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Citation for published version (APA):

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Department of Economics and Management

DEM Working Paper Series

Beliefs and Precedent: The Dynamics of Access to Justice

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# 84 (07-14)

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http://epmq.unipv.eu/site/home.html

July 2014
Beliefs and Precedent: 
The Dynamics of Access to Justice

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This version: 21 July 2014

Abstract

The entire system of legal remedies rests on the decision of prospective plaintiffs to commence actions before a court. This study focuses on how both plaintiffs’ beliefs and legal precedent affect access to justice. In turn, actual accesses to the judiciary result in judicial decisions, and then in the establishment of further legal precedent that is able to affect the behaviour of new plaintiffs. This dynamic model shows that precedent works as a rectification tool with regard to biased beliefs. However, the strength of the rectification power significantly depends upon both the merit of the case and stickiness of subjective beliefs. The results highlight that although plaintiffs learn from precedent through a Bayesian process, access to justice does not always follow a desirable path. In fact, under some circumstances, meritorious causes of action hardly proceed through the court system, even as frivolous claims continue to flourish.

JEL classifications: D83, K41, D81.
Keywords: access to justice, Bayesian learning process, lock-in, precedent.

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This research was supported by a Marie Curie Intra European Fellowship within the 7th European Community Framework Programme. We are grateful to Theodore Eisenberg, Antonio Lijoi, Francesco Parisi and Pietro Rigo for their precious suggestions. We also thank participants at the Workshop on Economics of Litigation (Catania, 2014), the economics seminars of the University of Modena (2014), and the SIDE-ISLE Annual Conference (Bozen, 2014). All errors are our own.
1. Introduction

Tort law and the entire system of legal remedies crucially rest on the decision of prospective plaintiffs to commence actions before a court (filing phase).

As explained by Graham (2008), a critical mass of plaintiffs who actually access the judiciary for a cause of action is necessary for the establishment and then the consolidation of a tort. Conversely, a systematic waiver of access to justice for a given cause of action determines its decline and can ultimately seriously jeopardize its survival – likewise its methodical exclusion by the courts or the abrogation of underlying substantive law.

People appeal to the judiciary both for meritorious and frivolous causes of action. On the other hand, those causes of actions that are frivolous – or are no longer consistent with the aims of the society – will, one hopes, tend to disappear from the court system. More problematic is the case of meritorious causes of action for which access to justice systematically fails to rise.

There are several explanations as to how prospective plaintiffs who are seeking legal remedies actually access the judiciary or instead give up. This study focuses on the dynamic interactions among plaintiffs’ beliefs, legal precedent, the costs of accessing the judiciary, and the rise or fall of access to justice itself. Legal precedent is considered in terms of its capacity to convey information on the legal merit of a case.

A few examples provided below can illustrate the issue, which is theoretically explored in the next sections.

a. No (more) access to justice: ‘Heart-balm’ torts

‘Heart-balm’ torts and insult torts are only a few examples of torts for which access to the judiciary was common in the past; but they have progressively disappeared from courts before any explicit abrogation – if one ever occurred – simply because plaintiffs ceased to file these kinds of lawsuits. Although some of these cases have been perhaps ‘absorbed’ by other emerging tort categories, for some disappeared causes of action, we can guess that at a certain time, prospective plaintiffs systematically decided to refrain from filing their cases to the courts (Graham 2008).

b. ‘Viral’ access to justice: Product liability, medical malpractice, and environmental torts
On the other hand, both the recognition of new rights or interests and the emergence of new legal theories can result in the flourishing of viral lawsuits. This happens when potential plaintiffs and (especially) their attorneys evaluate proceeding to court as an attractive opportunity to obtain redress, protect their rights, enforce rules, or even abuse the system by filing frivolous lawsuits. According to the literature (Hensler 1987; Galanter 1996), some trends can actually be identified in terms of different types of tort litigation. While litigation concerning ‘standard’ and consolidated torts – like car accidents – were substantially stabilized over time and became a sort of routine for courts, other torts – including product liability (Viscusi 1991), medical malpractice (Danzon 1986 and 1990; Hyman and Silver 2006), environmental and toxic torts (Betlem and Faure 1997), and privacy torts (Citron 2010) – are being discussed before the courts more and more frequently.

c. No access to justice: The case of gender/race-biased plaintiffs

An opposite situation occurs when, even in presence of favourable substantive rules, discouraged potential plaintiffs may forsake accessing the judiciary. This can make some torts viable in theory, but actually silent – at least for people who consider legal actions an unaffordable option. For instance, although substantive rules are unbiased and free of considerations of gender and race, women and minorities are often discouraged from legally proceeding. This can happen either because potential plaintiffs are not sufficiently aware of their rights or because they perceive – correctly or otherwise – an unfavourable bias with respect to the possibility of winning the case and/or being fairly compensated (Wriggings 2005; Chamallas and Wriggins 2010; UNWomen 2012).

d. Optimistic beliefs, precedent, and access to justice: ‘Hot-coffee’ cases

It is well known that pivotal cases promoted by ‘optimistic’ plaintiffs (or by their lawyers), when successfully judged, pave the way for access to justice for new groups of plaintiffs. A successful precedent can actually encourage prospective plaintiffs with similar claims to proceed. In fact, landmark cases, or a sufficient number of minor cases that are successfully decided by the court, constitute one the most significant drivers of subsequent actions, even in civil law systems where the role of precedent is usually more circumscribed than that in common law systems. Conversely, (a set of) unfavourable judicial decisions may dissuade future potential plaintiffs from filing similar claims.

On the other hand, the evolution of access to the judiciary for given types of claims would seem to be barely linked to the ‘true merit’ – however it is evaluated – of the underlying cause of action. Conversely, plaintiffs’ conjectures, precedents, and their interactions seem to have a crucial role in determining future access to the court.

An iconic example is provided by the famous ‘hot-coffee’ cases in both the United States and the United Kingdom. In the United States, where the burnt plaintiff Mrs Liebeck obtained a substantial compensation, the case Liebeck v. McDonald’s Restaurants, Bernalillo County, N.M. Dist. (1994) represented an important stimulus for further similar claims. Perhaps this success induced potential plaintiffs to overestimate the possibility of obtaining significant damages. Actually, U.S. hot-coffee cases contributed to an overheating of the American debate on tort reforms that aimed to restrain unmeritorious lawsuits. In the English case Bogle and others v. McDonald’s Restaurants Ltd., [2002] EWHC 490 (QB), the court decided against the plaintiff. This surely discouraged access to the English judiciary for similar cases.
The examples above illustrate the intimate relation between prospective plaintiffs’ conjectures concerning the outcomes of their legal actions and the actual decision to commence a lawsuit.

In addition, previous judicial decisions on identical cases – or, by extension, on comparable claims when a precedent is missing – affect expectations. This happens because prospective plaintiffs (and their lawyers) are uncertain about possible trial outcomes but can learn the dominant case law from past litigation.

In a Bayesian framework, cases that were previously discussed before the court represent the *weight of evidence* that can strengthen or conversely disconfirm personal conjectures about possible success in future lawsuits (Good 1985). Furthermore, given that new legal actions imply further precedent and the latter, in turn, influences the conjectures of future potential plaintiffs, the evolution of access to justice for a given cause of action must be viewed within a dynamic paradigm.

This paper theoretically explores the possible dynamic of access to justice for a given cause of action over time, depending on the evolution of both prospective plaintiffs’ conjectures and precedent that evolves time by time according to judicial decisions which follow actual access to the court.

As in Baker and Mazzetti (2012), we do not model litigation; we avoid explicitly considering the *intra-lawsuit* interaction between a specific plaintiff–defendant pairing. Instead, we focus on learning over time, with prospective plaintiffs engaging in cost–benefit analysis of going in court by interpreting precedent. Finally, we consider the implications of cases going to court as a driver for the evolution of access to justice itself. The focus of the study is, in fact, the evolution over time of access to justice - meaning the filing phase - as a necessary phase that precedes litigation (or settlement), and not dispute selection for trial as in Dari-Mattiacci et al. (2011). For simplicity, we assume that cases cannot be settled and all the cases that actually access the judiciary finally result in some kind of knowable judicial decision (in a broad sense, including summary judgments, dismissals, etc.).1
Nevertheless, the (clear) implications of our model for selecting disputes for litigation are discussed in Section 5. Furthermore, as explained in that section, this simplifying assumption does not jeopardize general results vis-à-vis access to justice dynamics; our implications confirm and support some results concerning dispute selection as already described in the literature.

On one hand, this study illustrates the virtuous ‘rectification effect’ of precedent on subjective beliefs. This can progressively discourage access to justice for unmeritorious claims, while increasing access for meritorious causes of action. However, this dynamic process takes time and the more widespread and deep-rooted the personal biases are, the harsher the desirable path of access to justice. Additionally, inconsistency among judicial decisions may obstruct the rectification effects of precedent. In particular, eradicating frivolous causes of actions from court may be particularly difficult when residual over-optimistic plaintiffs are staunch in their beliefs. Finally, lock-in phenomena may occur, even for meritorious causes of action. In fact, when all the prospective plaintiffs surrender and no case accesses court, the beneficial effect of precedent is immediately hindered.

The study also provides useful technical implications, since it shows that both priors and posteriors on the probability of success in trial distributed according to a Beta distribution are characterized by parameters whose distributions belong to the same Binomial family. This result might be useful for future research to model priors and posteriors of parties who are involved in litigation.

