UNIVERSITY OF AMSTERDAM

UvA-DARE (Digital Academic Repository)

Finite element analysis of levee stability for flood early warning systems

Melnikova, N.B.

Publication date
2014

Link to publication

Citation for published version (APA):
Melnikova, N. B. (2014). *Finite element analysis of levee stability for flood early warning systems*. [Thesis, fully internal, Universiteit van Amsterdam].

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (https://dare.uva.nl)

Download date:14 Feb 2025

Chapter 2 Principles of dike modelling

This chapter reviews mathematical models describing porous flow and deformations in earthen dikes, with discussion of advantages and drawbacks of each model. The models are introduced in chronological order, according to the history of their development. After careful testing of different models, we have chosen a coupled combination of plane Drucker-Prager linear elastic perfectly plastic model for modelling deformations in soil skeleton and Richards' model for modelling filtration through porous media. This choice provided the optimal balance between realism and adequacy of the models, on the one hand, and fast and reliable convergence of numerical solution procedure in real-time mode, on the other hand.

Dike collapse mechanisms vary from macro-instability failures, like slope sliding or simple shear in a multi-layered dike, to a number of erosion failure mechanisms, such as piping, internal overtopping or wave erosion. The conventional engineering methods for levees stability analysis include: (a) probabilistic breach analysis (available for all kinds of failure) based on empirical engineering criteria (Vorogushyn et al., 2012); (b) limit equilibrium methods (LEM) only suitable for slope sliding prediction.

The first limit equilibrium method (LEM) for slope stability analysis (Fellenius/Petterson method) was proposed in (Fellenius, 1927). It considers a balance of disturbing and stabilizing forces acting on vertical soil slices located above a circular slip surface. The ratio restoring forces/disturbing forces is termed the factor of safety. Later the method underwent numerous modifications, for instance by Taylor, Bishop, Morgenstern-Price, Spencer, Janbu and others (an extensive overview of existing LEM methods and their comparison can be found in (Fredlund and Krahn, 1977; Chen and Morgenstern, 1983; Duncan, 1996)). Some of those, like Fellenius' and Bishop's methods, assume a circular slip surface and do not satisfy horizontal equilibrium conditions for the soil slices, while Spencer's, Morgenstern-Price's and Janbu's generalized methods work with non-circular slip surfaces and satisfy all equilibrium conditions.

LEM is nowadays the most popular analysis tool in practical engineering: it is simple and robust. A serious limitation of LEM usage in the EWS design is small amount of the output data fed back from it into the EWS: basically, it generates tables with dike's scalar factors of safety, under the prescribed load levels and loads combinations. LEM does not really allow a deep insight into the dike failure processes modelling and can not simulate dynamics of sensor measured parameters (such as pore pressure, displacements, inclinations and strains) which are of much importance for safety monitoring and AI training.

The main drawback of LEM lies in the narrowness of its application: it only predicts slope sliding failures and does not capture a great variety of different failure mechanisms mentioned above.

In opposite to LEM, the finite element method (FEM) in application to partial differential equations of continuum soil mechanics (which will be presented below) was chosen as a perfect tool for the implementation of the *Virtual Dike* module aimed for a detailed dike stability analysis, simulation of arbitrary and complex failure mechanisms and feeding the AI. A FEM-based analysis is capable of modelling *any* failure mechanisms - naturally, on a macroscopic level.

In co-operation with HR Wallingford, we carried a cross-validation of LEM against FEM for the Boston test site (Chapter 5).

Below we review the existing mathematical models describing dike behaviour under hydraulic and mechanical loadings and select the models optimal for real-time functioning in the workflow of the EWS. The equations describe two sub-models involved in the analysis: a fluid sub-model for simulation of porous flow through the dike and a mechanical sub-model for dike deformations modelling.

2.1 Simulation of water flow through the dike

2.1.1 Governing equations

Fluid flow through fully saturated porous media is governed by Darcy's law (Darcy, 1856) which states that the filtration velocity $\vec{V}$ is linearly proportional to the pore pressure gradient and to the gravity acceleration:

$$\vec{V} = -\frac{K_S}{\mu}(\nabla p + \rho g) \tag{2.1}$$

Here K_S [m^2] is permeability (a characteristics of soil type), μ is dynamic viscosity of water, [Pa·s], ∇ is gradient operator, p is relative pore pressure [Pa] (which is absolute pore pressure minus atmospheric pressure), g, ρ are standard gravity and water density, respectively.

