



## UvA-DARE (Digital Academic Repository)

### Alternatives in Montague grammar

Ciardelli, I.; Roelofsen, F.

**Publication date**

2015

**Document Version**

Final published version

**Published in**

Proceedings of Sinn und Bedeutung 19

[Link to publication](#)

**Citation for published version (APA):**

Ciardelli, I., & Roelofsen, F. (2015). Alternatives in Montague grammar. In E. Csipak, & H. Zeijlstra (Eds.), *Proceedings of Sinn und Bedeutung 19* (pp. 161-178). Georg-August-University. <https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/view/227>

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

# Alternatives in Montague Grammar<sup>1</sup>

Ivano Ciardelli — *Institute for Logic, Language, and Computation, University of Amsterdam*

Floris Roelofsen — *Institute for Logic, Language, and Computation, University of Amsterdam*

**Abstract.** The type theoretic framework for natural language semantics laid out by Montague (1973) forms the cornerstone of formal semantics. Hamblin (1973) proposed an extension of Montague’s basic framework, referred to as *alternative semantics*. In this framework, the meaning of a sentence is not taken to be a single proposition, but rather a *set* of propositions—a set of *alternatives*. While this more fine-grained view on meaning has led to improved analyses of a wide range of linguistic phenomena, it also faces a number of problems. We focus here on two of these, in our view the most fundamental ones. The first has to do with how meanings are *composed*, i.e., with the type-theoretic operations of function application and abstraction; the second has to do with how meanings are *compared*, i.e., the notion of entailment. Our aim is to reconcile what we take to be the essence of Hamblin’s proposal with the solid type-theoretic foundations of Montague grammar, in such a way that the observed problems evaporate. Our proposal partly builds on insights from recent work on inquisitive semantics (Ciardelli et al. 2013), and it also further advances this line of work, specifying how the inquisitive meaning of a sentence, as well as the set of alternatives that it introduces, may be built up compositionally.

**Keywords:** alternative semantics, inquisitive semantics, compositionality, entailment.

## 1. Introduction

Alternative semantics (Hamblin 1973; Rooth 1985; Kratzer and Shimoyama 2002, among others) diverges from the standard, Montagovian framework for natural language semantics in that the semantic value of an expression is taken to be a *set* of objects in the expression’s usual domain of interpretation, rather than a single object. For instance, the semantic value of a complete sentence is not a proposition but a *set* of propositions, the semantic value of an individual-denoting expression is not an individual but a *set* of individuals, and so on. Alternative semantics has been fruitfully applied to a range of linguistic phenomena, including questions (Hamblin 1973),

---

<sup>1</sup>This paper has been presented at the Fifth Questions in Discourse Workshop in Stuttgart, the Logic and Language Seminar in Tilburg, and the Nineteenth Sinn und Bedeutung Conference in Göttingen. We are grateful to Reinhard Muskens, Daniel Gutzmann, Wataru Uegaki, and Yimei Xiang for helpful questions and comments on these occasions. We are also grateful to Lucas Champollion, Liz Coppock, Donka Farkas, Andreas Haida, Edgar Onea, Anna Szabolcsi, and Matthijs Westera for extensive discussion of the ideas presented here. We are especially grateful to Jeroen Groenendijk, with whom we have worked very closely over the past few years, and to Maria Aloni and Nadine Theiler, with whom we have collaborated to develop a fully compositional inquisitive semantics. The present paper was written in parallel with Nadine’s MSc thesis (Theiler 2014), as well as a joint paper (Roelofsen, Theiler, and Aloni 2014) developing an account of declarative and interrogative complements in inquisitive semantics. While the focus of these works is somewhat different, they all contribute to the same overall goal. Finally, financial support from the Netherlands Organization for Scientific Research (NWO) is gratefully acknowledged.

focus (Rooth 1985), indeterminate pronouns (Shimoyama 2001; Kratzer and Shimoyama 2002), indefinites (Kratzer and Shimoyama 2002; Menéndez-Benito 2005; Aloni 2007), and disjunction (Simons 2005; Alonso-Ovalle 2006; Aloni 2007).<sup>2</sup> While this wealth of applications shows that alternatives are a useful tool in the semantic analysis of natural language, the move from the basic Montagovian framework to an alternative-based one also raises some fundamental issues. In this paper, we will be concerned with two such issues, in our opinion the most basic ones. The first issue, which we will refer to as the *compositionality issue*, has to do with the fact that in alternative semantics, meanings can no longer be composed by means of the standard type-theoretic operations of function application and abstraction. The second, which we will refer to as the *entailment issue*, has to do with the fact that meanings in alternative semantics can no longer be compared by means of the standard type-theoretic notion of entailment. Both problems concern very fundamental features of the semantic framework, and moreover, as we shall see, neither of them has a straightforward direct solution.

Rather than looking harder for a solution, our strategy will be to take a step back and examine why these problems arise in the first place. We will argue that it is not the presence of alternatives *per se* that is to be held responsible, but rather some specific features of the architecture of alternative semantics. We argue that these features are not essential, and that by making different architectural choices it is possible to obtain a framework in which the observed problems do not arise.

The paper has a straightforward structure: Section 2 is concerned with the compositionality issue, Section 3 with the entailment issue, and Section 4 concludes.

## 2. Compositionality

In the standard Montagovian framework, the semantic value of an expression  $\alpha$  of type  $\tau$  (notation:  $\alpha : \tau$ ) relative to an assignment  $g$  is an object  $\llbracket \alpha \rrbracket_g$  in the corresponding domain  $D_\tau$ , where the basic types  $e$ ,  $t$ , and  $s$  correspond to primitive domains of individuals, truth-values and possible worlds, respectively, and a derived type  $\langle \sigma, \tau \rangle$  corresponds to the domain  $D_{\langle \sigma, \tau \rangle} = \{f \mid f : D_\sigma \rightarrow D_\tau\}$  of functions from objects of type  $\sigma$  to objects of type  $\tau$ . This setup allows us to compose meanings through the basic type-theoretic operations of function application and abstraction:

- (1) **Function Application:** if  $\alpha : \langle \sigma, \tau \rangle$  and  $\beta : \sigma$  then  $\llbracket \alpha(\beta) \rrbracket_g = \llbracket \alpha \rrbracket_g(\llbracket \beta \rrbracket_g) \in D_\tau$
- (2) **Abstraction:** if  $\alpha : \tau$  and  $x : \sigma$  then  $\llbracket \lambda x. \alpha \rrbracket_g$  is the function mapping any  $x \in D_\sigma$  to  $\llbracket \alpha \rrbracket_{g[x/x]}$   
In the meta-language we will use  $\lambda x. \llbracket \alpha \rrbracket_{g[x/x]}$  as a shorthand description of this function.<sup>3</sup>

<sup>2</sup>We will concentrate here on the role of alternatives at the level of *ordinary semantic values*; the use of alternative semantics to represent *focus semantic values* (Rooth 1985) is beyond the immediate scope of the paper.

<sup>3</sup>Our general typographic convention is to use boldface for expressions in the object language ('logical form'), and the standard font for meta-language descriptions of semantic objects.

By contrast, in alternative semantics the semantic value  $\llbracket \alpha \rrbracket_g$  of an expression  $\alpha : \tau$  is no longer a single object in  $D_\tau$ , but rather a *set* of such objects:  $\llbracket \alpha \rrbracket_g \subseteq D_\tau$ .<sup>4</sup> As a consequence, meanings can no longer be composed by means of the standard type theoretic operations. Let us see why.

