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Those Who Must Do It: The Agency of Language

Johan van Benthem, Amsterdam & Stanford

I reflect on my encounters with Frans Zwarts, and what makes language and logic meet. ¹

1 The magic of Frans Zwarts

Frans and I have been friends since our days at the Christelijk Gymnasium Sorghvliet in Den Haag. It was his winning smile that drew me to him from day one. Subsequently, I discovered many more talents, such as his literary gifts and success with the girls (not unrelated in our school), and later on, in our years at the University of Amsterdam, his talent for exact grammatical thinking, eventually enhanced by his international gloss of having studied at the MIT, the Delphi of linguistics in those days when oracles boomed.

Our professional collaboration started around 1980, as young colleagues in Groningen. Our perspectives as a logician and a linguist interested in quantifiers turned out highly congenial and complementary. I remember receiving frequent little handwritten notes from Frans who had made one more intriguing observation, and then rushing home to think about it, and find some logical explanation or background. It is triggers like this that set me on my path to generalized quantifier theory as developed in my Essays in Logical Semantics (1986). Frans asked me to be a co-supervisor for his dissertation, though with a fertile independent mind like his, this was not midwifery for book labors, but rather like sitting on the shore watching the coming of the rose-fingered dawn.

One aspect of Frans’ maturity was the range of big themes concerning language that occupied him, going far beyond the small puzzle solving and occasional tunnel vision common in those days. He was interested in the expressive power of the language system per se, its richness and limitations, he saw language use as a combined practice of stating things and making inferences (with little words like polarity items as subtle triggers), he embraced categorial grammar studies of the compositional mechanics that makes language tick, and early on, he saw fruitful connections between semantic theories and cognitive psychology, traveling frequently to the Max Planck Institute in Nijmegen long before this became a pilgrimage shrine for many people nationwide.

In all this, I was consistently struck by Frans’ easy mastery of wide arrays of linguistic facts. To me, these facts looked like a pack of wolves waiting to devour any logical theory – with Frans, they meekly licked his hands. I also admired his easy way of

¹ This is a light piece, not a scholarly effort, and I will document references only now and then.
deflecting criticism by the rearguard, complaining that his newfangled combinations with logic were not 'real linguistics'. I get the same nowadays from logicians who feel alienated by my agenda expansion to information dynamics and agency (see below), and what has helped me through many a difficult moment is the Zwarts Syllogism:

“Linguistics is those topics that are studied by prominent linguists. I am a prominent linguist. Therefore, what I am doing is linguistics.”

What also bound Frans and me together in those golden days were our interactions with the many excellent colleagues and students that were active in formal studies of natural language. The Groningen faculties of humanities, philosophy, and mathematics teemed with junior and senior talents that are still quite vivid before my mind today, such as Crit Cremer, Jan van Eijck, Petra Hendriks, Frank Heny, Jack Hoeksema, Alice ter Meulen, Sjaak de Mey, Reinhard Muskens, Victor Sanchez Valencia, Leonie ter Meulen, Henriëtte de Swart, Elias Thijssse, and Co Vet.

2 A prodigal son?

I left Groningen for Amsterdam in 1986, to succeed my own supervisor Martin Löb on the Beth Chair. Löb himself moved to rural Drenthe, telling me once that he admired people there for the absolute minimality of their speech: they never said anything redundant – though he was still not sure about "Good morning". This move offered me a unique opportunity to create an interdisciplinary environment with a group of like-minded colleagues that has now become the Institute for Logic, Language and Computation ILLC. In the decades since, I have strayed far from natural language, engaging in the mathematics of modal logic, time, space, and computation, and increasingly, the study of information-driven agency, recorded in my Norwegian-length trilogy Exploring Logical Dynamics, Logical Dynamics of Information and Interaction, and Logic in Games. ²

But nowadays, I find myself drawn increasingly to thinking about continuities with my earlier research on logic and natural language – if only, because my Ph.D. students insist on uncovering connections between my Yellow and Blue periods. I have not published much on this topic yet, partly because one treads on tiptoe when visiting the happy home of a previous spouse. However, in what follows, I offer a few thoughts. ³