The actual water flow velocity in pores equals to $\vec{V}/f$, where f is the porosity of a medium (a ratio of pores volume to the total porous domain volume). Filtration velocity $\vec{V}$ describes the flux in a homogenised, continuous medium.

The Darcy's low is suitable for slow filtration velocities. For fast turbulent pore flows, a quadratic extension was proposed in (Brinkman, 1949). In practice, turbulent flows occur in gases in porous media; this is not the case of soil-water filtration and the linear form (2.1) will be used below.

Ground water flow equation is derived from the Darcy's law and the mass conservation equation under the assumptions that (a) water is incompressible and (b) the time derivative of filtration velocity is negligibly small: $\frac{\partial \vec{V}}{\partial t} = 0$ (Bear, 1972):

$$\nabla \cdot \vec{V} = Q_s \Rightarrow \nabla \cdot [-\frac{K_S}{\mu}\nabla p] = Q_s$$

Here right-hand side component Q_s is volume intensity of external sources and sinks.

The non-stationary extension of ground water flow equation takes into account the change of water amount stored in pores with their compaction or extraction under local pore pressure fluctuations. The formulation is based on the mass conservation equation:

$$S\frac{\partial p}{\partial t}+\nabla\cdot[-\frac{K_S}{\mu}\nabla p]=Q_s \tag{2.2}$$

Here storage coefficient S [1/Pa] is the ratio of soil porosity f to the soil skeleton bulk modulus K: $S=f/K$; t is time.

Equations (2.1) and (2.2) are completely analogous to Fourier's law and Laplace's equation for heat conduction in solid bodies.

Equation (2.2) describes a so called confined class of pore fluid problems, which do not contain a phreatic surface in the simulation domain. A phreatic surface is defined as a surface in a porous medium where relative pore pressure is zero. Earthen dikes typically refer to the second, unconfined class, because ground water table is naturally located within the dike. Unconfined flow modelling employs specific techniques like moving mesh (Comsol manual, 2009) or using stationary mesh with elements having artificial nonlinear permeabilities, which "turn off" elements in the vadose (partially saturated) zone (Larabi and Smedt, 1997). In the both techniques, flow in a zone of capillary fringe is neglected. In clayey soils, capillary fringe height can reach ten meters (Verruijt, 2001); under rapid hydraulic loads like rainfalls or fast rise of river level capillary fringe zones become saturated and significantly change pore pressure distribution. The effect of wetting (and drying) is taken into account with a more comprehensive Richards' equation of variably saturated ground water flow.

Richards (1931) extended the usage of Darcy's law on partially saturated media; with water storage and permeability depending on pore water content. Pressure-based form of Richards' equation (Bear, 1972) is given by formula

$$(C+\theta_e S)\frac{\partial p}{\partial t}+\nabla\cdot[-\frac{K_S}{\mu}k_r\nabla p]=Q_S \tag{2.3}$$

Here C, θ_e are specific moisture capacity and effective water content, respectively; p is pore water pressure (negative for suctions in the partially saturated zone); K_S is permeability of saturated media; $k_r=k_r(p)$ is relative permeability.

Specific moisture capacity and relative permeability are non-zero in vadose (partially saturated) zones; they are defined as functions of volumetric water content $\theta=\frac{V_{water}}{V_{total}}$, where V_{water} is volume of water in porous media, V_{total} is total volume (which is soil volume + water volume + air volume).

In saturated zone $\theta=\theta_s\approx f$, where θ_s is saturated water content; f is porosity (pores are mostly filled with water except of small air bubbles locked inside).

Due to absorption, even at high suction pressures there is always residual water content θ_r in the soil. The amount of residual water depends on the type of soil; maximal residual water contents are observed in clays.

Effective water content θ_e is defined as follows: $\theta_e = \dfrac{\theta - \theta_r}{\theta_s - \theta_r}$.

A specific property of a given type of soil is *water retention curve*, which is expressed as a relationship between the effective water content θ_e and negative suction pressure *p*. Due to the hysteretic effect of water filling and draining the pores, different wetting and drying curves may be distinguished.

The very first water-retention curve was proposed in 1907 by Edgar Buckingham (Buckingham, 1907). A number of analytical water-retention curves were then proposed in (Brooks and Corey, 1964), (Brutsaert, 1966), (Vauclin et al., 1979), (Van Genuchten, 1980). Van Genuchten curve (Van Genuchten, 1980) is most widely adopted nowadays; it defines the dependence between suction pressure and effective water content in as follows:

$$\theta_e = \begin{cases} \dfrac{1}{(1+(a|p/\rho g|)^n)^m}, & p < 0 \\ 1, \quad p \geq 0 \end{cases} \tag{2.4}$$

Here p is suction pressure, α, n, $m = 1 - 1/n$ are Van Genuchten parameters specific for each type of soil.