## 2.1. Composition in alternative semantics

**Function application.** First consider the operation of function application. Suppose  $\alpha$  is an expression of type  $\langle \sigma, \tau \rangle$  and  $\beta$  an expression of type  $\sigma$ . In alternative semantics, we have that  $\llbracket \alpha \rrbracket_g \subseteq D_{\langle \sigma, \tau \rangle}$  and  $\llbracket \beta \rrbracket_g \subseteq D_\sigma$ . Now suppose we want to compute the meaning of  $\alpha(\beta)$ . We can no longer obtain  $\llbracket \alpha(\beta) \rrbracket_g$  by simply applying  $\llbracket \alpha \rrbracket_g$  to  $\llbracket \beta \rrbracket_g$ , because  $\llbracket \alpha \rrbracket_g$  is not a function.<sup>5</sup> Thus, the type-theoretic rule of function application cannot be used to compute  $\llbracket \alpha(\beta) \rrbracket_g$ .

Instead,  $\llbracket \alpha \rrbracket_g$  is now a *set* of functions from objects of type  $\sigma$  to objects of type  $\tau$ . Since  $\llbracket \beta \rrbracket_g$  is a set of objects of type  $\sigma$ , what we *can* naturally do is apply each function  $f \in \llbracket \alpha \rrbracket_g$  to each object  $d \in \llbracket \beta \rrbracket_g$ . The set of all objects  $f(d)$  obtained in this way is a subset of  $D_\tau$ , and thus a suitable semantic value for  $\alpha(\beta)$ . This operation, known as *pointwise function application*, is indeed taken to be the fundamental composition rule in alternative semantics.

- (3) **Pointwise function application:** if  $\alpha : \langle \sigma, \tau \rangle$  and  $\beta : \sigma$   
then  $\alpha(\beta) : \tau$  and  $\llbracket \alpha(\beta) \rrbracket_g = \{f(d) \mid f \in \llbracket \alpha \rrbracket_g \text{ and } d \in \llbracket \beta \rrbracket_g\}$

However, this rule has an important drawback. In computing the meaning of a complex expression  $\alpha(\beta)$  using pointwise function application, the functor  $\alpha$  only has access to each alternative for  $\beta$  in isolation; it does not have access to the whole set at once. But in fact, many functors in natural language *do* need access to the whole set of alternatives introduced by their argument at once. Take for instance negation. The standard treatment of sentential negation in alternative semantics is as follows (see, e.g., Kratzer and Shimoyama 2002):

- (4)  $\llbracket \text{not } \beta \rrbracket_g = \{\overline{\llbracket \beta \rrbracket_g}\}$       where  $\overline{\llbracket \beta \rrbracket_g}$  denotes the set-theoretic complement of  $\llbracket \beta \rrbracket_g$

To determine  $\overline{\llbracket \beta \rrbracket_g}$ , **not** clearly needs access to all the alternatives for  $\beta$  at once. This result is impossible to obtain by associating negation with a set of objects  $\llbracket \text{not} \rrbracket_g \in D_{\langle t, t \rangle}$  and letting them

<sup>4</sup>In some work on alternative semantics, the types that expressions are usually taken to have are systematically adapted: expressions that are usually taken to be of type  $\tau$  are now rather taken to be of type  $\langle \tau, t \rangle$  (see, e.g., Shan 2004; Novel and Romero 2010). The usual correspondence between the type of an expression and its semantic value is then preserved. In other work, the usual types are preserved: expressions that are usually taken to be of type  $\tau$  are still taken to be of type  $\tau$  (see, e.g., Kratzer and Shimoyama 2002; Alonso-Ovalle 2006). In this case, the correspondence between the type of an expression and its semantic value changes: the semantic value of an expression of type  $\tau$  is no longer a single object in  $D_\tau$ , but rather a set of such objects. The choice between these two options seems immaterial; for concreteness we assume the second, but our arguments do not hinge on this assumption.

<sup>5</sup>To be more precise, the problem is that  $\llbracket \alpha \rrbracket_g$  is not a function of the *right type*. We can of course construe the set  $\llbracket \alpha \rrbracket_g \subseteq D_{\langle \sigma, \tau \rangle}$  as a function from  $D_{\langle \sigma, \tau \rangle}$  to  $\{0, 1\}$ ; but since  $\llbracket \beta \rrbracket_g \notin D_{\langle \sigma, \tau \rangle}$ , this does not help.

combine with the alternatives for  $\beta$  by pointwise function application. Thus, negation needs to be treated *syncategorematically*, that is, by means of a tailor-made rule in the grammar.

This problem is not confined to a few exceptional cases: in fact, the class of operators that need access to the whole set of alternatives for their argument includes virtually all operators that are interesting from an alternative semantics perspective: modals (e.g., Simons 2005; Aloni 2007), conditionals (e.g., Alonso-Ovalle 2006), exclusive strengthening operators (e.g., Menéndez-Benito 2005; Alonso-Ovalle 2006; Roelofsen and van Gool 2010), existential and universal closure operators (e.g., Kratzer and Shimoyama 2002), and even question-embedding verbs. Adopting pointwise function application as our fundamental composition rule implies that none of these operators can be given a meaning of their own. Instead, they all have to be treated by means of tailor-made, syncategorematic composition rules. Clearly, this is undesirable: we would like our grammar to contain only a few, general composition rules, and we would like the contribution of a specific linguistic item to be derived from its lexical meaning, based on these general rules.

**Abstraction.** Now let us consider abstraction. Suppose  $\alpha : \tau$  contains a variable  $x : \sigma$ , and suppose we want to abstract over  $x$  to obtain an expression  $\lambda x. \alpha$  of type  $\langle \sigma, \tau \rangle$ . This is an operation that is often used in semantics, typically (though not exclusively) in order to deal with quantification. What semantic value should we assign to  $\lambda x. \alpha$ ? We cannot apply the standard abstraction rule, which would identify  $\llbracket \lambda x. \alpha \rrbracket_g$  with the function mapping every  $x \in D_\sigma$  to  $\llbracket \alpha \rrbracket_{g[x/x]}$ . For, that would be a function from  $D_\sigma$  to subsets of  $D_\tau$ . But what we need for  $\llbracket \lambda x. \alpha \rrbracket_g$  is a different object, namely, a *set of functions* from  $D_\sigma$  to  $D_\tau$ , since we want that  $\llbracket \lambda x. \alpha \rrbracket_g \subseteq D_{\langle \sigma, \tau \rangle}$ . Thus, standard abstraction cannot be applied in alternative semantics.

Is there an alternative-friendly version of the abstraction rule? In other words, is there a satisfactory way to define which functions should belong to the set  $\llbracket \lambda x. \alpha \rrbracket_g$ ? A natural candidate is the following, proposed by Kratzer and Shimoyama (2002):

$$(5) \quad \llbracket \lambda x. \alpha \rrbracket_g := \{f : D_\sigma \rightarrow D_\tau \mid \text{for any } x \in D_\sigma, f(x) \in \llbracket \alpha \rrbracket_{g[x/x]}\}$$

However, Shan (2004) has pointed out that this proposal, combined with the standard techniques for quantification, leads to problematic empirical predictions. He furthermore argued that it is impossible to obtain the right set of functions in a principled way, and that an alternative-based notion of meaning therefore calls for a variable-free approach to meaning composition (Szabolcsi 1989; Jacobson 1999), which does entirely without abstraction. Novel and Romero (2010) argue that the cases which Shan deemed problematic could in fact be dealt with by enriching the underlying type theory with a new basic type for assignments, following Poesio (1996), and making certain assumptions about the meaning of wh-indefinites. Charlow (2014), however, points out that this remedy still fails for cases where the abstraction operator binds into an indefinite in its scope.

We will not make a direct contribution to this debate. Instead, we will take a more conservative approach, and ask whether it is at all necessary to depart from the standard abstraction mechanism.

## 2.2. Composing alternatives using standard composition rules

In our view, the feature of alternative semantics that is responsible for its empirical success is the fact that sentences are taken to express *sets of propositions*, rather than single propositions. This yields a notion of sentence meaning that is more structured than the standard, truth-conditional notion, and this extra structure seems to play a key role in a range of linguistic phenomena.