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² These books on logical dynamics of agency appeared in 1996, 2011, and 2013, respectively.
³ A few sustained reflections on logic, language and agency can be found in van Benthem 2014.
3 Natural logic

One topic that shines through Frans' dissertation are the simple but powerful and ubiquitous monotonicity inference patterns that are so characteristic of major expressions in natural language. When all watches are broken, then all gold watches do not keep time: all $AB$ allows downward replacement by a stronger predicate in its left-hand position $A$ and upward replacement by a weaker predicate in its right-hand position $B$. Most quantifiers have such inferential behavior, as all educated semanticists know.

Monotonicity inferences are an instance of what I called ‘natural logic’ at the time: a system of simple inferences lying at the surface of natural language, readily available when using language, and requiring much less computational effort than deep pieces of consequence that have to come from the inner reaches of first-order logic. I eventually developed it in a categorial setting, with a calculus computing monotonicity markings for lambda terms for meanings of arbitrary expressions. This view made for interesting connections between the modern logical study of natural language and traditional logic, which can be viewed as a long series of attempts at finding monotonicity laws for an ever-expanding set of syntactic contexts. One can find sophisticated instances of this with the medieval Scholastics, but recently, it has become clear that monotonicity inferences were also recognized in classical Chinese logic around 500 B.C., and more systematically, in Avicenna’s logic in the Islamic tradition after the year 1000 A.D. 4

My view eventually became that natural logic had at least three different subsystems, that can be defined precisely in suitable categorial grammars. One is a monotonicity calculus for marking syntactic parses with the inferential potential of suitable subexpressions, the other a system for restricting and unrestricting predicate arguments to sub-domains or sub-contexts (the famous ‘conservativity’ for generalized quantifiers is one example), and the third a set of simple algebraic laws for suitable lexical items, such as associativity or symmetry, helping us to change the rhythm of statement for propositions. What is interesting here is not any conflict with standard formal systems such as first-order predicate logic. But natural logic draws the natural boundaries at different lines from the usual transitions from propositional logic to monadic predicate

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4 The monotonicity calculus and the natural logic program can be found in van Benthem 1986, 1987, van Eijck 1985, Sanchez Valencia 1991, while van Benthem 2008A is a survey of modern developments. Sommers 1982 connects to traditional logic, MacCartney 2009 to computational linguistics, Geurts & van der Slik 2005 to cognitive psychology, while Moss 2010 is a modern logical version of the program including detailed computational complexity results for fragments of natural language. Zhang & Liu 2007 have Chinese sources, and Hodges 2014 studies the Islamic tradition in depth.
logic to full predicate logic. Moreover, it makes sense just as well in higher-order logic, and does not care about crossing borders that logicians might consider dramatic leaps. Thus, what became an issue was the fine-structure of inference in natural logic, where we do not need a shovel, but the archeologist’s brush to detect the lie of the land.

Monotonicity calculus has been rediscovered independently many times, and that process is continuing up until today. There are sources in the history of traditional logic, or its attempted modern revivals, in the area of information retrieval allowing fast inferences rather than just word matching, and in cognitive studies of inferences in the brain. Recently, Larry Moss has revived the program systematically by axiomatizing a wide array of natural language fragments, determining their precise computational complexity to find thresholds where complexity jumps in the landscape of reasoning.

But an interplay remains with developments inside logic itself. Newer looks at such thresholds have also occurred inside logic itself, witness my own work with Andréka and Németi on the ‘guarded fragment’ (1998), a large decidable subset of first-order logic where quantification never ranges over arbitrary objects, but only over those objects that are reachable in some way from the current free objects of a formula through the use of ‘guard predicates’. Guarded patterns might make sense for natural language.

Thus, this encounter between logic and language from our Groningen days is far from over. We still have not charted the realm of natural reasoning in its entirety.

