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A CRITICAL SHOCK MACH NUMBER FOR PARTICLE ACCELERATION IN THE ABSENCE OF PRE-EXISTING COSMIC RAYS: $M = \sqrt{5}$

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ABSTRACT

It is shown that, under some generic assumptions, shocks cannot accelerate particles unless the overall shock Mach number exceeds a critical value $M > \sqrt{5}$. The reason is that for $M \leqslant \sqrt{5}$ the work done to compress the flow in a particle precursor requires more enthalpy flux than the system can sustain. This lower limit applies to situations without significant magnetic field pressure. In case that the magnetic field pressure dominates the pressure in the unshocked medium, i.e., for low plasma beta, the resistivity of the magnetic field makes it even more difficult to fulfill the energetic requirements for the formation of shock with an accelerated particle precursor and associated compression of the upstream plasma. We illustrate the effects of magnetic fields for the extreme situation of a purely perpendicular magnetic field configuration with plasma beta $\beta = 0$, which gives a minimum Mach number of $M = 5/2$. The situation becomes more complex, if we incorporate the effects of pre-existing cosmic rays, indicating that the additional degree of freedom allows for less strict Mach number limits on acceleration. We discuss the implications of this result for low Mach number shock acceleration as found in solar system shocks, and shocks in clusters of galaxies.

Key words: acceleration of particles – galaxies: clusters: intracluster medium – shock waves – Sun: coronal mass ejections (CMEs) – Sun: particle emission

Online-only material: color figures

1. INTRODUCTION

Collisionless shock waves occur in a wide variety of astrophysical settings, and involve a wide variety of length and energy scales. Examples are, on the scales of the solar system, the Earth' bow shock, and the solar wind termination shock; on parsec scales, supernova remnants shocks; and on megaparsec scales, the shocks in clusters of galaxies.

In many cases collisionless shocks are associated with particle acceleration. It is, for example, generally thought that the origin of Galactic cosmic rays, with proton energies up to 3×10^{15} eV, are high-Mach-number supernova remnant shocks (Helder et al. 2012), whereas the ultra-high energy cosmic rays, up to 10^{20} eV, are usually associated with relativistic shock waves caused by active galactic nuclei, or gamma-ray bursts (Kotera & Olinto 2011).

Low Mach number shocks are also associated with particle acceleration, but not always. For example, some shocks driven by coronal mass ejections (CMEs), which have magnetosonic Mach numbers $M_{\rm ms} \lesssim 4$, are accompanied by Type II radio burst (e.g., Gopalswamy et al. 2010), whereas others are not. Type II radio bursts are often considered a sign for particle acceleration. The solar wind termination shock has a similarly low Mach number, of around 2.5 (Lee et al. 2009), and is associated with particle acceleration (e.g., Florinski et al. 2009). On a much larger scale, some shocks in clusters of galaxies result in so-called radio relics, elongated structures that emit radio synchrotron emission (e.g., van Weeren et al. 2010). But not all cluster shocks identified in X-rays appear to be accompanied by radio emission. The typical shock velocities in clusters of galaxies are of the order of a few 1000 km s^{-1}. But due to the high temperatures, and hence high sounds speeds, of the plasma in which the shocks propagate, the Mach numbers are modest, with $M_{\rm ms} \lesssim 3$ (Markevitch & Vikhlinin 2007).

In many cases particle acceleration by shocks is attributed to diffusive shock acceleration (Malkov & Drury 2001, for a review). According to the diffusive shock acceleration theory, elastic scattering of energetic, charged particles on both sides of the shock causes particles to cross the shock front repeatedly. Each shock crossing results in an average increase in momentum of order $\Delta p/p \sim V_{\rm s}/c$, with $V_{\rm s}$ the shock velocity, and c the speed of light. The scattering of the particles is caused by magnetic field fluctuations/plasma waves. The interaction of these particles with the magnetic field fluctuations causes the accelerated particles to exert a pressure on the upstream plasma (i.e., the unshocked medium), which results in the formation of a shock precursor that compresses and slows down the plasma before it enters the actual shock (which is labeled subshock, in order to distinguish it from the total shock structure). This back-reaction of the shock-accelerated particles on the plasma flow has been observed in situ at the solar termination shock, as measured by *Voyager 2* (Florinski et al. 2009).

The purpose of this paper is to show that particle acceleration, under general assumptions, requires a minimum Mach number of $M = \sqrt{5}$, and somewhat higher if magnetic fields are dynamically important (i.e., for low plasma betas, with $\beta \equiv 8\pi n k_{\rm B} T/B^2 < 1$).

Note that the critical Mach number discussed here is distinct from the so-called first critical Mach number, $M_{\rm c}$, which is often mentioned in the literature on collisionless shocks (Marshall 1955; Edmiston & Kennel 1984; Treumann 2009). The first critical Mach number concerns the details of the shock formation process itself in the presence of magnetic fields. The magnetic pressure component prevents shocks with Mach numbers lower

than the critical Mach number to heat the post-shock plasma to temperature where the flow-speed is subsonic. Similar critical Mach numbers exist for shocks moving through a medium with pre-existing cosmic rays (Becker & Kazanas 2001).

The critical Mach number discussed in this paper concerns the overall thermodynamic properties of shocks with a precursor of accelerated particles. In order to explain it, we draw upon the two-fluid model of Vink et al. (2010). In this paper it was already noted that particle acceleration seemed impossible for low Mach numbers, but the exact Mach number was not given. In addition, we derive here the critical Mach number for acceleration for perpendicular shocks with $\beta = 0$, and discuss the more peculiar case when there are pre-existing cosmic rays.

2. A MINIMUM MACH NUMBER FOR DIFFUSIVE SHOCK ACCELERATION

2.1. The Rankine–Hugoniot Relations Extended with a Cosmic-Ray Component

Shock jump conditions are governed by the so-called Rankine–Hugoniot relations (e.g., Zel'dovich & Raizer 1966; Tidman & Krall 1971), which describe the state of the media on both sides of the shock, based on the equation of state and the conservation of mass-, momentum, and energy-flux. These equations assume, therefore, steady state conditions.

Nonlinear particle acceleration (Malkov & Drury 2001), however, may change shock-jump conditions in astrophysical shocks, as the pressure of particles in the shock precursor compresses the plasma flowing into the shock, and because the highest energy particles may escape the shock region. The escape of the highest energy particles does hardly affect mass- and momentum-flux conservation across the whole shock region, since only a very small fraction of the particles escape, but it does violate energy-flux conservation, as the escaping particles are typically particles that have gained considerable energy (Berezhko & Ellison 1999). Some of the physics of nonlinear particle acceleration can be captured by treating the accelerated particles as a separate component, which is referred to as a two-fluid model (e.g., Drury & Voelk 1981). The accelerated particles contribute to the pressure on both sides of the subshock. Since the length scale associated with the subshock is small compared to gradient over which the accelerated particle pressure changes, the accelerated particles do not change the properties of the subshock directly, as the pressures of the accelerated particles just upstream and downstream of the shock are equal. However, the pressure of the accelerated particles upstream of the subshock results in a compression and slowing down of the plasma flowing into the subshock. As a result the Mach number just upstream of the subshock is smaller than the overall Mach number as measured far upstream.

Vink et al. (2010) showed that one can incorporate an accelerated particle (cosmic-ray) component in the Rankine–Hugoniot relations by evaluating the Rankine–Hugoniot relations in three distinct regions: (0) the (undisturbed) far upstream medium, (1) in the shock precursor, just upstream of the subshock, and (2) downstream of the subshock. The solutions allow for energy to escape from the system, which in kinetic models for cosmic-ray acceleration is either a result of having particles remove once they reach a certain maximum momentum (e.g., Blasi et al. 2005), or by imposing a maximum length scale to which particles are allowed to diffuse upstream (Reville et al. 2009).

In Appendix A the results of the extended Rankine–Hugoniot relations of Vink et al. (2010) are summarized and extended by allowing also for pre-existing cosmic-rays. The input parameters of the extended Rankine–Hugoniot relations are the upstream gas Mach number ($M_{\rm g,0}$) and the fractional pressure upstream in cosmic rays, $w_0 = P_{\rm cr,0}/P_{\rm tot}$ (Equation (A1)). For the cosmic-ray component one has to assume an adiabatic index, $4/3 \leqslant \gamma_{\rm cr} \leqslant 5/3$. The extended Rankine–Hugoniot relations give the downstream pressure contribution of cosmic rays, w_2 (Equation (A11)), as a function of the cosmic-ray precursor compression ratio, $\chi_{\rm prec}$ (Equation (A2)). Note that like more elaborate cosmic-ray acceleration models (e.g., Caprioli et al. 2010, for an overview), and the classical two-fluid models (Drury & Voelk 1981; Becker & Kazanas 2001), the extended Rankine–Hugoniot relations assume a steady state situation.

2.2. A Minimum Mach Number for Acceleration

The gas flowing into the subshocks behaves like a standard, classical shock, but due to compression in the cosmic-ray precursor, the subshock Mach number, $M_{\rm g,1}$, is lower than the upstream Mach number $M_{\rm g,0}$. The compression ratio at the subshock is given by Equation (A12) in Appendix A. Since the basic parameter of the extended Rankine–Hugoniot relation is the precursor compression ratio $\chi_{\rm prec}$ the total compression ratio for a cosmic-ray accelerating shock is

$$\chi_{\rm tot} = \chi_{\rm prec}\chi_{\rm sub} = \frac{(\gamma_{\rm g}+1)M_{\rm g,0}^2\chi_{\rm prec}^{-\gamma_{\rm g}}}{(\gamma_{\rm g}-1)M_{\rm g,0}^2\chi_{\rm prec}^{-(\gamma_{\rm g}+1)}+2}. \quad (1)$$

According to Equation (1) the total compression ratio can be larger than that allowed by standard shock jump relation[3] as long as Equation (A13) is obeyed, with $\epsilon > 0$ (see also Berezhko & Ellison 1999).

The maximum value for the compression ratio can be found by solving $d\chi_{\rm tot}/d\chi_{\rm prec} = 0$, with $\chi_{\rm tot}$ given by Equation (1). This shows that the maximum total compression ratio occurs for

$$\chi_{\rm prec} = \left(\frac{(\gamma_{\rm g}-1)}{2\gamma_{\rm g}}M_{\rm g,0}^2\right)^{1/(\gamma_{\rm g}+1)} = \left(\frac{1}{5}M_{\rm g,0}^2\right)^{3/8}, \quad (2)$$

with $\gamma_{\rm g} = 5/3$. By inserting Equation (2) in Equation (1) one finds the corresponding sub-shock compression ratio

$$\chi_{\rm sub} = \frac{\gamma_{\rm g}}{\gamma_{\rm g}-1} = \frac{5}{2}, \quad (3)$$

which, according to Equation (A12) corresponds to $M_{\rm g,1} = \sqrt{5}$.