At suction pressures close to zero, a soil is close to saturation, and water is held in the soil primarily by capillary forces. As saturated water content θ_s decreases, binding of the water becomes stronger, and at high suction pressures water is strongly bound in the smallest of pores, at contact points between grains and as films bound by adsorptive forces around particles. Sandy soils will involve mainly capillary binding, and will therefore release most of the water at relatively low suction pressures. Clayey soils, with adhesive and osmotic binding, will release water at higher suction pressures. At any given pressure, peaty soils will usually display much higher moisture contents than clayey soils, which would be expected to hold more water than sandy soils. The water holding capacity of any soil is due to the porosity and the nature of the bonding in the soil.

Moisture capacity and relative permeability are described by the following expressions (Van Genuchten, 1980):

$$C = \frac{\partial \theta}{\partial p} = \begin{cases} \dfrac{\alpha m}{1-m}(\theta_s - \theta_r)\theta_e^{1/m}(1-\theta_e^{1/m})^m & , p < 0 \\ 0, \quad p \geq 0 \end{cases} \tag{2.5}$$

$$k_r = \begin{cases} \theta_e^{\,l}\left[1-(1-\theta_e^{\,1/m})^m\right]^2, p<0 \\ 1, \quad p \ge 0 \end{cases} \tag{2.6}$$

Here l is pore connectivity parameter.

2.1.2 Boundary conditions

Boundary conditions for (2.3) can be of two types:

(a) Pressure specified at the boundary S_1: $p\big|_{S1} = p_s$. Example: p_s=0 refers to seepage to atmosphere.

(b) Flow velocity across the boundary $S_2 : V_n\big|_{S_2} = \underline{n}\cdot[-K_S k_r \nabla(p+\rho g y)]$, where $\underline{n}$ is normal to the boundary surface. Examples: V_n=0 at impermeable walls or $V_n=V_n(t)$ for rainfall infiltration.

2.1.3 Initial conditions

For transient tasks with external loading, pore pressure dynamics is fully determined by external hydraulic loads. Initial pore pressure distribution dissipates within a finite period of time and its choice is only a technical item. A typical initial condition for fluid sub-model used in the present work was: hydrostatic pressure distribution below an estimated phreatic line connecting land side and sea side water tables, and suction pressure p=-5000 [Pa] above the phreatic line:

$$\begin{cases} p = -\rho g y, \; for \;\; y \le h_{gw}, \\ p = -5000[Pa], \; for \;\; y > h_{gw} \end{cases},$$

where y is vertical elevation coordinate, h_{gw} is local position of the phreatic line, estimated from land side and sea side boundary conditions.

This choice of initial condition provided fast convergence of numerical solution in the fluid sub-model.

2.2 Modelling soil deformations

2.2.1 Governing equations

Soil skeleton is considered as deformable continuum subjected to hydraulic and mechanical loads. There have been developed various constitutive soil models: Mohr-Coulomb, Drucker-Prager, hardening models: cam-clay family models, soft soil model and others. Mohr-Coulomb and Drucker-Prager models work well for stiff cohesionless soils like sands which exhibit plastic shear deformations and elastic volume deformations. Cam-clay and soft soil models have been developed for soft clays and peats, which generally produce significant nonlinear elastic deformations, with volume deformations becoming plastic at some load level. Plastic volume deformations are taken into account in cam-clay model by adding elliptic cap to a Mohr-Coulomb yield surface. The Cam-clay material model was developed at the University of Cambridge in the 1970s, and since then it has experienced different modifications. The modified Cam-clay model (Potts and Zdravkovic, 1999) is the most commonly used due to the smooth yield surface. The modified cam-clay model is a so-called critical state model, where the loading and unloading of the material follows different trajectories in stress space. The model also features hardening and softening of clays.

We have tested the Mohr-Coulomb, Drucker-Prager and cam-clay models for the *Virtual Dike* module, and finally choose the Drucker-Prager (DP) model by the following reasons:

- Drucker Prager soil model is quite simple and hence fast in computations, suitable for real-time work
- Drucker Prager function is smooth and does not cause numerical solution problems due to singularities in the flow rule, unlike the Mohr-Coulomb model which actually showed numerical convergence problems in our tests
- The cam-clay model is computationally heavy; moreover, it required model parameters which were not available from soil data on the Livedike and on the Boston levee (compression and swelling indices, initial consolidation pressure).

