hydrogen mass in the added material and that initially present in the star forming region: $\delta_{M_i} = M_{i\text{add}}/M_{i\text{ini}}$. Similarly, we define $\beta_i$ as the mass-ratio of element $i$, relative to hydrogen, of the added material and that of the star forming region: $\beta_i = (M_i/H)_{\text{add}} / (M_i/H)_{\text{ini}}$. Consequently, the logarithm of the ratio of the abundances of stars formed before and after mixing material to the star forming region, can be expressed as:

$$\alpha_i = 10 \log \left( \frac{1 + \beta_i \delta_{M_i}}{1 + \delta_{M_i}} \right) \quad (B1)$$

We tabulate values of $\alpha_i$ for different combinations of $\delta_{M_i}$ and $\beta_i$ in Table 5.5. We consider the observed abundance scatter of $\sim 0.6$ dex in $[O/H]$. We assume that the initial cloud abundances are equal to the mean stellar oxygen abundances observed at a given age. This implies variations in $\alpha_O$ of $\pm 0.3$ dex. Alternatively, we could have assumed that the initial cloud abundances are equal to the largest stellar oxygen abundances observed (i.e. variations in $\alpha_O$ down to $-0.6$ dex) or equal to the smallest abundances observed (i.e. variations in $\alpha_O$ up to $+0.6$ dex). Since the mean interstellar oxygen abundances in the SNBH are not well known, we simply illustrate the effect of mixing material that is needed to achieve stellar abundance variations of $\pm 0.3$ dex. Stellar abundance variations larger than $\pm 0.3$ dex will lead to more extreme values of $\delta_{M_i}$ and $\beta_i$ than discussed below.

B.1 Mixing of metal-deficient material

Table 5.5 illustrates that stellar abundance variations $\alpha_i$ of about $-0.3$ dex can be obtained by mixing metal-deficient material with abundance ratios $\beta_i \lesssim 0.1$ and hydrogen mass-ratios of $\delta_{M_i} \gtrsim 1$. In this case, mixing of material metal-poor by one order of magnitude is needed to explain abundance variations of $-0.3$ dex for comparable amounts of hydrogen within the added material and (initially present) within the star forming cloud. Similarly, abundance variations of $-0.3$ dex can be reached also by mixing very large amounts of material that is less metal-deficient with respect to the initial abundances in the star forming region. We conclude that in order to explain stellar oxygen abundances below the mean value of $[O/H]$ observed at a given age (see Fig. 5.1), mixing of gas with an underabundance of at least one order of magnitude relative to the ambient ISM would be required. Such mixing would imply infall (or accretion) of metal-deficient material and/or inefficient mixing of parcels of interstellar gas over extended periods of time during the lifetime of the disk.

We emphasize that the effect of adding more and more metal-deficient material to a star forming region strongly depends on the abundances within the diluting material. We illustrate this in Fig. 5.5 where we show the effect of mixing metal-poor material (with the lowest stellar abundance ratios observed, i.e. $[Fe/H] \sim -1$ and $[O/H] \sim -0.65$) to gas with the highest stellar abundance ratios observed (i.e. $[Fe/H] \sim -0.23$ and $[O/H] \sim 0.18$). Interestingly, the result of such dilution of the stellar abundances appears consistent with the mean $[Fe/H]$ vs. $[O/H]$ relation observed (see Fig. 5.11 below). As an example, Fig. 5.5 illustrates the maximum stellar abundance-abundance variations which may occur when metal-poor gas is mixed to the disk ISM. To this end, we consider a given mean $[P/H]$ vs. $[Q/H]$ evolution trajectory in the disk ISM for two elements P and Q (e.g. Fe and O). Fig. 5.5 shows the stellar abundance-abundance variations due to infall of metal-deficient gas for various metallicities. At a given ratio $[P/H]$, the separation between the abundance contour resulting from mixing metal-deficient gas and that the mean trajectory assumed in the

**Table 5.4** Theoretical abundance variations $\alpha_i$ as a function of $\delta_{M_i}$ and $\beta_i$

<table>
<thead>
<tr>
<th>$\delta_{M_i}$</th>
<th>$\beta_i = 0$</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.02</td>
<td>0.0</td>
<td>+0.04</td>
<td>+0.14</td>
<td>+0.26</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.18</td>
<td>-0.16</td>
<td>-0.08</td>
<td>0.0</td>
<td>+0.13</td>
<td>+0.37</td>
<td>+0.60</td>
</tr>
<tr>
<td>1</td>
<td>-0.30</td>
<td>-0.26</td>
<td>-0.13</td>
<td>0.0</td>
<td>+0.18</td>
<td>+0.48</td>
<td>+0.74</td>
</tr>
<tr>
<td>10</td>
<td>-1.04</td>
<td>-0.74</td>
<td>-0.26</td>
<td>0.0</td>
<td>+0.28</td>
<td>+0.67</td>
<td>+0.91</td>
</tr>
</tbody>
</table>
Appendix B: Basic effects of mixing metal-poor and metal-rich gas

5 Appendix B: Basic effects of mixing metal-poor and metal-rich gas

As expected, the resulting stellar abundance variations due to metal-poor gas infall are relatively large when: 1) the infall abundance ratios [P/Q]_{inf} that are far from the enrichment trajectory followed by the disk ISM, and 2) the abundance-ratios within the disk ISM are large (we assumed [P/H]_{disk}=[Q/H]_{disk}=+0.3). In general, the theoretical abundance-abundance contours shown in Fig. 5.10 are inconsistent with the observations (see Fig. 5.1). This suggests that the abundance ratios within the infalling gas are usually close to the enrichment trajectory followed by the disk ISM and/or that metal-poor gas infall is not entirely responsible for the abundance variations observed among similarly aged stars in the SNBH.

B.2 Mixing of metal-rich material

Similar to the case of mixing metal-deficient material, stellar abundance variations of +0.3 dex can be achieved by adding enriched material with $\beta_i > \sim 10$ and a mass of about 10% of total initial mass of the star forming region (cf. Table 5.5). Alternatively, variations of +0.3 dex can be explained also by mixing large amounts ($\delta M_H$ $\sim$ 10) of moderately enriched material (e.g. $\beta_i > \sim 2$). Since efficient mixing in the disk ISM probably rules out the latter possibility (see Sect. 5.5.3), we will concentrate on abundance variations which are caused by adding small amounts of very metal-rich material to a star forming region. Such concentrations of metal-rich material are most readily produced within the ejecta of massive stars (observations pointing to sequential stellar enrichment in the local ISM are discussed in Sect. 5.5).

We examine the stellar abundance variations $\alpha_i$ that result from mixing the ejecta of massive stars to a star forming region. Table 5.6 lists the theoretical stellar yields, defined as the total element mass ejected by a star of initial mass $m$, both for the progenitors of SNII, SNIa, and SNIb/c. For a detailed description of the yields we refer the reader to the references given in Table 2 and Chap. 3.

The final helium mass just before a star of initial mass $m$ becomes a supernova is listed as $m_{He}$. We consider helium stars surrounded by hydrogen-rich envelopes as the immediate progenitors of SNII, SNIa, and SNIb/c (see Woosley, Langer & Weaver 1994). These helium stars presumably leave a neutron star remnant of 1.4 $M_\odot$. SNIa are assumed to originate from accreting and/or coagulating WDs in a binary system. SNIa leave no remnant as the WD is assumed to disrupt completely during the explosion. Typical amounts of iron produced are $\sim$ 0.08 $M_\odot$ for SNII, $\sim$0.8 $M_\odot$ for SNIa, and $\sim$0.1 $M_\odot$ for SNIb/c (see Table 5.6).
Total oxygen and iron masses ejected by a star of initial mass $m$ with metallicity $Z = Z_\odot$ at birth are denoted by $\Delta M_O$ and $\Delta M_{Fe}$, respectively. Corresponding mean abundance ratios within the stellar ejecta (relative to solar) are given by $[O/H]_*$ and $[Fe/H]_*$, and are shown in Fig. 5.11. These were calculated assuming an initial hydrogen abundance $X = 0.68$ (cf. Anders & Grevesse 1989). In the last two columns of Table 5.6, we list the resulting abundance variations $\alpha_O$ and $\alpha_{Fe}$ when the ejecta of a star with initial mass $m$ are mixed to a gas cloud that has the same initial mass and abundances as the star. We note that the stellar abundance variations $\alpha_i$ are given relative to the abundances of the material out of which the progenitor star formed (cf. Eq. B1).

From Table 5.6 it can be seen that local enrichment of the ISM by SNII easily can account for abundance variations of $\alpha_O \gtrsim +0.3$ dex. However, in order to explain the mean $[Fe/H]$ vs. $[O/H]$ relation and to comply with the full range in $[Fe/H]$ and $[O/H]$ observed for F and G dwarfs in the SNBH (see Fig. 5.11), combined mixing of e.g. SNII and SNIa-ejecta as well as dilution of the star forming region by more metal-deficient material is required. We conclude from Table 5.6 that abundance variations of $\gtrsim 0.6$ dex in $[Fe/H]$ and $[O/H]$ easily can be achieved by local enrichment from massive stars.

To estimate the typical abundances in the ejecta of an entire generation of stars, the stellar yields in Table 5.6 (corrected for initial stellar abundances) need to be integrated over the initial stellar mass range after weighing by the stellar mass function (IMF) as well as weighing by the relative contributions of SNII, SNIa and SNIb/c. For this reason, we show in Fig. 5.11 the resulting $[Fe/H]$ vs. $[O/H]$ relation for a stellar generation in case of enrichment by SNII only.

This relation was computed after weighing the SNII yields by an IMF with slope $\gamma = -2.35$ while assuming an upper SNII mass limit of $m_{\text{SNII}}^{\text{u}} = 60 M_\odot$. The fact that the predicted relation drops substantially below that observed, clearly demonstrates that SNIa and/or SNIb/c are required to explain the observed abundance variations by means of sequential stellar enrichment. Although an extensive discussion of these IMF-weighed yields and relative contribution of SNII, SNIa, and SNIb/c on the resulting stellar abundance variations is beyond the scope of this paper, we give some examples below.

We estimate the effect of sequential enrichment on the relative abundances of two generations of stars, the second generation forming in part out of the material enriched by the first generation. For simplicity, we only consider the ejecta of SNI and compute the IMF integrated ($\gamma = -2.35$) stellar oxygen and iron yields for the first generation (assumed to have solar abundances initially). We assume that the enriched material returned by the first generation (formed with solar metallicity) is mixed homogeneously to a gas cloud (with initially solar abundances as well). Furthermore, we assume that the abundances in the enriched gas cloud are those available to the second stellar generation.

Resulting abundance differences $\alpha_i$ (see Eq. 1) between the two stellar generations are listed in Table 5.6 for various values of: 1) the cloud mass to which the stellar ejecta are mixed (defined by $M_{\text{cl}} = \vartheta M_*$ where $M_*$ is the total mass of gas initially converted into stars), 2) the upper mass limit $m_{\text{SNII}}^{\text{u}}$ of stars assumed to end as SNII, and 3) the least massive star $m_{\text{enr}}$ that is able to contribute to the enrichment of the gas cloud before cloud dispersal.

Figure 5.11 Theoretical abundances within SNII and SNI-ejecta. Data are shown for SNII (triangles, Hashimoto et al. 1993), SNIa (open circles; Nomoto et al. 1984), and SNIb/c (filled squares; Woosley et al. 1993). Error bars for the SNIb/c abundances indicate the spread in the theoretical predictions due to the fact that a wide range in initial stellar mass $m$ may result in roughly the same helium star mass (which presumably is the immediate progenitor of SNIb/c; cf. Woosley, Langer & Weaver 1994). The mean $[Fe/H]$ vs. $[O/H]$ relation observed for F and G main-sequence dwarfs in the SNBH is shown as a solid line and has been extrapolated above $[Fe/H]\sim -0.2$ dex. For comparison, theoretical curves are drawn in case of: 1) enrichment by SNIH only (—em dashed line), and 2) dilution by adding more and more metal-free material (thick dotted curve). Width and position of the horizontal line segments mark the range covered by the observational data (see also Fig. 5.1).
Table 5.5 Theoretical oxygen and iron yields of SNII and SNI (Z=Z⊙)

<table>
<thead>
<tr>
<th>m [M⊙]</th>
<th>mFe [M⊙]</th>
<th>ΔMO [M⊙]</th>
<th>ΔMFe [M⊙]</th>
<th>[O/H]</th>
<th>[Fe/H]</th>
<th>αO</th>
<th>αFe</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNII</td>
<td>10</td>
<td>2.5</td>
<td>0.15</td>
<td>0.02</td>
<td>+0.41</td>
<td>0.19</td>
<td>+0.20</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3</td>
<td>0.25</td>
<td>0.02</td>
<td>+0.67</td>
<td>0.21</td>
<td>+0.36</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>4</td>
<td>0.15</td>
<td>0.2</td>
<td>+0.70</td>
<td>+0.96</td>
<td>+0.41</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>6</td>
<td>1.2</td>
<td>0.08</td>
<td>+1.03</td>
<td>+0.67</td>
<td>+0.67</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>16</td>
<td>3.8</td>
<td>0.3</td>
<td>+1.63</td>
<td>+1.26</td>
<td>+0.95</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>32</td>
<td>1.5</td>
<td>0.45</td>
<td>+1.16</td>
<td>+1.34</td>
<td>+0.52</td>
</tr>
<tr>
<td>SNIa</td>
<td>2.5</td>
<td></td>
<td>0.13</td>
<td>0.78</td>
<td>+0.46</td>
<td>+1.98</td>
<td>+0.17</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td>0.13</td>
<td>0.78</td>
<td>+0.29</td>
<td>+1.80</td>
<td>+0.0</td>
</tr>
<tr>
<td>SNIb/c</td>
<td>≥35</td>
<td>4</td>
<td>0.05</td>
<td>0.08</td>
<td>−0.75</td>
<td>+0.15</td>
<td>−0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.18</td>
<td>0.13</td>
<td>−0.22</td>
<td>+0.37</td>
<td>−0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.44</td>
<td>0.16</td>
<td>+0.18</td>
<td>+0.49</td>
<td>+0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.79</td>
<td>0.12</td>
<td>+0.48</td>
<td>+0.33</td>
<td>+0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.91</td>
<td>0.14</td>
<td>+0.54</td>
<td>+0.53</td>
<td>+0.13</td>
</tr>
</tbody>
</table>


Notes: (1) mH2 refers to the pre-SN mass used in the SNII and SNIb/c calculations. (2) Binary WD scenario (assumed initial mass of secondary: 5 M⊙). (3) αO and αFe values for SNIb/c have been calculated assuming m=35 M⊙.

It can be seen that the IMF-integrated stellar abundance variations given in Table 5.6 are considerably below the individual stellar abundance variations listed in Table 5.5. This is due to the metal-poor material returned by intermediate mass stars relative to that returned by individual massive stars. First, we verify from Table 6 that the resulting abundance variations in [O/H] are substantially larger than those in [Fe/H] (as expected for SNII ejecta). Thus, in order to explain observed variations in [O/H] smaller than those in [Fe/H] by means of sequential enrichment, SNII and/or SNIb/c nucleo-synthesis products are required.

Secondly, for a given combination of m^SNII and m^enr, it can be seen that abundance variations αO and αFe rapidly increase with decreasing cloud masses to which the stellar ejecta are mixed (i.e. decreasing values of ϑ). Consequently, an abundance variation of ~0.6 dex in [O/H] due to sequential enrichment alone probably excludes values of ϑ ≥ 0.15.

Thirdly, for a given combination of ϑ and m^enr, abundance variations rapidly increase with m^SNII (this effect is more pronounced for oxygen; cf. Table 5.5). We assumed that stars more massive than m^SNII do not explode as supernova but presumably end as black hole (e.g. Maeder 1992). Consequently, such stars contribute to the ISM enrichment during their stellar wind phase only. However, even though such stars probably do not participate in the iron enrichment of the ISM, they still may affect the interstellar [Fe/H] abundance ratio by means of the amounts of hydrogen they consume. This is reflected by the abundance variations listed in Table 5.6 which are reasonably insensitive to m^enr except for values of m^enr ≥ 25 M⊙.

The abundance variations listed in Table 5.6 will be substantially larger in case of initial stellar abundances much below solar provided that element yields for SNI are insensitive to the initial abundances of their progenitors (see Sect. 3.3). In contrast, these abundance variations will be substantially reduced in case of partial mixing of the enriched stellar ejecta (i.e. λ < 1, see above) and/or when a steeper IMF towards low mass stars is considered. We note that the critical stellar mass m^enr able to enrich the dispersed cloud material, can be roughly related to the dispersal time of the star forming region using a theoretical main-sequence turnoff-mass vs. stellar age relation. In this manner, m^enr = 12, 15, 25, and 40 M⊙, approximately corresponds to sequential enrichment times of t^disp ≈ 2 10^7, 10^6, 7 10^6 yr, and 4 10^6 yr, respectively (for stars formed with solar abundances; see Schaller et al. 1992). Cloud dispersal times t^disp ≥ 10^7 Gyr, equivalent with m^enr ≤ 15 M⊙, are probably not supported by the observations (Sect. 5.5.2).

An argument often proposed against sequential stellar enrichment as explanation for the observed stellar abundance variations is based on inefficient mixing of the nucleo-synthesis products from different types of supernovae (e.g. in the case of mixing of the ejecta of SNII and SNIa that are associated with the one and the same stellar generation). This argument is primarily based on the different enrichment time scales for SNII and SNIa (e.g. Gilmore & Wyse 1991; Edvardsson et al. 1993) and states that in case of sequential stellar enrichment during a local burst of star formation, variations in e.g. [O/H] (predominantly
Table 5.6 Theoretical abundance variations $\alpha_{\text{O}}$ and $\alpha_{\text{Fe}}$ in case of sequential enrichment by SNII

<table>
<thead>
<tr>
<th>$\vartheta$</th>
<th>$m_u^{\text{SNII}} = 60$</th>
<th>$m_u^{\text{SNII}} = 40$</th>
<th>$m_u^{\text{SNII}} = 25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ox</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>1.05</td>
<td>1.12</td>
<td>1.18</td>
</tr>
<tr>
<td>0.05</td>
<td>0.84</td>
<td>0.87</td>
<td>0.81</td>
</tr>
<tr>
<td>0.1</td>
<td>0.69</td>
<td>0.70</td>
<td>0.61</td>
</tr>
<tr>
<td>0.2</td>
<td>0.52</td>
<td>0.52</td>
<td>0.42</td>
</tr>
<tr>
<td>0.5</td>
<td>0.31</td>
<td>0.30</td>
<td>0.23</td>
</tr>
<tr>
<td>1.0</td>
<td>0.19</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Fe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.79</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>0.05</td>
<td>0.61</td>
<td>0.58</td>
<td>0.50</td>
</tr>
<tr>
<td>0.1</td>
<td>0.48</td>
<td>0.44</td>
<td>0.35</td>
</tr>
<tr>
<td>0.2</td>
<td>0.33</td>
<td>0.30</td>
<td>0.22</td>
</tr>
<tr>
<td>0.5</td>
<td>0.19</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>1.0</td>
<td>0.11</td>
<td>0.09</td>
<td>0.06</td>
</tr>
</tbody>
</table>

caused by SNII) are expected to be larger than those in $[\text{Fe/H}]$ (which in part originate from SNIa). This is exactly the opposite to what is observed as the data provided by Edvardsson et al. (1993) suggest that different nucleosynthesis sites mixed their products together well (cf. Fig. 5.1).

However, apart from the uncertainties still involved with the enrichment time scales of SNII and SNIa, the effects of sequential stellar enrichment on abundance variations in the ISM heavily depend on the integrated, IMF-weighed stellar yields (i.e. are determined by the contributions of different types of SNe). First, SN Ib/c ejecta (in addition to that of SNIa) may help (and in fact may be required) to explain the observed variations in $[\text{Fe/H}]$ relative to those in $[\text{O/H}]$ (see Fig. 5.11). Compared to models which account for the enrichment by SNII only, the main effect of the inclusion of SN Ib/c is an enhancement of the iron enrichment. This results in a corresponding shift of the $[\text{Fe/H}]$ vs. $[\text{O/H}]$ relation and improves the agreement with the observations, in particular at solar abundances. Secondly, stellar abundance variations observed for elements such as Fe still may be explained by the IMF-weighed element contributions of SNII in spite of the fact that such elements are efficiently produced by SNIa (see Sect. 3.4). Thirdly, and probably most important, the effect of sequential enrichment on the stellar abundance variations for individual elements is strongly affected by the abundances in the ambient ISM to which the stellar ejecta are actually mixed. Thus, even though sequential stellar enrichment alone seems to be ruled out by theoretical arguments as the full explanation for the observed stellar abundance variations, the extent to which sequential enrichment may contribute to these variations strongly depends on both the details of the stellar enrichment process and the chemical evolution of the ambient ISM. This allows for an explanation of the observed stellar abundance variations in terms of combined sequential stellar enrichment and metal-deficient gas infall as discussed in Sect. 5.4.4.

We conclude that infall of metal-deficient material and/or sequential stellar enrichment provide plausible mechanisms to explain (in part) the observed stellar abundance variations among similarly aged stars in the SNBH. It is evident that a more quantitative investigation, of the effects of these processes on the stellar abundance variations, relies on the detailed chemical evolution of the disk ISM and on the typical time scales during which abundance inhomogeneities in the local disk ISM can exist (see Sect. 5.4).
Modelling the spectro-photometric and chemical properties of Low Surface Brightness galaxies

van den Hoek, L.B., de Blok, W.J.G., van der Hulst, J.M., and de Jong, T.

Abstract

We investigate the star formation history and chemical evolution of low surface brightness (LSB) disk galaxies by means of their observed spectro-photometric and chemical properties. To this end, we use a galactic chemical evolution model incorporating a detailed metallicity dependent set of up-to-date stellar input data covering all relevant stages of stellar evolution. Comparison of our model results with observations confirms the idea that LSB galaxies are relatively unevolved systems.

Based on extensive modelling, we find that for the majority of the LSB galaxies in our sample, observed Johnson-Cousins UBVRI magnitudes, [O/H] abundances, gas masses and fractions, and H\textsc{i} mass-to-light ratios, are best explained by galactic evolution models incorporating an exponentially decreasing global star formation rate (SFR) ending at a present-day gas-to-total mass ratio of $\mu = 0.5$ for a galaxy age of 14 Gyr. About 35% of the LSB galaxies in our sample exhibit properties that cannot be explained by exponentially decreasing SFRs alone. We argue that most of these systems experienced recent episodes of enhanced star formation superimposed on exponentially decreasing global SFR models. Only a small fraction ($\sim 10-15\%$) of the LSB galaxies have properties consistent with those resulting from linearly decreasing or constant SFR models.

We find evidence, from model point of view, for recent and ongoing star formation in the disks of LSB galaxies at rates of $\sim 0.1 \, \text{M}_\odot \, \text{yr}^{-1}$. In particular, we demonstrate that the occurrence of small amplitude star formation bursts in LSB galaxies is required to explain the contribution of the young (5-50 Myr old) stellar population to the galaxy integrated luminosity. This result suggests that star formation in LSB galaxies has proceeded in a stochastic manner from the moment star formation started in their disks. We argue that sporadic star formation in LSB galaxies is probably associated with local accretion and/or infall of matter.