However, alternative semantics does not just assume that sentences express sets of propositions: it goes on to assume that *every expression* denotes a set of objects in the corresponding domain. As we have seen, this stronger assumption forces us to depart from the standard composition rules.

There does not seem to be any particular conceptual motivation for the assumption that every expression denotes a set of objects. Moreover, in linguistic applications of the framework the assumption does not seem essential, as we will show in a moment for some concrete cases. Most importantly, if we discharge this stronger assumption, then it becomes apparent that the remaining, more fundamental assumption, i.e., that sentences express sets of propositions, is perfectly compatible with the standard type-theoretic operations of meaning composition. We will demonstrate this by sketching a framework that is based on the following three assumptions:

1. the semantic value of a complete sentence is a set of propositions;
2. the semantic value of an expression of type  $\tau$  is a single object in  $D_\tau$ ;
3. the fundamental composition rules are the standard type-theoretic ones.

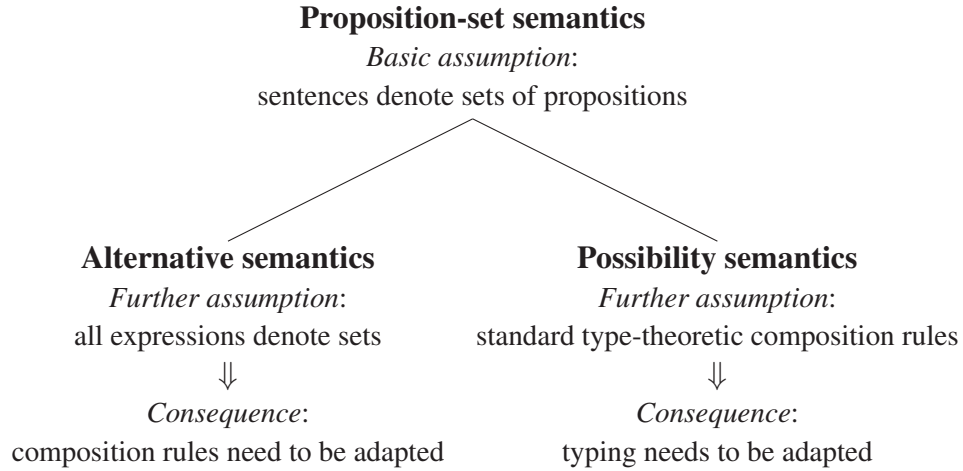
In this framework, which we will refer to as *possibility semantics*, it is not the compositional machinery, but rather the *typing of expressions* that needs to be adjusted. For instance, consider a complete sentence  $\alpha$ . By assumption (1), its semantic value  $\llbracket \alpha \rrbracket_g$  should be a set of propositions. Moreover, by assumption (2),  $\llbracket \alpha \rrbracket_g$  will be an object in the domain  $D_\tau$  of the corresponding type. Thus, we must take sentences to be of a type  $\tau$  such that the objects in  $D_\tau$  are sets of propositions: this is the type  $\langle \langle s, t \rangle, t \rangle$ , which we will abbreviate for convenience as  $T$ .

Assuming standard syntactic structures for sentences, we can then use assumption (3) to reverse engineer the types that should be assigned to various sorts of sub-sentential expressions. For instance, the following types suggest themselves for verbs, sentential operators, and quantifiers.

- |                                                             |                                                          |                                                                |
|-------------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------------|
| (6) a. <b>walks</b> : $\langle e, T \rangle$                | c. <b>not</b> : $\langle T, T \rangle$                   | e. <b>everyone</b> : $\langle \langle e, T \rangle, T \rangle$ |
| b. <b>likes</b> : $\langle e, \langle e, T \rangle \rangle$ | d. <b>or</b> : $\langle T, \langle T, T \rangle \rangle$ | f. <b>who</b> : $\langle \langle e, T \rangle, T \rangle$      |

Thus, the relation between alternative semantics and possibility semantics may be represented succinctly as follows.

(7)



Now let us consider the actual meanings that should be assigned to expressions in possibility semantics. In alternative semantics, a basic sentence like **John walks** is taken to express the singleton set  $\{|Wj|\}$ , which has as its unique element the proposition that John walks, i.e., the set  $|Wj| = \{w \mid j \text{ walks in } w\}$ . This treatment may be adopted in possibility semantics as well. Then, using assumption (3) again, we can work backwards to infer what meanings should be assigned to sub-sentential constituents. For instance, this procedure suggests the following entry for **walks**:

$$(8) \quad \llbracket \mathbf{walks} \rrbracket = \lambda x. \{|Wx|\} = \lambda x. \{\{w \mid x \text{ walks in } w\}\}$$

Suitable meanings for other sub-sentential constituents may be inferred similarly starting from the desired sentential meanings.

Let us now show by means of two examples how theories formulated in alternative semantics may be reproduced in possibility semantics. First, consider disjunction. The treatment advocated by Simons (2005), Alonso-Ovalle (2006), and Aloni (2007), which has it that  $\llbracket \alpha \text{ or } \beta \rrbracket = \llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket$ , makes disjunction an alternative-generating operation. E.g., for **John sings or Mary dances** we get two separate alternatives, one for each disjunct, rather than just one disjunctive alternative.

$$(9) \quad \llbracket \mathbf{John sings or Mary dances} \rrbracket = \llbracket \mathbf{J sings} \rrbracket \cup \llbracket \mathbf{M dances} \rrbracket = \{|Sj|\} \cup \{|Dm|\} = \{|Sj|, |Dm|\}$$

This may be reproduced *categorically* in possibility semantics simply by associating sentential disjunction with its familiar meaning:  $\llbracket \mathbf{or} \rrbracket = \lambda \mathcal{P}_T. \lambda \mathcal{Q}_T. \mathcal{P} \cup \mathcal{Q}$ .

Since we dropped the assumption that all expressions denote sets, one may wonder how disjunctions of sub-sentential constituents can be handled in possibility semantics. To see this, consider the sentence **John sings or dances**. In alternative semantics, for the disjunctive VP we have:

$$(10) \quad \llbracket \mathbf{sing or dance} \rrbracket = \llbracket \mathbf{sing} \rrbracket \cup \llbracket \mathbf{dance} \rrbracket = \{\lambda x. |Sx|\} \cup \{\lambda x. |Dx|\} = \{\lambda x. |Sx|, \lambda x. |Dx|\}$$

This set of properties then combines pointwise with  $\llbracket \mathbf{John} \rrbracket = j$ , yielding  $\{|Sj|, |Dj|\}$ . Notice that the disjunctive verb phrase expresses a *set* of properties here. Thus, the alternatives that eventually emerge at the sentential level are already clearly visible at the verb phrase level.

In possibility semantics, the final result is the same, but it is obtained in a different way. We simply assume that disjunction is given its standard cross-categorical meaning:  $\llbracket \mathbf{or} \rrbracket = \lambda \mathcal{P}_\tau. \lambda \mathcal{Q}_\tau. \mathcal{P} \cup \mathcal{Q}$  for any conjoinable type  $\tau$ .<sup>6</sup> The verb phrase is then interpreted as follows:<sup>7</sup>

$$\begin{aligned} (11) \quad \llbracket \mathbf{sing or dance} \rrbracket &= \llbracket \mathbf{sing} \rrbracket \cup \llbracket \mathbf{dance} \rrbracket = \lambda x. \{|Sx|\} \cup \lambda x. \{|Dx|\} \\ &= \{\langle x, p \rangle \mid p = |Sx|\} \cup \{\langle x, p \rangle \mid p = |Dx|\} \\ &= \{\langle x, p \rangle \mid p = |Sx| \text{ or } p = |Dx|\} \\ &= \lambda x. \{|Sx|, |Dx|\} \end{aligned}$$

This function then combines by means of standard function application with  $\llbracket \mathbf{John} \rrbracket = j$ , yielding  $\{|Sj|, |Dj|\}$ . Notice that in this case, the verb phrase does not express a set of properties, i.e., a set of functions from individuals to propositions, but rather a single function from individuals to sets of propositions. These sets of propositions only fully emerge at the sentential level. However, at the VP level they are already latently present, so to speak: the VP expresses an alternative-generating function, i.e., a function that, for any given input, produces a set of alternative propositions. Precisely because of this shift in perspective, there is no need for pointwise function application.