This result was obtained by Vink et al. (2010), but an important aspect for shocks without pre-existing cosmic-rays (i.e., $w_0 = 0$) was not recognized: Equation (2) indicates that the solution becomes unphysical for $M_{\rm g,0} < \sqrt{5}$ as it requires a rarefaction instead of a compression in the cosmic-ray precursor ($\chi_{\rm prec} < 1$). So below $M_{\rm g,0} < \sqrt{5}$ the only allowed solution is one in which there is no cosmic-ray precursor, and for which the compression ratio is given by the standard Rankine–Hugoniot relations.

We refer to this critical Mach number as $M_{\rm acc}$, in order to distinguish it from the first critical Mach number, $M_{\rm c}$ (Edmiston & Kennel 1984), and the related critical Mach

[3] See Equation (A12), but in this case changing the subscript "sub" by "tot."

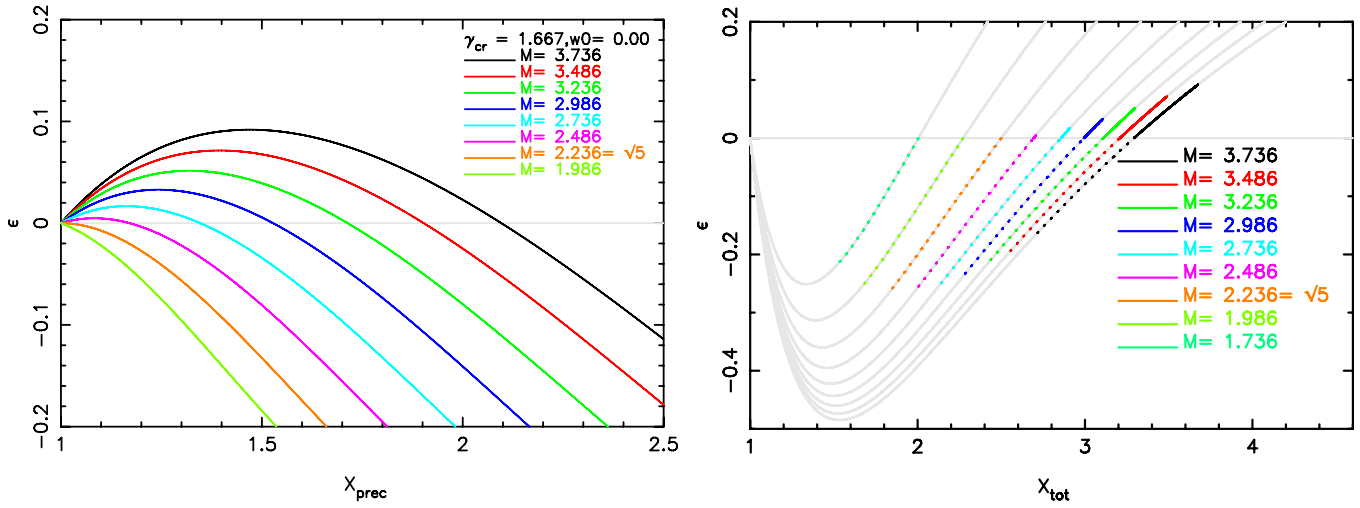

Figure 1. Left panel: the curves represent the solutions of the energy-flux escape parameter ϵ as a function of the precursor compression ratio $\chi_{\rm prec}$, for various values of the overall Mach number, with increments of $\Delta M_{\rm g,0} = 0.25$. The slopes at $\chi_{\rm prec} = 1$ are negative for $M_{\rm g,0} < \sqrt{5}$, resulting in negative values of ϵ, which is unphysical. For $M_{\rm g,0} > \sqrt{5}$ one does obtain physical solutions, but energy escape is required ($\epsilon > 0$). Right panel: the behavior of ϵ as a function of total shock compression $\chi_{\rm tot}$ (Equation (A15)) for the same Mach numbers as in the left panel. The total, light gray, curve shows a wide range of shock compression ratios, but only values $\epsilon \geqslant 0$ correspond to potentially physical solutions. The colored curves are solutions to the two-fluid model of Vink et al. (2010), with the unphysical solutions ($\chi_{\rm prec} < 1$) indicated with a dotted line. The highest values of $\chi_{\rm tot}$ of the colored lines correspond to the maximum compression ratios as given by Equation (1). The compression ratios with $\epsilon = 0$ correspond to the standard Rankine–Hugoniot solutions.

(A color version of this figure is available in the online journal.)

numbers investigated by Becker & Kazanas (2001). As we will describe below, for shocks moving through a magnetized medium (Section 2.4), or for a (partially) relativistic cosmic-ray population ($\gamma_{\rm cr} < 5/3$, Section 2.3) $M_{\rm acc} > \sqrt{5}$. However, as we will discuss in Section 2.5, a population of pre-existing cosmic rays, may result in cosmic-ray acceleration for values lower than $M_{\rm acc}$.

The maximum value for the energy flux escape, ϵ, is determined by solving $d\epsilon/d\chi_{\rm prec} = (d\epsilon/d\chi_{\rm tot})(d\chi_{\rm tot}/d\chi_{\rm prec}) = 0$. For $\gamma_{\rm cr} = 5/3$ this equation has two possible solutions. One corresponds to a minimum of ϵ, with $\epsilon < 0$. This minimum does not have a physical meaning. The other solution corresponds to $d\chi_{\rm tot}/d\chi_{\rm prec} = 0$, and is associated with a maximum value of ϵ, and hence with the maximum of $\chi_{\rm tot}$ (Equation (2)).

Figure 1 illustrates the properties of the energy flux equation for shocks with Mach numbers around $M_{\rm g,0} = \sqrt{5}$ and $\gamma_{\rm cr} = 5/3$, indicating that the accelerated particles are non-relativistic. The panel on the left shows that for $M_{\rm g,0} < \sqrt{5}$ and $\chi_{\rm prec} > 1$ one obtains $\epsilon < 0$, which is unphysical. A solution with $\epsilon = 0$ is always possible, and occurs for $\chi_{\rm prec} = 1$. This solution corresponds to the standard Rankine–Hugoniot relations.

The right-hand panel of Figure 1 shows the behavior of the energy escape (ϵ, Equation (A15)) as a function of total compression ratio. Note that this figure does not rely on the details of a two-fluid model, as only the total compression ratio is used, but an effective adiabatic index γ needs to be specified. The figure shows that higher compression ratios than the standard shock-jump conditions are allowed, but only if there is energy flux escape, i.e., $\epsilon > 0$. But in the context of a system with precursor compression and a subshock, there is a restriction on the total compression ratios that are possible, namely $\chi_{\rm prec} \geqslant 1$. As a consequence, physical solutions with higher compression ratios than the standard shock jump conditions are only possible for $M_{\rm g,0} > \sqrt{5}$. These physical solutions are indicated by solid colored lines.

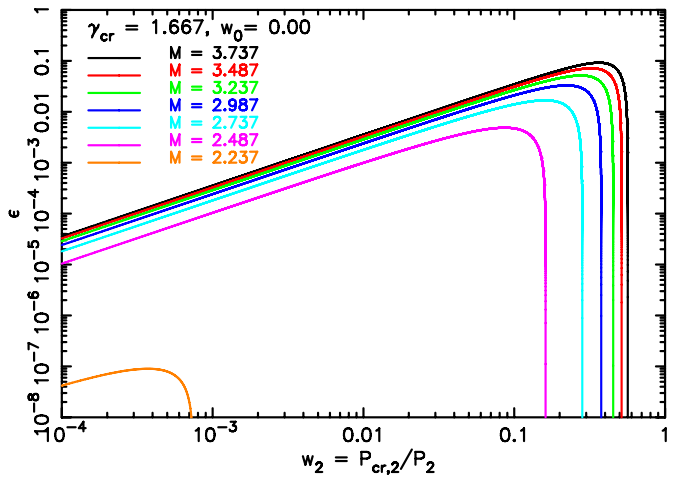

Figure 2. Solutions to two-fluid model of Vink et al. (2010). The values for the Mach number correspond to those in Figure 1, except that the orange curves correspond to $M_{\rm g,0} = \sqrt{5} + 0.001$, in order to show the behavior very close the critical Mach number.

(A color version of this figure is available in the online journal.)

Figure 2 shows the allowed combinations of the fractional downstream cosmic-ray pressure w_2 and ϵ. It illustrates that there is a dramatic change in the maximum possible particle acceleration efficiency going from a Mach number around $M_{\rm g,0} = 2.5$ to a Mach number very close to $M_{\rm acc} = \sqrt{5}$.

There are other potential effects that may shift the limiting Mach number to higher values. In Section 2.4, the effects of plasma-beta is treated. But another factor is non-adiabatic heating in the precursor. Up to now it was assumed that the accelerated particles compress the upstream plasma, and heats it only adiabatically. However, additional heating may occur in the precursor, for example through Coulomb collisions, wave damping, or through friction with neutral atoms (Ohira &

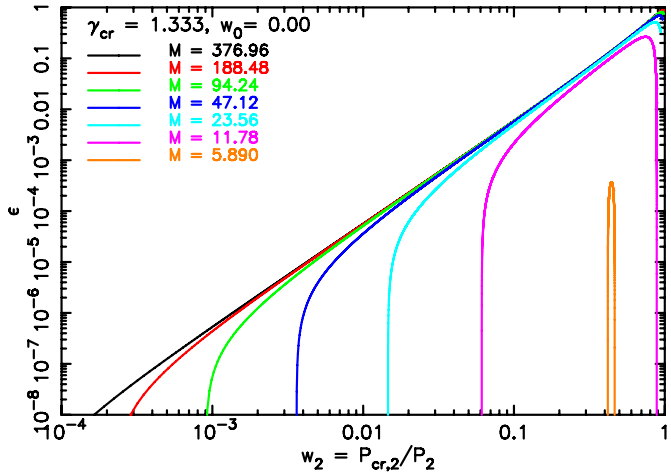

Figure 3. Same as Figure 2, but now for an accelerated particle component characterized by $\gamma_{\rm cr} = 4/3$, for logarithmically spaced intervals of the Mach number.