The presence of an old stellar population in many late-type LSB galaxies, as confirmed by our results, suggests that LSB galaxies roughly follow the same evolutionary history as HSB galaxies, except at a much lower rate. In particular, our results imply that LSB galaxies do not form late, or have a delayed onset of star formation, but evolve slowly. We show that the observed color differences between LSB and HSB galaxies can be interpreted almost entirely in terms of the relatively low extinction and metallicity in LSB galaxies. We propose that LSB galaxies are in an early stage of disk formation and probably are still in the accumulation phase of gas during which their current amount of star formation and chemical enrichment is regulated. In particular, the gas reservoir at the time of onset of main star formation in LSB galaxies may have been substantially less than that estimated from their present-day amounts of gas since accretion of matter is still very important in these systems.

The low evolutionary state of LSB galaxies relative to HSB galaxies suggests that LSB galaxies are just HSB galaxies in the making (except on time scales much longer than a Hubble time). We discuss our results in the context of the evolutionary history of LSB galaxies compared to that of HSB and dwarf irregular galaxies.

6.1 Introduction

Deep searches for field galaxies in the local universe have revealed the existence of a large number of galaxies with such low surface brightnesses that they, until recently, were hard to detect against the night sky (Schombert et al. 1988, 1992; Knezek 1993; Turner et al. 1993; Bergvall & Rönneback 1995; Schwartzzenberg et
al. 1995; Sprayberry et al. 1995). The low surface brightness galaxies detected in the field are predominantly late-type spirals which in general are disk-dominated, do not show any clear signs of a large bulge or strong bar, and have central surface brightnesses \( \geq 23 \, \text{mag arcsec}^{-2} \) in the B band (e.g. Römbrock & Bergvall 1994; McGaugh & Bothun 1994; de Blok et al. 1995, hereafter dB95; Vennik et al. 1996). A small fraction (~ 15 %) of the field LSB galaxies detected thus far comprises early type systems, ellipticals, and dwarf galaxies.

The sizes and luminosities of LSB galaxies can range from the small and faint Local Group dwarfs like GR8 (Hodge 1967) to that of the giant and luminous Malin-1 like systems (e.g. Impey & Bothun 1989; Knezek 1993; Sprayberry et al. 1995). This latter group of giant LSB galaxies, which comprises ~ 10 % of the LSB galaxy population, usually have a large bulge and are much different from the disk dominated LSB galaxies described above.

In this paper, we will concentrate on late-type LSB spirals. The main property which distinguishes these systems from their “normal” late-type spirals is their low surface brightness, not e.g. their luminosity or optical size. Observations show that LSB galaxies are neither dwarf systems nor just the fainter counterparts of HSB spirals (de Blok et al. 1996, hereafter dB96).

In many cases, LSB galaxies follow the trends in galaxy properties found along the Hubble sequence towards very late types. These trends, including increasingly blue colors (e.g. Römbrock 1993; McGaugh & Bothun 1994; dB95), decreasing oxygen abundances in the gas (e.g. McGaugh 1994; Römbrock & Bergvall 1995), and decreasing HI surface densities (from type Sc onwards; e.g. van der Hulst et al. 1993, hereafter vdH93) suggest that LSB galaxies must be in a low evolutionary state compared to HSB galaxies.

Atomic gas surface densities of LSB spirals are among the lowest known for disk galaxies (dB96). Notwithstanding, LSB spirals rank among the most gas-rich disk galaxies of a given total mass as their HI disks in general are rather extended (Zwaan et al. 1995; dB96). The fact that LSB galaxies still have large reservoirs of gas together with their low abundances suggest that their amount of star formation in the past cannot have been very large. Clearly, LSB galaxies are not the faded remnants of HSB spirals.

Current star formation rates in LSB galaxies are among the lowest known for late-type disk galaxies as well, as deduced from narrow-band H\( \alpha \) imaging (McGaugh 1992; vdH93). These observations reveal the presence of a few giant H\( \alpha \) regions which are ionized by OB associations formed during recent episodes of star formation (e.g. McGaugh 1992). Such sites of minimal star formation are, however, low in number, do not trace the spiral arms very well, and are usually found towards the outer parts of the galaxy (dB95). This suggests that local rather than global star formation is a common phenomenon in LSB galaxies.

The unevolved nature of LSB spirals as implied by their low gas abundances, unusually blue colors, low gas surface densities, large gas contents, and low current star formation rates, can be interpreted in many different ways. For instance, LSB spirals may be relatively young systems in which the main phase of star formation is still to occur. If the mean age of the stellar population in LSB galaxies is much younger than that in HSB spirals, this would imply different star formation histories for galaxies differing in surface brightness. Alternatively, the stellar population in LSB spirals might be as old as in their HSB counterparts but with a young population dominating the luminosity. This would imply similar star formation histories and further would suggest the existence of a distinct group of red LSB spirals undetected yet because of their absence of a young stellar population (e.g. McGaugh & Bothun 1994). Other explanations for the dissimilarities observed between LSB and HSB spirals may include differences in internal extinction and/or in the stellar mass function at birth.

The goal of this paper is to address these and other scenarios for the evolution of LSB galaxies by detailed modelling of their spectro-photometric and chemical properties. The model used incorporates a detailed metallicity dependent set of stellar input data covering all relevant stages of stellar evolution and is able to describe the evolution of low metallicity galaxies such as LSB spirals.

Model results are compared directly with the observed colors, gas phase abundances, gas contents, and current star formation rates of LSB galaxies to constrain the global star formation history and chemical evolution of these systems. In particular, we consider the important question whether LSB spirals do have an evolutionary history fundamentally different from that of HSB spirals and dwarf galaxies. We will show that models incorporating exponentially decreasing SFRs are in best agreement with the spectro-photometric and chemical properties of the majority of the LSB galaxies in our sample, provided that these LSB galaxies have turned about half of their present-day disk mass into stars.

This paper is organized as follows. We briefly compare observational data of LSB galaxies with those of face-on spirals and dwarf galaxies in Sect. 6.2. In Sect. 6.3, we describe the ingredients of the galactic evolution model developed to study the spectro-photometric evolution of LSB spirals. In Sect. 6.4, we compare and calibrate the model and describe the initial set of star formation histories studied. Model
results related to the chemical and spectro-photometric evolution of LSB galaxies are presented in Sect. 6.5. The impact of small amplitude star formation bursts in LSB galaxies is investigated in Sect. 6 and predicted star formation rates are compared with the observations in Sect. 6.7. We discuss our results in the context of the star formation history and dynamical evolution of LSB galaxies in Sect. 6.8.

6.2 Observational characteristics of LSB galaxies

To amplify the properties of LSB galaxies, we compare Johnson-Cousins UBVRI magnitudes, neutral hydrogen masses, gas fractions, and oxygen abundances of LSB galaxies, with those of HSB and dwarf galaxies.

6.2.1 Sample selection

We refer to de Blok et al. (1996) for an extensive description of the sample selection. In brief, their sample consists of 24 late-type LSB galaxies (inclinations up to $\sim 60^\circ$), taken from the lists by Schombert et al. (1992) and the UGC (Nilson 1973), which are representative for the LSB galaxies generally found in the field by Schombert et al. For these systems, optical data have been taken from dB95, HI data from dB96, and abundance data from McGaugh & Bothun (1994) and de Blok & van der Hulst (unpublished).

From this sample, we selected a subsample of 16 LSB galaxies for which high-quality data are available. We list the galaxy identification and UBVRI absolute magnitudes in columns (1) to (6) in Table 6.1 (a Hubble constant of $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$ was used). To allow for direct comparison with photometric evolution models Johnson UBV and Kron-Cousins RI magnitudes will be used throughout this paper. Corresponding luminosities and mass-to-light ratios in the B band, neutral hydrogen and dynamical masses, gas-to-total mass ratios $\mu$ (corrected for helium; see Sect. 2.6 for the definition of $\mu_{\text{dyn}}$ and $\mu_{\text{rot}}$), and mean [O/H] abundances are listed in columns (7) to (13).

In addition to the sample listed in Table 6.1, we consider a complementary set of LSB galaxies for which less data are available. For these systems, we use photometry data presented by McGaugh et al. (1995) and abundance data provided by McGaugh (1994) and Rönning & Bergvall (1994). Selection criteria for the McGaugh et al. (1995) sample were the same as those used by dB95. The LSB galaxies from Rönning & Bergvall (1994) were selected in a different manner. In general, both sets of complementary LSB galaxies show properties similar to those of the dB95 subsample.

Table 6.1 Observational data on LSB galaxies (de Blok et al. 1995, 1996)

<table>
<thead>
<tr>
<th>Name</th>
<th>U (mag)</th>
<th>B (mag)</th>
<th>V (mag)</th>
<th>R (mag)</th>
<th>I (mag)</th>
<th>L$<em>B$ [L$</em>\odot$]</th>
<th>M$_{\text{HI}}$ L$_B$</th>
<th>M$_B$</th>
<th>M$_{\text{dyn}}$</th>
<th>$\mu_{\text{dyn}}$</th>
<th>$\mu_{\text{rot}}$</th>
<th>[O/H]</th>
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<td>F561-1</td>
<td>-17.4</td>
<td>-17.2</td>
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<td>8.1(8)</td>
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<td>0.59</td>
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<td>-17.6</td>
<td>-16.4</td>
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<td>1.5(9)</td>
<td>3.8(10)</td>
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<td>0.9</td>
<td>2.8(8)</td>
<td>1.0(9)</td>
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<td>*</td>
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<td>3.4(8)</td>
<td>3.8(9)</td>
<td>0.12</td>
<td>0.33</td>
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<td>-16.9</td>
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<td>-17.3</td>
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<td>1.4(10)</td>
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<td>-17.8</td>
<td>*</td>
<td>9.8(9)</td>
<td>0.9</td>
<td>9.3(8)</td>
<td>3.2(9)</td>
<td>0.39</td>
<td>0.66</td>
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<td>-18.0</td>
<td>-18.1</td>
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<td>0.8</td>
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<td>*</td>
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<td>*</td>
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<td>4.9(10)</td>
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<td>*</td>
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<td>-18.5</td>
<td>-18.9</td>
<td>1.9(9)</td>
<td>1.7</td>
<td>1.2(9)</td>
<td>6.3(10)</td>
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<td>-17.5</td>
<td>-17.5</td>
<td>-17.5</td>
<td>-17.5</td>
<td>6.0(9)</td>
<td>1.0</td>
<td>1.0(9)</td>
<td>1.0(9)</td>
<td>0.5</td>
<td>*</td>
<td>-0.6±0.4</td>
</tr>
<tr>
<td>Typ. LSBG</td>
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<td>-17.5</td>
<td>-18.0</td>
<td>-18.5</td>
<td>-18.5</td>
<td>1.5(9)</td>
<td>1.3</td>
<td>2(9)</td>
<td>1(10)</td>
<td>0.2</td>
<td>0.5</td>
<td>-0.6±0.4</td>
</tr>
<tr>
<td>Typ. HSBG</td>
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<td>-19.5</td>
<td>-20.5</td>
<td>-21.0</td>
<td>-21.0</td>
<td>1.0(10)</td>
<td>0.4</td>
<td>4(9)</td>
<td>1(11)</td>
<td>0.04</td>
<td>*</td>
<td>~0.0</td>
</tr>
</tbody>
</table>

* not available, † uncertain

6.2.2 Magnitudes and colors

We compare in Fig. 6.1 magnitudes and broadband colors of LSB galaxies to those of HSB face-on spirals (de Jong & van der Kruit 1994) and dwarf galaxies (Melisse & Israel 1994; Gallagher & Hunter 1986, 1987). We note that the usual distinction between HSB and LSB galaxies is purely an artificial one since normal
galaxies along the Hubble sequence show a continuous range in central surface brightness, i.e. ranging from values around the Freeman value for early type systems to the very faint values observed for the late-type LSB galaxies in our sample (e.g. de Jong 1995; dB95).

LSB galaxies are usually much bluer, both in (B−V) and (R−I), and have fainter B magnitudes than their HSB counterparts (e.g. McGaugh 1992; vdH93; dB95). At a given gas mass, LSB galaxies are among the bluest disk galaxies observed (see below). Due to sample incompleteness, the distribution of galaxies in the (R−I) vs. B diagram (Fig. 6.1b) is limited to systems brighter than ∼−16 mag in the B band. Since photometric evolution models predict galaxies to evolve rapidly (i.e. within one Gyr) to colors (R−I)≥ +0.2 mag (see Sect. 6.4), one expects to observe only a few galaxies with colors dominated by a young stellar population, i.e. with (R−I)≤ +0.2 mag. Three galaxies with (R−I)≤ −0.1 mag (i.e. F563-I and two dwarfs) have been omitted from the data because their (R−I) colors are probably contaminated by nearby objects and/or suffer from large observational errors. Dwarf galaxies, on average, appear to be even bluer than LSB galaxies while spanning roughly the same range in luminosity.

The reason why LSB spirals are unusually blue compared to normal late type galaxies may be explained by the presence of a relatively young stellar population, the lack of internal dust extinction, and/or metallicity effects. Alternatively, the stellar mass function at birth in LSB galaxies may be different from that in HSB galaxies. We will discuss these possibilities in Sects. 6.4 and 6.5 below.

6.2.3 Abundances

Estimates of the ISM abundances in LSB galaxies predominantly rely on abundance determinations of their constituent HII regions. Within such regions, oxygen abundances are usually derived using an empirical relation for the line-ratio R23 = ([OII] λ3727 + [OIII] λ4959,5007) / Hβ as first discussed by Pagel et al. (1979) and later calibrated by e.g. McGaugh (1991). A full discussion of the method is given in McGaugh (1994) but we remark that for a given HII region observational errors in [O/H] can be as large as ±0.2–0.3 dex (apart from uncertainties due to internal reddening). For bright HII regions within the LSB galaxies listed in Table 6.1, abundances are taken both from McGaugh (1994) and de Blok & van der Hulst (unpublished).

We assume that the HII-region abundances on average are a reasonable indicator of the ISM abundances within a given LSB spiral. The intrinsic scatter in [O/H] among different HII region abundances within a given LSB galaxy is usually less than ±0.2 dex around the mean HII region abundance (e.g. McGaugh 1994). However, in LSB galaxies containing only a few bright HII regions for which abundances have been determined, abundances may be biased towards the physical properties (e.g. age, initial metallicity and amount of self-enrichment) of individual HII regions (e.g. Pilyugin 1992; Pettini & Lipman 1995).

In Fig. 6.1c we compare mean [O/H] abundances of HII regions in LSB galaxies with those in HSB spirals (Zaritsky et al. 1994) and dwarf galaxies (Melisse & Israël 1994; Gallagher & Hunter 1986, 1987). On average, LSB galaxies seem to follow the correlation between the characteristic gas-phase abundance and luminosity as found for HSB spirals (Zaritsky et al. 1994). However, the range in [O/H] at a given B magnitude is nearly one dex and large scatter in the correlation is present. This scatter is probably related to evolutionary differences among the LSB galaxies of a given B magnitude (e.g. in the ratio of old to young stellar populations) and/or the HII regions they contain. Clearly, LSB galaxies (and dwarf systems) on average show substantially smaller [O/H] abundances than HSB galaxies.

As LSB galaxies have HII surface densities about a factor of ∼3 lower than in normal late-type galaxies (vdH93; dB96), their low [O/H] abundances may be interpreted in terms of a strong dependence of the SFR on surface density (see also Kennicutt 1989). Such a dependence has been suggested for late-type HSB galaxies (e.g. Edmunds & Pagel 1984; Dopita 1990; Phillips and Edmunds 1991; Ryder & Dopita 1994) and may apply to LSB galaxies as well (see Sect. 6.5).

6.2.4 Extinction

Estimates of extinction are particularly important when photometric evolution models are applied to spiral galaxies. Statistical studies of variations of galaxy magnitudes with inclination support the traditional view that Sc galaxies are semi-transparent with most of the extinction concentrated in the inner regions (e.g. Huizinga & van Albada 1992; Giovanelli et al. 1994). These studies conclude that the outer parts of spiral galaxies in general are optically thin.

Measurements of extinction in foreground spirals obscuring a background galaxy range from ∼0.3 mag in B in the interarm regions and outer parts of a spiral (Keel 1983; Andredakis & van der Kruit 1992; White & Keel 1992), to ∼1.6 mag within the spiral arms itself (Keel 1983; James & Puxley 1993). These studies suggest that internal extinction in HSB spirals is concentrated towards the galaxy nucleus and spiral arms, while extinction in the outer galaxy and inter-arm regions is relatively low. This is consistent with
Figure 6.1 Observational data on LSB galaxies compared to that for normal spirals and dwarf galaxies. Symbols refer to LSB galaxies (triangles; data from de Blok et al. 1995, 1996), normal face-on spirals (full dots; de Jong & van der Kruit 1994), and dwarf galaxies (crosses; Mellise & Israel 1994). LSB galaxies which are probably dwarf systems are indicated by triangles with crosses overlayed. Typical error bars are indicated in the bottom left of each panel.
conclusions derived from independent extinction studies for large samples of spiral galaxies (see also Jansen et al. 1994; Peletier et al. 1995; Huizinga 1995; Beckman et al. 1996).

Dust radiative transfer models indicate an overall face-on extinction in spirals of $\lesssim 0.5$ mag in B (e.g. Knappen & van der Kruit 1991; Byun et al. 1994; Huizinga 1994), while estimates of the maximum face-on extinction in spirals based on far-IR measurements are in the range 1.5 to 2 mag in B (Disney et al. 1989). No support for optically thick disks has been found in a sample of nearby spirals from 60$\mu$m observations (Bothun & Rogers 1992). These studies support the idea that spiral galaxies have face-on extinctions of typically less than $\sim 0.5$−1 mag in B.

The observational finding by Bosma et al. (1992) and Byun (1992) that low luminosity spirals appear transparent throughout their edge-on disks, while more luminous spirals become optically thick at a given galactocentric distance, supports the idea that face-on extinction in LSB galaxies is relatively low, i.e. typically less than $E_{B-V} = 0.1$ mag (e.g. McGaugh 1994). This is consistent with observational evidence in support of low dust contents in galaxies having low gas abundances (e.g. Issa et al. 1991; van den Hoek & de Jong 1992). In addition, low column densities of HI imply a low dust content if the gas-to-dust ratio is the same as (or larger than) in the Galaxy.

We consider $E_{B-V} \sim 0.5$−0.6 mag as a plausible upper limit for the face-on extinction in spirals (assuming $R_V = A_V / E_{B-V} \sim 3$ in our own Galaxy; e.g. Johnson 1968). In normal HSB spirals, we estimate typical face-on reddenings of $E_{B-V} \sim 0.3$−0.4 and $\sim 0.1$−0.2 mag, in systems with prominent and conspicuous spiral arms in their outer disk, respectively. In LSB spirals, which usually do not show either a strong nucleus or well developed spiral arms, face-on internal extinction is expected to be rather low, i.e. less than $E_{B-V} \sim 0.1$ mag.

### 6.2.5 Gas masses

We compare in Fig. 6.1d the present-day amounts of atomic gas $M_\text{g} \sim 1.4 M_{\text{HI}}$ (corrected for helium) in LSB galaxies with those present in HSB galaxies and dwarfs. At a given B-band luminosity, LSB galaxies are usually found among the spirals containing the highest gas masses (dwarfs appear concentrated to somewhat smaller gas masses). Thus, since their HI surface densities are relatively low (dB96), LSB galaxies must have a larger, more extended disk of gas compared to that of HSB galaxies of the same luminosity (Zwaan et al. 1995).

The above strictly applies to the atomic gas content of galaxies only, since the inclusion of molecular gas may change the observed trend. If HSB spirals would be as gas-rich as LSB spirals of the same luminosity, HSB galaxies would need to contain at least $\sim 5$ times more molecular gas than atomic hydrogen. This seems exceedingly high. For instance, our own Galaxy contains only as much H2 as HI (Scoville & Sanders 1987). Furthermore, estimates of the amounts of molecular gas in normal Scd galaxies exclude H2 / HI ratios larger than $\sim 1$ (Young & Knezek 1989; Young & Scoville 1991).

So far no CO-emission has been detected in LSB galaxies (Schombert et al. 1990; de Blok & van der Hulst, in prep.). Therefore, the total amount of molecular gas in LSB galaxies is probably small even though relatively high CO/H2 conversion factors may apply in these low metallicity galaxies (Wilson 1995). We will assume that the atomic hydrogen masses listed in Table 6.1 (multiplied by 1.4 to correct for He) represent the total amounts of gas in LSB galaxies. We conclude that, on average, LSB galaxies are considerably more gas-rich (up to a factor $\sim 3$) than HSB galaxies of the same luminosity (see also Fig. 6.1g; and dB96). This implies that evolutionary differences between galaxies differing in surface brightness must exist (see Sect. 6.5).

### 6.2.6 Total masses and gas fractions

In principle, determination of the gas-to-total mass-ratio ($\mu_1 \equiv M_{\text{gas}} / (M_{\text{gas}} + M_{\text{stars}})$) of the matter contained within a given galactocentric radius (i.e. usually up to where the HI rotation curve can be measured), involves the conversion of the observed galaxy luminosity to stellar mass. However, the mass-to-light ratio of the underlying stellar population is generally not well known and, in fact, is an important quantity to determine. Alternatively, if one assumes a fixed value of the mass-to-light ratio, artificial trends will be introduced in the gas-fractions derived since this ratio is expected to vary among galaxies having different star formation histories. We note that, from theoretical point of view, $\mu_1$ is strictly related to the amount of gas that is associated with the star forming disk and is available for star formation.

Independent estimates of the gas fraction $\mu_1$ can be obtained from gas-to-dynamical mass-ratios $\mu_{\text{dyn}} \equiv M_{\text{gas}} / M_{\text{dyn}}$. However, since the dynamical masses of LSB galaxies usually include dark matter (i.e. matter not observed as gas or stars), values of $\mu_{\text{dyn}}$ provide lower limits to the actual gas fractions in LSB galaxies.
We determined $\mu_{\text{dyn}}$ for a dynamical mass corresponding to the outermost point of the HI rotation curve (see dB96; Table 6.1).

For late-type HSB galaxies, studies by Bosma (1978), Begeman (1987), and Broeils (1992) have shown that the ratio of dark to luminous matter at the edge of the optical disk is $\sim 50\%$. In this case, the true gas fraction is underestimated by a factor $\sim 2$. The discrepancy probably increases for LSB galaxies and dwarfs (i.e. up to factors $2-10$; see Broeils 1992), but is almost negligible in early-type HSB galaxies where the stellar population dominates the optical disk.

To get around the discrepancy for LSB galaxies, we also determined gas fractions $\mu_{\text{rot}} = M_{\text{gas}} / (M_{\text{gas}} + M_{\text{stellar, max}})$ where $M_{\text{stellar, max}}$ denotes the mass of the stellar component obtained from maximum disk fitting of the rotation curve (dB96). This method likely overestimates the contribution of the luminous stellar disk to the observed total mass distribution (e.g. Kuijken & Gilmore 1989; Bottema 1995) and, therefore, also provides a lower limit to the actual gas fraction.