4 Language, inference, and automata

As I said, Frans’ work and mine looked at two tasks entangled in language use: making statements and drawing inferences. On the usual logical division of labor, this involves two different stages. First we get the grammar right, then, once we know what the well-formed expressions are, we design a proof system for these. This is the standard set-up of logical systems. But monotonicity calculus questions this design: it marks the available inferences right on the grammatical parse tree, and hence inferences get a ‘free ride’ on the syntax, making them much faster and less complex. Many further entanglements of producing or interpreting language with inference exists, and they have been noticed by other authors, notably, by Hans Kamp in his ongoing program of discourse representation theory starting from his classic 1981 paper.

Interestingly for historians, even Tarski’s famous 1936 paper on the conception of truth still has this entanglement. It says that a formal system to be interpreted semantically consists of both syntax formation rules and proof rules, that could in principle work in
parallel. Proof rules take syntactically well-formed premises to well-formed conclusions, and so, no need to do a separate grammatical analysis for the conclusion first.

Many questions can be raised here of a general methodological impact for logical studies of natural language. Frans’ work on polarity items in his dissertation and beyond suggests that language can grammaticalize inferential positions, inviting us to draw inferences when needed. How far does this inferential marking go in general linguistic communication? But also, the above phenomenon of free rides on syntax has been looked at more systematically in terms of grammar design in Ed Stabler 2011, while Icard 2011 has recently looked at concrete extensions to many new patterns of Boolean reasoning, partly going back to earlier observations by Frans.

But here, I want to draw attention to another aspect, which represents a major shift in looking at the situation. We are all used to thinking of grammars as calibrated by the Chomsky Hierarchy, or equivalently, the hierarchy of automata that fine-structure all-powerful Turing machines in levels such as finite automata, push-down store automata, and many others that modulate memory and/or available actions – often tied to complexity classes for time or space complexity. 5 Many simple grammars require push-down storage only, and hence put their available monotonicity inferences at the same low polynomial complexity, whereas full predicate logic is undecidable, requiring the much stronger symbol-manipulation powers of Turing machines. But to me, automata do not stand for mere computational implementation devices for some extensionally given notion of well-formedness or validity, but rather for intensional procedures that can be used by real linguistic agents, perhaps even for those agents themselves. This theme of the agency behind language will be my main thread in what follows.

One early use of automata in my work in the 1980s were the semantic automata for testing the truth of natural language quantifiers on given finite domains. 6 These devices brought to light a procedural semantic hierarchy, where first-order quantifiers are computed by finite state automata, whereas for instance “most” required pushdown store automata, whose general theory is included in the decidable system of additive Pressburger Arithmetic. While this style of analysis tells us something about the agency needed for dealing with lexical meanings only, recent work in Icard & Steiner-Threlkeld

2013 on nested automata in this line has looked at the semantic automata complexity of iterated quantifiers, tackling the agency behind compositional semantics. I hasten to add that powerful automata-theoretic techniques exist by now in the area of computational logic for many languages, first-order, higher-order, with fixed-points, and so on. These automata typically also deal with infinite streams of data that allow perpetual behavior (a crucial notion in studies of agency) that need not terminate at all. This may also be relevant to natural language. The usual emphasis on finite terminating linguistic tasks may lose us sight of the infinite ‘operating system’ of communication.

The automata perspective, too, raises general issues, in particular, about different types of agents meeting different kinds of linguistic tasks. For instance, if we look at a powerful monolith like first-order logic, what parts of it can actually be used successfully by automata at various levels in the standard hierarchy? Say, what could a finite automaton or a push-down store automaton achieve when allowed the use of a complete first-order deduction system like Gentzen sequent calculus, or Beth tableaux? Of course, without further precisation, such questions cannot be answered. But I do find them highly suggestive as a way of thinking about language ‘in use’ – and they remind me of work in syntax on ‘finite state approximations’ of context-free grammars (Pereire & Wright 1997), approximating real recursions by Kleene star-type iterations.

Automata viewed as agents are an attractive general perspective on natural language. They allow us to think more precisely about many issues of architecture for linguistic agents, i.e., the know-how, whether implicit or explicit, of us language users. Do our powers of interpretation and inference reside in different subautomata, composed in some convenient mathematical way? But also, when do free rides allow the same automaton to perform tasks that are traditionally thought to lie at different language levels? And what if we increase the range of basic linguistic skills for single agents to include social interactive strategies in discourse or argumentation?