Below we describe the equations (2.7) suitable for classical isotropic linear elastic perfectly plastic soil models (including Drucker-Prager soil model), which assume that volume deformations of soil are always elastic, while plastic yielding occurs due to the shear deformations in soil, with sliding between material planes. According to Terzaghi's principle (Terzaghi, 1943), stressed state of soil skeleton is characterized by the effective stress tensor $\underline{\underline{\sigma}}_{eff}$ which is a sum of total stress tensor (obtained from equilibrium equation) and hydrostatic water pressure tensor $\underline{\underline{\sigma}}_{eff} = \underline{\underline{\sigma}} + p\underline{\underline{I}}$ (here compressive stresses are negative). The Terzaghi's principle means that pore water provides buoyancy effect on soil skeleton. Elasto-plastic deformations of the soil skeleton are described in terms of strains and effective stresses by the general equations of plastic flow theory (Potts and Zdravkovic, 1999):

$$\begin{cases} \nabla \cdot \underline{\underline{\sigma}} + \rho_S \underline{g} = 0 \\ \underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_{pl} + \underline{\underline{\varepsilon}}_{e} \\ \underline{\underline{\varepsilon}}_{e} = \dfrac{1+\nu}{E}\left[\underline{\underline{\sigma}}_{eff} - \dfrac{\nu}{1+\nu}\sigma_{eff}\underline{\underline{I}}\right] \\ \dot{\underline{\underline{\varepsilon}}}_{pl} = 0 \; if \; F < 0, \quad \dot{\underline{\underline{\varepsilon}}}_{pl} = q\dfrac{\partial P}{d\underline{\underline{\sigma}}_{eff}} \; if \; F = 0 \end{cases} \qquad (2.7)$$

where $\nabla = \underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}$ is gradient operator; ρ_s is soil density; $\underline{g}$ is gravity vector; $\underline{\underline{\sigma}}$ and $\underline{\underline{\sigma}}_{eff}$ are total and effective stress tensors, respectively (compressive stresses are negative); $\underline{\underline{\varepsilon}}_e$ and $\underline{\underline{\varepsilon}}_{pl}$ are elastic and plastic components of strain tensor, respectively; E is Young's modulus; v is Poisson's ratio; $\underline{\underline{I}}$ is unit tensor; $\underline{\underline{\varepsilon}}$ is total strain tensor; q is plastic multiplier; P is plastic potential function, F is plastic yield function (F <0 corresponds to elastic behaviour, F =0 refers to plastic yield); $K = \frac{E}{3(1-2\nu)}$ is bulk modulus, $\sigma_{eff} = I_1 = \sigma_{x\,eff} + \sigma_{y\,eff} + \sigma_{z\,eff}$ is the first effective stress invariant.

In general non-associated plastic flow rule, plastic potential P and plastic yield function F do not coincide. In associated plastic flow rule, they are equal: $F = P$.

Plastic flow rule $\dot{\underline{\underline{\varepsilon}}}_{pl} = q \frac{\partial P}{d\underline{\underline{\sigma}}_{eff}}$ determines the ratio between plastic strain rate tensor components. The magnitude of plastic strains is governed by a scalar plastic multiplier q determined from Prager's consistency condition, which closes the set of equations (2.7). The condition states that at yield $\dot{F}(\underline{\underline{\sigma}}_{eff}) = 0$ because $F(\underline{\underline{\sigma}}_{eff})$ =0:

$$\dot{F}(\underline{\underline{\sigma}}_{eff}) = \frac{dF}{d\underline{\underline{\sigma}}_{eff}} \cdot\cdot \dot{\underline{\underline{\sigma}}}_{eff} = 0 \qquad (2.8)$$

The mechanical sub-model (2.7)+(2.8) is quasi-static; inertia effects are not taken into consideration. Mechanical loadings are defined as functions of a pseudo-time parameter; differentiation in (2.7), (2.8) refers to pseudo-time.

2.2.2 Calculation of effective stresses in vadose zones

In saturated zones, effective stresses $\underline{\underline{\sigma}}_{eff}$ are calculated according to Terzaghi's classical effective stress principle (Terzaghi, 1943):

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_{eff} - p\underline{\underline{I}},$$

where $p\underline{\underline{I}}$ is taken with minus because compressive stresses are negative. The principle means that water resists to compressive load, unloading soli skeleton; while water does not resist shear.