In the following, we will use $\mu_{\text{rot}}$ for LSB galaxies whenever the data allows application of the maximum disk method (cf. Table 6.1). We estimate that $\mu_{\text{rot}}$ approximates the true gas fraction $\mu_{1}$ within a factor of $\sim 2$. For LSB galaxies, we find that $\mu_{\text{rot}}$ is $3-10$ times larger than $\mu_{\text{dyn}}$. This is consistent with the factors estimated by Broeils (1992) and suggests that LSB galaxies are dark matter dominated (see dB96).

For the HSB galaxies and dwarfs in our comparison samples, we are forced to use $\mu_{\text{dyn}}$ as estimate of the actual gas fraction $\mu_{1}$ as $\mu_{\text{rot}}$ has been derived for a few of these systems only. Consequently, the adopted gas fractions are hard lower limits for all galaxies considered. We note that for HSB galaxies, the differences between $\mu_{\text{rot}}$ and $\mu_{\text{dyn}}$ are usually small (e.g. dB96).

Figs. 1e and 1f show the distribution of the present-day gas fraction $\mu_{\text{rot}}$ and total gas mass vs. $(B-V)$. It can be seen that LSB galaxies and dwarfs exhibit much larger gas fractions (typically $\mu_{1} \sim 0.5$) than HSB spirals ($\mu_{1} \sim 0.05$). This implies that LSB galaxies are in a low evolutionary state with respect to HSB spirals consistent with our earlier findings.

### 6.2.7 Mass-to-light-ratios

We show in Figs. 1g and h the distribution of the mass-to-light ratio $M_{\text{HI}} / L_{\text{B}}$ vs. $(B-V)$ and vs. $\mu_{1}$, respectively. LSB galaxies exhibit considerably higher $M_{\text{HI}} / L_{\text{B}}$ ratios than HSB spirals (cf. Table 6.1). This is primarily due to the relatively large atomic gas contents of LSB spirals as discussed above. In addition, high values of $M_{\text{HI}} / L_{\text{B}}$ may originate from a less well developed (both old and young) stellar population. In either case, the high values of $M_{\text{HI}} / L_{\text{B}}$ observed for LSB spirals indicate a low evolutionary state of these galaxies. This is consistent with the fact that central surface brightnesses decrease with increasing values of $M_{\text{HI}} / L_{\text{B}}$ as found for LSB galaxies (dB96).

We conclude that the low surface densities and brightnesses, blue colors, low abundances, large scale lengths, inconspicuous spiral arms and nuclei, low rotation velocities (dB96), and high gas masses observed in LSB spirals, all provide evidence in support of the view that LSB galaxies are relatively unevolved systems compared to HSB spirals. This agrees well with our finding that LSB galaxies usually display properties intermediate to those of HSB spirals and dwarf galaxies.

### 6.3 Model description and assumptions

We describe the galactic evolution model developed to study the chemical and spectro-photometric evolution of LSB galaxies (for a more extensive description of the model see van den Hoek 1997). We concentrate on the stellar contribution to the total galaxy luminosity in a given passband (other contributions are neglected). For a given star formation history (SFR), we compute the chemical enrichment of a model galaxy by successive generations of evolving stars. To derive the stellar luminosity in a given passband at a given age, we use an up-to-date metallicity dependent set of theoretical stellar isochrones as well as a library of spectro-photometric data. The spectro-photometric properties of the model galaxy are calculated by integrating the stellar luminosities at a given galactic age weighted by the SFR at the time these stars were born.

#### 6.3.1 Chemical evolution model

We restrict ourselves to a brief outline of the basic assumptions and boundary conditions to the chemical evolution model used. We start from a model galaxy initially void of stars. We follow the chemical enrichment of this galaxy during its evolution assuming stars to be formed according to a given $\Phi_{\text{star}}$ formation rate (SFR) and initial mass function (e.g. a power law IMF: $dN/dm \equiv M(m) \propto m^{-7}$). Specific choices of the SFR will be described in Sect. 6.4. Both stellar and interstellar abundances as a function of galactic evolution
time \( t \) are computed assuming that the stellar ejecta are returned and homogeneously mixed to the ISM at the end of their lifetimes (i.e. relaxing the instantaneous recycling approximation; see Searle & Sargent 1972). A description of the set of galactic chemical evolution equations used can be found in e.g. Tinsley (1980) and Twarog (1980), see also van den Hoek (1997).

We follow the stellar enrichment of the star forming galaxy in terms of the characteristic element contributions of Asymptotic Giant Branch (AGB) stars, SNII and SNIa. This treatment is justified by the specific abundance patterns observed within the ejecta of these stellar groups (see e.g. Trimble 1991; Russell & Dopita 1992). A detailed description of the metallicity dependent stellar lifetimes, element yields, and remnant masses is given by van den Hoek (1997) and van den Hoek & Groenewegen (1997). We compute the abundances of H, He, O, and Fe, as well as the heavy element integrated metal-abundance \( Z \) (for elements more massive than helium), during the evolution of the model galaxy. Both the SFR, IMF and resulting element abundances as a function of galactic evolution time, are used as input for the spectro-photometric evolution model described below.

Boundary conditions to the chemical evolution model are the galaxy total mass \( M_{\text{tot}} \), its evolution time \( t_{\text{ev}} \), and the initial gas abundances. Unless stated otherwise, we assume \( M_{\text{tot}} = 10^{10} \, M_{\odot} \) and \( t_{\text{ev}} = 14 \, \text{Gyr} \). For a given value of \( M_{\text{tot}} \), we normalize the model SFR such that a gas-to-total mass-ratio \( \mu_1 \sim 0.1 \) is reached at \( t = t_{\text{ev}} \). Note that solutions of the galactic chemical evolution equations are independent of the ratio of the SFR normalisation and \( M_{\text{tot}} \). Primordial helium and hydrogen abundances are adopted as \( Y_p = 0.232 \) and \( X = 0.768 \) (cf. Pagel & Kazlauskas 1992). Initial abundances for elements heavier than helium are set to zero.

### Table 6.2 IMF related parameters and stellar enrichment

<table>
<thead>
<tr>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle m_1, m_a \rangle )</td>
</tr>
<tr>
<td>( (0.1, 60) , M_{\odot} )</td>
</tr>
<tr>
<td>slope of power-law IMF</td>
</tr>
<tr>
<td>( \langle m_{\text{AGB}}, m_{\text{AGB}} \rangle )</td>
</tr>
<tr>
<td>( (0.8, 8) , M_{\odot} )</td>
</tr>
<tr>
<td>stellar mass range at birth</td>
</tr>
<tr>
<td>( \langle m_{\text{SNII}}, m_{\text{SNII}} \rangle )</td>
</tr>
<tr>
<td>( (8, 30) , M_{\odot} )</td>
</tr>
<tr>
<td>progenitor mass range for SNII</td>
</tr>
<tr>
<td>( \langle m_{\text{SNIa}}, m_{\text{SNIa}} \rangle )</td>
</tr>
<tr>
<td>( (2.5, 8) , M_{\odot} )</td>
</tr>
<tr>
<td>progenitor mass range for SNIa</td>
</tr>
<tr>
<td>( \nu_{\text{SNIa}} )</td>
</tr>
<tr>
<td>0.015</td>
</tr>
<tr>
<td>fraction of progenitors ending as SNIa</td>
</tr>
</tbody>
</table>

We list the main input parameters in Table 6.2, i.e. the adopted IMF-slope, minimum and maximum stellar mass limits at birth as well as the progenitor mass ranges for stars ending their lives as AGB star, SNIa, and SNII, respectively. For simplicity, we assume the stellar yields of SNIbc to be similar to those of SNII. Furthermore, we assume a fraction \( \nu_{\text{SNIa}} = 0.015 \) of all white dwarf progenitors with initial masses between \( \sim 2.5 \) and \( 8 \, M_{\odot} \) to end as SNIa. These and other particular choices for the enrichment by massive stars are based on similar models recently applied to the chemical evolution of the Galactic disk (e.g. Groenewegen, van den Hoek & de Jong 1995; van den Hoek & de Jong 1997). We will adopt these values also when modelling the stellar enrichment in LSB galaxies. We emphasize that the detailed inclusion of the stellar enrichment in LSB galaxies is important for their spectro-photometric evolution and is relevant for the qualitative conclusions presented below. Quantities used in the chemical evolution model are identical to those used in the spectro-photometric evolution part of the model.

#### 6.3.2 Spectro-photometric evolution model

In principle, the total luminosity of a galaxy in a specific wavelength interval \( \Delta \lambda \) is determined by: 1) the contribution by its stellar content \( L_{\ast} \), 2) the contribution from the interaction between stars and gas \( L_{\text{ism}} \) (e.g. HII-regions, high-energy stellar outflow phenomena, etc.), and 3) the total amount of radiation absorbed \( L_{\text{ext}} \) (or scattered to wavelengths in- or outside \( \Delta \lambda \)) by gas and dust contained within the galaxy:

\[
L_{\text{gal}}^{\Delta \lambda}(t) = L_{\ast} + L_{\text{ism}} - L_{\text{ext}}
\]

where each term in general is a complex function of galactic evolution time. We concentrate on the stellar contribution and neglect the latter two terms in Eq. (6.1). In this case, the galaxy luminosity within a waveband \( \Delta \lambda \) at galactic evolution time \( t = T \) can be written as:

\[
L_{\text{gal}}^{\Delta \lambda}(t = T) = \int_{t=0}^{T} \int_{m_1}^{m_2} L_{\ast}^{\Delta \lambda}(m, Z(t), T - t) S(t) M(m) \, dm \, dt
\]
where \( m_1 \) denotes the lower stellar mass limit at birth, \( m_o(t) \) the turnoff mass for stars evolving to their remnant stage at evolution time \( t \), and \( L^{\Delta \lambda}_* \) the luminosity of a star with initial mass \( m \), initial metallicity \( Z(t) \), and age \((T-t)\). We assume a separable SFR: \( S(m,t) = S(t) M(m) \) where \( S(t) \) is the star formation rate by number \([yr^{-1}]\) and \( M(m) \) the IMF \([M_\odot^{-1}]\). By convention, we normalize the IMF as \( \int M(m) \, dm = 1 \) where the integration is over the entire stellar mass range \([m_1, m_o] \) at birth (cf. Table 6.2).

Starting from the chemical evolution model described above, we compute the star formation history \( S(m,t) \), gas-to-total mass-ratio \( \mu(t) \), and age-metallicity relations (AMR) \( Z(t) \) for different elements \( i \). Thus, at each galactic evolution time \( t \) the ages and metallicities of previously formed stellar generations are known. To derive the stellar passband luminosity \( L^{\Delta \lambda}_* \) we use a set of theoretical stellar isochrones, as well as a library of spectro-photometric data. Stellar evolution tracks provide the stellar bolometric luminosity \( L_{bol} \), effective temperature \( T_{eff} \), and gravity \( g \), as a function of stellar age for stars with initial mass \( m \) born with metallicity \( Z_\odot \). We compute Eq. (6.2) using a spectro-photometric library containing the stellar passband luminosities \( L^{\Delta \lambda}_* \) tabulated as a function of \( T_{eff} \), \( g \), and \( Z_\odot \) (see below).

We emphasize that the turnoff mass \( m_o(T-t) \) occurring in Eq. (6.2) depends on the metallicity \( Z(t) \) of stars formed at galactic evolution time \( t \). For instance, the turnoff mass for stars born with metallicity \( Z = 10^{-3} \) at a galactic age of \( t_{ev} = 14 \) Gyr is \( m_o \sim 0.8 \ M_\odot \) (e.g. Schaller et al. 1992). This value differs considerably from \( m_o \sim 0.95 \ M_\odot \) for stars born with metallicity \( Z = Z_\odot \). Such differences in \( m_o \) affect the detailed spectro-photometric evolution of a galaxy by constraining the mass-range of stars in a given evolutionary phase (e.g. horizontal branch) at a given galactic evolution time. In the models described below, we explicitly take into account the dependence of \( m_o(t) \) on the initial stellar metallicity \( Z_\odot \) (see van den Hoek 1997).

6.3.3 Stellar evolution tracks and spectro-photometric data

We use the theoretical stellar evolution tracks from the Geneva group (e.g. Schaller et al. 1992; Schaerer et al. 1993). These uniform grids are based on up-to-date physical input (e.g. opacities, nuclear reaction rates, mixing schemes, etc.) and cover large ranges in initial stellar mass and metallicity, i.e. \( m = 0.05 - 120 \ M_\odot \) and \( Z = 0.04 - 0.001 \), respectively. These tracks imply a revised solar metallicity of \( Z_\odot = 0.0188 \) with \( Y_\odot = 0.299 \), and \( \Delta Y/\Delta Z = 3.0 \) for a primordial He-abundance of \( Y_p = 0.232 \). For stars with \( m > 7 \ M_\odot \), these tracks were computed until the end of central C-burning, for stars with \( m = 2 - 5 \ M_\odot \) up to the early-AGB, and for \( m < 1.7 \ M_\odot \) up to the He-flash. For stars with \( m \lesssim 0.8 \ M_\odot \), we used the stellar isochrone program from the Geneva group (Maeder & Meynet, private communication).

To cover the latest stellar evolutionary phases (i.e. horizontal branch (HB), early-AGB, and AGB) for stars with \( m \lesssim 8 \ M_\odot \), we extended the tracks from Schaller et al. with those from Lattanzio (1991; HB and early-AGB) for \( m \sim 1 - 2 \ M_\odot \), and from Groenewegen & de Jong (1993; early-AGB and AGB) for \( m \sim 1 - 8 \ M_\odot \). These tracks roughly cover the same metallicity range as the tracks from the Geneva group. The synthetic AGB models from Groenewegen & de Jong were succesfully applied to AGB stars both in the Galactic disk and Magellanic Clouds (see also Groenewegen, van den Hoek & de Jong 1995). Special attention has been paid to smoothly fit together these distinct data sets. Corresponding isochrones were computed at a carefully selected logarithmic grid of stellar ages, well covering galactic evolution times up to \( t_{ev} \sim 14 \) Gyr. Isochrones are linearly interpolated in \( m, \log Z \), and \( \log t \).

The spectro-photometric data library that we use is based on the Revised Yale Isochrones and has been described extensively by Green et al. (1987). These data include stellar UBVRI Johnson-Cousins magnitudes covering the following ranges in \( T_{eff} [K] = 2800 \) to 20000, \( \log g \) \([cm \, s^{-2}]\) = -0.5 to 6, and \( \log (Z/Z_\odot) \sim -2.5 \) to +0.5. Corresponding spectro-photometric data for stars with \( T_{eff} > 20000 \) K have been adopted from Kurucz (1979) at solar metallicity, covering \( T_{eff} = 20000 - 50000 \) K.

6.4 Model tuning, uncertainties, and selection

6.4.1 Model calibration

For the photometric evolution model discussed in the previous section, we show in Fig. 6.2 the evolution of the \((U-B), (B-V), (V-R), (V-I)\) colors, and the stellar mass-to-light ratio \( M_*/L_\odot \), of a single stellar population formed at \( t = 0 \) with initial stellar metallicities \( Z = 0.02, 0.008, \) and \( 0.001 \), respectively. We note that our models are dust-free and have been computed at a time resolution of \( \Delta t \approx [yr] \).\sim 6\).

We compare the model adopted in this paper with the recent photometric evolution models presented by Worthey (1994) and Bressan et al. (1994). In brief, the model of Worthey includes the metallicity dependent stellar evolution tracks of van den Berg (e.g. 1985) up to the base of the red giant branch (RGB), the Revised Yale Isochrones (Green et al. 1987) up to the tip of the RGB, and post RGB tracks from
different literature sources. The model of Bressan et al. comprises a homogeneous library of metallicity
dependent stellar evolution tracks up to the end of the early AGB (or the onset of central carbon ignition)
presented by Alongi et al. (1993) and Bressan et al. (1994). Both models use the stellar spectral flux library
from Kurucz (1992) and determine colors and magnitudes by convolution of the integrated spectral energy
distribution of a stellar population with the UBVRI pass-band filters.

Comparison of the UBV colors predicted by the model used in this paper with those given Worthey
(1994) and Bressan et al. (1994) reveals that these models provide very similar results. Note that the
selected colors in general become bluer with decreasing initial metallicity (metallicity effects in the R and I
band are usually small; see also Worthey 1994). Particularly good overall agreement is found between the
photometric results of Worthey and that developed by us (even though distinct libraries of stellar isochrones
were used).

![Figure 6.2 Theoretical UBVRI colors and stellar mass-to-light ratio evolution of a single stellar population with
initial metallicity $Z = 0.02$ (left panels), $Z = 0.008$ (center), and $Z = 0.001$ (right panels). Photometric evolution
models refer to: Worthey (1994, full circles), Bressan et al. (1994, dashed lines), this paper (solid curves)](image)

In general, the Bressan et al. model predicts R and I band magnitudes that are somewhat brighter
($\sim 0.1-0.4$ mag) at ages $\gtrsim 1$ Gyr at $Z = 0.02$ (and at ages $\gtrsim 0.1$ Gyr at $Z \lesssim 0.008$) than predicted by the
other models. This is due to the fact that Bressan et al. used the Johnson RI filter passbands (as supplied
with the Kurucz 1992 distribution) which are known to result in $(V-R)_{J}$ and $(V-I)_{J}$ colors that are too
red for cool stars (see e.g. Worthey 1994). The stellar mass-to-total light ratios vs. log Age predicted by
the different models are in good agreement. Variations in $M/L_{tot}$ with metallicity are found negligible. In
contrast, mass-to-light ratios for different passbands show a strong metallicity dependence, which reverses
when going from blue to near-IR colors (Worthey 1994).
Although a detailed description of the tuning and calibration of the adopted photometric model is beyond the scope of this paper, we note that the model has been checked against various observations including integrated colors and magnitudes, luminosity functions, and color-magnitude diagrams of Galactic (and Magellanic Cloud) open and globular clusters covering a wide range in age and metallicity.

As an example, we show in Fig. 6.3 resulting color-magnitude diagrams for a Monte-Carlo simulation of the Galactic disk open cluster M67. We used the stellar photometry data (mainly from the Geneva group) described in Sect. 3.3. For M67, we assumed an age of 3.5 Gyr and metallicity $Z = 0.016$. These values are consistent with observations which suggest that M67 is a solar metallicity cluster with an age of $\sim 4$ Gyr (see Montgomery et al. 1993). Furthermore, a binary fraction of $\sim 75\%$ (see Sect. 3.3.6) was assumed (for binaries with mass-ratios $m_2/m_1 \lesssim 1$) and we adopted an extinction in the direction of M67 of $E(B-V) = 0.05$ mag as indicated by the observations.

Fig. 6.4 shows the corresponding color-magnitude diagrams observed for M67 presented by Montgomery et al. (1993). Very good agreement is found between the predicted and observed color-magnitude diagrams of M67 cluster stars. In particular, the precise location and shape of the main-sequence turnoff points as well as the locations of the red giant branches are well reproduced by the models. Note that the detailed positions of the cluster stars in the color-magnitude diagrams can be compared (i.e. not the relative number of stars of a given V magnitude since the model stars were not weighted by the IMF). Similar calibration tests were performed successfully for open and globular clusters covering a wide range in age and metallicity. In general, good agreement was found between the predicted and observed color-magnitude diagrams of the clusters studied. We note that analogue comparisons have been presented e.g. by Worthey (1994) and Bressan et al. (1994).

### 6.4.2 Uncertainties and limitations

Although the previous comparison demonstrates that the adopted stellar evolution data are essentially correct and reliable, several uncertainties and sources of errors are involved in the photometric evolution model. These are related to the detailed assumptions and interpolations made in the evolutionary tracks used (e.g. amount of overshooting, mixing lengths, convection, nuclear reaction rates, etc.), and to the calibration of the stellar fluxes, magnitudes, and colors in the spectral library (e.g. temperature, gravity, chemical composition). Both the adopted stellar evolution tracks and stellar spectro-photometric library (accuracy and input physics, grid-range and interspacing, included stellar evolutionary phases, spectral range) determine to a large extent the final galaxy magnitude and color evolution. In addition, errors may arise because of differences in the filter transmission curves used to calculate synthetic magnitudes and those used with observations of e.g. galaxies (e.g. Bessell 1979).

Apart from these sources of errors, which are inherent to any photometric model used to predict the spectral evolution of a galaxy according to a given star formation history and chemical evolution, there are several uncertainties involved with the importance of binary stars, extinction, the detailed stellar mass function at birth and lower mass cutoff, initial element abundances (e.g. helium, oxygen) at a given metallicity, and the inclusion of late stages of stellar evolution which are relatively uncertain (e.g. post AGB and Wolf-Rayet stages). Although a detailed discussion of the above uncertainties is beyond the scope of this paper (see e.g. Worthey 1994; van den Hoek 1997), we do not expect these to alter the qualitative conclusions presented below. Overall, the models above are in good agreement with many independent aspects of stellar evolution theory which provides confidence for their application to more complex systems such as galaxies.

Nevertheless, the influence of binaries and the adopted IMF on the photometric evolution results may be relevant for the photometric evolution of LSB galaxies compared to that of HSB galaxies. Therefore, we will briefly address these effects when discussing model results below.

### 6.4.3 Model selection and properties

We present results for the chemical and spectro-photometric evolution model discussed in the previous section. We start from a model LSB galaxy with initial mass $M_\odot(t = 0) = 10^{10}$ M$_\odot$, initially metal-free and void of stars. The chemical and photometric evolution of this galaxy are followed during evolution time $t_{\text{ev}} = 14$ Gyr, assuming one of the theoretical star formation histories discussed below. Unless stated otherwise, we assume that stars are formed according to a Salpeter (1955) IMF (i.e. $\gamma = -2.35$) with stellar mass limits at birth between 0.1 and 60 M$_\odot$ (cf. Table 6.2).

A basic set of star formation histories is used to see how these models behave with respect to the observed properties of LSB spirals discussed in Sect. 6.2. The following functions of the SFR with galactic age are considered: 1) constant, 2) exponentially decreasing, 3) linearly decreasing, 4) exponentially increasing, and 5) linearly increasing. Normalized SFRs and resulting age-metallicity relations are shown in Fig. 6.5.
Figure 6.3. Monte-Carlo simulation of the Galactic disk open cluster M67. Resulting $V$ vs. $(B-V)$ and $V$ vs. $(V-I)$ color-magnitude diagrams for all stars in the field of the open cluster. An extinction of $E(B-V)=0.05$ mag and a binary fraction of 75% were assumed. Stars were selected randomly in mass but with absolute visual magnitudes $V<\sim 20$ mag.

Figure 6.4. Observed $V$ vs. $(B-V)$ and $V$ vs. $(V-I)$ color-magnitude diagrams for all stars in the field of the open cluster M67 from Montgomery et al. (1993). Note that the main sequence is still visible down to the limit of the photometry (i.e. $V\sim 20$ mag). The binary sequence at $\sim 0.7$ mag above the main sequence can be distinguished. Stars below the main-sequence are presumably field stars.
### 6.4.3 Model selection and properties

**Figure 6.5** Basic set of star formation histories considered (left) and resulting \([\text{O}/\text{H}]\) vs. age relations (right): constant SFR model (thick solid line), linearly decreasing (thin solid), exponentially decreasing (dashed; \(t_{\text{sfr}} = 5\) Gyr), rapid exponentially decreasing (dotted; \(t_{\text{sfr}} = 4\) Gyr), linearly increasing (dash-dotted), and exponentially increasing (dot-dashed). SFRs have been normalized to a current gas-to-total mass-ratio \(\mu_1 = 0.1\).