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Takahara 2010; Raymond et al. 2011; Morlino et al. 2013). This leads to higher values of the critical Mach number. This can be easily seen by replacing Equation (A4) by

$$M_{\rm g,1}^2 = \frac{M_{\rm g,0}^2 \chi_{\rm prec}^{-(\gamma_{\rm g}+1)}}{(1+\alpha)}, \quad (4)$$

with $\alpha \geqslant 0$ a parameter that parameterizes the additional heating as an additional fraction of the adiabatic heating, resulting in a lower subshock Mach number. It can be easily seen that introducing the additional factor $1/(1+\alpha)$ in Equation (1) results in increasing $M_{\rm acc}$ by a factor $\sqrt{1+\alpha}$.

2.3. The Minimum Mach Number for Acceleration to a Relativistically Dominated Cosmic-Ray Population

In the previous section the limit for particle acceleration was obtained by assuming that the accelerated particles are non-relativistic ($\gamma_{\rm cr} = 5/3$). This gives the lowest limit on particle acceleration one can obtain. If instead the accelerated particles are dominated by relativistic particles ($\gamma_{\rm cr} = 4/3$), $M_{\rm acc}$ needs to be much higher. Deriving the value for $M_{\rm acc}$ is much more difficult as the overall equation of state of the two-fluid plasma depends now on the mixture of thermal particles and accelerated particles. Instead we give here the numerical value we obtained, $M_{\rm acc} = 5.882$.

Figure 3 shows the behavior of energy escape and downstream cosmic-ray pressure for $M_{\rm g,0} > M_{\rm acc} = 5.882$. It illustrates a peculiar feature of the solutions for $\gamma_{\rm cr} = 4/3$ as compared $\gamma_{\rm cr} = 5/3$. In the latter case (Figure 2) $\epsilon > 0$ for $w_2 > 0$, up to maximum possible value for w_2. However, for $\gamma_{\rm cr} = 4/3\epsilon$ first becomes negative for $w_2 > 0$, then reaches a minimum, and then crosses again the line $\epsilon = 0$. In other words for $\gamma_{\rm cr} = 4/3$ there are for some Mach numbers three solutions for $\epsilon = 0$, namely the standard shock solution (i.e., $w_2 = 0$), a solution that maximizes w_2 and for which $\chi_{\rm sub} = 1$, and a point somewhere in between these two limits. These solutions correspond to the solutions of the two-fluid model of Drury & Voelk (1981), which assumes energy flux conservation. $M_{\rm acc}$ corresponds to the Mach number where the two non-standard solutions coincide, for which the sub-shock compression ratio is $\chi_{\rm sub} = 5/2$ (Equation (3)).

For many astrophysical settings, especially in interplanetary shocks, for low Mach numbers the adiabatic index for the accelerated particle population will more closely resemble $\gamma_{\rm cr} = 5/3$. We illustrate this in Figure 4, which is not based on the extended Rankine–Hugoniot relations of Vink et al. (2010), but on the semi-analytical kinetic solutions of Blasi et al. (2005). It shows that as the Mach number decreases $\gamma_{\rm cr}$ approaches 5/3. However, the energy flux reaches $\epsilon = 0$ for $M_{\rm g,0} \approx 2.79$, with a corresponding $\gamma_{\rm cr} \approx 1.57$, and $w_0 \approx 0.15$. For lower Mach numbers $\epsilon < 0$. Figure 5 shows the critical Mach number for acceleration as a function of the assumed adiabatic index for cosmic rays.

2.4. Perpendicular, Magnetically Dominated Shocks

The best studied low Mach number shocks are arguably shocks in the solar system. But these shocks often have a low upstream plasma-beta ($\beta_0 < 1$). The presence of significant pressure from a magnetic field component will make the flow less compressible, and requires more work to be done by the shock in order to compress the plasma. As a result, there will be less energy available for accelerating particles. Including magnetic fields into the Rankine–Hugoniot solutions complicates the calculation of shock parameters (Tidman &

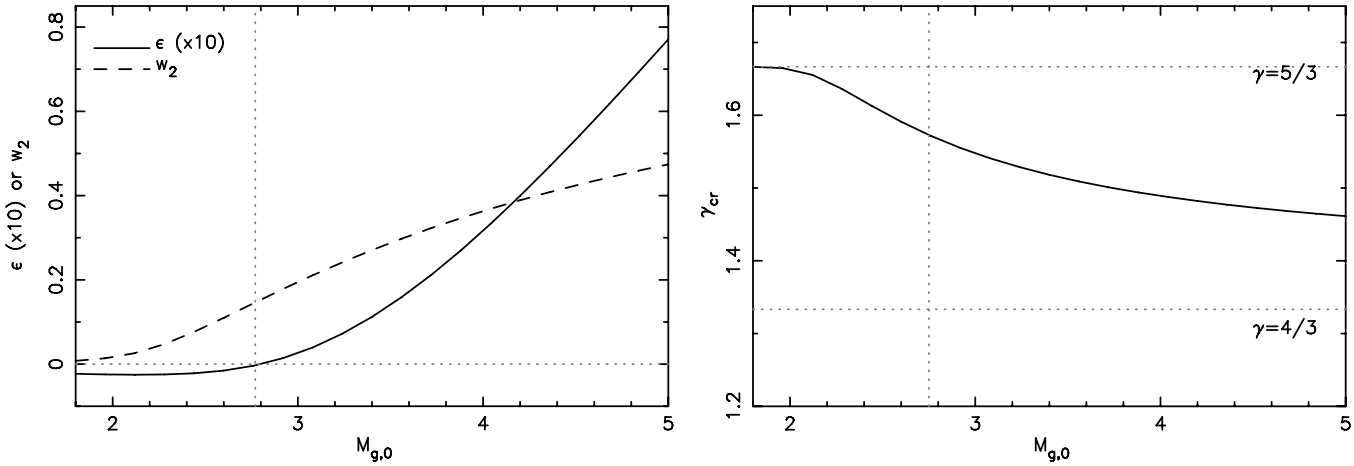

Figure 4. Shock solutions as obtained with the kinetic shock acceleration model of Blasi et al. (2005) for a shock velocity of $V_{\rm s} = 10$ km s^{-1}, and a maximum momentum of the accelerated particles of $p_{\rm max} = 100mc$. Left: the values for the escape flux, ϵ (multiplied by 10), and w_2. Right: effective adiabatic index $\gamma_{\rm cr}$ of the accelerated particles. The limiting Mach number for this case is $M_{\rm acc} \approx 2.79$.

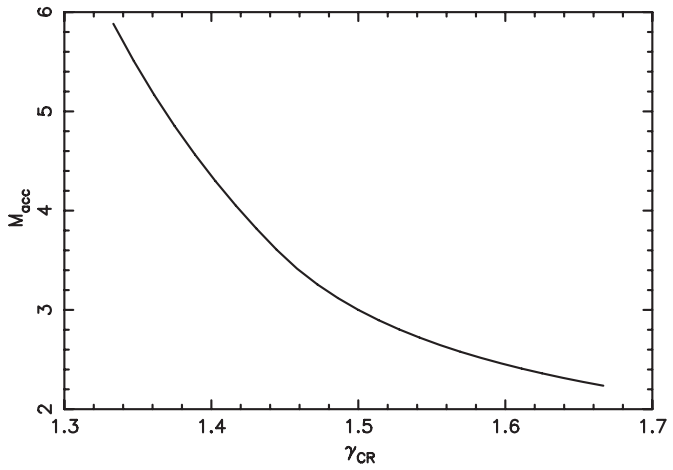

Figure 5. Critical Mach number as a function of assumed adiabatic index for the accelerated particle population ($4/3 \leqslant \gamma_{\rm cr} \leqslant 5/3$).

Krall 1971), but one can obtain some insights by considering the limiting case of a strictly perpendicular shock in which all the upstream pressure is provided by the magnetic field; so $\beta_0 = 0$, $B_0 = B_{0,\perp}$ and $P_{\rm g,0} = 0$, and $w_0 = 0$. The relevant shock equations are given in Appendix B, but here we list the main points.

For a strictly perpendicular shock with $\beta_0 = 0$, one finds for the shock compression ratio at the subshock (see Equation (B10))

$$\chi_{\rm sub} = -\left(M_{\rm A,1}^2 + 5/2\right) + \sqrt{D_1}, \tag{5}$$

with

$$D_1 \equiv M_{\rm A,1}^4 + 13M_{\rm A,1}^2 + \frac{25}{4}, \tag{6}$$

with the numerical values valid for $\gamma_{\rm g} = 5/3$. The subshock Alfvén Mach number is given by

$$M_{\rm A,1}^2 = M_{\rm A,0}^2 \chi_{\rm prec}^{-3}. \tag{7}$$

The maximum compression ratio can be found in analogy with the procedure that lead to Equation (2), namely by determining $d\chi_{\rm tot}/d\chi_{\rm prec} = 0$ in the limit of $\chi_{\rm prec} = 1$, with

$$\begin{aligned}\chi_{\rm tot} &= \chi_{\rm prec}\chi_{\rm sub} \\ &= -\left(M_{\rm A,0}^2\chi_{\rm prec}^{-2} + \frac{5}{2}\chi_{\rm prec}\right) + \sqrt{\chi_{\rm prec}^2 D_1}.\end{aligned} \tag{8}$$

After some algebra one finds that in the limit $\chi_{\rm prec} \rightarrow 1$, and $M_{\rm A,0} = M_{\rm A,1}$, the solution has to obey the relation

$$\left(8M_{\rm A,0}^2 - 10\right)\sqrt{D_1} - \left(8M_{\rm A,0}^4 + 26M_{\rm A,0}^2 - 25\right) = 0. \tag{9}$$

The solution to this equation is $M_{\rm A,0} = M_{\rm A,1} = 5/2$, which corresponds to a subshock compression ratio of $\chi_{\rm sub} = 5/2$ (Equation (5)). So the critical Mach number for acceleration for a perpendicular shock with $\beta_0 = 0$ and $w_0 = 0$ is $M_{\rm acc} = 5/2$.

Equation (B8) in Appendix B is the equivalent of Equation (A15), and shows which values of the compression ratio are allowed (i.e., $\epsilon \geqslant 0$). The relation between ϵ and the precursor compression strength around $M_{\rm acc}$ is illustrated in Figure 6, which is similar to Figure 1.