As a second example, consider Hamblin (1973)'s account of wh-questions, for which alternative semantics was originally developed. Hamblin assumes that **who** is of type  $e$ , but rather than denoting a single individual, it denotes the whole set of (human) individuals in the domain. By combining this denotation pointwise with, e.g. the meaning of **sing**,  $\lambda x. \{|Sx|\}$ , Hamblin obtains the meaning of **who sings**, namely,  $\{|Sx| \mid x \in D_e\}$ . The same result may be obtained in possibility semantics without assuming that all expressions denote sets. However, **who** cannot be taken to have type  $e$  in this setting, because that would mean that its semantic value is a specific individual. Instead, it has to be treated as a generalized quantifier, with type  $\langle\langle e, T \rangle, T \rangle$ :

$$(12) \quad \llbracket \mathbf{who} \rrbracket = \lambda P_{\langle e, T \rangle}. \bigcup_{x \in D_e} P(x)$$

In words, the function denoted by **who** takes a function  $P$  from individuals to sets of propositions, and returns the set consisting of all propositions which belong to the output of  $P$  for some input individual  $x$ . It is easy to see that applying this function to the meaning of **sing**, or to anything of the same semantic type, results precisely in the meaning that Hamblin obtained by pointwise

<sup>6</sup>The set of conjoinable types is defined by the following two clauses: (i)  $t$  is a conjoinable type; (ii) if  $\beta$  is a conjoinable type, then  $\langle \alpha, \beta \rangle$  is a conjoinable type. Essentially, conjoinable types are types whose objects may be identified with sets of some kind. See, e.g., Partee and Rooth (1983) for more details.

<sup>7</sup>As customary in type-theory, we identify a set with the corresponding characteristic function. Thus, an object of type  $\langle \sigma, \langle \tau, t \rangle \rangle$  may be equivalently regarded as a function from  $D_\sigma$  to sets of objects in  $D_\tau$ , or as a set of pairs in  $D_\sigma \times D_\tau$ . In particular, a function from individuals to sets of propositions may be identified with a set of individual-proposition pairs. Such functions may thus be combined by union, intersection, and other set-theoretic operations.

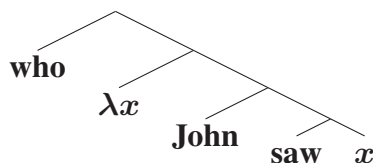


function application.

Finally, let us verify that the compositionality issues that we discussed above for alternative semantics no longer arise for possibility semantics. First, since meanings are composed by means of standard function application, rather than pointwise function application, in possibility semantics there is nothing that prevents a categorematic treatment of operators that need access to the whole set of alternatives generated by their argument. After all, the input to the functor is now the entire set of alternatives, rather than each alternative in isolation. To illustrate this, consider again negation. Sentential negation now has type  $\langle T, T \rangle$ , that is, it expresses a function that takes a set of propositions into a new set of propositions. We obtain the desired result simply by defining  $\llbracket \text{not} \rrbracket = \lambda \mathcal{P}_T. \{ \overline{\mathcal{P}} \}$ , and letting negation combine directly with its argument by standard function application. Thus, the problem with pointwise function application no longer arises.

Moreover, in possibility semantics there is no need to devise a special abstraction rule: the standard rule works fine. To see this, consider the following syntactic tree for **who did John see**:

(13)



Assume that  $\llbracket \text{John} \rrbracket_g = j$ ,  $\llbracket \text{saw} \rrbracket_g = \lambda x. \lambda y. \{ |S y x| \}$ , and  $\llbracket x \rrbracket_g = g(x)$ . By function application we get that  $\llbracket \text{John saw } x \rrbracket_g = \{ |S j g(x)| \}$ , a set containing a single proposition. Now,  $\lambda x$  is interpreted by means of the standard abstraction rule, which yields  $\llbracket \lambda x \text{ John saw } x \rrbracket_g = \lambda x. \{ |S j x| \}$ . This constituent is of type  $\langle e, T \rangle$ , i.e., it expresses a function from individuals to sets of propositions. Applying the above entry for  $\llbracket \text{who} \rrbracket_g$  to this function yields the set of propositions  $\{ |S j x| \mid x \in D_e \}$ , as desired. Abstraction is unproblematic here, because it needs to deliver a single function from individuals to sets of propositions, rather than a set of functions from individuals to propositions.

Although we only gave a very minimal sketch of a full-fledged Montagovian fragment, we hope it suffices to illustrate that theories which have been formulated in alternative semantics may generally be reproduced straightforwardly in possibility semantics.<sup>8</sup> This allows us to handle the same phenomena in a mathematically more well-behaved setting, and frees us from the problems de-

<sup>8</sup>An exception is the use of alternatives as a device for scope-taking (Shimoyama 2001), which derives the exceptional scoping ability of Japanese indeterminate pronouns as a consequence of pointwise function application. However, as Charlow (2014, p.149-150) points out, this strategy does not provide us with a general account of exceptional scope. For instance, it cannot deal with cases where multiple disjunctions/indefinites appear together in the same environment and only one of them takes exceptional wide scope (e.g., *Bill denied the rumor that a friend of his speaks a Bantu language*). A full-fledged alternative for Shimoyama's account of Japanese indeterminate pronouns, and exceptional scope-taking more generally, is beyond the scope of the present paper, but various options would be compatible with our general framework.

scribed above: first, since function application is no longer pointwise, operations that need access to the whole set of alternatives generated by their argument can be given a categorematic treatment; and second, we no longer need to look for a clever alternative-friendly abstraction rule.

### 3. Entailment

Type theory does not only come with the operations of function application and abstraction which are used to compose meanings; it also comes with a notion of entailment which is used to *compare* meanings. This notion amounts to set-theoretic inclusion, and it applies cross-categorically to expressions of any conjoinable type. This general notion of entailment also gives rise to a principled cross-categorial treatment of conjunction and disjunction. Namely, if  $\alpha$  and  $\beta$  are expressions of any conjoinable type, then their conjunction  $\alpha$  **and**  $\beta$  may be taken to denote the *meet*, i.e., the *greatest lower bound*, of  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  with respect to entailment. Dually, the disjunction  $\alpha$  **or**  $\beta$  may be taken to denote the *join*, i.e., the *least upper bound*, of  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  with respect to entailment.<sup>9</sup> It is easy to see that, for any two expressions  $\alpha$  and  $\beta$  of a conjoinable type, the meet of  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  with respect to  $\subseteq$  always exists, and amounts simply to the intersection  $\llbracket \alpha \rrbracket \cap \llbracket \beta \rrbracket$ ; and similarly, the join of  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  exists and amounts to the union  $\llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket$ .

Just like the composition rules of function application and abstraction, the cross-categorial treatment of entailment, as well as the cross-categorial treatment of conjunction and disjunction as meet and join operations that it gives rise to, are crucial features of the type-theoretic framework, which should not be lost in the process of moving to a more fine-grained notion of meaning.