Levees subjected to tidal oscillations or other fluctuations of ground water table contain variably saturated zones. It is well known that pore suctions in vadose zones stabilize slopes (see, for example, Krahn et al., 1989; Griffiths and Lu, 2005). The extension of Terzaghi's classical effective stress principle on unsaturated soils was first proposed by Bishop (1955a) and then was extensively developed (e.g., Fredlund et al., 1978; Vanapalli et al., 1996). In these extensions, soil matric suction is relaxed by a matric suction coefficient (which is a function of soil-water saturation) and contributes to the calculation of effective stresses from total stresses:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_{eff} - \alpha p \underline{\underline{I}} \text{ if } p < 0,$$

where α is matric suction.

Sensitivity of slope safety margin to the pore suctions depends highly on the water table elevation, soil type and infiltration conditions (Griffiths and Lu, 2005). Case studies of unsaturated slope failures can be found in (Tsaparas et al., 2002; Cho and Lee, 2001).

In many cases, especially for loose media, pore suctions are omitted when calculated effective stress in vadose zones:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_{eff} \text{ if } p < 0.$$

2.2.3 Boundary conditions

Possible boundary conditions for the mechanical sub-model are:

(a) displacements specified at the boundary S1: $\underline{U}\big|_{S1} = \underline{U}_s$;

(b) loading specified at the boundary S2: $\underline{n} \cdot \underline{\underline{\sigma}}\big|_{S2} = \underline{f}_{S2}$

2.2.4 Initial condition

Differentiation in the mechanical sub-model is done with respect to pseudo-time, which in general case does not have to coincide with physical time. Pseudo-time is a parameter used in plastic yielding theory to describe incremental growth of all loads and corresponding increments of stresses and strains in the dike. A natural initial condition for the set of equations (2.7)+(2.8) is zero plastic strain in the domain: $\underline{\underline{\varepsilon}}_{pl} = 0$.

2.2.5 Plasticity models

Below the classical associated plasticity models (Mohr-Coulomb and Drucker-Prager) are described in details. In all constitutive models, plastic yield function $F(\underline{\underline{\sigma}})$ is defined as a function of principal stresses or stress tensor invariants.

Mohr-Coulomb plasticity model

In Mohr-Coulomb plasticity model, yield function is a difference between magnitude of normal effective compressing stress $|\sigma_f|$ acting on a plane and multiplied by tangent of friction angle, and magnitude of total shear stress $|\tau_f|$ acting in the plane:

$$F = c + |\sigma_f| \cdot \tan\varphi - |\tau_f| ,$$

where c is effective cohesion [Pa], φ is effective friction angle [grad].

In 3D, Mohr-Coulomb yield function is expressed via maximum and minimum effective principal stresses σ_1, σ_3 as follows:

$$F = (-(\sigma_1 + \sigma_3) \cdot \sin\varphi + 2c \cdot \cos\varphi) - (\sigma_1 - \sigma_3)$$

Second effective principal stress is not active in Mohr-Coulomb plasticity model. The Mohr-Coulomb yield function has corners when plotted in principal effective stress space. For the associated plasticity modelling, the corners imply singularities in the yield function's partial derivatives with respect to the stress components. These derivatives appear in the plastic flow rule; they are needed to define the elasto-plastic constitutive matrix, and they are not unique at the corners. This numerical problem can be avoided either by implementing special numerical procedures to treat derivatives at corners or, alternatively, by using a smooth yield function which approximates Mohr-Coulomb yield function. This yield function defines a so called Drucker-Prager plasticity model, which gives a shape of circular cone in 3D space.

Drucker-Prager plasticity model

Below we describe a modification of the Drucker-Prager (DP) plasticity model, specially optimized for plane strain problems by providing the best smooth approximation of the Mohr-Coulomb surface in the stress space (Chen and Mizuno, 1990). Plastic yield function is independent of Lode angle (which is the third effective stress invariant). Unlike Mohr-Coulomb model, the second effective principal stress is active in the Drucker-Prager model:

$$F = \alpha \cdot I_1 + \sqrt{J_2} - F_{DP} \tag{2.9}$$

Here $I_1 = \sigma_1 + \sigma_2 + \sigma_3$ is the first effective stress invariant, $J_2 = I_1^2 / 3 - I_2$ is second deviatoric effective stress invariant, $I_2 = \sigma_1 \cdot \sigma_2 + \sigma_3 \cdot \sigma_2 + \sigma_1\sigma_3$ is second effective stress invariant; α and F_{DP} are constants: $\alpha = tg(\varphi) / \sqrt{9 + 12 \cdot tg^2(\varphi)}$, $F_{DP} = 3c / \sqrt{9 + 12 \cdot tg^2(\varphi)}$; c, φ are effective cohesion and internal friction angle, respectively.