The latter extension reinforces the idea of agency to a new level, as we shall see now.

5 From computation to games

In recent years, single-agent views of language have been under increasing pressure from game models stressing the interactive nature of language use. Receivers interpret messages from senders trying to achieve a stable equilibrium, general scenarios of communication are always between different parties, while even logical argumentation is naturally associated with multi-agent dialogue games. In line with this, language is
full of words and constructions that denote or trigger interactive events, with questions and answers as a key example. The result has been a wide variety of game models for many aspects of natural language use: from syntax to semantics and pragmatics. Likewise, in computer science, games have become a major model for computation, with interacting agents as the heart of what it means to produce system behavior (Abramsky 2008, Flum, Grädel & Wilke, eds., 2007).

Games are a generalization of automata that lends itself well to pursuing our earlier issues. For a concrete illustration consider the well-known ‘Ehrenfeucht-Fraïssé games’ played by two players Spoiler and Duplicator between two given models in order to investigate their analogies and differences. In each round, Spoiler picks an object in one of the models, and Duplicator responds with a matching object in the other model. Duplicator survives the round if the total match of objects so far is a partial isomorphism between the models, otherwise, Spoiler wins the round, and the game.

There is a tight match with logic here. Duplicator has a winning strategy in the Ehrenfeucht-Fraïssé game between models $M$ and $N$ over $n$ rounds iff $M, N$ satisfy the same first-order sentences up to quantifier depth $n$. Moreover, these games are ‘determined’ in the sense of game theory: if Duplicator has no winning strategy, then Spoiler has one. That strategy arises from taking any first-order formula $\varphi$ of depth $n$ that is true in one of $M, N$ and false in the other, and maintaining this invariant throughout the game by moving to suitable subformulas of $\varphi$ as the number of rounds decreases.

Now the hierarchy of automata is reflected in different restrictions we can place on players and their available strategies. Here is a concrete example. In so-called ‘pebble games’ each player gets a fixed supply of $k$ pebbles that can be placed on objects in the models to bring them into the current comparison. If one has placed all one’s pebbles, a new object can only be introduced by taking the pebble away from some earlier object. One can think of the pebbles as a Hans and Gretchen-style working memory of fixed size. Now the Duplicator has a winning strategy in the $n$-round game iff the two models agree on all first-order formulas up to depth $n$ that use only $k$ variables (free or bound).

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7 Games exist for semantic evaluation (Hintikka & Sandu 1997), lexical meaning formation (Lewis 1969, Jäger, Benz & van Rooij, eds., 2005, Clark 2012) or dialogue (Lorenzen 1955).
8 Two great sources for Ehrenfeucht-Fraïssé model comparison games are Doets 1996 and Väänänen 2011. The pebble games to be introduced in a moment come from Immerman & Kozen 1987. For computational complexity of various logical tasks, see Papadimitriou 1994.
The import of pebbling is that model checking for formulas in fixed $k$-variable fragments takes polynomial time, which is less than the general model-checking complexity of first-order logic which is $Pspace$-complete. As for satisfiability or validity, $k$-variable fragments are undecidable for $k \geq 3$, but for $k \leq 2$ they are decidable. Thus, if memory becomes very restricted, allowing us only to consider two objects together at any one time, reasoning becomes decidable. Automata with this restriction can only make their behavior depend on configurations of two objects: this is the case in many modal logics. It is easy to see then that 2-object memory players can only detect differences definable in the 2-variable fragment of first-order logic, and so we have made precise sense of the idea of a restricted agent accessing only part of a larger available formalism.

These are just some observations, and many questions remain about general uses of pebbling memory. For instance, one could also apply it to positions in proofs or stages in conversations that players are allowed to remember. Also, logicians have only looked at cases where both players get the same amount of pebbles. This uniformity is common in the study of language: agents can differ qua information or preferences, but they have the same computational powers for processing information or making inferences. What happens to language games when they are played, as often seems to be the case in practice, between agents with different powers, or with very different grasps of the language? I can think of many interesting issues here, especially given that communication does take place, even in settings with diversity of agents.