Unfortunately, in both alternative semantics and possibility semantics, the standard notion of entailment as set inclusion no longer gives sensible results. To see this, consider two basic sentences such as **John walks** and **John moves**: intuitively, the first sentence entails the second. In a classical semantic framework, this is captured by the type-theoretic notion of entailment:  $\llbracket \mathbf{John\ walks} \rrbracket$  is the set  $|Wj|$  of worlds where John walks, and  $\llbracket \mathbf{John\ moves} \rrbracket$  is the set  $|Mj|$  of worlds where John moves; since every world in which John walks is also a world in which John moves, we have  $|Wj| \subseteq |Mj|$ , and the entailment is predicted. However, in both alternative semantics and possibility semantics we have  $\llbracket \mathbf{John\ walks} \rrbracket = \{|Wj|\}$  and  $\llbracket \mathbf{John\ moves} \rrbracket = \{|Mj|\}$ ; since  $\{|Wj|\} \not\subseteq \{|Mj|\}$ , the entailment is not predicted.<sup>10</sup>

The general type-theoretic treatment of conjunction as intersection no longer gives desirable results in alternative/possibility semantics either. For instance, we would expect the conjunction **John sings and Mary dances** to express the singleton  $\{|Sj \wedge Dm|\}$ , which has as its unique alternative the proposition that John sings and Mary dances. However, treating conjunction as inter-

<sup>9</sup>Formally, the meet of  $a$  and  $b$  with respect to a partial order  $\leq$  is an element  $c$  such that (i)  $c \leq a$ ,  $c \leq b$  and (ii) for any  $d$  such that  $d \leq a$  and  $d \leq b$  it holds that  $d \leq c$ . Similarly for join. See, e.g., Keenan and Faltz (1985); Winter (2001); Roelofsen (2013a) for more background on these algebraic notions and their linguistic relevance.

<sup>10</sup>This problem was first pointed out by Groenendijk and Stokhof (1984), who gave it as an argument against Hamblin's theory of questions. But as argued here, the problem in fact concerns alternative semantics more generally.

section yields an absurd meaning:

$$(14) \quad \llbracket \mathbf{John\ sings\ and\ Mary\ dances} \rrbracket = \{|Sj|\} \cap \{|Dm|\} = \emptyset$$

Just like for the compositionality problem, there are two ways to react to this problem: we may try to replace the standard type-theoretic notions of entailment and conjunction with pointwise counterparts which make suitable predictions in the alternative/possibility semantics framework; or, alternatively, we may reconsider some assumptions of our setup so that the standard type-theoretic notions may be recovered. We will first consider the first option, i.e., to define *pointwise* notions of entailment and conjunction. We will find, however, that this is still problematic, and then turn to the second approach.

### 3.1. Pointwise entailment and conjunction

**Pointwise entailment.** Let us consider again the example illustrating the failure of standard entailment in alternative/possibility semantics: the problem is that the set of alternatives expressed by **John walks** is not a subset of the set of alternatives expressed by **John moves**; however, notice that the unique alternative for **John walks** is a subset of the unique alternative for **John moves**. This suggests that, instead of comparing the whole *set* of alternatives, in alternative/possibility semantics we should really be comparing the individual alternatives in the sets. More precisely, we may define entailment as pointwise inclusion:  $\alpha$  entails  $\beta$  in case every alternative for  $\alpha$  is included in some alternative for  $\beta$ :

$$(15) \quad \alpha \models \beta \stackrel{def}{\iff} \forall p \in \llbracket \alpha \rrbracket \exists q \in \llbracket \beta \rrbracket \text{ such that } p \subseteq q$$

This notion of entailment would indeed make the right predictions for basic cases: for instance, since the unique alternative for **John walks**,  $|Wj|$ , is included in the unique alternative for **John moves**,  $|Mj|$ , we would now correctly predict that **John walks**  $\models$  **John moves**.

However, as discussed in Roelofsen (2013a), there is a fundamental problem with this notion. Namely, entailment defined in this way does not amount to a *partial order* on the space of meanings. In particular, it is not *anti-symmetric*, which means that two expressions  $\alpha$  and  $\beta$  may be logically equivalent—that is, entail each other—and yet have different meanings. To see this, consider the following two sentences:<sup>11</sup>

$$(16) \quad \llbracket \mathbf{John\ moves} \rrbracket = \{|Mj|\}$$

$$(17) \quad \llbracket \mathbf{John\ moves\ or\ walks} \rrbracket = \{|Mj|, |Wj|\}$$

Since the proposition  $|Wj|$  that John walks is contained in the proposition  $|Mj|$  that John moves, every alternative for **John moves or walks** is contained in an alternative for **John moves**. Vice

<sup>11</sup>For concreteness, we assume in our examples that disjunction has the alternative-generating behavior argued for by Simons (2005), Alonso-Ovalle (2006), and Aloni (2007). Nothing hinges on this assumption, though, and the reader should feel free to replace disjunction with her favorite alternative-generating item.

versa, the unique alternative for **John moves** is clearly contained in one of the alternatives for **John moves or walks**. Thus, the two sentences entail each other, but they have different meanings.

It seems to us essential for the notion of logical equivalence that it *does* imply synonymy, i.e., identity of meaning. If a certain equivalence relation does not guarantee that, we would just not call it *logical equivalence*. As a consequence, the relation  $\models$  defined above does not really qualify as a satisfactory notion of *entailment* in the alternative semantics framework.

This conceptual problem also has practical repercussions. For instance, if entailment is not a partial order on the space of meanings, conjunction and disjunction can no longer be treated as meet and join operations with respect to entailment. Consider for instance conjunction: we would like to define  $\llbracket \alpha \text{ and } \beta \rrbracket$  as the meet of  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$ , i.e., as the weakest meaning entailing both  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$ . However, since pointwise entailment is not anti-symmetric, there is not a *unique* such meaning, but rather a whole *cluster* of them, and we have no principled way to single out one particular element from this cluster. This means that we lost our principled account of conjunction and disjunction in terms of cross-categorial *meet* and *join* operations.

Thus, for both conceptual and practical reasons, redefining entailment as pointwise inclusion is unsatisfactory.

**Pointwise conjunction.** Setting the general problem with entailment aside, we may still try to devise an alternative-friendly notion of conjunction that avoids the problematic predictions which result from treating conjunction as intersection. Recall our example: we have  $\llbracket \text{John sings} \rrbracket = \{|Sj|\}$ ,  $\llbracket \text{Mary dances} \rrbracket = \{|Dm|\}$ , and we want  $\llbracket \text{John sings and Mary dances} \rrbracket = \{|Sj \wedge Dm|\} = \{|Sj| \cap |Dm|\}$ . This suggests that, rather than intersecting two meanings directly, conjunction should be intersecting the individual alternatives within these meanings. More precisely, it suggests the following treatment of conjunction as pointwise intersection:

$$(18) \quad \llbracket \text{and} \rrbracket = \lambda P. \lambda Q. \{p \cap q \mid p \in P \text{ and } q \in Q\}$$

Again, for the most basic cases, this treatment makes the right predictions. For instance, we do indeed get that  $\llbracket \text{John sings and Mary dances} \rrbracket = \{|Sj| \cap |Dm|\} = \{|Sj \wedge Dm|\}$ ; and this extends more generally to all cases where both conjuncts have singleton meanings. However, with non-singleton conjuncts, pointwise intersection often yields spurious alternatives. For instance, we expect that conjoining a sentence with itself will make no difference to its meaning. But that is not generally the case. Consider a sentence with two alternatives, such as  $\alpha = \text{John sang or danced}$ . Besides the two expected alternatives  $|Sj|$  and  $|Dj|$ , the conjunction  $\alpha \wedge \alpha$  also generates a third alternative, namely the proposition  $|Sj \wedge Dj|$  that John sang and danced.

$$(19) \quad \llbracket \text{John sang or danced and John sang or danced} \rrbracket = \{|Sj|, |Dj|, |Sj \wedge Dj|\}$$

We see no reason why conjunction should give rise to this extra alternative, and we doubt that empirical support for this prediction may be found. Thus, even if the general problem concerning entailment and the usual characterization of conjunction as a meet operator is set aside, it is difficult, if not impossible, to devise a satisfactory alternative-friendly treatment of conjunction.<sup>12</sup>

### 3.2. Recovering standard entailment and conjunction

Given that adapting the notions of entailment and conjunction to alternative/possibility semantics is not a trivial affair, to say the least, it is worth considering once more the strategy we adopted in Section 2 to deal with the compositionality problem: identify exactly which features of the framework are responsible for the problem, and ask whether it is possible to modify these features so that the problem is avoided, while the desirable features of the framework are retained.