In order to illustrate the effects of the critical Mach number on particle acceleration, Figure 7 shows the possible three-fluid

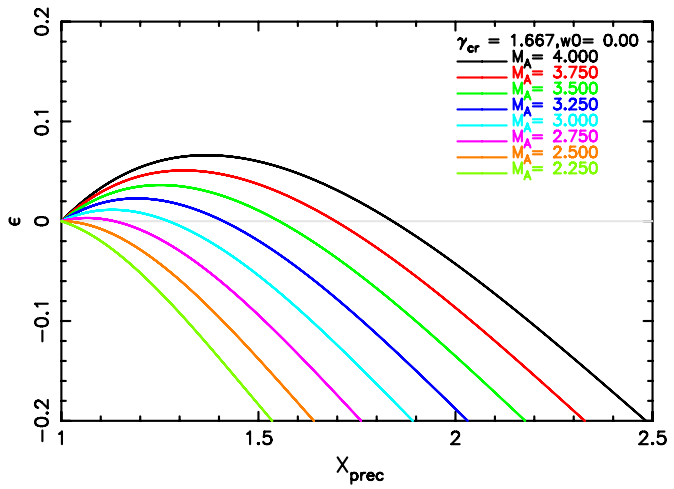

Figure 6. Same as Figure 1 (left), but now for perpendicular shocks with $\beta_0 = 0$, and with Mach numbers that include the appropriate critical Alfvén Mach number $M_{\rm A} = 2.5$ (orange).

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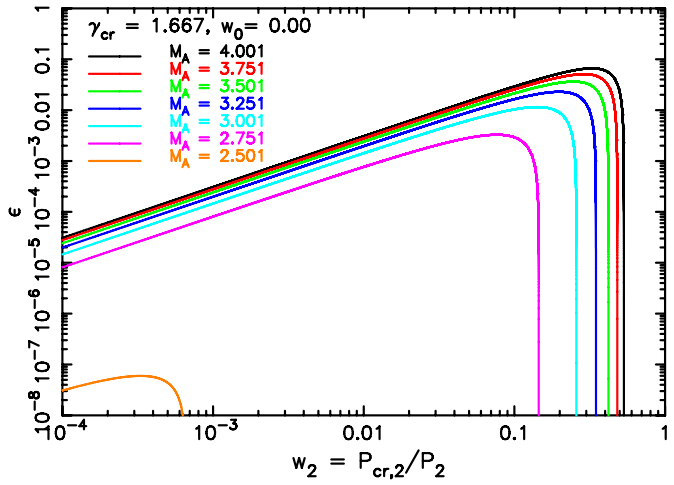

Figure 7. Same as Figure 2, but now for perpendicular shocks with $\beta_0 = 0$, and with Mach numbers that include a value close to the critical Alfvén Mach number $M_{\rm A} = 2.5$ (orange).

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solutions for the shock conditions and acceleration efficiency, with the third "fluid" being the magnetic field. These curves are calculated using the appropriate expression for the efficiency parameter w_2, which is now defined as

$$w_2 \equiv \frac{P_{\rm cr,2}}{P_{\rm g,2} + P_{\rm cr,2} + P_{\rm B,2}}. \tag{10}$$

The expression for w_2 as function of the Mach number, and the total and subshock compression ratios is

$$w_2 = \frac{\left(1 - \chi_{\rm prec}^2\right) + 2M_{\rm A,0}^2\left(1 - \frac{1}{\chi_{\rm prec}}\right)}{1 + 2M_{\rm A,0}^2\left(1 - \frac{1}{\chi_{\rm tot}}\right)}. \tag{11}$$

Note the similarity with Equation (A11): inserting $\gamma = 2$ and $w_0 = 0$ in that equation and replacing $M_{\rm g,0}$ with $M_{\rm A,0}$ gives the above expression.

The results in this section, therefore, show that due to a lower compressibility of plasmas with dominant magnetic field pressures, more work needs to be done to compress the plasma,

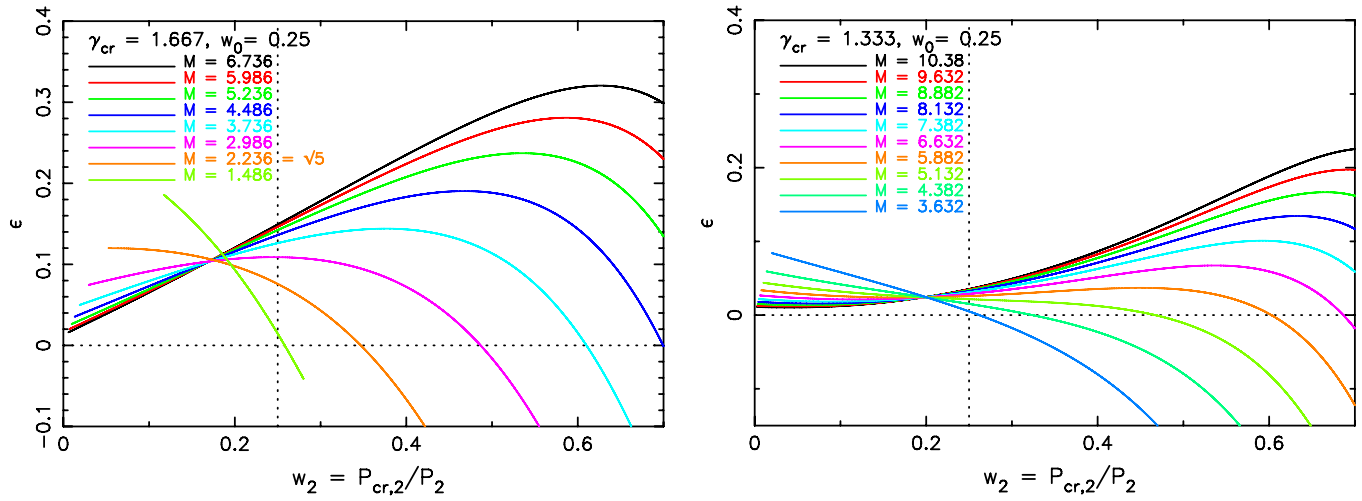

Figure 8. Solutions for the escape energy flux as a function of the downstream cosmic-ray pressure w_2 (similar to Figures 2 and 7), but now with the contribution of an additional upstream cosmic-ray pressure from pre-existing cosmic rays, $w_0 = 0.25$ (Equation (A15)). The left panel is for a non-relativistic accelerated particle population ($\gamma_{\rm cr} = 5/3$), the right panel is for relativistically dominated particles ($\gamma_{\rm cr} = 4/3$). The Mach numbers differ 0.75 (1.33) times an integer number from the critical Mach number, $M_{\rm acc} = \sqrt{5}$ for $\gamma_{\rm cr} = 5.3$ and $M_{\rm acc} = 5.882$ for $\gamma_{\rm cr} = 4/3$.

(A color version of this figure is available in the online journal.)

and, as a result, the critical (Alfvén) Mach number for forming a precursor is higher than for $\beta_0 \gg 1$, $M_{\rm acc} = 5/2$.

It is assumed here that the magnetic field is passive. If, however, the magnetic field is amplified due to cosmic-ray streaming, or some turbulent dynamo mechanism, the resulting value of $M_{\rm acc}$ will be higher, in a similar way as non-adiabatic heating in the precursor results in larger values for $M_{\rm acc}$.

2.5. Shocks with Pre-existing Cosmic Rays

In the solutions discussed above we assumed that there is no population of pre-existing cosmic rays. However, pre-existing cosmic rays can be incorporated in the extended Rankine–Hugoniot relations, by specifying the additional parameter $w_0 = P_{\rm cr,0}/P_0$, as explained in Appendix A. The solutions to the energy flux equation (Equation (A13)) are shown in Figure 8 for non-relativistic ($\gamma_{\rm cr} = 5/3$) and completely relativistic cosmic rays ($\gamma_{\rm cr} = 4/3$).

These figures show that for $w_0 > 0$ it is possible to find solutions with $\epsilon \geqslant 0$ even for $M_{\rm g,0} < M_{\rm acc}$. However, some of these solutions are unphysical. For example, the left most limit of all the curves in the figures correspond to no-precursor compression ($\chi_{\rm prec} = 1$). The continuity of the cosmic-ray pressure in that case implies that from far upstream to downstream the cosmic-ray pressure is constant ($P_{\rm cr,2} = P_{\rm cr,0}$). But it is impossible to have cosmic-rays take away energy flux from the system, if there is no cosmic-ray pressure gradient present.[4]

It is beyond the possibilities of the extended Rankine–Hugoniot relations to firmly state what parts of the curves with $w_0 > 0$ are physically possible. Analytic solutions in the framework of the two-fluid model and $w_0 > 0$ do exist for the case of conservation of energy flux ($\epsilon = 0$; Drury & Voelk 1981; Malkov & Voelk 1996; Becker & Kazanas 2001), which correspond to the zero points in Figure 8. These zero points are shown as a function of Mach number in Figures 9 and 10, for respectively $\gamma_{\rm cr} = 5/3$ and $\gamma_{\rm cr} = 4/3$. They illustrate the different behavior for relativistic and non-relativistic accelerated particles.

For the non-relativistic case ($\gamma_{\rm cr} = 5/3$), there is never more than one solution for $\epsilon = 0$, if pre-existing cosmic rays are present ($w_0 > 0$). For $w_0 = 0$ these solutions require $M_{\rm g,0} > M_{\rm acc} = \sqrt{5}$. The highest values for w_2 in case we take energy flux conservation ($\epsilon = 0$) provides an upper bound on w_2 for solutions with escape (see Figures 2 and 8 (left)). For completely relativistic cosmic rays ($\gamma_{\rm cr} = 4/3$) there are for $w_0 = 0$ two solutions with $\epsilon = 0$ and $w_2 > 0$. This leads to the bifurcation in $\chi_{\rm tot}$ and w_2 in the top panels of Figure 10 for $M_{\rm g,0} > M_{\rm acc}$. Figure 10 once more illustrates that there is no solution with $w_0 = 0$ and $w_2 > 0$ for Mach numbers $M_{\rm g,0} < M_{\rm acc} \approx 5.88$.

Increasing the pressure in pre-existing cosmic rays ($w_0 > 0$) changes the character of the solutions, as slowly the bifurcation disappears, and also viable solutions exist for $M_{\rm g,0} < M_{\rm acc} \approx 5.88$. The reason is that with a higher pressure in pre-existing cosmic rays, the shock solutions with $\epsilon = 0$ start approaching the standard Rankine–Hugoniot solutions for a relativistic gas, which for high Mach numbers approaches the compression ratio $\chi_{\rm tot} = 7$. Note that Figure 10 is similar to the figures in Malkov & Voelk (1996), showing that the extended Rankine–Hugoniot relations explored here encompass the two-fluid model with conservation of energy flux (Drury & Voelk 1981; Malkov & Voelk 1996; Becker & Kazanas 2001).