There is much more to interfaces between logic and game theory than the above topics, as one can see in my book *Logic in Games* that just came out this year. But my main point will have been made. Like automata, games show how we can place everything we studied in the 1980s in an interactive multi-agent setting, even though we never thought of it that way. And then we see all old questions in a new light.

6 Language as agency: less, but also more

What I have done so far is extrapolate some trends from work on automata and games to a view of language where the agents using it are an explicit part of the phenomenon to be described. In the earlier computer science terms: we are now after an account of linguistic ‘behavior’. But then we are not operating in a vacuum. Agents have been studied by logical means for decades by now in computer science and adjoining parts of
logic, philosophy, and game theory. And a full-fledged view of agents involves a much more systematic view of what they can do than what I have discussed so far.  

This piece is not the place to do all this in detail, but let me just summarize the view of agency taken in my own work on logical dynamics. First, agents can perform a wide range of basic informational acts, of which inferences are only one. In particular, they can also make observations, and engage in communication. A famous example is the ‘dark room’ from ancient Chinese logic (Zhang & Liu 2007), where you see an object of indeterminate color inside a dark room, you see a white object outside, and someone tells you that the two objects have the same color, after which you know that the object in the dark room is white. All three mentioned sources of information have to cooperate here – and the same is true, of course, in many of our ordinary activities.

But crucially, much of agency is not about indubitable hard information and knowledge at all, but rather about beliefs that we have, leading us to fallible soft information and transient opinions that we may have to revise. 10 Real agency is all about forming and modifying beliefs that guide our actions, perhaps even more than about accumulating knowledge – where the possibility of being wrong adds a creative element absent from mechanistic updates with indubitable information.

And even that is not a natural boundary: local steps of information update or belief revision, both in language use and in general agency, take place in a longer-term temporal setting of conversation, inquiry, and action, where we want to achieve goals over time, often with strategies for interacting with others. That brings in three more crucial dimensions of agency: its irreducibly social multi-agent character, its long-term strategic temporal perspective, and very importantly also, its entanglement between what we know, what we can do, and the essential driver for our behavior: how we evaluate the results of our actions, in other words, our preferences. 11
My own work over the last decade brings all of these ingredients together in lush 'dynamic-epistemic logics' of information and agency. 12 Such a study can be done in other frameworks, too, but my main point here is that today, we have a much richer set of tools for looking at the agency underlying natural language than anything dreamed off (at least, by me) in the 1980s. I submit that it would be of interest to develop the earlier themes concerning natural language in this broader setting, but in what follows, I will come down to earth again and just consider one aspect of agency as an illustration.

7 From correctness to correction

While traditional semantics, and our own work in the 1980s, was focused on truth and its adequate expression in rich enough languages, there have been many developments that brought information and agency into the picture. Examples are discourse representation theory and dynamic semantics, but one could cite many others. However, with few exceptions (such as the work on defaults in Veltman 1996), these approaches have focused on knowledge and what might be called hard information. But if I revisit my old themes now, one of the most striking features of natural language matches what I called one of the most telling aspects of agency in the above, our creative ability of forming beliefs beyond the given facts, the risk of being wrong, and then, crucially, our ability of correcting ourselves by changing beliefs and adapting to surprises.

In other work, I have described this in terms of a metaphor for what we expect logic to be or do. At traditional interfaces of logic and language, logic may seem to be the guardian of correctness, keeping all errors out of the way. But on the present view, the strength of our practices does not reside in perfect health, perhaps behind a screen of antibiotics, but rather in the workings of our immune system that deals with foreign invasions as they occur, and steps up body function accordingly – the way the Roman Army in its most successful years organized its flexible border defenses. I feel that the latter dynamic metaphor is much more revealing of rational agency and successful interaction, and hence my slogan: "logic is the immune system of the mind".