The paper is organized as follows. Section 2 provides a brief literature review; Section 3 sets up the model; Section 4 studies the access-to-justice dynamics over time, depending on both legal merit and litigation costs; the role of victims’ optimism and lock-in phenomena are also investigated. Section 5 concludes and suggests possible future extensions. Proofs are presented in the Appendix.
2. Literature review

The law and economics literature rarely focuses on access to justice issues: contributions on litigation generally investigate dispute selection for trial, provided that the access to justice (filing condition) for plaintiffs is verified. Moreover, to the best of our knowledge, no contributions to the literature deal with access to justice from a dynamic perspective.

To frame the present model, we identify two main fields related to our contribution, from which we nonetheless significantly depart: the literature on the efficient evolution of law, and that on the dynamic models on dispute selection.

- Efficient evolution of law. Several authors discuss precedent and trials as evolutionary forces that shape rules. Typically, dynamic models analyse whether laws improve in terms of efficiency, thanks either to the efficiency-oriented interventions of judges who set new precedents (Posner 1973) or through selective mechanisms that are implied by litigation itself (Priest 1977; Rubin 1977; Miceli 2009). Cooter and Kornhauser (1980) show that the law does not necessarily evolve towards efficiency. Parisi and Von Wangenheim (2006) and Carbonara et al. (2012) show how social norms and legal rules dynamically interact. Fon and Parisi (2006) analyse the impact of previous judicial decisions on subsequent judicial decision-making in Civil law systems.

According to other authors, litigation can affect rules (i) by augmenting law precision (Gennaioli and Shleifer 2007), (ii) by promoting biases that favour parties who are able to devote large resources to litigation itself (Hirshleifer 1982; Rubin et al. 2001), (iii) by expanding or contracting the extent of legal remedies in line with judges’ ideologies (Fon and Parisi 2003), and (iv) by allowing private information to percolate in legal rules (Hylton 2006). Law evolution driven by litigation may finally depend on the costs of the litigation system and on rule uncertainty (Baker and Mezzetti 2012). However, according to this stream of literature, the set of cases actually discussed before the court is a driver of law
evolution and not something that evolves in itself. Different from the literature on efficient precedent, the present study investigates how the rectification effects of precedent induce efficient access to courts.

- **Dynamic models on dispute selection.** Dari-Mattiacci and Deffains (2007) and Dari-Mattiacci et al. (2011) each provide dynamic models of the evolution of litigation and settlement over time and at an aggregate level. The authors show that precedent reduces uncertainty of law and therefore favours settlement. In turn, settlement does not determine any further precedent, and makes the legal system more uncertain. This uncertainty pushes again the aggregate level of trials, thus inducing a cyclical movement. In these models, all the plaintiffs are assumed to have access to justice, while the dynamics of the latter over time are not at all investigated. Moreover, in these contributions, the precedent helps to reduce the expectation gap between two parties who have asymmetric private information as described in static models on settlement. Here, instead, the precedent helps generate better conjectures of each individual potentially involved in a legal issue.5

The precedent, however, is crucially informative even before litigation, independently of any strategic interaction between the parties in conflict. When an individual plaintiff chooses whether to file a lawsuit or simply waive, the precedent represents an essential benchmark in decision-making. As argued above, plaintiffs’ subjective conjectures – interpreted as initial conditions (Hahn 1974) – and precedent – as the keystone of a Bayesian learning process – represent the two fundamental drivers in the access-to-justice path. Differences in the expectations of heterogeneous plaintiffs may generate different decisions on whether or not to commence a lawsuit. The progressive establishment of case law may support (or conversely discourage) access to justice, because plaintiffs progressively learn how courts decide a given type of case6.
Finally, this study is the first to use suggestions from the literature on the diffusion of innovation (Bogliacino and Rampa 2012) to frame access to justice evolution as a consequence of plaintiff uncertainty, the Bayesian learning of precedent, and heterogeneity in personal conjectures.

3. The model

3.1. Setup and hypotheses

3.1.1. Timing

Consider a society where individuals are characterized by a number of conjectures. At each date $t$, $K$ people face an identical justiciable problem. In particular, these $K$ prospective plaintiffs need to evaluate whether to file a lawsuit or to waive. Time is discrete. Every identical cause of action implies, for simplicity, a unitary value at stake. Once the cause of action occurs, prospective plaintiff $i \in \{1,2,\ldots,K\}$ can legally proceed at date $t$ itself, but not later, given the statute of limitation. This assumption means that potential plaintiffs cannot strategically use time: it is a sound assumption for non-professional plaintiffs who face everyday-life justiciable problems. The decision on the part of $i$ of whether or not to legally proceed at date $t$ depends on $i$’s conjecture formed at date $t-1$ (more on this below). At the end of $t$, judges decide all the cases that have been actually filed with the court, and judicial decisions are publicly released. As a consequence, the population updates its own conjectures on the basis of both their personal priors and the outcomes of cases previously decided in court. These updated conjectures will be used by prospective plaintiffs to decide whether or not to access court at date $t + 1$.

3.1.2. Court technology

To proceed in court, each plaintiff must bear a cost $c \in (0,1)$. The court can decide either in favour of or against the plaintiff (i.e. grant either a unitary relief or no relief). Before sentencing,
the outcome of a lawsuit is uncertain. Every single case accessing the judiciary has an outcome \( S \in \{0,1\} \) (‘failure’ or ‘success’), and the probability of success for every case is \( p \in (0,1) \).

Notice that \( p \) may be seen as the legal (objective) merit of a case. We can distinguish two possible classes of lawsuits: one characterized by \( p > c \), and the other by \( p \leq c \). Following part of the literature, the former class might be defined as comprising meritorious cases, while the latter comprises frivolous cases. Although debatable, this distinction provides a possible welfare criterion which will be useful later on.

Saying that the probability of success is \( p \) for every case means that each \( S \) is a Bernoulli variable of parameter \( p \). The interpretation of this is that every judge has the same background education and reasoning scheme, and is subject to the same ‘trembles’, when facing identical cases. In other terms, we are assuming that every judge’s decision is independent of any external circumstance other than the merit of the cases.

We assume, furthermore, that every judge is independent of other judges in sentencing, which implies that the different \( S \)s are i.i.d. Consider now any positive integer number \( m \) of cases that are actually filed, whose outcomes are \( S_1, S_2, \ldots, S_m \), and define \( Y(m) = \sum_{j=1}^{m} S_j \) : \( Y(m) \) is the number of successes in the \( m \) trials with i.i.d. outcomes. Define also \( N(m) = m - Y(m) \) (i.e. the number of ‘failures’ (no-relief outcomes)). It is well known that, under these assumptions, \( Y(m) \) is a Binomial variable of parameters \( m \) and \( p \), while \( N(m) \) is a Binomial variable of parameters \( m \) and \((1 - p)\). All this is summarized by the following notation:

\[
S \sim B(1, p) \quad Y(m) \sim B(m, p) \quad N(m) \sim B(m, 1 - p)
\] (1)

where \( B(\cdot, \cdot) \) is the symbol for the Binomial distribution and, as a particular case, \( B(1, \cdot) \) is the Bernoulli distribution.
3.1.3. Plaintiffs

Prospective plaintiffs are risk-neutral. They are always able to pay for litigation cost $c$; no budget constraint limits their choice between proceeding in court and abandoning the cause of action. What actually influences their decision of accessing the judiciary is the fact that they are uncertain about the outcome resulting from filing the lawsuit. The uncertainty of judicial outcomes forces plaintiffs to decide on the base of some conjecture. Although the legal merit $p$ would be the correct expectation for all the prospective plaintiffs, this is unknown to them by assumption. This hypothesis describes the pivotal reason why cases, even frivolous ones, are debated in court: the ‘true’ legal merit of a case is what the judge actually establishes through his judicial decision, and this is ex-ante uncertain; not only, but the true probability of success is unknown to individuals.

We model individuals’ uncertainty by assuming that for each of them, the parameter $p$ is a random variable distributed according to a Beta distribution. That is, for prospective plaintiff $i$ at time $t$:

$$p_{i,t} \sim \text{Beta}(\alpha_{i,t-1}, \beta_{i,t-1}),$$  \hspace{1cm} (2)

where the non-negative parameters $\alpha_{i,t-1}$ and $\beta_{i,t-1}$ characterize plaintiff $i$ at date $t$. Notice the date-suffixes of the parameters: as we said above, the conjecture used by individual $i$ in order choose at date $t$ itself is the one that she/he formed ad date $t-1$.

This allows for the development of an analysis that accounts for both the presence of multiple heterogeneous prospective plaintiffs and the dynamics of their conjectures over time according to a learning process.

Indeed, the advantage of our assumption is that the Beta distribution is a ‘conjugate prior’ for the Binomial likelihood of the observable sample: if the prior conjectured distribution of $p$ is Beta and the observed sample is Binomial, then the updated conjecture, or posterior, is also
Beta. A further advantage of the Beta distribution is that it provides an explicit measure of 'prior robustness', as explained in the details in the next subsection.\(^9\)

Note that lawyers are not explicitly included in our reasoning. Actually, lawyers are those individuals who help plaintiffs develop an opinion concerning the legal merit of a case. (See Hadfield (2000) on legal service as a credence good.) Here, plaintiffs and lawyers are perfectly aligned. Therefore, for the sake of simplicity, we focus only on the plaintiffs’ subjective priors of \(p\).