3. DISCUSSION

3.1. The Case for a Minimum Mach Number for Acceleration

We showed that the ability to accelerate particles relies a critical magnetosonic Mach number $M_{\rm acc}$, which depends on the presence/absence of perpendicular magnetic fields and the assumed adiabatic index of the population of accelerated particles. If there are no pre-existing cosmic rays ($w_0 = 0$), this critical Mach number is the minimum Mach number for which

[4] In fact, this could be a possible, but trivial solution, if the pre-existing cosmic-rays do not couple to the gas at all. In that case one should not write for the downstream enthalpy flux $H = [P_{\rm cr,2} + u_{\rm cr,2} + P_{\rm g,2} + u_{\rm g,2} + (1/2)\rho_2 v_2^2]v_2$, but associate the cosmic rays still with the velocity of the upstream medium, as there is no coupling, $H = [P_{\rm cr,2} + u_{\rm cr,2}]v_0 + [P_{\rm g,2} + u_{\rm g,2} + (1/2)\rho_2 v_2^2]v_2$. In that case no escape flux is necessary for $w_0 > 0$ and $\chi_{\rm prec} > 1$. The problem arises that for $\chi_{\rm prec} = 1$, $w_0 > 0$ the cosmic-ray pressure is continuous but leads nevertheless to an associated change in enthalpy flux, due to the change in frame velocity ($v_0 \rightarrow v_2$).

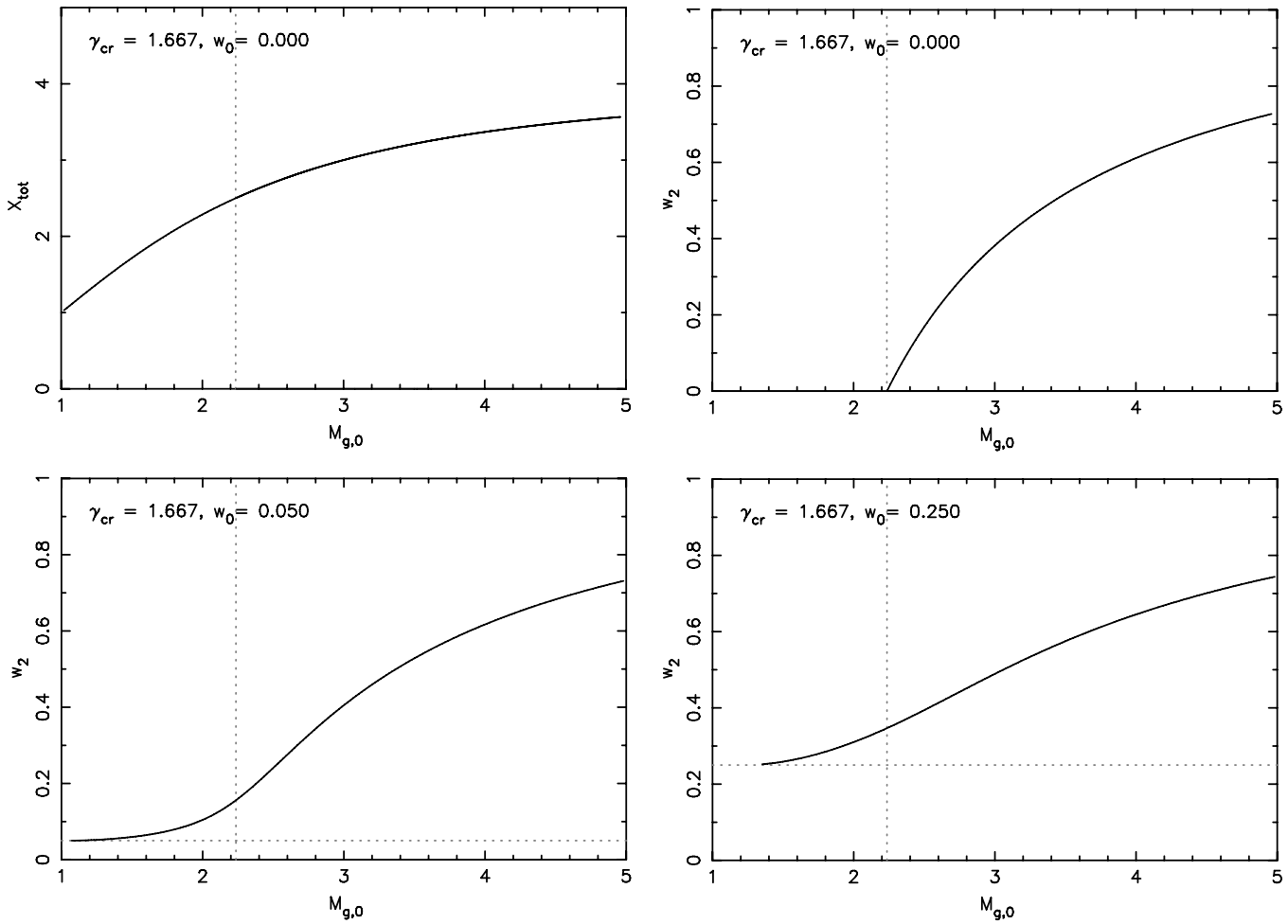

Figure 9. Shock solutions for $\gamma_{\rm cr} = 5/3$ as a function of Mach number $M_{\rm g,0}$ for the case in which no energy is escaping from the system ($\epsilon = 0$), corresponding to the two-fluid model of Drury & Voelk (1981). Top left panel: the total compression ratio, which follows the standard Rankine–Hugoniot relations for $\gamma = 5/3$. Other panels: the downstream fractional cosmic-ray pressure for increasing values of the pre-existing cosmic-ray fractional pressure: $w_0 = 0$, 0.05, 0.25. The vertical dotted line indicates the critical acceleration Mach number $M_{\rm acc} = \sqrt{5}$, whereas the horizontal dotted line indicates w_0. Note that the total compression ratios can be higher for $\epsilon > 0$, whereas the maximum values for w_2 provide upper bounds for $\epsilon > 0$.

sufficient energy flux is available to accelerate particles. In all cases the critical Mach number corresponds to a compression ratio at the sub-shock of $\chi_{\rm sub} = 5/2$, corresponding to a subshock Mach number $M_{\rm g,1} = \sqrt{5}$. For non-relativistically dominated cosmic-rays the critical Mach number lies in the range $\sqrt{5} \leqslant M_{\rm acc} \leqslant 5/2$, depending on whether plasma beta is large, or very low. The values of $M_{\rm acc}$ are higher if heating or magnetic field amplification are important, or if the non-thermal particles have a significant relativistic component. For completely relativistic cosmic rays $M_{\rm acc} \approx 5.88$.

The situation changes in case a pre-existing population of cosmic rays exist, in the sense that in that case the additional degree of freedom allows for cosmic-ray acceleration even for Mach numbers lower than $M_{\rm acc}$. However, not all the solutions found with the extended Rankine–Hugoniot relations employed here, may be physical possible, because in some cases escape of energy flux is required, even though there are no substantial pressure gradients in the cosmic rays.

The derivation of $M_{\rm acc}$ in the previous sections is based on only a few assumptions: like for the general shock-jump relations, it relies on the plane parallel shock approximation; it requires steady state conditions; and it requires the subshock to be governed by the standard Rankine–Hugoniot relations.

These assumptions are very generic and are common to most shock and diffusive shock acceleration models. However, the steady state assumption leaves open the possibility that particle acceleration is not a continuous phenomenon, but occurs irregularly or in bursts.

Another, more fundamental, issue is that if one observes the (sub)shock region in detail the distinction between what is a precursor and what is the subshock becomes more complicated. We followed here the convention of diffusive shock acceleration theories that refer to the main shock as the subshock. However, in collisionless shock theory the subshock refers to the steep gradient in density and pressure, as opposed to other quantities, like magnetic field that may change on slightly larger length scales. Indeed, collisionless shocks, even with ignoring diffusive shock acceleration, can have a complex structure (Treumann 2009). They have precompression in a so-called foot region, a steep shock ramp, a downstream overshoot region, which corresponds to a compression ratio higher than allowed by the Rankine–Hugoniot relations, followed by an undershoot region. Only further downstream the flow relaxes to the standard shock-jump conditions. The foot region is associated with ions reflected immediately back upstream by the shock. So the foot region could also be labeled a shock precursor. But, in the context of the discussion here, the precursor/foot region should still be