Consider what we would call intelligent mastery of a language. To me, that does not consists in always speaking or understanding correctly, but in the ability to detect when things go wrong, and then repair the situation. We do not need infallible mechanisms for making communication succeed all the time. What matters is that competent language users are able to perform corrections as needed. I myself would judge, for instan-

12 The earlier-cited sequence van Benthem 1996, 2011, 2014 documents this research program. The metaphor of the immune system to follow is from van Benthem 2008B.
ce, whether someone has really learnt a foreign language by how well they perform under such dynamic circumstances, not just on some test of grammaticality.

But then a lot starts shifting in what I said earlier. For instance, a capacity for revising beliefs allows us to live more dangerously, and use classically invalid default rules whose conclusions may turn out wrong. Thus, a wide spectrum of non-standard notions of consequence becomes relevant to natural language, as has been observed for many different reasons since the 1990s (cf. the Handbook chapter Thomason 1997). And this again raises new questions about my earlier enterprises.

First take the case of natural logic. If what I have just said is true, then we need to look, not just at locally available fast classical inferences, but also at fast default inferences from sentence structure. What is usually said about the structure of natural language based on Boolean inclusion should then also allow for inferences referring to default inclusion orderings where not all A are B, but A’s are normally B’s. What would be such a natural logic? I once tried some forays, but I have never found a definite answer.

But we could also look at natural logic in another place. The dynamic-epistemic logics of agency in my own work pile up new notions, and get pretty complex. On the other hand, natural language itself is our medium of choice for both engaging in agency (the ‘participating stance’) and reflecting on what we are doing (the ‘reporting sense’). Does natural language itself contain a natural surface logic of agency, far removed from the elaborate systems of computational logic or game theory, and can we extract it?

Similar points can be made about semantic automata and related devices. These originally served to find fine-structure of simpler or more complex agents inside universal devices for language. But now things appear in a different light. First, while many automata had fewer powers than idealized ones, reflecting the idea of bounded agency, we now saw that limited agents may also have more powers than have been traditionally acknowledged, namely, for revision and correction. What would be an automata theory for machines or procedures that can be wrong, correct themselves, and behave optimally, rather than ideally? I submit that such a theory would get closer to how we really use language – though I have no idea at present what it will look like.

8 Scientific conclusion

In this brief and yet both grandiose and rambling narrative, I have sketched a view where all of the logic-and-language themes that Frans and I worked on in the 1980s make eminent sense, but then in the light of modern perspectives on agency, not just of
linguistic structure as usually conceived in the Montagovian tradition we came from. I have outlined a number of issues that emerge then, whose exploration I find challenging and worth pursuing, and I have indicated a few concrete technical questions where these might lead, such as natural logic or automata theory with default rules.

I have also suggested that broader conceptual issues are worth thinking about here. One was the possible primacy of linguistic agents when we shift from what natural language can express to what agents can do. This reminds me of the old Chomsky-Piaget debate about what comes first: language structure, or general cognitive ability. One might even raise some new empirical questions, for instance about the core vocabulary of information flow and agency as described above. This might cut the cake very differently from existing classifications in semantics and pragmatics. For instance, I am working on a surface logic of agency where the key linguistic expressions in the natural logic of decision making are “good”, “bad”, “better”, “improve”, “hope”, and “fear”. More globally, the interplay of agency and language structure might give rise to empirical evolutionary questions: does every kind of agent get the language it deserves, or can handle – making its cognitive burden never more than it can bear?

I feel it enriches our view of language if we can look at matters from both these sides.

9 Personal conclusion

I have no idea where Frans’ thinking stands today on the above matters. I do know that he likes history the way it was, witness his supervising a thesis about the rise of formal semantics in The Netherlands (see van der Beek 2001). But I also see a forward-looking streak. His work has always been groundbreaking, innovative, and free from turf prejudices – opening up paths into the future for many others to follow. In all then, I am not sure that I can claim Frans’ blessing for the paths I have advertized here. But what I am hoping for at least is one typical Zwarts smile, youthful and warm as ever.

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