Drucker-Prager model has been used in the *Virtual Dike* module, as it is smooth and does not require specific treatment of numerical problems caused by cornered shape of Mohr-Coulomb function.

2.3 Dike stability assessment

In the limit equilibrium analysis, dike stability margin is assessed by computing the factor of safety (*FoS*), describing the capacity of a slope to withstand its own weight together with applied external loadings from surcharge and groundwater. *FoS* is defined as the ratio of restoring forces F_R (soil shear strength + externally applied restoring forces) to disturbing forces F_D (soil self weight + externally applied disturbing forces):

$$FoS = F_R / F_D, \tag{2.10}$$

where the sums of restoring and disturbing forces are calculated on all possible circular slip surfaces with arbitrary diameters and centre locations. The critical failure surface corresponds to that providing the minimal value of *FoS*.

Shapes of a slip surfaces in LEM are predefined and specific for each modification of the method. The classical limit equilibrium methods consider circular slip surfaces.

When solving partial differential equations of soil mechanics (e.g., by FEM, FD or any other appropriate method), no assumptions on the shape of critical surface are made – slip surfaces can be arbitrary and they are obtained during simulation. Moreover, slope sliding is not the only failure mechanism simulated by soil mechanics modelling by FEM. In 1975, Zienkiewicz proposed a shear strength reduction method to evaluate the dike stability margin, for arbitrary failure mechanisms. In the shear strength reduction method, soil strength parameters are gradually scaled down until the onset of slope instability is reached (which is detected by the divergence of numerical analysis iterations). A strength reduction factor *SRF* is then defined as the ratio of the original and scaled strength parameters.

$$SRF = \frac{c}{c_{margin}} = \frac{\tan\varphi}{\tan\varphi_{margin}}, \tag{2.11}$$

where c, φ are cohesion and friction angle of soil strata, $c_{margin}, \varphi_{margin}$ are cohesion and friction angle at the margin of instability, under given loadings.

There has been published a number of papers, comparing *SRF* obtained by the strength reduction method and *FoS* from the limit equilibrium analysis (Totsev and Jellev, 2009), (Griffiths and Lane, 1999). The discussion about the optimal method to assess stability margin is still open. Obviously, these two factors refer to different dike models (*FoS* is computed for the original dike, while *SRF* is computed for the dike with weakened soil strength) and they are not completely identical, as nature experiments have also proved (see our cross-validation of these two methods in Chapter 5). In physically non-linear structures, like elasto-plastic soils, response of structure to the external load does not depend linearly on the strength parameters of soil. Thus, we can not expect that scaling of soil properties with constant load in the strength reduction method would give absolutely the same result as computing ratio of disturbing and restoring forces in the original dike in the limit equilibrium analysis.

2.4 Virtual Dike model

2.4.1 Drained behaviour modelling

Equations (2.3), (2.7), (2.8) form a one-way coupled fluid-structure interaction system, describing behaviour of deformable porous media. In general case of bi-directional coupling, the term $C\frac{\partial p}{\partial t}$ in (2.3) is replaced with $\frac{\partial \varepsilon}{\partial t}$, where ε is volume deformation of soil skeleton which is squeezing/absorbing pore water. The *Virtual Dike* module employs a one-way coupled fluid-structure interaction system with non-stationary form of Richard's equation for fluid sub-model which is independent from structural deformations (eq. (2.12)). The porous flow sub-model generates pore pressure used to compute effective stresses in the mechanical sub-model, which in turn is described by eq. (2.13). This simplification was made due to the assumption that the soil skeleton does not undergo large volume deformations under mechanical loading. Having made this assumption, we loose possibility of modelling fully-coupled consolidation problems (where slow porous flow restricts solid skeleton deformations rate), but we improve numerical stability of solver and make the model more robust, as it is no longer fully coupled. Switching to one-directional coupling was conditioned by poor convergence of numerical solution for fully-coupled problems, in out tests (fully coupled systems were ill-conditioned).