In order to do this, let us look once more at the example illustrating the problem with entailment. Why is it that  $\llbracket \text{John walks} \rrbracket$  is not a subset of  $\llbracket \text{John moves} \rrbracket$  in alternative/possibility semantics? Well, because both meanings are singleton sets, consisting of the unique alternative for the sentence. The assumption that a basic sentence  $\alpha$  denotes the singleton  $\{|\alpha|\}$ , shared by alternative and possibility semantics, may seem quite innocent: after all, the standard meaning of a sentence  $\alpha$  is a single proposition,  $|\alpha|$ , and if we want to represent this meaning as a *set* of propositions, what better candidate than the singleton set containing just  $|\alpha|$ ? However, the problems with entailment and conjunction indicate that identifying classical propositions with the corresponding singleton sets may not be the best way of embedding classical semantics into alternative semantics after all.

It is certainly natural to regard a basic sentence like **John walks** as having a unique alternative, namely, the proposition  $|Wj|$ . But it does not follow from this that we have to construe the meaning of **John walks** in alternative/possibility semantics as the singleton set  $\{|Wj|\}$ . To enjoy the benefits of having alternatives in our semantics, it is not necessary to assume that the meaning of a sentence is *identical* with the set of alternatives that the sentence introduces; it is sufficient to assume that the meaning of a sentence *determines* the set of alternatives that it introduces.

What, then, should we take the meaning of a basic sentence like **John walks** to be? Let us examine carefully what the desiderata are. Suppose  $\alpha$  and  $\beta$  are two basic sentences, that is, two sentences having as their unique alternative the proposition that they classically express. For such sentences, we want the standard, truth-conditional notion of entailment to be preserved. That is,  $\alpha \models \beta$  should hold just in case  $|\alpha| \subseteq |\beta|$ . Moreover, we want to preserve the standard type-theoretic conception of entailment as meaning inclusion, so  $\alpha \models \beta$  should amount to  $\llbracket \alpha \rrbracket \subseteq \llbracket \beta \rrbracket$ . To satisfy these two desiderata, we need to make sure that  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  are construed in such a way that:

$$(20) \quad |\alpha| \subseteq |\beta| \iff \llbracket \alpha \rrbracket \subseteq \llbracket \beta \rrbracket$$

<sup>12</sup>A similar issue arises for universal quantification: if we take a universal quantifier to perform pointwise intersection, even a vacuous universal quantifier may introduce spurious alternatives.

This result is naturally obtained if we do not construe  $\llbracket \alpha \rrbracket$  and  $\llbracket \beta \rrbracket$  as the *singleton* sets  $\{|\alpha|\}$  and  $\{|\beta|\}$ , respectively, but rather as the *powersets*  $\wp(|\alpha|)$  and  $\wp(|\beta|)$ , i.e., the set of all subsets of  $|\alpha|$  and  $|\beta|$ , respectively. Clearly, if  $|\alpha| \subseteq |\beta|$ , then any subset of  $|\alpha|$  is also a subset of  $|\beta|$ . And conversely, if any subset of  $|\alpha|$  is a subset of  $|\beta|$ , then it follows that  $|\alpha| \subseteq |\beta|$ . Intuitively, we take the meaning of **John walks** to be the set of all propositions that contain *enough* information to establish that John walks, i.e., all propositions  $p$  such that John walks in every world in  $p$ , rather than just the proposition that contains *precisely* the information that John walks, i.e., the proposition consisting of *all* worlds in which John walks.

This does not mean that we give up the idea that **John walks** has a unique alternative: for, we can recover the unique alternative for **John walks** as the *maximal element* of its meaning. This is precisely the set of *all* worlds where John walks. Thus, by carefully distinguishing the meaning of a sentence from the alternatives it introduces, we can simultaneously retain the usual alternatives for the sentence on the one hand, and the standard type-theoretic notion of entailment on the other.

The reasoning just outlined for basic sentences with a single alternative can be generalized to sentences with multiple alternatives as well. In the spirit of Hamblin (1973) as well as more recent work on *inquisitive semantics* (see, e.g., Ciardelli et al. 2012, 2013) such sentences can be thought of as raising an *issue* as to which of the alternatives contains the actual world. Crucially, while Hamblin originally identified the meaning of a sentence with the alternatives it introduces, inquisitive semantics dissociates the two notions in precisely the way discussed above for basic sentences. That is, the meaning of a sentence in inquisitive semantics consists of all propositions that contain *enough* information to resolve the issue that the sentence raises. As a consequence, sentential meanings in inquisitive semantics are not unconstrained sets of propositions, as in alternative/possibility semantics, but rather sets of propositions that are *downward closed*: if  $\llbracket \alpha \rrbracket$  contains a proposition  $p$  then it also includes every stronger proposition  $q \subseteq p$ . After all, if  $p$  contains enough information to resolve the issue that  $\alpha$  raises, then any  $q \subseteq p$  will also contain enough information to do so.

We will refer to downward closed sets of propositions as *inquisitive meanings*. Given the inquisitive meaning  $\llbracket \alpha \rrbracket$  of a sentence  $\alpha$ , the *alternatives* that  $\alpha$  introduces can still be recovered as the *maximal elements* of  $\llbracket \alpha \rrbracket$ . Intuitively, these are propositions that contain enough information to resolve the issue raised by  $\alpha$ , and *not more information than necessary* to do so.<sup>13</sup>

$$(21) \quad \text{ALT}(\alpha) = \{p \in \llbracket \alpha \rrbracket \mid \text{there is no } q \in \llbracket \alpha \rrbracket \text{ such that } p \subset q\}$$

<sup>13</sup>Interestingly, this approach imposes some constraints on the kinds of alternative sets that may be associated with a sentence. In particular, if  $p$  and  $q$  are two alternatives associated with a sentence  $\alpha$ , we must have that  $p \not\subseteq q$  and  $q \not\subseteq p$ , neither one can be nested in the other. This has consequences, e.g., for the analysis of sentences like **Frege lived in Göttingen or in Germany** (cf., Hurford 1974; Chierchia et al. 2009). Appendix A of the handout version of this paper, available via [www.illc.uva.nl/inquisitivesemantics](http://www.illc.uva.nl/inquisitivesemantics), provides some preliminary remarks on this issue, suggesting that the more constrained notion of alternatives is in fact advantageous, but a comprehensive discussion must be left for another occasion. Due to space limitations, the appendix could not be included in the present paper.

It is easy to see how the compositional fragment outlined above for possibility semantics may be adjusted to yield downward closed sentential meanings. For instance, since we now want that  $\llbracket \text{John walks} \rrbracket = \wp(|Wj|)$ , we will let **walk** denote the function that maps any individual  $x$  to the set of propositions which contain enough information to establish that  $x$  walks.

$$(22) \quad \llbracket \text{walks} \rrbracket = \lambda x. \wp(|Wx|) = \lambda x. \{p \mid x \text{ walks in every } w \in p\}$$

For a detailed exposition of a compositional inquisitive semantics for an elementary fragment of English we refer to Theiler (2014) and Roelofsen et al. (2014).<sup>14</sup> Here, we will focus on showing how the problems with entailment and conjunction discussed above evaporate when meanings are taken to be downward-closed.