### 3.1.4. Heterogeneity in conjectures and access to justice

Call \(E_{i,t}(p)\) the subjective expected value of \(p\) on the part of individual \(i\) at date \(t\). Given that the relief in the case of success equals 1, and since plaintiffs are rational and risk-neutral, each of them decides to file a lawsuit in court if and only if

\[
E_{i,t}(p) > c. 
\]  
(3)

Inequality (3) describes the condition\(^{10}\) under which prospective plaintiff \(i\) actually accesses the judiciary at time \(t\) with the aim of obtaining a remedy to a claim, based on facts that occurred at \(t - 1\). Since conjectures are different among people with identical claims, heterogeneity makes potential plaintiffs with an identical cause of action different with respect to the choice of actually accessing the judiciary.

By assuming the heterogeneity of conjectures, we account for the fact that plaintiffs, although facing identical claims, behave differently, either because they have different degrees of awareness of their entitlements, are variously optimist/pessimist, or variously construe information to evaluate the legal merit of the case, etc. We submit that all this is reasonably summarized by (2). First of all, given this assumption it is known that\(^{11}\):

\[
E_{i,t}(p) = \frac{\alpha_{i,t-1}}{\alpha_{i,t-1} + \beta_{i,t-1}} 
\]  
(4)
Thus, various parameter configurations imply various degrees of optimism/pessimism on the part of various prospective plaintiffs. Second, the Beta parameters incorporate also the degree of a plaintiff’s confidence in his or her own conjectures, since the term $P_{i,t} = \alpha_{i,t} + \beta_{i,t}$ is a measure of the prior robustness of individual $i$ at date $t$: as we shall see in Subsection 3.1.5, a low (high) level of robustness means that the plaintiff is more (less) prone to modify his or her conjecture after observing a precedent sample.\textsuperscript{12} As a consequence, Beta parameters allow us to account for both heterogeneity in optimism (mean) and prior robustness.\textsuperscript{13}

At each date $t$, out of $K$ prospective plaintiffs who have a cause of action to proceed in court in $t$, only those for which (3) holds actually file the case. The number $m_t$ of plaintiffs who file a lawsuit in court at each date $t$ is then:

$$m_t = \# \{ i \mid E_{i,t}(p) > c \} \in [0, K]$$

(5)

The dynamics of $m_t$ over time is the core of the analysis offered by this paper.

3.2. Learning: from priors to posteriors

Since judicial decisions are public, at each date $t$ the population can learn from the $m_t$ cases actually filed to the court. Although prospective plaintiffs still ignore parameter $p$, they update their priors by taking into account the precedent represented by the number of successes and failures in the courts at $t$. The updating, or learning, mechanism follows a Bayesian scheme. In what follows, with respect to the decision at date $t$, we call ‘prior’ the conjecture formed up to date $t-1$, and call ‘posterior’ the updated conjecture after the outcomes of date $t$ have been observed. Of course, the posterior of date $t$ is to be interpreted as the prior of date $t+1$.

To keep the analysis as simple as possible, we need to make some further assumptions and to establish some facts.

New causes of actions emerge in society over time. New causes of actions are typically rooted in previous and different, but similar, cases that can justify the application of rules by analogy.
For a new cause of action, no specific precedent is available. Therefore, at this stage, plaintiffs have initial priors concerning the possibility of success for the new cause of action. The initial priors are formed on the available past cases that have been debated in the courts on the basis of already-consolidated causes of action being as much as possible similar to the new one.

Coherently with (2) and with the timing introduced above, we express plaintiff $i$’s initial prior as $p \sim \text{Beta}(\alpha_{i,0}, \beta_{i,0})$. We introduce the following:

**Assumption A.** For all $i$’s, $\alpha_{i,0} \sim \text{B}(m_0, p)$ and $\beta_{i,0} \sim \text{B}(m_0, 1 - p)$, with $m_0$ positive integer.

In addition, $\alpha_{i,0}$ is independent of $\alpha_{j,0}$, and hence $\beta_{i,0}$ is independent of $\beta_{j,0}$, for all $i$ and $j$.

The meaning of this assumption is as follows: it is as if our prospective plaintiffs, starting from an ‘uninformative’ aboriginal prior, could observe, independent of each other, $m_0$ similar, but different and independent, court decisions concerning causes of action similar but not equal to the new one and likewise characterized by a success probability $p$. As a consequence, as we see more precisely below, $\alpha_{i,0}$ and $\beta_{i,0}$ are set equal, respectively, to the number of ‘successes’ and ‘failures’ in those pre-play court decisions; therefore, $\alpha_{i,0} + \beta_{i,0} = m_0$.

Now, as explained in Subsection 3.1.4., each plaintiff $i$ has $E_{i,1}(p) = \frac{\alpha_{i,0}}{\alpha_{i,0} + \beta_{i,0}}$, and this plaintiff does access justice at date 1 if $E_{i,1}(p) > c$. Call $m_1$ the number of plaintiffs for which this condition holds; then, from (1), the ex-ante (before sentencing) numbers of successes and failures in the $m_1$ trials are the random variables $Y(m_1) \sim \text{B}(m_1, p)$ and $N(m_1) \sim \text{B}(m_1, 1 - p)$, respectively.

We now slightly simplify our notation by setting

$$ Y_1 \equiv Y(m_1) \quad \text{and} \quad N_1 \equiv N(m_1). $$

(6)
Call \( y_1 \) and \( n_1 \), respectively, the *actual realizations* of these two random variables at date 1, with \( y_1 + n_1 = m_1 \). That is, all plaintiffs can observe \( y_1 \) successes and \( n_1 \) failures, of the \( m_1 \) cases filed with the court.

Based on these observations, each plaintiff updates his or her prior, resulting in the posterior as of date 1, or equivalently the prior as of date 2. One can prove that:

**Fact 1.a (starting date).** \( p \sim \text{Beta}(\alpha_{i,1}, \beta_{i,1}) \), where \( \alpha_{i,1} = \alpha_{i,0} + y_1 \), and \( \beta_{i,1} = \beta_{i,0} + n_1 \).

More generally, consider what happens at the generic date \( t \): given the \( t \)-prior \( \text{Beta}(\alpha_{i,t-1}, \beta_{i,t-1}) \), each plaintiff decides whether to access justice, if condition (3) is satisfied.

Suppose that there are \( m_t \) plaintiffs accessing justice at date \( t \), and that the actual realizations of these trials are \( y_t \) successes and \( n_t \) failures. Then, one has:

**Fact 1.b (general case).** \( p \sim \text{Beta}(\alpha_{i,t}, \beta_{i,t}) \), where \( \alpha_{i,t} = \alpha_{i,t-1} + y_t \), and \( \beta_{i,t} = \beta_{i,t-1} + n_t \).

Before proceeding, a remark is in order. When presenting Fact 1, the number of people who access justice at date \( t \), \( m_t \), was considered a constant, since we were interested in studying the updating formula after an actual realization had been observed. However, putting oneself at the start of the story, i.e. at date 0, it is a *random variable*. In fact, the initial individual characteristics (priors) are themselves random variables (Assumption A); hence, the number of accesses to justice at any date \( t \geq 1 \), as seen from date 0, is a random variable as well. We call \( M_t \) this random variable: the evolution of \( M_t \) in time will be explored in Section 4.

Fact 1 implies that the subjectively expected value of \( p \) on the part of plaintiff \( i \) at date \( t+1 \) is

\[
E_{i,t+1}(p) = \frac{\alpha_{i,t}}{\alpha_{i,t} + \beta_{i,t}} = \frac{\alpha_{i,t-1} + y_t}{\alpha_{i,t-1} + \beta_{i,t-1} + m_t}.
\]

After some simple arithmetic passages, it turns out that we can write (7) as
\[ E_{i,t+1}(p) = E_{i,t}(p) \cdot \frac{P_{i,t-1}}{P_{i,t-1} + m_t} + \gamma_i \cdot \frac{m_t}{P_{i,t-1} + m_t}, \tag{8} \]

where \( P_{i,t-1} = \alpha_{i,t-1} + \beta_{i,t-1} \), as mentioned, is prior robustness at date \( t - 1 \), and \( \gamma_i = y_i / m_t \) is the share of cases decided in favour of the plaintiffs (favourable precedent) at \( t \).

The interpretation of (8) generates several implications. At each date \( t \), the new subjective expectation of success in trial involves two components: the previous expectation \( E_{i,t}(p) \) and the frequency of favourable precedent \( \gamma_i \). These two components have weights that depend on conjecture robustness and the number of cases being actually filed with the court. The higher the robustness \( P_{i,t-1} \) of the previous expectation \( E_{i,t}(p) \), the higher the weight of the latter in the updated expectation. The higher the number of cases to have had access to the court \( m_t \) is, the higher the weight of the successful precedent share \( \gamma_i \) in determining the new expectation.

In other words, the higher the prior robustness \( P_{i,t-1} \), for any given \( m_t \), the higher the stickiness of the updating process: the plaintiff will modify his or her opinion only marginally. Finally \( \gamma_i \) measures the marginal informative contribution that the new judicial decisions provide to the stock of case law.