**Table 6.3** Basic set of star formation models \((\mu_1 = 0.1\) and \(M_{\text{tot}} = 10^{10}\) \(M_\odot\), unless noted otherwise)

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>(&lt;\text{SFR}&gt;)</td>
<td>SFR(_1)</td>
<td>(\alpha)</td>
<td>([\text{O}/\text{H}])</td>
<td>(N_{\text{tot}})</td>
<td>(L_{\text{B}})</td>
<td>(M_{\text{HB}}/L_{\text{B}})</td>
<td>IMF/SFR</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>Exp. decreasing</td>
<td>0.93</td>
<td>0.17</td>
<td>0.18</td>
<td>+0.3</td>
<td>3.6(10)</td>
<td>3.6(9)</td>
<td>0.16</td>
<td>(\gamma = -2.35, (+))</td>
</tr>
<tr>
<td>A2</td>
<td>&quot;</td>
<td>0.52</td>
<td>0.09</td>
<td>0.18</td>
<td>-0.25</td>
<td>2.1(10)</td>
<td>2.3(9)</td>
<td>1.56</td>
<td>(\gamma = -2.35,(+, 1))</td>
</tr>
<tr>
<td>A3</td>
<td>&quot;</td>
<td>0.69</td>
<td>0.13</td>
<td>0.18</td>
<td>-0.9</td>
<td>4.7(10)</td>
<td>1.6(9)</td>
<td>0.45</td>
<td>(\gamma = -3, (+))</td>
</tr>
<tr>
<td>A4</td>
<td>&quot;</td>
<td>1.08</td>
<td>0.20</td>
<td>0.18</td>
<td>+0.5</td>
<td>2.2(10)</td>
<td>5.2(9)</td>
<td>0.16</td>
<td>IMF: *, (+)</td>
</tr>
<tr>
<td>B1</td>
<td>Constant</td>
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<td>0.89</td>
<td>1.</td>
<td>+0.3</td>
<td>3.4(10)</td>
<td>1.0(10)</td>
<td>0.03</td>
<td>(\gamma = -2.35)</td>
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<td>&quot;</td>
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<td>0.49</td>
<td>1.</td>
<td>-0.25</td>
<td>2.1(10)</td>
<td>6.4(9)</td>
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<td>(\gamma = -2.35,(+))</td>
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<td>0.68</td>
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<td>-0.9</td>
<td>4.7(10)</td>
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<tr>
<td>B4</td>
<td>&quot;</td>
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<td>1.08</td>
<td>1.</td>
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<td>2.2(10)</td>
<td>1.7(10)</td>
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<td>C</td>
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<td>0.07</td>
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<td>0.60</td>
<td>0.67</td>
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<td>0.03</td>
<td>(\gamma = -2.35)</td>
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<td>F</td>
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<td>+0.3</td>
<td>3.3(10)</td>
<td>1.8(10)</td>
<td>0.04</td>
<td>(\gamma = -2.35)</td>
</tr>
</tbody>
</table>

* Kroupa et al. IMF (1992) assumed; (+) \(t_{\text{sfr}} = 5\) Gyr; (x) \(t_{\text{sfr}} = 4\) Gyr; (I) \(\mu_1 = 0.5\)

For each model, the amplitude of the SFR is chosen such that a present-day gas-to-total mass-ratio \(\mu_1 = 0.1\) is achieved (indices 1 and 0 will be used to refer to current and initial values, respectively).

In columns (2) to (6) of Table 6.3, we list the functional form of the SFR, average past and current SFRs, the ratio of current and average past SFRs \(\alpha\), and the present-day oxygen abundance by mass. It can be verified that the average past SFR, \(<\text{SFR}>\) = 0.9 \(M_\odot\) yr\(^{-1}\), is roughly the same for all models ending at \(\mu_1 = 0.1\), assuming a Salpeter IMF and \(M_\text{g}(t = 0) = 10^{10}\) \(M_\odot\). In contrast, *present-day* SFRs range from \(\langle \text{SFR}\rangle_1 = 0.07\) to 3 \(M_\odot\) yr\(^{-1}\) and in fact determine the contribution by young stars to the integrated light of the model galaxy (see below). Current oxygen abundances predicted are \([\text{O}/\text{H}]_1 \sim +0.25\) and are mainly determined by \(\mu_1\), the IMF, and the assumed mass limits for SNII (cf. Table 6.3).

Before comparing different SFR models with available observational data on LSB galaxies, we consider the photometric evolution of the constant and exponentially decreasing SFR models in some more detail. Fig 6.6 shows the evolution of the total number of MS and post-MS stars for the exponentially decreasing SFR model. The current total number of MS stars is roughly \(4 \times 10^{10}\), compared to \(~10^8\) post-MS stars (all phases) and \(~3 \times 10^4\) (AGB stars only). Mean stellar luminosities for stars in distinct evolutionary phases decreased over the past 10–12 Gyr by about one order of magnitude for MS, RGB, and HB stars while remaining relatively constant for early-AGB (EAGB) and AGB stars. The present-day mean luminosity of stars on the MS is \(\sim 10^{-1}\) \(L_\odot\) compared to \(\sim 10^4\) \(L_\odot\) for AGB stars. Summing over all phases, the product
6.4 Model tuning, uncertainties, and selection

Figure 6.6 Exponentially decreasing SFR model: total number of stars formed (left) and average stellar bolometric luminosity (right) vs. galactic age for distinct evolutionary phases: MS (solid curve), RGB (dashed), HB (dot-dashed), EAGB (dotted), and AGB (dash-dotted).

Figure 6.7 Photometric evolution of exponentially decreasing SFR (top panels) and constant SFR (bottom) models. Left panels: B-mag contribution for stars in distinct evolutionary phases as in Fig. 6.6: MS (solid curve), RGB (dashed), HB (dot-dashed), EAGB (dotted), AGB (dash-dotted), and Total (thick solid). Right panels: Same as left panels but for I-mag.
of total number of stars and mean stellar luminosity, shows that the current bolometric galaxy luminosity is determined mainly by MS stars ($L_{\text{MS}} \sim 4 \times 10^9 L_\odot$). In particular, AGB stars ($L_{\text{AGB}} \sim 10^8 L_\odot$) are relatively unimportant. This is characteristic of the constant and exponentially decreasing SFR models discussed here.

For the exponentially decreasing star formation model discussed in Fig. 6.6, we show in Fig. 6.7 the B and I-band magnitudes of stars in distinct evolutionary phases. As for the total galaxy luminosity, MS stars generally dominate in the B-band. However, within the I-band, RGB and HB stars are nearly as important as MS stars, at least at late stages of galactic evolution. Due to the cooling of old, low-mass MS stars as well as the contribution by RGB and HB stars increasing with galactic age, the current total I-band magnitude is considerably brighter than that in the B-band. We emphasize that this qualitative model behaviour is insensitive to the adopted star formation history (for e-folding times larger than $3\sim5$ Gyr) but instead is determined by the assumed IMF and the stellar input data used (e.g. lifetime in each evolutionary phase, stellar evolution tracks, etc). Thus, constant star formation models exhibit a similar behaviour apart from being brighter by about one magnitude in all passbands at later evolution times (cf. Fig. 6.7).

![Figure 6.8 Broadband colours vs. galactic age for exponentially decreasing SFR (thick solid) and constant SFR (solid) models. Note the different magnitude scales in the upper two and bottom three panels, respectively.](image)

Fig. 6.8 illustrates the sensitivity of broadband colors to the galactic star formation history for constant and exponentially decreasing SFR models. It can be seen that the colors considered increase with galactic age (most rapidly in $U-I$). In general, differences between colors such as $(U-B)$, $(B-V)$, and $(R-I)$ for distinct SFR models are less than the variations of these colors with age for a given model, the largest differences occurring in $(U-I)$ and $(B-V)$. Also, assuming a galactic age since the onset of star formation of e.g. 8 instead of 14 Gyr has limited effect on the resulting galaxy colors (e.g. less than 0.1 mag in $B-V$), even though absolute magnitudes are substantially altered (cf. Fig. 6.7). We like to emphasize that both age and extinction effects can result in substantial reddening of the colors of a stellar population in almost the same manner and it is difficult to disentangle their effects on the basis of photometry data alone. Clearly, galaxy colors alone are not well suited to discriminate between distinct SFR models, even when internal extinction is low and other reddening effects are negligible (see below).

Fig. 6.9 demonstrates that the contribution by post-MS stars to the mean colors of a stellar population is usually limited to a few tenths of a magnitude. Therefore, the use of $(B-V)$ and $(R-I)$ colors as mean age indicator of the dominant stellar population is valid for the dust-free models discussed here (cf. Fig. 6.8). In particular, the mean age of an unreddened stellar population increases with $(R-I)$, provided that AGB stars are negligible contributors to this color. Note that the detailed $(R-I)$ vs. age relation given in Figs. 6.8 and 6.9 depends on the assumed IMF, SFR, and stellar evolution data used.
6.4 Model tuning, uncertainties, and selection

6.4.4 More detailed predictions

Fig. 6.10 illustrates in detail the properties (i.e. the luminosities, colors, effective temperatures, abundances, and masses) of the present-day stellar population predicted by the exponentially decreasing SFR model ($\tau_{\text{sfr}} = 5$ Gyr) ending at $\mu_1 = 0.1$. Results are shown for stars in different evolutionary phases from the main-sequence up to the AGB by means of a Monte-Carlo simulation. This figure serves to illustrate the main characteristics of the present-day stellar populations (with ages 14 Gyr) predicted by our models and can be compared both with observations and other spectro-photometric evolution models. Very similar results are obtained e.g. for exponentially decreasing SFR models ending at $\mu_1 = 0.5$ or constant SFR models.

Fig. 6.11 shows the corresponding luminosity, color, mass, and metallicity distributions for the present-day main-sequence stellar population shown in Fig. 6.10. Again, these distributions (such as the mass and luminosity function of main-sequence stars) can be compared directly to observations whenever such data is available. We emphasize that these distributions do not depend strongly on the adopted star formation history since the distributions are normalized to all stars (ever formed in the model galaxy) that are nowadays on the main-sequence. In contrast, these distributions are very sensitive to the stellar IMF at birth and, in principle, can be used to constrain the IMF.

In Fig. 6.12 we show the resulting present-day luminosity and number contributions of stars in different evolutionary phases as function of their age (corresponding to the results discussed in Figs. 6.11 and 6.10). For instance, Fig. 6.12 demonstrates that main-sequence stars younger than $\sim 1$ Gyr contribute nearly 75% to the total present-day U band luminosity of all main-sequence stars. Similarly, the contribution of main-sequence stars younger than 1 Gyr to the I band is less than $\sim 25%$. These results can be used to estimate the contribution of young stars to the present-day galaxy luminosity in a given wave band which, in principle, also can be determined observationally. Such observations provide valuable constraints to the underlying star formation history and stellar IMF at birth of the present-day stellar populations in galaxies. For comparison, Fig. 6.13 illustrates the same results for the constant SFR model ending at $\mu_1 = 0.1$ (see Fig. 6.5) and reveals the sensitivity of the present-day luminosity distributions of stars in different evolutionary phases to the underlying star formation history.
Figure 6.10 Predicted properties of individual stars in different evolutionary phases at galactic age $t_{\text{ev}} = 14$ Gyr in case of the exponentially decreasing SFR model ($\tau_{\text{sfr}} = 5$ Gyr) ending at $\mu_1 = 0.1$. Stars were selected according to their present-day evolutionary phase (i.e. no IMF or SFR weighting was applied). For each phase, the properties of $\sim 100$ stars were plotted as follows: MS (full circles), RGB (open circles), HB (crosses), EAGB (open triangles), and AGB (full triangles).
Figure 6.11 Resulting integrated properties of main-sequence stars at galactic age $t_{ev} = 14$ Gyr in case of the exponentially decreasing SFR model ($\tau_{sfr} = 5$ Gyr) ending at $\mu_1 = 0.1$. A total of $10^4$ main-sequence stars were selected according to the Salpeter IMF and SFR model adopted. Grey-scale corresponds to the fraction of all main-sequence stars selected within each bin (maximum bin content is indicated on top of legend).
Figure 6.12 Resulting present-day luminosity (and number) contributions as a function of stellar age for stars in different evolutionary phases. Results are shown in case of the exponentially decreasing SFR model ($\tau_{sfr} = 5$ Gyr) ending at $\mu_1 = 0.1$. Both normalized (solid curve) and cumulative (thick solid) distributions of (left to right) U, B, V, R, I, $L_{tot}$, and $N_{tot}$ are plotted for: (top to bottom) MS, RGB, HB, EAGB, AGB, and all (TOT) stars.
Figure 6.13 Resulting present-day luminosity and number contributions as a function of stellar age for stars in different evolutionary phases. Results are shown in case of the constant SFR model ($\tau_{sfr} = 5 \text{ Gyr}$) ending at $\mu_1 = 0.1$. Both normalized (solid curve) and cumulative (thick solid) distributions of (left to right) $U$, $B$, $V$, $R$, $I$, $L_{tot}$, and $N_{tot}$ are plotted for: (top to bottom) MS, RGB, HB, EAGB, AGB, and all (TOT) stars.
6.5 Results

We confront the basic set of star formation models discussed in the previous section with the complete set of observations available for LSB galaxies, including UBVRI broadband photometry, present-day HI masses, gas-to-total mass-ratios, and [O/H] abundances. We are primarily interested in global differences between the star formation history of LSB and HSB spirals. However, we attempt to extend some of our results to the sample of dwarf irregulars discussed in Sect. 6.2. In this section, we restrict ourselves to an investigation of model parameters related to the global star formation history for a few standard SFR models. In the next sections, we consider the impact of small amplitude bursts on the photometric evolution of LSB galaxies and discuss what our results imply for each of the galaxy samples.

Figure 6.14 Exponentially decaying SFR model: photometric and chemical evolution results. Curves have been drawn for initial galaxy masses $M_g(t=0) = 10^8$, $10^9$, and $10^{10}$ $M_\odot$. For the $M_g(t=0) = 10^9$ $M_\odot$ model, arrows indicate evolution times of 1, 2, 4, 8, and 14 Gyr, respectively. Symbols refer to the following galaxy samples: face-on HSB spirals (dots; de Jong & van der Kruit 1994), LSB spiral galaxies (triangles; de Blok et al. 1995), and dwarf irregulars (crosses; Melisse and Israel 1994). Typical observational errors are shown in the bottom left of each panel.

6.5.1 Star formation history of LSB vs. HSB galaxies

Predictions of exponentially decreasing SFR models are usually found in good agreement with observations of HSB galaxies with different $e$-folding times for different Hubble types (e.g. Larson & Tinsley 1978; Guiderdoni & Rocca-Volmerange 1987; Kennicutt 1989; Bruzual & Charlot 1993; Fritze-v. Alvensleben & Gerhard 1994).

In Fig. 6.14 we concentrate on the exponentially decreasing SFR model $SFR_x \propto \exp(-t/\tau_{sfr})$ with $\tau_{sfr} = 5$ Gyr ending at a present-day gas-to-total mass-ratio of $\mu_1 = 0.1$ (appropriate to the Galactic disk; e.g. Clayton 1988). For this model, the chemical and spectro-photometric evolution have been followed during the last 14 Gyr. A value of $\mu_1 = 0.1$ was achieved by scaling the amplitude of the SFR accordingly. Model
results are considered for the range of initial galaxy masses appropriate to LSB spirals, i.e. $M_g(t=0) \sim 10^8 - 10^{10} M_\odot$, in such a way that the ratio $SFR(t=0)/M_g(t=0)$ remains constant. Note that galaxy colors and abundances are not affected by such scaling while, in contrast, absolute magnitudes and final gas masses scale with the adopted value of $M_g(t=0)$ (as indicated by the distinct curves shown in each panel of Fig. 14).

**LSB galaxies**

Present-day ($B-V$) and ($R-I$) colors for the exponentially decreasing SFR model are 0.6 and 0.5 mag, respectively. These colors agree reasonably well with the reddest values observed for LSB galaxies. However, many LSB galaxies included in Fig. 6.14a exhibit present-day ($B-V$) colors $\lesssim 0.5$ mag that cannot be explained by the exponentially decreasing SFR model shown. The same appears true when considering the ($R-I$) colors (Fig. 6.14b) although these data are incomplete relative to the ($B-V$) data. Both the colors of ($B-V$) $\lesssim 0.5$ and ($R-I$) $\lesssim 0.4$ mag observed for many LSB galaxies indicate the presence of a bright stellar population probably younger than $\sim 5$ Gyr (cf. Fig. 6.2). Alternatively, metallicity, extinction, IMF, and/or $\tau_{sfr}$ effects may play an important role in determining the ($B-V$) and ($R-I$) colors of LSB galaxies. We will discuss these possibilities further on in this paper.

Fig. 6.14c demonstrates that the [O/H] abundances predicted by the exponentially decaying SFR model are much larger than observed in LSB galaxies. Since current metal-abundances are primarily determined by the present-day gas-to-total mass-ratio $\mu_1$, this is typical for SFR models ending at $\mu_1 = 0.1$ (cf. Table 6.3). The large range observed in [O/H] abundances at a given B magnitude (i.e. down to [O/H] $\sim -1.4$) indicates that substantial variations in stellar enrichment have occurred among LSB galaxies. There are several plausible explanations for this: 1) LSB galaxies show present-day gas fractions much larger than $\mu_1 = 0.1$, 2) the ISM in LSB galaxies has been diluted by metal-poor material (e.g. by metal-deficient gas infall), 3) LSB galaxies exhibit a considerable range in age between 8 and 14 Gyr since the onset of main star formation in their disks, and/or 4) LSB galaxies experienced various degrees of stellar enrichment (e.g. due to differences in the IMF and/or metallicities related to SNII enrichment). In the latter case, it would be necessary to explain why stellar enrichment in LSB galaxies would be distinct from that in HSB spirals for which exponentially decaying SFR models predict reasonable abundances (cf. Fig. 6.14c) assuming a Salpeter IMF and SNII enrichment as described Sect. 6.3.

Fig. 6.14d shows that exponentially decreasing SFR models ending at $\mu_1 = 0.1$ are also inconsistent with the total amount of gas $M_g \sim 1.4M_H$ (corrected for helium) observed in LSB galaxies of a given B magnitude, provided that the present amount of molecular gas in these systems is negligible (see Sect. 6.2). This is true for all SFR models in the case of $\mu_1 = 0.1$ (we exclude the possibility that LSB galaxies are younger than $\sim 1$ Gyr; cf. Fig. 6.14d). These results imply that the present-day gas-to-total mass-ratios in LSB galaxies are much larger than $\mu_1 = 0.1$ and/or that substantial amounts of gas have been accreted in these systems during their evolution. Both possibilities are consistent with our findings from the [O/H] abundances. We conclude that exponentially decreasing SFR models ending at $\mu_1 = 0.1$ are inconsistent with the observed properties of LSB galaxies.

**HSB galaxies**

Present-day colors of HSB galaxies with ($B-V$)$\gtrsim 0.6$ and ($R-I$)$\gtrsim 0.5$ mag can be explained by the exponentially decreasing SFR model ending at $\mu_1 = 0.1$ only when substantial amounts of internal dust extinction are incorporated. The reason is that, even though a single stellar population born at $Z=0.02$ may become as red as ($B-V$)$\sim 1.1$ and ($R-I$)$\sim 0.8$ mag (cf. Fig. 6.2), the luminosity contribution by young stellar populations (with ages less than a few Gyr) results in substantial bluing of the galaxy colors. For the same reason, values of ($B-V$) $\gtrsim 0.65$ mag are not predicted by our dust-free models independent of the adopted SFR or IMF (see below). Therefore, considerable reddening of, in particular, the emission associated with these younger stellar populations is required to explain the colors of HSB galaxies.

Since internal extinction in HSB galaxies up to $E(B-V) \sim 0.5$ mag (corresponding to $A_V \sim 1.5$ mag) is required, no matter what the detailed underlying star formation history of these systems is, extinction and metallicity may be one of the main explanations for the differences in color observed between LSB and HSB galaxies. This conclusion is consistent with the well established fact that most extinction in HSB galaxies originates from the galaxy nucleus and spiral arms (see Sect. 6.2), i.e. the sites where star formation and luminous young stellar populations in HSB galaxies are usually observed and which are weakly developed or absent in LSB galaxies. In this manner, LSB galaxies are not remarkably blue compared to HSB galaxies but instead the young stellar populations determining the colors of HSB galaxies appear exceptionally red due to relatively large amounts of internal extinction (and high metallicity). This view is consistent with independent arguments which suggest that reddening by dust in LSB galaxies is relatively unimportant.
(Sect. 6.2) and indicates that at least part of the color differences between HSB and LSB galaxies are due to effects of extinction. We emphasize that the underlying stellar populations in LSB and HSB galaxies are distinctly different due to marked differences in the chemical evolution of these systems (see below). Therefore, extinction effects, although important, cannot be the entire explanation for the color differences observed.

Apart from extinction, reddening effects due to the inclusion of binaries and/or preferential formation of low mass stars may play a significant role in determining the colors of individual galaxies. Although a detailed investigation of these effects is beyond the scope of this paper, we have verified that reasonable corrections for binaries (by means of an enhanced contribution by post-MS stars) and for variations in the stellar IMF are unable to account for the observed color differences between LSB and HSB galaxies (even though such corrections can result in substantial reddening up to \( \sim 0.2 \) mag in \((B-V)\) and \( \sim 0.15 \) mag in \((R-I)\); cf. van den Hoek 1997).

Figs. 6.14c and d show that both the [O/H] abundance ratios and present-day gas masses of HSB galaxies can be explained reasonably well by exponentially decreasing SFR models ending at \( \mu_1 \sim 0.1 \) (provided that the upper HSB galaxies in Fig. 6.14d have initial masses as large as \( M_g(t=0) \gtrsim 10^{11} M_\odot \)).

We conclude that, in contrast to LSB galaxies, exponentially decreasing SFR models ending at \( \mu_1 = 0.1 \) generally provide adequate explanations for the star formation history of late-type HSB galaxies when internal extinction is taken into account. This is consistent with the results from previous evolution models (with different stellar input data) for HSB galaxies (e.g. Larson & Tinsley 1978; Guiderdoni & Rocca-Volmerange 1987).

6.5.2 A more detailed comparison

In the previous section, we have argued that exponentially decreasing SFR models ending at a present-day gas-to-total mass-ratio \( \mu_1 = 0.1 \) are inconsistent with the observed B–V colors, abundances, gas fractions, and gas contents of LSB galaxies. We here extend the set of SFRs considered to models ending at values \( \mu_1 = 0.025, 0.1, 0.3, 0.5, 0.7, \) and 0.9, for the various star formation histories shown in Fig. 6.5.

Colors and magnitudes

Results for the exponentially decaying SFR models ending at values of \( \mu_1 \gtrsim 0.025 \) are shown in Fig. 6.15 (the amplitude of the SFR was scaled while maintaining the functional form of the SFR). While the age distribution of the stellar populations is the same for the models shown, present-day \((B-V)\) and \((R-I)\) colors are found to decrease by 0.2 and 0.1 mag, respectively, when going from models ending at \( \mu_1 = 0.025 \) to \( \sim 1 \) (cf. Figs. 6.15a,b). This blueing effect is due to the decrease of stellar initial metallicities for models ending at increasingly higher gas fractions. We emphasize that the abundance effect on the galaxy colors is substantial and originates from the metallicity dependent set of stellar evolution data used.