$$(C+\theta_e S)\frac{\partial p}{\partial t}+\nabla\cdot[-\frac{K_S}{\mu}k_r\nabla p]=0 \tag{2.12}$$

$$\begin{cases} \nabla\cdot\underline{\underline{\sigma}}+\rho_S\underline{g}=0 \\ \underline{\underline{\varepsilon}}=\underline{\underline{\varepsilon}}_{pl}+\underline{\underline{\varepsilon}}_{e} \\ \underline{\underline{\varepsilon}}_{e}=\frac{1+\nu}{E}\left[\underline{\underline{\sigma}}_{eff}-\frac{\nu}{1+\nu}\sigma_{eff}\underline{\underline{I}}\right] \\ \dot{\underline{\underline{\varepsilon}}}_{pl}=0\; if\; F<0 \\ \dot{\underline{\underline{\varepsilon}}}_{pl}=q\frac{\partial F}{d\underline{\underline{\sigma}}_{eff}}\; if\; F=0 \\ \underline{\underline{\sigma}}=\underline{\underline{\sigma}}_{eff}-p\; if\; p\geq 0 \\ \underline{\underline{\sigma}}=\underline{\underline{\sigma}}_{eff}\; if\; p<0 \\ \frac{dF}{d\underline{\underline{\sigma}}_{eff}}\cdot\cdot\dot{\underline{\underline{\sigma}}}_{eff}=0 \end{cases} \tag{2.13}$$

Here associated plastic flow rule is used; plastic yield function and plastic potential coincide and are equal to Drucker-Prager yield function (2.9). Pore suctions were omitted when calculating effective stresses above the phreatic line: $\underline{\underline{\sigma}}=\underline{\underline{\sigma}}_{eff}$ in vadose zones. This assumption was possible due to low suctions in highly permeable vadose zones of the dikes studied in the research (see Chapter 4- Chapter 6 for description of the test dikes).

Equations (2.12) and (2.13) form the *Virtual Dike* drained soil model employing Richards' equation for fluid flow and linear elastic perfectly plastic Drucker-Prager associated plasticity model for soil deformations simulation.

2.4.2 Undrained behaviour modelling

In undrained conditions, the rate of filtration is negligibly small compared to the rate of hydraulic and mechanical loads' change. Filtration is so slow that it can be assumed that it does not occur during loading period. Due to impossibility of pore water drain, pore water highly resists compressive loads, making media almost incompressible. Soil-water medium works as conglomerate which behaviour is described in terms of total stresses $\underline{\underline{\sigma}}$. The components of total stresses are expressed as sum of effective stresses and pore pressures, according to the Terzaghi's principle:

$$\begin{cases} \sigma_x = \sigma_{x\,eff} - p \\ \sigma_y = \sigma_{y\,eff} - p \\ \sigma_z = \sigma_{z\,eff} - p \\ \tau_{xy} = \tau_{xy\,eff} \\ \tau_{xz} = \tau_{xz\,eff} \\ \tau_{yz} = \tau_{yz\,eff} \end{cases}$$

For undrained behaviour, a distinction is made between steady state pore pressure and excess pore pressure:

$$p = p_{steady} + p_{excess},$$

where p_{steady} is input data (for dikes stability analyses, p_{steady} is generated on the basis of phreatic levels obtained from the drained gravity settlement problem solution).

Excess pore pressures and effective stresses are calculated from total stresses via coefficient matrices; the procedure of obtaining these matrices was obtained from (Vermeer, 1993)

Since the time derivative of p_{steady} is zero, it follows that

$$\dot{p} = \dot{p}_{excess}$$

Applying Hooke's law for elastic component of strains $\underline{\underline{\varepsilon}}_e$ gives us:

$$\begin{bmatrix} \dot{\varepsilon}_{e,x} \\ \dot{\varepsilon}_{e,y} \\ \dot{\varepsilon}_{e,z} \\ \dot{\gamma}_{e,xy} \\ \dot{\gamma}_{e,xz} \\ \dot{\gamma}_{e,yz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & 0 & 0 & 0 & 0 \\ -\nu & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \cdot \begin{bmatrix} \dot{\sigma}_{eff,x} \\ \dot{\sigma}_{eff,y} \\ \dot{\sigma}_{eff,z} \\ \dot{\sigma}_{eff,xy} \\ \dot{\sigma}_{eff,xz} \\ \dot{\sigma}_{eff,yz} \end{bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} \dot{\varepsilon}_{e,x} \\ \dot{\varepsilon}_{e,y} \\ \dot{\varepsilon}_{e,z} \\ \dot{\gamma}_{e,xy} \\ \dot{\gamma}_{e,xz} \\ \dot{\gamma}_{e,yz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & 0 & 0 & 0 & 0 \\ -\nu & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \cdot \begin{bmatrix} \dot{\sigma}_x + \dot{p} \\ \dot{\sigma}_y + \dot{p} \\ \dot{\sigma}_z + \dot{p} \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{xz} \\ \dot{\sigma}_{yz} \end{bmatrix} \qquad (2.14)$$

Considering slightly compressible water, the rate of excess pore pressure is written as:

$$\dot{p} = -\frac{K_W}{f}(\dot{\varepsilon}_{e,x} + \dot{\varepsilon}_{e,y} + \dot{\varepsilon}_{e,z}) \qquad (2.15)$$

where K_W is water bulk modulus, f is porosity.