Furthermore, since one can write \( P_{i,t} = \alpha_{i,t} + \beta_{i,t} = \alpha_{i,t-1} + \beta_{i,t-1} + m_t = P_{i,t-1} + m_t \), if at each period any case went to court, the robustness would continue to grow over time. Given that the number of cases that are decided in court cannot exceed \( K \), the capacity for further case law to be informative for prospective plaintiffs becomes smaller and smaller over time.

Observe finally that the term \( m_0 \), introduced in Assumption A, can be interpreted as the initial robustness of the plaintiffs: the higher the number of cases, similar to the new one, that they had observed in the past, the higher their initial confidence in the quality of their conjecture. Of course, the assumption that \( m_0 \) is equal across agents is a limitation inherent in our setup, but it is necessary for the tractability of our current analysis.
These implications can be summarized by the following proposition.

**Proposition 1.** For a given cause of action, the updated success expectation of prospective plaintiff $i$ is a weighted average of his or her previous expectation and of favourable precedent: the weights depend on both conjecture robustness and precedent robustness. The latter depends on the number of cases that actually went to court. Case law resulting from litigation allows for expectation updating, but the informative capacity of precedent decreases over time.

We would like to point out how standard assumptions and a canonical Bayesian setting allows us to model reliably the dynamic interaction between precedent and expectations via access-to-justice. Proposition 1 describes a sound dynamics; rooted beliefs are harder to eradicate, while precedent works as the Bayesian weight of evidence. Consistent judicial decisions on consolidated causes of actions do not add particularly useful information on future prospective plaintiffs. Conversely, when a new cause of action is emerging and a few related cases have been debated in court, every judicial decision becomes valuable as a pivotal precedent that orients beliefs.

4. The dynamic path of access to justice

When a new cause of action emerges ($t = 0$), the potential flourishing of related claims in court mainly depends on (i) the decisions of prospective plaintiffs to actually access the judiciary, (ii) the way access affects posteriors, and (iii) the interplay of these two sides. We wish to characterize the stochastic properties of these dynamics.

4.1. The emergence of a new cause of action: the first step in the dynamics

Given (3), (4), and Assumption A, the probability that prospective plaintiff $i$ actually accesses the judiciary to litigate the new cause of action for the first time in court at $t = 1$ is
\[
\Pr \left( \frac{\alpha_{i,0}}{\alpha_{i,0} + \beta_{i,0}} > c \right): \text{ given our definitions, after a couple of passages, it can be written as }
\]

\[
\Pr(\alpha_{i,0} > cm_0). \text{ Call } g_0 \text{ this probability, and observe that this number is the same for all }
\]

prospective plaintiffs.

Now, express the cumulative distribution function of \( \alpha_{i,0} \sim B(m_0, p) \) as \( F(x; m_0, p) \). Therefore, the ex-ante (in \( t = 0 \)) probability \( g_0 \) is given by:

\[
g_0 = 1 - F\left(\left\lfloor cm_0 \right\rfloor; m_0, p\right) \quad \forall i, \quad \text{(9)}
\]

where \( \left\lfloor cm_0 \right\rfloor \) is the ‘floor’ of \( cm_0 \) (i.e. the maximum integer that is lower or equal to \( cm_0 \)). This probability depends on the legal merit \( p \), the litigation cost \( c \), and the initial robustness of prospective plaintiffs’ conjectures. First of all, \( g_0 \) is strictly increasing in \( p \) and strictly non-increasing in \( c \): these facts depend on the standard properties of the Binomial distribution. In addition, we submit the following (Proof in Appendix):

**Fact 2.** If \( p > c \) (resp. \( p < c \)), then: (a) \( g_0 \) is bounded from above and from below by functions that are increasing (resp. decreasing) in \( m_0 \), for \( m_0 \) large enough; (b) as \( m_0 \) tends to infinity \( g_0 \) converges to 1 (resp. to 0); for \( p=c \), as \( m_0 \) tends to \( \infty \) \( g_0 \) converges to \( 1/2 \).

These properties show that access to justice for a new cause of action is discouraged by high litigation costs, and is favoured in terms of legal merit. Note that the latter is unknown; however, it is implicitly conveyed to the prospective plaintiffs through their pre-play signals, founding their initial priors, which are based on case law concerning different but analogous causes of action. Fact 2 tells us that for \( p > c \) (resp. \( p < c \)) there tends to be an increasing (resp. decreasing) relation between \( g_0 \) and \( m_0 \). Therefore, the robustness of initial priors encourages access to justice for meritorious causes of action while discouraging unmeritorious claims. As said in the proof of Fact 2, it can be shown that the greater the distance between \( p \) and \( c \) (either a highly
meritorious cause of action or strongly unmeritorious cases) is, the stronger the effect of robustness.

The above results allow us to state the following:

**Proposition 2.** When a new cause of action emerges \((t = 0)\), the ex-ante probability of access to justice \((in t = 1)\), \(g_0\), increases with legal merit but decreases with litigation costs. Prior robustness determines a favourable trend in the probability of access to justice for meritorious claims, while fosters a negative trend for unmeritorious claims.

Proposition 2 has two relevant implications. The first – which concerns the probability of success and litigation costs as opposing incentives for actions before the court – is in line with the standard literature. The second one is less standard. In fact, Proposition 2 points out the role of doctrine and jurisprudence as a corpus that people reference when new causes of action emerge. To rise in the courts, even very innovative legal actions should have roots in grounded case law by analogy.\(^{18}\)

We turn now to the question raised above – namely, that the number of accesses to justice at date 1 is not a constant, but is in fact a random variable. With regard to this point, we offer the following (Proof in Appendix):

**Fact 3.** Ex-ante \((in t = 0)\), the number of accesses to the judiciary at date \(t = 1\) is a random variable \(M_1\) distributed according to a Binomial distribution of parameters \(K\) and \(g_0\), that is, \(M_1 \sim B(K, g_0)\).

Once a particular realization \(m_1\) of the variable \(M_1\) has occurred (i.e. the number of accesses is determined at date 1), the numbers of successes and failures before the court are again random variables, and we wish to know the ex-ante distribution of these variables. The following Fact (Proof in Appendix) holds with respect to this problem.
**Fact 4.** Ex-ante (in \( t = 0 \)), the number \( Y_1 \) of successful cases at date \( t = 1 \) is a random variable distributed according to a Binomial distribution of parameters \( K \) and \( p g_0 \) – that is \( Y_1 \sim B(K, p g_0) \). In the same way, ex-ante, the number of unsuccessful cases at date \( t = 1 \) is the Binomial variable \( N_1 \sim B(K, (1 - p) g_0) \).

In order to proceed towards a characterization of the learning dynamics, we need to establish some more properties of the random variables involved. Thus, we have the following:

**Fact 5.** \( Y_1 \) is independent of the \( \alpha_{i,0} \)'s, and \( N_1 \) is independent of the \( \beta_{i,0} \)'s.

Proof: it simply follows from our assumptions on court functioning (see Subsection 3.1.2), in particular from the independence of judges’ decisions of anything different from the merit of the cause.

We can now characterize the distribution of the parameters \( \alpha_{i,1} \) and \( \beta_{i,1} \), as defined in Fact 1.a (i.e. the parameters of the posterior individual conjectures at date 1 or, equivalently, the prior conjectures at date 2). The following holds (Proof in Appendix):

**Fact 6.** Under mild assumptions on \( m_0, K, \) and \( p \), the distribution of \( \alpha_{i,1} \) is well approximated by the Binomial \( B(m_0 + K g_0, p) \), and the distribution of \( \beta_{i,1} \) is well approximated by the Binomial \( B(m_0 + K g_0, 1 - p) \).

The parameter restrictions that allow for a good approximation consist of a sufficiently high number of trials, and of a probability of success that is not overly close to the extremes. These constraints do not seem unreasonably restrictive in our setting. Actually, we are working on a large number of prospective plaintiffs who are affected by common legal needs, time by time: hence \( K \) and \( m_0 \) can be considered sufficiently large. In addition, cases being characterized by extremely polarized merit seem to be those less relevant to the functioning of the judiciary.
Roughly speaking, plaintiffs typically need to go before the court to obtain legal remedies when their case is sufficiently ‘debatable’.

Fact 6 is noteworthy, since it shows that plaintiffs’ posterior conjectures at date 1 (i.e. their prior conjectures at date 2) are characterised by parameters whose distributions belong to the same family as those of the previous date – namely, the Binomial family. In addition, the probabilities involved in these Binomials are the same as those of $t = 0$, and only the first terms in $B(.)$ do change. This result might be useful for future research that models priors and posteriors on the probability of success in trial.

Now, we can calculate $T_1 \equiv m_0 + \lfloor Kg_0 \rceil$, a constant. We can thus write: $\alpha_{i,1} \sim B(T_1, p)$ and $\beta_{i,1} \sim B(T_1, 1 - p)$. Therefore, we have $\alpha_{i,1} + \beta_{i,1} = T_1$, and this is the robustness of the plaintiff’s prior at date 2. Finally, the probability that plaintiff $i$ accesses justice at date 2 is $g_1 = 1 - F \left( \lfloor cT_1 \rceil ; T_1, p \right)$, $\forall i$.