Exponentially decreasing SFR models ending at values \( \mu_1=0.5–0.7 \) are in best agreement with the \((B–V)\) and \((R–I)\) colors of a typical LSB galaxy with \((B–V) \gtrsim 0.5 \) mag. Note that these results can be shifted towards brighter B magnitudes by assuming initial gas masses larger than \( M_g(t = 0) = 10^{10} M_\odot \) (and vice versa) while this leaves the resulting colors unaltered (cf. Fig. 6.14).

In principle, single burst SFR models in which all stars were formed about 5 Gyr ago with \( Z=0.001 \) predict colors of \((B–V) = 0.55 \) and \((R–I) = 0.4 \) mag that are consistent with the typical LSB galaxy colors observed as well (cf. Fig. 6.2). However, as recent star formation is observed in basically all the LSB galaxies in our sample (see below), a prominent stellar population much older than 5 Gyr must be present to compensate for the color contributions of the more recently formed stellar populations in these systems. Consequently, single burst models cannot be appropriate for the evolution of LSB galaxies and illustrate the need for roughly exponentially decreasing SFR models to fit the colors of LSB galaxies with \((B–V) \gtrsim 0.5 \) mag.

In contrast, relatively blue LSB galaxies with \((B–V) \lesssim 0.4 \) mag cannot be fitted by exponentially decreasing SFR models alone (assuming \( t_{ev} = 14 \) Gyr), regardless of their current gas fraction \( \mu_1 \). In these blue LSB galaxies a relatively young stellar population may contaminate the galaxy colors superimposed on an exponentially decreasing SFR. Alternatively, these LSB galaxies may be much younger than \( \sim 14 \) Gyr (e.g. 5–8 Gyr old, cf. Fig. 6.15a) and/or may have experienced linearly (i.e. more slowly than exponentially) decaying or constant SFRs which generally result in present-day colors of \((B–V) \lesssim 0.4 \) and \((R–I) \lesssim 0.35 \) mag.

This suggests that LSB galaxies can be distinguished in two major groups by means of their colors: one group with \((B–V) \gtrsim 0.5 \) mag whose colors can be explained by exponentially decreasing SFR models (ending at \( t_{ev} = 14 \) Gyr) and a second group with \((B–V) \lesssim 0.5 \) mag which shows clear evidence for the
6.5.2 A more detailed comparison

Figure 6.15 Photometric and chemical evolution results in case of exponentially decaying SFR models ending at $\mu_1 = 0.025, 0.3, 0.5, 0.7, \text{ and } 0.9$ for an initial galaxy mass of $M_g(t=0) = 10^{10} M_\odot$ (in Fig. 6.15d we show also results for $M_g(t=0) = 10^9 M_\odot$). Arrows on top of the $\mu_1 = 0.025$ model indicate evolution times of 1, 2, 4, 8, and 14 Gyr, respectively. Observational data as in Fig. 6.14

presence of a young stellar population that dominates the luminosity (e.g. these LSB galaxies may have: 1) experienced their onset of main star formation relatively recently, or 2) experienced a recent burst of star formation on top of an old stellar population; see Sect. 6.6).

Abundances

The observed range in $[O/H]$ abundances for LSB galaxies is well explained by exponentially decaying SFR models with $\mu_1 \gtrsim 0.3$ (cf. Fig. 6.15c). However, constant SFR models ending at $\mu_1 \gtrsim 0.3$ are also consistent as the abundances of elements predominantly produced in massive stars are in general determined by the present-day gas fraction $\mu_1$ and are insensitive to the detailed underlying star formation history (e.g. Tinsley 1980).

Metal-poor LSB galaxies with $[O/H] \lesssim -1$ probably have experienced low and sporadic star formation histories different from exponentially decaying or constant SFRs (low and constant SFRs appear to be excluded by the colors and gas fractions of these systems; see below). Alternatively, star formation may have turned on recently. In either case, such galaxies are relatively unevolved and usually have high gas-to-total mass-ratios. A substantial fraction of these metal-poor systems may have accreted considerable amounts of metal-poor gas since the onset of main star formation in their disks, maintaining the low abundances in the disk ISM (see Sect. 6.8).

Another effect which may play an important role for the enrichment of LSB galaxies is the stellar IMF. Although present observations are inconclusive, the very low gas surface densities observed in LSB galaxies may result in significantly lower low and high-mass cutoffs of the IMF, i.e. the preferential formation of low mass stars in LSB compared to that in HSB galaxies. For instance, a steep power law IMF with $\gamma = -3$ results in oxygen abundances of $[O/H] = -1$ at $\mu_1 = 0.1$ (cf. Table 6.3). On the other hand, steep IMF models are unable to explain LSB galaxies with the highest relative luminosities and abundances, regardless
of the assumed SFR history and value of $\mu_1$, so that a range of IMFs would be needed to explain the entire range of abundances observed in LSB and HSB galaxies.

We estimate that at least $\sim 30\%$ of the LSB galaxies in our sample have very low abundances (i.e. $\text{[O/H]} \lesssim -1$) with respect to their small present-day gas fractions (i.e. $\mu_1 \sim 0.5$) and probably experienced infall of substantial amounts of metal-poor gas and/or have formed stars relatively deficient in massive stars. We will return to these possibilities below.

**Gas contents**

Present-day gas masses observed in LSB galaxies can be well fitted by exponentially decaying SFR models ending at $\mu_1 = 0.3 - 0.5$ (note that the results can be shifted towards fainter B magnitudes and smaller present-day gas masses (and vice versa) by varying the adopted value for the initial mass $M_g(t=0)$ while leaving predicted colors and abundances unaltered; cf. Fig. 6.15d). This is consistent with the range of $\mu_1 > 0.3$ derived from the [O/H] data. Exponentially decreasing SFR models ending at $\mu_1 \gtrsim 0.6$ are clearly inconsistent with the observations if we exclude the possibility that LSB galaxies are extremely young systems with a stellar population younger than a few Gyr.

Present-day gas masses in combination with the colors and magnitudes observed in LSB galaxies, probably exclude e.g. constant or slowly decaying SFR models since such models predict present-day gas mass-to-light ratios that are too small compared to the observations (see below).

![Figure 6.16 Comparison of model predicted and observed gas-to-total mass-ratios and total gas masses vs. (B–V). Top panels: exponentially decreasing SFR models. Bottom panels: constant SFR models. Models with SFR normalizations according to $\mu_1 = 0.025, 0.1, 0.3, 0.5, 0.7,$ and $0.9$ are shown for an initial galaxy mass of $M_g(t=0) = 10^{10} M_\odot$. Arrows on top of the $\mu_1 = 0.025$ model indicate evolution times of 1, 2, 4, 8, and 14 Gyr, respectively. Observational data as in Fig. 6.14](image-url)
Gas fractions

Fig. 6.16 displays the present-day gas-fraction and total amount of gas vs. \( (B-V) \) for constant and exponentially decaying SFR models. Exponentially decreasing SFR models are able to explain simultaneously values of \( \mu_1 \sim 0.5 \pm 0.2 \), \( (B-V) \gtrsim 0.5 \) mag, and \( M_g \gtrsim 10^9 \, M_\odot \), as observed for the majority of the LSB galaxies in our sample. Over the entire range of possible gas fractions, constant (or increasing) SFR models are clearly inconsistent with the observations provided that: 1) internal extinction in these galaxies is low (i.e. \( E(B-V) \lesssim 0.1 \) mag; see Sect. 6.2.4), and 2) these galaxies have not recently accreted large amounts of gas, i.e. much larger than that presently observed within their optical disks. In fact, the inclusion of gas infall does not alter this conclusion since the colors predicted by such SFR models remain inconsistent with the observations.

For exponentially decreasing SFR models, we emphasize that (continuous) accretion of matter is consistent with the observations as long as: 1) the initial galaxy mass was substantially less than the typical dynamical mass \( M_{\text{dyn}} = 10^{10} \, M_\odot \) currently observed for LSB galaxies (see Table 6.1), and 2) a present-day gas fraction \( \mu_1 = 0.5 \pm 0.2 \) is predicted. Thus, the gas reservoir at the time of onset of main star formation in LSB galaxies may have been substantially less than that estimated from their present-day amounts of gas.

Similar conclusions can be reached when the gas mass-to-light ratio is considered (see below). Unfortunately, LSB galaxies for which reliable gas fractions are currently available are biased towards relatively red LSB galaxies with \( (B-V) \gtrsim 0.5 \) mag. A few LSB galaxies with \( (B-V) \lesssim 0.4 \) mag, however, may be best fitted by constant SFR models (or exponentially or more slowly decreasing SFR models with the contribution of an additional young stellar population superimposed).

![Figure 6.17](image-url) Evolution of \( M_{\text{HI}} / L_B \) vs. \( (B-V) \) (left) and \( M_{\text{HI}} / L_B \) vs. \( \mu_1 \) (right). Top panels: exponentially decreasing SFR models, bottom panels: constant SFR models. From bottom to top, curves are shown for models ending at increasing gas fractions \( \mu_1 = 0.1, 0.3, 0.5, \) and 0.7, respectively. Arrows on top of the \( \mu_1 = 0.1 \) model indicate evolution times of 1, 2, 4, 8, and 14 Gyr, respectively. Observational data as in Fig. 6.16
Gas mass-to-light ratios

Fig. 6.17 demonstrates that exponentially decreasing SFR models ending at $\mu_1 \sim 0.5$ are in good agreement with the observed atomic hydrogen mass-to-light ratios $M_{\text{HI}} / L_B$ as well, in contrast to constant (or slowly decreasing) SFR models.

Consequently, our earlier finding that it is possible to distinguish the SFR history of LSB galaxies on the basis of their colors probably can be interpreted only in a manner consistent with their mass-to-light ratios, if most of the LSB galaxies with $(B-V) \lesssim 0.4$ mag experienced exponentially (or somewhat less rapidly) decreasing SFRs with an additional luminosity and color contribution from a young stellar population. This finding essentially excludes constant SFR models for the majority of the LSB galaxies in our sample.

The high $M_{\text{HI}} / L_B$ ratios and the large values of $\mu_1$ observed for LSB galaxies can be interpreted in terms of slow evolution due to low rates of star formation and/or large amounts of gas infall since the onset of star formation. Infall would be consistent also with the relatively low abundances observed in some LSB galaxies with respect to their gas fractions.

As discussed in Sect. 6.2, the present-day gas fraction $\mu_1$ used in this paper provides a hard lower limit to the actual gas-to-total mass-ratio within the optical radius of a LSB galaxy. In fact, the total amount of gas available for star formation within LSB galaxies may be much larger than that present within their optical radii. This may be due to accretion of matter from beyond their optical disk and/or infall of matter from large scale heights onto the disk. In either case, this would shift data points to the upper left in the $M_{\text{HI}} / L_B$ vs. $\mu_1$ diagram. However, from the agreement between the $M_{\text{HI}} / L_B$ vs. $(B-V)$ data observed and that predicted by exponentially decreasing SFR models ending at $\mu_1 = 0.3$–0.7 for the majority of the LSB galaxies in our sample, we argue that the observationally determined gas fractions of $\mu_1 \sim 0.5$ are probably accurate within $\sim 50\%$. Some exceptions include the LSB and dwarf galaxies with $M_{\text{HI}} / L_B \gtrsim 2$ for which gas fractions of $\mu_1 \lesssim 0.5$ are likely underestimated by factors 2–3. This conclusion is insensitive to whether infall is involved or not (as long as $\mu_1 = 0.3$–0.7 and $M_{\text{tot}} \sim 10^{10}$ $M_\odot$ at present) but depends on our assumptions of $\tau_{\text{eff}} = 5$ Gyr, $t_{\text{ev}} = 14$ Gyr, and $E(B-V) \lesssim 0.1$ mag for LSB galaxies.

A possibility to reproduce the observed colors and $M_{\text{HI}} / L_B$ ratios of LSB galaxies by constant or slowly decreasing SFR models would be if the contribution by post main-sequence stars to the galaxy integrated colors is considerably underestimated in our models (e.g. due to IMF effects, binary inclusion, see Sect. 6.5.1). However, the predicted HI mass-to-light ratios for such models would lie far below those observed and large amounts of infall would be needed to compensate for this effect. Since the stellar evolution data used in our models is comparable to that of other recent photometric evolution models (see Sect. 6.4), a significant underestimate of the post-MS star contributions to the galaxy colors appears improbable. Therefore, it seems safe to conclude that constant and slowly (e.g. linearly) decreasing SFR models are appropriate only for a very small fraction ($\lesssim 10\%$) of the LSB galaxies in our sample.

The main reason to conclude that constant and slowly decreasing SFR models are inconsistent with the observed photometric and chemical properties of LSB galaxies is based on the assumption of negligible amounts of dust extinction in these systems. However, if a considerable fraction of the LSB spirals in our sample would suffer from extinctions of $E(B-V) = 0.1$ – 0.25 mag, the $M_{\text{HI}} / L_B$ ratios predicted by constant SFR models would increase by a factor 1.4–2.5 after correction for internal extinction. In this manner, constant and slowly decreasing SFR models ending at gas fractions $\mu_1 = 0.3$ – 0.7 after $\sim 14$ Gyr could also explain relatively red LSB galaxies with $(B-V) \gtrsim 0.5$ mag and $M_{\text{HI}} / L_B$ ratios as large as $\sim 1.5$. However, we have argued in Sect. 6.2 that internal extinction in LSB galaxies is unlikely to exceed $E(B-V) \sim 0.1$ mag so that slowly decreasing and constant SFRs appear inconsistent with most of the LSB galaxies in our sample.

To summarize, we find that exponentially decreasing SFR models ending at $\mu_1 = 0.3$ – 0.7 are in optimal agreement with the colors, magnitudes, [O/H] abundances, gas contents, and mass-to-light ratios observed for LSB galaxies with $(B-V) \gtrsim 0.45$ mag. For some of the LSB galaxies in our sample, the observed [O/H] abundances (as well as present-day gas contents) indicate that accretion of substantial amounts of metal-deficient gas has occurred since the onset of star formation in their disks. Blue LSB galaxies with $(B-V) \leq 0.45$ mag cannot be fitted by exponentially decreasing SFR models without an additional light contribution from a young stellar population. Alternatively, such LSB galaxies have experienced constant SFRs, SFRs more slowly decreasing than exponentially, or may be much younger than 14 Gyr. We presented arguments in favour of the possibility that such LSB galaxies experienced exponentially decreasing SFRs as well but with recent epochs of enhanced star formation. We will consider this possibility in more detail below.
6.6 Recent star formation in LSB galaxies

6.6.1 Luminosity contribution by \textsc{H} II regions in LSB galaxies

We investigate the contribution of young massive stars to the integrated colors and magnitudes of LSB galaxies. To this end, we select all \textsc{H} II regions that can be identified by eye, either from the \textsc{H}α or R-band CCD images, and add up their total luminosity in a given passband. We define $\eta_\lambda$ as the ratio of this \textsc{H} II region integrated luminosity and the luminosity of the hosting LSB galaxy. In this way, the brightest and largest \textsc{H} II regions within each LSB galaxy are easily traced although some \textsc{H} II regions may be missed due to extinction and selection effects (see below). The \textsc{H} II regions identified contain recently formed, massive OB stars that are hot and luminous enough to ionize the surrounding ISM.

<table>
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<tr>
<th>Name</th>
<th>#</th>
<th>B</th>
<th>I</th>
<th>$\eta_B$</th>
<th>$\eta_I$</th>
<th>$(B-V)_{\text{HII}}$</th>
<th>$(B-V)_{\text{gal}}$</th>
<th>SFR$_{\text{cont}}$</th>
<th>SFR$_{\text{burst}}$</th>
<th>SFR$<em>{\text{tot}}$ [M$</em>\odot$ yr$^{-1}$]</th>
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<td>-14.4</td>
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<td>0.59</td>
<td>0.013</td>
<td>0.068</td>
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<td>0.12†</td>
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<td>0.65</td>
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<td>1.7†</td>
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<td>0.04</td>
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<td>-10.1*</td>
<td>0.06</td>
<td>0.10*</td>
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<td>–</td>
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</tr>
<tr>
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<td>-12.2*</td>
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<td>0.04*</td>
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<td>0.19*</td>
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</tbody>
</table>

* values refer to R band magnitudes instead of I-band
† uncertain due to contamination by fore- or background objects
— $\mu_{\text{rot}}$ not determined

In columns (1) and (2) of Table 6.4, we list the LSB galaxy identification and number of \textsc{H} II regions selected. The number of \textsc{H} II regions identified within one LSB galaxy ranges from a few to $\sim 25$. For the ensemble of \textsc{H} II regions in each LSB galaxy, we tabulate the absolute B and I magnitudes as well as the corresponding ratios $\eta_B$ and $\eta_I$ of the \textsc{H} II region integrated luminosity and total LSB galaxy luminosity, in columns (3) to (6). Mean $(B-V)$ colors for the \textsc{H} II regions and for the LSB galaxy as a whole are given in columns (7) and (8), respectively.

Magnitudes and colours of the individual \textsc{H} II regions are found to vary over a wide range. This may be expected for \textsc{H} II regions differing in e.g. age, size, metallicity, IMF, etc. We find that there is a tendency of brighter magnitudes for LSB galaxies with larger number of identified \textsc{H} II regions. An exception is F564-V3 which is probably an extreme dwarf galaxy. We recall that the \textsc{H} II region integrated luminosity contribution may be strongly biased towards the properties of individual \textsc{H} II regions in LSB galaxies for which the total number of identified regions is low.

Substantial amounts of dust extinction within the \textsc{H} II regions in LSB galaxies are found to be present. First, $(B-V)$ colors of many \textsc{H} II regions are nearly the same as (or even redder than) observed for the hosting LSB galaxy. Second, the \textsc{H} II regions identified are usually brighter in I than in the B band (cf. Table 6.4). Third, selective extinction of $E_{B-V} = 0.3\pm0.1$ mag is suggested by the reddening coefficients based on the ratio of \textsc{H}α and $H_\beta$ lines towards the \textsc{H} II regions observed (e.g. McGaugh 1994; see also Osterbrock 1989). We note that a value of $E_{B-V} = 0.3$ mag is consistent with the typical extinction derived for \textsc{H} II regions in the Magellanic Clouds (e.g. Wilcots 1994). Also, very few \textsc{H} II regions display properties of an optically thin medium (McCall et al. 1985; Dopita & Evans 1986). Assuming a mean Galactic interstellar extinction curve, this corresponds to $A_B \sim 1.2\pm0.4$ and $A_I \sim 0.6\pm0.2$ mag. As the mean extinction curve for \textsc{H} II regions in low metallicity, low density LSB galaxies may be distinct from that in the Galactic disk, the actual extinction...
6.6.1 Luminosity contribution by H II regions in LSB galaxies

may be much lower.

For most of the LSB galaxies in our sample, the contribution by the H II regions to the total light emitted by LSB galaxies does not exceed \( \eta = 0.05 \pm 0.1 \) both in the B and I band. Corrected for extinction, the actual contributions may be larger, up to factors \( \sim 3-4 \) in B and \( \sim 2 \) in I, respectively, provided that the mean extinction in LSB galaxies is low compared to that in their constituent HII regions. Some HII regions may be missed due to dust extinction (values up to \( E(B-V) \sim 0.6-1.1 \) mag are observed; e.g. McGaugh 1994) and/or seeing effects. Thus, the values of \( \eta \) derived provide severe lower limits to the actual luminosity contributions of the HII regions.

For some LSB galaxies, e.g. F568-V1 and F577-V1, the HII region contribution is found as high as \( \eta = 0.2 \) both in the B and I band. These systems contain a modest number of HII regions so that their HII regions on average may be larger and/or brighter than those present in several other LSB galaxies. Alternatively, the old stellar population within F568-V1 and F577-V1 may be under-represented due to a relatively low past SFR. This may apply in particular to F577-V1 which has \( (B-V) \sim 0.4 \) mag and is among the bluest systems listed in Table 6.4.

Figure 12 shows the resulting HII region contributions \( \eta_\lambda \) in the B and I band for the SFR models discussed in Sect. 6.4. We assume a maximum age \( \tau_{\text{HII}} = 5 \) Myr for the HII regions observed in the LSB galaxies in our sample. This implies that stars more massive than \( \sim 25 \) M\(_{\odot}\) are associated with the ionized regions identified in H\( \alpha \), according to the stellar evolution tracks from the Geneva group (see below). Constant and increasing SFR models predict the HII region contribution \( \eta_\lambda \) to increase at recent epochs (both in the B and I bands). In contrast, exponentially decreasing SFR models predict \( \eta_\lambda \) to decrease. We note that both \( \eta_\lambda \) and the galaxy (R−I) color are independent of the initial galaxy mass assumed.

![Figure 6.18](image)

**Figure 6.18** Evolution of the HII region integrated luminosity contribution for distinct SFR models ending at \( \mu_1 = 0.1 \). Left panel: Ratio of HII region integrated and total luminosity in the B band vs. (R−I). Right panel: same as left panel but for I band. For the exponentially decreasing SFR model, solid triangles indicate evolution times of 1, 2, 4, 8, and 14 Gyr, respectively. Observational data on LSB galaxies is shown as open triangles. Typical errors in the data (upper right) and directions to which the data would shift after corrections for extinction (bottom left) are indicated. Top and bottom arrows indicate corrections for extinction within the HII regions and the LSB galaxy, respectively.

Corrects for dust extinction within the HII regions and LSB galaxy as a whole, respectively, will shift the observations in the directions as indicated in Fig. 6.18 (assuming a mean Galactic extinction curve). Clearly, the B band contributions by HII regions in LSB galaxies do not provide useful constraints on our models as extinction is too large at these wavelengths. However, from the I band observations we can conclude that the values of \( \eta_\lambda \) predicted by smoothly varying SFR models are systematically too low by large factors for most of the LSB galaxies in our sample.

This is true in particular for F568-V1, F577-V1, and U628 for which values of \( \eta_{RI} \gtrsim 0.2-0.25 \) suggest that star formation has been recently enhanced by factors \( \sim 5-10 \) relative to the SFRs predicted by exponentially decreasing or constant SFR models. Alternatively, the I band magnitudes of young stars used in our models may be considerably too low and/or the HII region lifetime assumed (i.e. \( \tau_{\text{HII}} = 5 \) Myr) may be too short. The former possibility can be excluded as synthetic color magnitude diagrams for young open clusters (e.g. V−I vs. I), based on the stellar evolution data adopted here, are in good agreement with the observations (van den Hoek 1997). The latter possibility can be excluded provided that stars older than \( \sim 20 \) Myr (i.e. initial mass \( < 10 \) M\(_{\odot}\)) do not contribute substantially to the HII regions identified (see below). Thus, star formation may be recently enhanced in LSB galaxies with \( \eta_{R,I} \gtrsim 0.2-0.25 \).
As an additional check, we investigated whether there is a correlation between the H\textsc{ii} region integrated contributions in the R band and $M_{\text{H}\textsc{ii}} / L$. Such a correlation is expected when enhanced star formation is the primary cause for the high H\textsc{ii} region contributions observed. Figure 6.19 shows that such a trend may be present, at least for the largest values of $\eta_R$ observed. However, the data are affected by extinction and selection effects and additional observations are needed to be conclusive.