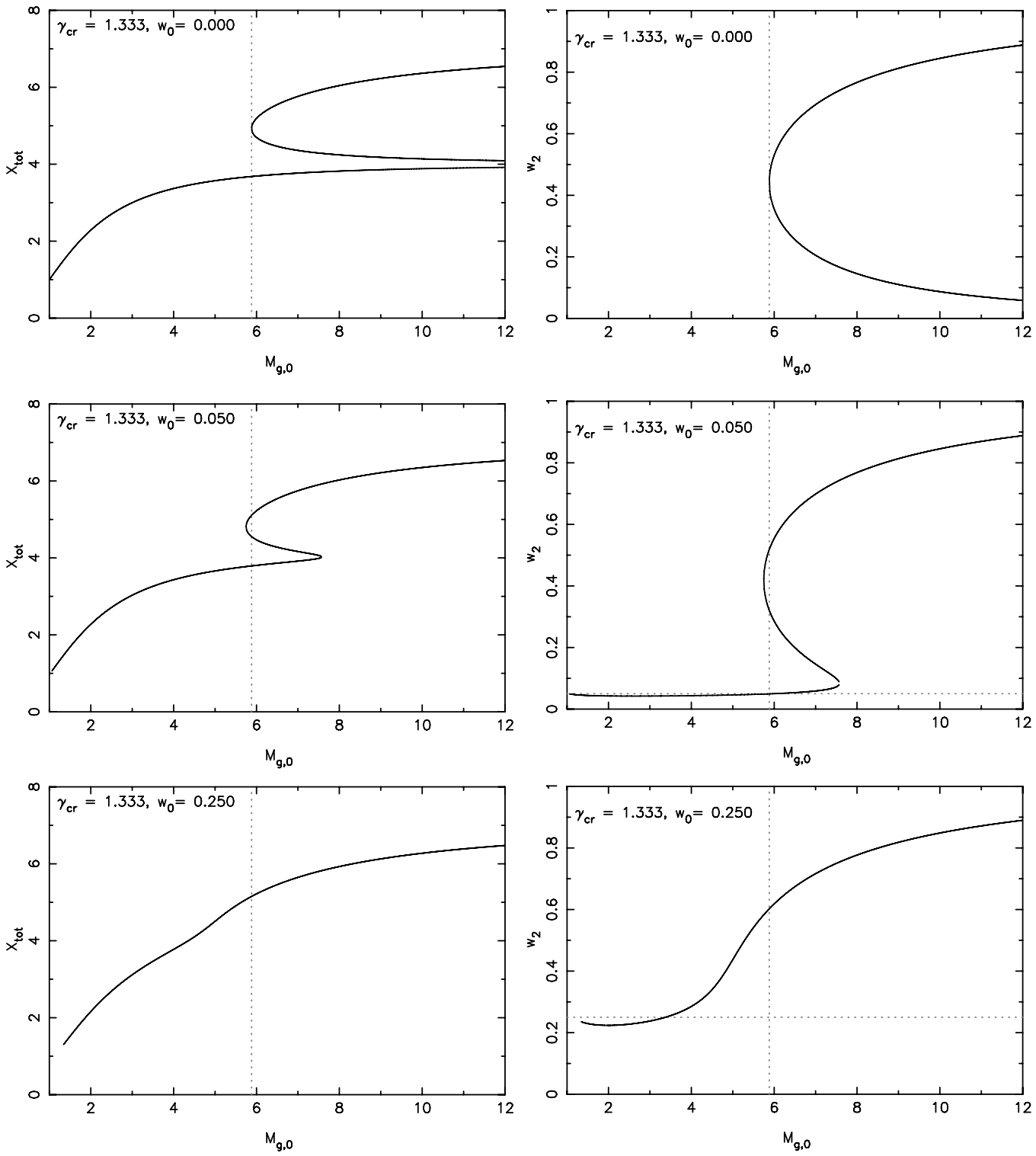

Figure 10. Similar to Figure 9, but now for $\gamma_{\rm cr} = 5/3$. The left hand panels show the total compression ratio $\chi_{\rm tot}$, and the right hand panels the downstream fractional cosmic-ray pressure w_2, for increasing values of w_0.

regarded as an integral part of the subshock itself. The reason is that across the total subshock structure the standard shock-jump relations are observed. The complex structure, and physical processes like ion reflection, are a means by which nature forces the flow to establish a shock and observe the Rankine–Hugoniot relations. In contrast, shocks with diffusive shock acceleration do not observe the Rankine–Hugoniot relations, and they can have compression ratios much higher than the standard shock-jump relations. This is possible due to the escape of high energy particles upstream.

Nevertheless, the distinction between an "accelerated particle precursor" and a "foot region" may not be that sharp. The distinction is more easily defined if shock acceleration is very efficient, and the accelerated particle precursor becomes very extended. But around $M = M_{\rm acc}$ the efficiency is low (Figures 2 and 7), and it may observationally be difficult, or even arbitrary

to distinguish between a precursor from diffusively accelerated particles and a foot region.

The appearance of foot regions, ion reflection, and overshoot regions is usually associated with another critical Mach number, the so-called *first critical Mach* number, M_c, which has a range of $1 \leqslant M_c \leqslant 2.76$, depending on the shock obliquity and plasma-beta (Edmiston & Kennel 1984), $M_c = 1$ corresponding to $\beta \gg 1$ and $M_c \approx 2.76$ corresponding to perpendicular shocks with $\beta_0 = 0$.

Below the first critical Mach number ordinary resistivity is sufficient to provide the necessary shock steepening, whereas for supercritical shocks anomalous dissipation mechanisms are necessary to force the shock to observe the Rankine–Hugoniot relations. Ion reflection is one of the ingredients by which the flow manages to acquire the required shock heating. Indeed, ion reflection is observationally associated with supercritical shocks, although some subcritical shocks also appear to have ion reflection and overshoot regions (Mellott & Livesey 1987). Note that the presence of an overshoot seems to violate the flux conservation laws (Equations (A2)–(A13)), but this may be an indication that energy flux is temporarily stored in the electrostatic oscillations, and therefore the equation of state is temporarily altered, corresponding to a lower specific heat ratio γ, and higher compression ratios (Eselevich 1984).

The idea that two critical Mach numbers may operate in the same Mach number regime is interesting and may have some observational consequences. For high beta shocks, the first critical Mach number is very low, $M_c \approx 1$, and lies below the critical Mach number for acceleration $M_{acc} = \sqrt{5}$, hence $M_c < M_{acc}$. In contrast, for very low beta, perpendicular shocks the first critical Mach number is $M_c \approx 2.76$, which is larger than $M_{acc} \approx 2.5$. The effects of the two different critical Mach numbers, M_c and M_{acc}, may therefore be observationally investigated by exploiting this difference between low and high beta shocks.

3.2. Comparison to Observations

Observationally the case for whether there is a critical Mach number for particle acceleration is not so clear. The Earth' bowshock is generally associated with Mach numbers above the critical regime ($M_{ms} \approx 5$; Bale et al. 2003). The solar wind termination shock has a Mach number in the range where one may expect to see critical behavior ($M_{ms} \approx 2.5$; Lee et al. 2009). Florinski et al. (2009) made a case for nonlinear particle acceleration at the solar wind termination shock, as *Voyager 2* data indicate the presence of a precursor induced by accelerated particles. The total compression ratio for that case was $\chi = 3.1$, which is above the critical value of $\chi_{tot} = 5/2$.

CMEs are also associated with particle acceleration, and Type II radio bursts are considered to be evidence for acceleration. Gopalswamy et al. (2010) showed that Type II radio bursts are associated with high velocity/high Mach number CMEs (with mean velocities of 1237 km s^{-1}) and the radio quiet CMEs with low velocities (with mean velocities of 537 km s^{-1}). The Mach numbers of the low velocity CMEs were still relatively high, with a median of $M_{ms} = 2.3$ and an average of $M_{ms} = 2.7$. The latter value is above the critical Mach number derived here, and close to the first critical Mach number M_c. But it should be noted that the errors on the Mach numbers are relatively high (systematic error $\Delta M \approx 0.55$; Gopalswamy et al. 2010). Pulupa et al. (2010) even concluded that the measured Mach numbers are not well correlated with the occurrence of Type II radio bursts, whereas there is a strong correlation with velocity.

Another measure for the compression ratio for shocks associated with Type II radio bursts is the bandwidth of the radio emission. The work by Mann et al. (1995) indicates that the minimum bandwidth is $\Delta f/f = 0.16$, which, according to Mann & Classen (1995), corresponds to a minimum shock-compression ratio of $\chi = 1.35$. This is clearly not in accordance with the critical Mach number M_{acc} derived in the present paper, which occurs for a compression ratio of 2.5 or more. However, it is not clear yet whether the bandwidth is indeed caused by the jump in the density caused by the shock, or whether density gradients in the upstream region are responsible. A joint analysis of the location of the radio emission and optical CME locations seems to suggest that the radio emission is in general coming from a region upstream of the shock (Ramesh et al. 2012).

Clearly, the uncertainty of the correlation between Type II bursts and Mach numbers could be resolved by more precise measurements of the Mach numbers, rather than the shock velocity, for those exact locations that emit in the radio. A recent analysis of *Solar and Heliospheric Observatory* observations by Bemporad & Mancuso (2011) shows that more precise Mach numbers can be obtained, indicating that the highest compression ratios, $\chi \approx 3$, are found near the center of the CME. A problem may remain that for CMEs the plasma beta is rather low, so that the determination which critical Mach number determines Type II bursts, M_c or M_{acc}, may be difficult to distinguish.

For this reason it is very interesting that recently Giacalone (2012) showed that all shocks that have high enough compression ratios show evidence for particle acceleration. Interestingly, this study uses as an indication of a strong shock a compression ratio of $\chi \geqslant 2.5$, which is exactly the compression ratio associated with lowest possible value for the critical Mach number $M_{acc} = \sqrt{5}$ in case of a sonic shock, and $M_{acc} = 2.5$ for a magnetically dominated, perpendicular shock.

Apart from Mach number, another factor that appears to influence the presence or absence of accelerated particles associated with CMEs is the occurrence of a CME preceding the event by less than a day (Kahler et al. 1999; Gopalswamy et al. 2004). This correlation has been attributed to the presence of non-thermal particle populations created by the first CME (Laming et al. 2013). Our theoretical results here indicate that the mere presence of accelerated particles may facilitate particle acceleration for Mach numbers lower than the critical Mach number. Note that both effects, the influence on the jump relations, and the presence of seed-particles, may play complementary roles.

In this context one should raise the question to what extent the omnipresent Galactic cosmic rays are important. This likely depends on the length scale of the coupling between cosmic rays and the plasma directly up- and downstream of the shock. If the length scale is much longer than the typical length scales over which the shock develops, these pre-existing cosmic rays are likely to not affect the shock structure. For that reason, for CMEs probably only low energy accelerated particles are important (keV to MeV energies). So particles from preceding CMEs are much more important than Galactic cosmic rays. However, these are subtleties that require further investigation.

The largest shocks observed in the universe are those in clusters of galaxies. Many of them are detected as discontinuities in the X-ray emission (Markevitch & Vikhlinin 2007). These shocks are caused by infalling subclusters or galaxy groups, or due to mergers of clusters. Some shocks are detected through

their non-thermal radio emission, clearly indicating that at these shocks electrons are accelerated (van Weeren et al. 2010; Hoeft et al. 2011). The radio detected shocks, often called radio relics, are usually located in the outskirts of the cluster. The shock velocities can be several thousand km s^{-1}, but due to the high plasma temperatures, $kT \approx 1$–10 keV, the Mach numbers are usually modest $M_{\rm ms} \lesssim 3$. The radio relics are mostly found in the periphery of the clusters where the density is lower than in the center, whereas the magnetic field may be as high as a few μG. The plasma betas are believed to be $\beta \approx 1$–10 (M. Hoeft 2013, private communication). The lack of radio emission from many X-ray detected shocks suggest that there is, indeed, a dependence of radio emission on Mach number, which could therefore hint at the existence of a critical Mach number for acceleration. It is usually assumed that the onset of radio emission happens in the range of $2 < M_{\rm acc} < 3$ (Hoeft et al. 2011).[5] This should be contrasted to the first critical Mach number, $M_{\rm c}$, which in clusters of galaxies is likely smaller than 2. Therefore, the critical Mach number derived in the present paper may be important for the presence or absence of radio emission from shocks in clusters of galaxies. However, the derived numbers for $M_{\rm acc}$ were for non-relativistic particles. The radio emission is caused by relativistic electrons. As long as the protons are non-relativistic and dominate the population of accelerated particles, $\gamma_{\rm cr} = 5/3$, may still be a reasonable approximation. If protons are accelerated to relativistic energies, with $E > 938$ MeV, $\gamma_{\rm cr}$ will decrease toward $\gamma_{\rm cr} = 4/3$, and $M_{\rm acc}$ will increase. As discussed in Section 2, it depends on the spectral energy distribution what the effective specific heat ratio of the accelerated particles is. But for a significant component of relativistic protons a limiting Mach number of $M_{\rm acc} \approx 3$ is likely. This could mean that many of the observed relics cannot accelerate protons to very high energies, and only the highest Mach number shocks ($M > 3$) contain significant fractions of relativistic protons.