4.2. The full-fledged dynamics of access to justice over time

The analysis on the ex-ante probability of access to justice for a new cause of action can now be generalized for all dates $t \geq 1$. At every date $t + 1$ every prospective plaintiff $i$ decides whether to access the judiciary on the basis of condition (3). The ex-ante probability $g_i$ that prospective plaintiffs will access the judiciary at date $t + 1$ is given by $g_i = 1 - F \left( \lfloor cT_1 \rceil ; T_1, p \right)$, $\forall i$, where we define:

$$T_i \equiv T_{i-1} + \lfloor Kg_{i-1} \rceil$$

and $T_0 \equiv m_0$. (10)

By applying Fact 1.b and Facts 3–6 recursively, the ex-ante probability of access to justice $g_t$ and the distribution functions of random variables $M_{t+1}$ (number of prospective plaintiffs accessing the judiciary at $t + 1$), $Y_{t+1}$ (number of successful trials at $t + 1$), and $N_{t+1}$ (number
of unsuccessful trials at $t + 1$ can be easily derived. The next Proposition summarizes the whole reasoning until now.

**Proposition 3.** Suppose that at date $t = 0$ prospective plaintiffs’ priors on the probability $p$ of prevailing in court are described by Beta distributions with parameters $\alpha_{i,0} \sim B(T_0, p)$ and $\beta_{i,0} \sim B(T_0, 1 - p)$ that are i.i.d. among plaintiffs, with $T_0 = \alpha_{i,0} + \beta_{i,0}, \forall i$. Suppose that prospective plaintiffs update their priors at each date $t > 0$ through the Bayesian learning process defined by Fact 1.b. Then, the ex-ante distribution functions of the Beta parameters $\alpha_{i,t}$ and $\beta_{i,t}$ can be approximated by Binomial distributions so that:

$$\alpha_{i,t} \sim B(T_t, p) \quad \text{and} \quad \beta_{i,t} \sim B(T_t, 1 - p) \quad \forall i, t$$

where $T_t$ is defined in (10).

We offer now some implications of this result. To start with, Corollary 1 (Proof in Appendix) describes the expected future path of access to justice, given that the expected number of accesses is greater than 0 at some date $t \geq 1$.

**Corollary 1.** Suppose that $[K_{g_{t-1}}] \geq 1$, with $t \geq 1$. Then the ex-ante probability that cases will access justice at each date $t' > t$ converges either to 1 or to 0 as $t'$ approaches infinity, according to whether $p > c$ or $p < c$.

Corollary 1 underlines the importance of prior-updating through the precedent, which is provided by cases discussed before the court, time by time. If prospective plaintiffs can update their priors on the basis of case law, the litigation dynamics follow a suitable path over time: access to justice for meritorious causes of action progressively increases, while unmeritorious petitions progressively disappear from courts. In addition, as stated in the proof of Fact 2, the greater the distance between $p$ and $c$, the faster the convergence. The corollary seems to effectively illustrate some rectification effects of precedent on priors. On one hand, the
aforementioned path might explain the progressive disappearance from the court of some causes of action, due to systematic abandonment by plaintiffs. This probably happens when those kinds of claims come to be viewed as ‘frivolous’ in an increasingly modern society (example \(a\) of Section 1). On the other hand, the portrayed dynamics might fit the evolution of some recent meritorious causes of actions that emerged as \textit{viral} (example \(b\) of Section 1).

All this sounds quite sensible; however, as we shall presently see, it is not always that meritorious access to justice will flourish, nor that all frivolous causes of actions will easily disappear.

4.3. Lock-in problems for meritorious causes of action

Although access to justice tends to favour a correct updating of the plaintiffs’ priors, people do not necessarily access the courts. From the viewpoint of both the fairness and efficiency of in-court litigation, it is particularly important to focus on the probability that a new meritorious cause of action (characterized by \(p > c\)) cannot access the judiciary when emerging. Actually, even if a new cause of action is meritorious, when all prospective plaintiffs surrender, no case goes in court at \(t = 1\); in such a case, there is no chance of setting precedent and boosting future meritorious accesses to the judiciary. We call this the ‘locking-in’ of meritorious access to justice.

Expression (3) informs us that nobody accesses the judiciary at date \(t = 1\) if \(E_{i,1}(p) \leq c, \forall i\).

The \textit{ex-ante} probability that prospective plaintiff \(i\) is not going in court at date 1 is

\[
 f_{i,o} = F\left(\left[cT_o\right];T_o, p\right),
\]

recalling that we defined \(T_o = m_0\). Since plaintiffs’ prior parameters are independent of each other at \(t = 0\), we can calculate the \textit{ex-ante lock-in probability} at \(t = 1\) as the joint probability that all the \(K\) prospective plaintiffs will not access the judiciary at \(t = 1\).

Therefore, the lock-in probability is 

\[ f_o = \left(f_{i,o}\right)^K \]

This leads to the following:
**Proposition 4.** Define the lock-in probability as the ex-ante probability that none of the prospective plaintiffs accesses the judiciary at date 1 to discuss a meritorious cause of action. Then, the lock-in probability is decreasing in the number of prospective plaintiffs, in the initial robustness, and in the legal merit, and is increasing in litigation costs.

Proposition 4 is proved by inspecting first of all \( f_{i,0} \), as defined above, that is unquestionably positive for \( [cm_0] > 0 \). *Ex-ante*, a cause of action, though meritorious, might remain away from the court because all prospective plaintiffs are discouraged by comparisons of expected success probability with litigation costs. In fact, given the definition \( f_{i,0} = F\left(\lfloor cT_0 \rfloor; T_0, p\right) \), this probability is increasing in \( c \) and is decreasing in \( p \); these are standard properties of the Binomial distribution. In addition, the smaller the number of prospective plaintiffs is, the higher the lock-in probability for meritorious claims: in fact, given \( f_{i,0} \in (0,1) \), \( (f_{i,0})^K \) is monotonically decreasing in \( K \). Finally, the smaller the initial robustness \( T_0 \equiv m_i \) of individual conjectures is, the higher the lock-in probability for meritorious access to justice tends to be (the proof for Fact 2 applies again here).

Proposition 4 seems to effectively grasp lock-in phenomena as those involving *systematically discouraged* potential plaintiffs (example \( c \) of Section 1) who forsake accessing the judiciary because legal actions are perceived as non-affordable options. As stated above, when people are fragile in their beliefs of being the party of a meritorious claim, the actions of a few individuals cannot urge an increase in access to justice. This is all the more true if several of the few who did commence action are accidentally given unfavourable decisions.

**4.4. Rectification of over-optimistic and over-pessimistic bias**

As seen in previous sections, precedent can be used by prospective plaintiffs to update their priors on possible trial outcomes. By consolidating precedent and case law, access to justice progressively allows for the rectification of priors.
In fact, over-optimistic biases, misunderstandings, and misrepresentations of case law may lead to excessive unmeritorious access to justice. On the other hand, consistent judicial decisions stating the ‘true’ legal merit of a given cause of action can progressively discourage unmeritorious court petitions. This corrective effect of precedent works more quickly and is stronger, depending upon the magnitude of the bias in conjecture, its robustness, and the degree of lack of merit.

Let us focus on an unmeritorious cause of action \( p < c \) that is brought to court at a generic date \( t \) by at least one plaintiff \( (m_t > 0) \). Call \( k \) the most optimistic plaintiff in the set of those plaintiffs who are ‘optimistic enough’ to access the judiciary at \( t \). That is, \( k \)'s prior concerning the probability of success in court at \( t \) is such that \( E_{k,t} \geq E_{j,t} > c, \forall j \in O_t \), where \( O_t \) is the set of cardinality \( m_t \) including all optimistic plaintiffs.

We can now derive the expected number \( \mu \) of consistent judicial decisions that are necessary in order to correct every optimistic bias, including that of \( k \). Prospective plaintiff \( k \) will not access further the judiciary, if and only if his or her posterior expectation becomes smaller than or equal to the litigation cost (i.e. if \( E_{k,t+1} (p) \leq c \)). Starting from expression (8), the latter condition can be rewritten as:

\[
E_{k,t+1} (p) = E_{k,t} (p) \frac{P_{k,t-1}}{P_{k,t-1} + m_t} + \frac{y_t}{m_t} \frac{m_t}{P_{k,t-1} + m_t} \leq c,
\]

where \( y_t \) and \( m_t \) are, respectively, the number of favourable decisions and the number of overall judicial decisions that are actually taken in court at date \( t \). \( P_{k,t-1} \) is \( k \)'s prior robustness at date \( t \).

Since we wish to calculate the expected number \( \mu \) of consistent judicial decisions needed to correct \( k \)'s over-optimism, in (11) we must use the expected frequency \( y_t/m_t \) of favourable
decisions; that, of course, is \( p \). Hence, \( \mu \) can be calculated by solving

\[
E_{k,t}(p) \frac{P_{k,t-1}}{P_{k,t-1} + \mu} + p \frac{\mu}{P_{k,t-1} + \mu} = c.
\]

This leads to

\[
\mu = P_{k,t-1} \frac{E_{k,t}(p) - c}{c - p} \tag{12}
\]

Equation (12) shows that \( \mu \) is increasing in both the magnitude \( E_{k,t}(p) - c \) and the robustness of \( k \)'s overoptimistic subjective bias (prior robustness \( P_{k,t-1} \)), while \( \mu \) is decreasing in the degree of lack of merit \( c - p \). All this sounds quite reasonable: in particular, observe that a higher prior robustness makes the correction of over-optimism more difficult.