![Figure 6.19](image)

Figure 6.19 Evolution of the H\textsc{ii}-region integrated luminosity contribution for exponentially decreasing SFR models. **Left panel:** Ratio of H\textsc{ii} region integrated and total luminosity in the R band vs. $M_{\text{H}\textsc{ii}} / L_B$ for models ending at $\mu_1 = 0.3$ (solid curve), 0.5 (dotted), and 0.7 (dashed). **Right panel:** same as left panel but vs. $M_{\text{H}\textsc{ii}} / L_R$. Symbols have the same meaning as in Fig. 6.18.

We conclude that smoothly varying SFRs disagree with the observations. This is true in particular for the exponentially decreasing SFR models ending at $\mu_1 = 0.3 - 0.7$ which, as discussed in the previous section, are favoured for LSB galaxies. Note that models ending at gas fractions $\mu_1 > 0.1$ provide results similar to models ending at $\mu_1 = 0.1$ (i.e. the ratio of the young-to-old stellar luminosity contributions is independent of the normalization of the SFR).

We expect that LSB galaxies with high H\textsc{ii} region contributions in the R and I band, such as F568-V1, F577-V1, and U628, are probably experiencing small amplitude bursts of star formation. Such bursts may be a common phenomenon in many LSB galaxies.

### 6.6.2 Effects of small amplitude star formation bursts

We investigate whether the large H\textsc{ii} region integrated contributions to the galaxy integrated luminosity of $\eta_I \sim 0.25$ in the I band, as observed for several LSB galaxies discussed above, can be explained by small amplitude bursts of star formation.

We assume a Gaussian star formation burst profile with a given amplitude and width. The burst amplitude is taken as a free parameter which can be adjusted to fit the observations. To study the effect of the burst width ($2\sigma$), we consider burst durations $\Delta t_b = 1, 5,$ and $10$ Myr, respectively. For each burst profile we follow the chemical and photometric evolution during $1$ Gyr with a time resolution of $\sim 0.1$ Myr at time of burst maximum and of $\sim 2$ Myr at roughly $5\sigma$ from burst maximum.

We superimpose the star formation burst on each of the star formation models discussed in Sect. 6.5.1 while assuming a galactic evolution time at burst maximum of $t_b = 13$ Gyr, burst maximum amplitude $A_b = 8 \ M_\odot \ yr^{-1}$, burst duration $\Delta t_b = 5$ Myr, maximum H\textsc{ii} region lifetime $\tau_{\text{H}\textsc{ii}} = 5$ Myr, and an initial galaxy mass of $10^{10} \ M_\odot$. In the following, we will refer to these burst parameters unless noted otherwise. For convenience, we neglect any influence of the burst on the chemical evolution of the model galaxy.

We show in Fig. 6.20a the resulting H\textsc{ii} region integrated I band contribution $\eta_I$ for an exponentially decreasing SFR plus burst model. Up to a galactic evolution time of $\sim 13$ Gyr the photometric evolution of the model galaxy is identical to that for the exponentially decreasing SFR model without burst (cf. Fig. 6.18). Thereafter, the contribution by young stars to the galactic integrated light increases rapidly. Simultaneously, the (R–I) colors become significantly bluer. After burst maximum, the contribution by young stars decreases and galaxy colors start to redden again until the effect of the burst becomes negligible and colors and magnitudes evolve as prior to the burst. We remark that the H\textsc{ii} integrated-to-total luminosity ratio short after the burst can be substantially below those during the pre-burst evolution. This is due to
6.6.2 Effects of small amplitude star formation bursts

Figure 6.20 Impact of small amplitude bursts on the evolution of the HIi region integrated luminosity contribution in the I-band vs. (R−I). Unless stated otherwise, we plot the exponentially decreasing SFR + burst model assuming $A_b = 8 \text{ M}_\odot \text{yr}^{-1}$, $\Delta t_b = 5 \text{ Myr}$, $\tau_{\text{HIi}} = 5 \text{ Myr}$, and $E_{B-V} = 0 \text{ mag}$. a) effect of varying burst amplitude: $A_b [\text{ M}_\odot \text{yr}^{-1}] = 8$ (solid curve), 4 (dotted), and 1.6 (dashed); b) effect of mean extinction within HIi regions: $E_{B-V} [\text{mag}] = 0.25$ (solid curve), 0.5 (dotted), and 1 (dashed); c) effect of global star formation history: exponentially decreasing SFR (solid curve), constant (dotted), and linearly increasing (dashed); d) effect of actual gas-to-total mass ratio: $\mu_1 = 0.1$ (dashed curve), 0.5 (solid), and 0.7 (dotted); e) effect of burst duration: $\Delta t_b [\text{Myr}] = 1$ (dashed curve), 5 (solid), and 10 (dotted); f) effect of HIi region age: $\tau_{\text{HIi}} [\text{Myr}] = 2$ (solid curve), 10 (dotted), and 50 (dashed; no burst shown). Symbols have the same meaning as in Fig. 6.18.

Burst amplitude and IMF: Fig. 6.20a demonstrates that high HIi region integrated I band contributions $\eta_I \sim 0.25$ can be well explained by bursts with amplitude $A_b = 8 \text{ M}_\odot \text{yr}^{-1}$ superimposed on an exponentially decreasing SFR model ending at $\mu_1 = 0.1$. Decreasing the burst amplitude by factors 2.5 and 5, respectively, results in the smaller loops shown in Fig. 6.20a. The actual burst amplitude required to explain the fact that stars somewhat older than $\tau_{\text{HIi}} = 5 \text{ Myr}$ dominate the galaxy light for a considerable time after the burst.

In this manner, a characteristic burst loop is completed as shown in Fig. 6.20a. The shape of this loop is determined by: a) the burst amplitude, b) the extinction within the HIi regions, c)+d) the contribution by the old stellar population to the integrated galaxy light, e) the duration (and profile) of the burst, and f) the maximum lifetime of the HIi regions during which young stars can be distinguished from the old stellar population. In addition, the IMF of the stars formed during the burst is of importance but we will argue that this effect is similar to that of varying the burst amplitude. Note that the shape of the burst loop is insensitive to the galactic age at which the burst occurs. This is because the contribution by young stars to the galaxy integrated colors change little over the past few Gyr for smoothly varying SFR models (cf. Fig. 6.18).
observations depend on many quantities as we will argue below.

The contribution by young stars to the galaxy integrated magnitudes and colors increases when considering burst IMFs that favour high mass star formation (e.g. \( M > 10 \, M_\odot \)) compared to the Salpeter IMF. For various IMFs we find that the shape of the burst loop remains intact while the burst amplitude varies between 0.2 (steep IMFs; cf. Sect. 6.4.2) and 1.5 (Kroupa et al. IMF) normalized to 1 for the Salpeter IMF. Since high-mass stars dominate the \( \text{H} \text{II} \) region integrated luminosity contribution, the effect of varying the burst IMF is similar to that of varying the burst amplitude.

**Dust extinction:** Fig. 6.20b illustrates the effect of dust extinction in \( \text{H} \text{II} \) regions on the burst contribution to the I band luminosity as well as to the galaxy (\( R-I \)) color. A selective extinction of \( E_{B-V} = 0.25 \, (0.5) \) mag results in a reduction of the \( \text{H} \text{II} \) region integrated I band contribution by a factor 1.6 (2.5) and a reddening of \( E_{R-I} = 0.14 \, (0.27) \) mag, assuming a Galactic extinction curve. For values of \( E_{B-V} \gtrsim 0.5 \) mag, we find that the bluing effect on (\( R-I \)) by young massive stars formed during the burst is neutralized almost entirely by extinction. We note that a maximum extinction of \( E_{B-V} \sim 1.1 \) mag within the \( \text{H} \text{II} \) regions in LSB galaxies (e.g. McGaugh 1994) would result in a reduction in \( \eta_1 \) by a factor \( \sim 6.5 \) (for intense bursts even reddening of the galaxy colors may occur). If variations in the mean extinction of the ensembles of \( \text{H} \text{II} \) regions among LSB galaxies are small (e.g. less than a factor two), it is difficult to see how extinction alone can provide an adequate explanation for the large variations observed in \( \eta_1 \). We will return to this point below.

**Global star formation history:** Fig. 6.20c shows the impact of the assumed star formation history of the old stellar population on the effect of the burst. The burst effect on the galaxy magnitudes and colors increases when the contribution by the old stellar population is decreased. Thus, colors and magnitudes are less affected by bursts imposed on constant or even increasing SFRs compared to those imposed on exponentially decreasing SFR models: the smaller loop sizes just reflect that the mean age of the old stellar population is relatively young.

**Current gas fraction:** Fig. 6.20d demonstrates how the assumed present-day gas fraction \( \mu_1 \) for an exponentially decreasing SFR model modifies the effect of the burst. The luminosity contribution by the old stellar population decreases for increasing values of \( \mu_1 \). We find that the effect of a star formation burst for galaxies with \( \mu_1 = 0.9 \) (i.e. unevolved systems) is as large as that of a ten times stronger burst for \( \mu_1 = 0.1 \) (i.e. highly evolved systems). Thus, the burst amplitude required to explain the observations strongly depends on the present-day gas-to-total mass-ratio.

**Burst duration:** Fig. 6.20e shows that the duration of the burst affects the impact of the burst as well. For \( \Delta t_\text{b} = 1, 5, \) and 10 Myr, the variation in \( \eta_1 \) is \( \sim 0.25 \) while the resulting galaxy (\( R-I \)) colors become bluer. Burst durations in excess of \( \Delta t_\text{b} \sim 5 \) Myr are unlikely since this would require dust extinctions \( E(B-V) > 1 \) mag in order to provide agreement with the observed (\( R-I \)) colors. Such large extinction in \( \text{H} \text{II} \) regions are probably excluded by the observations (e.g. McGaugh 1994). Thus, relatively narrow burst profiles are needed to explain adequately extreme values \( \eta_1 \sim 0.2 \) (as for F568-V1; cf. Table 6.4).

**Maximum age of \( \text{H} \text{II} \) regions:** Fig. 6.20f shows that when the maximum lifetime of the \( \text{H} \text{II} \) regions is increased from \( \tau_{\text{HII}} = 5 \) to 50 Myr, partial agreement with the observations can be achieved without invoking star formation bursts. Similarly, values of \( \tau_{\text{HII}} \gtrsim 200 \) Myr would be required to explain the large values of \( \eta_1 \gtrsim 0.2 \) observed (see below). We note that the resulting \( \text{H} \text{II} \) region contributions do not increase linearly with \( \tau_{\text{HII}} \) as short lived massive stars dominate the luminosity contribution of all stars formed during the past \( \tau_{\text{HII}} \) yr.

To explain the observed variations in \( \eta_1 \) by means of variations in \( \tau_{\text{HII}} \), the question arises why such large variations would occur in the mean value of \( \tau_{\text{HII}} \) among different LSB galaxies. First, this could be due to selection effects. In this case, fainter \( \text{H} \text{II} \) regions may be detected preferentially in regions of low surface brightness (e.g. the outer parts of LSB galaxies and/or LSB galaxies with relatively low surface brightnesses). We plot in Fig. 6.21 the average surface brightness of the \( \text{H} \text{II} \) regions within a given LSB galaxy vs. central surface brightness of the hosting LSB galaxy. Indeed there appears to be a trend of preferential detection of faint \( \text{H} \text{II} \) regions in LSB galaxies with low central surface brightnesses, but further observations are needed to clarify this interesting tendency.

Second, this could be due to physical effects. For instance, the formation of massive stars could be favoured in LSB galaxies with relatively high surface densities. Since fainter \( \text{H} \text{II} \) regions are probably ionized by less massive stars, as has been argued for \( \text{H} \text{II} \) regions in the Magellanic Clouds (e.g. Wilcots 1994), larger \( \text{H} \text{II} \) ages \( \tau_{\text{HII}} \) may be associated with LSB galaxies having relatively low surface brightnesses (or equivalently low surface densities; cf. dB96). This would imply that systematic variations in the least massive star \( \eta_{\text{HII}} \)
6.6.2 Effects of small amplitude star formation bursts

Figure 6.21 Mean surface brightness of HII regions vs. central surface brightness of the hosting LSB galaxy. Observational data refer to the U band (open squares), B (full squares), V (open stars), R (open circles), and I band (full circles)

ionizing the HII regions are present from one LSB galaxy to another (consistent with the data shown in Fig. 6.21). This would be consistent with observations of sites of intense star formation which support the idea that the IMF is biased towards more massive stars in high surface density regions (e.g. Rieke & Lebofsky 1985).

Values of $\tau_{\text{HII}} \gtrsim 50$ Myr would mean that stars down to masses of $m_{\text{ion}} \lesssim 7$ $M_\odot$ would contribute to the HII regions identified (e.g. Schaerer et al. 1992; Z=0.001). Observational estimates for $m_{\text{ion}}$ are usually in the range 10–15 $M_\odot$ (Wilcots 1994; García-Vargas et al. 1995) and correspond to $\tau_{\text{HII}} \gtrsim 15$ Myr. Even though these values would imply that our adopted value of $\tau_{\text{HII}} = 5$ Myr is too low, this probably excludes extreme values of $\tau_{\text{HII}} = 200$ Myr which would be required to explain the observed range in $\eta$ exclusively in terms of variations in $\tau_{\text{HII}}$ and/or $m_{\text{ion}}$.

Table 6.5 Effect of 5 Myr star burst model on galaxy magnitudes and colors for $M_g(0) = 10^{10}$ $M_\odot$

<table>
<thead>
<tr>
<th>Model</th>
<th>$A_v$ [ $M_\odot$ yr$^{-1}$]</th>
<th>$\Delta B$</th>
<th>$\Delta I$</th>
<th>$\Delta (B-V)$</th>
<th>$\Delta (R-I)$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1+burst</td>
<td>0.8</td>
<td>-0.34</td>
<td>-0.14</td>
<td>-0.18</td>
<td>-0.04</td>
<td>SFR$<em>1 = 0.17$ $M</em>\odot$ yr$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>-0.58</td>
<td>-0.30</td>
<td>-0.28</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>-1.05</td>
<td>-0.42</td>
<td>-0.43</td>
<td>-0.17</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>-1.72</td>
<td>-0.53</td>
<td>-0.56</td>
<td>-0.26</td>
<td>*</td>
</tr>
<tr>
<td>B1+burst</td>
<td>0.8</td>
<td>-0.14</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.02</td>
<td>SFR$<em>1 = 0.89$ $M</em>\odot$ yr$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>-0.32</td>
<td>-0.21</td>
<td>-0.11</td>
<td>-0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>-0.56</td>
<td>-0.32</td>
<td>-0.21</td>
<td>-0.11</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>-0.94</td>
<td>-0.41</td>
<td>-0.33</td>
<td>-0.14</td>
<td>*</td>
</tr>
</tbody>
</table>

* E(B$-$V) $\gtrsim 0.3$ mag in HII regions is required to provide agreement with observations

We conclude that variations in $\tau_{\text{HII}}$ (or equivalently $m_{\text{ion}}$) and/or extinction may provide an explanation only for part of the variations in the HII region integrated luminosity contributions observed among LSB galaxies. Therefore, the observations suggest that small amplitude bursts of star formation are important in at least several of the LSB galaxies for which accurate photometry data is available. Such recent episodes of enhanced star formation may play an important role in affecting the colors of the blue LSB galaxies discussed above.
6.6.3 Quantitative effect of small amplitude bursts on the galaxy colors and magnitudes

Table 6.5 lists the effect of a 5 Myr star formation burst on the galaxy integrated magnitudes and colors for various burst amplitudes. For an exponentially decreasing SFR (model A1 in Table 6.3; assuming $M_b(0) = 10^{10} \, M_\odot$, $\mu_1 = 0.1$, Salpeter IMF), we find that a burst with amplitude $A_b = 0.8 \, M_\odot \, \text{yr}^{-1}$ results in maximum color variations $\Delta(B-V)$ and $\Delta(R-I)$ of $-0.18$ and $-0.04$ mag, respectively. The effect of increasing the burst amplitude by a factor ten to $A_b = 8 \, M_\odot \, \text{yr}^{-1}$ results in corresponding shifts of $-0.56$ and $-0.26$ mag, respectively. This effect is similar to that when the initial galaxy mass is reduced by a factor ten while leaving the burst amplitude unaltered. We note that extinction in H$\alpha$ regions is likely to suppress the color and magnitude effects of the bursts (i.e. the values given in Table 6.5 are hard upper limits).

For bursts superimposed on exponentially decreasing SFRs models ending at $\mu_1 = 0.1$, the color and magnitude shifts predicted are consistent with the observations in case of burst amplitudes $A_b \lesssim 3 \, M_\odot \, \text{yr}^{-1}$, assuming a typical extinction of $E(B-V) = 0.3$ mag in the H$\alpha$ regions. In fact, the effect of the burst is determined mainly by the total luminosity of the young stellar populations formed according to the continuous SFR during e.g. the last Gyr. Since the amplitude of the SFR scales with $(1-\mu_1)$, for example, the impact of the burst for models ending at different gas fractions $\mu_1$ (see Appendix B, Chap. 3), the burst of the impact for models ending at $\mu_1 = 0.5$ is about twice that given in Table 6.5. Similarly, for models ending at $\mu_1 > 0.9$, the burst effect becomes roughly ten times stronger compared to the $\mu_1 = 0.1$ case (cf. Fig. 6.20).

We emphasize that the effect of the burst is substantially reduced when going from exponentially decreasing to constant SFRs (cf. Table 6.5). We will discuss in Sect. 6.8 how these results fit into a more general scenario for the star formation history of LSB galaxies.

### Table 6.6 Observational estimates of present-day SFRs $[M_\odot \, \text{yr}^{-1}]$ in LSB galaxies

<table>
<thead>
<tr>
<th>Name</th>
<th>Dist. [Mpc]</th>
<th>$m_1$</th>
<th>R$_{17}^I$ [kpc]</th>
<th>R$_{HI}$ [kpc]</th>
<th>$\mu_1$</th>
<th>M$<em>{HI}$ [M$</em>\odot$]</th>
<th>$&lt;c_{HI}&gt;$</th>
<th>SFR$_{fix}$</th>
<th>SFR$_1$</th>
<th>SFR$_{cont}$</th>
<th>SFR$_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F561-1</td>
<td>47</td>
<td>14.7</td>
<td>6.6</td>
<td>7.6</td>
<td>23.2</td>
<td>8.91</td>
<td>4.5</td>
<td>0.05</td>
<td>0.09</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td>F563-1</td>
<td>34</td>
<td>16.1</td>
<td>10.2</td>
<td>16.3</td>
<td>26.4</td>
<td>9.19</td>
<td>1.0</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>0.16</td>
<td>1.7</td>
</tr>
<tr>
<td>F563-V1</td>
<td>38</td>
<td>15.8</td>
<td>4.6</td>
<td>4.8</td>
<td>24.0</td>
<td>8.48</td>
<td>4.2</td>
<td>0.02</td>
<td>0.03</td>
<td>0.005</td>
<td>0.024</td>
</tr>
<tr>
<td>F567-2</td>
<td>56</td>
<td>15.9</td>
<td>8.4</td>
<td>10.6</td>
<td>24.6</td>
<td>9.09</td>
<td>3.5</td>
<td>0.04</td>
<td>0.06</td>
<td>0.029</td>
<td>0.15</td>
</tr>
<tr>
<td>F568-1</td>
<td>64</td>
<td>15.0</td>
<td>9.6</td>
<td>11.5</td>
<td>23.7</td>
<td>9.35</td>
<td>5.4</td>
<td>0.08</td>
<td>0.18</td>
<td>0.12</td>
<td>0.31</td>
</tr>
<tr>
<td>F568-3</td>
<td>58</td>
<td>14.7</td>
<td>8.7</td>
<td>11.4</td>
<td>23.5</td>
<td>9.20</td>
<td>3.9</td>
<td>0.09</td>
<td>0.14</td>
<td>0.06</td>
<td>0.33</td>
</tr>
<tr>
<td>F568-V1</td>
<td>60</td>
<td>15.7</td>
<td>8.4</td>
<td>10.7</td>
<td>24.3</td>
<td>9.14</td>
<td>3.8</td>
<td>0.05</td>
<td>0.07</td>
<td>0.05</td>
<td>0.58</td>
</tr>
<tr>
<td>U128</td>
<td>48</td>
<td>13.5</td>
<td>18.2</td>
<td>21.4</td>
<td>24.1</td>
<td>9.55</td>
<td>(2.0)</td>
<td>(0.16)</td>
<td>(0.21)</td>
<td>0.14</td>
<td>0.82</td>
</tr>
<tr>
<td>U1230</td>
<td>40</td>
<td>14.3</td>
<td>12.0</td>
<td>18.8</td>
<td>24.5</td>
<td>9.51</td>
<td>(4.3)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>0.05</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Notes: * theoretical values in columns (11) and (12) repeat those in columns (9) and (11) from Table 6.4; † probably contaminated by field galaxy; values between parentheses are uncertain.

6.7 Present-day star formation rates in LSB galaxies

6.7.1 Theoretical star formation rates

Global star formation rates are derived according to the best fitting models discussed in the previous section assuming an exponentially decreasing SFR, Salpeter IMF, and observational estimates for the current gas fraction $\mu_{rot}$ and total HI mass for each LSB galaxy individually according to:

$$\text{SFR}^{\text{cont}} [M_\odot \, \text{yr}^{-1}] \approx A(\mu_{rot}) \frac{M_{\text{tot}}^{\text{rot}}}{10^{10}} \tag{6.3}$$

where $A(\mu_{rot})$ is the model SFR amplitude required to end at a gas fraction $\mu_{rot}$ at a galactic evolution time of 14 Gyr (assuming an initial mass of $10^{10} \, M_\odot$) and $M_{\text{tot}}^{\text{rot}}$ the total galaxy mass as obtained from M$_{HI}$ and $\mu_{rot}$. We derived $A[M_\odot \, \text{yr}^{-1}] = 0.18$ ($\mu_{rot} = 0.1$), 0.13 (0.3), 0.09 (0.5), 0.06 (0.7), and 0.02 (0.9), respectively. Accordingly, we find that LSB galaxies show present-day SFRs (without bursts) between $\text{SFR}^{\text{cont}} \sim 0.01$ and 0.15 $M_\odot \, \text{yr}^{-1}$ (cf. Table 6.4). For a typical LSB galaxy (i.e. $M_{\text{tot}} = 10^{10} \, M_\odot$, $\mu_{rot} = 0.5$) we estimate $\text{SFR}^{\text{cont}} = 0.1 \, M_\odot \, \text{yr}^{-1}$.