First, as we have already seen, the standard type-theoretic notion of entailment as inclusion now *does* make the right predictions for basic sentences. For instance:

$$(23) \quad \llbracket \text{John walks} \rrbracket = \wp(|Wj|) \subseteq \wp(|Mj|) = \llbracket \text{John moves} \rrbracket$$

Moreover, unlike the pointwise notion of entailment considered above, entailment as inclusion constitutes a partial order on the space of inquisitive meanings. In particular, it is anti-symmetric, which means that any two expressions that are logically equivalent express the same meaning. Furthermore, as shown in Roelofsen (2013a), the space of inquisitive meanings ordered by entailment forms a complete Heyting algebra, just like the space of classical propositions ordered by entailment. This means in particular that two inquisitive meanings  $\mathcal{P}$  and  $\mathcal{Q}$  always have (i) a *meet*, i.e., a unique greatest lower bound w.r.t. entailment, given by  $\mathcal{P} \cap \mathcal{Q}$ , and (ii) a *join*, i.e., a unique least upper bound w.r.t. entailment, given by  $\mathcal{P} \cup \mathcal{Q}$ . This means that we can restore the standard treatment of conjunction and disjunction as meet and join operations; moreover, these operations still amount to intersection and union, just as in the classical Montagovian setup.

$$(24) \quad \begin{array}{l} \text{a. } \llbracket \text{and} \rrbracket = \lambda \mathcal{P}. \lambda \mathcal{Q}. \mathcal{P} \cap \mathcal{Q} \\ \text{b. } \llbracket \text{or} \rrbracket = \lambda \mathcal{P}. \lambda \mathcal{Q}. \mathcal{P} \cup \mathcal{Q} \end{array}$$

This result generalizes to arbitrary conjoinable types, yielding a cross-categorical account of conjunction and disjunction. For instance, for the  $\langle e, T \rangle$ -type disjunction **sing or dance** we get:

$$(25) \quad \begin{aligned} \llbracket \text{sing or dance} \rrbracket &= \llbracket \text{sing} \rrbracket \cup \llbracket \text{dance} \rrbracket = \lambda x. \wp(|Sx|) \cup \lambda x. \wp(|Dx|) \\ &= \{ \langle x, p \rangle \mid p \subseteq |Sx| \} \cup \{ \langle x, p \rangle \mid p \subseteq |Dx| \} \\ &= \{ \langle x, p \rangle \mid p \subseteq |Sx| \text{ or } p \subseteq |Dx| \} \\ &= \lambda x. (\wp(|Sx|) \cup \wp(|Dx|)) \end{aligned}$$

As in alternative semantics, disjunction typically generates alternatives. For instance:

<sup>14</sup>See also Appendix B of the handout version of this paper, available via [www.ilc.uva.nl/inquisitivesemantics](http://www.ilc.uva.nl/inquisitivesemantics).

$$(26) \quad \llbracket \mathbf{John\ sings\ or\ Mary\ dances} \rrbracket = \llbracket \mathbf{John\ sings} \rrbracket \cup \llbracket \mathbf{Mary\ dances} \rrbracket = \wp(|Sj|) \cup \wp(|Dm|)$$

This meaning has two maximal elements, namely, the proposition that John sings, and the proposition that Mary dances:

$$(27) \quad \text{ALT}(\mathbf{John\ sings\ or\ Mary\ dances}) = \{|Sj|, |Dm|\}$$

Thus, we recover the alternative-generating treatment of disjunction that was argued for on an empirical basis by Simons (2005), Alonso-Ovalle (2006) and Aloni (2007). However, now this behavior is not merely stipulated, but follows from a principled treatment of disjunction as a join operation in the given semantic framework (cf. Roelofsen 2013b).

Indefinites and *wh*-phrases could also be treated as join operators, which would give them the potential to generate alternatives as well.<sup>15</sup>

- (28) a. **someone, who** :  $\langle \langle e, T \rangle, T \rangle$   
 b.  $\llbracket \mathbf{someone} \rrbracket = \llbracket \mathbf{who} \rrbracket = \lambda P. \bigcup_{x \in D_e} P(x)$   
 c.  $\llbracket \mathbf{someone\ walks} \rrbracket = \llbracket \mathbf{who\ walks} \rrbracket = \bigcup_{x \in D_e} \wp(|Wx|)$   
 d.  $\text{ALT}(\mathbf{someone\ walks}) = \text{ALT}(\mathbf{who\ walks}) = \{|Wx| \mid x \in D_e\}$

Let us now consider conjunction. We have restored the standard treatment of conjunction as a meet operator. This does not only re-establish the link between entailment and conjunction, but also resolves the empirical problems pointed out above. First, performing intersection now yields the right results for the cases that were problematic in alternative and possibility semantics.

$$(29) \quad \begin{aligned} \llbracket \mathbf{John\ sings\ and\ Mary\ dances} \rrbracket &= \llbracket \mathbf{John\ sings} \rrbracket \cap \llbracket \mathbf{Mary\ dances} \rrbracket \\ &= \wp(|Sj|) \cap \wp(|Dm|) \\ &= \wp(|Sj| \cap |Dm|) = \wp(|Sj \wedge Dm|) \end{aligned}$$

As desired, **John sings and Mary dances** is predicted to have a unique alternative, namely, the proposition that John sings and Mary dances. Moreover, unlike the pointwise conjunction operation that we considered above, intersection is obviously *idempotent*, which means that the problem with spurious alternatives no longer arises:

$$(30) \quad \text{ALT}(\mathbf{John\ sings\ or\ dances\ and\ John\ sings\ or\ dances}) = \{|Sj|, |Dj|\}$$

More generally, since conjunction is treated again as performing the *meet* operation with respect to entailment, it regains its familiar, well-understood logical features.

<sup>15</sup>On this view, **someone** and **who** generate the same set of alternatives, as in, e.g., Kratzer and Shimoyama (2002). The difference between the two could be captured by means of constraints on what happens with these alternatives in the further derivation. Of course, different choices for these items are also compatible with the framework we propose.



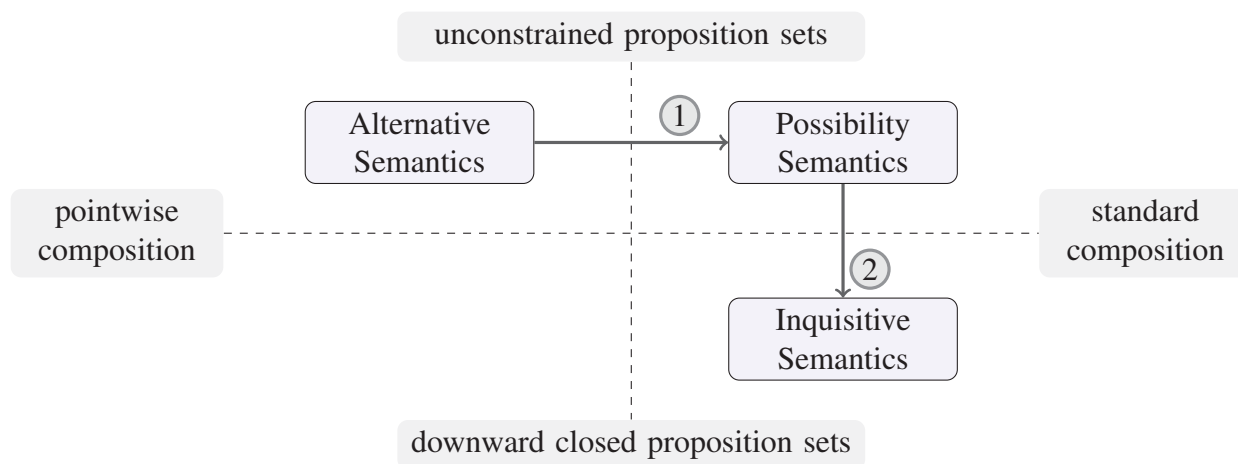


Figure 1: The proposal at a glance.