Notice that a similar reasoning can be made in the reverse case, namely, when deriving the expected number of consistent judicial decisions that have to occur at a given date \( t \) so that all the prospective plaintiffs, even the most pessimistic one, can access the judiciary to solve a meritorious claim. However, in this circumstance, we must assume that the pessimism is not so strong, nor so widespread, that \( \text{nobody} \) accesses justice; otherwise, we would fall back into the lock-in case of Proposition 4.

We can summarize the above argument in the following Proposition:

**Proposition 5.** Suppose that access to justice is non-null at date \( t \) \( (m_t > 0) \). Then, the expected number of accesses to justice that are necessary to have no prospective plaintiff (resp. all prospective plaintiffs) accessing the judiciary for an unmeritorious (resp. meritorious) cause of action is increasing (resp. decreasing) in the degree of merit \( p - c \), and is increasing in magnitude and robustness of the most optimistic (resp. pessimistic) bias.

The above considerations can be used to derive an additional Corollary (Proof in Appendix) concerning the expected time \( \tau \) that is needed to derive no access to justice for unmeritorious claims (or, analogously, to have full access to justice for meritorious claims).
**Corollary 2.** Starting from date \( t \), the expected time \( \tau \) that is needed to derive no prospective plaintiff (resp. all the prospective plaintiffs) accessing the judiciary for an unmeritorious (resp. meritorious) cause of action is bounded from below (resp. from above). The lower-bound (resp. upper-bound) of \( \tau \) is increasing (resp. decreasing) in the degree of merit \( p - c \), is increasing in the magnitude and robustness of the most optimistic (resp. pessimistic) bias, and is decreasing in the number of plaintiffs accessing justice at date \( t \).

Both Proposition 5 and Corollary 2 show how judicial decisions following access to justice can allow for bias rectification over time. This virtuous effect of precedent can lead to the disappearance from the courts of unmeritorious causes of actions and an increase in meritorious access to justice.

However, this dynamic process takes time and potentially several attempts in court, especially when there is a wide and strong bias inherent in the priors. The more widespread and deep-rooted the subjective biases, the harsher the suitable path of access to justice. Additionally, inconsistency of judicial decisions and prior heterogeneity may prevent a quick rectification of biases. In fact, a high number of favourable (unfavourable) random decisions on an unmeritorious (meritorious) cause of action can postpone bias rectification.

Our model appears to explain the divergent paths of hot-coffee cases in the United Kingdom and in the United States, described in our example \( d \) of Section 1 (provided that we can assume them to be comparable); in the United States a combination of staunch over-optimism and some favourable decisions have apparently fed further cases, to discuss a cause of action that in fact has had scarce favour in the United Kingdom.\(^\text{19}\)

A further implication is that, since access to justice is a necessary condition to set precedent that nourishes the learning process of prospective plaintiffs, the flourishing of meritorious access to justice and the disappearance of unmeritorious petitions to court are not perfectly symmetric. In fact, in the case of meritorious causes of action, rectification implies wider and
wider access to justice, which allows for even faster rectification. Conversely, when consistent decisions induce the rectification of priors for a group of optimistic plaintiffs – who then give up the cause of action – the residual over-optimistic plaintiffs will observe a more and more limited set of new judicial decisions, and this will weaken the correction of biases. Therefore, in the latter case, rectification slows down. Finally, the eradication of frivolous causes of action from the courts may be particularly difficult when residual over-optimistic plaintiffs are very staunch to their beliefs (i.e. are endowed with high prior robustness).

5. Conclusions and possible extensions

The uncertainty of judicial decisions and heterogeneity in conjectures certainly pose efficiency issues, since they substantially affect the decisions of prospective plaintiffs in accessing the judiciary or instead giving up. The system of legal remedies largely rests on the fact that potential plaintiffs commence lawsuits to solve their justiciable problems. If some people with meritorious causes of action give up because of a lack of right-awareness, pessimism, or even excessive court fees, legal enforcement may be insufficient and, finally, deterrence would be diluted. On the other hand, when people access the judiciary in good faith but with unmeritorious or improper claims – because either they are excessively optimist or they have a systematic bias towards a favourable outcome – relevant inefficiencies and wasteful burdens emerge for the judiciary. Case law and precedent can be decisive, since prospective plaintiffs (and their lawyers) can learn from cases that were previously discussed in the courts.

By extending the literature on innovation diffusion, the present study illustrates how, thanks to the learning of precedent, beliefs and consequently access to justice evolve over time. In particular, the higher the robustness of priors is, the higher the stickiness of the updating process. Conversely, the higher the number of cases accessing the court, the higher the weight of new judicial decisions in orienting new expectations correctly. Although the capacity of precedents to be informative decreases over time, case law drives a virtuous process of belief
updating that is potentially able to lead to the disappearance of unmeritorious causes of action from courts and the flourishing of meritorious access to justice.

Besides this suitable rectification effect of precedent, this paper also shows some possible limitations to this desirable path of access to justice. First, widespread and deep-rooted bias in priors and inconsistency of judicial decisions may prevent a quick correction of expectations. Second, since access to justice is a necessary condition to set precedent that nourishes the learning process, the lock-in phenomena for meritorious causes of action can significantly prevent the virtuous effects of precedent. In fact, when all the prospective plaintiffs of a new cause of action surrender, either because of the costs of accessing the judiciary or because they are over-pessimistic or insufficiently aware of their entitlements, the entire process of access–precedent–update can even fail to start.

Further problems may arise when very over-optimistic prospective plaintiffs are involved. Actually, although consistent decisions on frivolous cases certainly favour a prompt rectification of priors, unfavourable judgements might be unable to discourage residual over-optimistic plaintiffs who will observe a more and more limited case law that corrects their biases. This could explain why eradicating frivolous causes of action from court may be particularly difficult when over-optimistic plaintiffs (and lawyers) continue to be loyal to their biased beliefs.

Although possible extensions might be encompassed in the model, particularly by including the possibility of settlement for some of the prospective plaintiffs, the results seem to be quite robust. In fact, settlement may partially change the dynamics described above (i.e. rectification slows down because fewer cases are decided in the courts), however, without jeopardizing the main insights. In particular, a remark is in order: prior robustness might be seen also as a measure of precision (see footnote 10) and – as seen in Subsection 3.2 – it keeps growing with time. From the perspective of dispute selection for trial, thus admitting the possibility of settlement, learning would imply more room for settlement (both for meritorious and
unmeritorious causes of action). In turn, more settlement – provided a certain access to justice – would result in fewer judicial decisions and then in a slower learning process, as already suggested by Dari-Mattiacci et al. (2007).

Finally, from a policy perspective, this study sheds new light on access to justice as a dynamic process that is able to promote its own efficiency, provided prospective plaintiffs have sufficient primary access to justice and strong biases in initial priors are not too deeply rooted and widespread. Legal scholars, lawyers, and judges may positively contribute to the dynamics of access–precedent–update by guaranteeing consistent doctrine, well-informed legal advice, and fair judicial decisions.
Appendix

Proof of Fact 2

As regards part \((b)\) of Fact 2, it is a well-known fact that, as \(m_0\) goes to infinity, 
\[
1 - g_s = F\left(\left\lfloor \frac{c}{m_0} \right\rfloor; m_0, p\right)
\]
converges to 1 (resp. 0; resp. 1/2) if \(c > p\) (resp. \(p < c\); resp. \(p = c\)),
and it can be shown that the convergence rate increases with \(|c - p|\) in the inequality cases.

However, we can say more than this. We now study \(F\left(\left\lfloor \frac{c}{m} \right\rfloor; m, p\right)\) as \(m_0\) increases, or, more in general, \(F\left(\left\lfloor \frac{c}{m} \right\rfloor; m, p\right)\) as \(m\) increases. Notice in fact that we are going to study this problem for any \(m\), not only for \(m_0\), since Fact 2 is used subsequently in the text for any \(m_0, t \geq 0\) - see proof of Corollary 1.

Consider the following Lemma.

Lemma. Let \(F(k;m, p)\) be the CDF of the Binomial distribution with parameters \(m\) and \(p\); let \(H(x, p) = x \ln \left(\frac{x}{p}\right) + (1 - x) \ln \left(\frac{1 - x}{1 - p}\right), 0 < x < 1;\) and let \(\Phi(y)\) be the CDF of a standard normal variable with argument \(y\). Then, besides \(F(0; m, p) = (1 - p)^m\) and \(F(m; m, p) = 1,\) one has for \(1 \leq k < m - 1\) (\(k\) integer):

\[
\Phi\left(\text{sign}\left(\frac{k}{m} - p\right)\right) < F\left(\left\lfloor \frac{k}{m} \right\rfloor; m, p\right) < \Phi\left(\text{sign}\left(\frac{k + 1}{m} - p\right)\right)
\]

\[\text{(a.1)}\]

\[
F\left(m - 1; m, p\right) > \Phi\left(\text{sign}\left(\frac{m - 1}{m} - p\right)\right) \cdot \sqrt{2mH\left(\frac{k - m - 1}{m}, p\right)}
\]

\[\text{(a.2)}\]

Proof: See Serov and Zubkov (2013) who proved this Lemma as their main Theorem.