The gas reservoir at the time of onset of main star formation in LSB galaxies may have been substantially less than that estimated from their present-day amounts of gas since infall and accretion of matter
6.7.2 Empirical star formation rates

We have also derived current SFRs for LSB galaxies using the empirical method presented by Ryder & Dopita (1994; hereafter RD) based on CCD surface photometry of Galactic disks. These authors found an almost universal relationship between the Hα and I-band surface brightness at a given radius in the disks of a sample of southern spiral galaxies. From this relation, RD derived a constraint on the present-day SFR integrated over the entire stellar mass spectrum as:

$$\log SFR_1[M_{\odot} \text{pc}^{-2} \text{Gyr}^{-1}] = \log(0.26\mu + 0.92\log \sigma_{H_\alpha} + 5.3)$$

(6.5)

where $\mu_1$ is the I-band surface brightness and $\sigma_{H_\alpha}$ is the global mean Hα surface density [$M_{\odot} \text{pc}^{-2}$] within the star-forming disk. The relation between SFR$_1$ and $\mu_1$ is normalized by a term related to the mean surface density $\sigma_{H_\alpha}$ and by a constant which is partly related to the conversion of the massive star formation rate to total SFR depending on the adopted IMF (cf. Kennicutt 1983). It is unclear from the RD sample whether the relation is valid also for the lowest (stellar) I-band surface densities that are observed among LSB galaxies. However, since this relation holds over a wide range in surface brightness and massive star formation in the disks of spirals appears rather insensitive to galactic dynamics, extinction, and molecular gas content (RD), we expect this relation to be valid also in case of LSB galaxies. At the faintest surface brightnesses (i.e. $\mu_1 \gtrsim 25.6$ mag arcsec$^{-2}$), the relation may be flattening off although the effects of sky subtraction and small number statistics leave this open to question. If flattening indeed occurs, Eq. (6.3) provides lower limits to the actual SFRs in LSB galaxies.

Using Eq. (6.1), we estimate global present-day SFRs for all LSB galaxies in our sample with measured I band magnitudes and related data. For these LSB galaxies we list the distance, apparent I band magnitude, and radius $R_{27}$ at which the B band isophote is equal to $27$ mag arcsec$^{-2}$, in columns (2) to (4) in Table 6.6. This radius corresponds to the optical edge of the LSB galaxy and is more representative for the radius within which the old disk stellar population in LSB galaxies is contained than is $R_{25}$ as used by RD for HSB galaxies. Accordingly, we define an effective I band surface brightness as:

$$\mu_1^{\text{eff}} = m_I + 2.5 \log(\pi R_{27}^2)$$

(6.6)

and use this in Eq. (6.3). We tabulate the outermost radius of the measured Hα rotation curve $R_{H_\alpha}$, effective I band surface brightness $\mu_1$ and total Hα mass derived within $R_{H_\alpha}$, and the mean global surface Hα densities $\sigma_{H_\alpha}$ in columns (5) to (8), respectively. The mean global Hα surface densities $\sigma_{H_\alpha}$ derived from these values range between $2$ and $5.5$ $M_{\odot} \text{pc}^{-2}$ and are substantially smaller (i.e. by $20$–$60\%$) than those derived using $R_{27}$ instead. However, since $M_{H_\alpha}$ has been measured within $R_{H_\alpha}$, we expect the former values to be more representative of the average Hα surface density in the star forming part of the disk.

The empirically derived mean present-day SFRs for the sample LSB galaxies range from about $SFR_1 = 0.02$ to $0.2$ $M_{\odot} \text{yr}^{-1}$ (cf. column 10 of Table 6.6). Errors arising from the Hα normalisation are estimated to be within a factor of two. This is illustrated when the same SFRs are derived assuming a fixed Hα
surface density of 2 \, M_\odot \, pc^{-2} for all LSB galaxies (cf. SFR_{low} in Table 6.6). Other sources of errors include the conversion factor of massive-to-total SFRs as derived from the H\alpha luminosities (see Kennicutt 1983; RD) which may differ for HSB and LSB galaxies as this ratio is determined by the IMF and by theoretical estimates of the H\alpha radiation emitted by individual stars. For instance, the mean stellar population in LSB galaxies has a metallicity that is substantially lower than in HSB galaxies which will affect the stellar H\alpha emission. Probably the largest uncertainty arises from the possible flattening of the relationship between \mu_{H\alpha} and \mu_{I} found by RD towards low values of \mu_{I}. We estimate that the relative SFRs for LSB galaxies given in Table 6.6 are accurate within a factor of two while absolute SFRs may suffer from more substantial errors.

The empirically derived current SFRs in LSB galaxies range from SFR$_1 = 0.02$ to 0.2 \, M_\odot \, yr$^{-1}$. This is in particularly good agreement with the theoretically derived SFRs ranging from SFR$_{cont} \sim$0.01 to 0.15 \, M_\odot \, yr$^{-1}$ (cf. Table 6.6). As discussed before, the theoretically derived present-day SFRs of individual LSB galaxies lie probably between SFR$_{cont}$ and SFR$_{tot}$ where the latter values include the contribution of small amplitude bursts. In several cases, the values of SFR$_{tot}$ are considerably larger than the empirical values which suggests that the burst contribution is overestimated by the models and/or that the SFRs derived empirically do not trace well local enhancements of star formation at the faint surface brightnesses of LSB galaxies. We conclude that the predicted SFRs in LSB galaxies generally agree to within a factor 2–3 with those derived empirically.

6.7.3 Comparison of present-day SFRs in LSB and HSB galaxies

We have derived current star formation rates in LSB galaxies of typically \sim 0.1 \, M_\odot \, yr$^{-1}$. Several LSB galaxies in our sample are probably in a state of enhanced star formation and may experience small amplitude bursts of up to perhaps \sim 1 \, M_\odot \, yr$^{-1}$. What fraction of the LSB galaxies actually is in such an excited state is difficult to estimate from current observations. In either case, the present-day SFRs in LSB galaxies are found considerably below the \sim 5–10 \, M_\odot \, yr$^{-1}$ derived for their HSB counterparts (e.g. Kennicutt 1992 and references therein) but significantly larger than the \sim 0.001 \, M_\odot \, yr$^{-1}$ observed typically in dwarf irregular galaxies (e.g. Hunter & Gallagher 1986). Possible explanations for the marked differences in the present-day SFRs of LSB compared to HSB galaxies are discussed below.

6.8 Discussion

In the previous sections, we checked a comprehensive set of galactic photometric evolution models against a base of observations comprising the chemical and photometric properties of a representative sample of LSB galaxies. Here we discuss results of this comparison in the context of the early evolution and star formation history of such systems while focusing on the distinctly low evolutionary state of LSB relative to HSB galaxies.

6.8.1 Star formation history

For the majority of the LSB galaxies in our sample, observed UBVRI magnitudes, [O/H] abundances, gas masses and fractions, and HI mass-to-light ratios are best explained by galactic evolution models incorporating an exponentially decreasing global SFR ratios ending at a present-day gas-to-total mass-ratio of \mu_{I} \sim 0.5. When infall is involved to delay the chemical enrichment of the LSB galaxy disks, similar models ending at \mu_{I} \approx 0.3 may be appropriate as well. When small amplitude bursts are involved to decrease the predicted M_\odot/L ratios, models ending at \mu_{I} \approx 0.7 may also apply. In addition to exponentially decreasing SFR models, \sim 15\% of the LSB galaxies require modest amounts of internal extinction E(B−V)\leq 0.1 mag to explain the relatively red colors of (B−V) \sim 0.6 mag of these systems.

A substantial fraction (\sim 35\%) of the LSB galaxies in our sample have colors (B−V) \leq 0.45 mag which are inconsistently blue compared to the (B−V) colors predicted by exponentially decreasing global SFR models at galactic evolution times t_{ev} \sim 14 Gyr. Instead, these galaxies exhibit properties similar to those resulting from exponentially decreasing SFR models at evolution times of \sim 5–10 Gyr. Alternatively, recent episodes of enhanced star formation superimposed on exponentially decreasing SFR models may provide an adequate explanation for the colors of these systems (see Sect. 6.6). A small fraction of \sim 10–15 \% of the LSB galaxies have properties (such as (B−V) \leq 0.35 mag) consistent with those resulting from slowly decreasing or constant SFR models ending at \mu_{I} = 0.5. Although the current data are inconclusive to distinguish between the possibilities previously discussed, it is clear that the stellar population in these blue LSB galaxies must be relatively young.
Recent star formation is observed, at least at low levels, in essentially all the LSB galaxies in our sample. Hence, it seems justified to assume that the disks in LSB galaxies experienced continuous (i.e. frequent small amplitude bursts of) star formation, at least during the last few Gyr. Therefore, to explain the colors of LSB galaxies observed, the blue color contributions by stellar populations formed recently need to be compensated by the contributions of older stellar populations (provided that internal extinction is low; cf. Sect. 6.5.2). We have shown that star formation models which on average decay exponentially take such color compensation effects consistently into account. Thus, our result that these models are adequate for most of the LSB galaxies in our sample basically relies on two assumptions: 1) negligible amounts of internal extinction, and 2) more or less continuous star formation in LSB galaxies.

Comparison with HSB and dwarf galaxies

The presence of an old stellar population in many late-type LSB galaxies as indicated by their optical colors (e.g. vdH93; dB95) and as confirmed by the results obtained in this paper suggests that LSB galaxies roughly follow the same evolutionary history as HSB galaxies, except at a much lower rate.

First, this suggests that the mean age of the stellar populations in most LSB and HSB galaxies is similar even though the disks of LSB galaxies are in a relatively low evolutionary state (see below). Although we cannot justify a uniform $\tau_{sf} \sim 5$ Gyr for all LSB galaxies, our results indicate that, on average, the relative importance of the old (e.g. older than a few Gyr) and young stellar populations to the colors and luminosities of LSB galaxies are similar to that of late-type HSB galaxies. Values of $\tau_{sf} \ll 5$ Gyr would increase the colour contributions of the older stellar populations and would predict too red colors as compared to the colors observed for LSB galaxies. Such models probably can be excluded for LSB galaxies unless recent star formation in these systems would be more important than indicated by the observations. On the other hand, values of $\tau_{sf} \gg 5$ Gyr correspond to slowly decreasing or constant SFRs and would increase the colour contributions of the younger stellar populations. Such models would predict too blue colors and probably can be excluded for most of the LSB galaxies in our sample provided that extinction is low in these systems (i.e. $E(B-V) \lesssim 0.1$ mag).

Secondly, this implies that the combined effect of extinction and metallicity on galaxy colors is sufficient to explain the color differences observed between LSB and HSB galaxies. Since the amount of extinction depends strongly on the dust content which in turn is determined by the heavy element abundances in the ISM (see Sect. 6.2), metallicity is probably the main quantity directing the color differences between LSB and HSB galaxies. In this manner, the much lower rate of star formation in LSB galaxies, which implies relatively low metallicities and dust contents, indirectly determines the blue colors of LSB galaxies compared to HSB galaxies.

Recent models indicate that the stellar surface density is probably a fundamental quantity in determining star formation in the disks of spiral galaxies (Dopita & Ryder 1994). Such models imply a roughly exponential decrease of the SFR with time for disk galaxies (Dopita 1985) and suggest that the initial onset of main star formation has been much more intense in HSB galaxies compared to LSB galaxies. Provided that star formation proceeds from the center outwards in disk galaxies (e.g. Burkert et al. 1994), this is consistent with the insignificant bulges observed in LSB galaxies. Our results discussed above are in good agreement with this scenario.

In summary, our results confirm that the properties of most late-type HSB galaxies can be best explained by exponentially decreasing SFR models ending at present-day gas fractions $\mu_1 \lesssim 0.05$–$0.1$ (see e.g. Clayton 1985; Dopita 1985; Guiderdoni & Rocca-Volmerange 1987). In contrast, LSB galaxies are best fitted by similar SFR models ending at $\mu_1 \gtrsim 0.3$. This implies that LSB galaxies must be in a low evolutionary state compared to HSB galaxies, a result which is consistent with the relatively blue colors, large gas fractions, high gas mass-to-light ratios, and low current star formation rates observed in LSB galaxies. Also, this is consistent with observations which indicate that, for a given total mass or luminosity, LSB galaxies usually have gas contents and gas-to-total mass-ratios much larger than their HSB counterparts of the same Hubble type (dB96). For instance, LSB galaxies are more gas-rich than the late-type spirals in the Virgo cluster (Cayatte et al. 1994). Furthermore, relatively low rates of evolution are indicated by the observational fact that LSB galaxies usually show properties comparable to those found in the outer parts of HSB spirals. In addition, the absence of the formation of grand design spiral arms in the disks of LSB galaxies is an indication of the low evolutionary state of LSB compared to HSB galaxies (Elmegreen 1990).

In the above discussion, we have treated the bulge-disk system as a whole when comparing LSB with HSB spirals. Clearly, a more detailed comparison should include radial variations of the galaxy properties. For instance, the bulges of LSB galaxies are much less prominent with respect to their disks than in HSB galaxies. What fraction of the current stellar content in LSB galaxies actually formed in their central parts is unclear but is an important issue when considering evolutionary differences between LSB galaxies and
6.8 Discussion

HSB galaxies in more detail (van den Hoek & de Blok, in preparation).

In general, there appears a trend along the Hubble sequence of rapidly decaying SFRs for early type galaxies, to constant or even increasing SFRs for dwarf irregular galaxies (see e.g. reviews by Sandage 1986; Kennicutt 1992). On average, the observed trend corresponds to a decrease of the ratio of mean past to present SFR along the Hubble sequence. Most of the LSB galaxies in our sample belong to the group of late-type Scd/Sd galaxies for which exponentially decreasing SFR models are in good agreement with the observations. The remaining LSB galaxies, for which slowly decreasing or constant (sporadic) SFR models are more appropriate, belong to a group of galaxies with properties intermediate to those of disk galaxies with weak or absent spiral arms and Sm/Im galaxies. Thus, in general LSB galaxies comply well with the observed trend of SFR variation with Hubble type.

This picture is consistent with the finding that LSB galaxies usually cover the range of properties intermediate to that of HSB spirals and dwarf irregulars. Although not discussed in detail here, we have found that dwarf galaxies are best modelled using increasing or constant global SFR models. Since these systems are on average bluer and usually have larger gas fractions than LSB galaxies, their rate of evolution must have been very low (e.g. Gallagher et al. 1984) while star formation probably has occurred in bursts separated by quiescent periods in these systems (e.g. Searle & Sargent 1972; Sandage 1986; Hodge 1991; van Zee et al. 1996). In this case, the interburst period should be sufficiently long to allow young stars formed during the most recent burst to dominate the galaxy colors. To a lesser extent, the sporadic star formation scenario may apply to many of the LSB galaxies in our sample as well (cf. Sect. 6.6).

6.8.2 Present-day star formation

The presence of Hα regions observed in virtually all the Hα images of the LSB galaxies in our sample suggest that recent star formation is a common phenomenon in these systems. The star formation sites in LSB galaxies generally do not trace the spiral arms and are preferentially found towards the edges of their optical disks. In particular, very little of the Hα emission is associated with the nuclei of LSB galaxies (which is in marked contrast with HSB galaxies).

We found evidence for the occurrence of small amplitude bursts in several LSB galaxies for which accurate data is available. This result indicates that current star formation in virtually all the LSB galaxies in our sample is local both in time and space and suggests that sporadic star formation has been a continuous process from the time star formation started in the disks of LSB galaxies.

The low star formation rates of \( \sim 0.1 \, M_\odot \, \text{yr}^{-1} \) derived for LSB galaxies as well as the local nature of the star formation in these systems is consistent with the idea of a critical threshold for the onset of global star formation in disk galaxies (e.g. Skillman 1987; Kennicutt 1989; Davies 1990). The mean star formation threshold as suggested from observations of irregular galaxies and spirals (Guiderdoni 1987; Kennicutt 1989) is about \( 8 \, M_\odot \, \text{pc}^{-2} \). This is slightly below the average Hα peak surface densities of \( \sim 8.7 \, M_\odot \, \text{pc}^{-2} \) found for normal field Scd spirals (Warmels 1988; RD) and \( 10 \, M_\odot \, \text{pc}^{-2} \) for Sd galaxies (Cayatte et al. 1994). In LSB galaxies, typical Hα peak surface densities of \( \sim 3 \, M_\odot \, \text{pc}^{-2} \) are found (vdH93; cf. Table 6.6) which is well below the threshold. The idea of a star formation threshold is consistent with star formation models which state that star formation should increase with surface mass density (e.g. Dopita 1985; RD; see also Donas et al. 1987).

Even though LSB galaxies contain large amounts of gas, only very limited amounts participate in the process of star formation. If we assume that LSB galaxies maintain their current star formation rate of \( \sim 0.1 \, M_\odot \, \text{yr}^{-1} \), their typical present-day amount of gas \( M_g = 2.5 \times 10^9 \, M_\odot \) will be consumed within \( t_{\text{gas}} = \sim 30 \, \text{Gyr} \) (for a recycled fraction of 25%). Such gas consumption times for LSB galaxies are much larger than a Hubble time (e.g. Romanishin 1980). For comparison, \( t_{\text{gas}} = 2 - 4 \, \text{Gyr} \) in HSB galaxies, assuming typically \( M_g \sim 10^{10} \, M_\odot \) and SFR\(_1 \sim 5 \, M_\odot \, \text{yr}^{-1} \), which implies that HSB galaxies will run out of gas soon (see Kennicutt 1992).

Another reason for the low global star formation rates in LSB galaxies may be the fact that these systems are relatively isolated (Bothun et al. 1993) so that star formation is unlikely to be triggered by tidal interaction with nearby companions (e.g. Mo et al. 1994). Since the critical conditions for sporadic star formation can be reached only locally at the outer edges of the optical disks of LSB galaxies, this suggests that star formation may be associated with local accretion and/or infall of matter. This would naturally explain why the threshold can be reached in areas that are relatively void of stars and where the global gas surface density is relatively low.

The process of infall induced star formation is common in the Galactic disk (e.g. Lépine & Duvé 1994; van den Hoek & de Jong 1997) and may be the dominant mode of star formation in LSB galaxies in which low surface densities inhibit global star formation. In this manner, the LSB galaxies currently observed to undergo a small amplitude burst of star formation may have a relatively high gas infall rate.
6.8.3 On the formation of LSB galaxies

The present-day star formation rates derived here for LSB galaxies place an upper limit on the gas infall rate in these systems of $<0.3 \text{ ms yr}^{-1}$, provided that most of the infalling gas is associated with star formation. Such an infall rate would result in $<4.5 \times 10^9 \text{ M}_\odot$ accreted in the form of gas over a disk lifetime of $14 \text{ Gyr}$ and would imply that about half (up to all) of the total mass observed within the optical radius of LSB galaxies has been accreted and/or fallen in. More work is needed to settle the issue of gas infall and accretion in LSB galaxies.

Observations of gas-rich LSB galaxies and star-dominated HSB galaxies suggest that at least two star formation modes may be important in spiral galaxies: 1) a global, continuous star formation mode directed by accumulation of gas in the deep gravitational wells associated with old stellar population in the galactic nucleus and spiral arms (which basically involves the entire ISM on time scales of $>1 \text{ Gyr}$), and 2) a local, sporadic star formation mode associated with accretion and infall of matter interacting with gas already settled in the disk.

While the formation of low-mass stars (e.g. $m \lesssim 1 \text{ M}_\odot$) may be primarily related to the gas surface density, the formation of high-mass stars may be related to the total (both gas and stellar) surface density. This results in a SFR depending both on the total surface density and surface density of the gas (see e.g. Dopita 1989; Dopita & Ryder 1995). The low surface densities in LSB galaxies may directly affect the IMF in the sense that formation of massive stars may be suppressed in these systems. Substantial variations in the massive star IMF observed in our own Galaxy (e.g. Garmany, Conti & Chiosi; 1982), e.g. may be related to variations in surface density. Furthermore, massive star formation in low surface density regions may temporarily inhibit (massive) star formation by controlling the pressure in the disk ISM and maintaining the vertical velocity dispersion (scale height) of the gas (e.g. Kennicutt 1989). Especially in LSB galaxies, this self-regulating mechanism of star formation may prevent the gas from reaching the critical surface density soon after star formation has occurred. Observed differences between the star formation histories of the disks of LSB and HSB galaxies may be explained in terms of the relative importance of the high and low-mass star formation modes in these galaxies.

6.8.3 On the formation of LSB galaxies

Hierarchical theories of structure formation predict LSB galaxies to form at late times from small overdensities in the universe (Dalcanton et al. 1995). In this case, LSB galaxies form from the collapse of smaller amplitude overdensities than do HSB galaxies (Mo et al. 1994) since small amplitude peaks in the background density take longer to reach their maximum size and longer to recollapse than higher amplitude peaks. Furthermore, these small amplitude peaks are more likely to be found in underdense regions so that objects that collapse from small amplitude peaks will be less correlated than those that collapse from larger ones (Kaiser 1984; White et al. 1987). Another interesting prediction is that LSB galaxies should have lower total masses than HSB galaxies since low mass galaxies tend to form naturally with low surface densities according to hierarchical clustering theories.

This theoretical picture is consistent with observations which show that LSB galaxies are locally isolated systems (on scales $<2 \text{ Mpc}$; e.g. Bothun et al. 1993) and that their separation weakly tends to increase towards fainter surface brightnesses (McGaugh, private communication). As the initial structure of galaxies can be strongly affected by their large-scale environments (Hoffman et al. 1992), the formation and low evolutionary state of LSB galaxies may be directed by their environment which favours long collapse times.

Several aspects concerning the formation and evolution of LSB galaxies have not been considered here. For instance, it is unclear whether large amounts of undetected gas are present in the haloes of LSB galaxies. In fact, the hydrogen mass in spirals may be underestimated by more than a factor 10 owing to the very inhomogeneous nature of cold molecular gas which, therefore, is difficult to detect (Pfenniger et al. 1994). Alternatively, this gas may be in the form of ionized hydrogen since protogalaxies with low column densities of gas may experience a delay in the recombination of ionized hydrogen until the extragalactic background flux has dropped below a certain value (Babul & Rees 1992). This prevents star formation over large scales in these galaxies (Corbelli & Salpeter 1996).

The presence of large amounts of dark matter in the haloes of LSB galaxies compared to that in HSB galaxies is strongly suggested by the rotation curves of these systems (dB96). It is found that LSB galaxies do not only have diffuse disks but are diffuse also in their distribution of dark matter. If the dark matter ultimately turns out to be gas, this would strongly suggest that large amounts of dark matter are still to be accreted and converted into stars within LSB galaxies, implying that these systems will ultimately end as HSB galaxies.
6.9 Concluding remarks

The reason why LSB galaxies can maintain their gas surface densities below the star formation threshold for very long times is probably related to the settling of their disks and the onset of star formation therein. In fact, LSB galaxies may evolve at a low rate because they are relatively isolated systems (e.g. Bothun et al. 1993) and their disk gravitational potentials are relatively weak. Observations indicate that LSB galaxies are very extended compared to HSB galaxies and that LSB galaxies have truly low matter densities both in their disks and haloes (dB96) which implies slow contraction and accretion of matter. Furthermore, the large total gas masses and low rotation velocities suggest that LSB galaxies are still in the process of galaxy formation.

The inconspicuous spiral arms in LSB galaxies may be related to their low surface densities and/or to their low rotation velocities in combination with the gradual formation of their disks. Since there appear to be no HSB galaxies with weakly developed spiral arms, low surface brightness may imply weakly developed spiral arms. In addition, high rates of gas accretion over long evolution times may be required to form and maintain pronounced spiral structure.