Alternatively, the limiting Mach numbers for shocks moving through a medium containing cosmic rays is more relaxed (Section 2.5). So evidence for relativistic particles associated with low Mach number shocks, may indicate the presence of pre-existing cosmic rays in the intra-cluster medium. As is the case for CME induced shocks, for clusters the importance for pre-existing cosmic rays as seed particles for further acceleration has been pointed out. And also in this case it should be pointed out that pre-existing cosmic rays may have two, complementary, effects: it changes the degrees of freedom of the shock system, allowing for acceleration for lower Mach numbers (the present work), and it may help as a source of seed particles, which are injected in the shocked and then experience further acceleration (Pinzke et al. 2013).

Another effect could be that acceleration becomes discontinuous: for $\sqrt{5} < M < 3$ particles are being accelerated but once a significant number of protons become relativistic the acceleration efficiency goes dramatically down for some time, and then start up again. Clearly these effects need to be further investigated, both observationally in shocks close the critical Mach number, and with more elaborate kinetic shock-acceleration models.

[5] These exact Mach numbers are not easily measured, and either rely on interpreting the radio spectrum in the context of test particle acceleration, or on the detection of the shock in X-rays. However, it is not always clear whether the X-ray detected shock and the shock associated with the radio emission exactly coincide (Ogrean et al. 2013).

4. CONCLUSION

We presented in this paper a derivation of a critical Mach number for particle acceleration, $M_{\rm acc}$. The basic idea is that diffusive shock acceleration is inherently nonlinear, and results in the compression and slowing down of the upstream plasma, forming a so-called shock precursor. It turns out that adiabatic compression in the precursor followed by a shock, as given by the standard shock jump conditions, cannot be energetically sustained for Mach numbers smaller than a critical value $M_{\rm acc} = \sqrt{5}$. This limit is even higher for magnetic dominated plasmas, which in the extreme case of $\beta_0 = 0$ and purely perpendicular shock gives a critical Mach number of $M_{\rm acc} = 2.5$. In case there is substantial pre-existing cosmic-ray population the limits on further acceleration may be relaxed. This critical Mach number should not be confused with the so-called first critical Mach number $M_{\rm c}$, which depending on obliquity and β_0, lies in the range $1 \leqslant M_{\rm c} < 2.76$ (Edmiston & Kennel 1984).

We discussed the critical Mach number, $M_{\rm acc}$, in connection with observational evidence for particle acceleration at low Mach number shocks, such as in the solar system or in clusters of galaxies, and in conjunction with first critical Mach number. There is indeed observational evidence for a Mach number dependence of particle acceleration with Mach number, which agrees with the idea that between Mach numbers of 2–3 the acceleration properties of shocks change. However, the observational evidence is not precise enough to judge whether there is indeed a critical Mach number range for acceleration $\sqrt{5} < M_{\rm acc} < 2.5$, or whether the observed phenomenology of solar system shocks is governed by the first critical Mach number $M_{\rm c}$.

For shocks in clusters of galaxies, there is some indication that Mach numbers above 2–3 are needed to create a population of radio synchrotron emitting electrons. It is pointed out that the critical Mach number, $M_{\rm acc}$, increases if the energetics of the accelerated particles are dominated by relativistic particles, which could mean that there is a strong limit on the number fraction of relativistic protons in cluster shocks with Mach number $M < 3$.

It is a pleasure to thank Stefano Gabici for useful discussions. The writing of this paper was stimulated by discussions during the JSI Workshop "Nature's Particle Accelerators," held in 2012 October. I thank the organizers for inviting me to this stimulating workshop. I also thank Matthias Hoeft for discussions on shocks in clusters of galaxies.

APPENDIX A

THE EXTENDED RANKINE–HUGONIOT RELATIONS INCLUDING PRE-EXISTING COSMIC RAYS

Vink et al. (2010) described a version of the Rankine–Hugoniot relations extended with a component of accelerated particles. Like the Rankine–Hugoniot relations it evaluates the mass, momentum, and enthalpy flux, but with some modifications: Instead of applying the relations to two regions (upstream and downstream of the shock) the relations are evaluated at three specific locations: (0) the (undisturbed) far upstream medium, (1) in the cosmic-ray shock precursor, just upstream of the subshock (i.e., the actual gas shock), and (2) downstream of the subshock. The standard Rankine–Hugoniot relations only consider (0) and (2). Unlike the standard Rankine–Hugoniot relations we allow energy flux to escape from the overall system, which is a

standard outcome of kinetic models of cosmic-ray acceleration (Caprioli et al. 2010, for an overview). The system can be closed using the condition that the gas pressure does have a shock-jump at the sub-shock, but the cosmic-ray pressure (P_c) is continuous across the shock, which is a necessary consequence of diffusive shock acceleration (see, for example, the Appendix of Becker & Kazanas 2001), i.e., $P_{cr,1} = P_{cr,2}$. It is important to note that in the context of this model the continuity of cosmic-ray pressure across the subshock is what sets the cosmic-ray component apart from the gas component.

For a given upstream gas Mach number $M_{g,0}$, and an assumed adiabatic index, γ_{cr}, for the cosmic-ray component, the extended Rankine–Hugoniot relations give a range of solutions that can be parameterized by the cosmic-ray precursor compression ratio $\chi_{prec} \equiv \rho_1/\rho_0$. The standard Rankine–Hugoniot shock jump solutions are retrieved for $\chi_{prec} = 1$.

Here we summarize the solutions presented in Vink et al. (2010), but augmented with an additional parameter, namely the upstream cosmic-ray pressure ($P_{cr,0}$). We do this by extending the use of the fractional cosmic-ray pressure,[6]

$$w \equiv \frac{P_{cr}}{P_g + P_{cr}}, \tag{A1}$$

to the upstream region. The subscript "g" refers to the gas (thermal) component. So w in Vink et al. (2010) is now labeled w_2 and the upstream quantity is w_0.

The conservation of mass flux (ρv) and momentum flux ($P_{cr} + P_g + \rho v^2$) throughout the whole shock system can be made dimensionless by dividing pressure by the upstream ram pressure $\rho_0 V_s^2$, with $V_s(= v_0)$ the shock velocity, and using the compression factors

$$\chi_{prec} = \frac{\rho_1}{\rho_0} = \frac{v_0}{v_1}, \quad \chi_{sub} = \frac{\rho_2}{\rho_1} = \frac{v_1}{v_2},$$
$$\chi_{tot} = \chi_{prec}\chi_{sub} = \frac{\rho_2}{\rho_0} = \frac{v_0}{v_2}, \tag{A2}$$

which express mass flux conservation.

To make momentum flux conservation dimensionless it is convenient to use the definition of the gas Mach number

$$M_{g,0} \equiv \sqrt{\frac{\rho_0 V_s^2}{\gamma_g P_{g,0}}} = \frac{V_s}{c_{sound}}, \tag{A3}$$

$$M_{g,1} \equiv \sqrt{\frac{\rho_1 v_1^2}{\gamma_g P_{g,1}}} = \frac{v_1}{c_{sound}} = M_{g,0}\chi_{prec}^{-(\gamma_g+1)/2}, \tag{A4}$$

with Equation (A4) indicating that we assume that the compression of the gas in the precursor (region 1) is purely adiabatic.

The dimensionless pressures $\mathcal{P}_i$ ($i = 0, 1, 2$) are then given by the following relations

$$\mathcal{P}_0 \equiv \frac{P_{g,0} + P_{cr,0}}{\rho_0 V_s^2} = \frac{1}{\gamma_g M_{g,0}^2}\left(\frac{1}{1-w_0}\right), \tag{A5}$$

$$\mathcal{P}_1 \equiv \frac{P_{g,1} + P_{cr,1}}{\rho_0 V_s^2} = \frac{1}{\gamma_g M_{g,0}^2}\left(\frac{1}{1-w_0}\right) + \left(1 - \frac{1}{\chi_{prec}}\right), \tag{A6}$$

$$\mathcal{P}_{g,1} \equiv \frac{P_{g,1}}{\rho_0 V_s^2} = \frac{\chi_{prec}^{\gamma_g}}{\gamma_g M_{g,0}^2}, \tag{A7}$$

$$\mathcal{P}_2 \equiv \frac{P_2}{\rho_0 V_s^2} = \frac{1}{\gamma_g M_{g,0}^2}\left(\frac{1}{1-w_0}\right) + \left(1 - \frac{1}{\chi_{tot}}\right), \tag{A8}$$

$$\mathcal{P}_{g,2} \equiv \frac{P_{g,2}}{\rho_0 V_s^2} = (1-w_2)\mathcal{P}_2 = \frac{\chi_{prec}^{\gamma_g}}{\gamma_g M_{g,0}^2} + \left(1 - \frac{1}{\chi_{sub}}\right)\frac{1}{\chi_{prec}}, \tag{A9}$$

$$\mathcal{P}_{cr,2} = \mathcal{P}_{cr,1} = w_2\mathcal{P}_2. \tag{A10}$$

Equation (A9) follows from the relation $P_2 = P_1 + (1 - 1/\chi_{sub})\rho_1 v_1^2$, which is similar to Equation (A8).

The fractional pressure of cosmic-rays downstream w_2 can be derived from combining Equations (A8) and (A9),

$$w_2 = \frac{1 - (1-w_0)\chi_{prec}^{\gamma_g} + (1-w_0)\gamma_g M_{g,0}^2\left(1 - \frac{1}{\chi_{prec}}\right)}{1 + (1-w_0)\gamma_g M_{g,0}^2\left(1 - \frac{1}{\chi_{tot}}\right)}. \tag{A11}$$

Setting $w_0 = 0$ (i.e., no upstream cosmic rays) gives the expression found by Vink et al. (2010), and its asymptotic approximation ($M_{g,0} \to \infty, w_0 = 0$) is $w_2 \approx (\chi_{tot} - \chi_{sub})/(\chi_{tot} - 1)$.