The Lemma, and in particular its part (a.1), holds for \textit{integer} values of \(k\). In our case, the role of \(k\) is played by \(\left\lfloor \frac{c}{m} \right\rfloor\); hence, it might be difficult to study the behaviour of the bounds appearing in (a.1) as \(m\) increases, due to the fact that \(\left\lfloor \frac{c}{m} \right\rfloor\) undergoes jumps when it reaches the next integer (of course, this would not be a problem in the very special case \(\left\lfloor \frac{c}{m} \right\rfloor = c m\)).

Notice, however, that \(\text{sign}\left(x - p\right)\cdot \sqrt{2mH\left(x, p\right)}\) is increasing in \(x\). In fact, one sees that \(H(x, p)\) is the Kullback-Leibler divergence between two Bernoulli variables with respective
probabilities of success $x$ and $p$: one has $H(x, p) > 0$ for $x \neq p$, and $H(x, p) = 0$ for $x = p$. It is not difficult to see that $H(x, p)$ increases (resp. decreases) in $x$ if $\text{sign}(x - p) > 0$ (resp. $\text{sign}(x - p) < 0$). In addition, $\text{sign}(x_1 - p) \cdot \sqrt{2mH(x_1, p)} < \text{sign}(x_2 - p) \cdot \sqrt{2mH(x_2, p)}$ if $\text{sign}(x_1 - p) < \text{sign}(x_2 - p)$. Therefore, $\text{sign}(x - p) \cdot \sqrt{2mH(x, p)}$ is always increasing in $x$ for given $m$ and $p$.

Observing that $cm - 1 \leq \lfloor cm \rfloor \leq cm$, and given the properties of $\Phi$, we can thus replace the inequalities appearing in expression (a.1) of the Lemma with the following inequalities (a.3):

$$
\Phi \left( \text{sign} \left( c - \frac{1}{m} - p \right), \sqrt{2mH \left( c - \frac{1}{m}, p \right)} \right) < F \left( \lfloor cm \rfloor; m, p \right) < \Phi \left( \text{sign} \left( c + \frac{1}{m} - p \right), \sqrt{2mH \left( c + \frac{1}{m}, p \right)} \right)
$$

(a.3)

Given the definition of $H$, the bounds appearing in (a.3) are defined for $c - \frac{1}{m} > 0$ and $1 - c - \frac{1}{m} > 0$, so one requires $m > \max \left\{ \frac{1}{c}; \frac{1}{1 - c} \right\}$. We wish to study how those bounds behave as $m$ increases. We thus consider the derivatives of the bounds, in particular of $\gamma H \left( c \pm 1/y, p \right)$, with respect to $y$, where $y \in \mathbb{R}^+$ substitutes the positive integer $m$: the alternative “$\pm$” is related to the upper/lower bound. After some algebra, it turns out that

$$
\frac{d}{dy} \left( \gamma H \left( c \pm 1/y, p \right) \right) = c \ln \left( \frac{c \pm 1/y}{p} \right) + (1 - c) \ln \left( \frac{1 - c \mp 1/y}{1 - p} \right)
$$

(a.4)

One can check (at least via numerical simulation) that the following properties hold:

- **If** $c < p$ (resp. $p < c$), the derivative (a.4) of the upper (resp. lower) bound appearing in (a.3) is negative if $y < 1/(p - c)$ (resp. $y < 1/(c - p)$), and is positive if $y > 1/(p - c)$ (resp. $y > 1/(c - p)$).

- **If** $c < p$ (resp. $p < c$), the derivative (a.4) of the lower (resp. upper) bound appearing in (a.3) is negative if $y < y^- \approx \frac{1}{p - c} - 6 \left( c - \frac{1}{2} \right)$ (resp. $y < y^+ \approx \frac{1}{c - p} + 6 \left( c - \frac{1}{2} \right)$), and is positive if $y > y^-$ (resp. $y > y^+$).

- **If** $c = p$, the derivatives (a.4) of the lower and upper bounds appearing in (a.3) are always negative.
Substituting back $y \in \mathbb{R}^+$ with the positive integer $m$, one observes that:

- If $c < p$ (resp. $p < c$), $\text{sign} \left( c - \frac{1}{m} - p \right) < 0$ (resp. $\text{sign} \left( c + \frac{1}{m} - p \right) > 0$) for any $m$, while
  $$\text{sign} \left( c + \frac{1}{m} - p \right) < 0 \text{ for } m > \frac{1}{p - c} \quad \text{(resp. } \text{sign} \left( c - \frac{1}{m} - p \right) < 0 \text{ for } m < \frac{1}{p - c})$$

- If $c = p$, $\text{sign} \left( c + 1/m - p \right) > 0$ for any $m$, and $\text{sign} \left( c - 1/m - p \right) < 0$ for any $m$.

Collecting all the above material, after careful inspection one obtains that:

- For $c < p$, $F \left( \lfloor cm \rfloor; m, p \right)$ is bounded from above by a decreasing function of $m$, while it is bounded from below by a function of $m$ that is decreasing for $m > \frac{1}{p - c} - 6 \left( c - \frac{1}{2} \right)$; both functions converge to zero.

- For $c > p$, $F \left( \lfloor cm \rfloor; m, p \right)$ is bounded from below by an increasing function of $m$, while it is bounded from above by a function of $m$ that is increasing for $m > \frac{1}{c - p} + 6 \left( c - \frac{1}{2} \right)$; both functions converge to one.

- For $c = p$, $F \left( \lfloor cm \rfloor; m, p \right)$ is bounded from below (above) by an increasing (decreasing) function of $m$; both functions converge to $1/2$.

Finally, recalling that $g = 1 - F \left( \lfloor cm \rfloor; m, p \right)$, Fact 2 follows.

**Remark.** The requirements $m > \max \left\{ \frac{1}{c}, \frac{1}{1 - c} \right\}$, and $m > \frac{1}{|c - p|} \pm 6 \left( c - \frac{1}{2} \right)$ in the case $c \neq p$, motivating the phrase “for $m$ large enough”, are not overly restrictive. For instance, under the reasonable assumptions $0.1 \leq c \leq 0.9$ and $|c - p| \geq 0.1$ one requires $m \geq 13$ in the worst case.

**Proof of Fact 3**

Define the variable $A_{i,t} \in \{0, 1\}$ such that $A_{i,t} = 1$ (resp. $A_{i,t} = 0$) if plaintiff $i$ does (resp. does not) access justice at date $t$. From our definitions it follows that $A_1 \sim B(1, g_0)$. In addition,
Assumption A implies that the different \( A_{i,t} \)s are i.i.d. Define now \( M_1 = \sum_{i=1}^{K} A_{i,t} \) as the number of accesses to justice at date 1. Then clearly \( M_1 \sim B(K, g_0) \).

**Proof of Fact 4**

We prove only the first part, since the second one runs in a similar way. Recall that if \( X \) is a random variable \( X \sim B(n, q) \) and, conditional on \( X, Y \sim B(X, v) \), then \( Y \sim B(n, qv) \). Now, by Fact 3, \( M_1 \sim B(K, g_0) \). On the other hand, from expression (1) of the text we derive that, conditional on \( M_1, Y_1 \sim B(M, p) \). Hence \( Y_1 \sim B(K, pg_0) \).

**Proof of Fact 6**

In order to derive the distribution function of \( \alpha_{i,1} \) (and \( \beta_{i,1} \)) we resort to the approximation of Binomial distributions by Normal distributions and to the sum of normally distributed independent random variables (Mood et al., 1963).

**Approximation.** If \( X \sim B(n, q) \), and in addition \( n \) is sufficiently large and \( q \) is not close to 1 or 0, then the Binomial distribution can be well approximated by a Normal such that \( X \sim N(nq, nq(1-q)) \). The parameter restrictions to make possible a good approximation consist in a sufficient number of trials and a probability of success not to close to the extremes. Therefore, given that \( \alpha_{i,1} = \alpha_{i,0} + Y_1 \) where \( \alpha_{i,0} \sim B(m_0, p) \) and \( Y_1 \sim B(K, pg_0) \), we approximate the two independent Binomials as \( \alpha_{i,0} \sim N(m_0p, m_0p(1-p)) \) and \( Y_1 \sim N(Kpg_0, Kpg_0(1-pg_0)) \). As \( \alpha_{i,1} \) is the sum of two independent Normal variables, we have \( \alpha_{i,1} \sim N(m_0p + Kpg_0, m_0p(1-p) + Kpg_0(1-pg_0)) \).

Now, by solving some analytical steps, we can observe that the Normal distribution of \( \alpha_{i,1} \) can be easily approximated again by a Binomial distributions so that \( \alpha_{i,1} \sim B(m_0 + Kg_0, p) \). In particular,

\[
N(m_0p + Kpg_0, m_0p(1-p) + Kpg_0(1-pg_0)) = N(m_0p + Kpg_0, m_0p(1-p) + Kpg_0(1-p + p - pg_0)) = N(p(m_0 + Kg_0)(m_0 + Kg_0)p(1-p) + Kg_0^2(1-g_0))
\]

As we show below, the second term of the variance is negligible with respect to the first term. Thus, \( \alpha_{i,1} \sim N(p(m_0 + Kg_0), (m_0 + Kg_0)(1-p)) \), that can be approximated back by a Binomial distribution, so that: \( \alpha_{i,1} \sim B(m_0 + Kg_0, p) \).
About the negligibility of the second term of the variance with respect to the first one, note that their ratio is:

\[
\frac{Kp^2 g_0 (1 - g_0)}{(m_0 + Kg_0) p(1 - p)} = \frac{Kpg_0 (1 - g_0)}{(m_0 + Kg_0)(1 - p)} - \frac{m_0 p (1 - g_0)}{(m_0 + Kg_0)(1 - p)}
\]

For \(g_0 = 0\) or \(g_0 = 1\), the ratio is zero and the approximation is perfect. Moreover, the ratio tends to zero (and the approximation betters) as long as \(g_0\) tends either to zero or to one, or \(m_0\) increases.