Summing up, we have shown that the issues with entailment and coordination that arise in alternative semantics may be avoided by carefully reconsidering one of the basic features of the framework, namely, the identification of the *meaning* of a sentence with the set of *alternatives* that it introduces. By teasing the two notions apart, construing the meaning of a sentence as a *downward closed* set of propositions, and viewing the maximal elements of this set as the alternatives that the sentence introduces, we obtain a semantic framework which allows us to retain on the one hand an alternative-inducing notion of meaning, and on the other hand, the principled type-theoretic account of generalized entailment and coordination that is characteristic of Montague grammar.

#### 4. Conclusion

While it clearly seems that alternatives have an important role to play in semantics, the specific architecture of Hamblin-style alternative semantics forces us to give up two crucial features of the standard Montagovian framework, namely, (i) the type-theoretic composition operations of function application and abstraction and (ii) the type-theoretic treatment of cross-categorical entailment and coordination. This leads to a number of problems, both empirical and theoretical.

We have tried to identify precisely which features of alternative semantics are responsible for these issues, and how they could be modified in order to avoid the resulting problems. Our proposal is summarized in Figure 1. First, we argued that the compositionality issue stems from the assumption that *all* expressions denote sets of objects of the corresponding type. This assumption does not seem to have strong conceptual or empirical motivation, and dropping it does not seem to undermine the general spirit of the framework, nor the empirical coverage of the theories that are formulated within it. This step, marked ① in Figure 1, led us to the framework of *possibility semantics*, where sentences still denote unconstrained sets of propositions, but meanings are composed by means of the standard type-theoretic operations.

However, like alternative semantics, possibility semantics still faces the entailment issue, which also leads to problems in the treatment of conjunction. We argued that this issue stems from the assumption that the meaning of a sentence is *identical with* the set of alternatives that it introduces. Once again, this assumption does not seem strictly necessary, neither from a conceptual point of view nor from an empirical one. Conceptually, there is another natural perspective on sentential meanings, motivated in recent work on inquisitive semantics, under which they are construed as sets of propositions that are *downward closed*. Empirically, all that is required for applications is that the meaning of a sentence *determine* a set of alternatives. If the meaning of a sentence is a downward closed set of propositions, the maximal elements of this set are naturally viewed as the alternatives that it introduces. This step, marked ② in Figure 1, resolves the entailment issue: the general type-theoretic notion of entailment is recovered, and conjunction and disjunction can again be treated as *meet* and *join* operations w.r.t. entailment. Notice that the two proposed steps are independent, that is, one of them could in principle be adopted without the other.

The resulting framework retains a fine-grained notion of meaning, which associates with every sentence a set of alternatives, but has a much more solid type-theoretic foundation than Hamblin-style alternative semantics: as in the usual Montagovian framework, meanings are composed by means of the standard type-theoretic composition rules, and compared by means of the standard notion of entailment as meaning inclusion. In this way, the empirical coverage of the analyses formulated in alternative semantics is preserved, while the problematic predictions stemming from the observed framework issues are avoided.

## References

- Aloni, M. (2007). Free choice, modals and imperatives. *Natural Language Semantics* 15(1), 65–94.
- Alonso-Ovalle, L. (2006). *Disjunction in Alternative Semantics*. Ph. D. thesis, University of Massachusetts, Amherst.
- Charlow, S. (2014). *On the semantics of exceptional scope*. Ph. D. thesis, New York University.
- Chierchia, G., D. Fox, and B. Spector (2009). Hurford’s constraint and the theory of scalar implicatures: evidence for embedded implicatures. In *Presuppositions and implicatures*, pp. 47–62. MIT Press Cambridge, MA.
- Ciardelli, I., J. Groenendijk, and F. Roelofsen (2012). Inquisitive semantics. NASSLLI lecture notes, available via [www.ilc.uva.nl/inquisitivesemantics](http://www.ilc.uva.nl/inquisitivesemantics).
- Ciardelli, I., J. Groenendijk, and F. Roelofsen (2013). Inquisitive semantics: a new notion of meaning. *Language and Linguistics Compass* 7(9), 459–476.
- Groenendijk, J. and M. Stokhof (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph. D. thesis, University of Amsterdam.
- Hamblin, C. L. (1973). Questions in Montague English. *Foundations of Language* 10(1), 41–53.
- Hurford, J. (1974). Exclusive or inclusive disjunction. *Foundations of Language* 11(3), 409–411.

- Jacobson, P. (1999). Towards a variable-free semantics. *Linguistics and Philosophy* 22(2), 117–185.
- Keenan, E. L. and L. M. Faltz (1985). *Boolean semantics for natural language*. Springer.
- Kratzer, A. and J. Shimoyama (2002). Indeterminate pronouns: The view from Japanese. In Y. Otsu (Ed.), *The Proceedings of the Third Tokyo Conference on Psycholinguistics*, pp. 1–25.
- Menéndez-Benito, P. (2005). *The grammar of choice*. Ph. D. thesis, University of Massachusetts, Amherst.
- Montague, R. (1973). The proper treatment of quantification in ordinary English. In J. Hintikka, J. Moravcsik, and P. Suppes (Eds.), *Approaches to natural language*, pp. 221–242. Springer.
- Novel, M. and M. Romero (2010). Movement, variables and Hamblin alternatives. In *Proceedings of Sinn und Bedeutung 14*, pp. 322–338.
- Partee, B. H. and M. Rooth (1983). Generalized conjunction and type ambiguity. In R. Bäuerle, C. Schwarze, and A. von Stechow (Eds.), *Meaning, Use and Interpretation of Language*, pp. 115–143. Berlin: De Gruyter.
- Poesio, M. (1996). Semantic ambiguity and perceived ambiguity. In K. van Deemter and S. Peters (Eds.), *Semantic Ambiguity and Underspecification*, pp. 231–250. Stanford, CA: CSLI.
- Roelofsen, F. (2013a). Algebraic foundations for the semantic treatment of inquisitive content. *Synthese* 190(1), 79–102.
- Roelofsen, F. (2013b). An inquisitive perspective on meaning: the case of disjunction. Stanford Linguistics Colloquium.
- Roelofsen, F., N. Theiler, and M. Aloni (2014). Embedded interrogatives: the role of false answers. Presented at the Seventh Questions in Discourse workshop, Göttingen, September 2014.
- Roelofsen, F. and S. van Gool (2010). Disjunctive questions, intonation, and highlighting. In M. Aloni, H. Bastiaanse, T. de Jager, and K. Schulz (Eds.), *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*, pp. 384–394. Springer.
- Rooth, M. (1985). *Association with Focus*. Ph. D. thesis, University of Massachusetts, Amherst.
- Shan, C.-C. (2004). Binding alongside Hamblin alternatives calls for variable-free semantics. In *Proceedings of Semantics and Linguistic Theory (SALT 14)*, pp. 289–304.
- Shimoyama, J. (2001). *Wh-constructions in Japanese*. Ph. D. thesis, University of Massachusetts, Amherst.
- Simons, M. (2005). Dividing things up: the semantics of *or* and the modal/*or* interaction. *Natural Language Semantics* 13(3), 271–316.
- Szabolcsi, A. (1989). Bound variables in syntax: Are there any? In R. Bartsch, J. van Benthem, and P. van Emde Boas (Eds.), *Semantics and Contextual Expression*, pp. 294–318. Foris.
- Theiler, N. (2014). A multitude of answers: embedded questions in typed inquisitive semantics. MSc thesis, University of Amsterdam.
- Winter, Y. (2001). *Flexibility principles in Boolean semantics: The interpretation of coordination, plurality, and scope in natural language*. MIT press.