To complete the set of equations we give here the sub-shock compression ratio, which is simply the standard Rankine–Hugoniot relation, applied to the gas component in region 1 (Malkov & Drury 2001; Becker & Kazanas 2001; Blasi et al. 2005):

$$\chi_{sub} = \frac{(\gamma_g + 1)M_{g,1}^2}{(\gamma_g - 1)M_{g,1}^2 + 2}. \tag{A12}$$

Equations (A2)–(A12) are sufficient to predict all shock relations, and cosmic-ray contributions, for a given value of the main variable, χ_{prec}, the precursor compression ratio. In case that $w_0 = 0$, or $w_2 \gg w_0$, w_2 provides a direct measure for the cosmic-ray acceleration efficiency. But in order to see whether the solutions are physically possible we need to evaluate whether the enthalpy flux ($[P + u + (1/2)\rho v^2]v$) is either conserved, or energy is leaking out of the system by escaping cosmic rays. In dimensionless form (i.e., dividing enthalpy by $(1/2)\rho_0 V_s^3$) we can express enthalpy (non-)conservation as

$$\left\{\frac{\gamma_g}{\gamma_g - 1}\mathcal{P}_{g,2} + \frac{\gamma_{cr}}{\gamma_{cr} - 1}\mathcal{P}_{cr,2} + \frac{1}{2}\frac{1}{\chi_{tot}}\right\}\frac{1}{\chi_{tot}}$$
$$= \left\{\frac{\gamma_g}{\gamma_g - 1}\mathcal{P}_{g,0} + \frac{\gamma_{cr}}{\gamma_{cr} - 1}\mathcal{P}_{cr,0} + (1-\epsilon)\frac{1}{2}\right\}, \tag{A13}$$

with $\epsilon \geqslant 0$, with $\epsilon = 0$ indicating enthalpy conservation (cf. Berezhko & Ellison 1999; Malkov & Drury 2001).[7]

[6] This is denoted N in Drury & Voelk (1981). Note that Becker & Kazanas (2001) uses the upstream cosmic-ray Mach number, defined as $M_{cr,0} = \sqrt{\rho_0 V_s^2/\gamma_{cr} P_{cr,0}}$. The relation between w_0 and $M_{cr,0}$ is $w_0 = 1/(1 + \gamma_{cr} M_{cr,0}^2/\gamma_g M_{g,0}^2)$.

[7] We take here that the escaping energy flux cannot exceed the free energy flux of the system ($(1/2)\rho V_s^3$).

If we write for convenience[8]

$$G_0 \equiv w_0 \frac{\gamma_{\rm cr}}{\gamma_{\rm cr}-1} + (1-w_0)\frac{\gamma_{\rm g}}{\gamma_{\rm g}-1},$$
$$G_2 \equiv w_2 \frac{\gamma_{\rm cr}}{\gamma_{\rm cr}-1} + (1-w_2)\frac{\gamma_{\rm g}}{\gamma_{\rm g}-1}, \quad \text{(A14)}$$

Equation (A13) can with the help of Equation (A8) be rewritten as

$$\epsilon = 1 + \frac{2}{\gamma_{\rm g} M^2_{\rm g,0}}\left(\frac{1}{1-w_0}\right)\left[G_0 - \frac{G_2}{\chi_{\rm tot}}\right] - \frac{2G_2}{\chi_{\rm tot}} + \frac{1}{\chi^2_{\rm tot}}(2G_2 - 1). \quad \text{(A15)}$$

APPENDIX B

SHOCK SOLUTIONS FOR PERPENDICULAR SHOCKS

In the limit of an upstream plasma that is dominated by magnetic pressure, i.e., $\beta_0 \approx 0$ and $w_0 = 0$, one can ignore the upstream gas pressure $P_{\rm g,0}$ and precursor gas pressure $P_{\rm g,1}$ in Equations (A8) and (A13), but instead one has to introduce the pressure caused by the perpendicular magnetic field component. Hence, the momentum flux conservation equation for a perpendicular, magnetically dominated, shock is approximated by

$$\frac{B^2_{\perp,0}}{8\pi} + \rho_0 V_{\rm s}^2 = P_1 + \frac{B^2_{\perp,1}}{8\pi} + \rho_1 v_1^2 = P_2 + \frac{B^2_{\perp,2}}{8\pi} + \rho_2 v_2^2, \quad \text{(B1)}$$

with $P = P_{\rm g} + P_{\rm cr}$ referring to particle induced pressure only (thermal and non-thermal).

These equations can be normalized using the Alfvén Mach number $M_{\rm A,0} \equiv V_{\rm s}/V_{\rm A} = V_{\rm s}/(B_{\perp,0}/\sqrt{4\pi\rho_0})$, using the relation

$$\mathcal{P}_0 = \frac{P_0}{\rho_0 V_{\rm s}^2} = \frac{1}{2M^2_{\rm A,0}}. \quad \text{(B2)}$$

Here and in what follows $\mathcal{P}$ refers to the total pressure, including the contribution of the magnetic field. Using the above relations, we find that

$$\mathcal{P}_2 = \frac{1}{2M^2_{\rm A,0}} + \left(1 - \frac{1}{\chi_{\rm tot}}\right). \quad \text{(B3)}$$

The pressure of the accelerated particles is on both sides of the subshock assumed to be equal, hence $P_{\rm cr,2} = P_{\rm cr,1} = w_2(P_2 + B^2_{\perp,2}/(8\pi))$, with w_2 defined in Equation (10). Together with Equation (B1) this means that

$$\mathcal{P}_{\rm cr,1} = \mathcal{P}_{\rm cr,2} = w_2\left[\frac{1}{2M^2_{\rm A,0}} + \left(1 - \frac{1}{\chi_{\rm tot}}\right)\right]. \quad \text{(B4)}$$

Assuming only adiabatic compression of the magnetic field, with $B_{\perp,1} = \chi_{\rm prec} B_{\perp,0}$ and $B_{\perp,2} = \chi_{\rm tot} B_{\perp,0}$,[9] and using the fact that $P_{\rm cr,1} = P_{\rm cr,2}$ one can relate the downstream thermal pressure to the pressure in the precursor, which gives

$$\mathcal{P}_{\rm g,2} = \frac{\chi^2_{\rm prec} - \chi^2_{\rm tot}}{2M^2_{\rm A,0}} + \frac{1}{\chi_{\rm prec}}\left(1 - \frac{1}{\chi_{\rm sub}}\right). \quad \text{(B5)}$$

[8] In principle the adiabatic index of the cosmic rays upstream may differ from that downstream, but we assume the cosmic rays are characterized by a unique number, $4/3 \leqslant \gamma_{\rm cr} \leqslant 5/3$.

[9] Note that magnetic field amplification may be important for strong Mach number shocks (see Helder et al. 2012; Schure et al. 2012, for observational and theoretical reviews).

Comparing this with Equation (B3) shows that this should be equal to

$$\mathcal{P}_{\rm g,2} = \mathcal{P}_2 - \frac{\chi^2_{\rm tot}}{2M^2_{\rm A,0}} - \mathcal{P}_{\rm cr,2}$$
$$= -\frac{\chi^2_{\rm tot}}{2M^2_{\rm A,0}} + (1-w_2)\left[\frac{1}{2M^2_A} + \left(1 - \frac{1}{\chi_{\rm tot}}\right)\right], \quad \text{(B6)}$$

which states that the downstream thermal pressure is the total pressure minus the partial pressures of the magnetic field and the accelerated particles (Equation (B4)). Combining Equations (B6) and (B5) one arrives at Equation (11), given in the main text.

Finally, in order to complete the set of equation one needs to know the compression factor of a perpendicular, $\beta_0 = 0$, shock as a function of Alfvén Mach number.

In order to determine the shock compression ratio for a perpendicular shock with $\beta_0 = 0$ one has to solve the enthalpy flux equation,

$$\frac{1}{2}\rho_2 v_2^3 + G_2 P_2 v_2 + v_2 \frac{B^2_{\perp,2}}{4\pi} = (1-\epsilon)\frac{1}{2}\rho_0 V_{\rm s}^3 + \frac{V_S B^2_{\perp,0}}{4\pi}. \quad \text{(B7)}$$

Substituting Equation (B1) into Equation (B7), one can find the following expression for energy escape

$$\epsilon = 1 + \frac{2}{M^2_{\rm A,0}} - \frac{2\chi_{\rm tot}}{M^2_{\rm A,0}} - \frac{G_2}{\chi_{\rm tot} M^2_{\rm A,0}}\left(1 - \chi^2_{\rm tot}\right)$$
$$- \frac{2G_2}{\chi_{\rm tot}}\left(1 - \frac{1}{\chi_{\rm tot}}\right) - \frac{1}{\chi^2_{\rm tot}}, \quad \text{(B8)}$$

with G_2 as defined under Equation (A15). This equation is the equivalent for Equation (A15), but now for perpendicular shocks, with $\beta = 0$.

The standard Rankine–Hugoniot solution, corresponding to $\epsilon = 0$, can be found by solving the following cubic equation

$$(G-2)\chi^3 + \left(M_{\rm A}^2 + 2\right)\chi^2 - G\left(2M_{\rm A}^2 + 1\right)\chi + (2G-1)M_{\rm A}^2 = 0, \quad \text{(B9)}$$

where the subscripts have been dropped, as this is a general shock-jump condition for a perpendicular shock with $\beta_0 = 0$. Equation (B9) has one trivial solution, $\chi = 1$, which helps to transform the cubic equation into a quadratic equation, which has one non-negative solution

$$\chi = \frac{-\left(M_{\rm A}^2 + G\right) + \sqrt{D}}{2(G-2)} = -\left(M_{\rm A}^2 + \frac{5}{2}\right) + \sqrt{D}, \quad \text{(B10)}$$

with

$$D \equiv M_{\rm A}^4 - 18G_2 M_{\rm A}^2 + 8G^2 M_{\rm A}^2 + 8M_{\rm A}^2 + G^2 = M_{\rm A}^4 + 13M_{\rm A}^2 + \frac{25}{4}, \quad \text{(B11)}$$

with the numerical values found by using $\gamma = 5/3$, which gives $G = 5/2$. Asymptotically $\chi \rightarrow 4$ for $M_{\rm A} \rightarrow \infty$, which is the shock jump condition for a strong shock.

This solution can also be used for the subshock, using $G = \gamma_{\rm g}/(\gamma_{\rm g} - 1) = 5/2$ and the Alfvénic Mach number at the sub-shock (cf. Equation (7)),

$$M^2_{\rm A,1} = \frac{1}{2}\frac{\rho_1 v_1^2}{B^2_{\perp,1}/8\pi} = M^2_{\rm A,0}\chi^{-3}_{\rm prec}. \quad \text{(B12)}$$

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