Following analogous reasoning the approximation of the distribution function of \(\beta_{i,1}\) can be derived (steps available upon request).

**Quality of the Normal approximation.** According to Box et al. (1978), the approximation of a Binomial by a Normal distribution — i.e. \(X \sim B(n, q) \leftrightarrow X \sim N(nq, nq(1 - q))\) — generally improves as \(n\) increases (at least 20) and is better when \(q\) is not near to 0 or 1. A possible rule to select parameter values able to generate a good approximation is that both \(nq\) and \(n(1-q)\) are greater than 5 (a more conservative rule fixes the threshold to 10). Let us provide some examples to have a good approximation. Concerning distribution approximations of \(\alpha_{i,1}\) and \(\beta_{i,1}\), assuming \(m_0 \geq 50\) is enough to work with \(p\) ranging from 0.1 to 0.9. Concerning \(Y_1\) and \(N_1\), assuming for instance \(K \geq 100\) is enough to work with \(p_{g_{0}}\) ranging from 0.05 to 0.95, etc.

Note that, given the above argument, the Binomial-Normal approximation might be very unsatisfactory for \(g_0 = 0\). However, this is not a problem since the approximation is necessary only to model the update process of parameters alphas and betas. Without any access to justice (\(g_0 = 0\)), no update occurs. This is true at every date \(t>1\) and for every \(g_t\).

Note that, assuming a sufficiently high \(m_0\) sweeps away any concern about the quality of the approximation that are necessary to derive the distribution functions of alphas and betas for all the next periods. In fact, the number of trials in the distributions functions of alphas and betas are always greater or equal to \(m_0\).
Proof of Corollary 1

The proof of Corollary 1 follows directly from the analyses offered to arrive at Proposition 2. Notice in fact that the series \( \{T_{t+1}, T_{t+2}, \ldots, T_{t+k} \} \) is strictly increasing if \( \left[ K_{S_{t+k-1}} \right] \geq 1 \), and non-decreasing if \( \left[ K_{S_{t+k-1}} \right] = 0 \); hence we can use Fact 2, provided that we apply it to the general case \( t \geq 1 \). The condition \( \left[ K_{S_{t+k-1}} \right] \geq 1 \) involves the fact that at least one case is expected to access justice at \( t \), allowing priors’ updating for the next period \( t+1 \). Notice however that, according to Fact 2 itself, the non-meritorious case \( p < c \) implies that \( g_t \) tends to be decreasing in \( T_t \). As a consequence, a time \( t^* \) will be reached such that \( \left[ K_{g_{t^*}} \right] = 0 \); at that point we shall have \( T_{t^*+1} = T_{t^*} \), which implies \( g_{t^*+1} = g_{t^*} \), and hence \( \left[ K_{g_{t^*+1}} \right] = 0 \). At that point non further access to justice will be experienced, and that cause of action will disappear from courts.

Proof of Corollary 2

Corollary 2 can be easily derived starting from expression (12), and observing that if the number \( m_t \) stays constant from \( t \) onwards, the expected time that we are currently discussing is simply measured by \( \tau = \mu / m \). Hence, for given \( m_t \), \( \tau \) inherits all the properties of \( \mu \), and in addition it is decreasing in \( m_t \). However \( m_t \) does not stay constant in time: in the non-meritorious case with over-optimistic plaintiffs, \( m_t \) is expected to decrease in time after \( t \), since more and more plaintiffs will become less optimistic. On the contrary, in the meritorious case with over-pessimistic plaintiffs, \( m_t \) is expected to increase in time. Hence, in the former case we have \( \tau < \mu / m \), while in the latter case we have \( \tau > \mu / m \).
As will be made clear in Section 2, the only assumption that we really need to make is that some pieces of information concerning the legal merit of a case percolate across society as a consequence of each legal action commenced by a plaintiff. Examples of settings where, in fact, settlement is not an option are represented by cases involving passive defendants, or – in some countries – non-disposable rights. Finally, it is important to note that the literature indicates that several legal actions are terminated neither by litigation nor by settlement, but rather by dismissal/termination by the court. See, among others, Kritzer (1986).

In the considerable body of literature to model litigation, numerous authors highlight that plaintiffs usually require positive expected outcomes from trial in order to commence a legal action (Shavell 1982; Nalebuff 1987; Hay and Spier 1998; Spier 2007; Daughety and Reinganum 2012; Saraceno 2014). Both the capability and the commitment of the plaintiff in accessing the judiciary are *conditio sine qua non* to have either litigation or settlement. Additionally, as proved by Schwartz and Wickelgren (2009), when a plaintiff cannot access the judiciary, cases are typically neither litigated nor settled.

For an overview of the topic, see Hathaway (2001). On the complexity of the legal system and on rulemaking as a dynamic process, see Allen (2013).

See also Blume and Rubinfeld (1982).

The role of precedent in litigation has been accounted for in static models that involve homogenous multiple parties (one defendant against many homogeneous plaintiffs, or vice versa), both in sequential litigation (Che and Yi 1993; Briggs et al. 1996; Yang 1996; Spier 2002; Kim 2004; Daughety and Reinganum 2011) and in joint/collective litigation (Che 2002; Che and Spier 2008). In fact, in these asymmetric information models, precedent has strategic value, because the parties can learn some private information concerning the type of counterparty. Wickelgren (2012) for example considers the effect of sequential litigation on settlement when multiple heterogeneous plaintiffs with positively correlated probabilities of success are involved.

Galanter (1983) describes courts as teachers that can to convey (partial) judicial information to the actors involved in the judicial system.
In common-law systems, statutes of limitations set the maximum time after an event that legal proceedings based on that event may be commenced. Similarly, civil-law systems define periods of prescription. The length of the statute of limitation/prescription varies both across countries and across the areas of law (torts, personal injuries, contracts, etc.).

On the debate about meritorious vs. frivolous cases, on negative-expected-value vs. negative-expected-value lawsuits, and finally on the welfare effects of litigation, see Shavell (1982), Nalebuff (1987, footnote 3), Spier (2007), and Saraceno (2014).

On the Beta family as a conjugate family for samples from Binomial distributions, see e.g. De Groot (1970). Dari-Mattiacci (2007) assumes trial outcome beliefs are drawn from a Beta distribution, because they are unimodal and characterized by the single-crossing property. Here we provide further justifications for this assumption.

This assumption corresponds to the positive-expected-value hypothesis that is typical in the litigation literature.

Consider a generic Beta distribution of a stochastic variable \( y \sim Beta(a, b) \); the mean of \( y \) is \( \mu = \frac{a}{a+b} \).

See Ferrari and Cribari-Neto (2004). With reference to footnote 10, for fixed \( \mu \), the larger \( a+b \), the smaller the variance of \( y \); and precision – as per Ferrari and Cribari-Neto, for example – or robustness is inversely measured by the variance. Precision of the individual subjective prior is the term typically used in Bayesian statistics. However, we opt for robustness both because it is more meaningful for the implications of this study and also to avoid a deceptive overlapping with the precision of law – a term that in the literature usually influences the intra-population variance of the information distribution (Dari-Mattiacci et al. 2011).

A numerical example helps clarify the point. We could have different types of prospective plaintiffs: the staunch optimists, characterized by \( \alpha = 6, \beta = 2 \); the doubtful optimists, characterized by \( \alpha = 3, \beta = 1 \); and the staunch pessimists, characterized by \( \alpha = 2, \beta = 6 \). It is easy to check whether each optimist has exactly the same expectation about \( p (3/4) \), but with a different degree of robustness (8 vs. 4), and whether staunch plaintiffs have different expectations concerning \( p (3/4 \text{ vs. } 1/4) \), but same robustness (8).
This notion is related to that of the ‘improper prior’: see De Groot (1970), chap. 10. Some authors propose Haldane’s ‘Beta(0,0)’ distribution as a convenient and uninformative prior distribution. This completely aligns with our current argument.

See Fact 1 below, and recall the ‘Beta(0,0)’ assumption, mentioned in footnote 13, about the ‘aboriginal prior’.

With reference to the example in footnote 11, we could compare two possible emerging causes of action: one very innovative, and not deeply rooted in doctrine and jurisprudence; and a second both strongly related to case law and very similar to previous existing causes of action. Prospective plaintiffs – heterogeneous in their beliefs – will be characterized by lower prior robustness in the first case, and a higher one in the second case.

See De Groot (1970), chap. 9, particularly pp. 159–161.

Graham (2008:405) suggests that new cause of actions may suffer from the ‘novelty paradox [which] suggest(s) how a fledgling tort can be especially vulnerable to attack, particularly if the tort is not perceived as the organic product of well-established tort principles’.

Once again, debating the merit of hot-coffee cases is beyond the scope of this paper. This example was selected because it amply illustrates the interactions among precedent, beliefs, and subsequent legal actions.
References