$\Rightarrow$ Hooke's law (2.14) can be written as:

$$\begin{bmatrix} \dot{\varepsilon}_{e,x} \\ \dot{\varepsilon}_{e,y} \\ \dot{\varepsilon}_{e,z} \\ \dot{\gamma}_{e,xy} \\ \dot{\gamma}_{e,xz} \\ \dot{\gamma}_{e,yz} \end{bmatrix} = \frac{1}{E_u} \begin{bmatrix} 1 & -\nu_u & -\nu_u & 0 & 0 & 0 \\ -\nu_u & 1 & 0 & 0 & 0 & 0 \\ -\nu_u & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu_u & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu_u \end{bmatrix} \cdot \begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_z \\ \dot{\sigma}_y \\ \dot{\sigma}_z \\ \dot{\sigma}_{yz} \end{bmatrix} \qquad (2.16)$$

Here

$$E_u = 2G(1+\nu_u), \quad \nu_u = \frac{\nu + \mu(1+\nu)}{1+2\mu(1+\nu)}, \qquad (2.17)$$

$$\mu = \frac{1}{3f}\frac{K_W}{K}, \quad K = \frac{E}{3(1-2\nu)}$$

High values of K_W produce undrained Poisson's ratio ν_u close to 0.5, which leads to numerical instability. To avoid this, ν_u is assumed to equal to 0.495, which gives $K_W \approx 20K$.

Taking into account (2.15), effective stress $\underline{\underline{\sigma}}_{eff}$ is calculated as

$$\dot{\underline{\underline{\sigma}}}_{eff} = \dot{\underline{\underline{\sigma}}} + \dot{p}\underline{\underline{I}} = \underline{\underline{\sigma}} - \frac{K_W}{f}\dot{\varepsilon}\underline{\underline{I}} \tag{2.18}$$

We simulate the undrained plastic flow on the basis of Mohr-Coulomb effective strength parameters (the procedure was proposed in (Vermeer, 1993) as undrained model *A*). In undrained zones, the plastic flow rule is formulated for total stresses using the effective stress parameters (namely, effective friction angle and cohesion):

$$\begin{cases} \nabla \cdot \underline{\underline{\sigma}} + \rho_S \underline{g} = 0 \\ \underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_{pl} + \underline{\underline{\varepsilon}}_e \\ \underline{\underline{\varepsilon}}_e = \dfrac{1+\nu_u}{E_u}\left[\underline{\underline{\sigma}} - \dfrac{\nu_u}{1+\nu_u}\sigma\underline{\underline{I}}\right] \\ \dot{\underline{\underline{\varepsilon}}}_{pl} = 0 \; if \; F < 0 \\ \dot{\underline{\underline{\varepsilon}}}_{pl} = q\dfrac{\partial F}{d\underline{\underline{\sigma}}} \; if \; F = 0 \\ \dfrac{dF}{d\underline{\underline{\sigma}}} \cdot \cdot \dot{\underline{\underline{\sigma}}} = 0 \end{cases} \tag{2.19}$$

where $E_u, \nu_u = 0.495$ are undrained stiffness parameters, E_u determined from (2.17), *F* is Drucker-Prager yield function (2.9).

Total stresses $\underline{\underline{\sigma}}$ are calculated from (2.19), and effective stresses and pore pressures are derived from total stresses using relations (2.15), (2.18).

2.5 Conclusions

At the initial stage of this research, we have collected, tested and compared existing mathematical models for earthen dikes analysis, including models of filtration through porous media and soil mechanics. After a series of numerical tests, we selected the two-dimensional, unidirectional fluid-structure coupled model with linear elastic perfectly plastic associated flow defined by the Drucker-Prager yield function and with Richards' model for simulation of porous flow in variably saturated media. This choice provided the optimal balance between realism and adequacy of the computational model, on the one hand, and high speed of numerical convergence in real-time, on the other hand.