Recent theories of galaxy formation predict that continuous infall is very important in the outer parts of disk galaxies over long evolution times (e.g. Gunn 1987; Steinmetz & Mülter 1995). In this manner, galaxies continue to build up during their evolution instead of being formed at one preferred epoch. This is probably true for LSB galaxies which contain large fractions of their total atomic hydrogen content beyond the edges of their optical disks and perhaps also beyond the outermost points of their rotation curves measured. This implies intrinsic age differences between the gaseous disks of LSB and HSB galaxies even though the mean age of the stellar populations formed in these disks may be similar.

From theoretical computations it has been found that the formation time scale of Galactic disks strongly depends on the surface density (Burkert et al. 1992) which implies that the disk forms from inside out and that star formation progresses outwards in the disk. In particular, infall time scales are much longer at the outer radii of the gaseous disk (e.g. Larson 1976) while the inner disk depletes gas at short time scales. This is consistent with the fact that the major portion of the HI lies outside the star forming disk in many late-type disk galaxies (e.g. Kennicutt 1992; dB96). The low surface densities in the extended disks of LSB galaxies enable long dynamical time scales so that LSB galaxies are likely to represent an early stage of disk formation in which large amounts of gas are left-over after the initial collapse (i.e. with respect to that in HSB galaxies).

This picture is consistent with observations which show that the outer parts of LSB galaxies are usually bluer than their inner parts (which appear more evolved; e.g. van der Hulst et al. 1987; Bothun et al. 1990; Knezek 1993). Furthermore, color gradients in LSB galaxies are generally large compared to those in HSB galaxies (e.g. de Jong 1995; dB95) which is consistent with the low evolutionary state of LSB compared to HSB galaxies.

Overall, the low evolutionary state of LSB galaxies relative to HSB galaxies suggests that LSB galaxies are just HSB galaxies in the making (except on time scales much longer than a Hubble time). The low evolutionary state of LSB galaxies as confirmed by the results in this paper indicates that the process of collapse and secular contraction of the disk is still going on and proceeds in a regular fashion. Since LSB galaxies probably formed in relatively low density regions of the universe, infall and accretion of matter from their primordial gas reservoir is likely to play a major role in the evolution of their disks.

In the near future, near-IR observations will provide valuable information tracing the old red, metal-poor stellar population in LSB galaxies. Also, extension of the LSB galaxy observations towards smaller HI contents and lower HI surface densities will be very useful. In addition, many LSB galaxies may be still undetected at the faint end of the galaxy luminosity function (i.e. fainter than B ∼ −15 mag). Such observations surely will provide many new insights concerning the evolution of low surface brightness galaxies.

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Publications

Part of the results presented in this thesis are based on the following articles:

Chapter 3


Chapter 4


Chapter 5


Chapter 6

Dust


http://star-www.st-and.ac.uk/ccp/News_22/

• ‘Dust formation around evolved stars’ van den Hoek L.B., 1995, 1st Franco-British conference on the 'Physics and Chemistry of the Interstellar Medium’, Ed. C.S. Jeffery, Université des Sciences et Technologies de Lille, Villeneuve d’Ascq, France

http://star-www.st-and.ac.uk/ccp/News_22/


Miscellaneous


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Summary

The chemical enrichment of the interstellar medium (ISM) by successive generations of stars is a key issue in understanding the chemical evolution of galaxies in general, and the formation history and abundance distributions of the stellar populations in our Galaxy in particular. The goal of galactic chemical evolution modelling is to predict reliable abundances in our Galaxy as well as in other galaxies both as a function of time and location. From such modelling, we can learn what processes determine the chemical enrichment history of different galactic regions (e.g. disk, bulge, halo) and deduce how the formation and evolution of galaxies in general may have proceeded according to their chemical properties.

In this thesis, we concentrated on the star formation history and chemical evolution of the Galactic disk. Using a wide range of observational constraints, most of which have become available during the last few years, we aimed at reconstructing the Galactic star formation history by modelling simultaneously various aspects of Galactic chemical evolution. In the second part of this thesis, we investigated the spectro-photometric and chemical evolution of nearby galaxies by means of a photometric evolution model. In particular, this model was applied to the stellar populations of Low Surface Brightness galaxies, a class of very faint galaxies for which a wealth of observational data have recently become available.

We summarize the main results obtained in this thesis as follows:

In Chapter 3, considerable effort was made to discuss and to emphasize the assumptions and uncertainties involved with the current generation of galactic chemical evolution models. In particular, a thorough comparison was made between the two state-of-the-art models for the core-collapse and chemical evolution of massive stars, i.e. the model presented by the group of Woosley, Timmes, and Weaver (1996) on one hand, and that presented by the group of Nomoto, Thielemann, and Hashimoto (1996) on the other. This comparison clearly reveals for the first time the magnitude and origin of the uncertainties and differences between the yields of massive stars predicted by the two groups and allows for a more reliable interpretation of several discrepancies between the actual models and specific observational constraints to the chemical evolution of the Galactic disk.

In Chapter 4, we modelled a large set of observational data related to the chemical evolution of the Galactic disk and halo using a comprehensive and up-to-date galactic evolution model which incorporates metallicity dependent stellar yields, lifetimes, and remnant masses. A new, iterative solution procedure was applied to solve the galactic chemical evolution equations in a self-consistent manner with the freedom to study complex relations between e.g. the IMF and the SFR. We made a distinction between the enrichment contributions by Asymptotic Giant Branch stars, Supernovae Type Ia (SNIa), SNII, and SNII while using state-of-the-art evolution models for the chemical evolution of these final stages of stellar evolution.

First, we addressed the abundance inhomogeneities observed among similarly aged stars and open clusters in the Galactic disk. We analysed in detail the possibility suggested by e.g. Francois & Matteucci (1993) and Wielen et al. (1996) that stellar orbital diffusion in combination with radial abundance gradients in the disk ISM are the main explanation for these abundance inhomogeneities. We showed that in case of large errors in the derived ages and
orbital parameters of the stars in the Edvardsson et al. (1993) sample, orbital diffusion as described by Wielen et al. (1996) can provide an adequate explanation for the majority of the observed stellar abundance variations. However, at the same time, we argued that this requires several specific assumptions which may be unjustified and not appropriate to the chemical evolution of the Galactic disk.

Second, we investigated the sensitivity of the age-metallicity relation (AMR) to specific galactic chemical evolution model assumptions and we selected a set of models that can explain the mean [Fe/H] vs. age relation observed in the local Galactic disk. We studied the sensitivity of the AMR to the main parameters and assumptions involved in our models. We demonstrated that a wide range of enrichment scenarios is consistent with the observed AMR, i.e. there exists no unique model that is in best agreement with the observed AMR. Conversely, the observed AMR alone is found insufficient to constrain tightly Galactic chemical evolution models and additional constraints are needed.

Third, we confronted the models selected on their ability to fit the observed AMR with observational constraints related to the ISM abundances and stellar content of the disk:

- the present-day stellar mass function (PDMF) and IMF;
- the total number and formation rates of (post) main-sequence stars;
- the gas depletion, infall, and star formation rates in the disk ISM;
- the enrichment history of the Galactic disk as recorded by the abundance-abundance variations (i.e. the variation of the abundance of a given element as a function of the abundance of another element) and the present-day abundances observed. We investigated the impact of: 1) the adopted stellar yields, 2) the star formation history, 3) the IMF, 4) the delay time of SNIa, and 5) the upper mass limit for SNII, on the resulting abundance-abundance variations of the most abundant elements in the disk ISM including C, N, O, Mg, Al, Si, and Fe.
- the luminosity function of white dwarf (WD) remnants;
- the mass distribution of WD remnants;
- the age and metallicity distributions of long-living stars in the local disk (i.e. the classical G-dwarf problem).

By means of this comparison, we attempted to converge to a set of models for the chemical evolution of the Galaxy consistent with the above constraints and we traced back eventual discrepancies between our results and the observations.

In particular, we aimed to deduce the star formation history of the Galaxy both from the abundance-abundance variations observed and other independent observational constraints to the chemical evolution of the Galaxy. As a shortlist of interesting results we like to emphasize the following ones: 1) we found that evolution scenarios in which the SFR gradually increases up to a given maximum in the disk and thereafter decreases exponentially are clearly favoured by the observations. We argue that models which incorporate infall of gas regulating this kind of behaviour of the SFR with age in the Galactic disk are preferred over models which do not incorporate gas infall; 2) we demonstrated that the ejecta of SNIa, associated with stars formed early in the evolution of the Galaxy and with initial masses in the range $\sim 2.5 - 8 \, M_\odot$, need to be delayed over at least 3–5 Gyr after the formation of their WD progenitors in order to fit the observations. Instead of such a time delay, SNIa
may be associated with considerably less massive stars than previously thought, i.e. with masses between \( \sim 1.5 \) and \( 2 \, M_\odot \); and 3) we showed that models in which the upper mass limit of SNII increases as a function of galactic age during early epochs of star formation in the Galaxy are consistent with the observations for variations of \( m_u \) between \( \sim 20 \) and \( \sim 30 - 40 \, M_\odot \) if these variations occur delayed with respect to the variation of the SFR. Such a behaviour of the upper mass limit of SNII may be supported by the formation of massive stars both in the Galactic disk and in external galaxies.

Fourth, we briefly compared our main results with those presented in several other recent investigations dealing with Galactic chemical evolution. We summarized the type of chemical evolution models that are in best overall agreement with the observations and we discussed what this may imply for the chemical evolution of the Galaxy as a whole. Combined with the detailed description in Chap. 3 of the galactic chemical evolution model assumptions and ingredients involved, the extensive results for a wide range of observations presented in this thesis make that our model is one of the best documented Galactic chemical evolution models currently available.

It’s beyond the scope of this summary to list all the results obtained in Chapter 4. Instead, we prefer to highlight some of the results obtained in Sect. 4.3.4 where we modelled the abundances and abundance-abundance relations observed among Galactic disk and halo stars.

- overall, reasonable agreement was found between the predicted and observed abundance-abundance variations for stars in the Galactic disk and halo. In detail, however, none of the SFR models selected on their ability to fit the observed AMR of iron could provide an adequate explanation of the abundance-abundance relations observed, unless additional variations of the element productions by (massive) stars with galactic age are taken into account;

- our results support: 1) a gradual increase of the SFR up to a maximum several Gyr after the onset of star formation in the Galaxy, 2) an exponentially decrease of the SFR past its maximum, and 3) an SFR in the disk ISM regulated by gas infall/accretion of matter. Gas infall onto the Galactic disk seems to be required to explain the stellar abundance-abundance variations observed. Infall time scales between 0.5 and 3 Gyr appear in best agreement with the observations when exponential decaying gas infall is assumed. The agreement of the SFR models above with the observed abundance-abundance variations is very sensitive to the contraction time of the disk ISM before the maximum SFR in the disk ISM is reached;

- our models suggest that the ejecta of SNIa associated with intermediate mass stars formed at early epochs in the evolution of the Galaxy have been delayed over at least \( 3 - 5 \) Gyr after the formation of their WD progenitors. It is difficult to extract information about the detailed SNIa delay time profile from the observed abundance-abundance variations. However, a substantial delay of a large number of SNIa over at least several Gyr after the major period of star formation is needed to strongly affect the slope of the \([\text{O}/\text{Fe}] \) vs. \([\text{Fe}/\text{H}] \) variation at values of \([\text{Fe}/\text{H}] \gtrsim -1 \). The WD delay time effect on the iron enrichment by SNIa leads to an underestimate of the iron abundance at early epochs in the evolution of the Galaxy. Although there are several ways out to compensate for this effect, we favour the possibility that the ages of stars in the Edvardsson et al. (1993) sample are systematically too large by at least \( 3 - 4 \) Gyr;
• if the time delay of SNIa would not be the primary cause for the change in slope of the variation of \([O/Fe]\) with \([Fe/H]\), it appears difficult to explain the observed abundance-abundance trends for these elements unless different processes have initiated and regulated the star formation history in the Galactic halo and disk ISM. This may involve corresponding differences in e.g. the IMF, lower stellar mass limit, and/or upper mass limit for SNII;
• our models combined with the Geneva/Nomoto yields are unable to explain adequately the \([O/Fe]\) and \([C/O]\) ratios observed in Galactic halo stars. This conclusion is independent of the SFR and IMF model used and is insensitive to the parameter values assumed;
• we have argued that the amount of carbon produced during the SNII explosion of massive stars as predicted both by the Geneva/Nomoto and Woosley/Weaver yield sets is considerably too large. This may be e.g. related to the \(^{12}\text{C}(\alpha, \gamma)\) rates adopted;
• we find that nitrogen is overproduced in our models by \(~0.3–0.4\) dex. This suggests that: 1) too many stars reach the AGB, and/or 2) the effect of hot bottom burning is too large in our models. This result needs further investigation;
• in general, an IMF distinct from the Salpeter IMF results in a shift of the abundance-abundance variations predicted, while the shape of these variations is predominantly determined by the underlying star formation (and infall) history;
• the agreement with the observations for oxygen and the \(\alpha\)-elements is improved when IMFs are considered that flatten towards low-mass stars as compared to the Salpeter IMF. However, elements such as C and N formed in intermediate mass AGB stars, probably are overproduced in case of such flat IMFs. Therefore, such flat IMFs are excluded by the observed abundance-abundance variations unless the formation rate of intermediate mass AGB stars is suppressed at the same time. Alternatively, the carbon and nitrogen yields of stars with metallicities \(Z \lesssim 0.001\) may be substantially in error;
• no observational support is found for large variations of the stellar lower mass limit at birth over the lifetime of the Galaxy. If such variations did occur, episodes of relatively massive star formation must have been very short with respect to the lifetime of the Galaxy and/or simultaneous variations in the enrichment contributions by massive stars must have occurred to prevent overproduction of heavy elements in the disk ISM;
• models for which the stellar upper mass limit at birth increases substantially with the SFR are not supported by the observations (unless e.g. simultaneous variations in the lower stellar mass limit at birth did occur);
• models for which the upper mass limit of SNII increases as a function of galactic age during early epochs of star formation in the Galaxy are consistent with the observations for variations of \(m_u\) with between \(~20\) and \(~30–40\) \(M_\odot\), if these variations occur delayed with respect to the variation in the SFR with age. We emphasize, however, that the precise value and variation of \(m_u^{\text{SNII}}\) favoured by the observed abundance-abundance variations is rather sensitive to e.g. the IMF, and the contribution by SNIa to the iron enrichment;
• the Dopita SFR (e.g. Dopita 1989; Dopita & Ryder 1994) and Salpeter IMF models are found in best agreement with the observed stellar abundance-abundance variations in the Galaxy for values of \(m_u^{\text{SNII}}\) between 20 and 25 \(M_\odot\) at the early epoch of
star formation in the Galaxy, a SNIb/c fraction $F_{\text{SNIb/c}} \sim 0.2$ for stars with masses between $\sim 8$ $\text{M}_\odot$ and $m_u^{\text{SNI}}$, a SNIa fraction $F_{\text{SNIa}}$ between 0.01 and 0.02 for stars with masses between $\sim 2.5$ and $\sim 8$ $\text{M}_\odot$, and a SNIa delay time after formation of the WD progenitor of $\sim 3$–5 Gyr;

- for these models, we find that AGB stars roughly account for $\sim 40\%$ of the present-day stellar consumption rate of hydrogen, and contribute $\sim 50\%$ and $\sim 90\%$ to the present-day ejection rates of newly synthesized helium and nitrogen, respectively. SNIa are found to contribute $\sim 80\%$ to the current stellar ejection rate of newly synthesized oxygen. When oxygen initially present in stars at time of their formation is included in the total stellar ejection rate of oxygen, we find that the contribution by SNIa is reduced to $\sim 50\%$ and that AGB stars contribute $\sim 35\%$ to this rate;

- models in best agreement with the observations and computed with the Woosley/Weaver stellar yields, the Salpeter IMF, and parameters as listed hereabove imply typical contributions by AGB stars, SNIa, SNIb/c, and SNIa, to the total present-day stellar ejection rates of C, O, and Fe as follows (normalized to one):

<table>
<thead>
<tr>
<th>Element</th>
<th>AGB</th>
<th>SNIa</th>
<th>SNIb/c</th>
<th>SNII</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.45</td>
<td>–</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>O</td>
<td>0.35</td>
<td>–</td>
<td>0.15</td>
<td>0.50</td>
</tr>
<tr>
<td>Fe</td>
<td>0.25</td>
<td>0.50</td>
<td>0.10</td>
<td>0.15</td>
</tr>
</tbody>
</table>

- the present-day abundances observed in the Galactic disk ISM are not suited to distinguish between different SFR models. However, the interstellar abundances for a large number of elements at distinct epochs in the evolution of the Galaxy may provide the most stringent constraint to models for the star formation history and chemical evolution of the Galaxy. A first confrontation of this kind was made in Sect. 4.3.4.

- unfortunately, the present-day abundances predicted by our models deviate strongly from the mean abundances observed in HII regions in the SNBH and in Canopus. We propose that the abundances of young objects in the solar vicinity are not representative for the mean present-day abundances in Galactic disk stars;

- the possibility that the enrichment in the Galaxy proceeded at a relatively rapid rate during the transition phase in the $[\text{O/Fe}]$ vs. $[\text{Fe/H}]$ relation may imply that the formation of massive stars during this phase has not been accompanied by a corresponding enhancement in the formation of low and intermediate mass stars (i.e. not many of such stars are nowadays observed). This may point to a difference in the IMF of stars formed before and after the transition phase as compared to the bulk of disk stars nowadays observed;

- we suggest that the large spread in abundances observed among Galactic halo stars is related to small-scale spatial variations in the nucleosynthesis of intermediate mass stars ($m = 2$ – $8$ $\text{M}_\odot$) which do not produce both at the same time iron and e.g. oxygen in substantial amounts. These abundance variations may be primarily due to the local enrichment of the halo ISM by SNIa.

In Chapter 5, we investigated in detail the origin of the abundance variations observed among similarly aged F and G dwarfs in the local Galactic disk. We argued that orbital diffusion of stars in combination with radial abundance gradients is probably insufficient to explain these variations. We showed that episodic and local infall of metal-deficient gas can
provide an adequate explanation for iron and oxygen variations as large as $\Delta[M/H] \sim 0.6$ dex among stars formed at a given age in the solar neighbourhood (SNBH). However, such models appear inconsistent with the observations because they: 1) result in current disk ISM abundances that are too high compared to the observations, 2) predict stellar abundance variations to increase with the lifetime of the disk, and 3) do not show substantial scatter in the [Fe/H] vs. [O/H] relation. Notwithstanding, our results do suggest that metal-deficient gas infall plays an important role in regulating the chemical evolution of the Galactic disk. We demonstrated that sequential enrichment by successive stellar generations within individual gas clouds can account for substantial abundance variations as well. However, such models are inconsistent with the observations because they: 1) are unable to account for the full magnitude of the observed variations, in particular for [Fe/H], 2) predict stellar abundance variations to decrease with the lifetime of the disk, and 3) result in current abundances far below the typical abundances observed in the local disk ISM.

We presented arguments in support of combined infall of metal-deficient gas and sequential enrichment by successive stellar generations in the local Galactic disk ISM. We showed that galactic chemical evolution models which take into account these processes simultaneously are consistent with both the observed abundance variations among similarly aged F and G dwarfs in the SNBH and the abundances observed in the local disk ISM. For reasonable choices of parameters, these models can reproduce $\Delta[M/H]$ for individual elements M = C, O, Fe, Mg, Al, and Si as well as the scatter observed in abundance-abundance relations like [O/Fe]. For the same models, the contribution of sequential stellar enrichment to the magnitude of the observed abundance variations can be as large as $\sim 50\%$. We discussed the impact of sequential stellar enrichment and episodic infall of metal-deficient gas on the inhomogeneous chemical evolution of the Galactic disk.

In Chapter 6, we investigated the star formation history and chemical evolution of low surface brightness (LSB) disk galaxies by means of their observed spectro-photometric and chemical properties. To this end, we used a galactic chemical and spectro-photometric evolution model incorporating a detailed metallicity dependent set of up-to-date stellar input data covering all relevant stages of stellar evolution. Comparison of our model results with the observations confirms the idea that LSB galaxies are relatively unevolved systems. Based on extensive modelling, we found that for the majority of the LSB galaxies in our sample, observed Johnson-Cousins UBVRI magnitudes, [O/H] abundances, gas masses and fractions, and H\textsc{i} mass-to-light ratios, are best explained by galactic evolution models incorporating an exponentially decreasing global star formation rate (SFR) ending at a present-day gas-to-total mass ratio of $\mu_1 = 0.5$ for a galaxy age of 14 Gyr. About 35 % of the LSB galaxies in our sample exhibit properties that cannot be explained by exponentially decreasing SFRs alone. We argued that most of these systems experienced recent episodes of enhanced star formation superimposed on exponentially decreasing global SFR models. Only a small fraction ($\sim 10\%-15\%$) of the LSB galaxies have properties consistent with those resulting from linearly decreasing or constant SFR models.

We found evidence, from model point of view, for recent and ongoing star formation in the disks of LSB galaxies at rates of $\sim 0.1$ $M_\odot$ yr$^{-1}$. In particular, we demonstrated that the occurrence of small amplitude star formation bursts in LSB galaxies is required to explain the contribution of the young (5-50 Myr old) stellar population to the galaxy integrated luminosity. This result suggests that star formation in LSB galaxies has proceeded in a
stochastic manner from the moment star formation started in their disks. On the basis of this result, we argued that sporadic star formation in LSB galaxies is probably associated with local accretion and/or infall of matter.

The presence of an old stellar population in many late-type LSB galaxies, as confirmed by our results, suggests that LSB galaxies roughly follow the same evolutionary history as HSB galaxies, except at a much lower rate. In particular, our results imply that LSB galaxies do not form late, or have a delayed onset of star formation, but evolve slowly. We showed that the observed color differences between LSB and HSB galaxies can be interpreted almost entirely in terms of the relatively low extinction and metallicity in LSB galaxies. We proposed that LSB galaxies are in an early stage of disk formation and probably are still in the accumulation phase of gas during which their current amount of star formation and chemical enrichment is regulated. In particular, the gas reservoir at the time of onset of main star formation in LSB galaxies may have been substantially less than that estimated from their present-day amounts of gas since accretion of matter is still very important in these systems.

The low evolutionary state of LSB galaxies relative to HSB galaxies suggests that LSB galaxies are just HSB galaxies in the making (except on time scales much longer than a Hubble time). We discussed our results in the context of the evolutionary history of LSB galaxies compared to that of HSB and dwarf irregular galaxies.

Apart from the results presented in this thesis, I have been and/or am currently working on: evolutionary population synthesis models for the stellar contents in the nuclei of elliptical galaxies (in cooperation with Paul Goudfrooij, Pascale Jablonka, and Danielle Alloin), the star formation history and chemical evolution of the Magellanic Clouds (with Martin Groenewegen and Ken’ichi Nomoto), the luminosity function of AGB stars in the Galactic disk and Magellanic Clouds (with Martin Groenewegen), the formation and evolution of interstellar dust grains in the Galactic disk and Magellanic Clouds (with Teije de Jong), and the radial distribution of the stellar content in spiral galaxies with high and low central surface brightnesses (with Roelof de Jong and Erwin de Blok). Part of the results of these investigations will be presented elsewhere.

The road to wisdom?
Well, it’s plain and simple to express
Err, err, and err again
but less, and less, and less

Piet Hein
Niemand kijkt naar wat zich voor zijn/haar voeten bevindt. We staren allemaal naar de sterren ...

Quintus Ennius